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DYNAMIC PROGRAMMING APPROACH TO THE OPTIMIZATION OF NAVAL AIRCRAFT REWORK AND REPLACEMENT POLICIES[1]

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ABSTRACT

This paper describes a method for determining optimal repair and replacement policies for aircraft, with specific reference to the F-4. The objective of the analysis is to choose the set of policies from all possible alternatives over a finite planning horizon which minimizes the cost of operations. A dynamic program is presented which seeks an optimal path through a series of decision periods, when each period begins with the choice of keeping an aircraft, reworking it before further operation, or buying a new one. We do not consider changes in technology. Therefore, when a replacement does occur, it is made with a similar aircraft. Multivariate statistical techniques are used to estimate the relevant costs as a function of age, and time since last rework.

I. INTRODUCTION

There is a strong effort within the Navy today to improve the aircraft maintenance program. This interest resulted from the rapid rise in the number of naval aircraft unavailable for squadron use during the current Vietnam conflict for reasons of maintenance. In 1966 the Chief of Naval Operations requested the Center for Naval Analyses to investigate the Naval maintenance program for the purpose of reducing the number of aircraft out of service. Initial findings were so appealing that the scope of the study was expanded. One ensuing task was for the development of a rationale for retirement of aircraft based on economic considerations.

Previously, for the most part, aircraft rework and replacement decisions involved historical evidence, planning factor costs, and immediate operational requirements. Selection of these elements of decisions were to some extent arbitrary, and the effects of time on these elements were rarely considered. Nevertheless, the desirability for a methodology to predict the cost impact of present and future decisions had been recognized, and the results of CNA's preliminary study attested to the feasibility of developing such methodology.

PRINCIPAL FACTORS

There are 3 principal factors which interact to establish the service life of an aircraft:

- Material condition;
- Mission effectiveness;
- Economic considerations.

Material condition

The material condition of an aircraft tends to decline as the aircraft ages: major components of the aircraft approach a fatigue life limit, unscheduled maintenance actions occur more frequently, and the underlying mechanical wear and tear factors along with corrosion effects all accumulate to produce the aging effect. This effect can be deterred somewhat through an intensive maintenance program. Nevertheless, it is rarely appropriate to combat the aging effect "at all costs," so in reality some aging does take place even if an economically optimal maintenance program is operative. The aging effect manifests itself, as the aircraft ages, as a decreasing readiness rate, which influences mission effectiveness adversely and thereby affects the aircraft service life.

Mission effectiveness

In general, owing to its material condition, the ability of an aircraft to perform a stipulated mission declines with the passing of time. Moreover, the succession of stipulated mission requirements tends to be increasingly difficult for a type-model-series of aircraft to achieve. In sum, the mission effectiveness of an aircraft declines as the gap widens between mission requirements and attainable aircraft performance. Consequently, this mission effectiveness factor becomes increasingly important throughout time in its effect on aircraft service life determination. In fact, under some circumstances it could dominate the other 2 factors -- material condition and economic cost -- but ordinarily all 3 factors influence that determination.

Economic considerations

Many variables contribute to a given level of aircraft mission effectiveness. Some of the significant ones are: (1) depth of maintenance, (2) length of the planned maintenance cycle, (3) aircraft type, and (4) aircraft age. Resources available to attain any level of aircraft mission effectiveness are limited. Axiomatically the controllable factors should be adjusted to minimize resource use compatibly with attainment of desired effectiveness for a given planning horizon. In that way economics influences the aircraft maintenance plan and the service life determination. In the past, most carrier-based aircraft were assigned a 7-year service life. This decision was based primarily on subjective judgment, because an effective planning tool was lacking for determining the service life of an aircraft based on a proper integrated consideration of material condition, mission effectiveness, and economics.

MAINTENANCE CONCEPTS

The Navy's concept of aircraft maintenance has long included a periodic processing at industrial facilities which have capabilities and skills exceeding those of the fleet maintenance activities. This periodic high-level maintenance became known as depot level maintenance. From the 1940's through the 1950's and 1960's there was an evolutionary development of this depot level maintenance from an overhaul program, to an Interim Rework program, and then to a PAR (Progressive Aircraft Rework) program.

Aircraft overhaul

Overhaul consists of a complete disassembly of an aircraft to permit inspection of all operating components and all basic elements of the aircraft structure. This is followed by repair, replacement or servicing, the incorporation of changes required by technical directives, and flight test to a mission ready status, i.e., all systems in the aircraft in normal operating condition. An integral part of the overhaul concept involves a change of aircraft custody from the fleet operating unit to the shore establishment during overhaul. Simultaneously, a replacement aircraft which has just completed overhaul is delivered from the shore establishment to the fleet unit. Consequently, the recipient becomes accustomed to receiving a virtually remanufactured aircraft. This overhaul concept is still operative for a limited number of naval aircraft.

The length of service tour between overhauls varied by aircraft model. During the 1940's, however, when overhaul applied to virtually all naval aircraft, the service tour averaged 26 months. This time interval was the prime reason for the next evolution in the depot maintenance concept. About 1952, with the introduction in the fleet of the new jet aircraft, aircraft technology was advancing so rapidly that aircraft needed frequent updating. In the 26-month period from one overhaul to another, a wide gap developed between the actual configuration/capabilities of the aircraft and the potential level that technology would permit. Such a gap was not acceptable to the fleet, and the Interim Rework concept evolved. With Interim Rework in operation, the overhaul and the 26-month service tour were both maintained, but in the middle of the tour a 30-day interim rework was performed at the industrial facility. The interim rework concentrated on updating the aircraft through modifications.

Aircraft interim rework

The Interim Rework concept met the fleet need for frequent updating of the aircraft configuration, but generated an adverse effect because of increased out-of-service time. The Progressive Aircraft Rework (PAR) program was initiated to obviate this new problem.

Progressive aircraft rework

The PAR concept became possible owing to rapid technological advances in developing basic aircraft materials. The result was improvements in basic structure which precluded the need to rework an airplane to the exhaustive depth of earlier years. For example, the useful life of aircraft wiring was so greatly increased that the old practice of rewiring during overhaul was no longer required. The technological replacement of "wood, dope and glue" by metal, sophisticated fasteners, and honeycomb reduced the scope of active processing and turned the PAR emphasis away from routine remanufacturing tasks toward a detailed inspection of the aircraft and correction of defects discovered during inspection.

