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DYNAMICS OF WHEELED VEHICLES

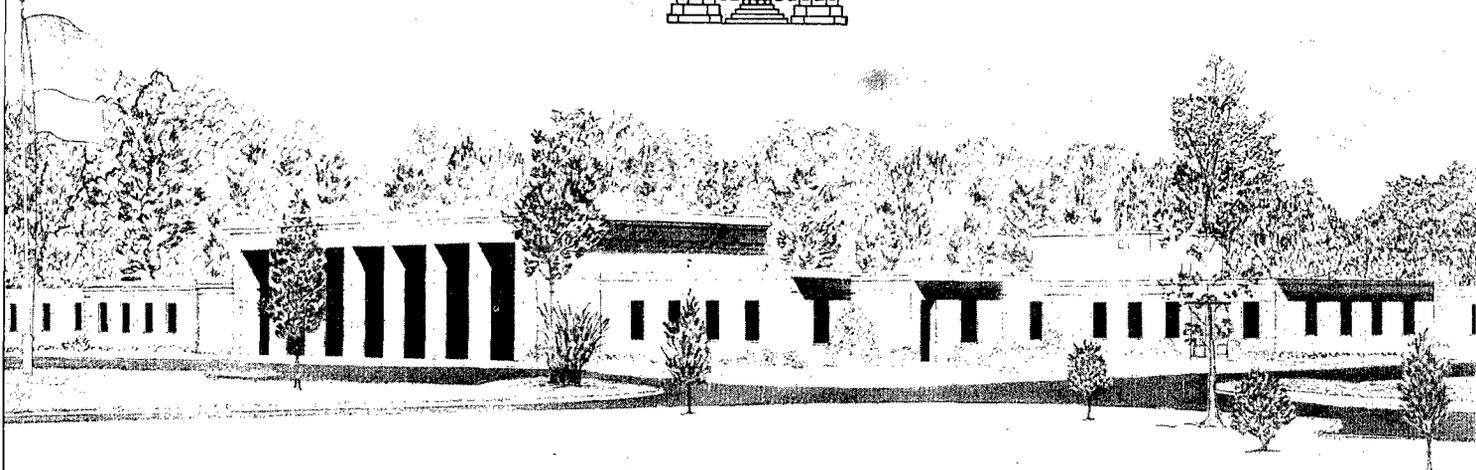
Report 3

A STATISTICAL ANALYSIS OF TERRAIN-VEHICLE-SPEED SYSTEMS

by

N. R. Murphy, Jr.

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DA Project IT062103A046, Task 03

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ARMY-MRC VICKSBURG, MISS.

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FOREWORD

This report was prepared by Mr. Newell R. Murphy, Jr., of the Research Projects Group, Mobility Research Branch, Mobility and Environmental Division, U. S. Army Engineer Waterways Experiment Station. The report is essentially a thesis submitted by Mr. Murphy in partial fulfillment of the requirements for the degree of Master of Science in Engineering to the Faculty of Mississippi State University, and is a study concerned with vehicle dynamics. The study described herein was conducted under DA Project 1T062103A046, "Trafficability and Mobility Research," Task O3, "Mobility Fundamentals and Model Studies," under the sponsorship and guidance of the Research, Development and Engineering Directorate, U. S. Army Materiel Command. The study was accomplished under the general direction of Messrs. S. J. Knight and W. G. Shockley.

COL Levi A. Brown, CE, and COL Ernest D. Peixotto, CE, were Directors of the Waterways Experiment Station during the period of preparation and publication of this report. Mr. F. R. Brown was Technical Director.

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Last but not least, the author wishes to express his sincere appreciation to his wife, Althea, and children, Debra, George, and Rogers, for the patience and understanding they exhibited during this study and for whom this work is dedicated.

N. R. M.

Vicksburg, Miss.
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CHAPTER I: INTRODUCTION

In recent years the concept of speed made good has come to be recognized as one of the most meaningful measures of the off-road mobility of a vehicle. Speed made good is simply the average speed that results when the straight-line distance between two points is divided by the time required to get from one point to the other, irrespective of the actual path taken by the vehicle. The maximum speed that a vehicle can attain in a given terrain is affected by the number and character of the terrain deterrents it must overcome or, if preferable, avoid. Terrain deterrents include soft soils, slopes, vegetation, obstacles, riverine situations, rough terrain, etc.

Experience has shown that one of the most significant single features influencing a vehicle's speed is that of ride dynamics, that is, the vibratory activity of a vehicle caused by terrain roughness. Because of this, a great deal of interest has been shown in the ride problem. However, to date, there is no satisfactory means of relating a measure of ride to a measure of terrain roughness. There has actually been much work done in the area of random vibration studies, and the literature available is quite abundant. However, for the most part, this work is largely of an academic nature and oriented toward analysis of linear systems using the concepts of transfer functions. The results are very general and nebulous in nature, and an adequate understanding of them by an individual would require a substantial background of education and experience in engineering, physics, mathematics, statistics, and electronics. It is for this reason that widespread practical application of these results has been stymied.

The General Problem

The description and evaluation of vehicle dynamic responses involves essentially a study and interpretation of waveforms. Generally, an arbitrary waveform (the terrain) serves as an input to excite some deterministic system (a vehicle or a model), and the outputs are generated as continuous waveforms representing the resulting motions of the system. Obviously, these continuous traces or waveforms must be analyzed in accordance with some specified standard to provide a basis for comparisons. This means simply that numbers must be determined that adequately describe the pertinent characteristics of each waveform. Generally, these waveforms are not well-defined mathematical functions, such as sinusoids, sawtooth waves, etc., but vary in seemingly erratic fashions. However, regardless of how erratic these waveforms may appear, their peculiar characteristics can, possibly, be described adequately by statistical methods.

Most engineers are familiar enough with the fundamentals of statistics to know that statistical quantities evolve theoretically from idealized mathematical relations, such as infinite integrals, that serve as approximations to the real world. Therefore, when statistics is applied to real-world problems, only estimates of the true values can be obtained, and the deviation of these estimates from the true values depends on how well certain criteria have been met. For example, when a coin is flipped a sufficient number of times, about as many heads as tails result. This relation improves as the number of flips is increased, and theoretically the number of heads and tails is the same after an infinite number of flips.

The nature of the quantities one might choose to describe vehicle-terrain responses and how they may be interpreted to define certain properties that have physical meaning in mobility studies are discussed here without rigorous details of probability and sampling theory.

Averages, measures of mean, and dispersion

An average is a value that is typical or representative of a set of data. Since such typical values tend to lie centrally within a set of data arranged according to magnitude, averages are also called measures of central tendency. Some of the most commonly used averages are the mean, the median, and the mode.

Variation or dispersion is a measure of the degree that numerical data tend to spread about some reference value. Generally, this reference is chosen to be the mean. Several commonly used measures of variation or dispersion are the mean deviation, the standard deviation, the variance (which is simply the square of the standard deviation), the mean square, and the root mean square (RMS). As an example of how such values represent data, examine the four groups of numbers in the following illustration:

(1)	(2)	(3)	(4)
7.5 56.25	7 49	5 25	2 4
7.5 56.25	8 64	10 100	13 169
7.5 56.25	7 49	6 36	4 16
7.5 56.25	8 64	7 49	8 64
7.5 56.25	7 49	9 81	10 100
<u>7.5 56.25</u>	<u>8 64</u>	<u>8 64</u>	<u>8 64</u>
45.0 337.50	45 339	45 355	45 417
$\frac{45.0}{6} = 7.5$	$\frac{45}{6} = 7.5$	$\frac{45}{6} = 7.5$	$\frac{45}{6} = 7.5$
(arithmetic mean)	(arithmetic mean)	(arithmetic mean)	(arithmetic mean)
$\frac{337.5}{6} = 56.25$	$\frac{339}{6} = 56.50$	$\frac{355}{6} = 59.16$	$\frac{417}{6} = 69.50$
(mean square)	(mean square)	(mean square)	(mean square)
RMS = 7.50	RMS = 7.52	RMS = 7.69	RMS = 8.34

All four groups have the same arithmetic mean, and obviously, the numbers in groups (1) and (2) are more uniform than those corresponding in (3) and (4). It is, therefore, apparent that the arithmetic mean gives only some measure of central tendency and provides no indication of the uniformity of numbers. However, the numbers in the right-hand columns in the above groups reflect the squared values of the left-hand numbers, and the mean of these numbers (the mean square) is not the same in all columns, but increases in magnitude with an increase in nonuniformity. Hence, the mean square value gives some measure of nonuniformities. Thus, the square root of these mean square values (RMS) would yield a similar descriptor of nonuniformity.

Suppose now that the four left-hand columns of numbers have different mean values. To provide a number with a more consistent comparison capability, the deviation about each mean value can be found. This quantity is referred to as the mean deviation. To illustrate this, consider again the four left-hand columns, each with an arithmetic mean of 7.5. The mean deviation for group 1 is

$$\frac{|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|+|7.5-7.5|}{6} = 0 \quad (1)$$

i.e., there is zero dispersion or variation of the numbers about the arithmetic mean. However, for group 4

$$\frac{|2-7.5|+|13-7.5|+|4-7.5|+|8-7.5|+|10-7.5|+|8-7.5|}{6} = \frac{5.5 + 5.5 + 3.5 + 0.5 + 2.5 + 0.5}{6} = \frac{18.0}{6} = 3 \quad (2)$$

This indicates that there is less dispersion in group 1 than in group 4, as it should.

A somewhat similar measurement, called the standard deviation, denotes the root mean square of the deviations from the mean and is sometimes called the RMS deviation. The standard deviation of the numbers in the left-hand column of group 1 is obviously zero since each of the deviations from the mean is zero. However, the standard deviation for group 4 is

$$\sqrt{\frac{(2-7.5)^2 + (13-7.5)^2 + (4-7.5)^2 + (8-7.5)^2 + (10-7.5)^2 + (8-7.5)^2}{6}} =$$

$$\sqrt{\frac{79.50}{6}} = \sqrt{13.25} = 3.64 \quad (3)$$

The basic difference between the standard deviation and the mean deviation is the greater influence of extreme values on the standard deviation than on the mean deviation. If the nature of the problem is such that the influence of extreme values are deemed important, then the standard deviation is the preferable measure of dispersion. The square of the standard deviation is called the variance. For normally distributed data, the standard deviation and variance play a significant part in probability and statistical estimation theory.

Applicability to terrain roughness

To see how such measures might be used to describe terrain roughness, suppose a terrain elevation profile along a line is obtained and plotted in the continuous waveform as follows:



The dispersion measurements used previously will yield a number that describes the deviation of the elevation amplitudes from the mean value or from the level datum, if desired. This quantity should then provide a good measure of roughness. With this method, a large value of the standard deviation or variance indicates a large dispersion of the elevations and, therefore, a large amount of terrain roughness; while a low value indicates a relatively smooth terrain. If the mean value is zero, then the RMS and standard deviation are the same, and the mean squared value and the variance are the same.

However, this means for describing terrain roughness is not quite adequate for evaluating the effect terrain roughness has on vehicle response because a vehicle is a finely tuned, frequency-dependent system with a different response for each different frequency of a sinusoidal displacement. This implies that a correlation of a vehicle's response to an input terrain requires a knowledge of the dispersion of the terrain amplitudes per unit frequency. This will yield a spectrum of dispersions which when summed will yield a single number representing the total dispersion. Such a spectrum, called a power spectral density function, serves the more flexible, dual purpose of describing terrain roughness in a manner that permits an evaluation of vehicle response. A cross-country terrain profile can be thought of as a random function of elevation values versus horizontal distance, i.e. there is no way to predict an exact elevation at some given distance. This randomness

becomes even more evident when consideration is given to the travel of a vehicle across an off-road area in which no specific path of travel is defined. In the same sense that a sinusoid can be defined by its amplitude and frequency, a random profile can be defined by its distribution of amplitudes and distribution of frequencies. Representative measurements have shown that the amplitudes of many terrain and road profiles can be reasonably approximated with a Gaussian or normal distribution.⁷ Although such a distribution is not a stringent requirement for spectral analysis of profiles, it enhances the statistical analysis by permitting the use of the many concepts defined for normal distributions. Regarding the distribution of frequencies for most profiles, the height of the profile is approximately proportional to the wavelength of the frequency being considered. However, in determining a power spectral density from a profile, estimating the true function from a finite sample and determining the validity of the estimates are problems. A major consideration which is dealt with later in the text is the Weiner-Khintchine relation which relates the autocorrelation function and the power spectral density as Fourier transform pairs, and is valid only for random, stationary profiles.² Stationary means that the statistical properties of the profile do not change with position. While most profiles are not stationary in the strictest sense, valid estimates of power spectral density can still be made from profiles that are quasi-stationary.

The general problem of the study reported herein was to investigate the validity and practicality of applying spectral density methods and the inherent statistics to the problem of describing terrain roughness

and vehicle responses in terms of vehicle speed.

Objective

The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness. Using this methodology, reliable estimates can be made of the average speed a given vehicle can attain over given terrain as well as of its probabilities of exceeding certain specified levels of vibration.

This may provide a more practical means of including the "ride" problem in computerized analytical models used to quantify the terrain-vehicle-driver system, and thus provide the military with a reliable analytical tool for (a) developing more effective vehicles at less cost and (b) rating them on a competitive basis in accordance with the tasks to be performed.

Scope of Study

This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with the desired power spectrum, variance, mean value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness.

A two-dimensional, nonlinear mathematical model of an M37 truck served as the vehicle in these simulations. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stops, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs.⁶

The analysis is presented chiefly in the form of graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity.

A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness.

CHAPTER II: DESCRIPTION OF COMPUTER CALCULATIONS

Digital Simulation of Low-Pass Gaussian Profiles

An essential requirement for this study was the capability to generate artificial terrain profiles composed of random, normally distributed amplitudes with a given degree of stationarity and control of the frequency content and pertinent statistics. Specifically, the first aim was to construct a group of terrain profiles that were different deterministically but were the same statistically, and then to systematically vary certain statistics.

A series of computer programs and subroutines, appropriately interwoven, generated these low-pass, Gaussian profiles with a zero mean and specified standard deviation by the following procedures: A random-number chain, generated by the power residue method, was entered at an arbitrary starting point, and 12 uniform, random numbers were computed and summed. By the central limit theorem, a single random, normal number was then computed by the formula:

$$V = (A - 6)(\sigma_n)$$

where

V = the random, normal number

A = the sum of 12 uniform, random numbers

σ_n = the standard deviation desired for the sequence of random, normal numbers.

This technique for computing random, normal numbers is often referred to as the "sum of the uniform deviates" method.⁵

Generally, the calculations were repeated until a sequence of 1000 random, normal numbers was obtained. Although the mean of the resulting sequence was always very nearly zero, a shifting operation was performed to insure a mean value of absolute zero. The frequency distribution of the sequence at this point approximates that of white Gaussian noise in that its spectral density is uniformly distributed over all frequencies with an average mean square level of σ_n^2 . To obtain the desired spectrum, the sequence was then passed through a numerical system that simulated an analog low-pass filter with a certain "time constant" $T = RC$ and cutoff frequency α .¹ The analog equivalent of this system is shown in Fig. 1. This system is formulated numerically by the following formula:

$$Y_{i+1} = X_{i+1}(0, \sigma_N) + Y_i e^{-\alpha \tau}$$

where

$X(0, \sigma_N)$ = sequence of random, normal numbers previously calculated

α = the system's cutoff frequency (temporal or spatial)

τ = the time (or distance) interval between points in the sequence

(NOTE: The points are assumed to be equidistant.)

Y = the resulting sequence.

The $\alpha\tau$ product gives complete control over the spectrum shaping. It was determined after several trials that $\alpha\tau \approx 0.055$ gave the best normality condition. The desired density level is achieved by specifying the appropriate standard deviation σ of the resulting sequence. This value is obtained by the formula:

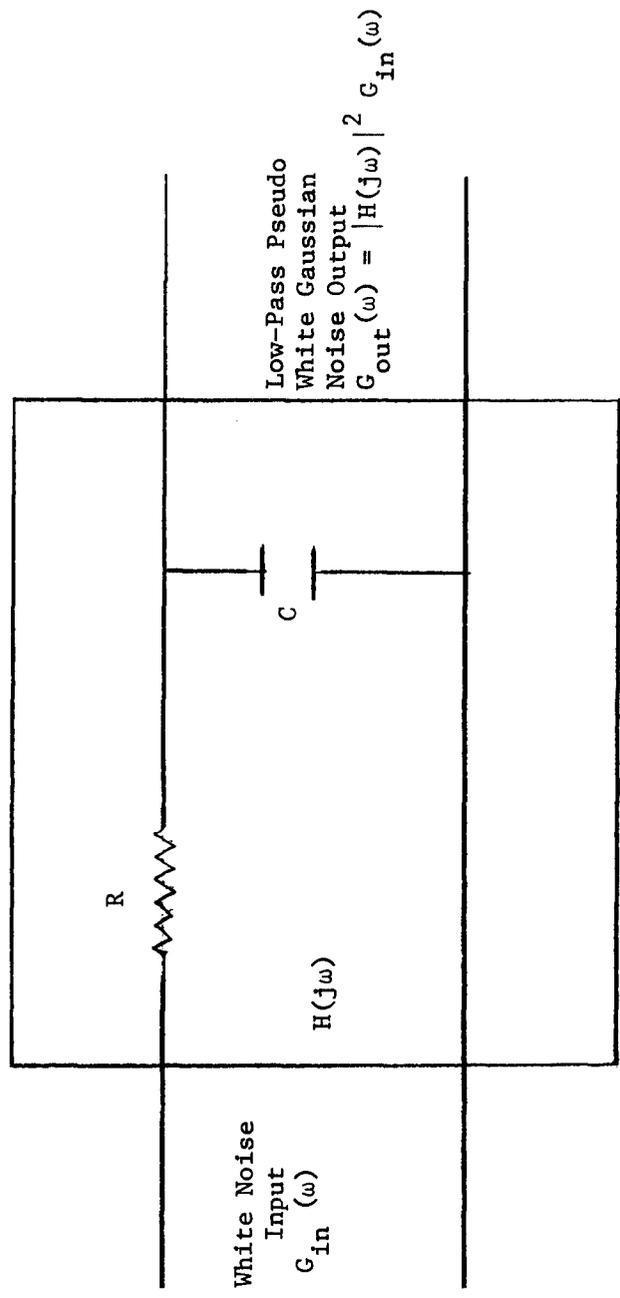


Figure 1. Low Pass RC Filter

$$\sigma = \frac{\sigma_N}{\sqrt{1 - e^{-2\alpha\tau}}}$$

The computer program entitled "NOISE," listed in Appendix A, performs the operations for generating the random, normal sequences.

Tests of Fundamental Assumptions

Tests for stationarity

The sequence of numbers representing a random terrain profile was divided into N equal intervals, where the data in each interval were considered to be independent of those in any of the other intervals. The mean and mean square values were computed for each interval. The resulting sequences of the mean and mean square values were tested for underlying trends by two tests, the run test and the trend test.³ Generally, the run test is more powerful for detecting fluctuating trends, and the trend test is more powerful for detecting underlying monotonic trends. Stationarity required that the sample pass both the run and the trend tests at the 95 percent confidence level. The sequence of mean square values were each assumed to be independent samples of a random variable with a true mean μ and mean square value ψ^2 . If this hypothesis were true, the variations in the sequence of sampled values would be random and display no trends. Hence, the number of runs in the sequence relative to some value (the mean) would be as expected for a sequence of independent random observations of the random variable, as given in Table 1. Moreover, the number of reverse arrangements in the sequence would be as expected for a sequence of

Table 1
Percentage Points of Run Distribution

<u>n = N/2</u>	<u>α</u>					
	<u>0.99</u>	<u>0.975</u>	<u>0.95</u>	<u>0.05</u>	<u>0.025</u>	<u>0.01</u>
5	2	2	3	8	9	9
6	2	3	3	10	10	11
7	3	3	4	11	12	12
8	4	4	5	12	13	13
9	4	5	6	13	14	15
10	5	6	6	15	15	16
11	6	7	7	16	16	17
12	7	7	8	17	18	18
13	7	8	9	18	19	20
14	8	9	10	19	20	21
15	9	10	11	20	21	22
16	10	11	11	22	22	23
18	11	12	13	24	25	26
20	13	14	15	26	27	28
25	17	18	19	32	33	34
30	21	22	24	37	39	40
35	25	27	28	43	44	46
40	30	31	33	48	50	51
45	34	36	37	54	55	57
50	38	40	42	59	61	63
55	43	45	46	65	66	68
60	47	49	51	70	72	74
65	52	54	56	75	77	79
70	56	58	60	81	83	85
75	61	63	65	86	88	90
80	65	68	70	91	93	96
85	70	72	74	97	99	101
90	74	77	79	102	104	107
95	79	82	84	107	109	112
100	84	86	88	113	115	117

independent random observations of the same variable, as given in Table 2. If the number of runs or reverse arrangements was significantly different from the expected number given in the tables, the hypothesis of stationarity would be rejected. The details of these two tests are presented here to give an understanding of the principles involved in the calculations.

Run test. Consider the sequences of mean values X_i and mean square values Y_i for a given profile to be classified into mutually exclusive categories identified by plus (+) or minus (-), where $X_i > \bar{X} = +$, $X_i < \bar{X} = -$; and likewise for the Y_i . A run is defined as a sequence of identical observations. The number of runs that occur in a sequence of observations indicates whether or not results are independent random observations.

Let it be hypothesized that there is no trend in the sequences, that is, no variations in the mean and mean square values, which implies stationarity. If the sequences of mean and mean square values are independent observations and the number of (+) observations equals the number of (-) observations, then the number of runs in the sequence will have a sampling distribution as given in Table 1. The hypothesis can be tested to any desired level of significance α by comparing the observed runs in the interval between $r_{n; 1 - \alpha/2}$ and $r_{n; \alpha/2}$, where $n = N/2$, N being the total number of observations. If the observed runs fall outside the interval, the hypothesis is rejected at the α level of

Table 2

Percentage Points of Reverse Arrangement Distribution

N	α					
	<u>0.99</u>	<u>0.975</u>	<u>0.95</u>	<u>0.05</u>	<u>0.025</u>	<u>0.01</u>
10	9	11	13	31	33	35
12	16	18	21	44	47	49
14	24	27	30	60	63	66
16	34	38	41	78	81	85
18	45	50	54	98	102	107
20	59	64	69	120	125	130
30	152	162	171	263	272	282
40	290	305	319	460	474	435
50	473	495	514	710	729	751
60	702	731	756	1013	1038	1064
70	977	1014	1045	1369	1400	1437
80	1299	1344	1382	1777	1815	1360
90	1668	1721	1766	2238	2283	2336
100	2083	2145	2198	2751	2804	2865

significance. For an example, consider the sequence of the observations below:

(1) 5.2	(6) 5.7	(11) 6.2	(16) 5.4
(2) 5.1	(7) 5.0	(12) 6.4	(17) 6.4
(3) 5.7	(8) 6.5	(13) 4.7	(18) 5.8
(4) 5.2	(9) 5.4	(14) 5.4	(19) 6.7
(5) 4.3	(10) 5.8	(15) 5.9	(20) 5.2

The mean value of the observations is 5.6. Let all observations with a value greater than 5.6 be (+) and those less than 5.6 be (-). The result is

$\underbrace{--}_{1} \quad + \quad \underbrace{--}_{3} \quad + \quad - \quad + \quad - \quad \underbrace{+++}_{8} \quad \underbrace{--}_{9} \quad + \quad - \quad \underbrace{+++}_{12} \quad -$
 1 2 3 4 5 6 7 8 9 10 11 12 13

There are 13 runs and 20 observations. Let it be hypothesized that the observations are independent. The acceptance region for this hypothesis is

$$r_{10; 1 - \alpha/2} < r < r_{10; \alpha/2}$$

From Table 1 for $\alpha = 0.05$, $r_{10; 1 - \alpha/2} = r_{10; 0.975} = 6$, and $r_{10; \alpha/2} = r_{10; 0.025} = 15$. The hypothesis is accepted because 13(r) falls between 6 and 15. Therefore, the run test is passed and there is no evidence of an underlying fluctuating trend.

