A STUDY OF THE ARMY HOT DAY DESIGN HOVER CRITERION

By Robert Bellaire and Lt. William Bousman

AUGUST 1970

DIRECTORATE OF FLIGHT STANDARDS AND QUALIFICATION
US ARMY AVIATION SYSTEMS COMMAND

ST. LOUIS, MISSOURI

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PREFACE

Under AMC Regulation No. 70-32, the Directorate of Flight Standards and Qualification, US Army Aviation Systems Command is responsible for the development, promulgation and application of Aeronautical Design Standards (ADS) for US Army Aircraft Systems. As part of this responsibility, a series of Aeronautical Design Standards Technical Notes will be published to provide substantiation for proposed standards, revisions and related studies. This report constitutes a portion of this series.

CHARLES C. CRAWFORD, JR.
Director of Flight Standards and Qualification
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SUMMARY

This study was conducted in order to determine an optimum or near-optimum design hover criterion for Army helicopters. Performance and relative cost were determined as functions of design altitude and temperature, and then evaluated with respect to a world environment as represented by nineteen countries/regions contiguous or nearly so to the Soviet-Sino block.

The mission requirement established for the analytical rotorcraft was to hover out of ground effect (OGE) with a fixed useful load. The effectiveness of the rotorcraft was measured by the probability that it could hover OGE with its useful load within the world environment.

It was determined that instead of there being one optimum design point criterion for all rotorcraft, there is a range of design points. This range of design points from about 5500 feet/95 °F to about 6500 feet/95 °F represents an uncertainty in predicting performance degrading factors that the rotorcraft can be expected to meet in its operational environment and the exact behavior of a particular design's empty weight as a function of design altitude and temperature. The best point for a design criterion within the range is a matter of judgment. For this reason, there is no justification for changing the present criterion of 6,000 feet, 95 °F. However, there is a need for a maneuverability requirement at the hover design point and therefore it is recommended that the Army Hot Day Design Hover Criterion be:

The rotorcraft shall be capable of hovering out-of-ground effect (OGE) at its design or primary mission gross weight with pressure altitude and temperature conditions of 6,000 feet, 95 °F using normal rated power. At this hover ceiling the rotorcraft must also achieve following a full directional control input a yaw angular displacement in either direction from trim in one second of $330/[GW + 1000]^{1/3}$ degrees, where GW is the gross weight in pounds.
INTRODUCTION

The out-of-ground effect (OGE) design hover criterion of 6,000 feet, 95 °F has been used in the procurement of Army helicopters with only a few exceptions since the mid-50's. The lack of a substantive basis for this design point has resulted in questioning as to the need for such a standard. In recent years, this questioning has resulted in a lowering of the standard to 5,000 feet, 90 °F in the case of the Advanced Aerial Fire Support System (AAFSS) procurement and a standard of 4,000 feet, 95 °F in the draft Qualitative Materiel Requirements (QMR) and Qualitative Materiel Design Objectives (QMDO) of a number of proposed systems.

The confusion surrounding the choice of a design point, and the need to understand the interrelationships of aircraft performance, environment and cost have made it necessary that the design point criterion be based upon an understanding of all the factors that affect aircraft procurement and operation. It is the purpose of this study to investigate these interrelationships and from them to determine if practical, an optimum or near-optimum design hover criterion.

This study is limited in scope to the discussion of hovering aircraft that derive their thrust from a rotor system or the equivalent, whose disk loading is on the order of 15 lb/ft² or less. No attempt has been made to extrapolate the study to represent a technology ten to twenty years away; rather it is the intent of the study to generalize current technology. However, it is felt that many features of the study will be applicable to other types of VTOL aircraft and future technologies.

Previous investigations into the factors affecting a hover criterion have been somewhat limited. Dodd (reference 1) at the US Army Natick Laboratories has gathered temperature data for elevations between 3,000 and 7,000 feet from various parts of the world in order to show the frequency of occurrence of density altitudes in excess of 6,000 feet, 95 °F. This presentation of the regions of the world with high density altitudes offers a quick guide to those areas that could someday create operational difficulties, although discussion of a constant density altitude is not directly related to a helicopter's hover capability unless its power output to the rotor is limited by the transmission rating at that density altitude.

The Boeing Company-Vertol Division (reference 2) has also examined the hover design criterion. They selected temperature data from various "trouble spots" throughout the world and examined the capability of an aircraft to hover upon encountering the mean daily maximum temperature of the hottest month. They concluded that the 6,000 feet, 95 °F design hover criterion would allow a reasonable expectation of operation for the "trouble spots" they had selected.
The problem of establishing a design hover criterion was approached indirectly by the Combat Operations Research Group (CORG) at Fort Belvoir, Virginia in separate studies related to UH-1D horsepower requirements and the Utility/Tactical Transport Aerial System (UTTAS) (references 3 and 4). In these studies, they examined aircraft hover capability relative to the environment of fifteen countries/regions. The important feature of this examination was that they described the environment in probabilistic terms. They concluded that based on a distribution of mean daytime temperature a design point of 4,000 feet, 95 °F would result in a probability of OGE hover of 0.95 for the countries/regions examined.

It should be noted that all these studies have related the design hover criterion to some representation of a world environment. This approach unfortunately ignores other features that will affect operation in the world environment such as in-service weight growth, power degradation, maneuvering requirements, etc. The need to consider these other features is one of the major reasons for the present study.

Initial work on this study was performed as part of the US Army's Aeronautical Design Standards (ADS) program under contract DAA-J01-68-C-0157(31) with Aerostructures, Inc. of Menlo Park, California, and is reported in references 5 and 6. The additional work that constitutes this report is dependent upon the original Aerostructures' study for its approach and method of analysis.
SYMBOLS

\( A_B \)  blade area, \( \text{ft}^2 \)

\( A_e \)  effective disk area, \( \text{ft}^2 \)

\( A_v \)  vertical gap area between tandem rotors, \( \text{ft}^2 \)

\( A_i \)  aircraft equivalent flat plate area, \( \text{ft}^2 \)

\( a \)  airfoil section lift-curve slope, \( \text{rad}^{-1} \)

\( B \)  tip loss correction factor

\( C \)  cost

\( C_{LR} \)  mean blade lift coefficient

\( C_p \)  power coefficient

\( C_T \)  thrust coefficient

\( C_T^* \)  dummy thrust coefficient to account for ICE hover

\( \text{conhp} \)  control power, HP

\( D_{\phi} \)  yaw damping, slug-\( \text{ft}^2/\text{sec} \)

\( \text{ERP} \)  engine rated power, HP

\( F_{vd} \)  fuselage downwash drag factor

\( GW \)  gross weight, lb.

\( H_p \)  pressure altitude, ft.

\( \text{hp}_{\text{acc}} \)  accessory power, HP

\( \text{hp}_c \)  engine cooling power, HP

\( \text{hp}_{\text{gen}} \)  generator power, HP

\( I_R \)  rotor inertia, slug - \( \text{ft}^2 \)

\( I_x \)  yaw moment of inertia, slug - \( \text{ft}^2 \)

\( \text{ihp} \)  induced power, HP
K_u  forward flight induced velocity correction factor
K_p  forward flight profile power correction factor
k_e  normalized power available
k_e  engine installation loss correction
k(t)  component weight group cost/lb., dollars/lb.
\ell_{tr}  distance between main rotor and tail rotor shafts, ft.
MRP  Military Rated Power, HP
NRP  Normal Rated Power, HP
n  number of rotors
P(H)  probability of hover
php  parasite drag power, HP
R  rotor radius, ft.
RALI  Relative Autorotational Landing Index
\bar{php}  profile drag power, HP
SHP  shaft horsepower, HP
SHP_a  available power, HP
SHP_r  power required, HP
T_{tr}  tail rotor thrust, lb.
t  temperature, °F
Q_{lim}  transmission limit as a fraction of difference between
design point power required and engine rated power
available at sea level, standard day
TRP  transmission rated power, HP
u_H  induced velocity, ft/sec
V  velocity, ft/sec
\( V_t \)  
rotor tip speed, ft/sec

\( V_T \)  
true air speed, knots

\( W \)  
component group weight, lb.

\( W_B \)  
blade weight, lb.

\( W_E \)  
an aircraft weight empty, lb.

\( W_U \)  
useful load, lb.

\( w \)  
disk loading, lb/ft²

\( x_g \)  
fractional vertical gap for tandem rotors

\( x_o \)  
fractional rotor overlap for tandem rotors

\( Z/D \)  
ratio of height of rotor hub above ground to rotor diameter

\( \gamma_1 \)  
Lock number based on blade inertia about rotational axis

\( \delta \)  
rotor drag coefficient

\( \eta \)  
transmission mechanical efficiency

\( \mu \)  
advance ratio

\( \rho \)  
density, slugs/ft³

\( \sigma \)  
solidity ratio

\( \psi \)  
yaw angle, rad

\( \Omega \)  
rotor angular velocity, rad/sec

Subscripts:

\( \text{tr} \)  
tail rotor

\( \text{o} \)  
sea level, standard day
DISCUSSION

Method of Analysis

The power required by the rotor system of a helicopter is density dependent (ignoring Mach number effects) and therefore is a function of pressure and temperature only as they affect density. The power available from the engine, however, is a function of pressure and temperature separately (except for the case when the engine power is limited by a transmission), and therefore, discussion relating to the development of a design hover criterion must concern itself with both the design pressure altitude and the design temperature (collectively referred to as the design point).

In order to establish a design hover criterion, it is necessary to relate a rotorcraft's performance and cost to its expected operational environment. This requires that these various factors and their interrelationships be represented by some form of analytical model. The way in which this has been done is illustrated below.
As can be seen, design pressure altitude and temperature are treated as independent input variables in order to determine their effect upon aircraft effectiveness and cost. The development of the various parts of this model are the subject of the subsequent sections.

Performance Model

The effect of an increase in the altitude or temperature used for the design of a particular aircraft is to increase size, installed power, performance, empty weight and cost. In order to select an optimum or near-optimum design point for rotorcraft, it is necessary to evaluate the performance at the design point relative to the Army mission as well as the price that is paid to achieve that performance.

**Primary measure of performance.** - For the purposes of this study, the prime measure of an aircraft's performance will be its capability to hover out-of-ground effect (OGE) at its design gross weight. In this study design gross weight and primary mission gross weight are considered synonymous. This capability is expressed by the aircraft's OGE hover ceiling which can be considered a locus of altitude and temperature points which represent the environment in which the aircraft can hover. This is illustrated below.

![OGE Hover Ceiling Diagram](image)
For any one design, temperatures and altitudes to the left of the hover ceiling represent possible environments in which the aircraft can hover at its design gross weight at a reduced power setting while temperatures and altitudes to the right represent environments which are not within the capabilities of the aircraft unless it uses a higher power setting, reduces its useful load or employs ground effect. In this sense, the hover ceiling represents the boundary of hover capability. The hover ceilings for five parametric aircraft are presented in Figure 1. As can be seen, an increase in the design point increases the environmental capability of the aircraft.