PAR is based on a precept of adjusting rework content and frequency as necessary to preclude the need for overhaul, and to assure, within high confidence limits, continuance of a material condition which will sustain the aircraft through a subsequent operating tour. To minimize out-of-service time, fleet maintenance actions are avoided at the depot. If minor aircraft discrepancies are discovered during rework, they are left uncorrected if they do not interfere with the rework process or affect the safe flight of the aircraft. This policy on minor maintenance items is flexible. The extent to which this policy is implemented reflects a compromise between minimal in-process time and the natural fleet desire to accept an aircraft with all systems in a normal operating condition.

In tracing out the evolution of Navy depot-level maintenance concepts, several factors influencing the maintenance decisions are identified, but economic costs associated with each maintenance concept are conspicuous by their absence. This shortcoming in decision making under maintenance policies, existing and abandoned, is dealt with by providing a dynamic program which introduces a systematic consideration of costs for all alternative decisions that might be undertaken.

II. THE PROBLEM

The problems to be solved are:

1. The age at which the F-4A should be replaced with the F-4J.
2. The determination of an optimal rework schedule.

III. METHOD OF SOLUTION

Suppose that at the beginning of a planning period the decisionmaker can choose to keep an aircraft operating or purchase a new one. If we consider N_o planning periods, then the total number of alternatives available over this planning horizon is 2^{N_o}. In general, the total number of alternatives is equal to the number of choices available at the beginning of a planning period raised to the power N_o, the number of planning periods. If there are 2 alternatives, the decision to keep denoted by (K), and the decision to purchase denoted by (P), and the planning horizon is 3 periods, then the total number of alternatives will be 8. The alternatives are:

1:	(P_1, P_2, P_3)	5:	(K_1, P_2, P_3)
2:	(K_1, K_2, K_3)	6:	(K_1, P_2, K_3)
3:	(P_1, K_2, P_3)	7:	(P_1, K_2, K_3)
4:	(P_1, P_2, K_3)	8:	(K_1, K_2, P_3)

The costs associated with a decision to keep an aircraft operating for a planning period are the maintenance costs, consisting of labor and material costs, and an imputed cost of downtime. If the decision is to purchase, then the costs will include the purchase price of the replacement aircraft less its residual value plus the maintenance costs of a new aircraft for the planning period. It is assumed that all costs are incurred at the beginning of a planning period and that these costs can be related to the age of the aircraft.

If C_i^J represents the cost in the i^{th} period for the J^{th} alternative and N_o is the number of periods, then the decision rule is to select that alternative, J^M, which results in minimum costs:

$$\text{Cost}_{\text{minimum}} = C_1^{J^M} + \sum_{i=2}^{N_o} \frac{C_i^{J^M}}{(1+r)^{(i-1)}}, \qquad (1)$$

where r is the appropriate discount rate. The least cost alternative will indicate the period(s) when purchases should be made.

It is evident that as both the number of choices available at the beginning of a planning period and the number of periods increase, the number of alternatives that have to be evaluated become extremely large. The dynamic programming algorithm derived by Bellman and Dreyfus [2], illustrated below, is an efficient method for selecting the least cost alternative with the use of a digital computer. It was used to solve the 2 problems considered here. In the Appendix we show the equivalence of the "present value" solution, using equation 1, with the dynamic programming solution.

The example with 2 choices available at the beginning of a planning period was used to simplify the discussion of the decision rule that is used to determine when to purchase a new aircraft. For purposes of determining optimal replacement and rework policies, a third choice is added. The decisionmaker can now choose to keep (K), purchase (P), or rework and continue operating (R). If the decision is to rework, then the costs will include the cost of a Progressive Aircraft Rework plus the maintenance cost of the reworked aircraft for the portion of the planning period that the aircraft is not in rework.

The dynamic programming formulation consists of the following set of recurrence relations:

$$f_N(t_1,t_2) = \text{Minimum} \begin{cases} P: U_N(o,o) + C_N(t_1) + \frac{1}{(1+r)} f_{N+1}(1,1) \\ K: U_N(t_1,t_2) + \frac{1}{(1+r)} f_{N+1}(t_1+1,t_2+1) \\ R: U_N(t_1,o) + R_N(t_1,t_2) + \frac{1}{(1+r)} f_{N+1}(t_1+1,1) \end{cases} \quad (2)$$

where

t_1 is the age of the aircraft.

t_2 is tour length.

r is the discount rate.

$f_N(t_1,t_2)$ is the cost at year N of the overall cost of an aircraft where an optimal replacement policy is employed for the remainder of the process.

$C_N(t_1)$ is the net replacement cost as a function of age.

$R_N(t_1,t_2)$ is the cost of a Progressive Aircraft Rework as a function of age and tour length.

$U_N(t_1,t_2)$ is the maintenance costs as a function of age and tour length.

N_O is total number of periods being considered.

Because the process lasts N_O stages and then stops, $f_{N_O+1}(t_1,t_2) \equiv 0$.

This algorithm is begun by evaluating all admissible values of the function $f_{N_O}(t_1,t_2)$ in the last period, and then using these results to determine all admissible values of the function $f_{N_O-1}(t_1,t_2)$. This procedure continues through to the first period where the minimum cost of the optimal policy is determined. Once the algorithm is completed, the optimal path can be traced out by following the minimum cost decisions beginning with the first period. This indicates the age at which the aircraft should be replaced and the age at which the aircraft should be reworked. In the Appendix we show how this algorithm is used in the solution of a simple problem where the decisionmaker can choose to keep or replace over a planning horizon of 3 periods.

A computer program capable of evaluating 80 planning periods was written for this algorithm by Mr. William Pierce of the Center for Naval Analyses. The program is described in [8] . The program has been programmed in 3400 FORTRAN for use on a Control Data Corporation Model 3400 computer, and requires approximately 20,000 words of storage. The running time for the program is about 28 minutes.

Flexibility was added to the program by allowing the decisionmaker to suppress alternatives that are not available to him along the optimal path. For example, if the decision at the beginning of the N^{th} period is to purchase, but no funds will be available at that time for procurement, it is possible to suppress the purchase decision at this period and continue on a sub-optimal path. The cost of deviating from the optimal path can readily be determined. If engineering considerations require certain types of aircraft to be reworked within specified intervals, the decisionmaker can force reworks even though the optimal path indicates that reworks do not occur. Again, the cost of deviating from the optimal path will be determined.