Trend test. The trend test counts the number of times that $x_i > x_j$ for $i < j$. Each such inequality is called a reverse arrangement. Let the total number of reverse arrangements be denoted by A . Test

the same sequence of 20 observations used in the previous example for a trend at the $\alpha = 0.05$ level of significance. For example, the first number, 5.2, is greater than four of the subsequent numbers. The number of reverse arrangements would therefore be 4. The second number, 5.1, is greater than three of the subsequent numbers, with the first number excluded from this comparison. The number of reverse arrangements is 3. The third number, 5.7, is greater than eight of the subsequent numbers, with the first two numbers excluded. Thus, the number of reverse arrangements is as follows:

$$\begin{array}{cccc}
 A_1 = 4 & A_6 = 6 & A_{11} = 6 & A_{16} = 1 \\
 A_2 = 3 & A_7 = 1 & A_{12} = 6 & A_{17} = 2 \\
 A_3 = 8 & A_8 = 10 & A_{13} = 0 & A_{18} = 1 \\
 A_4 = 3 & A_9 = 2 & A_{14} = 1 & A_{19} = 1 \\
 A_5 = 0 & A_{10} = 4 & A_{15} = 3 &
 \end{array}$$

The total number of reverse arrangements is 62. Now let it be hypothesized that the observations are independent observations where there is no trend. The acceptance region for this hypothesis is

$$A_{20; 1 - \alpha/2} < A < A_{20; \alpha/2}$$

From Table 2 for $\alpha = 0.05$, $A_{20; 1 - \alpha/2} = A_{20; 0.975} = 64$ and $A_{20; \alpha/2} = A_{20; 0.025} = 125$. Hence, since $A = 62$ falls outside this interval, the hypothesis is rejected at the 5 percent level of significance. A plot of the 20 observations (Fig. 2) shows a slight monotonic increasing trend that was not detected by the run test. The straight line delineating the trend was determined by a least squares fit of the data. The sensitivity of the test at this level of significance is

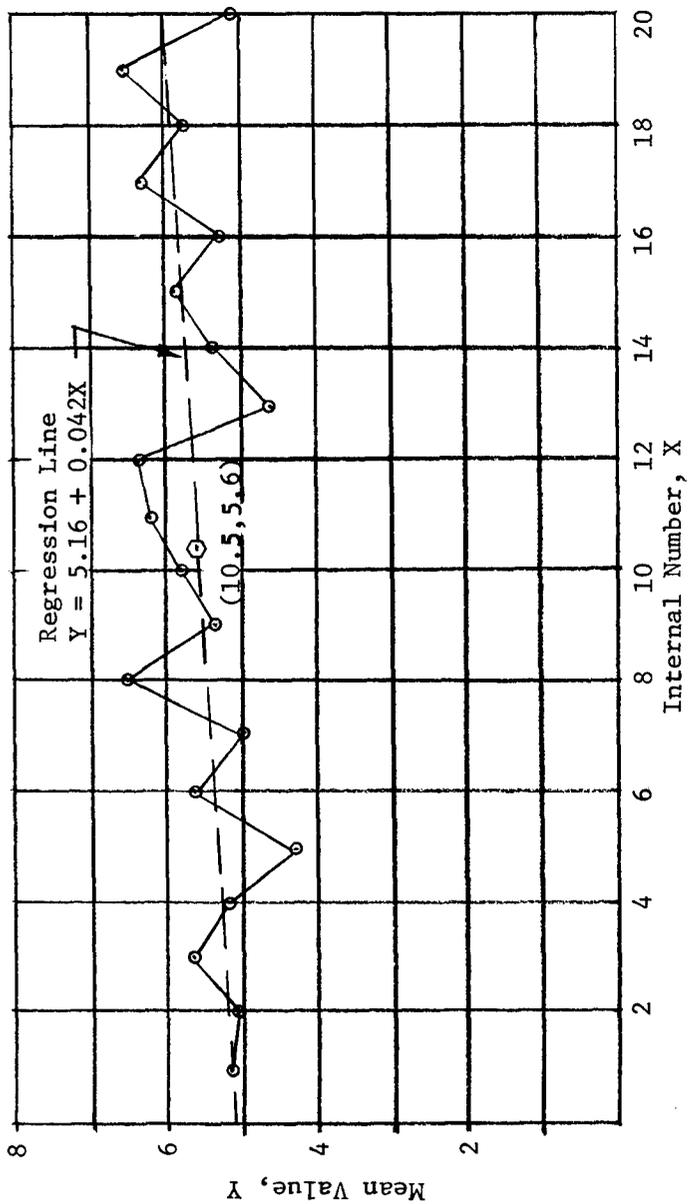


Figure 2. Mean Value Versus Interval Number

perceived from the small value of the slope of the regression line. Therefore, for a sequence such as this, the hypothesis of stationarity would be rejected.

Test for randomness

Randomness, the most fundamental concept underlying all the theory of estimation and testing of hypotheses, is far from easy to understand. It is generally interpreted as a lack of bias, implying no preference for any single element or group of elements. The determination of this lack of bias is centered around certain specific elements of bias (non-randomness) that can be detected from an analysis of the internal structure of the sample. This can be accomplished by recording the individual measurements that make up a sample in the precise order in which they were originally obtained and performing the run test used previously to test stationarity. However, this method is not practical (or necessary) for samples containing a large number of points. Having passed the test for stationarity, a test for randomness effectively reduces to a test for the presence of sinusoids due to periodic or almost periodic components in the data.⁴ The most powerful method of detecting sinusoidal components in otherwise random data is presented by an autocorrelogram, i.e. a plot of the autocorrelation function. For any purely random data, the autocorrelation function will always approach a value equal to the square of the mean value as the lag becomes large. On the other hand, the autocorrelation function for a sine wave, or collection of sine waves, will continue to oscillate no matter how large the lag becomes. Details concerning the autocorrelation function and its computation are presented later in this report.

Test for normality

Normality is tested conveniently by the "Chi-square Goodness-of-the-Fit-Test." The general procedure of the test involves the use of a statistic with an approximate Chi-square distribution as a measure of the discrepancy between an observed probability density function and the theoretical probability density function. The approximate Chi-square value of the given data is obtained by:

$$\chi^2 = \sum_{i=1}^K \frac{(f_i - F_i)^2}{F_i}$$

Where f_i is the frequency of occurrence in the subgroup interval of the original sequence, K is the total number of subgroups into which the sequence is divided, and F_i is the number of expected occurrences of the theoretically normal distribution. The Chi-square goodness-of-fit is influenced by the choice of the number of subgroups, K . For the case where the test will be performed at $\alpha = 0.05$ level of significance, William⁹ suggests that the minimum number of subgroups K be determined by the formula:

$$K = 1.87(N - 1)^{2/5} \quad (4)$$

where N is the total number of points in the sequence (1000 for this study). To visualize the magnitude of K for a given N , a limited tabulation is as follows:

<u>N</u>	<u>K</u>
100	12
200	16
300	18
400	21
500	22
600	24
700	26
800	27
900	28
1000	30
.	
.	
.	
1500	35
.	
.	
.	
2000	39

The region of acceptance at the $\alpha = 0.05$ level of significance is

$$\chi^2 < \chi^2_{n:\alpha}$$

where $\chi^2_{n:\alpha}$ is the Chi-square value of the theoretically normal distribution function at the significance level α .

A computer subroutine program was developed to test the normality of sequences of any length by means of this Chi-square concept. This program is a part of the STANOR program (Appendix A) that performs all the tests of fundamental assumptions previously mentioned.

Autocorrelation Function

Generally, the principal application for an autocorrelation function measurement of physical data is to establish the dependence of values upon one another. However, the main interest here is that the autocorrelation function affords a means of computing the power spectral density function. The autocorrelation function is found by taking two

identical waveforms, repeatedly shifting one with respect to the other, and then calculating the correlation coefficient for each shift. The correlation coefficient for a given shift or lag is obtained by multiplying the ordinates of the two waveforms at corresponding points on the horizontal scale and averaging the products over a sufficiently long range, usually several periods of the lowest frequency present, to provide a steady average. This is illustrated in Fig. 3a. The products a_1b_1 , a_2b_2 , a_3b_3 , etc., are computed, added together, then divided by the total number of products in the sample. The correlation function is obtained from a plot of the correlation coefficient as a function of the shift or lagged values (Fig. 3b). The shift values may be in terms of time or distance, depending upon whether the original waveforms are functions of time or distance. The autocorrelation function has a maximum value for zero shift, which is equal to the mean square value of the waveform, and decreases to zero, if the mean value of the waveform is assumed to be zero, for infinite value of the shift unless the waveform contains periodic components. If the waveform has periodic components, the autocorrelation functions also has periodic components of the same period. The autocorrelation functions for several common waveforms are shown in Fig. 3c.

The autocorrelation function of a discrete sequence is defined by

$$R_{\chi}(\tau) = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \chi(k) \cdot \chi(k + \tau) \quad (5)$$

where $R_{\chi}(\tau)$ is the value of the autocorrelation function when the lagged distance is τ and $\chi(k)$ is the original sequence. The discrete form of equation 5 is a direct translation of the autocorrelation

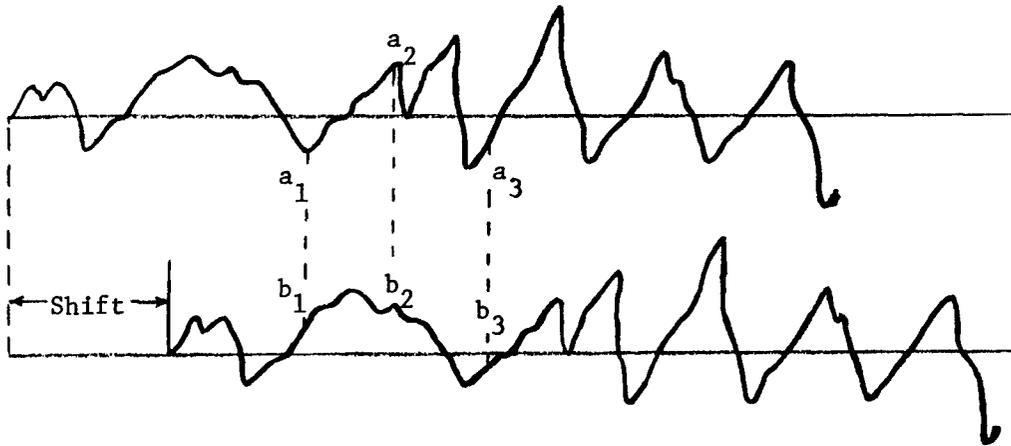


Figure 3a. Calculation of Correlation Coefficient

At corresponding points on horizontal scale, the ordinates of the two waveforms are multiplied. The correlation coefficient is the average of the $a_i b_i$ products.

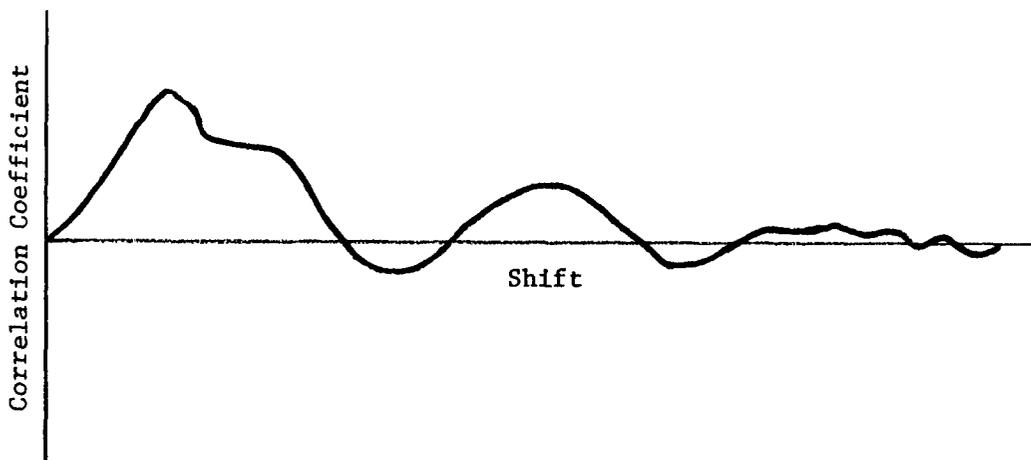


Figure 3b. The Correlation Function

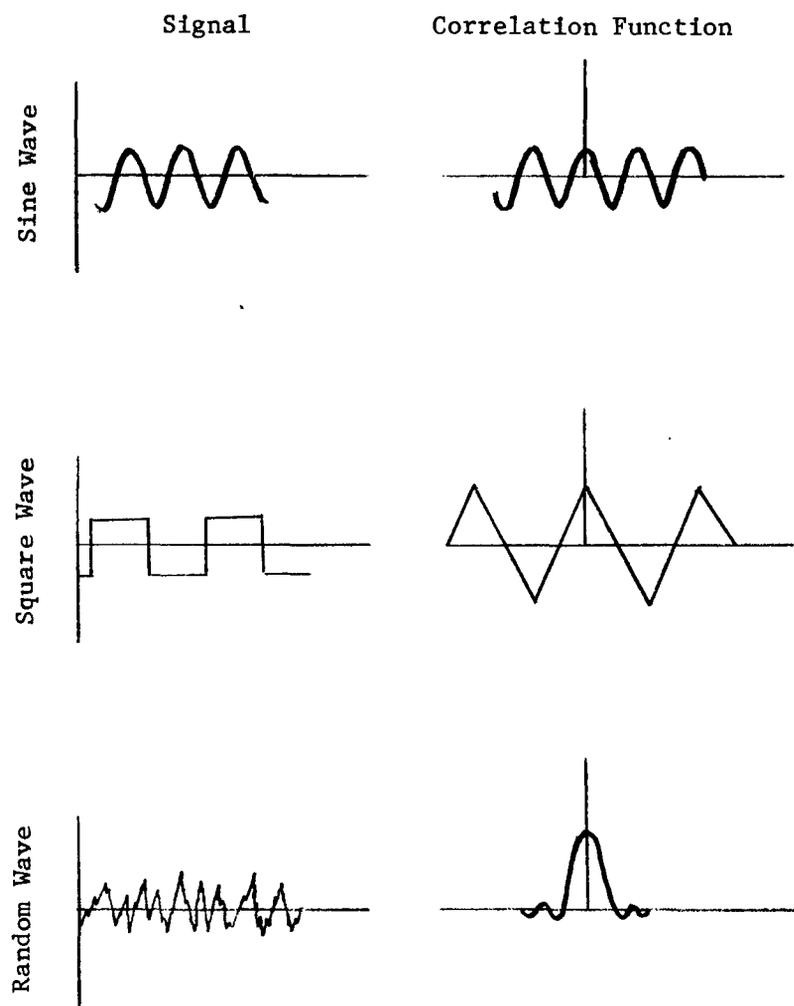


Figure 3c. Correlation Functions of Several Common Waveforms

function for a continuous and infinite sequence given by equation 6:

$$R_{\chi}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \chi(t) \chi(t + \tau) dt \quad (6)$$

This equation is true for a sequence that has an infinite length. However, for sequences with finite length, consistent results require that the autocorrelation function (equation 5) be modified to the form

$$R_{\chi}(\tau) = \frac{1}{N-\tau} \sum_{k=1}^{N-\tau} \chi(k) \cdot \chi(k + \tau) \quad (7)$$

This equation differs from equation 5 in that the quantity $\chi(k) \cdot \chi(k + \tau)$ is averaged by dividing by the actual length over which $\chi(k)$ and $\chi(k + \tau)$ are multiplied.

For a random sample with a zero mean, this function should approach zero and remain at zero as the lag values increase, and the function should not be periodic. (See Fig. 3c for random wave.) The extent to which the function fulfills these conditions indicates whether the waveform (terrain profile) is sufficiently random to be described by its power spectral density. However, an unsatisfactory autocorrelation function can frequently be improved by performing a filtering operation (detrending) on the original data to remove the influence of longer wavelengths. A most useful and very significant property of the autocorrelation function is that it happens to be the Fourier transform of the power density spectrum of a random, stationary waveform. Therefore, having once obtained the autocorrelation function, it is then possible to compute the power spectral density function.

Power Spectral Density

The power spectral density (PSD) is a second moment or variance density spectrum. PSD of a random, stationary function is defined as the Fourier transform of its autocorrelation function as follows:

$$P(\Omega) = 2 \int_{-\infty}^{\infty} R_x(\tau) e^{-i2\pi\Omega\tau} d\tau \quad (8)$$

where $R_x(\tau)$ is the value of the correlation function for shift τ and Ω is the frequency in cyc/in. if τ is measured in inches and cyc/sec if measured in seconds. The integral over the frequency range $0 \rightarrow \infty$ of the PSD is the variance.* The raw estimate of the power spectrum is obtained by the following numerical method:

$$P_x(h) = \frac{2\Delta t}{\pi} \sum_{\tau=0}^m \epsilon_{\tau} R_x(\tau) \cos \frac{h\tau\pi}{m} \quad h = 0, 1, \dots, m$$

where

$$\epsilon_{\tau} = \begin{cases} 1 & 0 < \tau < m \\ 1/2 & \tau = 0, m \end{cases}$$

$R_x(\tau)$ = autocorrelation value of series x (the waveform) at lag τ

$P_x(h)$ = raw power spectral estimate of series x at frequency

$$\frac{h\pi}{m\Delta t}$$

* This is true if the function has a zero mean. It was shown previously that for functions with a zero mean, the mean square and variance are the same.

$$\text{NOTE: } P_x^{(h)} = P_x \left(\frac{h\pi}{m\Delta t} \right)$$

Δt = constant sampling interval (may be time or distance)

m = maximum lag used in autocorrelation function not to exceed 199.

The raw estimates are then smoothed by using "Hamming" coefficients as follows:

$$SP_x^{(0)} = 0.54 P_x^{(0)} + 0.46 P_x^{(1)}$$

$$SP_x^{(h)} = 0.23 P_x^{(h-1)} + 0.54 P_x^{(h)} + 0.23 P_x^{(h+1)}, \quad 0 < h < m$$

$$SP_x^{(m)} = 0.54 P_x^{(m)} + 0.46 P_x^{(m-1)}$$

where $SP_x^{(h)}$ is the smoothed power spectral estimate of series x at frequency $\frac{h\pi}{m\Delta t}$. Smoothing is necessary because the raw estimate is an inefficient estimate of the true spectral density because the finite record length precludes identifying frequencies exactly. A more reliable statistical estimate can be obtained by averaging over neighboring frequencies. Trial-and-error methods have proved more powerful in obtaining sets of averaging coefficients that produce reliable spectral estimates than has any particular theory. There are several well-established sets of coefficients available, and in general, the Hamming method is quite satisfactory. The computer program "PSD" (Appendix A) computes both the autocorrelation and the PSD estimates.

A plot of two terrain power spectra is shown in Fig. 4. The ordinates are in terms of $\text{ft}^2/\text{cyc}/\text{ft}$, and the abscissa are in units of cyc/ft , the reciprocal of wavelength. These spectra measure the rate of change of the mean squared elevation at each frequency with frequency and are primarily geometric quantities that do not contain any

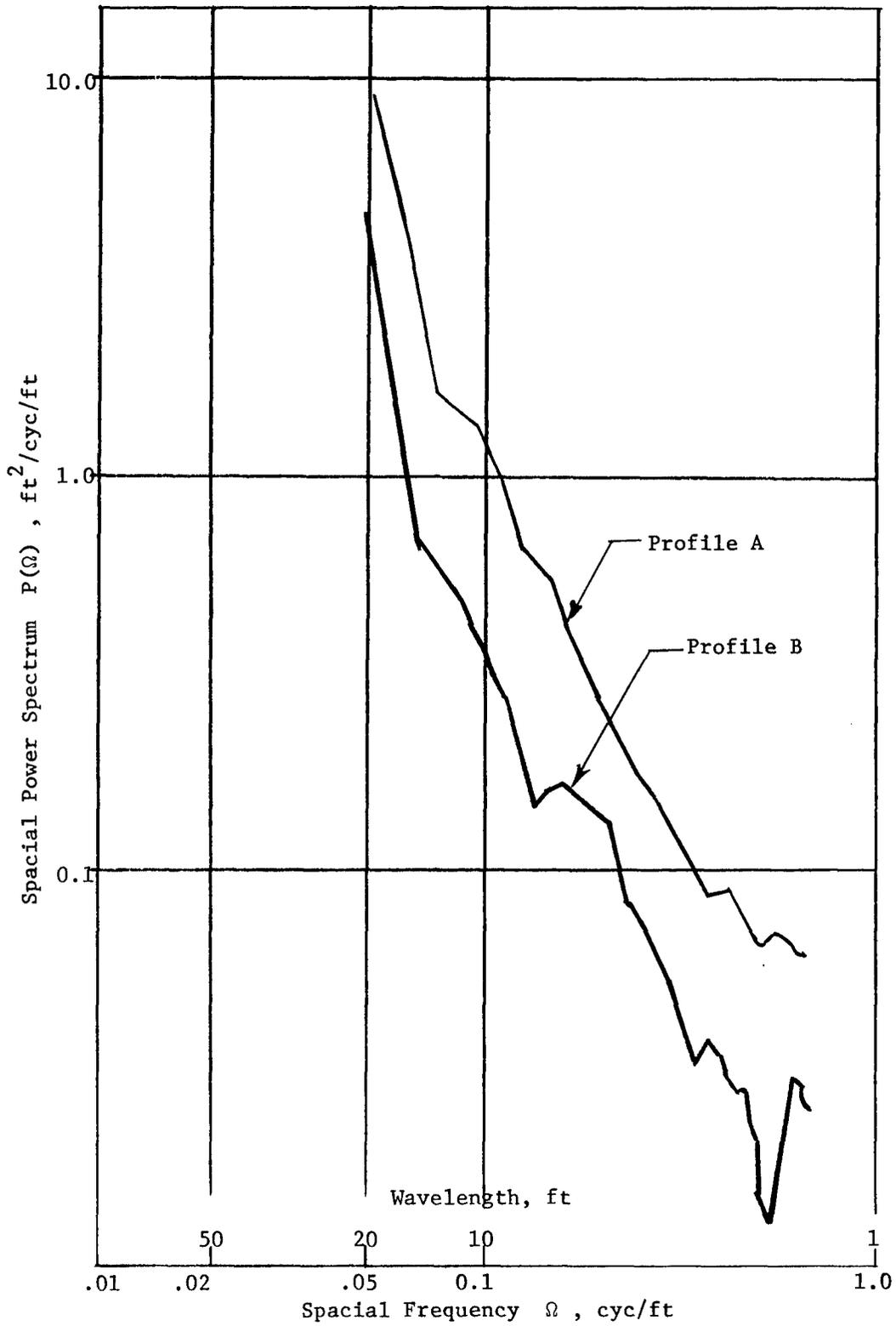


Figure 4. Examples of Terrain Power Spectra

units of time. The area under each power spectrum represents the mean square value of the total roughness for the respective profile. The area under the curve between any two selected wavelengths will indicate the contribution to the total roughness that this range of wavelengths produces. Thus, from the terrain power spectrum curve, the wavelengths that make significant contributions to the variation in the elevation measurements can be determined.

The terrain elevation power spectrum can be used also with certain vehicle characteristics to determine wavelengths and frequencies that induce significant forces and motions if the velocity of the vehicle is introduced. This is done by multiplying the abscissa of the power spectrum curve by the vehicle velocity to obtain the frequency cycles per second (cps), and by dividing the ordinates of the power spectrum by the vehicle velocity to obtain the units of $\text{ft}^2/\text{cyc}/\text{sec}$. A different curve obviously results for each velocity because changes in velocity tend to stretch or shorten the curve. Now, take a hypothetical case where a vehicle is driven onto a sinusoidal exciter platform with a controlled frequency, adjustable amplitude, and electronic transducers to measure the resulting reaction forces and motions. By varying only the frequency, a response of the force/unit displacement versus frequency (Fig. 5) can be obtained. Note the two predominant frequencies at about 2 and 15 cps. These are the basic natural frequencies of the center of gravity and the axles. Variations in the terrain profile that will excite frequencies of 2 and 15 cps will thus generate large forces and motions between the vehicle and the terrain. These frequencies are, therefore, of interest in determining the influence of

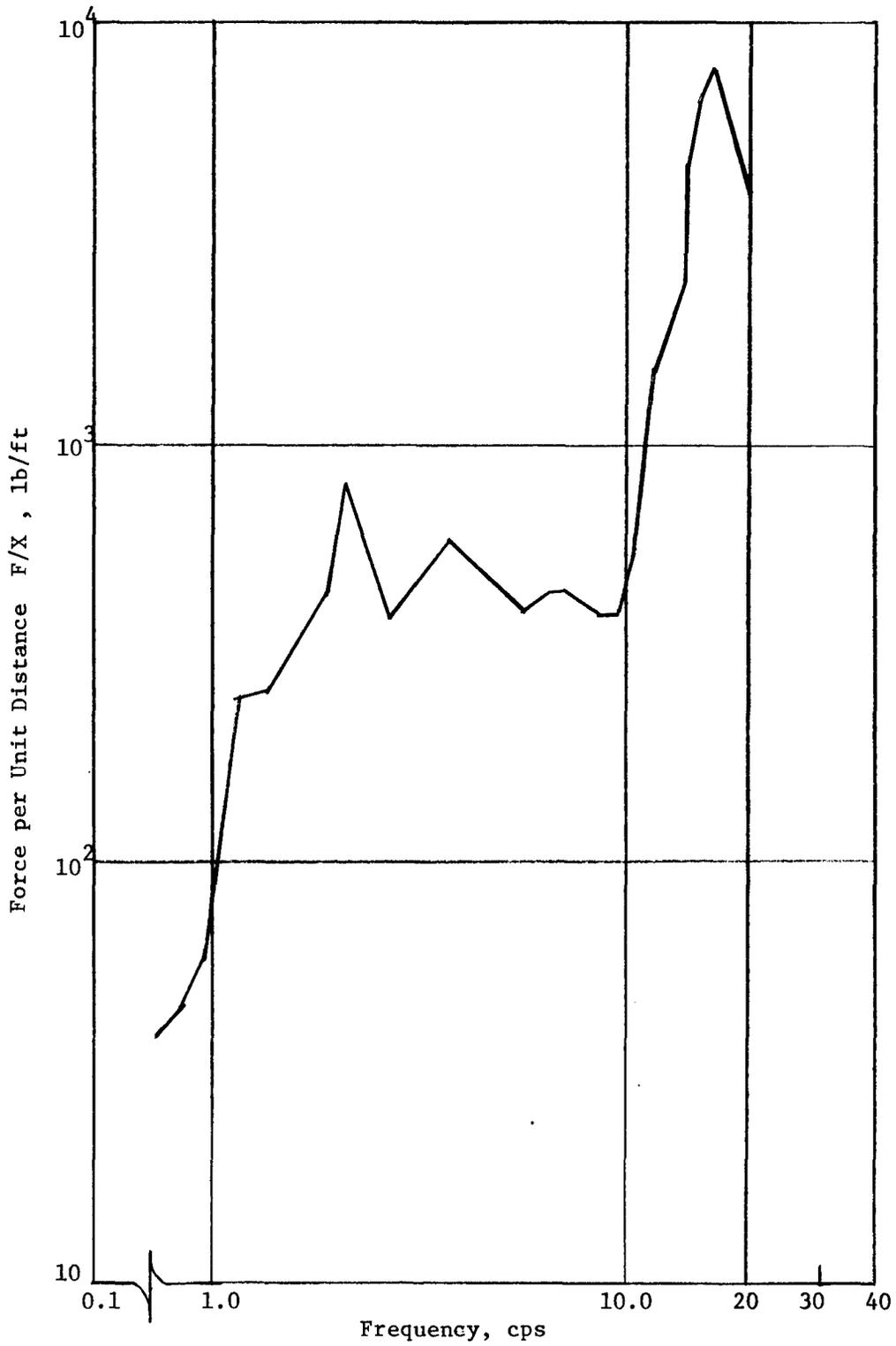


Figure 5. Force-Frequency Vehicle Response

the terrains on the reaction of the vehicle. This relation between forces (or motion if so desired) and frequency can now be combined with the terrain power spectrum to obtain an estimate of the dynamic force or motion that the vehicle will exert on the terrain, and vice versa. To do this, the terrain spectrum must first be modified by introducing the vehicle velocity, as previously described. If this is done, then to determine a power spectrum of the dynamic force that the vehicle will exert on the terrain is simply a matter of multiplying the two spectra. It is worth mentioning that the vehicle characteristics (Fig. 5) must be squared when this mathematical operation is performed to give the resulting units of $\text{lb}^2/\text{cyc}/\text{sec}$ in the ordinates of the response spectrum. A typical result of these computations can be perceived from the plots in Figs. 6 and 7 for the 44 ft/sec speed. The influence of the vehicle's natural frequencies (2 and 15 cps) is vividly portrayed by the dynamic response spectrum (Fig. 7). This response spectrum also has the same characteristics as previous spectra, i.e. the area under the curve represents the mean square value of the response. With this value, it is easy to determine the root mean square (effective) value. This RMS response value is the effective force (or motion) that the terrain exerts on the vehicle or vice versa, for that velocity. Such a spectral analysis provides a great deal of information on and insight into just how the frequencies of the terrain influence a vehicle's response and further pinpoints critical speeds and terrain wavelengths. However, this analysis is based on the fact that the vehicle is a linear system; that is, if the amplitude of the sinusoidal shaker is doubled, the only effect on the vehicle's response is the doubling of the output

