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FREQUENCY-DEPENDENT AMPLITUDE-DISTANCE CURVE
FOR P-WAVES FROM 87° TO 110°

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Summary

This work is an attempt to clarify the nature of the amplitude-distance curve for P-waves between 87° and 110°, using the spectral amplitudes of earthquakes in the Indonesian region recorded at the Swedish and Finnish seismograph stations. At the present time the results are inconclusive, because even after allowing for station and source terms there is a large unexpected scatter.

Introduction

The amplitude decrease of P-waves which are incident at the surface beyond 95° and which have passed through the deeper part of the mantle has been observed for a long time. As the quality and distribution of seismographs have increased, it has been possible to make more detailed studies. Gutenberg (1960) with limited data demonstrated the frequency dependence of the amplitude decrease, while Sacks (1966) with better data illustrated the effect very clearly. These results, however, are more qualitative than quantitative and cannot be used to test hypotheses on the nature of the core-mantle boundary region which produces the shadowing effect. Also these studies use a few earthquakes and do not consider the effects of station geology on the results. More recently, Alexander and Phinney (1966) have worked with long-period waves in the shadow region, but their data has large scatter, they do not consider station effects and they do not combine data from different earthquakes.

Recently, Carpenter, Marshall and Douglas (1966) and Cleary (1967) have worked on the amplitude-distance curve between 30° and 102° and have
used a joint analysis method described by Carpenter et al. This method allows the combination of different earthquakes, finds corrections for the station effect and produces an amplitude-distance curve independent of earthquakes and stations. These amplitude-distance curves are valid for short-period vertical-component records of about 1 sec period, but there is some disagreement between them at distances beyond 90° probably because of the different methods of measuring amplitudes. Both authors have little data over 95° and neither investigate the effect of frequency on the amplitude-distance curve. The present work has been done to attempt to clarify the frequency dependence of the amplitude-distance curve using the joint analysis technique to combine data from many earthquakes and many stations.

The observational material used in the present investigation consists of short-period vertical-component records of P-waves from the network of Swedish and Finnish stations for a number of earthquakes in the Indonesian archipelago.

Analytical method

At teleseismic distances we can express the frequency-dependent amplitude A of the body waves in the form

\[ A = B R S \]  

(1)

where B is the frequency-dependent source function which includes the effect of the crust and upper mantle at the source, R is the transmission coefficient for passage through the mantle which includes the effects of reflection and diffraction by the core-mantle boundary region (if the wave concerned is affected by this region), the effect of transmission at any boundaries and the effect of geometrical spreading of the waves, and
finally S is the receivei function which includes the effects of the
station seismographic response curve and the crust and the upper mantle
below the station. Each of B, R and S also includes the effect of the
anelastic dissipation and scattering by inhomogeneities in the regions
concerned. B and S vary with azimuth and also with angle of incidence on
the surface. To the extent that the lower mantle is inhomogeneous R is
dependent on the particular path through the mantle.

In our problem then, we have selected the stations and earthquakes
such that we make the assumption that B and S vary little over the small
variation of azimuthal angles and small variation of angles of incidence
involved (this assumption may not be valid!). R will apply to the mantle
between Indonesia and Fennoscandia and to the core-mantle boundary region
under Central Asia. All of A, B, R and S are frequency-dependent and
complex.

If we use the base ten logarithms of these quantities then we have

\[ a = b + r + s \]  \hspace{1cm} (2a)

where \( a = \log_{10}|A|, b = \log_{10}|B| \) etc. (\(|A|\) is the amplitude of the complex
A) and

\[ \text{phase}(A) = \text{phase}(B) + \text{phase}(R) + \text{phase}(S) \]  \hspace{1cm} (2b)