IV. ASSUMPTIONS AND LIMITATIONS

ASSUMPTIONS

1. A planning period will be a calendar quarter, and the number of planning periods will be 80.

2. The annual discount will be 10 percent.

3. When estimates are derived with the use of regression equations, the relationships will be valid beyond the observed range of data.

4. The functions used for estimating costs are the same for the F-4A and the F-4J.

LIMITATIONS

It is assumed that the process stops at the end of the 80th period; this implies that all costs beyond the 80th period are zero. It is possible that 80 periods are not sufficient to indicate a recycling of policies; we are not in a position at this time to fully evaluate this end effect.

V. DEVELOPMENT OF COST ESTIMATES

THE REGRESSION MODEL USED IN THE COST ANALYSES

The regression model we used to estimate the cost of a Progressive Aircraft Rework, $R_N(t_1, t_2)$, and the cost of unscheduled maintenance, $U_N(t_1, t_2)$, combined arithmetic and logarithmic variables. The function has the following form:

$$\text{Log}_e Y_i = \alpha + \sum_{j=1}^{n} B_j \text{Log}_e X_{ji} + \sum_{j=1}^{m} a_j W_{ji} + \text{Log}_e \epsilon_i, \qquad i = 1, \ldots, N$$

This relationship can also be conveniently written as a multiplicative function:

$$Y_i = e^{\alpha} \left[\prod_{j=1}^{n} X_{ji}^{B_j} \right] \left[e^{\sum_{j=1}^{m} a_j W_{ji}} \right] \epsilon_i, \qquad i = 1, \ldots, N$$

where:

N is the number of observations

n is the number of logarithmic variables

$B_j = \frac{\partial Y}{\partial X_j} \left(\frac{X_j}{Y} \right)$, which is the percentage change in Y for a given percentage change in X_j, $j = 1, \ldots, n$.

m is the number of arithmetic variables. If the arithmetic variable is a dummy variable with discrete values 0, and 1, then

$e^{a_j W_j}$ is the constant percentage multiplier e^{a_j} when $W_j = 1$.

ϵ_i is the disturbance

ESTIMATING THE COST OF A PROGRESSIVE AIRCRAFT REWORK AS A FUNCTION OF AGE AND TOUR LENGTH FOR THE F-4

Estimating the man-hours expended for a progressive aircraft rework

It is reasonable to expect the following relationships to exist between man-hours and various measures of flight activity and age:

- Aircraft with more flight hours and arrested landings during a tour require more maintenance man-hours.
- Aircraft with longer tour lengths will require more maintenance man-hours.
- Older aircraft require more maintenance man-hours.
- Maintenance man-hours differ by assignment, i.e., aircraft operating in a combat environment require more maintenance than similar aircraft used in training.

Statistical analysis

Data was collected from individual aircraft undergoing standard reworks at the Naval Air Rework Facility at North Island during the period October 1963 to March 1967. A log-linear regression equation was used to relate man-hours expended on a Progressive Aircraft Rework to:

- Number of arrested landings
- Age
- Tour length
- Flight hours
- Custodian[2]

---

[2]Dummy variables were used to distinguish Navy-deployed aircraft from Marine and training aircraft.

The regression equation is summarized as follows:

$$
\begin{aligned}
\text{Log}_e(\text{man-hours}) = {} & 7.8401 \qquad (5) \\
& + \underset{(3.4)}{.1645}\ \text{Log}_e\ (\text{tour length}) \\
& + \underset{(12.8)}{.2523}\ \text{Log}_e\ (\text{age}) \\
& + \underset{(6.4)}{.0006}\ (\text{arrested landings}) \\
& - \underset{(6.2)}{.0646}\ \text{Log}_e\ (\text{flight hours}) \\
& + \underset{(0.8)}{.0184}\ (\text{if training}) \\
& - \underset{(1.4)}{.0369}\ (\text{if Marine})
\end{aligned}
$$

Correlation coefficient = 0.83

Standard error of estimate = 0.1182

Degrees of freedom = 210

The t values for the coefficients, which are calculated by dividing the coefficient by its standard error, are shown below the coefficients. An arithmetic variable operates as a multiplier of the form $e^{\alpha_j W_j}$, while the coefficient of the logarithmic variable shows the proportional change in the dependent variable for a proportional change in the independent variable. For example, equation (5) indicates that a 10 percent change in tour length will increase man-hours by 1.645 percent. Marine aircraft require approximately 4 percent fewer man-hours than Navy-deployed aircraft.

An imputed cost of downtime equal to the estimated cost of an aircraft day times the expected days-in-process is added to the cost of a Progressive Aircraft Rework. The days-in-process is estimated with the following equation:

$$\text{Log}_e \text{ (days in process)} = -1.9288 + \underset{(8.3)}{0.6526} \text{ Log}_e \text{ (man-hours)} \quad (6)$$

Correlation coefficient	= 0.45
Standard error of estimate	= 0.2538
Degrees of freedom	= 210

Estimating material costs for a progressive aircraft rework

The accounting procedure used by the Naval Air Rework Facilities separates material costs into 2 categories: Navy Industrial Fund Material, and Government Furnished Material. These costs were collected by individual aircraft that underwent standard reworks at the Naval Air Rework Facility located at North Island. The data is summarized in table I.

TABLE I

MEANS AND STANDARD DEVIATIONS OF NAVY INDUSTRIAL FUND MATERIAL AND GOVERNMENT FURNISHED MATERIAL FOR F-4 PROGRESSIVE AIRCRAFT REWORKS

Variable	Means and standard deviations
Navy industrial fund material	\$3565. (1117)
Government furnished material	\$23,718 (12,962)
Sample size	147
Time Period	Oct. 1963 - Jan. 1967

Statistical analysis

Using a log-linear equation to relate Government Furnished Material to the same variables that were specified for equation (3), we have:

$$
\begin{aligned}
\text{Log}_e \text{ (government furnished material)} &= 7.98 \\
&+ \underset{(1.85)}{0.7451} \text{ Log}_e \text{ (tour length)} \\
&+ \underset{(0.82)}{0.1461} \text{ Log}_e \text{ (age)} \\
&+ \underset{(1.46)}{0.0008} \text{ (arrested landing)} \\
&- \underset{(2.26)}{0.3447} \text{ Log}_e \text{ (flight hours)} \\
&- \underset{(2.00)}{0.3974} \text{ (if Marine)} \\
&- \underset{(1.18)}{0.1815} \text{ (if deployed)} \\
&+ \underset{(4.5)}{0.0337} \text{ (time)}
\end{aligned}
\qquad (7)
$$

Correlation coefficient = 0.54

Standard error of estimate = 0.3015

Degrees of freedom = 139

A trend variable, time, was included because material costs are measured in dollars. This could be interpreted as a proxy for price and specification changes that have occurred during the observed period.