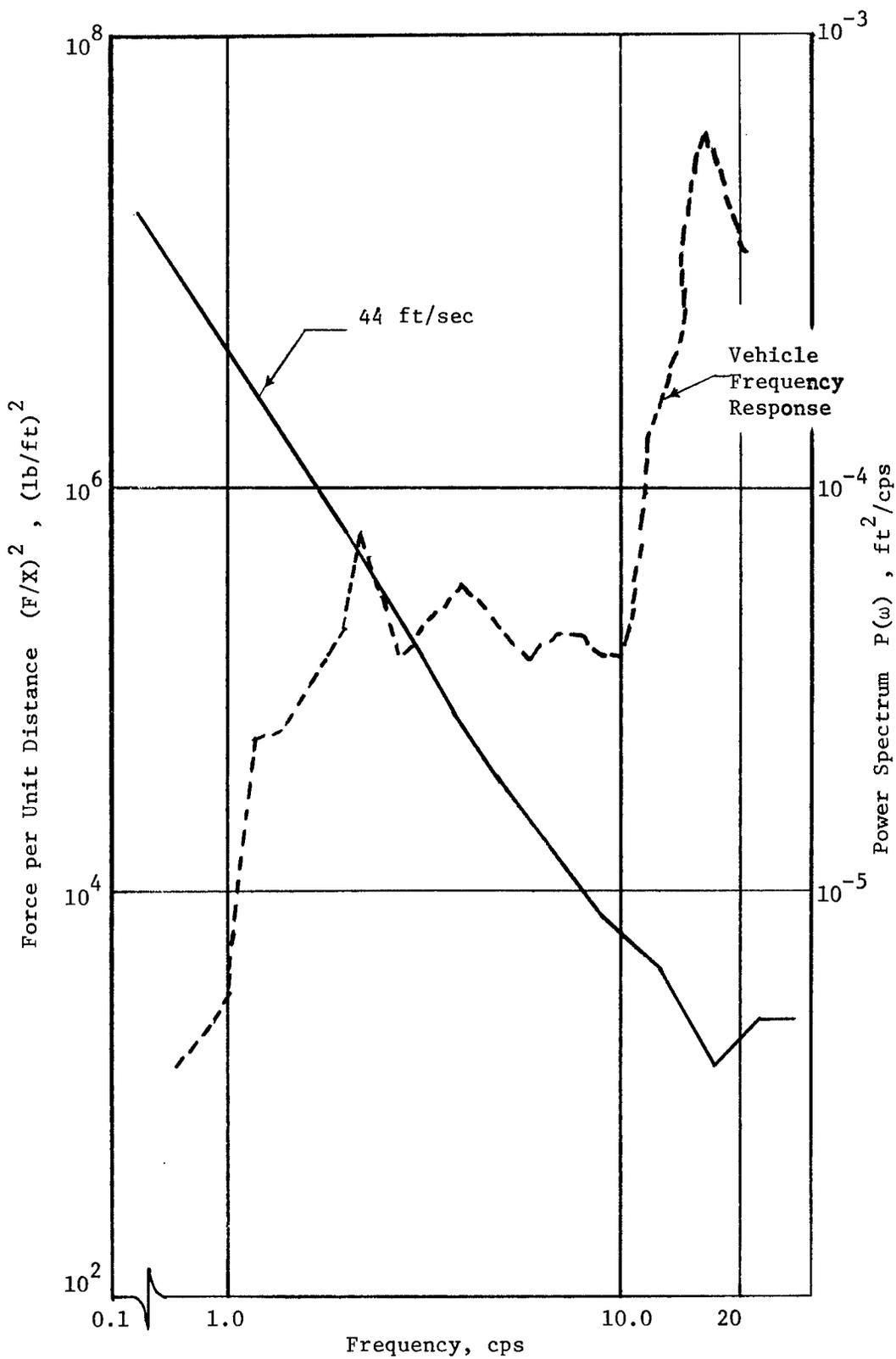


Figure 6. Terrain and Vehicle Characteristics as a Function of Frequency

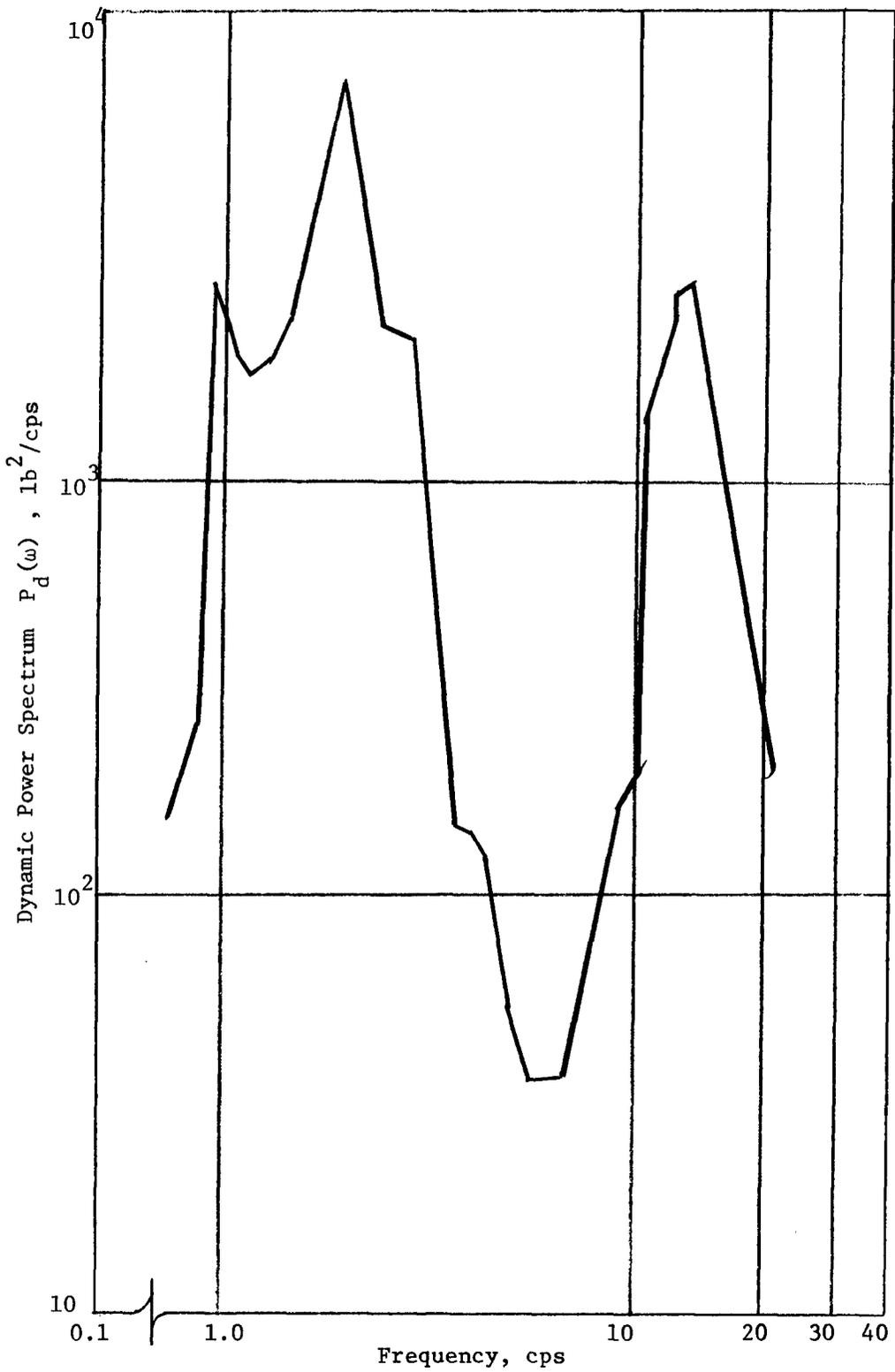
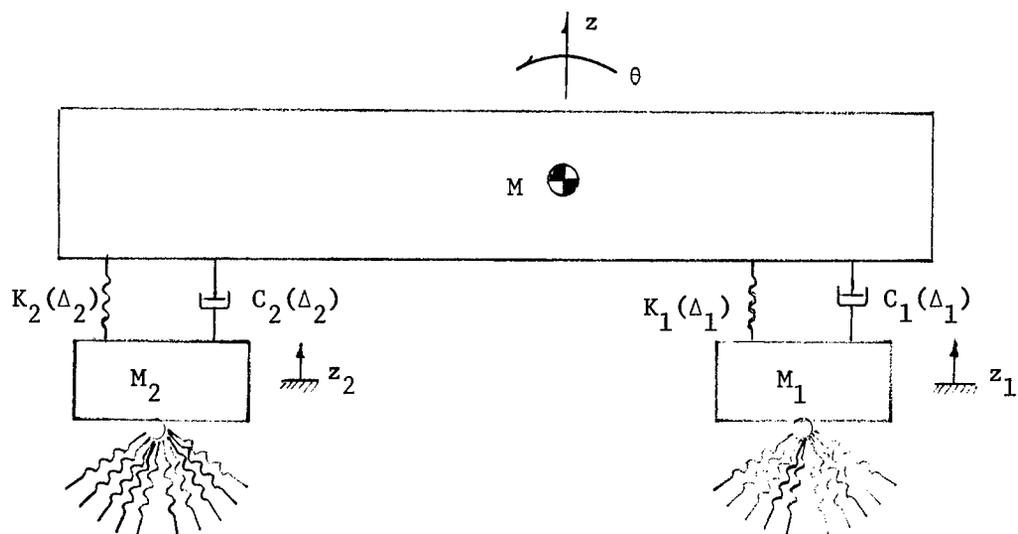


Figure 7. Vehicle Response-Frequency Relation

for all frequencies. This, however, is not the case for actual vehicles which, unfortunately, are nonlinear.

Vehicle Model

A two-dimensional, 4-deg-of-freedom mathematical model for the M37, 3/4-ton truck was developed for this study. The truck was modeled in terms of a series of coupled second-order ordinary differential equations that are solved simultaneously to describe the motions of the truck as it traverses the selected terrain profiles. The frame of the truck was considered rigid, and only the pneumatic tires and suspensions were considered to contribute to the sprung motion of the frame. The model included all the pertinent nonlinearities in the suspensions, those inherent in large rotational motions and loss of ground contact. Also, the segmented wheel concept⁶ was employed to describe tire-terrain compliance. This added still another nonlinear feature to the model. The equations and a schematic diagram for the model are as follows:



a. Forces on body.

$$M\ddot{z} = K_1(\Delta_1)(z_1 - z - a \sin \theta) + C_1(\dot{\Delta}_1)(\dot{z}_1 - \dot{z} - a \dot{\theta} \cos \theta) \\ + K_2(\Delta_2)(z_2 - z + b \sin \theta) + C_2(\dot{\Delta}_2)(\dot{z}_2 - \dot{z} + b \dot{\theta} \\ \cos \theta) - Mg$$

$$I\ddot{\theta} = K_1(\Delta_1)a(z_1 - z - a \sin \theta) + C_1(\dot{\Delta}_1)a(\dot{z}_1 - \dot{z} - a \dot{\theta} \cos \theta) \\ - K_2(\Delta_2)b(z_2 - z + b \sin \theta) - C_2(\dot{\Delta}_2)b(\dot{z}_2 - \dot{z} + b \dot{\theta} \\ \cos \theta)$$

b. Forces on front axle.

$$M_1\ddot{z}_1 = -K_1(\Delta_1)(z_1 - z - a \sin \theta) - C_1(\dot{\Delta}_1)(\dot{z}_1 - \dot{z} - a \dot{\theta} \\ \cos \theta) + \sum_{i=1}^{10} \gamma_{i1}(p_{i1} - z_1) - M_1g$$

c. Forces on rear axle.

$$M_2\ddot{z}_2 = -K_2(\Delta_2)(z_2 - z + b \sin \theta) - C_2(\dot{\Delta}_2)(\dot{z}_2 - \dot{z} + b \dot{\theta} \\ \cos \theta) + \sum_{i=1}^{10} \gamma_{i2}(p_{i2} - z_2) - M_2g$$

where

$$\Delta_1 = z_1 - z - a \sin \theta \quad \dot{\Delta}_1 = \dot{z}_1 - \dot{z} - a \dot{\theta} \cos \theta \\ \Delta_2 = z_2 - z + b \sin \theta \quad \dot{\Delta}_2 = \dot{z}_2 - \dot{z} + b \dot{\theta} \cos \theta$$

For this study, the suspension spring coefficients were presented by third-order polynomials, as shown in Fig. 8. These polynomials were obtained by curve fitting the actual force-deflection relations, and they are reasonable approximations. The suspension damping is as follows:

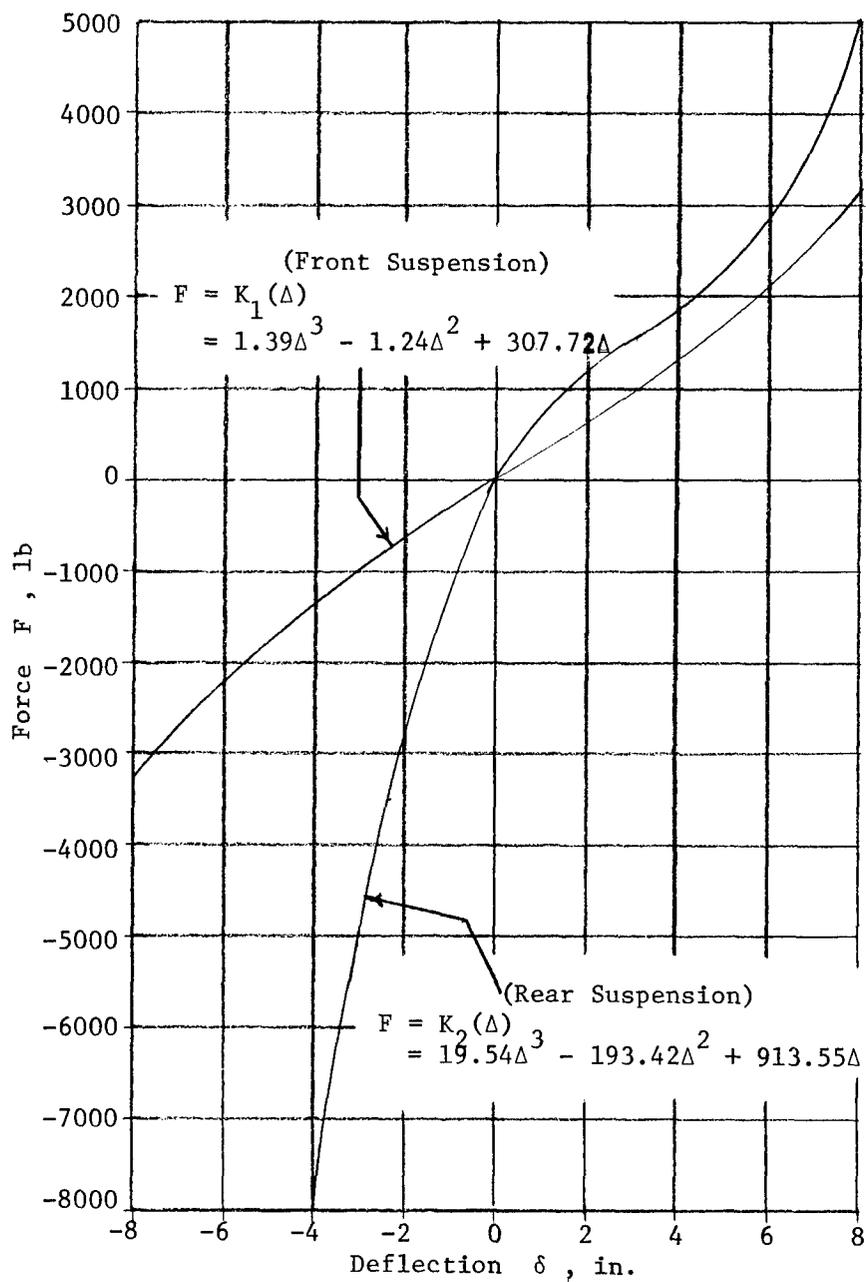


Figure 8. Force Versus Deflection for Front and Rear Suspensions of M37 Truck Model

$$C_1(\dot{\Delta}_1) = 11.8 \frac{\text{lb} - \text{sec}}{\text{in.}} \text{ (compression)}$$

$$= 22.8 \frac{\text{lb} - \text{sec}}{\text{in.}} \text{ (extension)}$$

$$C_2(\dot{\Delta}_2) = 12.0 \frac{\text{lb} - \text{sec}}{\text{in.}} \text{ (compression)}$$

$$= 49.0 \frac{\text{lb} - \text{sec}}{\text{in.}} \text{ (extension)}$$

No damping was incorporated in the tire compliance since in actual vehicles this damping is negligible compared to that of the suspensions. This truck model was forced to traverse each terrain condition at a constant speed, and the outputs consisted of motions of the main frame and axles in terms of displacement-, velocity-, and acceleration-time histories. The differential equations were programmed for solution on the GE-400 computer by the Runge-Kutta-Gill numerical integration scheme (Appendix A).

Similarity of Input and Output Statistics

The response of linear systems to random excitations is well defined in the form of mathematical solutions employing the concept of transfer functions. For example, if a random input forcing function can be assumed to have a normal probability distribution about a mean value of zero, then the response of this input will also be a random quantity with a normal probability distribution about a mean value of zero. In other words, the input statistics (the terrain) are transformed in a linear fashion by the transfer characteristics of the system (the vehicle) to produce response statistics that are related by the expression

$$P(\omega)_{\text{out}} = |H(\omega)|^2 P(\omega)_{\text{in}}$$

where

$P(\omega)_{\text{out}}, P(\omega)_{\text{in}}$ = the output and input spectral density functions, respectively

$H(\omega)$ = the system transfer function.

Hence, for a given input spectral density function, the output spectral density function is defined as the product of this spectral density function and the square of the system's transfer function.

Unfortunately, the topic of nonlinear response to random excitations does not readily lend itself to analytical treatment. Consequently, the most productive investigations in this area have been empirical in nature. Very little is known about how random inputs are transformed by nonlinear, multi-degree-of-freedom systems.

Before any effort was made to develop a method for defining terrain-vehicle-speed relations in a statistical fashion, an important question had to be answered: "Will constant statistical inputs to a nonlinear system produce constant statistical outputs?" A negative answer to this question is equivalent to saying that there is little hope for assembling a rational scheme for statistical analysis of nonlinear systems. To answer this question, three terrain profiles were generated (Fig. 9) that were different deterministically, but were the same statistically. That is, they had essentially the same PSD, distribution function and identical RMS values, although the elevation sequences composing their waveforms were quite different. The construction of these profiles was influenced by the fact that a number of investigators have observed that the power spectra (PSD) of natural surfaces and most man-made surfaces, such as runways and highways, can be expressed by the equation:

$$P(\Omega) = C\Omega^{-n} \quad (9)$$

Published data further indicate that the exponent n for both natural and man-made surfaces is approximately two.⁸ Therefore, a requirement for constructing these profiles was that their power spectra would be approximated by equation 9 with an exponent of two. Although the nearly 20-to-1 ratio between the vertical and horizontal scales produces distorted plots, the differences in features of the profiles are apparent. The RMS elevation was calculated for each profile, and in each case, a value of 8.5 was obtained. The RMS-distance plots are shown in Fig. 10. These profiles were purposely made extremely rough