For any particular measurement of \( a \) we have

\[ a = b + r + s + \epsilon \]  \hspace{1cm} (3)

where \( \epsilon \) is an error term which includes the inaccuracies of measurement of
\( a \) and the effect of inadequacies of the model we have set up. This
formulation is the same as derived by Carpenter et al. (1967) except that
our \( a \) is the log of the spectral amplitude and not \( \log_{10}(\frac{A}{T}) \) and in our
model the azimuths and angles of incidence are very limited in range.
Following Carpenter et al., we find that if we make a number of observations of \( a \) at a number of stations for a number of earthquakes we can obtain estimates of \( b, r \) and \( s \). If we denote by subscript \( i \) the particular earthquake considered then \( b_i \) is the source term of the \( i \)th earthquake. Similarly, if we denote by subscript \( j \) the particular station considered then \( s_j \) is the station (crustal + seismograph) function for the \( j \)th station. Finally, if we divide the distance range into intervals over which the amplitude-distance curve is assumed constant and we denote by \( k \) the \( k \)th such interval, then \( r_k \) is the mantle transfer function for this distance range. If some estimate \( r_e \) of the amplitude-distance curve is available, then we can subtract \( r_e \) from both sides of equation (3) and \( r_k \) is considered as the actual difference of \( r \) and \( r_e \). When \( r_e \) is a reasonable approximation to \( r \), the constancy of \( r_k \) over a distance interval is a less imposing condition and yet we retain the flexibility of the histogram representation.

Thus if earthquake \( i \) is observed at station \( j \) and the separation of the two is in distance range \( k \), the observed amplitude \( a_{ijk} \) may be expressed in the form

\[
a_{ijk} = b_i + r_k + s_j + e_{ijk}
\]  

For \( N_{re} \) observations of \( a_{ijk} \) from \( N_{ep} \) epicentres at some or all of \( N_{st} \) stations using \( N_{pa} \) distance ranges, we have a set of \( N_{re} \) linear equations for \( a_{ijk} \). We have \( N_{ep} \) unknowns \( b_i \), \( N_{pa} \) unknowns \( r_k \), \( N_{st} \) unknowns \( s_j \) and \( N_{re} \) unknowns \( e_{ijk} \). If we remove from \( b_i \), \( r_k \) and \( s_j \) their respective averages so that

\[
\begin{align*}
C &= \bar{b}_i + \bar{r}_k + \bar{s}_j \\
b_i - \bar{b}_i &= b'_i \\
r_k - \bar{r}_k &= r'_k \\
s_j - \bar{s}_j &= s'_j
\end{align*}
\]  

(5)
then the new $b_i'$, $r_k'$ and $s_j'$ averaged over $i$, $k$ and $j$ respectively are zero. Hence we have $N_{re} + 3$ equations:

$N_{re}$ equations

$$a_{ijk} = C + b_i' + r_k' + s_j' + \epsilon_{ijk}$$

and 3 equations

$$\sum_i b_i' = 0, \sum_k r_k' = 0 \text{ and } \sum_j s_j' = 0$$

We can henceforth drop the primes on $b_i'$, $r_k'$ and $s_j'$.

We can represent these equations in the matrix formulation

$$P = QX + E$$

where $P$ is the row vector of $a_{ijk}$ in some order, $E$ is the error row vector of $\epsilon_{ijk}$ in the same order as $a_{ijk}$, $X$ is the column vector $(C, b_1, b_2, \ldots, b_{N_{re}}, r_1, r_2, \ldots, r_{N_{re}}, s_1, s_2, \ldots, s_{N_{re}})$ and $Q$ is the matrix of indicator variables such that if the $n^{th}$ element of $P$ is $a_{ijk}$ then the $n^{th}$ row of $Q$ multiplied by $X$ gives $C + b_i' + r_k' + s_j'$ and the last three rows of $Q$ when multiplied by $X$ give equations (6).

It is possible to solve this matrix equation by the least squares method to minimise $|E|$ and get an estimate of $X$ and hence of $C$ and of $b_i'$, $r_k'$ and $s_j'$.