The hover ceiling as described above is in fact the simultaneous solution of power required (as a function of density) and power available (as a function of temperature and pressure). For the special case where power available is limited by the transmission, the aircraft will hover at the density that corresponds to the transmission limited power. In this case, the hover ceiling is simply a line of constant density altitude. This feature is shown below.
It is important to note that there are certain assumptions implicit in the use of the OGE hover ceiling as the prime measure of performance. These are summarized below:

(1) The requirement that the hover ceiling reflect constant gross weight means that there can be no trade-off in useful load to gain increased hover performance. The inability to hover with full useful load is considered a mission failure.

(2) It is recognized that an increased useful load to empty weight ratio will result by designing an aircraft to hover IGE, however, the particular usefulness of a rotorcraft is just in those areas where the ground effect is reduced or non-existent, i.e. in jungle, tall grass, hilly, rocky terrain, etc. In addition, the OGE requirement provides a power margin to allow take-off from an IGE hover or landing in a restricted area under more severe conditions.

(3) It is assumed that the capability to hover is the major influencing design requirement. If a particular aircraft's capability to hover is only of secondary importance to its mission, then discussion of the design hover point may become irrelevant.

It has not been the intent of the above discussion to negate the importance of other aircraft characteristics and performance capabilities. Many of these, however, are not explicit functions of the design point under question. For instance, to insure good handling qualities will be a problem regardless of design point.

Range was determined to be only a weak function of the design point by Randall, et. al. in reference 6 who investigated aircraft cruise characteristics for design points from sea level, 95°F to 8,000 feet, 95°F.

Parametric aircraft model. - In order to determine an aircraft's performance and physical characteristics as a function of the design altitude and temperature, it is necessary to have an analytical representation of the aircraft. For this purpose, a generalized parametric model was developed. No specific mission requirement was placed upon the parametric model in order to keep it as general as possible; rather possible missions were lumped into a useful load requirement. Three useful load classes were considered as defined in the table below.

<table>
<thead>
<tr>
<th>Type Aircraft</th>
<th>Useful Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation</td>
<td>1000</td>
</tr>
<tr>
<td>Utility</td>
<td>5000</td>
</tr>
<tr>
<td>Medium Cargo</td>
<td>15000</td>
</tr>
</tbody>
</table>
Each parametric aircraft was defined by the simultaneous solution of four non-linear equations relating gross weight, power available, power required and disk loading. Parameters, such as mean blade lift coefficient, rotor tip speed, etc. were selected to be representative of current aircraft. The effect of engine flat rating through the use of a transmission limit was examined where the transmission rating was somewhere between the power required at the design altitude and temperature and the engine power available at sea level, standard day. Details of the parametric model are included in Appendix A.

Climatology Model

As long as the prime function of a rotorcraft is to hover then the measure of an aircraft's performance will be its hover ceiling. To evaluate this hover ceiling requires a climatology model that will be representative of expected environments.

It is important in any discussion of a climatology model to realize that the world is essentially probabilistic in nature. Thus, in developing a model that is representative of the world environment, it is necessary to speak of expected environments rather than possible environments. What is meant by this distinction is that an extremely severe environment although possible may be highly improbable when compared to all possible environments. To give this one possible, severe environment an importance simply because it is severe, without considering its likelihood of occurrence is unreasonable.

Problems involved in modeling climatology. - A number of problems that are inherent in the development of a model are treated in the following paragraphs. Each of these problems must be rationalized in one fashion or another in order to make a climatology model credible.

Knowledge of world climatology is imperfect. Meteorological observations have been widely made in areas of high population density and hence an adequate statistical basis exists in these areas. However, in areas of low population density much less data is available and for this reason the confidence level in the data is reduced. There is no way to avoid the present imperfections in available climatology data. In time, additional meteorological observation and improved techniques and facilities will broaden the statistical basis of any climatology model. For the present, it must be recognized that for some areas of low or zero population density the temperature distribution at any elevation is based on limited observation.

To develop a model representative of expected environments requires a rational selection and employment of countries and regions as a statistical base. In selecting specific countries for this base, it is both impractical and unrealistic to consider the entire world. Impractical simply because of the size of the problem, unrealistic because many of the countries of the world cannot
be considered expected environments under current strategic thinking. References 3 and 4 developed climatology data for fifteen countries/regions (broken down to nineteen countries/regions in this study) that are contiguous or nearly so to the Soviet/Sino block and hence, can be considered expected environments. These countries represent 25% of the European-Asian land mass. Their choice is continued in this study in part due to its obvious convenience, although it should be recognized that the rationale used in this selection has some merit.

It is also necessary to speak of expected environments in the discussion of terrain distribution within countries or regions. If an algorithm could be formulated that would assign a probability of operation to values of elevation, distance from a hostile border, population density, vegetation, terrain roughness or some other parameter that could affect rotorcraft operation or usefulness, then this probability of operation could be used to give greater weight to some portions of a country, i.e. those areas that could be considered environments with the greatest expectation. The desirability of such an algorithm is clear. The problem with this approach is the difficulty in quantitizing such factors as population density, terrain roughness, etc. so that they can be used to define probabilities of operation. Even if such a quantitative approach were feasible, changes that will occur in air mobile doctrine in the future might destroy the algorithm's validity.

In examining temperature distributions, a major problem exists in that temperature fluctuate on a number of periodic scales, i.e. daily, annually, etc. Hence, a totally suitable temperature distribution in a model is difficult to achieve. It is not necessary to include long period time scales (in excess of a year) in the climatology model as their temperature fluctuations are either too slight to cause an effect or their period is so long as to be beyond the scope of this study. However, to include both annual and diurnal time scales requires that each be treated as a separate dimension. The complexity involved in going from a two-dimensional model (elevation and annual temperature variation) to a three-dimensional model (the diurnal temperature variation added) is considered too great for the purposes of this study. Instead the maximum daily temperature will be used in lieu of a diurnal variation.

Model assumptions. - The assumptions that have been made relative to the development of a climatology model are stated below.

(1) The world environment may be reasonably represented by a limited number of countries, in this case nineteen countries/regions contiguous or nearly so to the Soviet-Sino block.

(2) There is an equal likelihood of operating anywhere within the selected countries at any time of the year.

(3) A distribution of maximum daily temperatures encountered over a year will provide an adequate model of temperature variation.
Climatology model methodology. - The methodology used in this study is that developed and reported in reference 3. This methodology may be understood by reference to the accompanying figures which apply to an imaginary country.

The left-hand figure shows the distribution function of terrain for this country. This is obtained from topographical information and assumes that the pressure altitude ($H_p$) and elevation are equivalent. The right-hand figure illustrates the distribution function of temperature at various elevations for the country. These two figures may be conveniently combined as illustrated on the next page where the ordinate is a linear plot of altitude probability (with the corresponding elevations) while the abscissa is a linear plot of temperature probability.

The value of this form of representation rests upon the linear probability scales for altitude and temperature. For instance, if the 2000 foot elevation line is followed to the 80 °F line and then this line is followed to the sea level elevation, approximately one fourth of the plot has been enclosed. If the assumption is made that the probability of being anywhere in the country at...
any time is the same, then the shaded area is the probability that an altitude less than 2,000 feet and a temperature less than 80 °F will be encountered. In this case, the probability is approximately 0.25. A formal explanation of why this representation results in the probability of occurrence for any set of altitude and temperature conditions is presented in Appendix B.

The example above sought to establish the probability of occurrence of altitudes at or below 2,000 feet and temperatures at or below 80 °F. Any set of adequately defined altitude/temperature conditions will also result in a probability of occurrence. In this way, the climatology model is used to evaluate an aircraft's performance as determined by its hover ceiling. The hover ceilings for the parametric aircraft plotted in figure 1 have been replotted on the climatology model (in this case Afghanistan) in figure 2. The elevations and temperatures below any one hover ceiling represent an environment in which the aircraft can hover. The area beneath the hover ceiling is the probability that the aircraft is able to hover in this country. To determine the aircraft's probability of hover, \( P(H) \), in more than one country, it is necessary to weight this probability with respect to each country's land area. In this way, an aircraft's \( P(H) \) relative to the world is established and its behavior as a function of design point can be determined.

Two temperature distributions were used in this study - one based upon mean daytime temperatures (references 3 and 4) and the other based upon the mean daily maximum temperatures (reference 7). The latter data is included in this report in Appendix B. Initially, the distribution of mean daytime temperatures was used as a climatology base. It soon became apparent, however, that this created a significant uncertainty in discussing the probability of hover, in that the amount of the day in which the temperature exceeded the daytime mean was unknown. For this reason, this data was used only to determine the general functional behavior of the hover probability. In developing the hover criterion, the distribution of mean daily maximum temperatures was used as a base.

Cost Model

An increase in design point will increase empty weight and hence cost. To determine what this increase will be requires that cost be determined as a function of empty weight. In general terms, cost can be expressed as:

\[
C = \sum_{i=1}^{n} K(t_i) W_i
\]
where \( k(t) \) is a cost factor that describes the cost per pound for the \( i \)th component weight group and may be a function of the type aircraft; \( W_i \) is the weight of \( i \)th component group and \( n \) is the number of groups used in the cost model.

Two cost models were used in this study. The first was based upon a UH-1D cost proposal which treated four weight groups - rotor group, propulsion group (less engine), engine group and the remainder. The second cost model assumed that cost would be a linear function of empty weight, i.e.

\[
C = K \cdot W_E \sim \sum_{i=1}^{n} K(t) W_i
\]

A discussion of these cost models and their validity relative to the determination of a hover criterion is included in the next section.

**Criterion Determination**

As has been discussed in the previous sections, the prime measure of a rotorcraft's performance is its hover ceiling, which represents the boundary of the environment in which the aircraft can operate. This boundary is a function of the altitude and temperature to which the aircraft has been designed; as the design point is raised the boundary encompasses a larger environment. By representing the environment in a probabilistic fashion, it is then possible to evaluate the increased capability that results from an increase in design point.