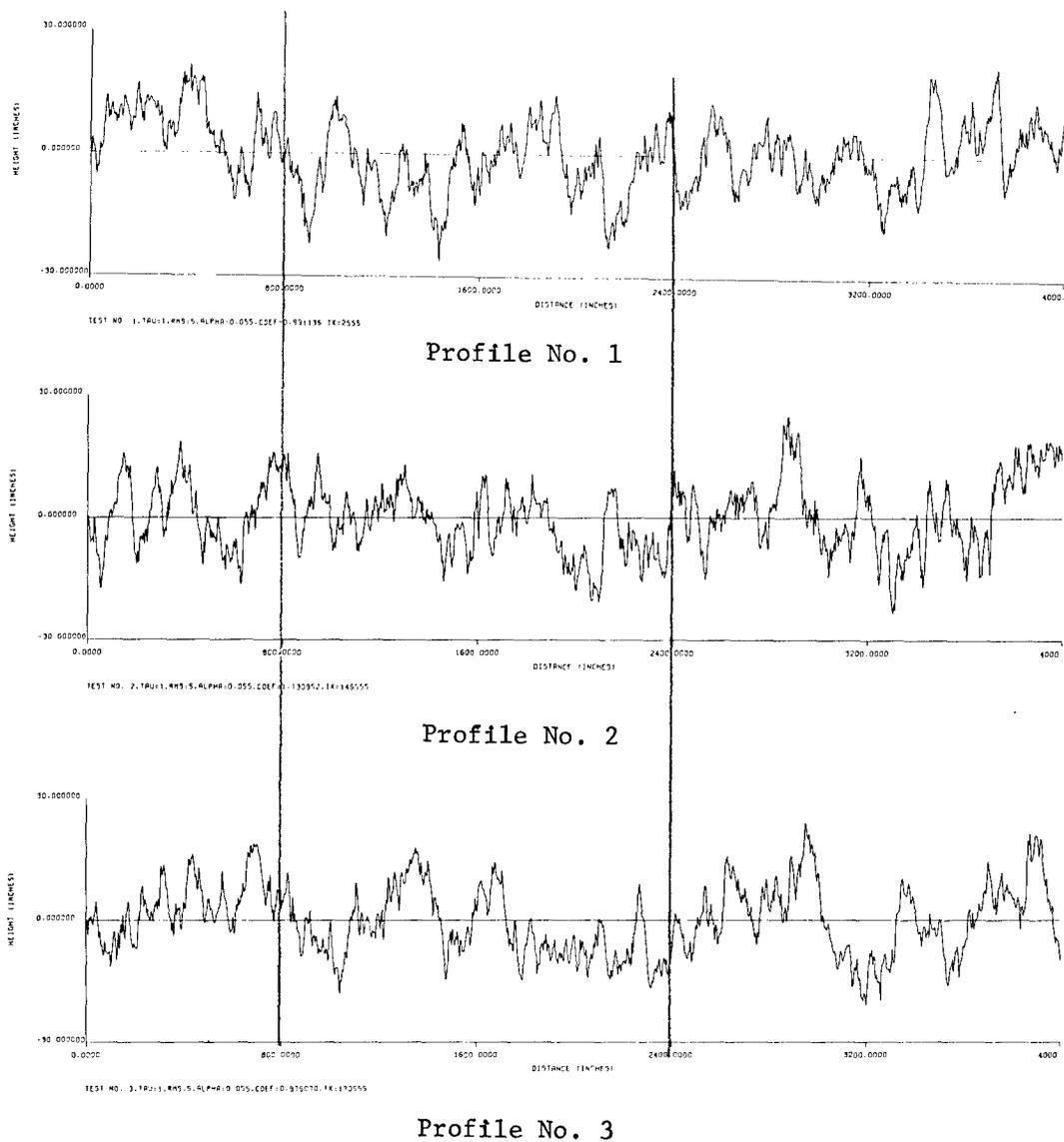


Figure 9. Computer-Generated Profiles with Same Statistical Content

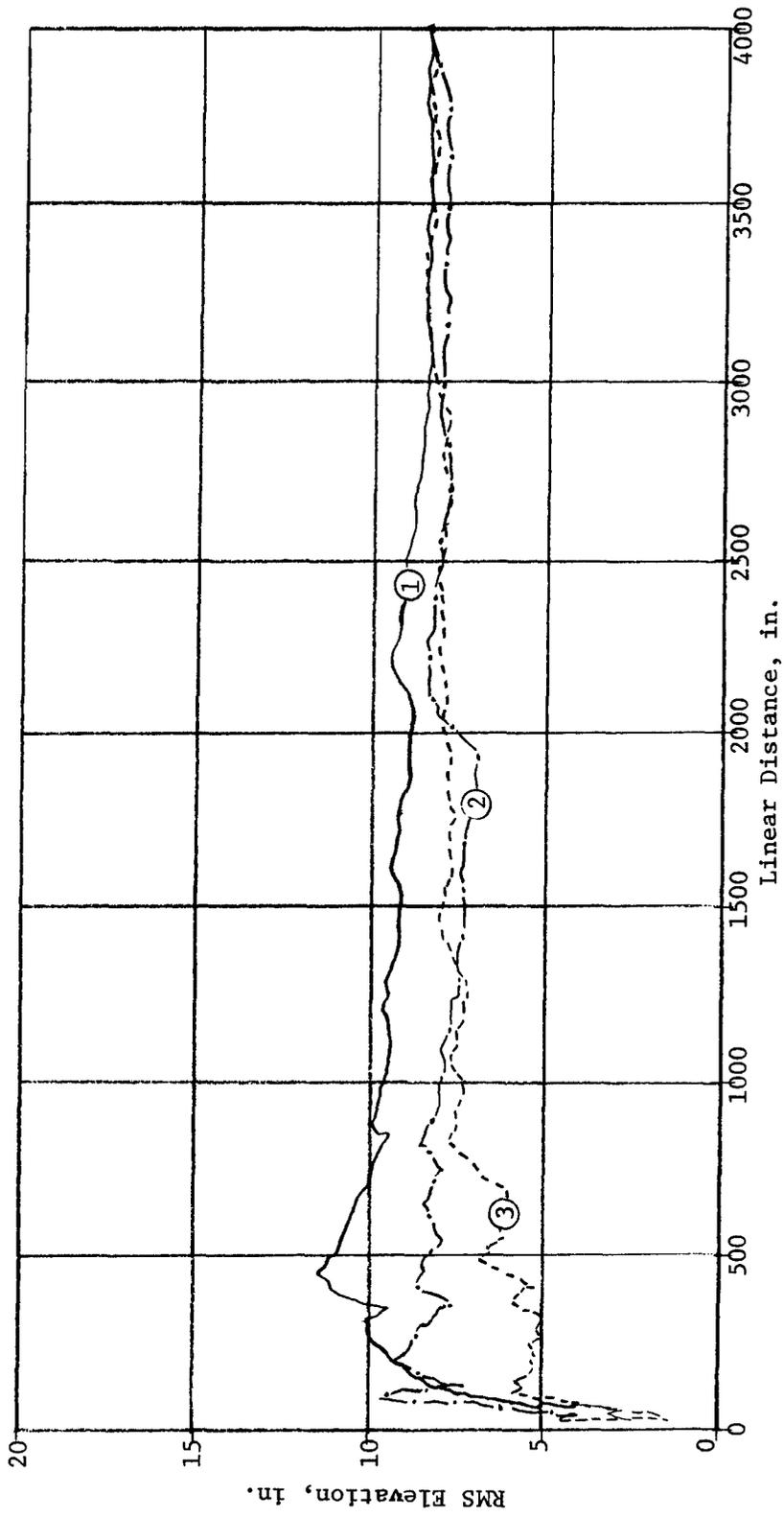
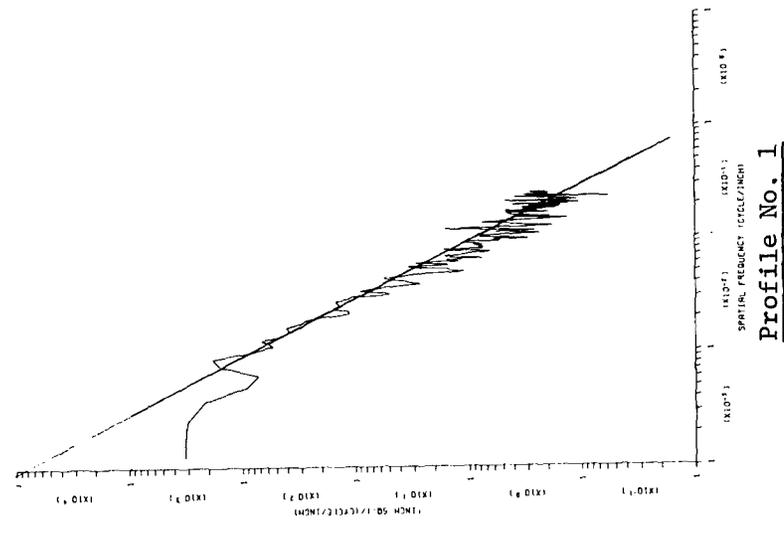
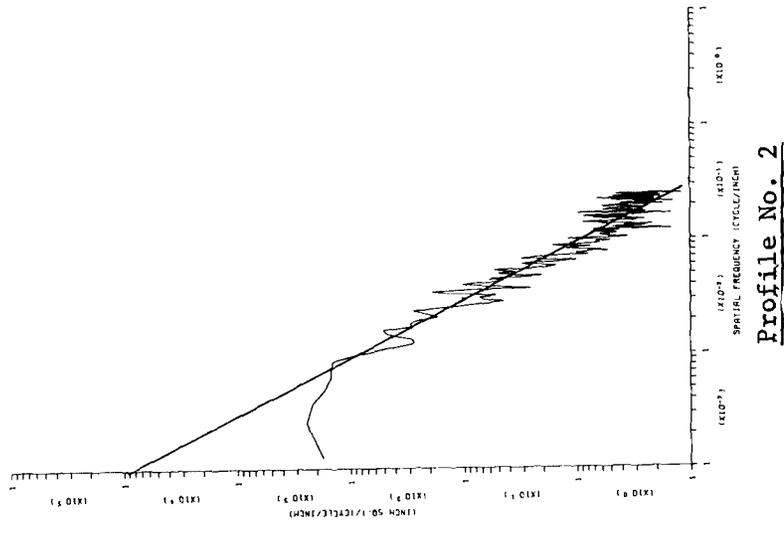
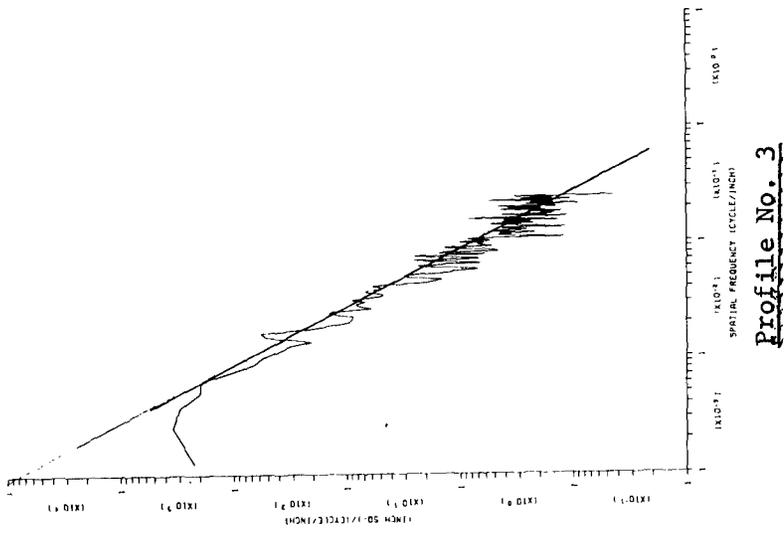


Figure 10. RMS Elevation Versus Distance for Computer-Generated Terrains

to assure that all the vehicle's nonlinearities would be influenced by the maximum degree. Logarithmic plots of the profile spectra are shown in Fig. 11. The straight line, whose equation is $P(\Omega) = 0.0785\Omega^{-2}$, drawn through each plot is to enhance a visual comparison of the similarity among the three spectra and illustrate the capability available for approximating given spectral relations.

The truck model traversed each of the three profiles at a speed of 5 mph (88 in./sec). The vertical acceleration-time histories of the vehicle's CG and front axle are shown in Figs. 12 and 13, respectively. Their respective power spectra are given in Figs. 14 and 15. The corresponding acceleration-time histories appear similar in form, but it is obvious from the distribution of the peaks that the waveforms are not truly stationary. However, if a quasi-stationary condition is considered for each case, the response spectra can be computed. Close examination of the corresponding response spectra reveal similar frequency distributions, but slight deviations in the low frequency range are noted for the CG motion. Unfortunately, at this speed this low frequency range corresponds closely to the basic natural frequency of the vehicle CG. Also, the axle response spectra for the test on terrain No. 3 is noticeably lower than that on the other two spectra. These deviations in the output spectra are reflected in the RMS acceleration-distance histories (Figs. 16 and 17). In Fig. 16, which shows the RMS-distance history of the CG motion, the RMS traces tend to converge and reach values of 1.02, 0.95, and 0.91 g, respectively, after about 4000 in. of travel. The RMS deviations at this point are really quite small, and indications are that they would converge to a common value



NOTE: The equation for the lines drawn on each plot is $P(\Omega) = 0.0785\Omega^{-2}$

Figure 11. Logarithmic Plots of the Profile Spectra for the Computer Generated Profiles with Similar Statistical Content

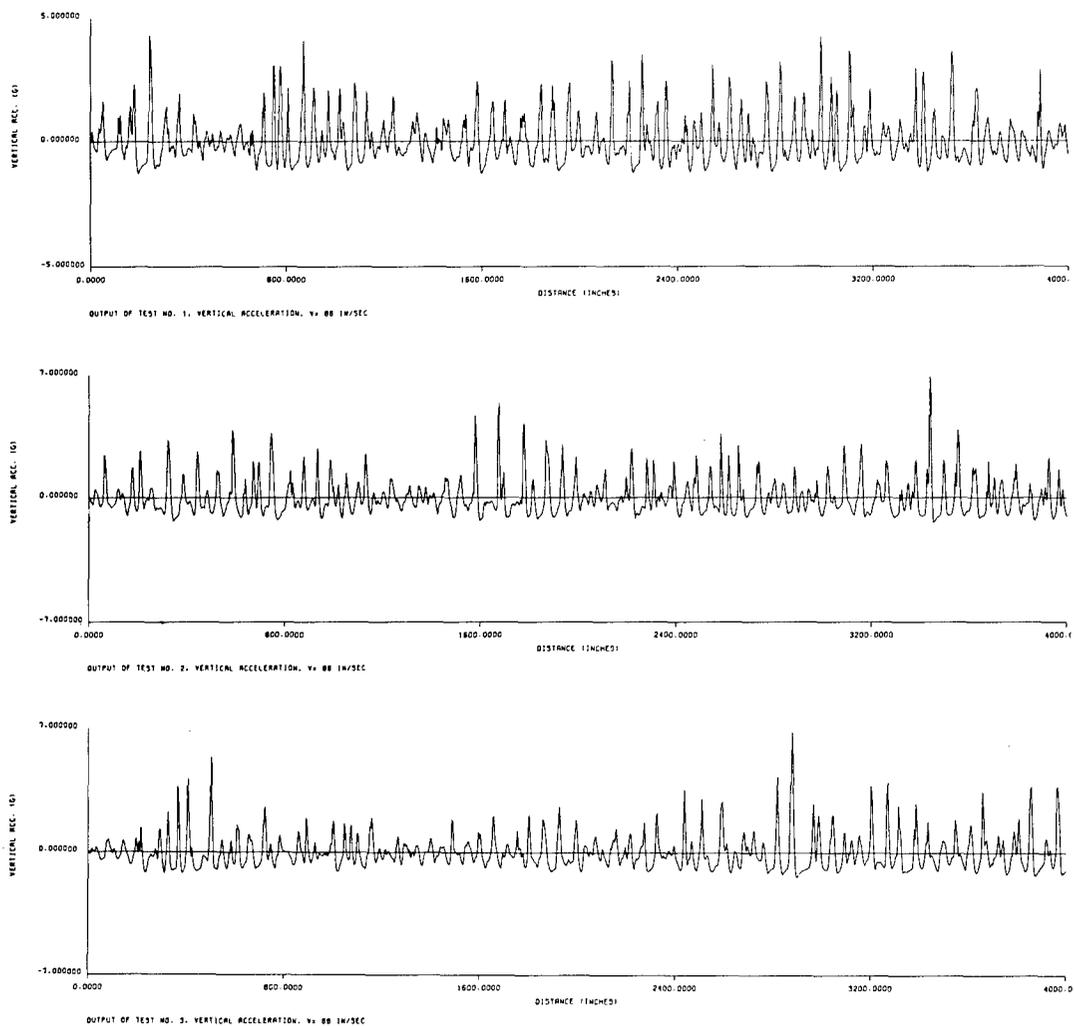


Figure 12. Acceleration-Time Histories of Vehicle's Center of Gravity Traversing Three Profiles with Same Statistical Content at 5 mph

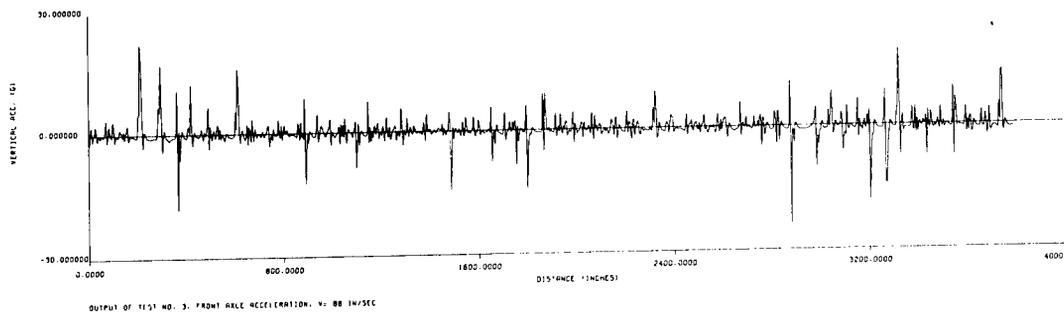
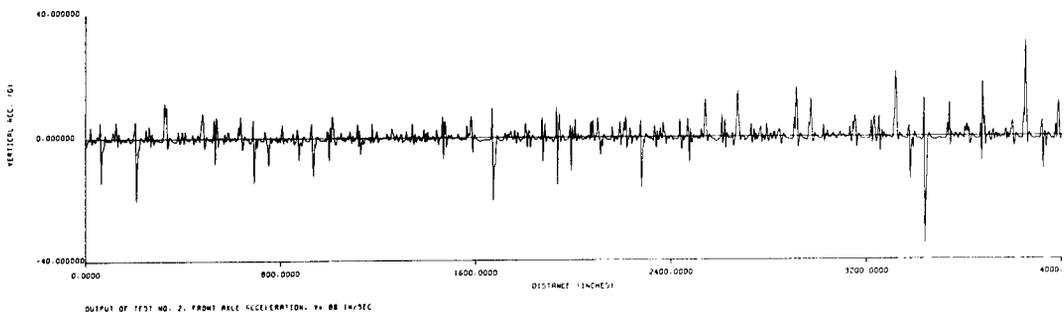
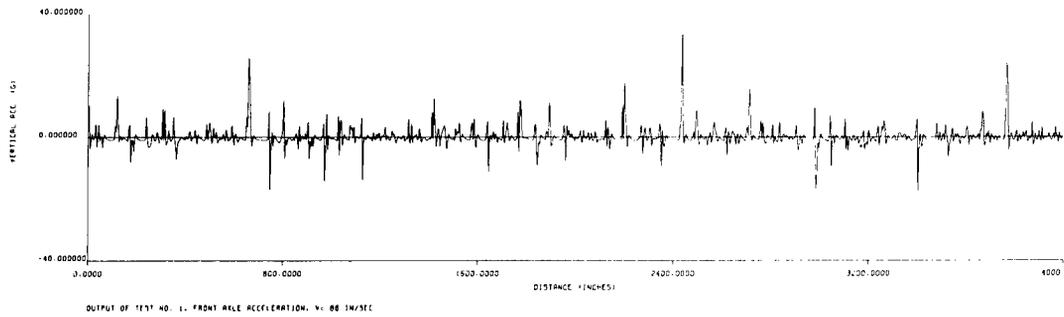


Figure 13. Acceleration-Time Histories of Vehicle's Front Axle Traversing Three Profiles with Same Statistical Content at 5 mph

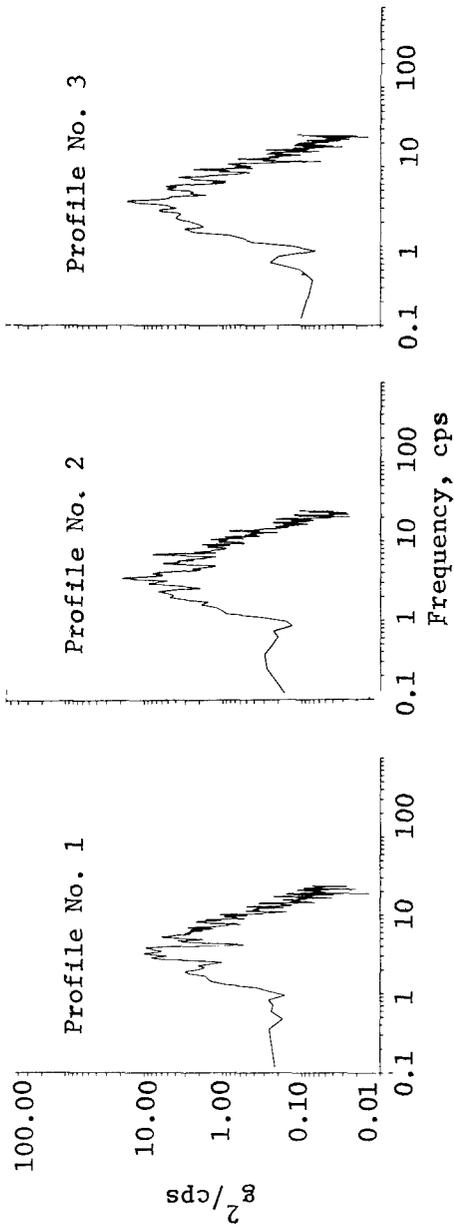


Figure 14. Response Spectra for CG Acceleration

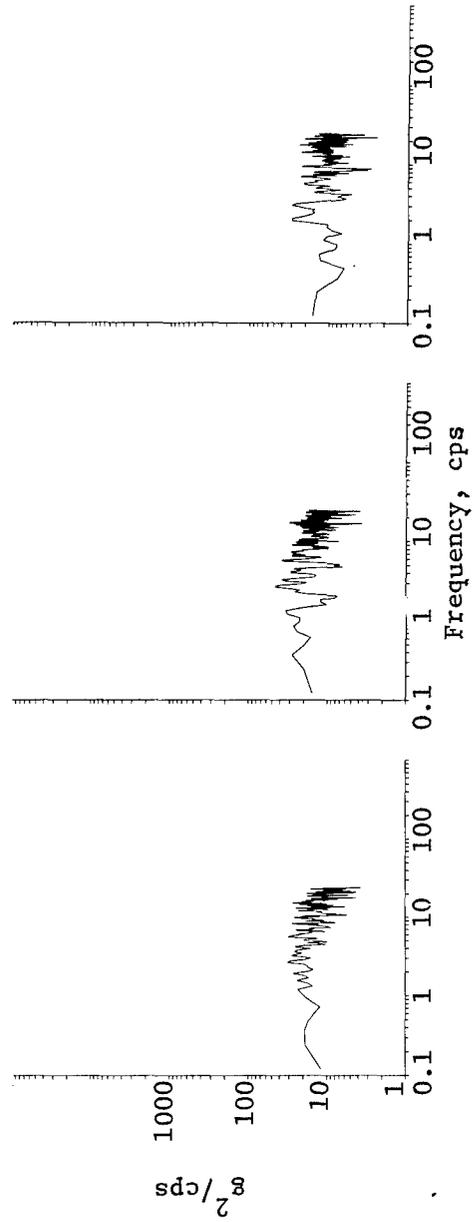


Figure 15. Response Spectra for Front Axle Acceleration

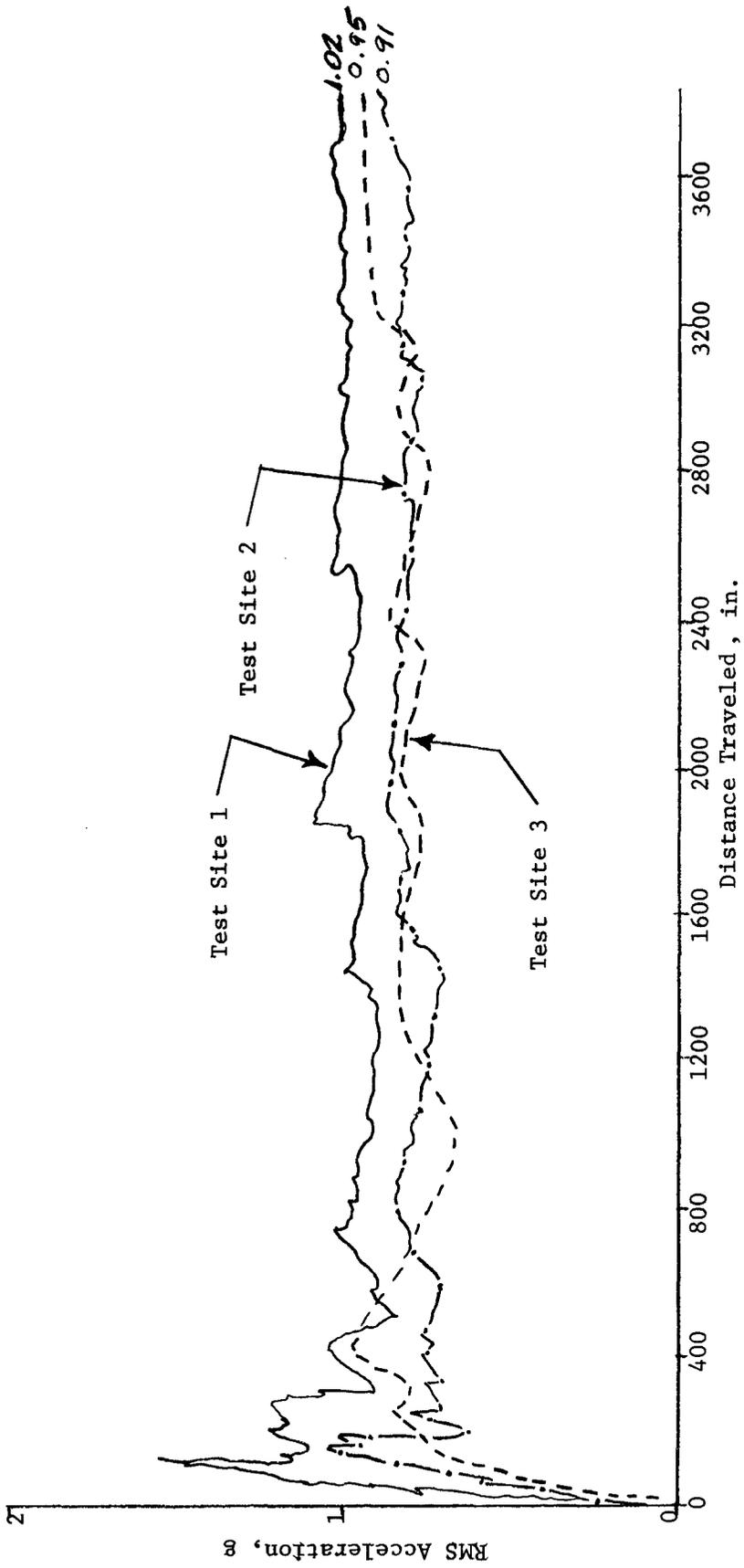


Figure 16. RMS Acceleration versus Distance of CG Response

if the test were evaluated over a longer distance.

A somewhat similar occurrence is noted for the axle motions (Fig. 17), where terminal RMS values of 3.55, 3.38, and 3.06 g were recorded. As indicated by the response spectra, the RMS acceleration at the axle for the test on terrain No. 3 was lower than that recorded for the other two. The noticeable deviations in the low frequency range of the CG response are believed to further indicate an insufficient length of sampling. Each of these tests represented only about 330 ft of travel for a vehicle with a wheel base of over 9 ft.