The least squares estimate for $X$ is given by

$$X = (Q^TQ)^{-1}(Q^TP)$$

where $Q^T$ is the transpose of $Q$ and $(Q^TQ)^{-1}$ is the inverse of $Q^TQ$, i.e. $(Q^TQ)^{-1}(Q^TQ) = I$, the identity matrix. Problems may arise with calculation of $(Q^TQ)^{-1}(Q^TP)$ and these problems are discussed by Anderssen (1969). In the present work straightforward matrix inversion
was used to form \((Q^T q)^{-1}\) and the difference \((q^T q)^{-1} (q^T q) - I\) was used as a
guide to the accuracy of the inversion of \(Q^T q\). Since \(q\) is composed of
integer indicator variables, \(Q^T q\) can be calculated exactly and is not
affected by computational rounding errors. If there are a sufficient
number \(N_i\) of linearly independent equations (4), then a solution \(X\) of
equation (7) can be found. \(N_i + 3\) must be greater than \(1 + N_{ep} + N_{pa} + N_{st}\),
and the greater \(N_i\) the better the statistical estimate of \(X\).

**Observational material**

The stations used are those of the high quality Swedish and Finnish
networks situated on the relatively homogeneous Fennoscandian shield
(table 1). The earthquakes used occurred in the Indonesian area between
the beginning of 1963 and the end of 1968 (table 2). See also figure 1.
The particular earthquakes selected were such that the signal-to-noise
ratio was generally good, the amplitude of the signal was sufficient to
make further analysis worthwhile and the energy of the signal was
concentrated near the onset. Any selection of the data will affect the
final result as the criteria used are subjective. If, for example, a record
is rejected because of low signal-to-noise ratio - then it may be that the
noise level is high or that the amplitude level is low. However, some
selection must be made and the criteria used seem reasonable.

The stations and earthquakes are related so that for any one earth-
quake the stations cover an azimuthal range of less than \(10^\circ\) and that for
one station the earthquakes cover a back azimuthal range of less than
about \(20^\circ\) (cf figure 2). For the range \(90^\circ - 110^\circ\) epicentral distance, the
angle of approach of the seismic P-wave changes little. So for each earth-
quake the station net covers a small solid angle and for each station the
earthquake epicentres cover a small solid angle. As we shall see later, these conditions should make the joint analysis method suitable for analysing the data.

The eleven Swedish and Finnish stations originally chosen are given in table 1. Of these SOD was later rejected, because of the nonstability of its amplification curve, and UDD, which because of its later construction recorded only four of the earthquakes (two on the earlier Grenet instrument and two on the later installed Benioff).

Sixteen earthquakes were initially selected, listed in table 2. Eleven of these earthquakes lie in a narrow back azimuthal range from Scandinavia and the other five are outside this band. The latter five are treated as suspect, as the station terms may vary too much with large changes in back azimuth. Of the original 176 possible records, 109 were selected and digitised. 14 records were not available, 20 were at too short epicentral distances and 33 were rejected because the signal-to-noise ratio was too low or the record amplitude was not large enough to make Fourier analysis worthwhile. Figure 3 shows a typical record.

For each earthquake the epicentral distance, azimuth and back azimuth to the stations of the net were calculated. The epicentral distances were corrected for depth of focus using the results of Buchbinder (1968). These corrections are such that all the earthquakes can be considered as surface focus events with regard to the amplitude-distance curve.

Experimental method

For the records from each earthquake a suitable record length was chosen, either 20, 30 or 40 sec, and this length was such that the main
part of the energy was in the earlier portion of the record and at the end of the record the amplitude was much smaller or reduced to near noise level. This selection of record lengths should minimise the effects of truncation of the record. The start of the record was taken just before the apparent onset of the arriving P-wave.

The records were photographically enlarged four or five times. Then the top and bottom of the trace were digitised on a DMac pen follower and the data were converted to cards. They were then interpolated to the desired interpolation interval: \( \frac{20}{256} \) sec for 20 sec records, \( \frac{30}{512} \) sec for 30 sec records and \( \frac{40}{512} \) sec for 40 sec records. The average of the two traces was taken and the Fourier transform of the average computed in the form of amplitude and phase spectra. The theory of the spectral analysis of digitised seismic data is well covered by Huang (1966). If the seismic trace is the time function \( f(t) \), then the computed Fourier spectrum is given by \( F(v_n) \) where

\[
F(v_n) = \frac{1}{m} \sum_{k=0}^{m-1} f(k\Delta t) e^{-\frac{2\pi i n k \Delta t}{T}}
\]

(9)

\( T \) is the length of the record, \( v_n \) is the \( n^{th} \) frequency in cycles per sec and \( v_n = \frac{n}{T} \), where \( n \) runs from 0 to \( m \), \( m \) is the number of digitised points, \( \Delta t \) is the digitising interval and \( m \Delta t = T \), and finally for most efficient computation \( m \) is a power of 2 (in our case \( m = 256 \) or 512). We avoid aliasing by using a digitising interval sufficiently small so that the amplitude spectra are negligible for frequencies above the folding frequency \( \frac{1}{2\Delta t} \).