Navy Industrial Fund material costs were found to be relatively stable over the period observed. Therefore, no significant relationship was found between these costs and our various measures of flight activity and age. When estimating the cost of a Progressive Aircraft Rework, the average cost of Navy Industrial Fund material will be used.

We can now proceed to estimate the cost of a Progressive Aircraft Rework with the use of equations (3), (4), and (5). Since we are interested in the cost in relation to age and tour length, we can collapse our multidimensional relationship into three dimensions by holding all other variables constant at the mean. Equations (3), (4) and (5) now have the following form:

$$\text{Man-hours} = e^{7.6043}(\text{age})^{0.2523}(\text{tour length})^{0.1645} \quad (8)$$

$$\text{Government furnished material} = e^{8.0191}(\text{age})^{0.1461}(\text{tour length})^{0.7451} \quad (9)$$

$$\text{Days in process} = e^{-1.9288}(\text{man-hours})^{0.6526} \,. \quad (10)$$

Using \$4.60 per hour as a direct labor charge and \$2.10 per hour as an indirect charge for applied production expense, we can estimate the man-hour cost by multiplying equation (6) by \$6.70.[3] Added to this man-hour cost is the Government Furnished Material from equation (7), Navy Industrial Fund Material of \$3560, \$17,000 for General Expense, and an imputed cost of downtime. The imputed cost of downtime is derived by multiplying the number of days in process by \$5940, which is the cost of an aircraft day.

[3]This rate was derived from North Island production and financial statements for the fourth quarter FY-1966.

The final equation for the cost of a Progressive Aircraft Rework, given age, t_1, and tour length, t_2, will be:

$$
\begin{aligned}
R_N(t_1, t_2) = {} & (e^{7.6043}\ t_1^{0.2523}\ t_2^{0.1645})\ \$6.70 \\
& + e^{8.0191}\ t_1^{0.1461}\ t_2^{0.7451} \\
& + \left(e^{-1.9288}\left(\text{man-hours } (t_1, t_2)\right)^{0.6526}\right)\ \$5{,}940 \\
& + \$17{,}000 + \$3{,}560
\end{aligned}
\tag{11}
$$

Figure 1 shows the estimated total cost of a Progressive Aircraft Rework as a function of age and tour length. Figure 2 shows the days in process as a function of man-hours.

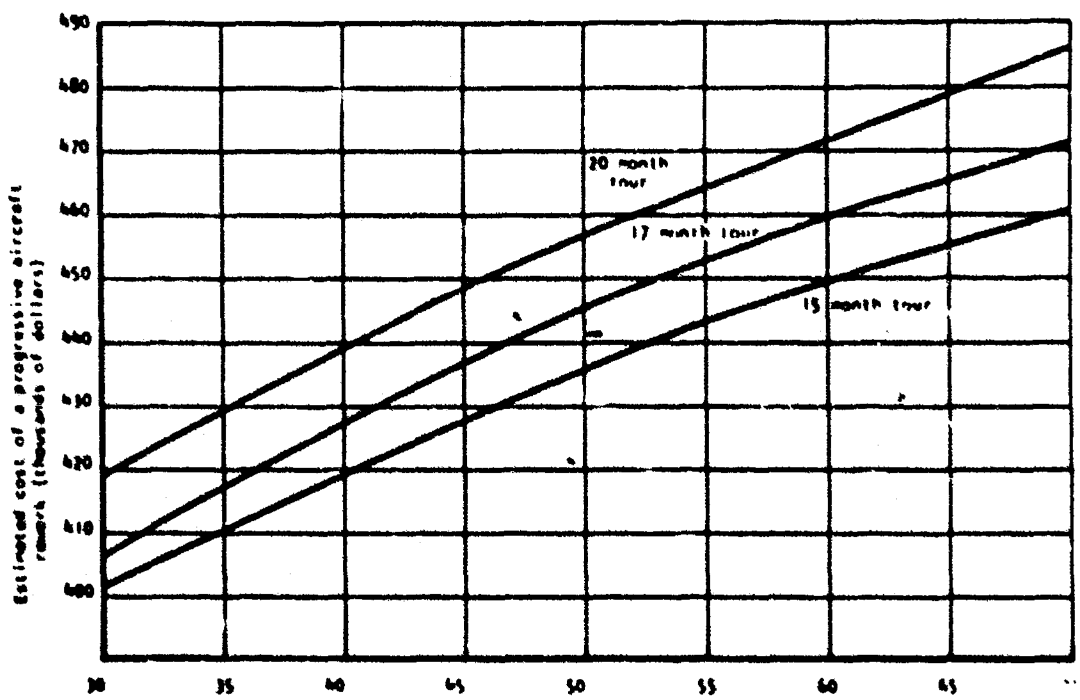

FIG. 1: THE ESTIMATED COST OF A PROGRESSIVE AIRCRAFT REWORK AS A FUNCTION OF AGE AND TOUR LENGTH (INCLUDING IMPUTED COST OR DOWNTIME)

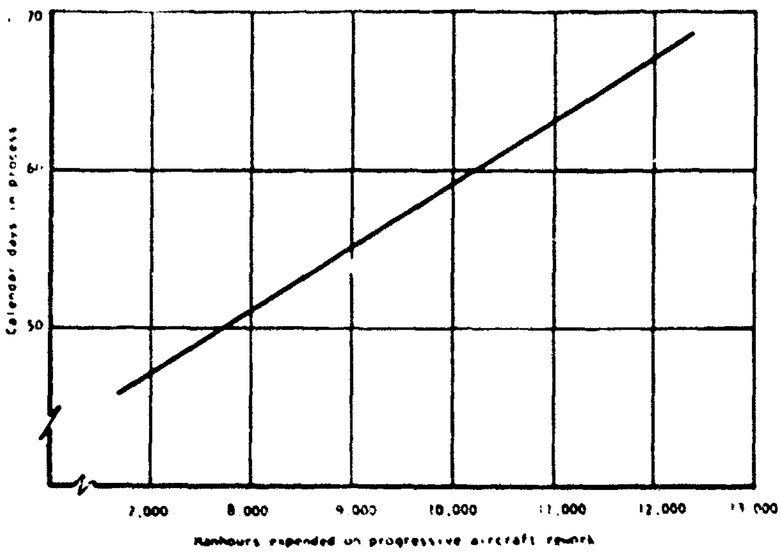

FIG. 2: ESTIMATED CALENDAR DAYS IN PROCESS AS A FUNCTION OF MANHOURS EXPENDED ON PROGRESSIVE AIRCRAFT REWORK

ESTIMATING REPLACEMENT COST AS A FUNCTION OF AIRCRAFT AGE FOR THE F-4

The general form of th equation that is used to estimate replacement cost is:

$$\text{Replacement cost} = C(1 - ke^{-\alpha' t})$$

where

C = Purchase price of the new plane

t = Age

α' = Value of the coefficient for a specified percentage rate of annual decline in the salvage value

k = Fraction of C remaining as trade-in value after purchasing.

Figure 3[4] shows the salvage or residual value as a function of age for specified rates of annual decline; table II shows various values of the coefficient, α' , for an arbitrary range of rates of annual decline.

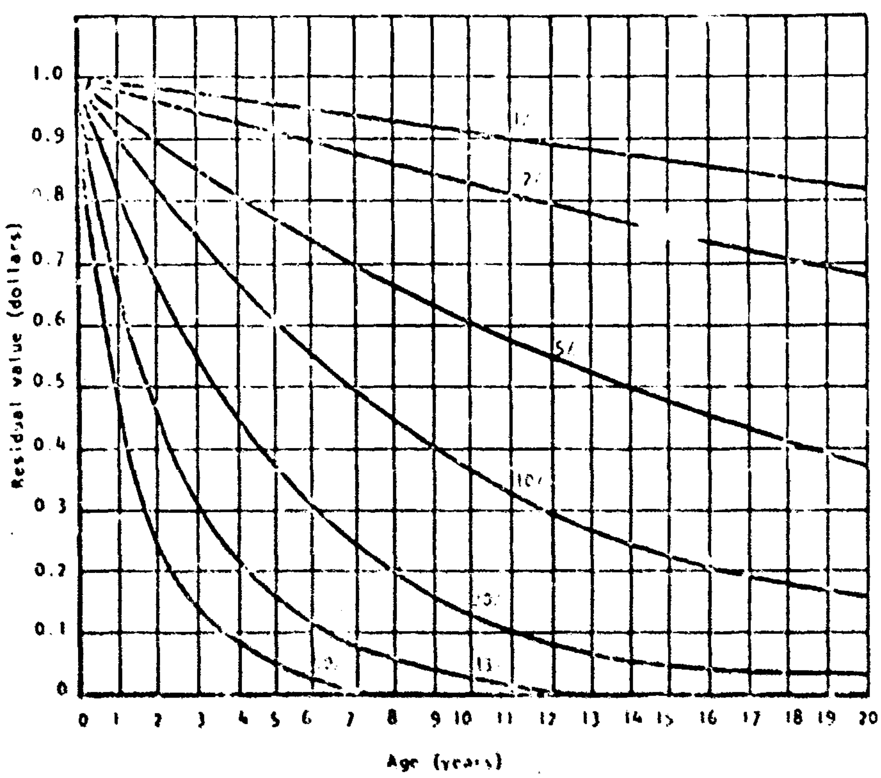

FIG. 3: RESIDUAL VALUES RELATED TO VARIOUS ANNUAL RATES OF DECLINE

[4]Extracted from [1].

TABLE II[5]

ANNUAL PERCENTAGE RATES OF CHANGE AND CORRESPONDING VALUES FOR COEFFICIENTS EXPRESSING THAT RATE OF CHANGE

Percentage rate of annual decline	α'
.00	.00
.01	.00
.02	.02
.03	.03
04	.04
.05	.05
.06	.06
.07	.07
.08	.08
.09	.09
.10	.10
.11	.12
.12	.13
.14	.15
.20	.22
.25	.30
.30	.35
.33	.40
.40	.50
.45	.60
.50	.70
.65	1.00
.70	1.20
.80	1.60
.90	2.30
.95	3.00
.999	6.00

ESTIMATING MAINTENANCE COSTS AS A FUNCTION OF AGE AND TIME SINCE LAST REWORK FOR THE F-4

The maintenance performed at the squadron for aircraft consists of the following:

- Unscheduled maintenance -- labor and material expended repairing random failures in the aircraft;
- Support maintenance -- labor expended in nonrepair actions, such as washing the aircraft, preparing it for flight, corrosion control, etc.;

[5]Extracted from [1]

- Scheduled maintenance -- labor and material expended as necessary inspecting and repairing specified aircraft components on a periodic basis; scheduled maintenance, which is also called a calendar inspection, occurs every 30 weeks for the F-4;

- Field modification -- labor and material expended making engineering changes by field teams sent from the appropriate rework facilities; field teams are used when the work exceeds the capability of the squadron.

Estimating unscheduled maintenance man-hours

Unscheduled direct maintenance man-hours were collected from individual aircraft on a quarterly basis from VF-121. Assuming that older aircraft require more maintenance man-hours at the squadron and that maintenance man-hours will increase with the time since last rework, we regressed direct maintenance man-hours on age and time since last rework. This produced the following relationship:

$$\text{Log}_e \text{ (unscheduled maintenance man-hours)} = 6.5515 + \underset{(2.4)}{0.1401 \log_e \text{ (time since last rework)}} + \underset{(1.0)}{0.0443 \log_e \text{ (age)}} \quad (12)$$

Correlation coefficient = 0.52

Standard error of estimate = 0.2165

Degrees of freedom = 21

Estimating support man-hours

Support man-hours are reported only for the squadron. The squadron allocates the support man-hours to individual aircraft on the basis of its flight activity. No attempt was made to relate support man-hours to age and time since last rework because the man-hours reported by individual aircraft were not the actual hours expended. Therefore, for all future cost calculations only the average support man-hours per F-4 will be used; the average support man-hours per F-4 for the month of June 1967 for VF-121 was 503.

Estimating man-hours for calendar inspections

Direct maintenance man-hours expended on 19 F-4's for calendar inspections during the period October 1966 to June 1967 were collected from VF-121. It was not possible to relate these man-hours to age and time since last rework because of the insufficient number of observations obtained from this sample of 19. The average man-hours per calendar inspection is 805, and the number of days the aircraft is down is approximately 8.