The autocorrelation functions of the CG motions were plotted for each of the three tests, and the results are shown in Fig. 18. In each test, a noticeable periodicity occurs. This is believed due largely to the periodic influence of the front and rear axles hitting the same profile elevations with a time delay dependent on the wheel base and the vehicle speed. This is probably the chief contributor to the nonstationarity of the output. Therefore, in all probability if the test had been conducted long enough to allow these periodic influences to achieve stationarity, it is conceivable that at this point and beyond the output RMS results would coincide. If this were the case and since this periodic phenomenon is a function of the wheel base, it appears logical that a minimum distance criterion could be established in terms of the vehicle's wheel base for evaluating vehicle responses in cross-country environments. For example, one might say that to statistically evaluate the dynamic response of cross-country vehicles requires travel over a profile that is suitably stationary and whose length is at least n times that of the wheel base of the

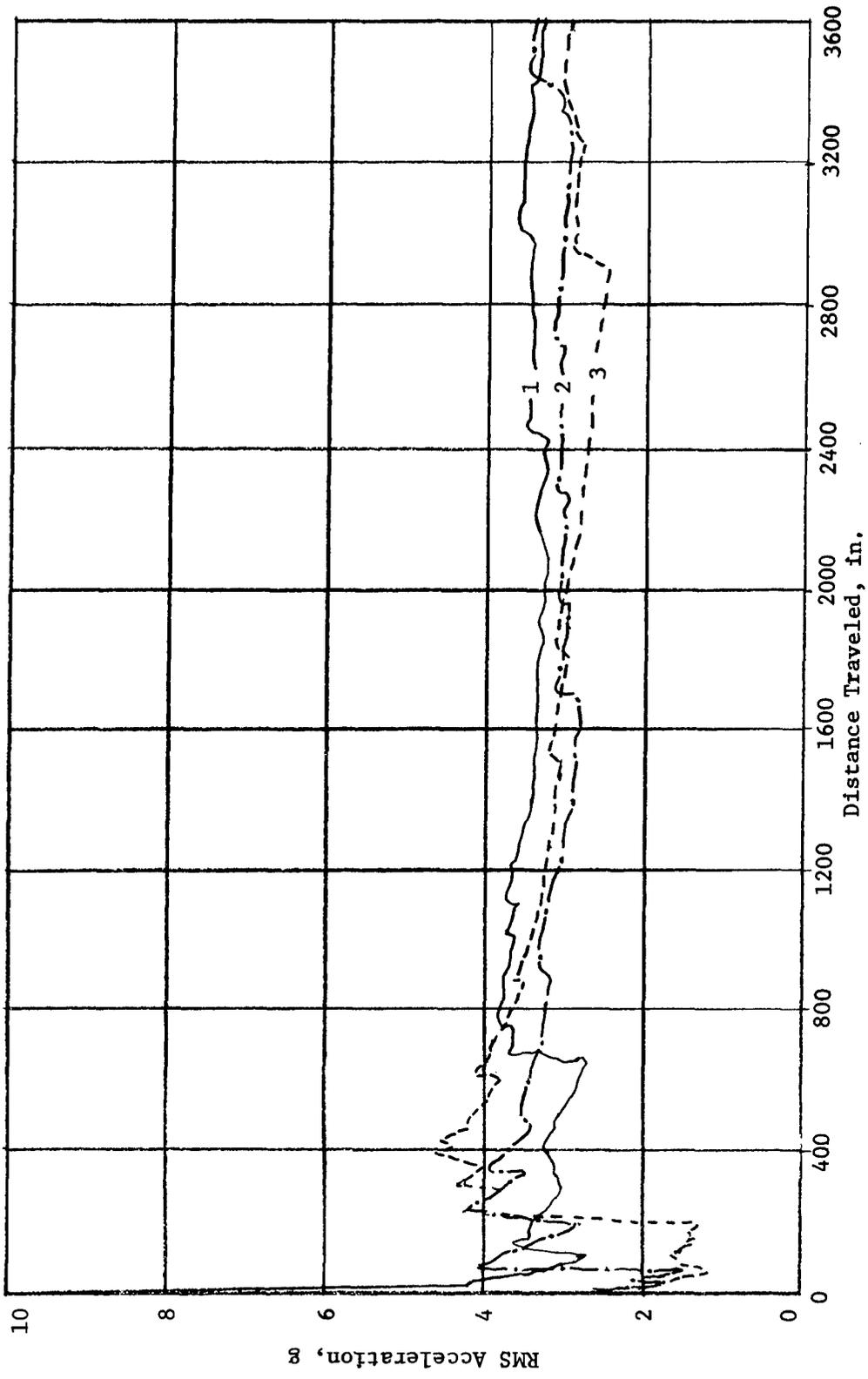


Figure 17. RMS Acceleration Versus Distance for Front Axle Motion

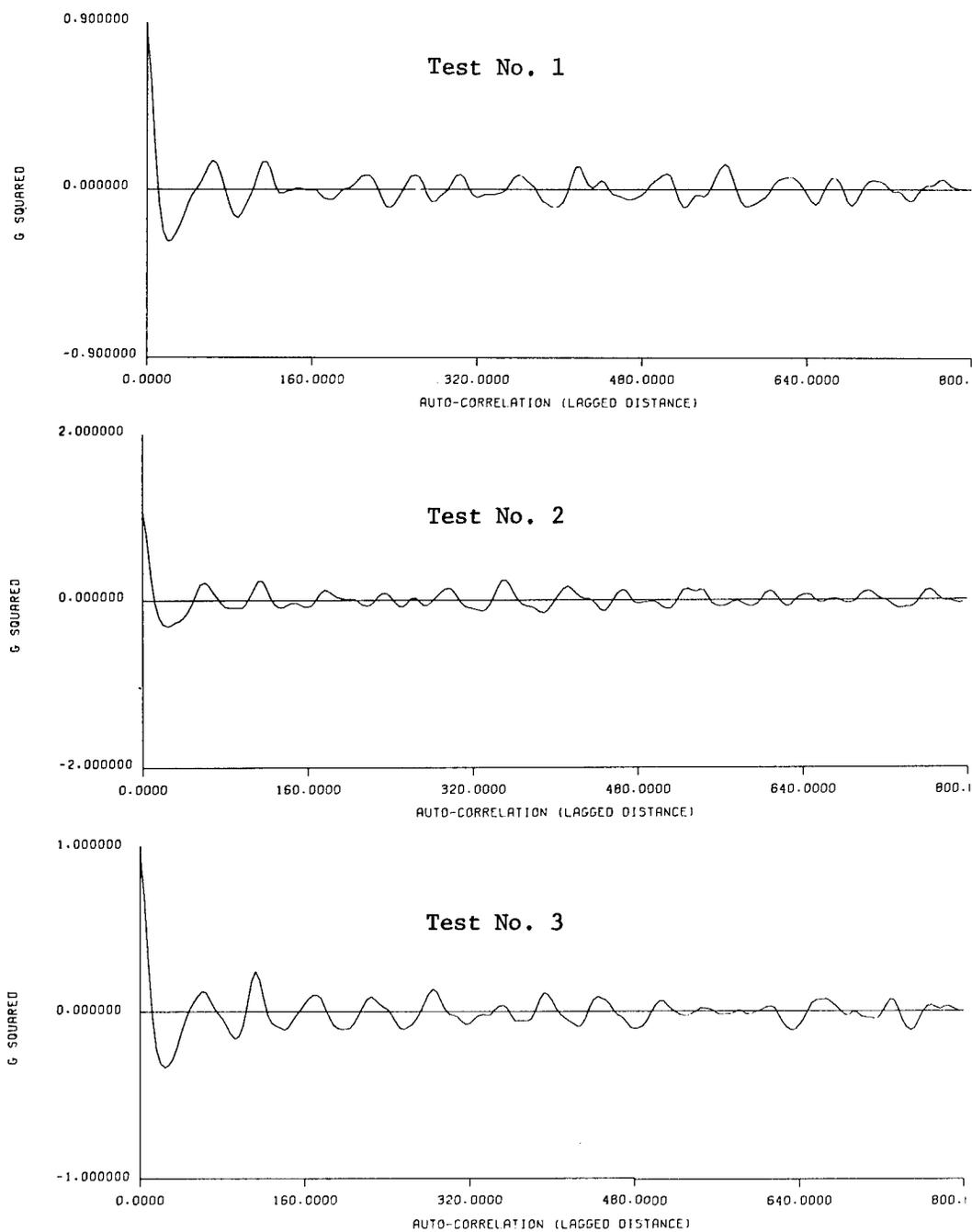


Figure 18. Autocorrelation Functions of Vehicle's CG Acceleration

vehicle. The value for n would have to be established from either experimental data or simulations.

Development of TVS Relations

It was concluded from the previous tests that the vehicle response statistics obtained from traversing terrains with constant statistics and a given terrain power spectra will be similar. This fact allows progression to the next step, which is the main objective in this study: to determine the effect of terrain roughness and vehicle speed on the vehicle's statistical response. Simulations with the truck model were conducted over eight profiles of varying levels of terrain roughness at speeds of 5, 7.5, 10, 15, and 20 mph over each of the eight profiles. Only the RMS elevation of the profile was varied, while the shape of the spectra was maintained constant. The nature of this variation is illustrated in Fig. 19, which shows logarithmic plots of the terrain profile spectra for three levels of roughness. The RMS elevations for these three profiles were 2.8, 8.5, and 63.7, respectively. The profile with $RMS = 63.7$ would preclude any type of vehicular traffic and is shown only to illustrate that the profile roughness can be changed without altering the profile's frequency distribution. This method of changing only the profile RMS simply alters each elevation point by a proportionate amount. That is, if the original profile had an RMS of 5 and the RMS was changed to 10, the features of the resulting profile would be identical to those of the original, but the elevations would be twice those of the original.

The straight lines drawn through the three spectra each represent

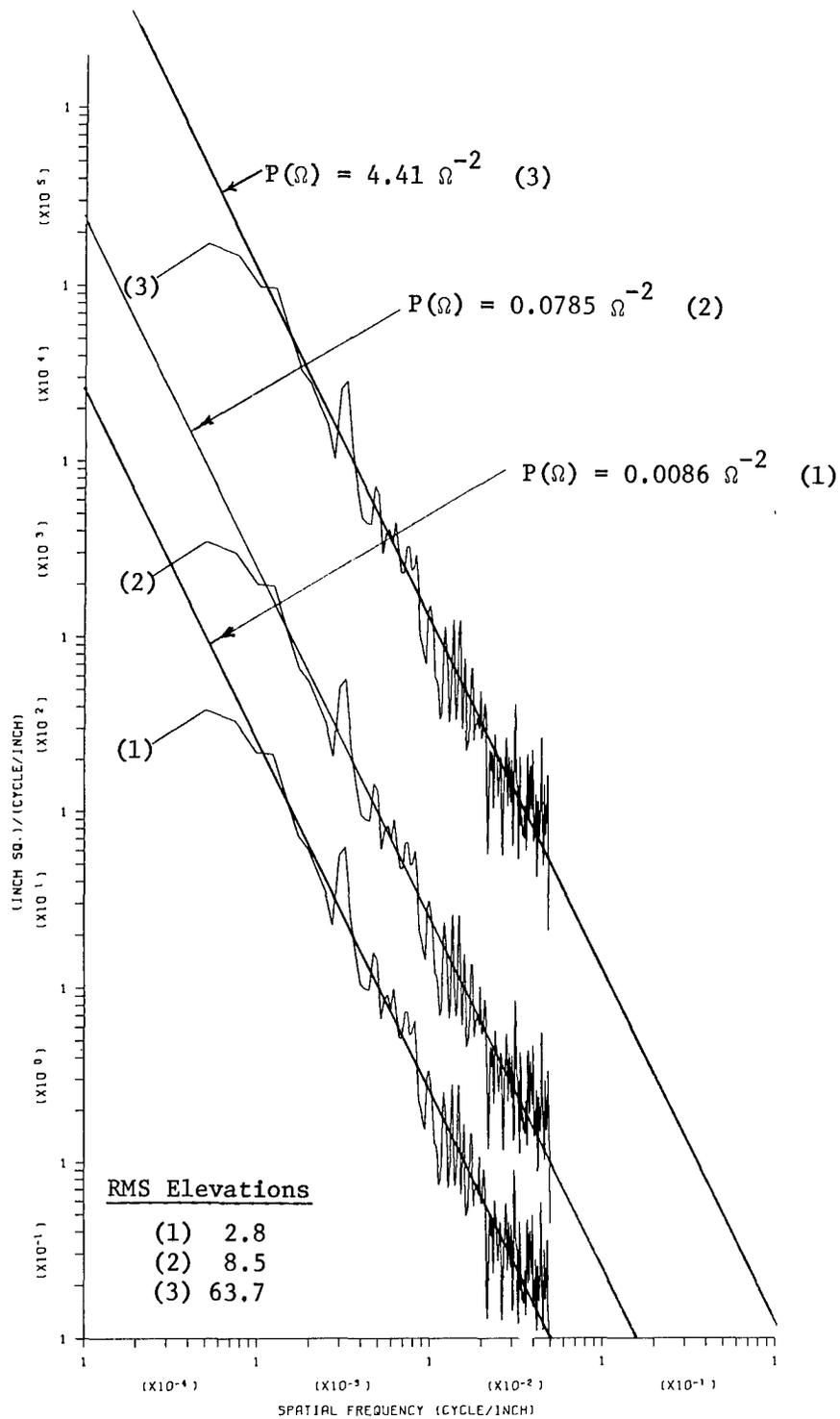


Figure 19. Power Spectra for Computer Generated Terrains of Three Levels of Roughness

a power function with a negative exponent of two. The coefficients of these power functions are related to the RMS by means of another power expression:

$$\text{RMS} = CA^{1/2}$$

where A is the coefficient for a particular spectra and C is a constant whose magnitude depends on the frequency range considered.

The results of the simulations are shown in Fig. 20 in the form of RMS acceleration of the vehicle's CG versus RMS roughness. There is a distinct response pattern for each speed, but the influence of the system's nonlinearities at the higher speeds on the rougher profiles caused pronounced deviations from a well-behaved pattern. This is typical of frequency-dependent systems; and, because of this, several more profiles and speeds should be inserted to adequately define the response pattern. To get some idea of the response sensitivity, an extra profile (RMS = 5.0) was included for the 20-mph speed and two (RMS = 4.5, 5.2) for the 10-mph speed. The most noticeable aspect of the additional profiles was a pronounced peak that occurred in the response pattern at 20 mph when the profile roughness changed from RMS = 4.9 to RMS = 5.0. This is undoubtedly due to the occurrence of resonant conditions that caused excessive contact between the axles and the bump stops and produced accelerations that would be harmful to the vehicle as well as its occupants in an actual situation. Perhaps for a more realistic range of speeds and terrain roughness the response-roughness patterns would all be well behaved.

Since speed and ride severity are of chief interest in cross-country ride dynamics, a more usable graph was obtained by cross

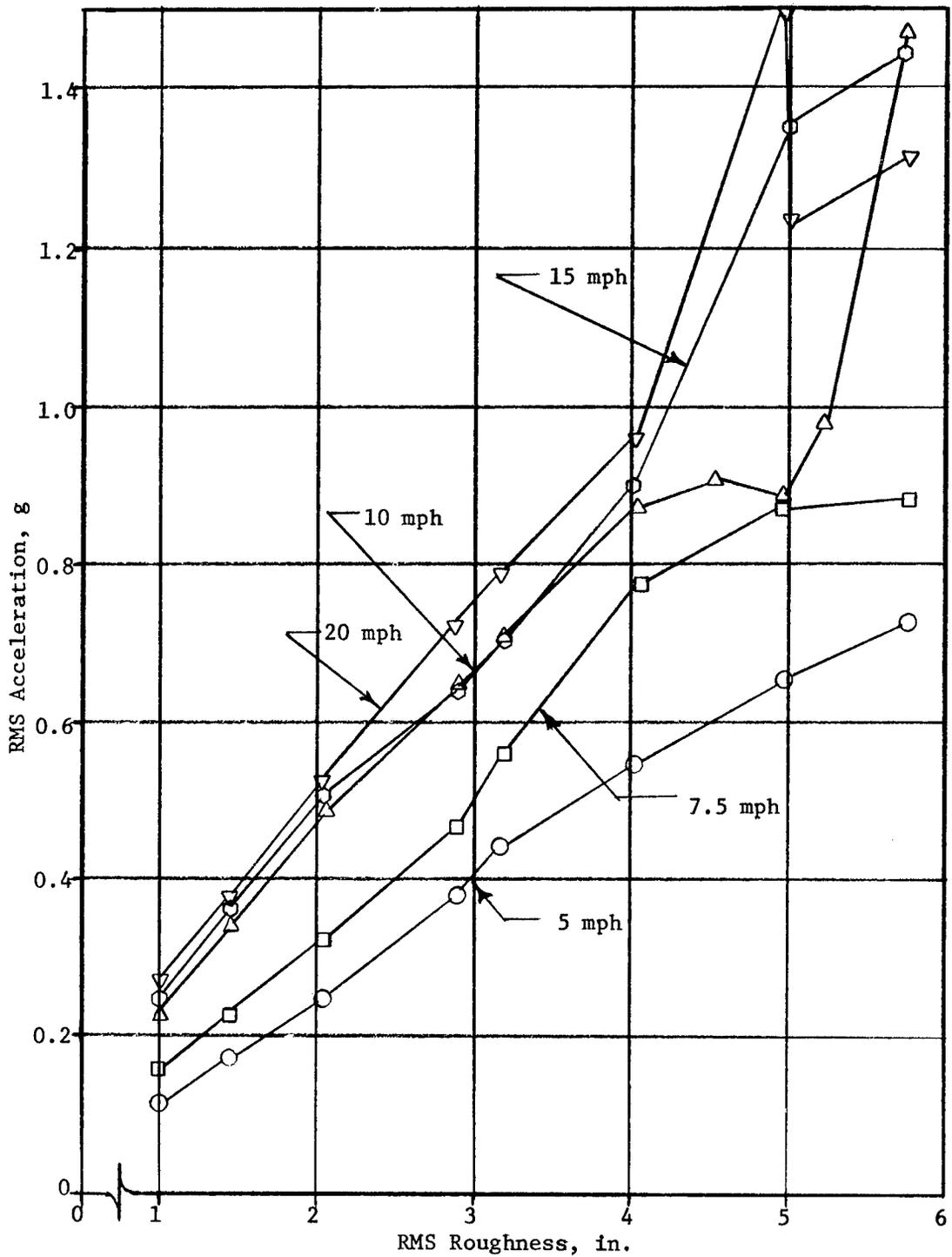


Figure 20. RMS Acceleration Versus RMS Terrain Roughness;
Vertical Motion at Vehicle's CG;
M37 Truck Model

plotting these results and relating the vehicle response to vehicle speed. These results are shown in Fig. 21. More tests are needed at speeds below 5 mph to adequately define the initial portions of the response-speed curves in Fig. 21. This range, shown by dashed lines, would most likely be the critical range for an actual TVS system, particularly for the rougher profiles. The dashed lines were made to intercept at a small common value at zero speed. This represents the small RMS level due to engine vibration that occurs in an actual vehicle when the vehicle is idle. These two graphs almost completely define the TVS characteristics for this particular vehicle model. For example, for a given level of ride severity based on RMS acceleration and a given level of terrain roughness based on RMS elevations, estimates of the average speed that the vehicle can attain and stay below the selected ride level can be made.

However, because of the statistical nature of the development, a complete definition of the TVS characteristics should provide a determination of the probability of exceeding the criteria of interest. This is attained by means of the amplitude probability distribution (APD), which is the probability that the amplitudes composing a waveform will exceed a given level. For digital data, an estimate of the APD is determined by simply counting the number of points above the given level and calculating the ratio of this number to the total number of available points.

For the terrain profile, the APD serves as a means for estimating the occurrence of elevations above a given level. For vehicle response, the APD serves to estimate the probability of exceeding a given g-level. Such a distribution function was constructed from the absolute values

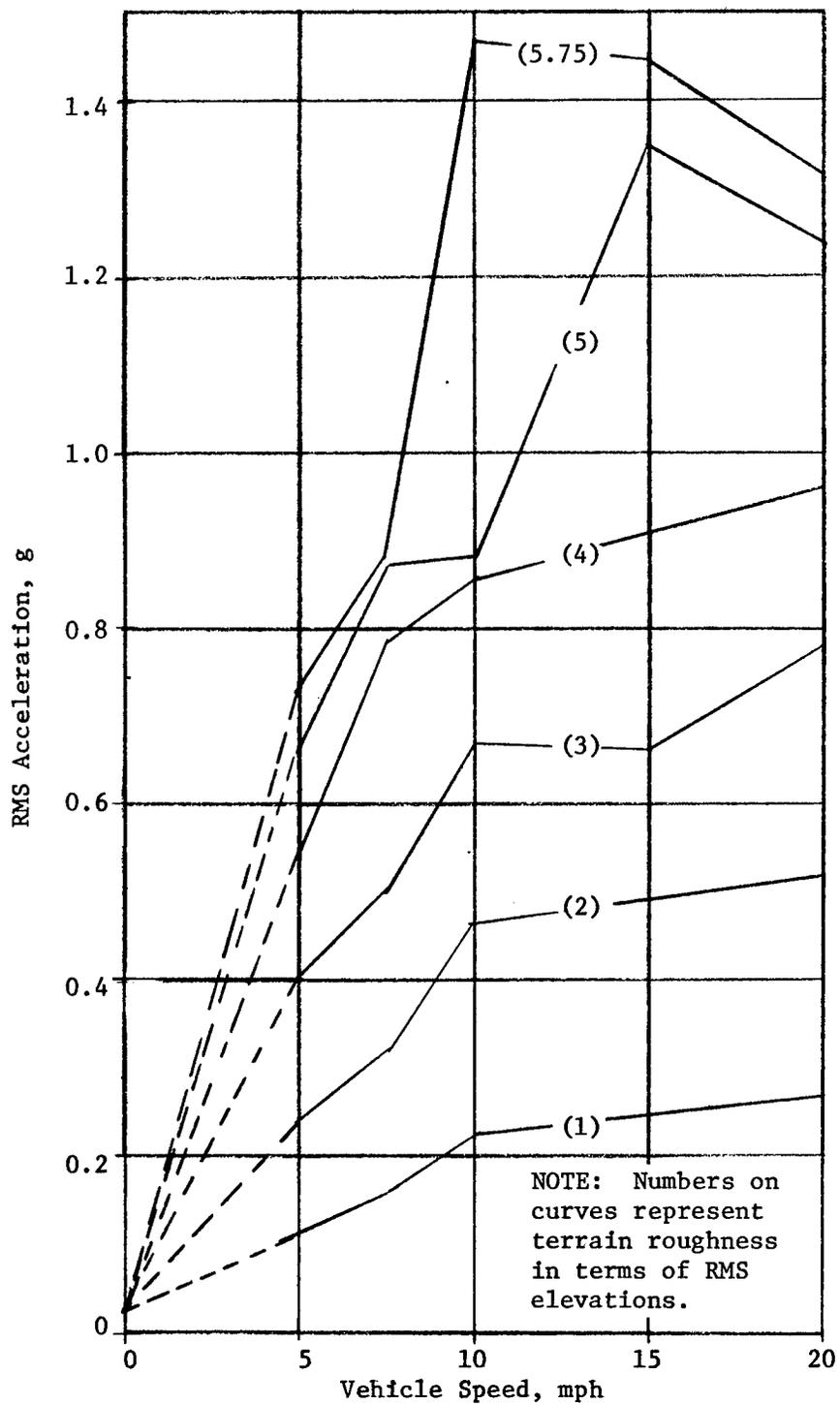


Figure 21. RMS Acceleration Versus Vehicle Speed;
Vertical Motion at Vehicle's CG;
M37 Truck Model

of the CG accelerations for the test at 7.5 mph over the terrain with an RMS roughness of five, the results are shown in Fig. 22. The maximum acceleration that occurred during this test was 3.85 g. According to amplitude distribution, the probability of exceeding 2.0 g is about 0.90, while the probability of exceeding 3.0 g is only about 0.26. Looked at in another way, when a vehicle is traversing a terrain of this nature at 7.5 mph, it can be expected to exceed 3 g for about 26 percent of the duration of the run. The response spectra will give the frequency of occurrence of these g levels. This leads quite naturally into a possibility for quantifying the intensity of a vibrational environment in terms of human tolerance. For example, such quantities as the frequency of occurrence of accelerations above a certain g level, or the area under the RMS acceleration-time history curve (which is a sort of impulse measure or measure of vibrational energy) should correlate with appropriate measures of human tolerance.

Figs. 20, 21, and 22 thus completely define the ride dynamics characteristics for the vehicle model operating on such random undulating profiles. However, since these profiles were constructed from a basic spectral shape that has been shown to represent most natural and man-made surfaces, it is conceivable that such TVS relations could be developed for any desired vehicle and serve to describe its ride dynamics characteristics for both on-road and off-road environments.