Various methods of windowing were considered but none was applied.
as none seemed suitable. Using longer record lengths with low cut-off amplitudes should minimise the effect of truncating the records.

One record of average quality was photographically enlarged and digitised separately three times. The agreement between the three amplitude spectra is very good as shown in figure k. The maximum variation throughout most of the frequency range was 0.25 units compared with the maximum amplitude of 4 units. For the larger amplitude components the difference is less than 8%. For better quality records the agreement should be better and the opposite for poorer quality records.

The amplitude spectra have been smoothed using a 3-point smoothing with \( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \) weighting for the 20 sec records, a 3-point smoothing with \( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \) weighting for the 30 sec records and a 5-point smoothing with \( \frac{1}{6}, \frac{1}{6}, \frac{1}{4}, \frac{1}{6}, \frac{1}{6} \) weighting for 40 sec records. This smoothing should improve the consistency of the results and also make the comparison of the spectra from different length records more meaningful. This smoothing is not windowing but an attempt to smooth the insignificant fluctuations in the amplitude spectra.

All the spectra obtained are divided by the instrument magnification factor at 1 sec, and so the amplification curves are normalised at 1 sec. Also the inverse of the magnification factors for the photo enlargement is applied such that the amplitude is in units of 0.1 microns and after we have taken the \( \log_{10} \) of the amplitude spectra, we add one to the results, i.e. the \( \log \) (amplitude) of the spectra is such that the amplitude is measured in 0.01 microns. PcP is always included in the pulse, and if the earthquake is shallow, pP is included in the record. If
the earthquake is deeper, pP either does not affect the record or is small and appears near the end of the record.

Computations and results

In the experimental work we find estimates of $a_{ijk}$ for various earthquakes and stations. Then we apply the analytical method of joint analysis to estimate the amplitude-distance curve and the station terms. A computer program to solve equation (7) and find $X$ in the form of equation (8) has been developed. The program calculates the station and source terms $s_j$ and $b_i$ and also the amplitude-distance curve using a histogram of $2^\circ$ intervals. See Appendix.

The results are very poor. In figure 5 we show a plot of the raw data from which is subtracted the appropriate source and station terms and the constant introduced in equation (5). Even though the station and source terms are allowed for, the scatter is very high and certainly no amplitude decrease is seen beyond $90^\circ$ - as would be expected. This behaviour seems to come from the data and not from the inversion program. So far no explanation has been found for the anomalous behaviour of the results. The data has been divided into smaller groups of earthquakes with narrower azimuthal and distance ranges but there is no substantial improvement in the results. It is possible that the model we have set up is based on invalid assumptions on the nature of the source and station functions.

As an example we present the amplitude data for 1 sec period for all stations which recorded the earthquake on the 29 July, 1968. We list the stations in order of azimuth (epicenter to station) and epicentral distance. It is obvious that no clear pattern emerges.
Again for the earthquake on the 15 July, 1965, we have the following results at 1 cycle per sec frequency. (Obviously the data is not accurate to four decimal places!)