**Optimum rotorcraft probability of hover.** - At the time of its design a rotorcraft can be considered to be optimum to the extent that it meets all of its design objectives. The capability of this rotorcraft to hover in the world environment is measured by its probability of hover, \( P(H) \). This is the probability that an aircraft will hover in the world environment as represented by nineteen countries/regions assuming that there is an equally likely chance of being anywhere in this environment at any time. In this sense, the probability of hover provides a measure of effectiveness. The behavior of the optimum aircraft hover probability as a function of design altitude and temperature is shown in figure 3 for a 5,000 lb. useful load, Military Rated Power (MRP) design aircraft. This figure illustrates that there are equivalent design points i.e. altitudes and temperatures that result in equivalent performance and therefore give the same hover probability. The most significant feature of this curve is the rapid rise in \( P(H) \) with an increase in design altitude until between probabilities of 0.80
and 0.90 the slope of the function drops off and approaches zero as \( P(H) \) approaches one. What this means is that aircraft that are not designed to the more restrictive criteria will suffer a larger loss in probability for a fixed performance degradation because of the steeper slope of the probability function at the lower design points. This is shown graphically in figure 4 which is a plot of the slope of the probability function \( \text{versus} \) design pressure altitude. A more physical interpretation of the figure is indicated by the right hand ordinate which shows the \( \Delta P(H) \) loss that would occur for a performance degradation equivalent to a 250 foot drop in design pressure altitude (1 to 2% installed power loss). As can be seen, the loss in capability encountered by the aircraft designed to lower criteria is more severe than that encountered by the aircraft designed to a higher standard.

To gain some insight into the behavior of the aircraft probability of hover as a function of rotorcraft parameters, the mean blade lift coefficient, rotor tip speed and the Relative Autorotational Landing Index (RALI) were varied around their selected values. To the extent of the variation examined which was \( \pm 10\% \) for mean blade lift coefficient and RALI and \( +10\% \) for tip speed, there was no effect on the aircraft hover ceiling and hence the \( P(H) \) for aircraft designed to the same altitude and temperature conditions. These aircraft did exhibit small changes in weight and power from one design to another, but for the most part these changes did not exceed \( \pm 5\% \). These results indicate that these parameters can be optimized to provide other characteristics or performance capabilities without changing the form of the hover ceiling and thereby compromising the design's probability of hover.

The behavior of \( P(H) \) as a function of design point was also investigated as to the effect of the type and weight class of the rotorcraft. Hover probabilities similar to figure 3 were generated for five aircraft as tabulated below:

<table>
<thead>
<tr>
<th>ROTOR TYPE</th>
<th>NUMBER OF LIFTING ROTORS</th>
<th>USEFUL LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>teetering</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>articulated</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>teetering</td>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>articulated</td>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>articulated</td>
<td>2</td>
<td>15000</td>
</tr>
</tbody>
</table>
At identical design altitudes and temperature, variation in the probability of hover was slight, being on the order of 2% for the less severe design points and negligible at the higher altitudes and temperatures. From this it was concluded that for rotorcraft, weight class and rotor type and number have only a limited effect upon the hover ceiling.

The requirement that an aircraft exhibit maneuverability at its hover ceiling has no effect upon its performance if it is both designed and evaluated with the maneuverability requirement as a portion of its power requirement. However, if the aircraft is designed solely for hover, the addition of a maneuver requirement at the hover ceiling severely degrades its performance.

The effect of a transmission limit on an aircraft's hover performance, can be understood by examining the figure below.

When transmission limited, an aircraft's hover ceiling follows a line of constant density as indicated by the dashed line. As can be seen, the effect of the transmission limit will be to reduce the environment in which the aircraft can operate. However, for aircraft whose transmission limit exceeds the power required at the design point (as shown in the figure) this effect is negligible relative to world climatology. This was demonstrated by calculating the probability of hover for aircraft designed with values of Qlim (see Appendix A) ranging from 0.2 to 1.0. For all cases, the P(H) values were the same for the aircraft designed to the same altitude and temperature. However, aircraft weight and power increased rapidly as Qlim approached unity (no flat rating). Unless an alternate mission exists for an aircraft to carry a large overload gross weight at sea level, the design should be transmission limited to save weight.
Performance degradation. - Up until now this discussion has been limited to factors that have only a limited effect on an optimum aircraft's hover ceiling. If an adequate design criterion is to be obtained, then it is also necessary to understand those factors that will degrade performance to the extent that an aircraft can no longer be considered optimum.

In general, these factors will fall into the categories of power loss (due to aging, dust and sand injection, etc.), power transmission inefficiency (blade erosion, etc.) and in-service weight growth (new avionics, increased mission essential equipment, etc.). The specific cause of any of these performance degrading factors is not within the scope of this study; for simplicity they will all be grouped into two terms - one for installed power loss and the other for weight growth.

Performance as measured by the hover ceiling exhibits a complex behavior as a function of the degrading factors. This can be seen by examining the figure below.

The simplest case would be a performance degrading factor shifting the hover ceiling a constant $\Delta t$ from the original position. This is illustrated by the shift of the dash-dot line (representing the degraded hover ceiling) from the solid line (the original hover ceiling). However, this behavior is more complicated in most cases since the $\Delta t$ shift varies with altitude. This is shown by the dotted line. The somewhat irregular behavior in the hover ceiling characteristics is due to the effect of the performance degrading factors on power required to hover and power available. Why this affects the hover ceiling can be understood by examining the figure on the next page.
The functions illustrated in this figure are power required (SHP,) and power available (SHPa), plotted as a function of temperature for two pressure altitudes. The powers have been normalized relative to the engine rating at sea level, standard day to allow comparison of different aircraft. As temperature increases at any altitude, the power required to hover also increases while the power available from the engine decreases. For some temperature the power required and the power available are the same and this then is a solution point on the hover ceiling curve. The figure above presents two such solution points.

A shift in either the power required or power available characteristics will shift the temperature of the solution point, and hence the hover ceiling. If this shift is caused by a loss in installed power then this will behave as a fixed percentage drop in the power available as illustrated in the top figure on the next page. The shift in the hover ceiling will then depend upon both the power loss and the slopes of the power available and power required curves. As can be seen, the shift in the power available characteristic has also shifted the hover ceiling characteristic and thereby reduced the aircraft's hover capability. Similarly, an increase in aircraft empty weight will cause a shift in the power required curves upward as shown in the second figure on the next page. The change in the hover ceiling characteristic will depend upon the weight increase and also the slopes of the power available and power required characteristics. Again this shift has decreased the aircraft's hover capability.
HOVER CEILING SOLUTION
AS AFFECTED BY LOSS OF AVAILABLE POWER

\[ H_p = 10,000 \text{ ft.} \]

\[ H_p = 0 \text{ ft.} \]

NOTES——
- DESIGN SOLUTION POINT
- \( \Delta \) OFF-DESIGN SOLUTION POINT
- \( \Delta P \) = LOSS IN AVAILABLE POWER AT SEA LEVEL, STD.

TEMPERATURE, \( ^\circ F \)

EFFECT OF INCREASE IN GROSS WEIGHT

\[ H_p = 10,000 \text{ ft.} \]

\[ H_p = 0 \text{ ft.} \]

NOTES——
- DESIGN SOLUTION POINT
- \( \Delta \) OFF-DESIGN SOLUTION POINT
Previous discussion of the optimum designed aircraft concluded that the hover ceiling characteristic was insensitive to design factors other than altitude and temperature. Once an aircraft has been designed and has encountered performance degrading factors it is not intuitively clear how much the new hover ceiling characteristic will depend upon type aircraft, useful load or other design factors. A measure of this dependence is indicated in figures 5 and 6 which show the off-design probabilities of hover for two aircraft: a 1,000 lb. useful load, single, teetering rotor helicopter and a 15,000 lb. useful load, tandem, articulated rotor helicopter. Both of these aircraft have suffered a 5% installed power loss and 5% increase in empty weight. As can be seen, the behavior of \( P(H) \) is similar to that of the optimum aircraft in figure 3, although the absolute values have decreased. A comparison of figure 5 and 6 is presented in figure 7 which shows that the greatest difference in \( P(H) \) for the two cases is 0.035 which occurs at a design point of sea level, 115 °F. The difference is considerably less over most of the range of design altitudes and temperatures considered which gives some confidence that the aircraft probability of hover will be relatively insensitive to the type and class of aircraft regardless of the performance degradation.

A further indication of the dependence of \( P(H) \) on performance degrading factors is illustrated in figures 8 and 9. Figure 8 shows the shift in the probability curve as installed power goes from 100% of design power to 95% of design power. In a similar fashion, figure 9 illustrates the shift in the curve and the corresponding drop in \( P(H) \) as empty weight growth goes from zero to 10%, for an aircraft that has already suffered a 5% installed power loss.

Although some idea has been gained as to the effect of performance degrading factors on the effectiveness of an aircraft as measured by the probability of hover, the problem of selecting a design criterion has not been resolved. To do this requires the consideration of aircraft cost.

Effectiveness/cost. - An increase in design altitude and temperature will affect cost through the increase in aircraft weight. This increase is illustrated in figure 10 for a 5,000 lb. useful load design where the empty weight has been normalized relative to the useful load. Cost will increase with design point in a similar fashion. It should be apparent that if effectiveness is measured by the hover probability and this is divided by cost then a maximum occurs in their ratio. This is illustrated in figures 11 and 12 for a 5,000 lb. useful load, MRP designed, optimum aircraft. In the case of figure 11 cost is based upon a UH-1D cost proposal where the cost figures have been normalized with respect to an arbitrary dollar value ($200,000) assigned to the useful load. In figure 12 a linear cost model has been used and the cost has been normalized to the value of the useful load which for convenience has been assigned a value per pound that is equal to the aircraft cost per pound. Therefore, what is referred to as \( \text{cost} \) in this figure is actually
WE/WU. It is also noted that the location of the maximum of this ratio as a function of design altitude and temperature in both figures is independent of the use of constant terms such as useful load and value per pound of useful load, rather the employment of such constant terms is to formulate the effectiveness/cost in a fashion that can be used to compare different classes of rotorcraft.

Figures 11 and 12 present a maximum in the effectiveness/cost ratio as a function of design altitude and temperature for the two cost models presented. The locus of maximum effectiveness/cost points is plotted over the P(H) curves for the 5,000 lb. useful load, MRP design in figure 13. Previous to this discussion it was concluded that although a high probability of hover was desirable in its own right, there was no inherent characteristic that allowed the selection of a design point. Through the use of the effectiveness/cost ratio it is now possible to determine when the increase in hover probability is no longer worth the cost and in this way determine the best design point. In addition, figure 13 shows that the use of this method is relatively insensitive to the particular details of the cost model, so that the simpler linear cost model can be used in the determination of a design criterion.

Before this method can be used to determine an adequate design point, it is necessary to gain some insight into the factors that will influence this selection. A considerable amount of discussion has already been presented showing the effect of performance degrading factors on aircraft effectiveness. It was concluded then that the aircraft probability of hover is relatively insensitive to type or class of aircraft, but quite sensitive to any performance degradation. The converse is true in discussing cost, as performance degrading factors will have no effect on the original aircraft procurement, but type aircraft and class will have an effect through the behavior of aircraft empty weight as a function of design altitude and temperature. This is illustrated in figure 14 which shows the change in WE/WU for five aircraft as a function of design altitude for a constant design temperature of 95 °F. As can be seen, the tandem rotor aircraft's empty weight increases more slowly with design altitude than the single rotor aircraft and hence the best effectiveness/cost point occurs at a higher altitude as is shown in figure 15. The value of the maximum effectiveness/cost point depends upon the size of the WE/WU ratio for the particular aircraft in question and this is a function of both aircraft type and useful load class. The selection of a design hover criterion will therefore depend upon both the influence of performance degradation on hover probability and the effect of aircraft type and weight class on cost. This is illustrated in figures 16 through 20 for a variety of performance degrading factors and different aircraft types and classes.