This graphical scheme will allow rating of vehicles, on a standard competitive basis of ride, as to their capability to negotiate cross-country terrains at various speeds. Such a comparison could take the form of the hypothetical relation in Fig. 23, which shows the

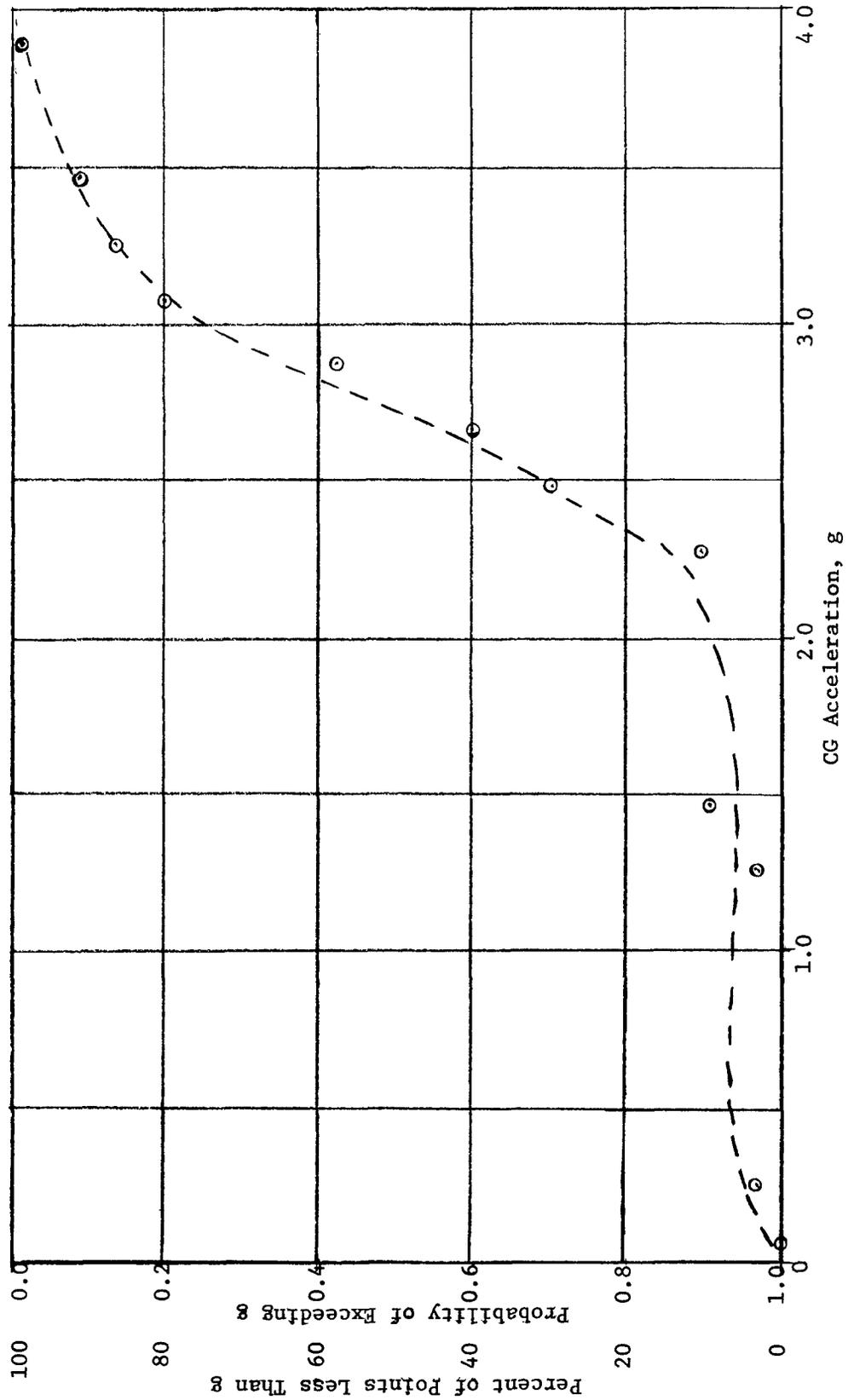


Figure 22. Amplitude Probability Distribution Function for CG Accelerations Incurred at 7.5 mph Over Terrain Profile with RMS Elevation = 5

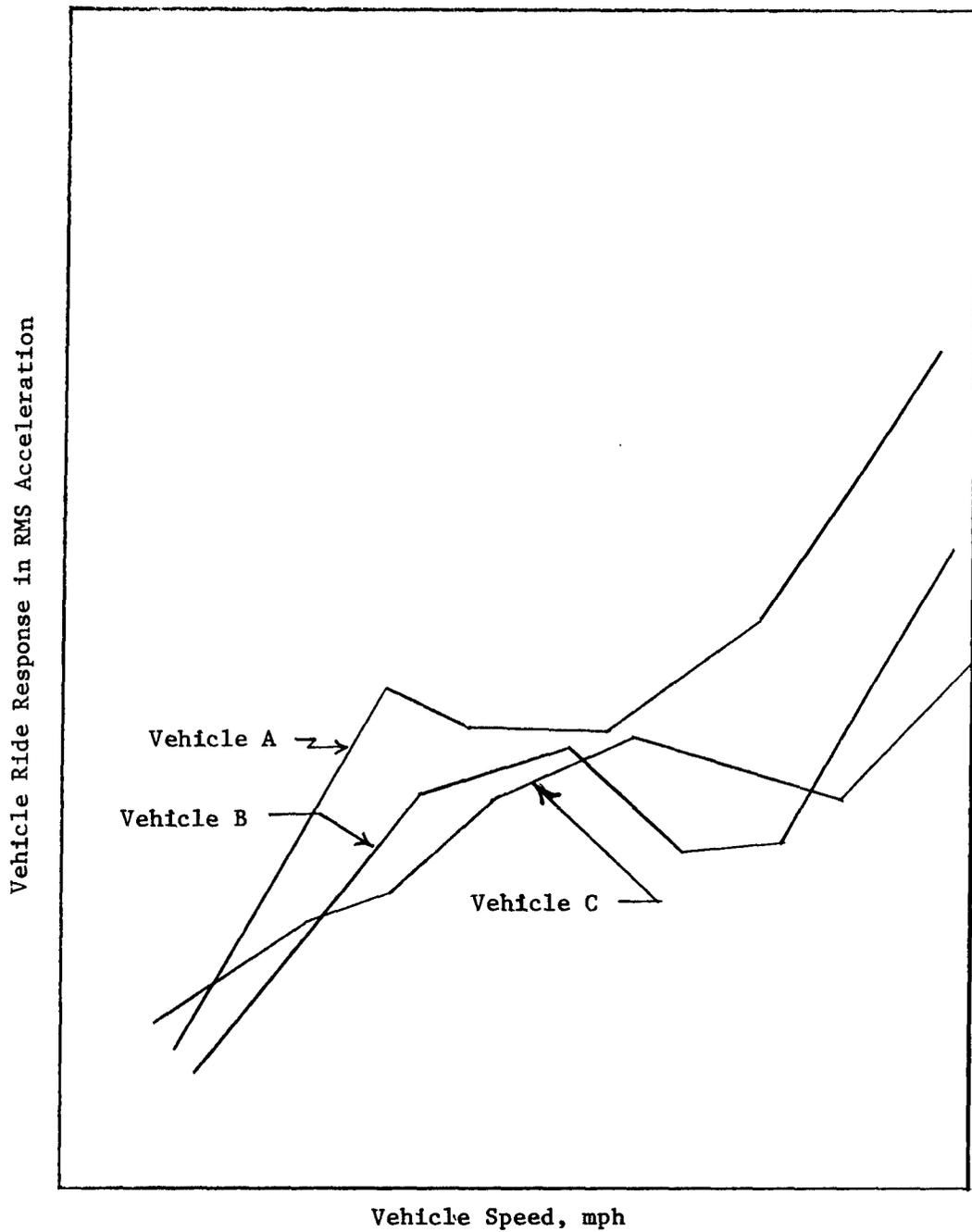


Figure 23. Comparison of Dynamic Response of Three Vehicles Operating in a Given Terrain Environment

response-speed relations for three vehicles operating in a given off-road environment. As before, the complete evaluation as to which vehicle was best suited to the environment would require the APD for each vehicle.

Also, the capability for constructing artificial profiles to conform to any desired power spectrum will permit vehicle response evaluations for remote areas without physical testing of the vehicle in that area. This analysis capability greatly enhances the utility of a comprehensive analytical model for describing the overall mobility aspects of off-road vehicles. It also can serve as an aid in the design of off-road vehicles by permitting the study of simulated motions of candidate vehicles operating on generalized terrain types. The effects of suspension components, for example, can be studied in terms of their vibrational isolation capabilities by examining the ratios of the RMS accelerations of the axles to those at various parts of the vehicle's main frame, or in regard to various independent suspension types by studying the ratios of the RMS fore and aft accelerations.

Application to Real-World Situations

This has been a computer development of the terrain-vehicle-speed relations for a computer model that is representative of an actual vehicle in the military's Table of Organization and Equipment (TO&E) traversing computer-constructed terrain profiles whose power spectra were representative of those of actual profiles. However, the application of this scheme of analysis to actual vehicles traversing actual terrains is necessary to establish its success and practicality. The

main problems that might occur will probably be associated with the terrain input. The terrain profiles in this study were constructed to fulfill certain essential requirements of randomness and stationarity. Tests need to be conducted to establish the degree of stationarity exhibited by representative terrains. A permissible degree of terrain stationarity, which has little effect on the TVS relation, needs to be established. This may turn out to be a uniform standard applicable to all vehicles, or may need to be specified according to vehicle classes, such as wheeled, tracked, etc.

The effect on both the input and output statistics, as well as the frequency distribution, of detrending (removing the longer terrain wavelengths that do not affect ride) the original profile to achieve this adequate degree of stationarity needs to be thoroughly studied. These results should establish the requirements necessary to develop actual TVS relations.

CHAPTER IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

On the basis of the results of this study, it is concluded that:

- a. Vehicle response statistics obtained from traversing terrains with constant statistics and a given terrain power spectra will be similar.
- b. Three basic graphs can be constructed that will completely define the ride dynamics characteristics for a vehicle operating in both on-road and off-road environments.
- c. This graphic scheme will rate vehicles, on a standard competitive basis of ride, as to their capability to negotiate cross-country terrains at various speeds.
- d. The capability for constructing artificial profiles to conform to any desired power spectrum will (1) permit vehicle response evaluations for remote areas through simulation methods, (2) enhance the utility of a comprehensive analytical model for predicting overall mobility aspects of vehicles, and (3) aid in the design of off-road vehicles.

Recommendations

It is recommended that:

- a. The results of this study be applied to describe actual TVS relations.
- b. Tests with actual vehicles in real terrain environments

be conducted to determine:

- (1) The degree of stationarity exhibited by representative terrains.
- (2) The effect of detrending the original terrain to achieve adequate stationarity.
- (3) Minimum length of terrain required to insure adequate stationarity of the output response.
- (4) The applicability of such TVS evaluations to comprehensive analytical cross-country mobility models.

CHAPTER V: ABSTRACT

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Major: Engineering Mechanics, College of Engineering

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Speed Systems

Directed by: Professor Walter R. Carnes

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Abstract

The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle-speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness and cross-country speed. This methodology should enable the determination of reliable estimates of the average speed a given vehicle can attain under given vibrational constraints as well as the probabilities of exceeding certain specified levels.

This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with a desired power spectrum, variance, mean

value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness.

A comprehensive, nonlinear mathematical model of an M37 truck served as the vehicle. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stops, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs.

The results are presented in the form of three basic graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity. A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness.

The use of such relations to catalog vehicles in terms of their ride dynamics capabilities is suggested. The TVS scheme is readily adaptable to computer models for evaluating and rating the cross-country mobility characteristics of vehicles traversing large, non-homogeneous terrain areas.

CHAPTER VI: APPENDIX

APPENDIX A: COMPUTER PROGRAMS

The study discussed in this report consisted entirely of a computer effort. The digital computer programs employed in this study are listed on the following pages in the proper sequence for developing the relations established in this thesis. The programs are written in FORTRAN IV programming language for use on a GE-430 computer system in the time sharing mode. A brief description of each program is given in the following paragraphs.

NOISE

The first program listed, NOISE, was developed specifically to compute the random terrain profiles with certain specified statistics. It first calls for a name for the output file of terrain profile points. The input variables are TAU, RMS, ALPHA, FACT1, IX, and III. The variables TAU and ALPHA control the PSD shaping and it was found that setting the ALPHA-TAU product = 0.055 gave the best normalities condition based on results of the Chi-square test. The variable RMS is used to determine the standard deviation of the sequence of profile points (see line No. 1180 in program listing). FACT1 is used to control the RMS roughness level of the resulting profile points. For example, a value of 0.1168 for FACT1 produced a sequence with an RMS = 1.0. To obtain a profile sequence with an RMS roughness of 4.0 requires a value for FACT1 which is $(4)(0.1168)$. The input variable IX is the arbitrary starting point in the random number chain; III is the number of desired profile points.

The NOISE program utilizes two library subroutines: GAUSS and

RANDU. The subroutine RANDU generates a sequence of uniform, random numbers by the conventional congruence method. The subroutine GAUSS generates a sequence of normal, random numbers from the uniform sequence by a technique referred to as the "sum of uniform deviates." A "shift" routine adjusts the numbers to provide an absolute zero mean. The normal, random sequence is passed through a numerical, low-pass filter to yield the final, proper frequency weighted sequence.

STANOR

The next program, STANOR, provides all the necessary tests of the fundamental assumptions. The input to this program is the output file of profile points generated by the NOISE program.

This program first performs the trend test and run test to check the stationarity of the points at the 0.05 level of significance. The sequence of numbers is divided into N equal intervals for the two tests and the mean value computed for each interval. The resulting sequence of the means is checked for the appropriate number of zero crossings (run test) and reverse arrangements (trend test). The Chi-square test is then conducted to check the normality condition. This is also accomplished at the 0.05 level of significance. For this test, the profile sequence is again divided into an appropriate number of groups determined by a formula that depends on the total number of points in the sequence. Three statistical tables, TABK1, TABK2, and TABK3, are used in conjunction with this program.

PSD

The program, PSD, first computes the autocorrelation function from which it obtains the PSD estimate via the Fourier transform of the

autocorrelation function. The input file is the file of numbers of which the PSD is to be obtained. Output files of the correlation coefficient versus lag and PSD estimates versus frequency are generated. The basic input variables are: KKK, LAG, DELTAT, and FACT, where KKK is the number of points in the input file, LAG is the maximum lag (199 for this study) used in computing the autocorrelation function. DELTAT is a value that determines the spacing between the input points and may represent either time or distance. FACT is a coefficient that multiplies each input number (see line No. 1064 in listing). This serves as a means of altering the input numbers by a factor. If no altering is desired, set the value of FACT to unity.

The frequency ω is determined by the formula

$$\omega = \frac{h\pi}{m\Delta t} \quad h = 0, 1, 2, \dots, m$$

where

m = maximum lags used not to exceed 199

Δt = the equal intervals between points (may be time or space units)

This frequency ω is in units of radians/unit time or distance and must be divided by 2π to obtain frequency in cycles/unit time or distance.

For example, calculate the maximum obtainable spacial frequency for $m = 199$ and $\Delta t = 1$ ft. The maximum frequency is obtained when $h = m = 199$, i.e.

$$\omega_{\max} = \frac{\pi}{\Delta t} = \frac{3.14}{1} = 3.14 \text{ rad/ft}$$

or in terms of cycle/ft

$$\Omega_{\max} = \frac{3.14}{2\pi} = \frac{3.14}{6.28} = 0.5 \text{ cyc/ft}$$

It is worthy to mention that the raw spectral estimates are smoothed with Hamming coefficients.

TRUCK

The TRUCK program, which represents a two-dimensional mathematical model for the M37, 3/4-ton truck, is a special purpose program written specifically for this study. The program is composed basically of an executive program and two subprograms; DIFFEQ, which contains the Runge-Kutta-Gill solution to the differential equations and ALGEBR, which performs all the required algebraic operations.

The input variables required by the program are clearly defined in the program listing. However, some noteworthy comments concerning the segmented wheel implementation are given to enhance an understanding of the method used to simulate the tire-terrain compliance.

Each wheel, which represented a 9.00x16, 8-PR tire at 45-psi inflation pressure, was divided into ten segments; five on each side of the tire's centerline as shown in Fig. A1. The measured load-deflection relation (Fig. A2) for the 9.00x16 tire at 45-psi inflation pressure was such that a centerline deflection of 1.5 in. required a load of 2860 lb. At this deflection, four spring segments are influenced, two on each side of the centerline (Fig. 1). The spring constant K can be determined from the statics equation:

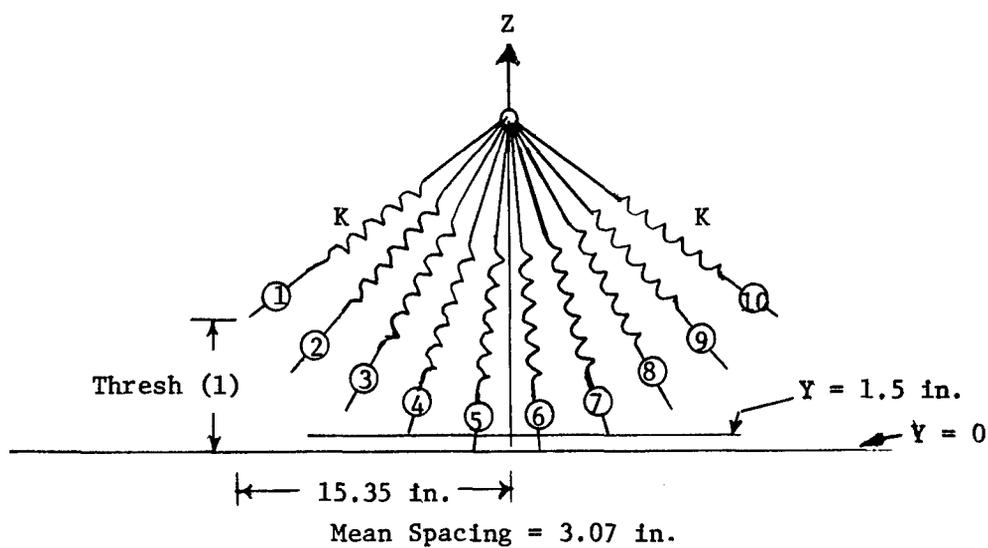


Figure A1. Schematic of Segmented Wheel

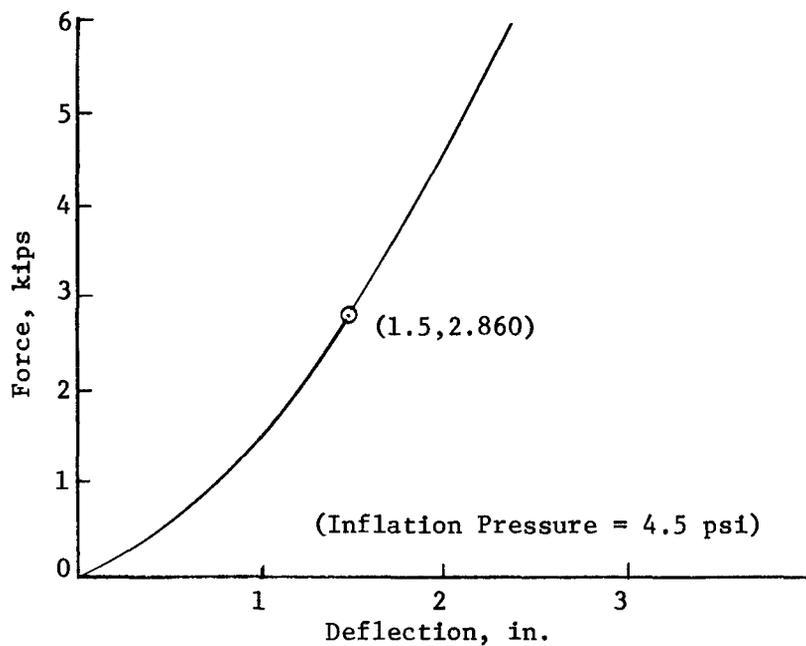


Figure A2. Force Versus Deflection for the 9.00-16, 8-PR Tire

$$F = \sum_{i=1}^{10} K \cos \phi_i \Delta_i$$

where

$$\begin{aligned} \Delta_i &= Y_i - \text{THRESH}(i) - Z, & Y_i - \text{THRESH}(i) - Z &\geq 0 \\ &= 0, & Y_i - \text{THRESH}(i) - Z &< 0 \end{aligned}$$

Y_i = vertical height of terrain profile beneath i^{th} segment

Z = vertical displacement of axle

$\text{THRESH}(i)$ = height from the zero reference to the i^{th} spring of the undeflected wheel (see Fig. 1).

For this case and due to the symmetry of the segments about the center-line the equation reduces to

$$2860 = 2K \sum_{i=1}^2 1.4 \cos \frac{11.7^\circ}{2} + 0.7 \cos 11.7^\circ + \frac{11.7^\circ}{2}$$

where the effective radial deflections are:

$$\Delta_5 = \Delta_6 = 1.4 \text{ in.}$$

$$\Delta_4 = \Delta_7 = 0.7 \text{ in.}$$

Solving for K yields

$$K = 675 \text{ lb/in.}$$

Defining $\text{GAMMA} = K \cos \phi_i = 675 \cos \phi_i$ yields the following relations

for the segments of the front and back wheels:

$$\text{GAMMA}(1) = \text{GAMMA}(10) = \text{GAMMA}(11) = \text{GAMMA}(20) = 411.75$$

$$\text{GAMMA}(2) = \text{GAMMA}(9) = \text{GAMMA}(12) = \text{GAMMA}(19) = 506.25$$

$$\text{GAMMA}(3) = \text{GAMMA}(8) = \text{GAMMA}(13) = \text{GAMMA}(18) = 587.25$$

$$\text{GAMMA}(4) = \text{GAMMA}(7) = \text{GAMMA}(14) = \text{GAMMA}(17) = 648.00$$

$$\text{GAMMA}(5) = \text{GAMMA}(6) = \text{GAMMA}(15) = \text{GAMMA}(16) = 668.25$$

A similar relation is derived for the threshold heights of each segment THRESH(i) and these relations are defined in the program.

A mean spacing of 3.07 in. was determined to be adequate for portraying the projected spacing of the springs. As a result, all profile points were spaced 3.07 in. apart and no interpolation scheme was employed to estimate elevation between adjacent points.

The truck was then advanced point by point along the profile and at each advance the resulting motions were computed.

NOISE

```

900$NDM
1000$LIB,GAUSS,,,**
1010C *****
1020C * LOW-PASS GAUSSIAN RANDOM NUMBER GENERATOR *
1030C *****
1110 DIMENSION X(1000),Y(1000)
1130C ++++++
1132 PRINT,"OUTPUT FILE NAME"
1134 READ 108,XNAME
1136 108 FORMAT(A6)
1138 PRINT,"TAU,RMS,ALPHA,FACT1,IX,III"
1140 READ,TAU,RMS,ALPHA,FACT1,IX,III
1180 SIGMAN=RMS*SQRT(1.-EXP(-2.*ALPHA*TAU))
1190 PRINT 102,SIGMAN
1200 102 FORMAT(8HSIGMAN= E16.6)
1210C ++++++
1220C GENERATE THE RANDOM NORMAL NUMBERS HAVING STANDARD
1230C DEVIATION SIGMAN.
1235C *****
1249C *****
1250 AM=0.
1260 DO 200 I=1,III
1262 SIGSQ=SIGMAN*SIGMAN
1270 CALL GAUSS(IX,SIGSQ,AM,V)
1280 200 X(I)=V
1281 CALL SHIFT(X,III,XBAR)
1282 PRINT 210,XBAR
1283 210 FORMAT("XBAR AFTER GAUSS="E16.6)
1285 CALL STDEV(X,III,DEV)
1286 RMSRAD=SQRT(DEV)*3.1415926536*2.
1287 PRINT 220,DEV,RMSRAD
1289 220 FORMAT("DEV="E16.6,"RMSRAD="E16.6)
1290 PRINT 106
1300 106 FORMAT("RANDOM NORMAL NUMBERS WITH STAND. DEV. SIGMAN")
1320 103 FORMAT(E20.10)
1332C ++++++
1334C STANDARD DEVIATION IS COMPUTED TO SEE WHETHER
1336C SUBROUTINE GAUSS(IX,SIGMAN,AM,V) GENERATES CORRECT
1338C RESULTS.
1340 CALL STDEV(X,III,DEV)
1342 PRINT 330,DEV
1344 330 FORMAT("COMPUTED STANDARD DEVIATION=" E16.8)
1350C STANDARD DEV. BLOCK IS TO BE INSERTED.
1360C ++++++
1370C GENERATE THE LOW-PASS GAUSSIAN
1375 CALL SHIFT(X,III,XBAR)
1379C *****
1380 AA=EXP(-ALPHA*TAU)
1381C *****
1390 Y(1)=0.0

```

NOISE CONTINUED

```

1400      DO 300 I=2,III
1410 300 Y(I)=X(I)+Y(I-1)*AA
1412      DO 401 I=1,III
1414 401 Y(I)=FACTI*Y(I)
1420      WRITE(2,103)(Y(I),I=1,III)
1430      CALL CLOSEF(2,XNAME)
1435      PRINT 107
1437 107 FORMAT("LOW-PASS GAUSSIAN IS GENERATED AND STORED")
1440      END
1990C    ++++++
2000      SUBROUTINE RANDU(IX,IY,YFL)
2010      IY=IX*4099
2020      IF(IY)5,6,6
2030      5 IY=IY+8388607+1
2040      6 YFL=IY
2050      YFL=YFL*1.192093E-7
2060      RETURN
2070      END
2080C    ++++++
2090      SUBROUTINE STDEV(X,III,DEV)
2095      DIMENSION X(1)
2100      SUM=0.0
2110      DO 420 I=1,III
2120 420 SUM=SUM+X(I)
2130      SUM1=SUM*SUM
2140      SUM2=0.0
2150      DO 440 I=1,III
2160 440 SUM2=SUM2+X(I)*X(I)
2165      AIII=III
2170      SD=SQRT((SUM2-SUM1/AIII)/(AIII-1.))
2180      DEV=SD
2190      RETURN
2200      END
3000C    ++++++
3010      SUBROUTINE SHIFT(X,III,XBAR)
3020      DIMENSION X(1)
3030      SUM=0.0
3040      DO 300 I=1,III
3050      SUM=SUM+X(I)
3060 300 CONTINUE
3065      AIII=III
3070      XBAR=SUM/AIII
3080      PRINT 310,XBAR
3090 310 FORMAT("THE TIME SERIES IS SHIFTED BY XBAR="E16.8)
3100      DO 320 I=1,III
3110      X(I)=X(I)-XBAR
3120 320 CONTINUE
3130      RETURN
3140      END

```

GAUSS

```

1750C
1762C .....
1774C
2006C SUBROUTINE GAUSS
2020C
2032C PURPOSE
2044C     COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN
2056C     MEAN AND STANDARD DEVIATION
2070C
2102C USAGE
2114C     CALL GAUSS(IX,S,AM,V)
2126C
2140C DESCRIPTION OF PARAMETERS
2152C     IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR
2164C     LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER
2176C     IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM
2210C     NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT
2222C     ENTRY TO THE SUBROUTINE.
2234C     S -THE DESIRED STANDARD DEVIATION OF THE NORMAL
2246C     DISTRIBUTION.
2260C     AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION
2272C     V -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE
2304C
2316C REMARKS
2330C     THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC
2342C
2354C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
2366C     RANDU
2400C
2412C METHOD
2424C     USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM
2436C     NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN
2450C     ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.
2462C     THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE
2474C     ARE FOUND BY THE POWER RESIDUE METHOD.
2506C
2520C .....
2532C
2544 SUBROUTINEGAUSS(IX,S,AM,V)
2556 A=0.0
2570 DO50I=1,12
2602 CALLRANDU(IX,IY,Y)
2614 IX=IY
2626 50A=A+Y
2640 V=(A-6.0)*S+AM
2652 RETURN
2664 END

```

RANDU

```

1750C
1762C .....
1774C
2006C SUBROUTINE RANDU
2020C
2032C PURPOSE
2044C   COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN
2056C   0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND
2070C   2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER
2102C   AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.
2114C
2126C USAGE
2140C   CALL RANDU(IX,IY,YFL)
2152C
2164C DESCRIPTION OF PARAMETERS
2176C   IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER
2210C   NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY,
2222C   IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS
2234C   SUBROUTINE.
2246C   IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT
2260C   ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS
2272C   BETWEEN ZERO AND 2**31
2304C   YFL- THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT,
2316C   RANDOM NUMBER IN THE RANGE 0 TO 1.0
2330C
2342C REMARKS
2354C   THIS SUBROUTINE IS SPECIFIC TO THE GE-427
2366C   THE SUBROUTINE SHOULD PRODUCE (2**22)-1 TERMS
2400C   BEFORE REPEATING
2412C
2424C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
2436C   NONE
2450C
2462C METHOD
2474C   POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011,
2506C   RANDOM NUMBER GENERATION AND TESTING
2520C
2532C .....
2544C
2556 SUBROUTINERANDU(IX,IY,YFL)
2570 IY=IX*4099
2602 IF(IY)5,6,6
2614 5IY=IY+8388607+1
2626 6YFL=IY
2640 YFL=YFL*1.192093E-7
2652 RETURN
2664 END

