AD

Research Note ETL-RN-70-2

A RELATION BETWEEN THE SPECTRUM OF SURFACE SLOPES AND THE SPECTRUM OF SURFACE ELEVATIONS AND ITS USEFULNESS IN THE THEORY OF ELECTROMAGNETIC WAVE SCATTERING FROM ROUGH SURFACES

by
Richard A. Hevenor

July 1970

U.S. ARMY ENGINEER TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VIRGINIA

9

Destroy this research note when no longer needed. Do not return it to the originator.

The findings in this research note are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

USAETL Research Notes are published to record and disseminate significant results of research tasks or portions of overall programs during the course of the work.

U. S. ARMY ENGINEER TOPOGRAPHIC LABORATORIES
FORT BELVOIR, VIRGINIA

Research Note ETL-RN-70-2

A RELATION BETWEEN THE SPECTRUM OF SURFACE SLOPES AND THE SPECTRUM OF SURFACE ELEVATIONS AND ITS USEFULNESS IN THE THEORY OF ELECTROMAGNETIC WAVE SCATTERING FROM ROUGH SURFACES

Project 4A062112A854

July 1970

Distributed by

The Commanding Officer
U. S. Army Engineer Topographic Laboratories

Prepared by

Richard A. Hevenor
Geographic Information Systems Branch
Geographic Sciences Division

SUMMARY

The purpose of this research note is to show that a simple relation exists between the spectrum of surface elevations and the spectrum of surface slopes. This relation is useful when the problem of electromagnetic wave scattering from rough surfaces is being worked on. When the small perturbation theory is being used to compute the scattered fields from a rough surface, both the spectrum of surface elevations and the spectrum of the surface slopes can appear in the final equations. The spectrum of the surface elevations is easy to work with and can be determined from measurements made on the surface. The spectrum of the surface slopes, however, is more difficult to work with. It would be beneficial if a relation between the two spectrums could be obtained.

FOREWORD

The relationships derived in this research note were discovered by the author when he was working on the problem of electromagnetic wave scattering from rough surfaces. The final expression derived for the spectrum of the surface slopes is to be used in a rigorous analysis of wave scattering from composite rough surfaces. This work was done in the Geographic Information Systems Branch under the supervision of Mr. Abraham Anson and under the general supervision of Dr. Kenneth R. Kothe.

CONTENTS

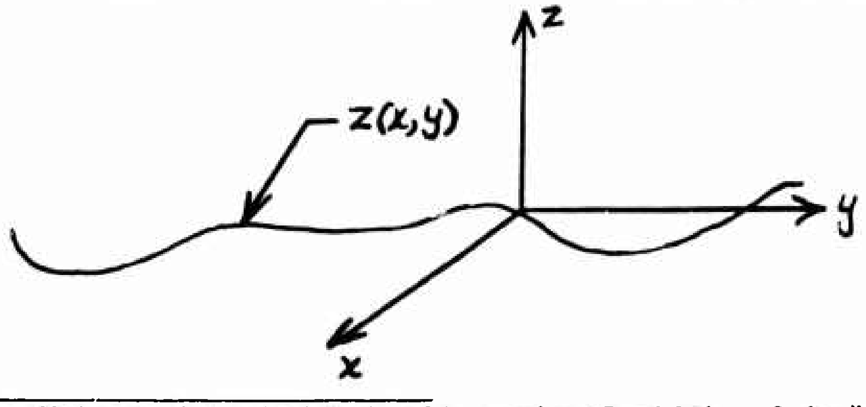
A RELATION BETWEEN THE SPECTRUM OF SURFACE SLOPES AND THE SPECTRUM OF SURFACE ELEVATIONS AND ITS USEFULNESS IN THE THEORY OF ELECTROMAGNETIC WAVE SCATTERING FROM ROUGH SURFACES

I. INTRODUCTION

The problem of the scattering of electromagnetic waves from rough surfaces has been the subject of many papers in recent years. A recent theory by Fung* has used the small perturbation technique along with the Fourier transformation to obtain expressions for the scattered fields from a slightly rough surface. In the final expression for the scattered fields for the vertical polarization case, one finds the spectrum of the surface elevations and the spectrum of the surface slopes. The spectrum of the surface elevations is easy to work with and can be determined from measurements made on the surface. The spectrum of the surface slopes, however, is more difficult to work with. It would be beneficial if a relation between the two spectrums could be developed. The purpose of this technical note is to show that a simple relation exists between the spectrum of the surface elevations and the spectrum of the surface slopes.