<table>
<thead>
<tr>
<th>Station</th>
<th>Azimuth</th>
<th>( \log_{10}(\text{Amp}) ) at 1 sec</th>
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</thead>
<tbody>
<tr>
<td>KLS</td>
<td>329.1°</td>
<td>-0.1664</td>
</tr>
<tr>
<td>GOT</td>
<td>331.5</td>
<td>-0.1292</td>
</tr>
<tr>
<td>NUR</td>
<td>331.7</td>
<td>-0.1053</td>
</tr>
<tr>
<td>UPP</td>
<td>332.2</td>
<td>+0.3614</td>
</tr>
<tr>
<td>UDD</td>
<td>334.0</td>
<td>-0.4820</td>
</tr>
<tr>
<td>KJN</td>
<td>334.8</td>
<td>-0.0336</td>
</tr>
<tr>
<td>UME</td>
<td>335.5</td>
<td>-0.3136</td>
</tr>
<tr>
<td>SKA</td>
<td>336.8</td>
<td>-0.4511</td>
</tr>
<tr>
<td>KIR</td>
<td>339.3</td>
<td>+0.2392</td>
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<table>
<thead>
<tr>
<th>Station</th>
<th>Distance (reduced to zero focus)</th>
<th>( \log_{10}(\text{Amp}) ) at 1 sec</th>
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<tr>
<td>KJN</td>
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<td>-0.0336</td>
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<tr>
<td>KIR</td>
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<tr>
<td>Station</td>
<td>Distance</td>
<td>$\log_{10}$(Amp) at 1 sec</td>
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<td>----------</td>
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In the first example the back azimuths vary from 61° to 75° and in the second example from 67° to 74°. The change in back azimuths between the two examples is about 5.5° for each station.

The Fourier spectral program from seismogram to amplitude spectrum has been checked against an independent program. The spectral estimates seem therefore to be valid.

The problem remains - which of the assumptions we have made is not valid? Possible it is that the source function can vary very rapidly over very small azimuthal angles. In both the above examples the smallest and the largest amplitudes are next to each other in the distribution of azimuth. (This effect has not been checked on other data sets). If the source spectrum does vary so much with such small angles, then spectral analysis of short-period P-waves from earthquakes could only be done on a statistical basis. Explosions provide much more symmetrical sources, but their limited distribution prohibits their application to our present problem.
References


Acknowledgements

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The author is grateful to Professor Markus Bäth, Uppsala, for helpful discussions and a critical reading of this report, also to the Director of the Seismological Institute, Helsinki, Finland, for the loan of records from the Finnish stations.

The CDC 3600 computer of the Uppsala University Computer Centre was used for the calculations.
Table 1

Stations used (see also figure 1)

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<th>Station Name</th>
<th>Location</th>
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Table 2

Earthquakes used (see also figure 1)

Data from USCGS

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<th>Epicentre</th>
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<th>Magnitude (m)</th>
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<tr>
<td>26.8.1967</td>
<td>00.36.42.1</td>
<td>12.2</td>
<td>140.7</td>
<td>33</td>
</tr>
<tr>
<td>24.5.1968</td>
<td>15.43.54.2</td>
<td>-6.8</td>
<td>118.9</td>
<td>605</td>
</tr>
<tr>
<td>29.7.1968</td>
<td>23.52.15.0</td>
<td>-0.2</td>
<td>133.4</td>
<td>12</td>
</tr>
<tr>
<td>27.9.1968</td>
<td>03.58.55.1</td>
<td>-6.8</td>
<td>129.1</td>
<td>127</td>
</tr>
</tbody>
</table>
Figure captions

Fig. 1. Mercator projection of area of interest (showing Fennoscandian stations and Indonesian epicentres used).

Fig. 2. Cross-section of the earth showing diagrammatically the ray paths from Indonesia to Fennoscandia (the diagram is merely suggestive and not to scale).

Fig. 3. Short-period vertical-component P-wave recorded at Umeå from the Banda Sea earthquake of 21 March, 1964.

Fig. 4. The effect on the spectral amplitude of treating the record at Kiruna from the earthquake of 21 March, 1964, three times as a separate unit.