```

STANOR

```

120 $NDM
130C *****
140C * STATIONARITY AND NORMALITY TEST *
150C *****
160 DIMENSIONX(1000)
165 DIMENSIONZ(50),T(50)
170 DIMENSIONTF(50)
180 DIMENSIONTABLE1(31,7),TABLE2(15,7),TABLE3(2,7)
185 COMMONMTEST
186 MTEST=0
190 100FORMAT(8I10)
195 109FORMAT(7F9.3)
200 101FORMAT(8F10.3)
210 102FORMAT(8E12.4)
220 103FORMAT(4H IP=I5,8E12.4)
230 104FORMAT(8F10.5)
240C TABLE1(I,J)---PERCENTAGE POINTS OF RUN DISTRIBUTION
250C TABLE3(I,J)---GROUP NUMBER DECISION TABLE
260C TABLE2(I,J)---PERCENTAGE POINTS OF REVERSE ARRANGEMENT DISTRIBUTION
270C READ THE TIME SERIES
271 PRINT,"FILE NAME"
272 READ839,XDNAME
273 839FORMAT(A6)
275 PRINT,"NO. POINTS, FACTOR"
280 READ,III
290 AIII=III
300 IP=1000
310 PRINT745,III
311 745FORMAT(5H III=I10)
315 CALLOPENF(1,XDNAME)
318 999FORMAT(E20.10)
320 READ(1,999)(X(I),I=1,III)
321C FACTOR IS MULTIPLIED FOR THE DATA X444
360C * * * * *
370C READ THE PERCENTAGE POINTS OF RUN TEST TABLE
380 CALLOPENF(2,"TABKA1")
390 READ(2,)((TABLE1(K1,K2),K2=1,7),K1=1,31)
440C * * * * *
450C READ THE PERCENTAGE POINTS OF REVERSE ARRANGE DISTRIBUTION TABLE
460 CALLOPENF(3,"TABKA2")
470 READ(3,)((TABLE2(K1,K2),K2=1,7),K1=1,15)
520C * * * * *
530C READ THE GROUP NUMBER DECISION TABLE
540 CALLOPENF(4,"TABKA3")
550 READ(4,)((TABLE3(K1,K2),K2=1,7),K1=1,2)
600C NUMBER = NUMBER IN SUBSET
610 NUMBER=1
620C * * * * *
630 600CONTINUE
640C DETERMINE NUMBER OF GROUP NG

```

STANOR CONTINUED

```

650 IF(200-III)339,339,340
660 340ANG=AIII*(16.0/200.)
670 IP=3001
680 GOTO390
690 339IF(III-2000)349,350,350
700 350ANG=1.87*(AIII-1.0)**(2.0/5.0)
710 IP=3002
720 GOTO390
730 349CONTINUE
740 K2=1
750 360CONTINUE
760 IF(AIII-TABLE3(1,K2))361,362,363
770 361ANG=((TABLE3(2,K2+1)-TABLE3(2,K2))/(TABLE3(1,K2+1)-TABLE3(1,K2))
780&)
790&*(AIII-TABLE3(1,K2))+TABLE3(2,K2)
800 IP=3003
810 GOTO390
820 362ANG=TABLE3(2,K2)
830 IP=3004
840 GOTO390
850 363K2=K2+1
860 GOTO360
870 390NG=ANG
880 PRINT801,NG
885 801FORMAT(17H NUMBER OF GROUP=15)
890 CALLGROUP(X,III,ZSTEP,NG,XMIN,XMAX,Z)
900 CALLAGROUP(X,III,T,NG,XXMIN,XXMAX,ASTEP)
910 CALLTRTEST(T,NG,XXMIN,XXMAX,TABLE2,JPASS)
920 CALLRNTEST(T,NG,XXMIN,XXMAX,TABLE1,JPASS)
930C  * * * * * TEST FOR NORMALITY * * * * *
940 CALLCHITN(X,III,NG,XMIN,XMAX,ZSTEP,Z,TF,XBAR,S,CHI2,PCHI)
950 PRINT401,PCHI
960 401FORMAT(19H CHI-SQUARE VALUE =E12.4)
970 IF(PCHI-0.05)409,410,410
980 410PRINT420
990 420FORMAT(30H NORMALITY TEST PASSED )
1000 GOTO499
1010 409PRINT430
1020 430FORMAT(30H NORMALITY TEST NOT PASSED )
1240 499CONTINUE
1250 END
1260 SUBROUTINEGROUP(X,III,ZSTEP,NG,XXMIN,XXMAX,Z)
1265 COMMONMTEST
1270 DIMENSIONX(2000),Z(50)
1280 104FORMAT(I5,4E12.4)
1290 ANG=NG
1300 AIII=III
1310 XXMAX=-10.E10
1320 XXMIN=10.E10
1330 DO200I=1,III

```

STANOR CONTINUED

```

1340 TEMP1=AMAX1(XXMAX,X(I))
1350 XXMAX=TEMP1
1360 TEMP2=AMIN1(XXMIN,X(I))
1370 200XXMIN=TEMP2
1380 IP=4000
1390 XXMAX=XXMAX+10.E-10
1400 XXMIN=XXMIN-10.E-10
1420 ZSTEP=(XXMAX-XXMIN)/ANG
1430 TEMP1=XXMIN
1440 TEMP2=TEMP1+ZSTEP
1445 IF(MTEST.NE.0)GOTO444
1446 107FORMAT(/,6H GROUP)
1449 444CONTINUE
1450 DO300N=1,NG
1460 Z(N)=0.0
1470 DO310I=1,III
1480 IF(X(I)-TEMP1)310,310,311
1490 311IF(TEMP2-X(I))310,310,315
1500 315Z(N)=Z(N)+1.0
1510 310CONTINUE
1520 IP=4008
1530 AN=N
1535 IF(MTEST.NE.0)GOTO445
1540 PRINT104,N,TEMP1,TEMP2,Z(N)
1545 445CONTINUE
1550 TEMP1=TEMP2
1560 300TEMP2=TEMP2+ZSTEP
1565 MTEST=MTEST+1
1570 RETURN
1580 END
1590 SUBROUTINE TRTEST(Z,NG,XXMIN,XXMAX,TABLE2,IPASS)
1600 DIMENSIONZ(50),A(50),TABLE2(15,7)
1610 104FORMAT(4H IP=I5,8E12.4)
1615 108FORMAT(15,F10.0)
1620C  * * * * * * * * * * * * * * * * * REVERSE ARRANGEMENT COMPUTATION
1625 PRINT109
1626 109FORMAT(/,21H REVERSE ARRANGEMENT )
1630 NGM1=NG-1
1640 ANG=NG
1650 DO200N=1,NGM1
1660 A(N)=0.0
1670 NP1=N+1
1680 DO210J=NP1,NG
1690 IF(Z(J)-Z(N))220,210,210
1700 220A(N)=A(N)+1.0
1710 210CONTINUE
1730 PRINT108,N,A(N)
1740 200CONTINUE
1750C  * * * * * * * TOTAL OF REVERSE ARRANGEMENT
1760 TREV=0.0

```

STANOR CONTINUED

```

1770 D0250N=1,NGM1
1780 250TREV=TREV+A(N)
1790 PRINT773
1792 773FORMAT(/,"SUMMARY OF TREND TEST")
1794 PRINT774,TREV
1796 774FORMAT(" TOTAL REVERSE ARRANGEMENT = "E12.4)
1810C     NOTE THAT IT IS TEMPORARY ASSUMED THAT K2L=3,AND K2U=5 IN THE
1820C     TABLE2(I,J) ARRAY
1830C     * * *COMPUTE THE LOWER LIMIT BY LINEAR INTERPOLATION. (ASSUME B
1840C     IS GREATER THAN 10 AND LESS THAN 100)
1850 IF(NG-10)309,309,301
1860 301IF(100-NG)309,309,302
1870 309PRINT308,NG
1880 308FORMAT(48H NG IS GREATER THAN 100 OR LESS THAN 10.   NG =I10)
1890 STOP
1900 302CONTINUE
1910 D0641K1=2,15
1915 IF(ANG.GT.TABLE2(K1,1))GOTO641
1920 IF(ANG.EQ.TABLE2(K1,1))GOTO642
1925 XX1=TABLE2(K1-1,1)
1930 XX2=TABLE2(K1,1)
1935 Y1=TABLE2(K1-1,3)
1936 Y2=TABLE2(K1,3)
1940 ALOW=(Y2-Y1)*(ANG-XX1)/(XX2-XX1)+Y1
1945 YY1=TABLE2(K1-1,6)
1946 YY2=TABLE2(K1,6)
1950 AUP=(YY2-YY1)*(ANG-XX1)/(XX2-XX1)+YY1
1953 GOTO643
1955 642ALOW=TABLE2(K1,3)
1960 AUP=TABLE2(K1,6)
1965 GOTO643
1970 641CONTINUE
1975 643CONTINUE
2034 PRINT775,ALOW
2036 775FORMAT(" LOWER LIMIT OF REVERSE ARRANGEMENT = "E12.4)
2038 PRINT776,AUP
2040 776FORMAT(" UPPER LIMIT OF REVERSE ARRANGEMENT = "E12.4)
2045 510FORMAT(6H ALOW=E12.4,5H AUP=E12.4,6H TREV=E12.4)
2050C     TEST WHETHER ANG IS BETWEEN THE LIMIT
2060 IF(TREV-ALOW)390,400,400
2070 400IF(AUP-TREV)390,410,410
2080 390IPASS=0
2090 PRINT391
2100 391FORMAT(" TREND TEST IS NOT PASSED")
2110 GOTO420
2120 410IPASS=1
2130 PRINT411
2140 411FORMAT(" TREND TEST IS PASSED ")
2150 420CONTINUE
2160 RETURN

```

STANOR CONTINUED

```

2170 END
2180 SUBROUTINEAGROUP(X,III,T,NG,XXMIN,XXMAX,ASTEP)
2190C   THE PURPOSE OF AGROUP IS TO GROUP THE SERIES X(I) AND AVERAGE I
2200C   AND STOR THEM IN T(I) ARRAY. NOTICE THE DIFFERENCE BETWEEN GROB
2210C   AND AGROUP.
2220 DIMENSIONX(2000),T(50)
2230 104FORMAT(4H IP=I10,8E12.4)
2240 AIII=III
2250 ANG=NG
2260C   COMPUTE THE INTERVAL IN FLOATING MODE
2270 TEMP1=AIII/ANG
2280 ASTEP=TEMP1
2290C   THESE CODES MAY BE TAKEN OUT * * * * *
2350C
2360 152FORMAT(7F8.2)
2370C   * * * * *
2380 TEMP2=XXMIN
2390 DO200N=1,NG
2400 T(N)=0.0
2410 AN=N
2420 NT=0
2430 TEMP3=TEMP1*(AN-1.0)
2440 TEMP4=TEMP1*AN+1.0E-10
2450 DO210I=1,III
2460 AI=I
2470 IF(AI-TEMP3)219,219,220
2480 220IF(TEMP4-AI)219,219,230
2490 230T(N)=T(N)+X(I)
2500 NT=NT+1
2510 219CONTINUE
2520 210CONTINUE
2530C   * * * * * PRINT THE INTERMEDIATE STEP
2560C   * * * * *
2570 ANT=NT
2580 T(N)=T(N)/ANT
2610 200CONTINUE
2615 PRINT310
2616 310FORMAT(/,23H AVERAGED TIME HISTORY )
2617 DO311N=1,NG
2618 311PRINT312,N,T(N)
2619 312FORMAT(I5,É12.4)
2620 RETURN
2630 END
2640 SUBROUTINERNTEST(X,NG,XXMIN,XXMAX,TABLE1,JPASS)
2650C   THE MAIN PROGRAM CALL RNTEST(Z,NG,XXMIN,XXMAX,TABLE1,JPASS)
2660 DIMENSIONX(50),TABLE1(31,7),X1(50)
2670 104FORMAT(4H IP=I5,4E12.4)
2680C   *****COMPUTE MEAN VALUE
2683 189CONTINUE
2684 187FORMAT(I5,3E12.4)

```

STANOR CONTINUED

```
2690 TEMP=0.0
2700 DO800N=1,NG
2710 800TEMP=TEMP+X(N)
2720 ANG=NG
2730 XMEAN=TEMP/ANG
2760C *****DETERMINE NUMBER OF RUN
2770 NRUN=1
2780 IF(X(1)-XMEAN)267,268,268
2790 267ICHG=-1
2800 GOTO270
2810 268ICHG=1
2820 270CONTINUE
2830 DO260I=2,NG
2840 IF(X(I)-XMEAN)262,261,261
2850 261K=1
2860 GOTO263
2870 262K=-1
2880 GOTO263
2890 263IF(ICHG-K)265,260,265
2900 265NRUN=NRUN+1
2910 ICHG=K
2920 260CONTINUE
2950C *****COMPUTE LOWER LIMIT BY LINER INTERPOLATION
2960C (ASSUME NG/2 IS GREATER THAN 5 AND LESS THAN 100)
2970 NNG=NG/2
2980 IF(NNG-5)309,309,301
2990 301IF(100-NNG)309,309,302
3000 309PRINT308,NNG
3010 308FORMAT(45H NNG IS GREATER THAN 100 OR LESS THAN 5. NG=I10)
3020 STOP
3030 302CONTINUE
3040 ANNG=NNG
3050 DO641K1=2,31
3055 IF(ANNG.GT.TABLE1(K1,1))GOTO641
3060 IF(ANNG.EQ.TABLE1(K1,1))GOTO642
3065 XX1=TABLE1(K1-1,1)
3070 XX2=TABLE1(K1,1)
3075 Y1=TABLE1(K1-1,3)
3080 Y2=TABLE1(K1,3)
3085 ALOW=(Y2-Y1)*(ANNG-XX1)/(XX2-XX1)+Y1
3090 YY1=TABLE1(K1-1,6)
3095 YY2=TABLE1(K1,6)
3100 AUP=(YY2-YY1)*(ANNG-XX1)/(XX2-XX1)+YY1
3105 GOTO643
3110 642ALOW=TABLE1(K1,3)
3115 AUP=TABLE1(K1,6)
3120 GOTO643
3125 641CONTINUE
3130 643CONTINUE
3174 PRINT790
```

STANOR CONTINUED

```

3176 790FORMAT(/,"SUMMARY OF RUN TEST")
3178 PRINT791, NRUN
3180 791FORMAT(" TOTAL NUMBER OF RUN = "110)
3181 PRINT793, AUP
3182 PRINT792, ALOW
3184 792FORMAT(" LOWER LIMIT OF RUN ="E12.4)
3185 510FORMAT(6H ALOW=E12.4,5H AUP=E12.4,6H NRUN=E12.4)
3186 793FORMAT(" UPPER LIMIT OF RUN = "E12.4)
3190C     TEST WHETHER ANNG IS BETWEEN THE LIMITS
3200 IF (ANNG-ALOW)390,400,400
3210 400IF (AUP-ANNG)390,410,410
3220 390IPASS=0
3230 PRINT391
3240 391FORMAT(" RUN TEST IS NOT PASSED ")
3250 GOTO420
3260 410IPASS=1
3270 PRINT411
3280 411FORMAT(" RUN TEST IS PASSED ")
3290 420CONTINUE
3300 RETURN
3310 END
3320 SUBROUTINECHITN(X,N,NG,XMIN,XMAX,STEP,F,TF,XBAR,S,CHI2,PCHI)
3330 DIMENSIONF(1),TF(1),X(1)
3340 2FORMAT(4X10F10.6)
3350 SX=0.
3352 PRINT791
3354 791FORMAT(/,"SUMMARY OF NORMALITY TEST")
3360 SX2=0.
3370 DO200I=1,N
3380 SX=SX+X(I)
3390 SX2=SX2+X(I)*X(I)
3400 200CONTINUE
3410 XBAR=SX/N
3420 S=SQRT((SX2-(SX*SX/N))/(N-1))
3430 CALLGROUP(X,N,STEP,NG,XMIN,XMAX,F)
3440 X1=(XMIN-XBAR)/S
3450 TEMP=STEP/S
3460 DO210I=1,NG
3470 X2=X1+TEMP
3480 A1=RNORM(X1)
3490 A2=RNORM(X2)
3500 TF(I)=(A2-A1)*N
3510 PRINT2,X1,X2,A1,A2,TF(I)
3520 X1=X2
3530 210CONTINUE
3540 CHI2=0.
3550 DO220I=1,NG
3560 CHI2=CHI2+((F(I)-TF(I))**2)/TF(I)
3570 220CONTINUE
3580 DF=NG-1

```

STANOR CONTINUED

```
3590 PCHI=CHIPR(DF,CHI2)
3600 RETURN
3610 END
3620 FUNCTIONRNORM(Z)
3630 DIMENSIONA(6)
3640 A(1)=4.9867347E-2
3650 A(2)=2.11410061E-2
3660 A(3)=3.2776263E-3
3670 A(4)=3.80036E-5
3680 A(5)=4.88906E-5
3690 A(6)=5.383E-6
3700 IF(Z)100,110,120
3710 100CONTINUE
3720 ASSIGN160TOJEL
3730 Z1=-Z
3740 GOTO130
3750 110CONTINUE
3760 B=.5
3770 GOTO160
3780 120CONTINUE
3790 ASSIGN150TOJEL
3800 Z1=Z
3810 130CONTINUE
3820 B=1.
3830 DO140I=1,6
3840 B=B+A(I)*Z1**I
3850 140CONTINUE
3860 B=.5/(B**16)
3870 GOTOJEL,(150,160)
3880 150CONTINUE
3890 B=1.-B
3900 160CONTINUE
3910 RNORM=B
3920 RETURN
3930 END
3940CCHIPR      CHISQUARE PROBABILITY
3950 FUNCTIONCHIPR(DF,CHSQ)
3960 A=.5*DF
3970 X=.5*CHSQ
3980 IF(X)90,90,100
3990 90CHIPR=1.
4000 GOTO170
4010 100TERM=1.
4020 SUM=0.
4030 COFN=A
4040 IF(13.-X)110,110,120
4050 110IF(A-X)140,140,120
4060C      CONVERGENT SERIES FOR X .LT. A OR .LT. 13.
4070 120CON=1.
4080 FACT=-A
```

STANOR CONTINUED

```

4090 130TEMP=SUM
4100 SUM=SUM+TERM
4110 COFN=COFN+1
4120 TERM=TERM*X/COFN
4130 IF(SUM-TEMP)160,160,130
4140C   ASYMPTOTIC SERIES FOR X .GTE. A AND X .GTE. 13.
4150 140CON=0.
4160 FACT=X
4170 150TEMP=SUM
4180 SUM=SUM+TERM
4190 COFN=COFN-1
4200 RATIO=COFN/X
4210 TERM=TERM*RATIO
4220 IF(SUM-TEMP)160,160,150
4230 160CHIPR=CON+EXP(ALOG(SUM)-X+A*ALOG(X)-ALOG(GAMMA(A,0.)))/FACT
4240 170CONTINUE
4241C   *****
4247C   *****
4248 RETURN
4250 END
5000 FUNCTIONGAMMA(U,V)
5010 DIMENSIONB(8)
5020 B(1)=.035868343
5030 B(2)=-.193527818
5040 B(3)=.482199394
5050 B(4)=-.756704078
5060 B(5)=.918206857
5070 B(6)=-.897056937
5080 B(7)=.988205891
5090 B(8)=-.577191652
5100 A=U
5110 X=V
5120 XN=15.0
5130 1IF(A)4,2,3
5140 2IF(X-1.0)4000,4000,5000
5150 4Z=AIN(T(A)
5160 Y=ABS(Z-A)
5170 EPS=3.0E-8
5180 IF(Y-EPS)21,21,5
5190 21A=Z
5200 GOTO22
5210 5Y=1.0-Y
5220 IF(Y-EPS)6,6,3
5230 6A=Z-1.0
5240 22IF(X-1.0)4000,4000,5000
5250 3IF(X)7,1000,7
5260 7IF(X-1.0)9,9,8
5270 8IF(A)5000,10,10
5280 10IF((X/A)-1.0)1000,5000,5000
5290 9IF(ABS(A)-1.)11,1000,1000

```

STANOR CONTINUED

```
5300 11IF(X-.1)1000,12,12
5310 12IF(X-.2)13,14,14
5320 13XN=140.0
5330 GOTO5000
5340 14IF(X-.4)15,16,16
5350 15XN=80.0
5360 GOTO5000
5370 16IF(X-.6)17,18,18
5380 17XN=60.0
5390 GOTO5000
5400 18IF(X-.8)19,20,20
5410 19XN=40.0
5420 GOTO5000
5430 20XN=20.0
5440 5000XFT=X
5450 N=XN
5460 D050011=1,N
5470 XFT=(XN/XFT)+1.0
5480 XFT=((XN-A)/XFT)+X
5490 5001XN=XN-1.0
5500 TEM=(ALOG(X)*A)-X
5510 XFT=EXP(TEM)/XFT
5520 ANS=XFT
5530 GOTO6000
5540 4000SUM=0.0
5550 EPS=1.0E-8
5560 XM=1.0
5570 XMT=1.0
5580 EX=-1.0
5590 TEM=X
5600 4001Y=TEM/(XMT*XM)
5610 4011SUM=SUM+EX*Y
5620 IF(ABS(Y)-EPS)4003,4002,4002
5630 4002TEM=TEM*X
5640 XM=XM+1.0
5650 XMT=XMT*XM
5660 EX=-EX
5670 GOTO4001
5680 4003E=-ALOG(X)-.57721566-SUM
5690 IF(A)4005,4004,4005
5700 4004ANS=E
5710 GOTO6000
5720 4005IF(A+2.0)4010,4009,4008
5730 4008ANS=-E+EXP(-X)/X
5740 GOTO6000
5750 4009SUM=(1.0/X)-(1.0/(X*X))
5760 ANS=(E-EXP(-X)*SUM)*.5
5770 GOTO6000
5780 4010EX=-1.0
5790 N=-A-1.0
```

STANOR CONTINUED

```
5800 XMT=1.0
5810 SUM=0.0
5820 TEM=X
5830 DO 4006 I=1,N
5840 TEM=TEM*X
5850 XM=I+1
5860 Y=XMT/TEM
5870 SUM=SUM+EX*Y
5880 EX=-EX
5890 4006XMT=XMT*XM
5900 SUM=SUM+1.0/X
5910 Z=E-EXP(-X)*SUM
5920 Y=ABS(A)
5930 EX=-1.0
5940 XMT=1.0
5950 N=Y
5960 DO 4007 I=2,N
5970 XM=I
5980 XMT=XMT*XM
5990 4007EX=-EX
6000 ANS=Z*EX/XMT
6010 GOTO 6000
6020 1000IF(A-1.0)1001,1003,1004
6030 1004IF(A-2.0)1003,1003,1002
6040 1003AT=A-1.0
6050 ATM=B(1)*AT
6060 DO 1005 I=2,8
6070 1005ATM=(ATM+B(I))*AT
6080 ANS=ATM+1.0
6090 GOTO 2000
6100 1001IF(A)1007,1006,1006
6110 1006BN=A
6120 BS=A
6130 GOTO 1010
6140 1007BN=A
6150 BS=A
6160 1011IF(BS)1009,1010,1010
6170 1009BS=BS+1.0
6180 BN=BS*BN
6190 GOTO 1011
6200 1010ATM=BS*B(1)
6210 DO 1012 I=2,8
6220 1012ATM=(ATM+B(I))*BS
6230 ANS=(ATM+1.0)/BN
6240 GOTO 2000
6250 1002BN=A-1.0
6260 BS=BN
6270 1013IF(2.0-BS)1015,1014,1014
6280 1015BS=BS-1.0
6290 BN=BS*BN
```

SIANOR CONTINUED

```
6300 GOTO1013
6310 1014BS=BS-1.0
6320 ATM=BS*B(1)
6330 DO1016I=2,8
6340 1016ATM=(ATM+B(I))*BS
6350 ANS=(ATM+1.0)*BN
6360 2000IF(X)2001,6000,2001
6370 2001W=1.0
6380 XM=1.0
6390 SUM=1.0
6400 EPS=1.0E-8
6410 2002W=(X/(A+XM))*W
6420 SUM=SUM+W
6430 XM=XM+1.0
6440 IF(ABS(W)-EPS)2003,2002,2002
6450 2003T=A*ALOG(X)
6460 T=EXP(T-X)/A
6470 ANS=ANS-(T*SUM)
6480 6000GAMMA=ANS
6490 RETURN
6500 END
```

TABKA 1

100	0.,0.99,0.975,0.95,0.05,0.025,0.01,
110	5.,2.,2.,3.,8.,9.,9.,
120	6.,2.,3.,3.,10.,10.,11.,
130	7.,3.,3.,4.,11.,12.,12.,
140	8.,4.,4.,5.,12.,13.,13.,
150	9.,4.,5.,6.,13.,14.,15.,
160	10.,5.,6.,6.,15.,15.,16.,
170	11.,6.,7.,7.,16.,16.,17.,
180	12.,7.,7.,8.,17.,18.,18.,
190	13.,7.,8.,9.,18.,19.,20.,
200	14.,8.,9.,10.,19.,20.,21.,
210	15.,9.,10.,11.,20.,21.,22.,
220	16.,10.,11.,11.,22.,22.,23.,
230	18.,11.,12.,13.,24.,25.,26.,
240	20.,13.,14.,15.,26.,27.,28.,
250	25.,17.,18.,19.,32.,33.,34.,
260	30.,21.,22.,24.,37.,39.,40.,
270	35.,25.,27.,28.,43.,44.,46.,
280	40.,30.,31.,33.,48.,50.,51.,
290	45.,34.,36.,37.,54.,55.,57.,
300	50.,38.,40.,42.,59.,61.,63.,
310	55.,43.,45.,46.,65.,66.,68.,
320	60.,47.,49.,51.,70.,72.,74.,
330	65.,52.,54.,56.,75.,77.,79.,
340	70.,56.,58.,60.,81.,83.,85.,
350	75.,61.,63.,65.,86.,88.,90.,
360	80.,65.,68.,70.,91.,93.,96.,
370	85.,70.,72.,74.,97.,99.,101.,
380	90.,74.,77.,79.,102.,104.,107.,
390	95.,79.,82.,84.,107.,109.,112.,
400	100.,84.,86.,88.,113.,115.,117.,

TABKA 2

100 0., 0.99, 0.975, 0.95, 0.05, 0.025, 0.01,
 110 10., 9., 11., 13., 31., 33., 35.,
 120 12., 16., 18., 21., 44., 47., 49.,
 130 14., 24., 27., 30., 60., 63., 66.,
 140 16., 34., 38., 41., 78., 81., 85.,
 150 18., 45., 30., 54., 98., 102., 107.,
 160 20., 59., 64., 69., 120., 125., 130.,
 170 30., 152., 162., 171., 263., 272., 282.,
 180 40., 290., 305., 319., 460., 474., 489.,
 190 50., 473., 495., 514., 710., 729., 751.,
 200 60., 702., 731., 756., 1013., 1038., 1067.,
 210 70., 977., 1014., 1045., 1369., 1400., 1437.,
 220 80., 1299., 1344., 1382., 1777., 1815., 1860.,
 230 90., 1688., 1721., 1766., 2238., 2283., 2336.,
 240 100., 2083., 2145., 2198., 2751., 2804., 2866.