Estimating man-hours for field team modifications

The man-hours expended on individual F-4 aircraft for field modification were collected from the Naval Air Rework Facility located at North Island for the period October 1965 to February 1968. Since man-hours expended for field team modifications are a function of the time since last rework, regression analysis was used to estimate the functional form of the equation. The equation is:

Log_e (man-hours) = 4.0075 (13)

+ 0.2699 Log_e (time since last rework)
(3.84)

Correlation coefficient = 0.20

Standard error of Estimate = 1.2048

Degrees of freedom = 355

An imputed cost of downtime will be added to the maintenance costs. Therefore, the next section discusses the procedure used for estimating the time the aircraft is not operationally ready as a function of age and time since last rework.

<u>Estimating the percentage of time the aircraft is not operationally ready</u>

The percentage of time the aircraft is not available for use is composed of Not Operationally Ready Maintenance (NORM) and Not Operationally Ready Supply (NORS). These percentages were collected for individual F-4 aircraft for the month of October 1967. Regressing these percentages against age and time since last rework resulted in the following equation:

Log_e (percentage of time not operationally ready) = 1.7782 (14)

+ 0.3539 Log_e (age)
(3.92)

+ 0.. $_e$ (time since last rework)
(2.74)

Correlation coefficient = 0.30

Standard error of estimate = 0.7459

Degrees of freedom = 217

Costing of material and man-hours

Because we cannot cost military man-hours or measure the material usage by individual aircraft, labor and material costs have been derived on the basis of the costs estimated at the Naval Air Rework Facility at North Island. The cost of labor and material has been estimated at $14 per man-hour and is derived by dividing the average cost of a Progressive Aircraft Rework by the average man-hours expended. The cost of support man-hours is taken to be $6.70 because virtually no material is consumed washing the aircraft, preparing it for flight, etc.

The imputed cost of downtime

The cost of an aircraft day will be taken as $5940. The imputed cost of downtime at the squadron for a quarter will therefore be:

Imputed cost of downtime

$$= \frac{[e^{1.7782}(\text{age})^{0.3539}(\text{time since last rework})^{0.207}]}{100} \qquad (15)$$

$$\times\ [90\ \text{days}][\$5940]\ .$$

The equation that is used to estimate the maintenance costs in relation to age, t_1 and tour length, t_2, on a quarterly basis is:

Maintenance cost = (unscheduled man-hours)$14 (16)

+ (support man-hours)$6.70

+ (field team man-hours)$14

+ imputed cost of downtime

+ [(man-hours expended for calendar inspection)$14 +

(days in process)$5940] added in every 30 weeks.

Finally we have

$$U_N(t_1, t_2) = (e^{6.5515}\, t_1^{0.0443}\, t_2^{0.1401})\$14 \tag{17}$$
$$+ \quad (3 \times 503)\$6.70$$
$$+ \quad (e^{4.0075}\, t_2^{0.2699})\$14$$
$$+ \left(\frac{(e^{1.7782}\, t_1^{0.3539}\, t_2^{0.207})}{100}\right) (90 \text{ days}) (\$5940)$$
$$+ \left((805)\$14 + (8)\$5940\right) \text{added in every } 30 \text{ weeks}$$

The following assumptions will be made because the cost equations are logarithmic.

- $U_N(o,o) \equiv U_N(1,1)$
- $U_N(t_1,o) \equiv U_N(t_1,1)$.

Figure 4 shows the maintenance costs as a function of age and time since last rework excluding the cost of a calendar inspection. The continuous curve in figure 5 indicates the percent of time not operationally ready for an aircraft that has never been reworked. The discontinuous curve in figure 5 shows the effect on the percent of time not operationally ready of periodic rework and the time since last rework.

The rework cost function in the dynamic programming formulation is:

$$U_N(t_1,o) + R_N(t_1,t_2) + \frac{1}{(1+r)} f_{N+1}(t_1+1,1) \; .$$

The costs associated with a decision to rework are: the maintenance costs for the planning period, $U_N(t_1,o)$; the cost of the rework, $R_N(t_1,t_2)$; and the discounted future costs, $\frac{1}{(1+r)} f_{N+1}(t_1+1,1)$. Because the aircraft is not being utilized for the entire planning period when a rework does occur, the total maintenance costs for the planning period should not be incurred. Therefore, only a portion of the maintenance costs, $U_N(t_1,o)$, will be allocated to the costs associated with a decision to rework. For a planning period of 90 days, $U_N(t_1,o)$ will be multiplied by

$$\frac{90 - (\text{days in process for a progressive aircraft rework})}{90}.$$

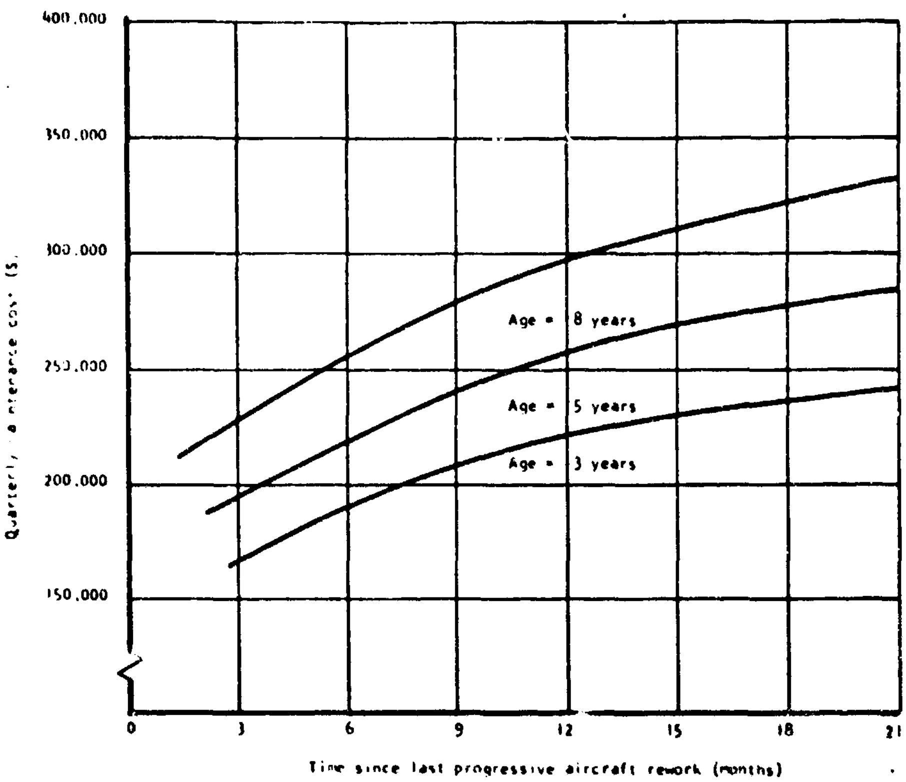

FIG. 4: ESTIMATED QUARTERLY MAINTENANCE COSTS (INCLUDING THE IMPUTED COST OF DOWNTIME) AS A FUNCTION OF TIME SINCE LAST PROGRESSIVE REWORK AND AGE