II. ANALYSIS

A surface is assumed to exist which has its elevations distributed in some random manner with respect to a single plane. A right-handed rectangular Cartesian coordinate system is set up with its origin near the surface and with the z axis representing the surface elevation dimension. The xy plane is the mean surface elevation plane. The surface elevations can be described by the function z(x,y) as shown in the figure. The assumption is made that z(x,y) is Fourier transformable over an arbitrary region of interest.



* A. K. Fung, "Mechanisms of Polarized and Depolarized Scattering from a Rough Dielectric Surface," *Journal of the Franklin Institute*, Vol. 285, No. 2 (February 1968).

The spectrum of the surface elevations $Z(k_x,k_y)$ can be obtained by taking the two dimensional Fourier transform of $z(x,y)$, where k_x and k_y are the two Fourier variables.

$$Z(k_x,k_y) = \frac{1}{2\pi} \iint z(x,y) \exp(-jk_x x - jk_y y)\, dy\, dx \tag{1}$$

The limits on all integrals in this note are from minus infinity to plus infinity. The surface elevations $z(x,y)$ can be obtained from the spectrum by an inverse Fourier transform relation:

$$z(x,y) = \frac{1}{2\pi} \iint Z(k_x,k_y) \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{2}$$

When equation (2) is differentiated with respect to x, an expression is obtained which is representative of the surface slopes in the x direction.

$$\frac{\partial z(x,y)}{\partial x} = \frac{1}{2\pi} \iint jk_x\, Z(k_x,k_y) \exp(jk_x x + jk_y y)\, dk_x\, dk_y \tag{3}$$

The spectrum $Z_x(u,v)$ of the surface slopes can be written as:

$$Z_x(u,v) = \frac{1}{2\pi} \iint \frac{\partial z(x,y)}{\partial x} \exp(-jux - jvx)\, dy\, dx \tag{4}$$

The Fourier variables u and v have been used here to keep them separate from k_x and k_y. When equation (3) is substituted into equation (4) the spectrum of the surface slopes in x becomes:

$$Z_x(u,v) = \frac{1}{(2\pi)^2} \iiiint jk_x\, Z(k_x,k_y) \exp[jx(k_x-u) + jy(k_y-v)]\, dk_x\, dk_y\, dy\, dx \tag{5}$$

Integrating equation (5) with respect to x and y, the Dirac delta function is obtained, which is defined as follows:

$$\delta(\tau) = \frac{1}{2\pi} \int e^{j\phi\tau}\, d\phi$$

The spectrum of the surface slopes in x then becomes:

$$Z_x(u,v) = \iint jk_x\, Z(k_x,k_y)\, \delta(k_x - u)\, \delta(k_y - v)\, dk_x\, dk_y \tag{6}$$

Equation (6) can only be defined at the point $k_x = u$ and $k_y = v$. Then the spectrum of the surface slopes in x can be related to the spectrum of surface elevations as follows:

$$Z_x(u,v) = juZ(u,v) \tag{7}$$

If we represent the spectrum of the surface slopes in y by $Z_y(u,v)$ then it can be shown by a similar derivation that:

$$Z_y(u,v) = jvZ(u,v)$$

III. CONCLUSIONS

It is concluded that:

a. A simple relation has been shown to exist between the spectrum of surface slopes and the spectrum of the surface elevations.

b. This relation is useful in the analysis of electromagnetic wave scattering from rough surfaces when the small perturbation technique is being used.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) U. S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
A RELATION BETWEEN THE SPECTRUM OF SURFACE SLOPES AND THE SPECTRUM OF SURFACE ELEVATIONS AND ITS USEFULNESS IN THE THEORY OF ELECTROMAGNETIC WAVE SCATTERING FROM ROUGH SURFACES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Research Note

5. AUTHOR(S) (First name, middle initial, last name)
Richard A. Hevenor

6. REPORT DATE July 1970	7a. TOTAL NO. OF PAGES ~~24~~ 9	7b. NO. OF REFS
8a. CONTRACT OR GRANT NO. b. PROJECT NO. 4A662707A854 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) ETL-RN-70-2 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY U. S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060

13. ABSTRACT

The purpose of this research note is to show that a simple relation exists between the spectrum of surface elevations and the spectrum of surface slopes. This relation is useful when working on the problem of electromagnetic wave scattering from rough surfaces. When using the small perturbation theory to compute scattered fields from a rough surface, both the spectrum of surface elevations and the spectrum of surface slopes can appear in the final equations. The spectrum of the surface elevations is easy to work with and can be determined from measurements made on the surface. The spectrum of the surface slopes, however, is more difficult to work with. It would be very beneficial if a relation between the two spectrums could be obtained.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fourier Transforms Electromagnetic Waves Surface Spectrums						