Fig. 5. Plot of all raw data for 1 cps with station and epicentre terms removed (shows failure of method to give the expected result and also shows large scatter).
Appendix

Program SOLVE is used to invert the system of linear equations described in the section "Analytical method". The program is not particularly efficient but is effective. Subroutine DIST 1 puts the earthquake-station pairs into their distance ranges. Subroutine SHUFFLE is used to vary the input without changing the coding of the original seismograms - earthquake 3 at station 6 has code 1316. Subroutine SELECT selects the frequencies required from the 20 frequencies of the input. Subroutine DIST 2 is here only formal but may be used to remove any prior estimate of station, distance or source terms.
PROGRAM SOLVE
CALL WORK
CALL EXIT
END

SUBROUTINE WORK

C THIS PROGRAM COULD BE MORE EFFICIENT
C MAXIMUM Dimensions 110 RECORDS, 15 FREQUENCIES AND
C NST STATIONS, NPA PARAMETERS AND NLF EPICENTERS WHERE
C NST+NPA+NLP IS LESS THAN OR EQUAL TO 49
DIMENSION A(113,50),B(113,15),DISTANCE(110),FNQ(20),ANUM(60)
DIMENSION X(151,15),AS(50,50),AT(125,20)
DIMENSION TDR(113),ATCOL(113),TITLE(6)
EQUIVALENCE(A,AT)
COMMON/10/AA(50,50)
READ 99,NST,NFR,NPA,NRE,NFR
901 FORMAT(515)
PRINT 992,NST,NFR,NPA,NRE,NFR
912 FORMAT(1X,15,F8.2) STATIONS, 15, EPICENTRES, 15, PARAMETERS,
1 15, RECORDS, 15, FREQUENCIES,*
NCOL=NST+NPA+NLP+15 NKOW=NRE+3
READ 7, (EPC(I),I=1,NFR)
1 FORMAT(15F5.2)
KST=1+NST*NPA=1+NST+NPA JST=2SJP=2+NST+JEP=2+NST+NPA
KEP=NCOL
TITLE(I)=EPC(I)
DO 555,I=1,3,6
555 TITLE(I)=PH
C PUT A(I,J)=0.0
DO 3,J=1,NROW
DO 2,I=1,NCOL
A(I,J)=0.0
3 CONTINUE
2 CONTINUE
C PUT IN VALUES OF FIRST COLUMN O A(I,J) AND BOTTOM THREE ROWS OF
C A AND B
DO 4,J=1,NOF
4 A(I,J)=1.0
NRET=NFR+1
DO 5,J=JST,KST
5 A(NRET+1)=1.0
NRET=NRET+2
DO 6,J=JEP,NCOL
6 A(NRET+1)=1.0
NRET=NRET-1
DO 9991,I=JPA,KPA
9991 A(NRET+1)=1.0
NRET=NRET+4
DO 7,J=1,NFR
ANUM(1) GIVES THE NUMBER OF NON-ZERO ELEMENTS IN EACH COLUMN OF A
PRINT 12
12 FORMAT(4X,'INPUT DATA')
PRINT 14
14 FORMAT(* *)
PRINT 15,'(FREQ(I),I=1,NFR)
15 FORMAT(9X,'FREQUENCY *,15F7.2)
PRINT 16
16 FORMAT(1X,'PST DISTANCE')
DO 9,1=2,NCOL
9 ANUM(1)=.0
ANUM(1)=FLOAT(NRF)