Viewing this multitude of possible design points in another fashion, figures 21 through 23 present the locus of equivalent performance degrading factors that would result in a best effectiveness/cost aircraft if it had originally been designed to 5,500 feet, 95 °F; 6,000 feet, 95 °F; or 6,500 feet, 95 °F. These figures show that if an aircraft is designed to the indicated altitude and temperature, and if it encounters performance degrading
factors equivalent to those on the locus, then the aircraft will be a best effectiveness/cost aircraft. If the anticipated performance degradation never occurs then the aircraft will have been designed to an altitude beyond that required for a best effectiveness/cost ratio. This is shown below. However, there is some consolation in that the aircraft although

\[
\text{EFFECT OF DEGRADATION ON DESIGN HOVER CRITERION}
\]

\[
\text{PERFORMANCE DEGRADATION} \leq \text{EXPECTED DEGRADATION}
\]

\[
\text{EXPECTED DEGRADATION}
\]

\[
\text{PERFORMANCE DEGRADATION} \geq \text{EXPECTED DEGRADATION}
\]

overdesigned will have an increased effectiveness. If the degrading factors are greater than anticipated then the aircraft will have been underdesigned and will also have a reduced effectiveness as is shown above. In this sense then the aircraft best effectiveness/cost loci in these figures 21 through 23 represent the aircraft's latitude to performance degradation. An acceptable design criterion must provide sufficient latitude for the operational environment, with the recognition that this latitude has a price.

To determine the best hover design criterion, therefore, requires an accurate estimate of the performance degradation that will be encountered in the field. An indication of the difficulty in estimating this performance degradation is illustrated in figure 24 which shows the in-service weight growth that has occurred during the design cycle of a number of military aircraft. A design cycle is defined by an aircraft change that affects hover
capability, i.e. an increased power engine, new rotor, etc. The weight
growth data was obtained from actual weight reports on production aircraft
and therefore does not represent changes that may occur in aircraft weight
through the design process to production, or weight that may be added to
specific aircraft in the field due to repairs or additional equipment require-
ments. This data shows that weight growth is unpredictable in nature, but
reliable in occurrence. To obtain an adequate estimate of installed power
losses is even more difficult, but it is noted that the Military Specifica-
tion for Qualification of Turboprop Aircraft Engines, MIL-E-8593(ASG) speci-
ifies that an engine is acceptable as long as its calibrated power after
the 150-hour endurance test does not decrease by more than five per cent.

Figures 21 through 23 were presented for three design points. It is
clear that other design points could be portrayed in a similar fashion. Be-
low a design point of 5500 feet, 95 °F aircraft will not have sufficient
latitude to allow them to capably meet the stresses of the operational en-
vironment. Somewhere around or above 6500 feet, 95 °F it appears that the
capability of the aircraft to tolerate performance degrading factors will
become more than adequate and the aircraft will be overdesigned. There
is no true optimum design point for all rotorcraft, but rather a region.
The best design point within this region is a matter of judgment.

Effect of design rated power. - The previous sections have considered
the aircraft designed for hover at Military Rated Power (MRP). The effect of
designing aircraft to hover at Normal Rated Power (NRP) was investigated in
the same fashion.

Figures 25 and 26 present the probability of hover for an optimum design
and one that has suffered a 5% installed power loss and a 5% empty weight
increase. The P(H) characteristics for the optimum designed aircraft are very
similar to those of the NRP design shown in Figure 3 although there are minor
deviations (see below). The off-design P(H) characteristics are significantly
different as can be seen by comparing figure 26 to figure 6. The difference
in P(H) is as large as 0.09 at some of the hover design points. The explana-
tion of this difference in hover probabilities is because the NRP engine has
a steeper slope in available power as a function of ambient temperature. This
is shown in Figure 27. Why this changes the hover ceiling characteristic can
be understood by examining the figure at the top of the next page. In the
 optimum case both aircraft will have the same hover ceiling solution at the
design point. Above this point the steeper NRP characteristic will match the
required power expression at a higher ambient temperature than the MRP charac-
teristic. Below the design point the NRP characteristic intersects at a lower
temperature. This results in a hover ceiling characteristic that appears in
the second figure on the next page. As can be seen the NRP design is better
at elevations above the design point and worse at elevation below. This is
why the optimum aircraft P(H)'s are similar for the two designs. The explana-
tion for the large difference in the off-design case can be seen by a similar
EFFECT OF RATED POWER ON HOVER CEILING SOLUTION

NOTES
---
- MRP
- NRP

DESIGN POINT

TEMPERATURE, °F

HOVER CEILINGS FOR MRP AND NRP DESIGNS

NOTES
---
- MRP
- NRP

DESIGN POINT

PRESSURE ALTITUDE, FEET

TEMPERATURE, °F
analysis. The figure below shows the hover ceiling solution for the weight
growth case and illustrates why the shift in hover ceiling is less for the
NRP design.

EFFECT OF WEIGHT
GROWTH ON MRP AND
NRP HOVER CEILING SOLUTIONS

This next figure shows why the hover ceiling shift is less for the NRP design
that has suffered an installed power loss.

EFFECT OF POWER LOSS
ON MRP AND NRP HOVER
CEILING SOLUTIONS

\[ \Delta P = \text{LCAS IN AVAILABLE POWER AT SEA LEVEL, STD} \]
Regardless of whether these degrading factors occur together or separately, the effect is a significant shift in the hover ceiling as shown below.

The presentation of the NRP design hover probabilities and the short discussion above have shown how the NRP design has a greater tolerance for performance degrading factors than the MRP design when both carry the same useful load. However, this increase in effectiveness is accompanied by an increase in empty weight as shown in figure 28. The overall effect is one of reduced effectiveness/cost which can be seen by comparing the effectiveness/cost curves for an NRP design in figures 29 and 30, with their counterparts in figure 15 and 18. This reduction in effectiveness/cost is approximately 6 to 8% at the best effectiveness/cost point.

Since the NRP design aircraft experiences a faster growth in empty weight as a function of design altitude and temperature over the MRP design this causes a lowering of the design altitude for best effectiveness/cost. This change in altitude, however, is within the range of the design point boundaries as discussed previously.
CONCLUSIONS

The parametric model described in this study provides a reasonable estimate of rotorcraft performance and cost. Through the use of a representation of the world climatological environment based upon the distribution of terrain and maximum daily temperatures for nineteen countries/regions contiguous or nearly so to the Soviet-Sino block, it is possible to determine the probability that a rotorcraft can hover OGE in these countries assuming that there is an equally likely chance of having to hover anywhere at any time. This probability of hover provides a measure of rotorcraft effectiveness which although useful has no inherent characteristic that allows the selection of a design hover criterion.

The determination of the best design point requires the consideration of cost as well as effectiveness. This is done by plotting the ratio of effectiveness to cost as a function of design altitude and temperature. The design altitude and temperature at which this ratio achieves a maximum results in a design hover criterion. It is apparent that this hover criterion must exceed 5500 feet, 95 °F if the aircraft is to be able to have sufficient tolerance for performance degrading factors due to operational usage. Higher design points will give a greater tolerance, although eventually the additional tolerance will be superfluous. Selecting an optimum design point is difficult simply because obtaining exact knowledge as to a likely performance degradation is difficult. Within the region of design points starting at 5500 feet, 95 °F, the best one point is a matter of judgment. For this reason a reduction of the present 6,000 feet, 95 °F standard design hover criterion is not justified.

While the aircraft is hovering OGE at the design point, it must also exhibit maneuverability. The requirement for maneuver used in this study is considered to be realistic and therefore a statement of the OGE hover design criterion should include a stipulation for maneuver that is at least as stringent as the requirement for yaw displacement to be achieved in one second after a full directional control input that is contained in the "General Requirements for Helicopter Flying and Ground Handling Qualities," MIL-H-8501A.

The use of Military Rated Power (MRP) or Normal Rated Power (NRP) does influence the selection of the design point, but this influence is considerably less than the uncertainty present in predicting performance degradation. The MRP designed aircraft exhibits an improved effectiveness/cost over the NRP design when based upon acquisition costs. However, AMC has pointed out that the present Army investment in engine spares and maintenance will overshadow this improvement for MRP designs and for this reason NRP should be used for the design hover criterion in order to lessen the life cycle costs.
It is recommended that the Army Standard Hot Day Design Hover Criterion for rotorcraft be:

The rotorcraft shall be capable of hovering out-of-ground effect (OGE) at its design or primary mission gross weight with pressure altitude and temperature conditions of 6,000 feet, 95 °F using normal rated power. At this hover ceiling the rotorcraft must also achieve following a full directional control input a yaw angular displacement in either direction from trim in one second of $330/\left[GW + 1000\right]^{1/3}$ degrees, where GW is the gross weight in pounds.
Appendix A

Parametric Solution

Introduction - In their initial work on the Hot Day Standard reported in references 5 and 6, Herda, et al. at Aerostructures, Inc. developed a generalized parametric model to determine the effect of design altitude and temperature on a helicopter's physical characteristics and performance. They treated power required, power available, gross weight and disk loading as implicit functions of each other, i.e.

\[
\begin{align*}
\text{SHP}_r &= f_1 (\text{SHP}_r, \text{SHP}_a, \text{GW}, w) \\
\text{SHP}_a &= f_2 (\text{SHP}_r, \text{SHP}_a, \text{GW}, w) \\
\text{GW} &= f_3 (\text{SHP}_r, \text{SHP}_a, \text{GW}, w) \\
w &= f_4 (\text{SHP}_r, \text{SHP}_a, \text{GW}, w)
\end{align*}
\]

while other parameters such as tip speed and mean blade lift coefficient were chosen to be representative of current aircraft. A fixed useful load was used in the parametric model to determine gross weight rather than any specific mission requirement.

The Aerostructures parametric model was followed for the most part in this subsequent study and is summarized in the following sections. Because of the complexity of the formulation, the parametric model was programmed for a digital computer. The solution of the equation set (1) was obtained using a secant method as described in reference 8.

The parametric model as developed has a number of limitations that should be recognized. One of these limitations is due to the statistical basis of the parametric data. The weight relationships, physical characteristics and power available characteristics have been determined by correlating statistical data from current aircraft. Thus, the model reflects current technology and does not anticipate advances in the state of the art. Because of this, improvements in engine technology, materials or structures are not represented in the expressions for gross weight or installed power. However, for the purpose of this study, the model is adequate.