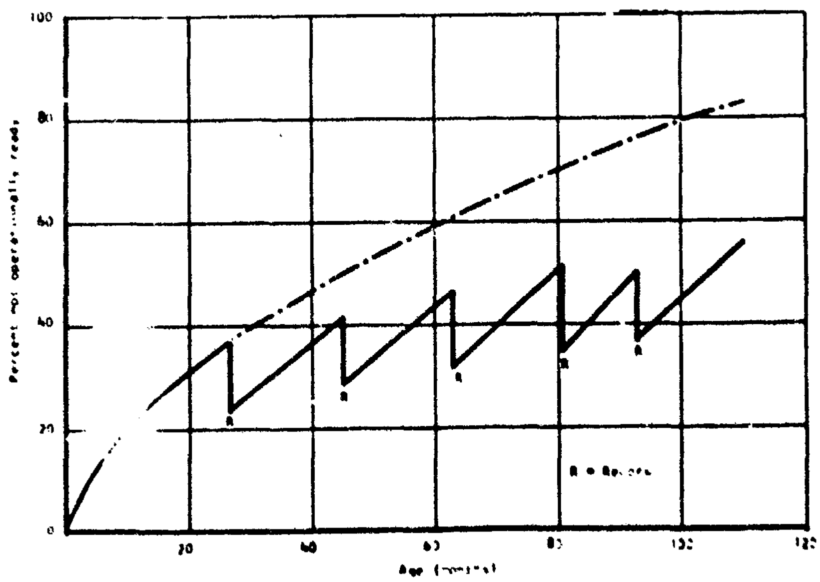

FIG. 5: THE EFFECT OF AGE AND TIME SINCE LAST REWORK ON THE PERCENT NOT OPERATIONALLY READY

VI . SOLUTION TO THE PROBLEM USING THE METHOD OF DYNAMIC PROGRAMMING

REPLACEMENT OF THE F-4A WITH THE F-4J

For this problem, the residual value is held constant over the entire planning horizon. This is based on the assumption that the value of the components that could be salvaged from the F-4A will remain relatively stable for the length of time the F-4A remains in the inventory.

The residual value of the F-4A components, excluding engines, was estimated at $950,800 by Aviation Supply Office, Philadelphia, Pa. If the cost of two new engines is added to this figure, the residual value could be as high as $1,400,000. There will be a cost incurred disassembling the F-4A's and refurbishing their components. Therefore, $950,800 could be considered a lower limit and $1,400,000 an upper limit. An intermediate value of $1,100,000 will also be considered. The figure was derived by taking 80 percent of $1,400,000, based on the assumption by the Naval Air Rework Facilities that the cost of refurbishing components is approximately equal to 20 percent of their value.

Three computer runs were made using these residual values and the derived maintenance cost functions. The results are shown in tables III, IV, and V. All runs began with the purchase of a new aircraft.

TABLE III

Residual value = $950,800

Tour	Decision	Length of tour (months)
I	Rework	27
II	Rework	18
III	Rework	18
IV	Rework	18
V	Rework	12
VI	Purchase	18

Purchase at the end of 9 years

TABLE I V

Residual value = $1,100,000		
Tour	Decision	Length of tour (months)
I	Rework	27
II	Rework	18
III	Rework	18
IV	Purchase	27
Purchase at the end of 7.5 years		

TABLE V

Residual value = $1,400,000		
Tour	Decision	Length of tour (months)
I	Rework	27
II	Rework	18
III	Purchase	27
Purchase at the end of 6 years		

Another run was made using no residual value; for this case the purchase decision occurred at 11.7 years. This indicated that using residual values below $950,800 will not have a marked effect on the replacement age; the rate of increase in the replacement age appears to be very slow relative to a decrease in the residual value below $950,800.

THE OPTIMAL REWORK CYCLE

What appears to be extremely interesting and intuitively appealing is the relationship between tour length and aircraft age. The data showed that new aircraft require less maintenance and are operationally ready a larger percentage of the time during a tour relative to older aircraft. Therefore, the results indicated that new aircraft should have longer tours. Older aircraft are more expensive to maintain, require frequent major reworks, and have a lower availability than the newer aircraft. As the aircraft approaches the end of its service life, expensive repairs such as a Progressive Aircraft Rework would not be undertaken as long as the aircraft is still safe for flight. The tour length would tend to increase before the aircraft is scrapped.

SENSITIVITY ANALYSIS

The tour length and the replacement age vary considerable with the cost of an aircraft day and the percentage of time the aircraft is not operationally ready. A relatively slow rise in the percentage of time the aircraft is not operationally ready as a function of age and time since last rework will tend to delay reworks and purchase decisions. Decreasing the cost of an aircraft day below $5,940, the figure used in this study, will also have the same effect, i.e., stretch out tour lengths and delay purchases. For example, using a residual value of $1,100,000 and $5,000 for the cost of an aircraft day, the purchase decision occurred at 9.75 years; using $7,000 for the cost of an aircraft day forced a purchase at 6.75 years.[6]

Two computer runs were made, one without discounting future costs and one discounting them at 20%. Increasing the discount rate postpones the replacement decision but does not effect the tour lengths. Undiscounted future costs decrease the replacement age of the aircraft and produce the same tour lengths as the computer runs using a 10 percent and 20 percent discount rate. For example, using a residual value of $1,100,000 and a 0 discount rate, the

[6] An iterative process should be used so that the cost of an aircraft day is compatible with the program service life. Convergence was tested and assured.

replacement decision while decreasing the discount rate decreases the replacement age of the aircraft.