READ IN DATA
DO 100,1=1,NRF
READ 10,MEPIC,MSTAT,DISTA
10 FORMAT(1X,212,F6,1)
DISTANCE(I)=DISTA
MEPIC=MEPIC-10 SMSTAT=MSTAT-1
MNUM=MSTAT
CALL SHUFFLE(MEPIC,NFR,MSTAT,NST)
M=MSTAT+1 %MEPIC+KPA
A(I,M)=1.0
ANUM(4)=ANUM(4)+1.0
ANUM(N)=ANUM(N)+1.0
CALL DIST1(DISTA,NPA,KDIST)
KA=KDIST+1+NST
DIR(I)=KA
A(I,KA)=1.0
ANUM(KA)=ANUM(KA)+1.0
READ 23,(FRO(K),K=1,20)
23 FORMAT(9F12.4)
CALL SELECT(FRO,NFR)
FRO IS HERP THE INPUT AMPLITUDE DATA
PRINT 24,MEPIC,MSTAT,DISTA,(FRO(K),K=1,NFR)
24 FORMAT(1X,215,FR,,1.1F10.6)
Z=DISTANCF(I)
CALL DIST2(FRO,Z,NFR,MSTAT,NST)
DO 9981,1=1,NFR
9981 B(I,K)=FRO(K)
100 CONTINUE
HAVE HOW FED DATA INTO B AND PARAMETERS INTO A, AND SUBTRACTED
FITTED CURVE AND INST EFFECTS FROM B
DO 1000 I = 1, NCOL
PRINT 1001, (A(I,J), J = 1, NCOL)
1001 CONTINUE
DO 1002 I = 1, NROW
PRINT 1004, (B(I,J), J = 1, NFP)
1005 CONTINUE
DO 1006 I = 1, 15 = 8.3
PRINT 14
C       WF NOW FORM ATA AND ATB DATA IS IN AA AND ATB IS IN X
DO 105 J = 1, NCOL
AA(I,J) = 0.0
DO 105 K = 1, NROW
AA(I,J) = AA(I,J) + A(K,J) # A(K,J)
106 CONTINUE
AA(I,J) = AA(I,J)
DO 110 J = 1, NCOL
DO 110 K = 1, NFP
X(I,J) = X(I,J)
112 CONTINUE
110 CONTINUE
PRINT 1010
1010 FORMAT (*.  MATIX ATA*)
DO 1011 I = 1, NCOL
PRINT 1002, (AA(I,J), J = 1, NCOL)
1011 CONTINUE
DETERM = 0.0
NMAX = 0
CALL MATINV(NCOL, X, NFR, DETERM, NMAX)
PRINT 3992, DETERM
DO 1002 I = 1, NCOL
PRINT 2001, (AA(I,J), J = 1, NCOL)
1002 CONTINUE
DETERM = E12*4
CDE = 0.0
DO 2001 J = 1, NCOL
DO 2002 I = 1, NCOL
VAP = 0.0
2001 CONTINUE
NOT REPRODUCIBLE
DO 2037, K=1, NCOL
2037 VAR=VAR+AA(I,K)*AS(K,J)
IF(I.EQ.J) VAR=VAR*1.0
VAR=VAR*VAR
DO 2002 IF(VAR.GT.0.1) GRE=VAR
2001 CONTINUE
PRINT 14
GRE=SORTE(GRE)
PRINT 2004, GRE
20 5 FORMAT(* GRATEST ERROR IN PRODUCT OF MATRIX AND ITS INVERSE = *),
1E12.4/
DO 120 I=1, NROW
DO 121 J=1, NFR
120 CONTINUE
DO 122 J=1, NCOL
122 DUM=DUM+AT(I,J)*X(K,J)
AT(I,J)=DUM-B(I,J)
B(I,J)=AT(I,J)*AT(I,J)-AT(I,J)
121 CONTINUE
120 CONTINUE
DO 550 I=1, NFR
DO 551 J=1, NFR
550 ATCOL(J)=AT(I,J)
551 ITIT=(2)*AH
CALL :NCODE(ITITLE)
CALL FMTS(R)
CALL FMTI(J,1)
CALL GRAPH1(DISTANCE,ATCOL,-NRE,3H7X8,4HAUTO,ITITLE,10HDISTANCE,: ,
15HAMP,,52606060606060606060606)
550 CONTINUE
C X CONTAINS THE SOLUTIONS AND B CONTAINS THE ERROR SQUARED
C WE FIND NOW THE STANDARD DEVIATIONS
SOAN1=SORTE(ANUM(1)-1.0)
SOAN2=SORTE(ANUM(1)-1.0-1.0)
ANNN=ANUM(1)-1.0*(NPA+NST+NEP+1)
ANNN=MAX1F(ANNN*1.0)
SOAN3=SORTE(ANNN)
DO 1510 I=1, NFR
AA(I,1)=0.0
DO 151 J=1, NFR
151 AA(I,J)=AA(I,1)*B(J,1)
DUM=SORTE(AA(I,1))
AA(I,J)=DUM/SOAN1
AA(I,J)=DUM/SOAN2
AA(I,J)=DUM/SOAN3
1510 CONTINUE
C EXCEPT FOR FIRST COL ANUM(I) IS NO IN COL MINUS ONE
DO 150 I=1, NCOL
K=J+3
L=J+1
DO 150, I=1, NFR
AA(I,K)=0
DO 161 I=1,M
DO 160 J=1,N
161 AA(I,K)=AA(I,K)+B(M,J)*A(M,L)
160 AA(I,K)=SORTF(AA(I,K)/ANUM(L))