In a similar fashion, some of the assumptions made in the analytical expressions for power required are of necessity naive. For instance, the lift and drag characteristics assumed for the rotor airfoil section have only a limited validity in regions of high tip Mach number, extreme angles of attack, etc. Hence, the validity of the parametric model in these regimes will also be limited. However, as a general representation of present rotorcraft technology the parametric model is satisfactory.
Power required - The power required expressions summarized below for the most part have been extracted from the work of Herda, et.al. at Aerostructures (reference 5). References 5 and 9 should be consulted for details as to their derivation. Changes to the Aerostructures derivation have been indicated where they occur.

Figures 31 and 32 illustrate two examples of the parametric model's ability to predict the power required by actual aircraft in hover and forward flight. Figure 31 shows the non-dimensional hover performance of the CH-47A along with the parametric model prediction. This prediction is good except at high gross weights at which point the model underestimates the power required. Figure 32 shows the computer prediction of forward flight power required for the UH-1B. Again the prediction is reasonable although for high speed flight the estimation of drag area is inadequate.

The power required for the aircraft system is expressed as

\[ \text{SHP}_r = \frac{1}{\eta} \left[ \text{ihp} + \text{Rhp} + \text{php} + \text{ihp}_{\text{tr}} + \text{Rhp}_{\text{tr}} + \text{conhp}_{\text{tr}} + \text{hp}_c + \text{hp}_{\text{acc}} + \text{hp}_{\text{gen}} \right] \]  

where the first three terms, representing induced, rotor profile and parasite power constitute the largest portion of the total aircraft power requirement. The next three terms represent tail rotor induced and profile drag power plus a control horsepower requirement. The last three terms represent aircraft subsystems' power required, which are assumed constant for a particular weight class aircraft. \( \eta \) represents the mechanical efficiency of the transmission system. Each of these terms is discussed in the paragraphs following.

Rotor induced power:

\[ \text{ihp} = \frac{1.13 \, F_{\text{vd}}^{3/2} \, GW \, u_H \, K_u}{550} \]  

The factor of 1.13 is a correction to account for a nonuniform inflow distribution.

The second term, \( F_{\text{vd}} \) is the fuselage vertical drag factor and accounts for the power loss due to rotor downwash drag on the fuselage. For the present formulation, it is assumed to have a discrete behavior as a function of...
aircraft weight class, i.e.

\[ F_{Vd} = 1.0 \quad GW < 30,000 \text{ lb.} \]
\[ F_{Vd} = 1.05 \quad GW \geq 30,000 \text{ lb.} \] (4)

In order to account for ground effect in hover, a dummy gross weight is used for calculating power required. In order to do this the nondimensional thrust co-efficient is calculated initially using the true gross weight,

\[ C_T = \frac{GW}{\sigma A_e V_t^2} \] (5)

From this a dummy thrust co-efficient is calculated as a function of the Z/D ratio.

\[ C_T^* = \frac{K_1 (Z/D) + K_3}{\left( \frac{1}{C_T} - \frac{K_2}{\sigma} \right) (Z/D) - \frac{K_4}{\sigma}} \] (6)

From this the dummy gross weight for use in equation (3) is calculated.

\[ GW = \rho A_e V_t^2 C_T^* \] (7)

The four constants of equation (5) have been determined from flight test data as

\[ K_1 = 1.0726 \]
\[ K_2 = -0.13841 \]
\[ K_3 = -0.0988 \]
\[ K_4 = 0.40333 \] (8)

The above formulation of power required as a function of ground effect is the subject of a soon-to-be-published ADS Technical Note by Harold Y. H. Law, and is a departure from the Aerostructure's treatment.
The induced velocity of the rotor in hover, $u_H$, is expressed as

$$u_H = \frac{1}{B} \sqrt{\frac{w}{2\rho}}$$

where the tip loss factor is assigned a constant value

$$B = 0.97$$  \hspace{1cm} (10)

The factor $K_u$ is used to correct the induced velocity for the effect of forward velocity

$$K_u = \left[ -\frac{1}{2} \frac{A_v}{A_e} \left( \frac{V}{u_H} \right)^2 + \frac{1}{2} \sqrt{\frac{A_v}{A_e} \left( \frac{V}{u_H} \right)^4 + 4} \right]^{1/2}$$  \hspace{1cm} (11)

where $A_e$ and $A_v$ account for the effects of tandem rotors

$$A_e = nR^2 \left[ \pi - \sin^{-1} \left( \frac{x_o}{4} \right) + \frac{1}{2} x_o \sqrt{x_o - \frac{x_o^2}{4}} \right]$$  \hspace{1cm} (12)

$$A_v = R^2 (\pi + 2x_g)$$  \hspace{1cm} (13)

and $x_o$ is the fractional rotor overlap and $x_g$ is the fractional vertical gap between rotors. Typically these values are

$$x_o = 0.705$$

$$x_g = 0.05$$  \hspace{1cm} (14)

For single rotor aircraft, the ratio $A_v/A_e$ is one.
Rotor profile power:

\[ R_{hp} = \frac{n \delta \rho A_B V_t^3 K_{\mu}}{4400} \]  \hspace{1cm} (15)

The drag co-efficient, \( \delta \) was taken from the drag characteristics of a 632-075 airfoil and is expressed as

\[ \delta = 0.009 + 0.00913 (C_{LR})^2 \]  \hspace{1cm} (16)

where

\[ C_{LR} = \frac{\rho \alpha}{\rho} C_{LR_0} \]  \hspace{1cm} (17)

The total blade area, \( A_B \) is expressed as

\[ A_B = \sigma A_e \]  \hspace{1cm} (18)

where

\[ \sigma = \frac{G_W}{\rho \alpha C_{LR_0} V_t^2} \]  \hspace{1cm} (19)

The term \( K_{\mu} \) corrects the profile drag expression for forward flight

\[ K_{\mu} = 1 + 3u^2 + 30u^4 \]  \hspace{1cm} (20)

where

\[ u = V/V_t \]  \hspace{1cm} (21)

Parasite power:

\[ P_{hp} = \frac{\rho V^3 A_{\pi}}{1100} \]  \hspace{1cm} (551)
The drag area of the aircraft is approximated by an equivalent flat plate area, $A_n$. Aerostructures treated this value as a constant for each useful load class. For the present study $A_n$ was expressed as a function of gross weight

$$A_n = 0.0376 \ (GW)^{0.665} \quad (23)$$

where this relationship was determined from a correlation of present aircraft equivalent flat plate areas.

**Tail rotor induced power:**

$$ihp_{tr} = \frac{1.13 \ T_{tr} \ u_{tr} \ K_u}{550} \quad (24)$$

The tail rotor thrust must be sufficient to counteract the torque of the main rotor.

$$T_{tr} = \frac{550 \ (ihp + Rhp + php)}{\Omega \ \epsilon_{tr}} \quad (25)$$

where the main rotor angular velocity is expressed in terms of the rotor radius and tip speed

$$\Omega = \frac{V_t}{R} \quad (26)$$

the rotor radius is a function of disk loading and gross weight

$$R = \sqrt[3]{\frac{GW}{\pi \ \omega}} \quad (27)$$

for single rotor aircraft, and the tail rotor moment arm is assumed to be

$$\epsilon_{tr} = 1.19R \quad (28)$$
The induced velocity through the tail rotor \( u_{H_{tr}} \) considers the tail rotor disk loading

\[
u_{H_{tr}} = \frac{1}{B_{tr}} \sqrt{\frac{T_{tr}}{2\pi \rho R_{tr}^2}} \quad (29)
\]

where

\[
B_{tr} = 0.9 \quad (30)
\]

and

\[
R_{tr} = 0.16 \, R \quad (31)
\]

The correction factor \( K_u \) for forward flight is the same as specified for the main rotor in equation (12). The induced velocity distribution correction for non-uniform flow is again 1.13.

Tail rotor profile power:

\[
R_{hp_{tr}} = \delta_{tr} \rho A_{B_{tr}} V_{t_{tr}}^3 K_u 4400 \quad (32)
\]

An NACA 0012 section airfoil was assumed for the tail rotor which gives a drag co-efficient of

\[
\delta_{tr} = 0.013 + 0.0168 \, (C_{LR_{tr}})^2 \quad (33)
\]

The tail rotor blade lift co-efficient \( C_{LR_{tr}} \) is expressed as

\[
C_{LR_{tr}} = \frac{ihp + Rhp + php}{2.25 \, TRP \, (\rho/\rho_0)} \quad (34)
\]
The tail rotor blade area is

\[ A_{\text{tr}} = \frac{6T_{\text{tr}}}{C_{L_{\text{tr}}0} \rho V_{\text{ttr}}^2} \tag{35} \]

where

\[ C_{L_{\text{tr}}0} = 0.4 \tag{36} \]

The tail rotor tip speed is assumed to be the same as the main rotor tip speed

\[ V_{\text{ttr}} = V_t \tag{37} \]

The forward flight correction \( k_f \) is identical to that specified for the main rotor in equation (16).

Control power: It is essential that an aircraft have sufficient excess power at its hover ceiling to enable it to perform turns, transition to forward flight or to overcome gusts. Without this capability the result of any of the above situations could be a loss in altitude and hence the possibility of mission failure.

The requirement for maneuverability at the hover ceiling is met in this study by adding an incremental tail rotor power (control power) to provide for yaw control per MIL-H-8501A (reference 10). No corresponding requirement was placed upon the tandem configurations investigated.

Paragraph 3.3.5 of MIL-H-8501A states that the yaw angular displacement to be achieved in one second after full directional control displacement will be

\[ \psi(1) = \frac{330}{3\sqrt{GW + 1000}} \text{ degrees} \tag{38} \]

where \( GW \) is the maximum overload gross weight in pounds. For the purposes of this study and the resulting design criterion, \( GW \) shall be the design gross weight instead and the requirement shall be demonstrated at the aircraft's hover ceiling. In addition, paragraph 3.3.19 requires that the
aircraft exhibit yaw angular velocity damping of at least

\[ D_\psi = 27 I_z^{0.7} \text{ slug - ft}^2/\text{sec} \]  \hspace{1cm} (39)

where \( I_z \) is the yaw moment of inertia in slug - ft^2.