SUMMARY AND CONCLUSIONS

The results show that the replacement age is sensitive to the residual value of the aircraft that is being replaced, the cost of an aircraft day, and the percent of time the aircraft is not operationally ready at the squadron. For example, using a fast rising curve to describe the percent of time the aircraft is not operationally ready will tend to decrease tour lengths and decrease replacement age. Increasing the cost of an aircraft day above $5,940, the figure used in this study, will have the same effect. Increasing the discount rate postpones the replacement decision while decreasing the discount rate decreases the replacement age of the aircraft.

The analysis strongly indicates that older aircraft require more resources to maintain, are less available during a tour, and should be reworked more frequently than newer aircraft. This implies that it is unrealistic to plan for fixed tour lengths, to allocate resources to aircraft without considering their age and condition, and to assume that all aircraft are equally capable.

VII . AREAS FOR FURTHER STUDY

Other problems that could be solved with the use of the dynamic program

The dynamic program could be used to determine whether a severely damaged aircraft should be restored or replaced. This could simply be accomplished by entering the program with the age and tour length of the damaged aircraft, and substituting the cost of restoring the aircraft as the cost of the next rework; the residual value would be the salvage value of the damaged aircraft. The keep decision would be suppressed because this would not be a feasible alternative.

If the aircraft were to be considered for use in the Naval Reserves or in a training squadron, the residual value would have to be adjusted to reflect the value of the aircraft in these secondary missions. The dynamic program would determine the age at which the aircraft would be retired from first-line status.

The finite planning horizon

A critical assumption made was that the process stopped at the end of the 80th period; this assumes that all costs beyond the 80th period are zero. Carying these zero future costs into the analysis will affect the decisions in the later stages of the program. How strong this effect is, and how far into the present it penetrates, are areas that need further investigation.

APPENDIX

AN EXAMPLE SHOWING THE EQUIVALENCE OF THE PRESENT VALUE SOLUTION WITH THE DYNAMIC PROGRAMMING SOLUTION

Suppose we are considering two alternatives, purchase (P) or keep (K), and a planning horizon of three years. The relevant costs are shown in the following matrixes.

TABLE A-I

COSTS FOR MACHINE MADE IN YEAR 1

Age	0	1	2
Maintenance	20	20	25
Replacement	200	220	240

TABLE A-II

COSTS FOR MACHINE MADE IN YEAR 2

Age	0	1	
Maintenance	15	20	
Replacement	210	230	

TABLE A-III

COSTS FOR MACHINE MADE IN YEAR 3

Age	0		
Maintenance	15		
Replacement	240		

Some examples clarify the use of the tables. If we enter period 3 with a machine that is 2 years old, the operating and maintenance costs will be 25 (table A-I). The cost of replacing a one-year old machine at the beginning of period 3 is 230 (table A-II). The cost of a new machine in year 3 is 240 (table A-III).

Present value solution consists of evaluating all possible combinations of alternatives and then selecting the minimum cost combination. We begin the enumeration with the purchase of a new machine.

If the decision in year N is purchase, then the cost associated with this action is the maintenance costs of a new machine in year N plus the cost of replacing a machine in year N that is of age t . If the decision is keep (K), then the only cost in year N will be the maintenance costs of a machine of age t .

1: (P_1, P_2, P_3)
$220+235(1.10)^{-1} + 245(1.10)^{-2} = 636.1$

2: (P_1, P_2, K_3)
$220+235(1.10)^{-1} + 20(1.10)^{-2} = 450.17$

3: (P_1, K_2, P_3)
$220+20(1.10)^{-1} + 255(1.10)^{-2} = 448.9$

4: (P_1, K_2, K_3)
$220+20(1.10)^{-1} + 25(1.10)^{-2} = 258.8$

The minimum cost path is 4.

Dynamic programming solution

The recurrence relations for this problem are:

$$f_N(t) = \text{Minimum} \begin{cases} P: U_N(o)+C_N(t) + \frac{1}{(1+r)} f_{N+1}(1) \\ K: U_N(t) + \frac{1}{(1+r)} f_{N+1}(t+1), \end{cases}$$

where:

$U_N(t)$ is the operating and maintenance cost of a machine in year N that is t years old.

$U_N(0)$ is the maintenance costs of a new machine purchased in year N .

$C_N(t)$ is the net cost of replacing a machine in year N that is t years old.

$f_N(t)$ is the cost at year N of the overall cost from a machine which is t years old, where an optimal replacement policy is employed for the remainder of the process.

N_0 is the total number of periods that are being considered.

$f_{N_0+1}(t) \equiv 0$

r is the discount rate.

We begin the algorithm by evaluating all admissible values of the function $f_{N_0}(t)$ in the last period and then use these results to determine all admissible value of the function $f_{N_0-1}(t)$. This procedure continues through the first period where the minimum cost of the optimal policy is determined. Once the algorithm is completed, the optimal path can be traced out by following the minimum cost decisions beginning with the first period.

$$f_3(1) = \text{Minimum} \begin{bmatrix} P: U_3(o)+C_3(1) = 15+230 \\ \\ K: U_3(1) \quad = 20 \end{bmatrix} = 20$$

$$f_3(2) = \text{Minimum} \begin{bmatrix} P: U_3(o)+C_3(2) = 15+240 \\ \\ K: U_3(2) \quad = 25 \end{bmatrix} = 25$$

$$f_2(o) = \text{Minimum} \begin{bmatrix} P: U_2(o)+C_2(1)+f_3(1)\cdot\frac{1}{(1+r)} = \\ 15+220+20[1.10]^{-1} \\ K: U_2(1)+f_3(2)\cdot\frac{1}{(1+r)} = \\ 20+25[1.10]^{-1} \end{bmatrix} = 42.73$$

$$f_1(o) = \quad P: U_1(o)+C_1(o)+42.7[1.10]^{-1} = $$

$$20+200+38.8 \quad = 258.8$$

We purchase in year 1 and move into year 2 wit' a machine that is 1 year old. The minimum cost decision in year 2 is keep (K). We move into year 3 with a 2 year-old machine where the decision is keep (K). As a check we can add up the discounted cost from this policy in the following way:

Year	Policy	Cost $(f_N^* - f_{N+1}^*)$
1	P	216.07
2	K	17.73
3	K	25
		258.8

where f_N^* is the cost in year N following the optimal path.

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