TABKA 3

100 200., 400., 600., 800., 1000., 1500., 2000.,
 110 16., 20., 24., 27., 30., 35., 39.

PSD

```

900$NDM
1000C *****
1010C * AUTO-CORRELATION USING FILED DATA *
1020C *****
1030 DIMENSION X(1000)
1031 DIMENSION RX(200),PX(200),SPX(200)
1032 PRINT ,INPUT FILE, AUTO-CORR. PLOT FILE, PSD PLOT FILE"
1034 READ 910, XNAME1,XNAME2,XNAME3
1036 910 FORMAT(3A6)
1038 PRINT ,KKK,LAG,DELTAT,FACT"
1040 READ,KKK,LAG,DELTAT,FACT
1042 LLAG=LAG+1
1050 CALL OPENF(1,XNAME1)
1060 READ(1,920)(X(K),K=1,KKK)
1061 920 FORMAT(E20.10)
1062 DO 144 K=1,KKK
1064 144 X(K)=X(K)*FACT
1070 PRINT 110
1080 110 FORMAT(20H RAW TIME SERIES )
1095 104 FORMAT(5E12.4)
1100C COMPUTE THE MEAN VALUE AND SHIFT THE DATA TO MEAN ZERO
1110 SUM=0.0
1120 DO 150 K=1,KKK
1130 150 SUM=SUM+X(K)
1140 AKKK=KKK
1150 XBAR=SUM/AKKK
1155 PRINT 107,XBAR
1156 107 FORMAT(6H XBAR=E12.4)
1160 DO 160 K=1,KKK
1170 160 X(K)=X(K)-XBAR
1180 PRINT 103
1190 103 FORMAT(32H TRANSFORMED INPUT DATA )
1210 NPOINT =KKK
1220 DO 300 I=1,LLAG
1230 SX=0.0
1240 M=I-1
1250 NX=NPOINT -M
1260 FNX =NX
1270 DO 305 J=1,NX
1280 K=M+J
1290 305 SX=SX+X(J)*X(K)
1300 300 RX(I)=SX/FNX
1310 PRINT 400
1315 400 FORMAT(20H AUTO-CORR. FT. )
1330 DO 401 I=1,LLAG,4
1332 J=I
1334 J3=I+3
1340C PRINT 222,((J,RX(J)),J=I,J3)
1349 401 CONTINUE
1350 222 FORMAT(4(I4,E12.4))

```

PSD CONTINUED

```

1352 PRINT 520,RX(1)
1354 520 FORMAT(7H RX(1)=E12.4)
1355 XNAME=XNAME2
1400C *****POWER SPECTRAL DENSITY COMPUTATION*****
1405 DELF=1./(DELTAT*AKKK)
1410 PI=3.1415926536
1420 FLAG=LAG
1430 Q=PI/FLAG
1440 CONST=2.*DELTAT/PI
1450 RX(1)=0.5*RX(1)
1460 RX(LLAG)=0.5*RX(LLAG)
1470 S1=-Q
1480 DO 1401 IH=1,LLAG
1490 PX(IH)=0.0
1500 S1=S1+Q
1510 S=-S1
1520 DO 1402 JP=1,LLAG
1530 S=S+S1
1540 1402 PX(IH)=PX(IH)+RX(JP)*COS(S)
1550 1401 PX(IH)=PX(IH)*CONST
1560 SPX(1)=0.54*PX(1)+0.46*PX(2)
1570 SPX(LLAG)=0.54*PX(LLAG)+0.46*PX(LLAG-1)
1580 KK=LLAG-1
1590 DO 1415 J=2,KK
1600 1415 SPX(J)=0.54*PX(J)+0.23*(PX(J+1)+PX(J-1))
1602 WRITE(5,920)(SPX(I),I=1,LLAG)
1610 PRINT 500
1620 500 FORMAT(30H POWER SPECTRAL DEN. )
1625 XNAME=XNAME3
1626 IDEV=5
1627 CALL PLOT4(SPX,LLAG,XNAME,IDEV)
1640 DO 550 I=1,LLAG
1641 AI=I
1642 TEMP13=AI*DELF
1650 PRINT 554,I,TEMP13,SPX(I)
1676 550 CONTINUE
1677 554 FORMAT(I5,E12.4,E16.6)
1680 PSUM=0.0
1690 DO 510 I=1,LLAG
1700 510 PSUM=PSUM+SPX(I)*DELF
1710 PRINT 511,PSUM
1720 511 FORMAT(6H PSUM=E12.4)
1999 END
2000 SUBROUTINE PLOT4(X,KKK,XNAME,IDEV)
2010 DIMENSION X(1)
2015 DIMENSION Y(71),ISYM(71)
2020 DATA IBLANK/3H /
2030 DATA MINUS/3H- /
2040 DATA IAI/3HI /
2050 DATA IDOT/3H. /

```

PSD CONTINUED

```

2055 558 FORMAT(A6)
2060C   FIND THE MAXIMUN VALUE IN ARRAY X(I)
2070   TEMP1=0.0
2080   DO 300 K=1,KKK
2090   TEMP2=ABS(X(K))
2100   YAXISL= AMAX1(TEMP1,TEMP2)
2102C   ::::::::::::::::::::::::::::::::::::
2110 300 TEMP1=YAXISL
2115   YAXISL=YAXISL+YAXISL/35.
2140   DELY = YAXISL/35.
2150   DO 200 IY=1,71
2160   AIY=IY
2170   Y(IY)=-YAXISL+DELY*(AIY-1.)
2190 200 CONTINUE
2200C   PRINT 215,(Y(IY),IY=1,71)
2210 215 FORMAT(5E12.4)
2220 202 FORMAT("*****")
2230   PRINT 202
2235   WRITE(IDEV,202)
2250C   INITIAL PLOT SET
2260   DO 401 J=1,71
2270 401 ISYM(J)=MINUS
2280   ISYM(35)=IAI
2290   DO 402 K=1,KKK
2300   DO 403 J=2,71
2310   IF(Y(J).GT.X(K).AND.Y(J-1).LT.X(K)) GO TO 410
2320   GO TO 403
2330 410 ISYM(J-1)=IDOT
2335 403 CONTINUE
2340C   PRINT 420,(ISYM(I),I=1,71)
2350 420 FORMAT(71A1)
2355C   ++++++
2356   WRITE(IDEV;420)(ISYM(I),I=1,71)
2357C   ++++++
2360   DO 440 I=1,71
2370 440 ISYM(I)=IBLANK
2380   ISYM(35)=IAI
2381 402 CONTINUE
2382   PRINT 558,XNAME
2391   PRINT 202
2392   WRITE(IDEV;202)
2393C   ++++++
2394   CALL CLOSEF(5,XNAME)
2395C   ++++++
2399   RETURN
2400   END

```

TRUCK

```

1$LIB,DIFFEQ
2$LIB,ALGEBR
3$RPC
4$NDM
5$ITY,120
100 COMMON NSTEPS,DELTAT,A,B,MASS,MASS1,MASS2,CPOS1,CPOS2,CNEG1,CNEG2
110 COMMON C11,C21,INRTIA,FORCW1,FORCW2,SPDEF1,SPDEF2,DSPDF1,DSPDF2
120 COMMON FORCK1,FORCK2,VAR1,VAR2,VAR3,VAR4,VAR5,VAR6,VAR7,VAR8
130 COMMON ZETA1,ZETA2,DZETA2,ETA1,ETA2,DETA2,NU1,NU2,DNU2
140 COMMON MU1,MU2,DMU2
150 COMMON THRESH(20),GAMMA(20),SEGDEF(20),Y(47)
160 REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
165 IBELL=458752
170 PRINT 1
180 1 FORMAT(18X,33(1H*),/,18X,1H*,31X,1H*,/,
190& 18X,33H* MURPHY'S M-37 BACK-UP MODEL *,/,
200& 18X,1H*,31X,1H*,/,18X,33(1H*),///)
210C-----DATA INITIALIZATION-----
220 A=64.8
230 B=47.2
240 INRTIA=25446.
250 MASS=8.05
260 MASS1=.94
270 MASS2=.8
280 CPOS1=11.8
290 CNEG1=23.8
300 CPOS2=10.2
310 CNEG2=44.
320 THRESH(1)=7.02
330 THRESH(2)=4.5
340 THRESH(3)=2.34
350 THRESH(4)=.81
360 THRESH(5)=.09
370 THRESH(6)=.09
380 THRESH(7)=.81
390 THRESH(8)=2.34
400 THRESH(9)=4.5
410 THRESH(10)=7.02
420 THRESH(11)=7.02
430 THRESH(12)=4.5
440 THRESH(13)=2.34
450 THRESH(14)=.81
460 THRESH(15)=.09
470 THRESH(16)=.09
480 THRESH(17)=.81
490 THRESH(18)=2.34
500 THRESH(19)=4.5
510 THRESH(20)=7.02
520 GAMMA(1)=411.75
530 GAMMA(2)=506.25

```

TRUCK CONTINUED

```

540     GAMMA (3)=587.25
550     GAMMA (4)=648.
560     GAMMA (5)=668.25
570     GAMMA (6)=668.25
580     GAMMA (7)=648.
590     GAMMA (8)=587.25
600     GAMMA (9)=506.25
610     GAMMA (10)=411.75
620     GAMMA (11)=411.25
630     GAMMA (12)=506.25
640     GAMMA (13)=587.25
650     GAMMA (14)=648.
660     GAMMA (15)=668.25
670     GAMMA (16)=668.25
680     GAMMA (17)=648.
690     GAMMA (18)=587.25
700     GAMMA (19)=506.25
710     GAMMA (20)=411.25
720     ZETA 1=-5.189
730     ZETA 2=0.
740     ETA 1=.028
750     ETA 2=0.
760     NU 1=-1.08
770     NU 2=0.
780     MU 1=-1.245
790     MU 2=0.
800     NSP=0
810     POINT1=0.
820     POINT2=0.
830     SDZ 2SQ=0.
840     T=0.
850     NSTOP=0
860     NPL=12
870     JJ=1
880     DO 10 I=1,47
890     10 Y(I)=0.
900C -----DATA READ IN-----
910     PRINT, "*****TYPE IN THE DATA FOR THE FOLLOWING"
920     PRINT,
930     PRINT, "EXTERNAL INPUT DATA"
940     PRINT,
950     PRINT, "NAME OF PROFILE INPUT FILE"
960     READ 2,FINAME
970     2 FORMAT(A6)
980     PRINT, "DATA VARIABLES"
990     PRINT,
1000    PRINT, "TRUCK VELOCITY IN INCHES PER SECOND [REAL]"
1010    READ,VEL
1020    PRINT, "PROGRAM VARIABLES"
1030    PRINT,

```

TRUCK CONTINUED

```

1040 PRINT,"NUMBER OF SPACES BETWEEN PROFILE POINTS [INTEGER]"
1050 READ,NSPACE
1060 PRINT,"NUMBER OF STEPS IN RKG [INTEGER]"
1070 READ,NSTEPS
1080 PRINT,"EXTERNAL OUTPUT DATA"
1090 PRINT,
1100 PRINT,"NAME OF OUTPUT DATA FILE"
1110 READ 2,FNAME2
1120 PRINT,
1130 PRINT,"*****END OF DATA INPUT*****"
1140 PRINT 3
1150 XMPH=VEL/17.6
1160 3 FORMAT(////,"T",10X,"Y(1)",7X,"AZETA2",5X,
1170& ARMS",/)
1180 WRITE(2;6)
1190 6 FORMAT(//,37X,45(1H*),/,37X,1H*,43X,1H*,/,37X,
1200& 45H* MURPHY'S M-37 TRUCK PROGRAM OUTPUT FILE *,/,
1210& 37X,1H*,43X,1H*,/,37X,45(1H*),//)
1220 WRITE(2;11)XMPH,NSTEPS
1230 WRITE(2;9)
1240 11 FORMAT(/,9HVELOCITY=,F6.2,15H MILES PER HOUR,50X,
1250& 35HNUMBER OF STEPS IN RKG INTEGRATION=,I4)
1260 9 FORMAT(//,13X,9(1H-),12HDISPLACEMENT,8(1H-),3X,10(1H-),
1270& 8HVELOCITY,11(1H-),3X,9(1H-),12HACCELERATION,
1280& 8(1H-),2X,7HC-G RMS,/,X,4HTIME,2X,4HY(1),X,
1290& 3(2X,3HC-G,4X,5HPITCH,3X,5HFR-AX,3X,5HRE-AX,2X),
1300& X,5HACCEL,//)
1310 DELTAT=3.07/VEL
1320 CALL OPENF(1,FNAME)
1330 4 FORMAT(E20.10)
1340 50 POINT1=POINT2
1350 IF(JJ-2)55,40,55
1360 55 READ(1,4)POINT2
1370 CALL EOFST(1,JJ)
1380 GO TO(60,40)JJ
1390 40 POINT2=POINT1
1400 NSTOP=NSTOP+1
1410 NSPACE=1
1420 60 PSTEP=(POINT2-POINT1)/NSPACE
1430 NSP=0
1440 100 I=46
1450 70 Y(I+1)=Y(I)
1460 I=I-1
1470 IF(I)70,75,70
1480 75 Y(I)=POINT1
1490 POINT1=POINT1+PSTEP
1500 NSP=NSP+1
1510 CALL DIFFEQ
1520 SDZ 2SQ=SDZ 2SQ+DZETA 2*DZETA 2
1530 RMSDZ 2=SQRT((SDZ 2SQ*DELTAT)/T)

```

TRUCK CONTINUED

```
1540 5 FORMAT(6(X,G10.4))
1550 NPL=NPL+1
1560 AZETA2=DZETA2/386.
1570 ANU2=DNU2/386.
1580 AMU2=DMU2/386.
1590 RMSAZ2=RMSDZ2/386.
1600 WRITE(2;7)I,POINT1,ZETA1,ETA1,MU1,NU1,ZETA2,ETA2,MU2,NU2,
1610& AZETA2,DETA2,AMU2,ANU2,RMSAZ2
1620 7 FORMAT(F6.1,F5.1,13F8.3)
1630 IF(NPL-54)120,110,120
1640 110 WRITE(2;9)
1650 PRINT 5,I,Y(1),AZETA2,RMSAZ2
1660 NPL=0
1670 120 IF(NSTOP-47)80,90,80
1680 80 I=I+DELTAT
1690 IF(NSP-NSPACE)100,50,100
1700 90 CALL CLOSEF(2,FNAME2,2)
1702 PRINT 12,(IBELL,I=1,40)
1704 12 FORMAT(40A1)
1710 PRINT 8,FNAME2
1720 8 FORMAT(///,"DETAILED OUTPUT IS IN FILE",X,A6,/)
1730 CALL EXIT
1740 END
```

DIFFEQ

```

100   SUBROUTINE DIFFEQ
110   REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
120   H=DELTA T/NSTEPS
130   RH=1./H
140   INDEX=0
150 100 INDEX=INDEX+1
160   VAR1=ZETA 1
170   VAR2=ZETA 2
180   VAR3=ETA 1
190   VAR4=ETA 2
200   VAR5=NU 1
210   VAR6=NU 2
220   VAR7=MU 1
230   VAR8=MU 2
240   PZETA 2=ZETA 2
250   PETA 2=ETA 2
260   PNU2=NU 2
270   PMU2=MU 2
280   CALL ALGEBR
290   FK 11=H*VAR 2
300   FK 12=H*(1/MASS)*(FORCK 1+C 11*DSPDF1+FORCK 2+C 21*DSPDF2-MASS*386.0)
310     FK 13=H*VAR 4
320   FK 14=H*(1/INRTIA)*(A*FORCK 1+A*C 11*DSPDF1-B*FORCK 2-B*C 21*DSPDF2)
330     FK 15=H*VAR 6
340   FK 16=H*(1/MASS1)*(-FORCK 1-C 11*DSPDF1+FORCW 1-MASS 1*386.0)
350   FK 17=H*VAR 8
360   FK 18=H*(1/MASS 2)*(-FORCK 2-C 21*DSPDF2+FORCW 2-MASS 2*386.0)
370   VAR 1=ZETA 1+FK 11*.5
380   VAR 2=ZETA 2+FK 12*.5
390   VAR 3=ETA 1+FK 13*.5
400   VAR 4=ETA 2+FK 14*.5
410   VAR 5=NU 1+FK 15*.5
420   VAR 6=NU 2+FK 16*.5
430   VAR 7=MU 1+FK 17*.5
440   VAR 8=MU 2+FK 18*.5
450   CALL ALGEBR
460   FK 21=H*VAR 2
470   FK 22=H*(1/MASS)*(FORCK 1+C 11*DSPDF1+FORCK 2+C 21*DSPDF2-MASS*386.0)
480     FK 23=H*VAR 4
490   FK 24=H*(1/INRTIA)*(A*FORCK 1+A*C 11*DSPDF1-B*FORCK 2-B*C 21*DSPDF2)
500     FK 25=H*VAR 6
510   FK 26=H*(1/MASS 1)*(-FORCK 1-C 11*DSPDF1+FORCW 1-MASS 1*386.0)
520   FK 27=H*VAR 8
530   FK 28=H*(1/MASS 2)*(-FORCK 2-C 21*DSPDF2+FORCW 2-MASS 2*386.0)
540   VAR 1=ZETA 1+.29289322*FK 21+.20710678*FK 11
550   VAR 2=ZETA 2+.29289322*FK 22+.20710678*FK 12
560   VAR 3=ETA 1+.29289322*FK 23+.20710678*FK 13
570   VAR 4=ETA 2+.29289322*FK 24+.20710678*FK 14
580   VAR 5=NU 1+.29289322*FK 25+.20710678*FK 15
590   VAR 6=NU 2+.29289322*FK 26+.20710678*FK 16

```

DIFFEQ CONTINUED

```

600   VAR 7=MU 1+.29289322*FK 27+.20710678*FK 17
610   VAR 8=MU 2+.29289322*FK 28+.20710678*FK 18
620   CALL ALGEBR
630   FK 31=H*VAR 2
640   FK 32=H*(1/MASS)*(FORCK 1+C 11*DSPDF 1+FORCK 2+C 21*DSPDF 2-MASS*386.0)
650     FK 33=H*VAR 4
660   FK 34=H*(1/INRTIA)*(A*FORCK 1+A*C 11*DSPDF 1-B*FORCK 2-B*C 21*DSPDF 2)
670     FK 35=H*VAR 6
680   FK 36=H*(1/MASS 1)*(-FORCK 1-C 11*DSPDF 1+FORCW 1-MASS 1*386.0)
690   FK 37=H*VAR 8
700   FK 38=H*(1/MASS 2)*(-FORCK 2-C 21*DSPDF 2+FORCW 2-MASS 2*386.0)
710   VAR 1=ZETA 1-.70710678*FK 21+1.70710678*FK 31
720   VAR 2=ZETA 2-.70710678*FK 22+1.70710678*FK 32
730   VAR 3=ETA 1-.70710678*FK 23+1.70710678*FK 33
740   VAR 4=ETA 2-.70710678*FK 24+1.70710678*FK 34
750   VAR 5=NU 1-.70710678*FK 25+1.70710678*FK 35
760   VAR 6=NU 2-.70710678*FK 26+1.70710678*FK 36
770   VAR 7=MU 1-.70710678*FK 27+1.70710678*FK 37
780   VAR 8=MU 2-.70710678*FK 28+1.70710678*FK 38
790   CALL ALGEBR
800   FK 41=H*VAR 2
810   FK 42=H*(1/MASS)*(FORCK 1+C 11*DSPDF 1+FORCK 2+C 21*DSPDF 2-MASS*386.0)
820     FK 43=H*VAR 4
830   FK 44=H*(1/INRTIA)*(A*FORCK 1+A*C 11*DSPDF 1-B*FORCK 2-B*C 21*DSPDF 2)
840     FK 45=H*VAR 6
850   FK 46=H*(1/MASS 1)*(-FORCK 1-C 11*DSPDF 1+FORCW 1-MASS 1*386.0)
860   FK 47=H*VAR 8
870   FK 48=H*(1/MASS 2)*(-FORCK 2-C 21*DSPDF 2+FORCW 2-MASS 2*386.0)
880   ZETA 1=ZETA 1+FK 11*.166667+.09763107*FK 21+.56903559*FK 31+FK 41*.1666
890     ZETA 2=ZETA 2+FK 12*.166667+.09763107*FK 22+.56903559*FK 32+FK 42*.1666
900     ETA 1=ETA 1+FK 13*.166667+.09763107*FK 23+.56903559*FK 33+FK 43*.1666
910     ETA 2=ETA 2+FK 14*.166667+.09763107*FK 24+.56903559*FK 34+FK 44*.1666
920     NU 1=NU 1+FK 15*.166667+.09763107*FK 25+.56903559*FK 35+FK 45*.166667
930     NU 2=NU 2+FK 16*.166667+.09763107*FK 26+.56903559*FK 36+FK 46*.166667
940     MU 1=MU 1+FK 17*.166667+.09763107*FK 27+.56903559*FK 37+FK 47*.166667
950     MU 2=MU 2+FK 18*.166667+.09763107*FK 28+.56903559*FK 38+FK 48*.166667
960     DZETA 2=(ZETA 2-PZETA 2)*RH
970   DETA 2=(ETA 2-PETA 2)*RH
980   DNU 2=(NU 2-PNU 2)*RH
990   DMU 2=(MU 2-PMU 2)*RH
1000   IF (INDEX-NSTEPS) 100,200,200
1010 200 RETURN
1020   END

```

ALGEBR

```
100     SUBROUTINE ALGEBR
110     REAL NU1,NU2,MU1,MU2,MASS,MASS1,MASS2,INRTIA
120     DO 100 I=1,10
130     SEGDEF(I)=Y(I)-VAR5-THRESH(I)
140     IF(SEGDEF(I))50,100,100
150 50  SEGDEF(I)=0.
160 100 CONTINUE
170     DO 200 I=11,20
180     SEGDEF(I)=Y(I+26)-VAR7-THRESH(I)
190     IF(SEGDEF(I))150,200,200
200 150 SEGDEF(I)=0.
210 200 CONTINUE
220     FORCW1=0.
230     FORCW2=0.
240     DO 300 I=1,10
250     FORCW1=FORCW1+GAMMA(I)*SEGDEF(I)
260 300 CONTINUE
270     DO 400 I=11,20
280     FORCW2=FORCW2+GAMMA(I)*SEGDEF(I)
290 400 CONTINUE
300     SPDEF1=VAR5-VAR1-A*SIN(VAR3)
310     SPDEF2=VAR7-VAR1+B*SIN(VAR3)
320     DSPDF1=VAR6-VAR2-A*VAR4*COS(VAR3)
330     DSPDF2=VAR8-VAR2+B*VAR4*COS(VAR3)
340     FORCK1=19.54*(SPDEF1**3)-192.42*SPDEF1*SPDEF1+913.55*SPDEF1
350     FORCK2=1.39*(SPDEF2**3)-1.24*SPDEF2*SPDEF2+307.72*SPDEF2
360     IF(DSPDF1)600,500,500
370 500 C11=CPOS1
380     GO TO 700
390 600 C11=CNEG1
400 700 IF(DSPDF2)800,900,900
410 900 C21=CPOS2
420     RETURN
430 800 C21=CNEG2
440     RETURN
450     END
```

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13. ABSTRACT The specific objective of this study was to define and formulate the procedures for implementing a practical, compact description of terrain-vehicle-speed (TVS) systems and thus establish a methodology for relating measures of ride to measures of terrain roughness and cross-country speed. This methodology should enable the determination of reliable estimates of the average speed a given vehicle can attain under given vibrational constraints as well as the probabilities of exceeding certain specified levels. This study was entirely a computerized effort. Digital computer simulations were conducted in which a vehicle was run at several selected speeds over terrain profiles with various levels of roughness. The terrain profiles were generated by computer programs that constructed and shaped sequences of random normal numbers to provide representative profiles with a desired power spectrum, variance, mean value, standard deviation, and RMS. These programs also performed the necessary statistical checks for normality, stationarity, and randomness. A comprehensive, nonlinear mathematical model of an M37 truck served as the vehicle. This model allowed for the nonlinearities inherent in large rotational motions, the suspension elements, bump stop, loss of ground contact, and the tire compliance, which was represented by a cluster of radially projecting springs. The results are presented in the form of three basic graphs comparing input and output statistics and distribution functions. The input statistics consisted of terrain roughness as measured by RMS elevation. The output is represented as the RMS vertical acceleration of the vehicle's center of gravity. A family of curves was developed relating vehicle response to terrain roughness for each speed of vehicle traversal. Cross plots of these established relations provided a more useful relation between vehicle response and speed for various degrees of terrain roughness. The use of such relations to catalog vehicles in terms of their ride dynamics capabilities is suggested. The TVS scheme is readily adaptable to computer models for evaluating and rating the cross-country mobility characteristics of vehicles traversing large, nonhomogeneous terrain areas.		

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