Using the two requirements stated above, it is possible to formulate the additional power needed to provide yaw maneuverability. The yaw angular equation of motion following a directional control input may be expressed as

\[ I_z \ddot{\psi}(t) + D_\psi \dot{\psi}(t) = \Delta T_{tr} \dot{z}_{tr} \]  \hspace{1cm} (40)

where \( \Delta T_{tr} \) is the incremental tail rotor thrust needed to provide the required yaw displacement. The boundary conditions that apply to the case of a sudden input are

\[ \psi(0) = \dot{\psi}(0) = 0 \]  \hspace{1cm} (41)

The solution for these conditions then is

\[ \psi(t) = \frac{\Delta T_{tr} \dot{z}_{tr}}{27^2 I_z^{0.4}} \left[ \exp(-27 I_z^{-0.3}) + 27 I_z^{-0.3} - 1 \right] \]  \hspace{1cm} (42)

The requirement for a yaw displacement in one second allows the solution for \( \Delta T_{tr} \)

\[ \Delta T_{tr} = \frac{B^2 I_z \psi(1)}{(\exp(-B) + B - 1) \dot{z}_{tr}} \]  \hspace{1cm} (43)

where

\[ B = 27 I_z^{-0.3} \]  \hspace{1cm} (44)
The yaw moment of inertia, $I_z$, is approximated with the expression

$$I_z = 0.0004353 \times (GW)^{1.87}$$

which was obtained through correlation of current aircraft data. Having determined the required incremental thrust required for yaw control, the control power may be expressed as

$$\text{conhp}_{\text{tr}} = \text{hp}_{\text{r}} \times (T_{\text{tr}} + \Delta T_{\text{tr}}) - \text{hp}_{\text{tr}} \times T_{\text{tr}}$$

where $\text{hp}_{\text{tr}}$ is the sum of the tail rotor induced and profile powers. It should be noted that the above formulation of yaw control power is different from that of Aerostructures.

Cooling, Accessory and Generator Powers: Cooling power is considered to be a function of both density altitude and engine size. Accessory and generator power are considered as functions of aircraft gross weight. In order to treat these simply, each is assigned a constant value depending upon its weight class. These values are assigned in the table below.

### SUBSYSTEM POWERS AS A FUNCTION OF WEIGHT CLASS

<table>
<thead>
<tr>
<th>GROSS WEIGHT</th>
<th>ENGINE COOLING POWER</th>
<th>ACCESSORY POWER</th>
<th>GENERATOR POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBS</td>
<td>Hpc</td>
<td>Hpacc</td>
<td>Hpgen</td>
</tr>
<tr>
<td>&gt; 3000</td>
<td>$\rho/\rho_0$</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\geq$ 3000</td>
<td>$4 \rho/\rho_0$</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$\leq$ 10000</td>
<td>$16 \rho/\rho_0$</td>
<td>20.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

42
Mechanical Efficiency: The power loss in the gear train was determined for representative cases as

\[ \eta = 0.98 \quad \text{single rotor} \]
\[ \eta = 0.965 \quad \text{tandem rotor} \]

Power Available - The data and methodology of Herda, et. al. (reference 5) were used to develop generalized power available characteristics as a function of altitude and temperature. In this work, they examined the power available characteristics of six current turbine engines; the T63-A-5A, T53-L-13, T55-L-7, T55-L-11, T64-GE-16 and the JFTD12A-4A. Each engine's power available was normalized with respect to its sea level, standard day performance and a generalized power available was determined by a least squares fit of the data. These least squares fitted curves are illustrated in figures 33 and 34 for the Military Rated Power (MRP) and Normal Rated Power (NRP) cases.

The MRP and NRP sea level, standard day ratings were related with an average value

\[ \frac{\text{MRP}_0}{\text{NRP}_0} = 1.1252 \] (48)

Installation losses were accounted for by subtracting a fixed loss from the generalized characteristic, i.e.

\[ \text{SHP}_a(H_p, t) = \text{ERP}_0 \left[ k_e(H_p, t) - k_k \right] \] (49)

where \( \text{ERP}_0 \) is the sea level, standard day rating of the engine, \( k_e \) is the normalized available power characteristics and \( k_k \) is the correction for installation losses and was assigned the values through a survey of current aircraft.

\[ k_k = \begin{cases} 
0.008 & \text{OGE hover} \\
0.015 & \text{IGE hover} 
\end{cases} \] (50)

The OGE hover value for installation loss was used for all forward flight conditions.
Weight Equations. - Aircraft empty weight was estimated using an equation of the form

\[ WE = \sum_{i=1}^{n} W_i \]

(51)

where

\[ W_i = A_i (P_i)^{k_i} \]

(52)

\( W_i \) represents a component weight group, there being a total of \( n \) weight groups making up weight empty. \( P_i \) represents some parameter or group of parameters with which good weight estimation can be achieved. The constants \( A_i \) and \( k_i \) are determined from statistical data. The weight estimating relationships used in this study were originally derived in reference 5 and were adjusted as more statistical data became available. Tables 1 and 2 list the weight estimating relationships for single and tandem rotor aircraft. Figure 35 demonstrates the correlation obtained for the rotor group.

Better weight estimation could possibly be achieved by expressing \( W_i \) as a function of more than one variable, however, for the purposes of this study the present relations were found to be acceptable. An illustration of estimated component weights is shown in Table 3 for the UH-1B along with the actual reported component weights.

The design gross weight used in determining the aircraft's performance then is the sum of weight empty and useful load,

\[ GW = WE + WU \]

(53)

Disk Loading. Disk loading has a significant effect on an aircraft's weight, power requirements, performance and autorotational capability. In reference 6, Randall and Talbot concluded that any criterion used to select disk loading must account for the aircraft's capability to autorotate safely. The fashion in which they did this was to use a qualitative index of autorotational capability derived by Katzenberger and Rich in reference 11 and referred to as the Relative Autorotational Landing Index (RALI). This index relates the energy stored in the rotor during autorotation, the kinetic energy of the sinking aircraft and rotor speed decay. Thus, it provides a measure of the energy available to decrease the aircraft's momentum and
the rate of conversion of this energy. The RALI is expressed as

\[ \text{RALI} = \frac{1000 \ I_R}{GW \ \omega \ \gamma_1} \]  \hspace{1cm} (54)

where \( \gamma_1 \) is similar to the Lock number and is expressed as

\[ \gamma_1 = \frac{\sigma \ \alpha \ \rho \ \pi \ R^5}{I_R} \]  \hspace{1cm} (55)

The rotor inertia, \( I_R \) can be approximated by the blade inertia alone. Assuming a uniform blade section and a constant mass factor, then

\[ I_R = 0.01034 \left[ \frac{W_B \ GW}{K_1 \ \omega} \right] \]  \hspace{1cm} (56)

where \( W_B \) is the blade weight and

\[ K_1 = \begin{cases} \pi & \text{single rotor} \\ 2.872 & \text{tandem rotor} \end{cases} \]  \hspace{1cm} (57)

If the blade weight is expressed

\[ W_B = 4.872 (\sigma \ A_e) \]  \hspace{1cm} (58)

then

\[ \text{RALI} = \frac{1.04 \times 10^6}{K_1 \ 1.44} \left[ \frac{GW^{0.44}}{\omega^{3/2} \ C_{LRo} \ 0.96 \ V_t \ 1.88} \right] \]  \hspace{1cm} (59)

This index assumes that other factors affecting aircraft autorotation such as airfoil characteristics and collective pitch input are the same. Therefore, the RALI can not be used as an accurate comparator of aircraft. For the purposes of this study RALI values were selected that correspond to those of a number of current aircraft as shown in the table on the next page.
ESTIMATED RALI FOR CURRENT AIRCRAFT

<table>
<thead>
<tr>
<th>AIRCRAFT</th>
<th>GROSS WEIGHT</th>
<th>RALI</th>
</tr>
</thead>
<tbody>
<tr>
<td>OH-5A</td>
<td>2530</td>
<td>15.1</td>
</tr>
<tr>
<td>UH-1B</td>
<td>6600</td>
<td>9.2</td>
</tr>
<tr>
<td>CH-47A</td>
<td>33000</td>
<td>9.55</td>
</tr>
<tr>
<td>CH-54A</td>
<td>38000</td>
<td>5.62</td>
</tr>
<tr>
<td>OH-6A</td>
<td>2155</td>
<td>9.53</td>
</tr>
</tbody>
</table>

To specify disk loading, RALI values of 15 were selected for the 1000 lb. useful load aircraft and 9.5 for all other classes.

Independent Parameters. A number of parameters were considered as arbitrary constants in the parametric solution. To measure their effect a number of solutions were repeated with slight changes in these constants.

A value for blade tip speed of 700 feet/sec was selected for the aircraft model. This value is reasonably representative of current aircraft design and reduces the difficulty of having to estimate compressibility (high tip speed) and blade stall (low tip speed) effects.

The mean blade lift coefficient, \( C_{LRp} \) was selected to be representative of blade loadings on current aircraft. A value of 0.41 was used for the 1000 lb. useful load class, and a value of 0.54 for the larger class aircraft.

Instead of selecting an arbitrary ratio of transmission limited power to engine rated power at sea level standard day, the effect of transmission limiting available power was examined by assuming that this limit was somewhere between the power required for hover at the design point and the power available at sea level, standard day.

\[
Q_{lim} = \frac{TRP - rhp|_{design \ point}}{ERP_o - rhp|_{design \ point}} \quad (60)
\]

This type of representation was required due to the treatment of design altitude and temperature as independent variables. Values of \( Q_{lim} \) ranged from 0.0 to 1.0.
Range of Solutions. - To keep the parametric model as general as possible, a wide number of type aircraft, design altitudes, design temperatures, etc. were considered. The table below gives some idea as to the range of aircraft examined.

<table>
<thead>
<tr>
<th>Rotor Type</th>
<th>Design Pressure Altitude</th>
<th>Design Temperature (°F)</th>
<th>Q_{lim}</th>
<th>Useful Load (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>single, articulated</td>
<td>0</td>
<td>75</td>
<td>0.0</td>
<td>1000</td>
</tr>
<tr>
<td>single, teetering</td>
<td>1000</td>
<td>85</td>
<td>0.2</td>
<td>5000</td>
</tr>
<tr>
<td>tandem, articulated</td>
<td>2000</td>
<td>95</td>
<td>0.4</td>
<td>15000</td>
</tr>
<tr>
<td></td>
<td>3000</td>
<td>105</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>115</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5000</td>
<td></td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table represents 2430 aircraft alone without considering the various sensitivity investigations. However, not all of these aircraft solutions were actually obtained, as once trends were established various portions of the solution matrix were ignored.
Appendix B

Derivation of Hover Probability

The climatological data available yields two probability distribution functions directly. The first is the conditional probability distribution function,

\[ P(T \leq t \mid H_p = x) \]  \hspace{1cm} (1)

which is the probability that the temperature, \( T \), is less than or at the most equal to a specified value \( t \) given that the pressure altitude, \( H_p \), is equal to a particular value, \( x \). The second is the probability distribution function

\[ P(H_p \leq x) \]  \hspace{1cm} (2)

which is the probability that the pressure altitude is less than or at the most equal to some specified value, \( x \).