50 CONTINUE
PRINT 14
PRINT 200
20 FORMAT(* RESULTS*)
PRINT 201
201 FORMAT(1X,16FR,4)
DO 204 J=1,3
PRINT 202,((X(I,J),AA(I,J),I=1,NFR)
204 CONTINUE
PRINT 14
PRINT 205
205 FORMAT(1X,#STATIONS*)
206 FORMAT(1X,#STATIONS*)
DO 206 I=1,2,KST
J=I-1 $L=I+2
PRINT 207,J
PRINT 202,((X(I,K),AA(K,L),K=1,NFR)
206 CONTINUE
PRINT 14
PRINT 206
208 FORMAT(1X,#STATIONS*)
DO 208 I=JDA,KDA
J=I-1-NST $L=I+2
PRINT 207,J
PRINT 202,((X(I,K),AA(K,L),K=1,NFR)
208 CONTINUE
PRINT 14
PRINT 211
211 FORMAT(1X,#CENTRES*)
DO 211 I=JFP,KFP
J=I-1-NST-NPA $L=I+2
PRINT 207,J
PRINT 202,((X(I,K),AA(K,L),K=1,NFR)
211 CONTINUE
PRINT 212
212 CONTINUE
OPTION
END

SUBROUTINE DIST1(DISTA,NPA,KDIST)
 X IS DISTANCE TO RIGHT OF FIRST INTERVAL,N IS NUMBER OF FIRST LONG
 INTERVAL,Y IS THE DISTANCE BETWEEN THE TOP OF FIRST AND BOTTOM
 1 OF FIRST LONG INTERVAL, W IS WIDTH OF SHORT INTERVAL, LONG IS 5.
 X=101.0 $N=4 $Y=4.0 $W=2.0
 Z=DISTA-X
 IF(Z.LT.0.0)Z=0.125
IF(Z GT Y) GO TO 1
7 = Z/W
KDIST = INTF(7) + 1
RETURN

1 Z = (Z - Y) / 5.0
KDIST = INTF(7) + N
RETURN
END

SUBROUTINE DIST2(FREQ, NFR, MSTAT, NST)
DIMENSION FREQ(1)
GO TO 2

CONTINUE
RETURN
END

SUBROUTINE SHUFFLE(MEPIC, NEP, MSTAT, NST)
DIMENSION NUTT(16), NURT(11)
SORTS EPICENTRES AND STATIONS INTO NUMBERD ORDER
DATA(NUTT(I), I = 1, 16) = 7, 0, 1, 1, 0, 2, 3, 0, 5, 0, 4
((NURT(I), I = 1, 11) = 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 10)
MEPIC = NUTT(MEPIC)
MSTAT = NURT(MSTAT)
RETURN
END

SUBROUTINE SELECT(FREQ, NFR)

DIMENSION FREQ(1), FL(20)
SELECTS THE FREQUENCIES REQUIRED FROM THE 20 READ
DATA(NUM(I), I = 1, 10) = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
DO 1 I = 1, 20
1 FREQ(I) = FREQ(I)
DO 2 I = 1, NFR
2 FREQ(I) = FL(NUM(I))
RETURN
END

LOAD,
(1, 12)
FREQUENCY-DEPENDENT AMPLITUDE-DISTANCE CURVE FOR P-WAVES FROM 87° TO 110°

This work is an attempt to clarify the nature of the amplitude-distance curve for P-waves between 87° and 110°, using the spectral amplitudes of earthquakes in the Indonesian region recorded at the Swedish and Finnish seismograph stations. At the present time the results are inconclusive, because even after allowing for station and source terms there is a large unexpected scatter.

14. Key Words

Indonesian earthquakes
Fennoscandian stations
Diffracted P-waves
Spectra, spectral analysis
Amplitude-distance curve
Computer program SOLVE