We wish to determine the joint probability distribution function

\[ P(T \leq t, H_p \leq x) \]  \hspace{1cm} (3)

which is the probability that the temperature, \( T \), is less than or at the most equal to \( t \) and the pressure altitude, \( H_p \), is less than or at the most equal to \( x \). An expression for this probability distribution function can be derived using the familiar Riemann Integral of differential calculus. (The notation used in this derivation is essentially that of reference 12).

\[
P(T \leq t, H_p \leq y) = \int_{-\infty}^{y} P(T \leq t, H_p = x) \, dx
\]

\[
= \int_{-\infty}^{y} P(T \leq t \mid H_p = x) \, p_{H_p}(x) \, dx
\]

\[
= \int_{-\infty}^{y} P(T \leq t \mid H_p = x) \, \frac{dP(H_p \leq x)}{dx} \, dx
\]

48
We now wish the joint probability mass function $p_{T,H_p}(t, y)$. It is

$$p_{T,H_p}(t, y) = \frac{\partial^2}{\partial t \partial y} P(T \leq t, H_p \leq y)$$  \hspace{1cm} (5)$$

Substitution of the above expression and using Leibnitz's Rule for the differentiation of an integral with respect to a parameter we obtain the following.

$$p_{T,H_p}(t, y) = \frac{\partial}{\partial t} P(T \leq t \mid H_p = y) \frac{dP(H_p \leq y)}{dy}$$  \hspace{1cm} (6)$$

Now then the helicopter hover ceiling depicted below

states that the helicopter can hover at any pressure altitude, $y$, provided that the temperature, $t$ is at or below the ceiling.

$$t \leq f(y)$$  \hspace{1cm} (7)$$
We know that the probability of hover, \( P(H) \), is that of finding climatic conditions at or below the hover ceiling.

\[
P(H) = \int_{-\infty}^{\infty} \int_{-\infty}^{f(y)} P_{T,H_p}(t, y) \, dt \, dy \tag{8}
\]

Substituting and simplifying in the following manner, we obtain the desired result.

\[
P(H) = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{f(y)} \frac{\partial}{\partial t} P(T \leq t \mid H_p = y) \, dt \right) \frac{dP(H_p \leq y)}{dy} \, dy
\]

\[
= \int_{-\infty}^{\infty} P(T \leq f(y) \mid H_p = y) \frac{dP(H_p \leq y)}{dy} \, dy \tag{9}
\]

It is tempting to cancel the "dy" term in the integrand. This is permissible provided we interpret the result as a Stieltjes Integral.

\[
P(H) = \int_{-\infty}^{\infty} P(T \leq f(y) \mid H_p = y) \, dP(H_p \leq y) \tag{10}
\]

This equation is of interest as it produces a simple method of evaluating the probability of hovering.

The Stieltjes Integral of a function \( g(\cdot) \) with respect to the function \( F(\cdot) \) over the interval \((a, b]\), written

\[
\int_{a+}^{b} g(y) \, dF(y) \tag{11}
\]

is defined as follows. Partition the interval \((a, b]\) into \(n\) subintervals \((y_{i-1}, y_i]\) in which \(y_0, y_1, \ldots, y_n\) are \(n + 1\) points chosen such that

\[
a = y_0 < y_1 < \ldots < y_n = b \tag{12}
\]
Choose a second set of \( n \) points \( y_1', y_2', \ldots, y_n' \) one in each subinterval, such that

\[
y_{i-1} < y_i' < y_i
\]  
(13)

For \( 1 \leq i \leq n \). The above integral is by definition

\[
\int_a^b g(y) \, dF(y) = \text{limit} \sum_{n+ \to 1} g(y_i') [F(y_i) - F(y_{i-1})]
\]  
(14)

in which the limit is taken over all partitions of the interval \( (a, b) \), as the length of the subinterval of maximum length tends to zero.

If we now assume that \( F(y) \) is a monotone increasing function of \( y \) ( \( P(H_0 \leq y) \) certainly is) and that \( g(y) \) is a continuous function of \( y \) ( \( P(T \leq g, H_0 = y) \) certainly is) then \( g \) may be plotted as a piece-wise continuous function of \( F \).

Consider the Riemann Integral of \( g(F) \) given by

\[
\int_{F(a)}^{F(b)} g(F) \, dF = \text{limit} \sum_{n+ \to 1} g(F_i') [F_i - F_{i-1}]
\]  
(15)
where

\[ F(a) = F_0 < F_1 < \ldots < F_n = F(b) \]  \hspace{1cm} (16)

and

\[ F_{i-1} \leq F_i' \leq F_i \] \hspace{1cm} (17)

and the limit is taken over all possible subdivisions of the interval \([F(a), F(b)]\) as the length of the largest subinterval tends to zero. It is obvious then that the Stieltjes Integral can be evaluated by merely reploting \(g\) as a function of \(F\) and then calculating the Riemann Integral between the limits \(F(a)\) and \(F(b)\).

Returning to the problem at hand, we know that we must first replot \(P(T \leq f(y) \mid H_p = y)\) as a function of \(P(H_p \leq y)\). This we shall do by first plotting \(P(T \leq t \mid H_p = y)\) as a function of \(P(H_p \leq y)\) and then superimposing the hover ceiling \(f(y)\). By initially selecting a range of pressure altitudes, \(\{y_i\}\), and determining the associated values of \(P(H_p \leq y)\), \(\{P(H_p \leq y_i)\}\), we may then select a range of temperatures, \(\{t_k\}\). For these temperature and pressure altitude ranges we may construct a matrix of values of \(P(T \leq t \mid H_p = y)\), \(\{P(T \leq t_k \mid H_p = y_i)\}\).

The plot on the following page is drawn for \(P(T \leq t \mid H_p \leq y)\) as a function of \(P(H_p \leq y)\). It can be seen that the graph and the lower scale have been rotated once and an altitude scale added to arrive at the plot as it is drawn in practice. The hover ceiling may be directly superimposed on the temperature-pressure altitude grids. The resulting curve is precisely \(P(T \leq f(y) \mid H_p = y)\) as a function of \(P(H_p \leq y)\). We note that the limits of integration are

\[ P(H_p \leq \infty) = 1 \]

\[ P(H_p \leq -\infty) = 0 \] \hspace{1cm} (18)

Thus the probability of hover can be seen to be the shaded area in the figure. If a planimeter or some other mechanical device is used to calculate this area, then the number of square inches of shaded area should be divided by the number of square inches covered by the entire pict to obtain the probability of hover.
\[ P(T \leq t | H_p = y) \]
Maximum Daily Temperature

Climatology Data

The United States Air Force Environmental Technical Applications Center (ETAC) was asked to provide distributions of mean daily maximum temperatures in nineteen countries/regions. This was done and is reported under Project #5935 (reference 7).

The mean daily maximum temperature distributions developed by ETAC were combined with the terrain distribution for each country and plotted in the same fashion as the data presented in the CORG study (reference 3). The plots for the nineteen countries/regions are presented in Figures 36 through 54.

Adequate temperature data was not available for the higher altitudes in some countries, hence extrapolation was required for these altitudes assuming a lapse rate similar to the lower elevations. This is indicated by dashed lines on these figures.
TABLES
Table 1: Weight Estimating Relationships* for Single Rotor Aircraft

<table>
<thead>
<tr>
<th>WEIGHT</th>
<th>TRANSMISSION</th>
<th>PROPELLATION</th>
<th>LANDING GEAR</th>
<th>BODY</th>
<th>TAIL ROTOR</th>
<th>Rotor(s)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_a$</td>
<td>$W_s$</td>
<td>$W_p$s</td>
<td>$W_g$</td>
<td>$W_b$</td>
<td>$W_{tg}$</td>
<td>$W_r$</td>
<td>$k_i$</td>
</tr>
<tr>
<td>$550$</td>
<td>$N_{RP}$</td>
<td>$G_{W}$</td>
<td>$G_{W}$</td>
<td>$G_{W}$</td>
<td>$G_{W}$</td>
<td>$G_{W}$</td>
<td>$550$</td>
</tr>
<tr>
<td>.144</td>
<td>.8346</td>
<td>.954</td>
<td>.884</td>
<td>.914</td>
<td>.995</td>
<td>.915</td>
<td>$k_i$</td>
</tr>
<tr>
<td>Includes engine weight</td>
<td>skid</td>
<td>GW &gt; 6,000 lb.</td>
<td>GW &lt; 6,000 lb.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*For NRP designs, rated power is multiplied by 1.1252 to obtain $N_{RP}$. 

\[
W = A(k_i)N_{RP}
\]
### TABLE 1 - CONTINUED

**WEIGHT ESTIMATING RELATIONSHIPS FOR SINGLE ROTOR AIRCRAFT**

<table>
<thead>
<tr>
<th>WEIGHT GROUP</th>
<th>$W_i$</th>
<th>PARAMETER(S)</th>
<th>$A_i$</th>
<th>$k_i$</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROTOR DRIVE SHAFT</td>
<td>$W_{rd}$</td>
<td>$W_{rg}$</td>
<td>.0705</td>
<td>1.034</td>
<td></td>
</tr>
<tr>
<td>FLIGHT CONTROLS</td>
<td>$W_{fc}$</td>
<td>$W_{rG}$</td>
<td>.224</td>
<td>1.114</td>
<td></td>
</tr>
<tr>
<td>INSTRUMENT</td>
<td>$W_1$</td>
<td>$GW$</td>
<td>.0822</td>
<td>.731</td>
<td></td>
</tr>
<tr>
<td>ELECTRICAL</td>
<td>$W_{el}$</td>
<td>$GW$</td>
<td>.00105</td>
<td>1.425</td>
<td></td>
</tr>
<tr>
<td>FURNISHINGS</td>
<td>$W_f$</td>
<td>$GW$</td>
<td>.000144</td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>HYDRAULICS</td>
<td>$W_{hy}$</td>
<td>$GW$</td>
<td>.011</td>
<td>.96</td>
<td></td>
</tr>
<tr>
<td>AIR CONDITIONING</td>
<td>$W_{ac}$</td>
<td>$GW$</td>
<td>.00018</td>
<td>1.44</td>
<td></td>
</tr>
<tr>
<td>AVIONICS</td>
<td>$W_a$</td>
<td>$GW$</td>
<td>.035</td>
<td>1.0</td>
<td>$GW \leq 10,000 \text{ lb.}$</td>
</tr>
<tr>
<td>ENGINE</td>
<td>$W_{eg}$</td>
<td>$\text{MRP}_o$ (2)</td>
<td>2.168</td>
<td>.714</td>
<td>information only</td>
</tr>
</tbody>
</table>
For NWP designs, total power is multiplied by 1.1252 to obtain NWP

<table>
<thead>
<tr>
<th>Weight (includes rotor)</th>
<th>0.857</th>
<th>1.047</th>
<th>1.446</th>
<th>2.764</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NWP (2)</td>
<td>1.549</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M_p</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Landing Gear</td>
<td>0.03725</td>
<td>1.146</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Body</td>
<td>2.276</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M_r</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remarks</td>
<td>2.764</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter(s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M_t</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight Group</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

TABLE 2
**TABLE 2 - CONTINUED**

WEIGHT ESTIMATING RELATIONSHIPS (1) FOR TANDEM ROTOR AIRCRAFT

<table>
<thead>
<tr>
<th>WEIGHT GROUP</th>
<th>$W_i$</th>
<th>PARAMETER(S)</th>
<th>$A_i$</th>
<th>$k_j$</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FURNISHINGS</td>
<td>$W_f$</td>
<td>GW</td>
<td>$2.63 \times 10^{-5}$</td>
<td>1.689</td>
<td></td>
</tr>
<tr>
<td>HYDRAULICS</td>
<td>$W_{hy}$</td>
<td>GW</td>
<td>.011</td>
<td>.96</td>
<td></td>
</tr>
<tr>
<td>AIR CONDITIONING</td>
<td>$W_{ac}$</td>
<td>GW</td>
<td>.00018</td>
<td>1.44</td>
<td></td>
</tr>
<tr>
<td>AVIONICS</td>
<td>$W_a$</td>
<td>GW</td>
<td>.035</td>
<td>1.0</td>
<td>$GW \leq 10,000$ lb.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.01</td>
<td>$GW &gt; 10,000$ lb.</td>
</tr>
<tr>
<td>ENGINE</td>
<td>$W_{eg}$</td>
<td>MRP$_o$ (2)</td>
<td>2.168</td>
<td>.714</td>
<td>information only</td>
</tr>
</tbody>
</table>

(1) $W = A \cdot (\text{PARAMETER})^{k}$

(2) for NRP designs, rated power is multiplied by 1.1252 to obtain MRP$_o$. 
<table>
<thead>
<tr>
<th>GROUPS</th>
<th>ACTUAL WEIGHT</th>
<th>ESTIMATED WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Group</td>
<td>755.3</td>
<td>754.2</td>
</tr>
<tr>
<td>Tail Rotor Group</td>
<td>56.0</td>
<td>57.6</td>
</tr>
<tr>
<td>Body Group</td>
<td>873.3</td>
<td>797.1</td>
</tr>
<tr>
<td>Landing Gear Group</td>
<td>106.4</td>
<td>133.8</td>
</tr>
<tr>
<td>Propulsion Group:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propulsion</td>
<td>919.9</td>
<td>860.2</td>
</tr>
<tr>
<td>Transmission</td>
<td>484.1</td>
<td>509.1</td>
</tr>
<tr>
<td>Rotor Drive</td>
<td>65.5</td>
<td>66.6</td>
</tr>
<tr>
<td>Flight Controls Group</td>
<td>385.3</td>
<td>359.6</td>
</tr>
<tr>
<td>Instrument Group</td>
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NOTE: Actual weight from BHC Report 204-099-477
FIGURES
Figure 1. OGE Hover Ceilings For Five Parametric Aircraft
Figure 2. Hover Capability of Four Parametric Aircraft in Afghanistan
Figure 3. OGE Hover Probability for an Optimum Aircraft
Figure 4. Loss in Hover Probability Due to Performance Degradation as a Function of Design Altitude and Temperature
Figure 5. OGE Hover Probability for an Off-design Aircraft
Figure 6. OGE Hover Probability for an Off-design Aircraft
Figure 7. Comparison of Off-design Probabilities of Hover
Figure 8. OGE Hover Probability for an Off-design Aircraft
Figure 9. OGE Hover Probability for an Off-design Aircraft
AIRCRAFT WEIGHT EMPTY AS A FUNCTION OF DESIGN ALTITUDE AND TEMPERATURE

NOTES -
MRP DESIGN
5000 LB. USEFUL LOAD
SINGLE, ARTICULATED ROTOR
QLIM = 0.2

Figure 10. Aircraft Empty Weight as a Function of Design Altitude and Temperature
EFFECTIVENESS/COST FOR AN OPTIMUM AIRCRAFT BASED ON UH-1D COST PROPOSAL

NOTES -

PARAMETRIC DESIGN -
MRP
5000 LB. USEFUL LOAD
SINGLE, ARTICULATED ROTOR
QLIM = 0.2
COST MODEL -
CN = AIRCRAFT COST/ARBITRARY VALUE OF $200,000 FOR USEFUL LOAD

Figure 11. Effectiveness/Cost for an Optimum Aircraft Based on UH-1D Cost Proposal
NOTES -
PARAMETRIC DESIGN -
MRP
5000 LB. USEFUL LOAD
SINGLE, ARTICULATED ROTOR
QLIM = 0.2

COST MODEL -
WE_N = WE/WU

Figure 12. Effectiveness/Cost for an Optimum Aircraft
Based on a Linear Cost Model
Figure 13. OGE Hover Probability of Optimum Aircraft with Locus of Maximum Effectiveness/Cost Points
Figure 14. Weight Empty as a Function of Design Altitude for Different Type and Class Aircraft
NOTES
PARAMETRIC DESIGN
95°F DESIGN TEMPERATURE
MRP
PERFORMANCE DEGRADATION - NONE

Figure 15. Effectiveness/Cost
NOTES -
PARAMETRIC DESIGN -
95°F DESIGN TEMPERATURE
MRP
PERFORMANCE DEGRADATION
2.5% INSTALLED POWER LOSS

Figure 16. Effectiveness/Cost

DESIGN PRESSURE ALTITUDE, 1000 FEET

0 2 4 6 8
0 .10 .20 .30 .40 .50 .60 .70
P(H)/MEL, N

- 5000 LB. USEFUL LOAD, SINGLE ROTOR MAXIMUM
- 1000 LB. USEFUL LOAD, SINGLE ROTOR MAXIMUM
- 15000 LB. USEFUL LOAD, TANDEM ROTOR MAXIMUM
Figure 17. Effectiveness/Cost
NOTES -
PARAMETRIC DESIGN -
95°F DESIGN TEMPERATURE
MRP
PERFORMANCE DEGRADATION -
5% INSTALLED POWER LOSS
5% EMPTY WEIGHT CROWTH

Figure 18. Effectiveness/Cost
NOTES -
PARAMETRIC DESIGN -
95°F DESIGN TEMPERATURE
MRP
PERFORMANCE DEGRADATION -
10% EMPTY WEIGHT GROWTH

Figure 19. Effectiveness/Cost
NOTES -

PARAMETRIC DESIGN - 95°F DESIGN TEMPERATURE
MRP
PERFORMANCE DEGRADATION - 5% INSTALLED POWER LOSS
10% EMPTY WEIGHT GROWTH

![Graph showing effectiveness/cost against design pressure altitude, 1000 feet. The graph includes lines for 5000 lb. useful load, single rotor maximum, 1000 lb. useful load, single rotor maximum, and 15000 lb. useful load, tandem rotor maximum.](image)

Figure 20. Effectiveness/Cost
Figure 21. Locus of Equivalent Performance Degrading Factors for Best Effectiveness/Cost Aircraft Designed to 5500 Feet, 95°F
Figure 22. Locus of Equivalent Performance Degrading Factors for Best Effectiveness/Cost Aircraft Designed to 6000 Feet, 95°F
Figure 23. Locus of Equivalent Performance Degrading Factors for Best Effectiveness/Cost Aircraft Designed to 6500 feet, 95°F.
Figure 2a. In-service Empty Weight Growth for Military Aircraft
Figure 25. OGE Hover Probability for Optimum Type Aircraft
PARAMETRIC DESIGN -
NRP
YAW REQ'TS PER MIL-H-8501A
15000 LB. USEFUL LOAD
TANDEM, ARTICULATED ROTOR
QIM = 0.2

CONTINGENCIES -
5% NRP0 DEGRADATION
5% EMPTY WEIGHT INCREASE

CLIMATOLOGY MODEL -
MAXIMUM DAILY TEMPERATURE
DISTRIBUTION IN NINETEEN
COUNTRIES BORDERING SOVIET-
SINO BLOCK

Figure 26. OCE Hover Probability for an Off-design Aircraft
Figure 27. Comparison of Power Available Characteristics for Military and Normal Rated Power
NOTES -
PARAMETRIC DESIGN -
95°F DESIGN POINT
NRP
PERFORMANCE DEGRADATION -
NONE

DESIGN PRESSURE ALTITUDE, 1000 FEET

Figure 29. Effectiveness/Cost
NOTES -

PARAMETRIC DESIGN -
95°F DESIGN TEMPERATURE
NRP
PERFORMANCE DEGRADATION -
5% INSTALLED POWER LOSS
5% EMPTY WEIGHT GROWTH

--

5000 LB. USEFUL LOAD, SINGLE ROTOR
MAXIMUM

15,000 LB. USEFUL LOAD, TANDEM ROTOR
MAXIMUM

DESIGN PRESSURE ALTITUDE, 1000 FEET

Figure 30. Effectiveness/Cost
NOTES -
FLIGHT TEST PERFORMANCE FROM AFFTC-TR-66-2
TEST POINTS INDICATED BY CIRCLES
MODEL PREDICTION -
- HD = 3,000 FT.
- HD = 11,000 FT.

Figure 31. Prediction of Non-dimensional Hover Performance of CH-47A
NOTES -

FLIGHT TEST PERFORMANCE FROM AFFTC-TDR-62-21
GW = 6600 LB.
H₀ = 10,000 FT.
ROTOR SPEED = 323 RPM

--- FAIRED LINE THROUGH TEST DATA
--- MODEL PREDICTION

Figure 32. Prediction of Forward Flight Performance for UH-1B
Figure 33. Military Rated Power Available Characteristics
NOTES -

ENGINES USED IN LEAST SQUARES FIT -
T63-A-5A
T53-L-13
T55-L-7
T55-L-11
T64-GE-16

PRESSURE ALTITUDE, 1000 FEET

Figure 34. Normal Rated Power Available Characteristics

95
Figure 35. Rotor Group Weight as a Function of Torque Divided by Gross Weight
Figure 36. West Germany
Figure 37. France
Figure 38. Italy
Figure 39. Spain
Figure 40. Turkey
Figure 41. Syria - Lebanon (Cumulative Probability of Temperature)
Figure 42. Saudi Arabia

Cumulative Probability of Temperature

Cumulative Probability of Altitude

Elevation Feet

Sea Level

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

115°F 110°F 105°F 100°F 95°F 90°F 85°F 80°F 75°F 70°F 65°F 60°F 55°F
Figure 43. Iraq
Figure 44. Iran
Figure 46. West Pakistan
Figure 47. India
Figure 48. East Pakistan

CUMULATIVE PROBABILITY OF TEMPERATURE
Figure 49. Burma
Figure 50. Thailand
Figure 51. Laos
Figure 52. Cambodia
Figure 53. Viet Nam
Figure 54. South Korea
REFERENCES


A study is presented of the Army hot day design hover criterion. Models are developed to represent rotorcraft, cost, environment and operation, and these models are integrated to examine the effectiveness and cost of the rotorcraft as a function of design altitude and temperature. Although no optimum design hover criterion can be derived, the effects of rotorcraft type and size, and performance degradation to be expected in the field are identified, and the appropriate range of design altitudes and temperatures are obtained.
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