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A SIMPLE THEORY FOR SHOCK PROPAGATION  
IN HOMOGENEOUS MIXTURES  
AFIT TR 70-3

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**A SIMPLE THEORY FOR SHOCK PROPAGATION  
IN HOMOGENEOUS MIXTURES**

by

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## ABSTRACT

A Hugoniot pressure density relationship for homogeneous mixtures is calculated from the Hugoniot pressure density relationship of the constituents. The calculated values are found to be in reasonable agreement with available experimental data on mixtures of solids and porous materials.

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## I. INTRODUCTION

The motivation for the study here reported is the intersection of two trends in technology; one being the increasingly wide applications of new composite materials; the other being the rapid growth of interest in the response of structures and structural elements to extremely strong disturbances.

Shock properties of mixtures have been computed [1] from the properties of the constituents by a method using the complete equation of state. In this report, a means of computing approximate shock properties from the Hugoniot relationships alone is proposed. The results of this study will be compared with experimental data for an unstructured or homogeneous composite. The simplified theory will also be investigated as a means of predicting the shock response of porous materials.

In an earlier study, a theory for predicting the response of a layered composite to a propagating shock was given [2]. It is to be expected that the particular geometrical arrangement of the two constituents assumed in that study would strongly influence the propagation. In the present study, the propagation of a shock wave through a composite material having no geometrical structure is of interest. It is anticipated that a comparison of the results from these two theories will provide at least a qualitative indication of the influence of structure on the propagation.

A related simple theory for the speed of propagation of an acoustic disturbance is also considered and is found to display certain interesting features. The condition under which the speed of sound in a mixture can be less than the sonic velocity of either constituent is established.

## II. THE HUGONIOT RELATIONSHIP

We wish to determine the shock response of a homogeneous composite, such as would result if powders of several constituents are prepared, thoroughly mixed, and then pressed so as to form a solid having the desired mass (or volume) fractions of the constituents. The resulting composite is assumed to be macroscopically homogeneous, i.e., it behaves as a homogeneous and isotropic solid and, to the scale of the objects which might be made from the composite material, no structure can be seen. This criterion, it should be noted, may not be met in a shock process, for the distance over which the load is applied is the thickness of the shock, the product of the rise time of the shock and the speed of propagation.

### a. Theory for a Mixture of Two Constituents

We assume that any element of volume  $\Delta V_0$  at one atmosphere of pressure contains a volume  $\Delta V_1$  of one constituent and  $\Delta V_2$  of the other. If the two constituents have initial densities  $\rho_{01}$  and  $\rho_{02}$ , then the masses of each of the two constituents are

$$\begin{aligned} M_1 &= \Delta V_1 \rho_{01} & \text{and} \\ M_2 &= \Delta V_2 \rho_{02} \end{aligned} \quad (1)$$

The initial average density of the composite is

$$\rho_{0 \text{ ave}} = \frac{M_1 + M_2}{\Delta V_1 + \Delta V_2} = f_{V1} \rho_{01} + f_{V2} \rho_{02} \quad (2)$$

or

$$\frac{1}{\rho_{0 \text{ ave}}} = \frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}} \quad (3)$$

where  $f_{V1}$  and  $f_{V2}$  are initial volume fractions of the two components, and  $f_{m1}$  and  $f_{m2}$  are the mass fractions. It is implicitly assumed that the mixture is simple, or non-reacting.

The average density of the composite after the passing of the shock is

$$\rho_{\text{ave}} = \frac{M_1 + M_2}{\Delta V_{1f} + \Delta V_{2f}} \quad (4)$$

where  $M_1$  and  $M_2$  are the masses of each constituent and  $\Delta V_{1f}$  and  $\Delta V_{2f}$  the volumes occupied by these masses at the shock pressure. Since  $M_1$  and the mass fractions  $f_{m1}$  are assumed to be unchanged by the shock, the possibility of phase change under pressure is excluded.

Thus

$$\frac{1}{\rho_{\text{ave}}} = \frac{f_{m1}}{\rho_{f1}} + \frac{f_{m2}}{\rho_{f2}} \quad (5)$$

Let us now assume that the response of each constituent to a one dimensional shock (the Hugoniot or shock adiabat) is known. For moderate pressures (hundreds of kilobars) this relationship can be approximated for many materials by the expression [3]

$$P = A \left\{ \left( \rho / \rho_0 \right)^n - 1 \right\} \quad (6)$$

where  $P$  is the pressure behind the shock, measured above one atmosphere,

$A$  and  $n$  are parameters of the material

$\rho$  is the density behind the shock and

$\rho_0$  is the density in front of the shock.

A relationship of this form is usually satisfactory only if no phase changes occur over the pressure range where it is to be applied. Assuming that the density of each constituent of the mixture at a given shock pressure is the same as would be found in a homogeneous sample at the same pressure, and that each constituent can be described by Equation (6),

$$\frac{\rho f_1}{\rho_{01}} = \left\{ \frac{P}{A_1} + 1 \right\}^{\frac{1}{n_1}} \quad \text{and} \quad (7a)$$

$$\frac{\rho f_2}{\rho_{02}} = \left\{ \frac{P}{A_2} + 1 \right\}^{\frac{1}{n_2}} \quad (7b)$$

where the pressure,  $P$ , is the same in each constituent, and the  $f_1$  are densities behind the shock. Substituting (7a) and (7b) and (3) into (5)

$$\frac{\rho_{0\text{ave}}}{\rho f_{\text{ave}}} = \frac{\frac{f_{m1}}{\rho_{01}} \left\{ \frac{P}{A_1} + 1 \right\}^{-1/n_1} + \frac{f_{m2}}{\rho_{02}} \left\{ \frac{P}{A_2} + 1 \right\}^{-1/n_2}}{\frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}}} \equiv \frac{1}{\eta} \quad (8)$$

which provides a means of predicting the average density vs pressure relationship for a homogeneous mixture of two constituents, and requires only the knowledge of an approximate Hugoniot relationship for each.

The assumed pressure density relationship, Equation (6), was selected only as a matter of convenience. Any form giving Hugoniot pressure as a function of density which can be solved explicitly for density as a function of pressure could as well be used and would lead to a closed form expression analogous to Equation (8).

For many materials, Hugoniot data is readily available [4] in the form

$$D = C + SU \quad (9)$$

where

$D$  is the shock speed

$U$  the particle velocity, and

$C$  and  $S$  are empirical constants

It has been demonstrated that such data [5] may be put in the form of Equation (4) through setting

$$A = \frac{\rho_0 C^2}{4S-1} \quad \text{and} \quad n = 4S-1 \quad (10)$$

It has also been shown [3] that "universal" values of  $A$  and  $n$  may be used with good accuracy for a wide variety of materials. These values are  $n = 5$  and  $A = \rho_0 C_0^2 / 5.5$ , where  $\rho_0$  is the initial density and  $C_0$  is the sonic velocity at standard conditions. By using these properties of the constituents, Equation (8) can be used to predict the Hugoniot pressure volume relationship even if Hugoniot information for the constituents is not available.

b. Response of a Hypothetical Copper-Polyethelene Mixture

The family of predicted shock pressure vs density ( ) relationships for a hypothetical copper-polyethelene composite was determined through Equation (8). A linear Hugoniot relationship

$$D = 3.92 + 1.488 U \quad (11)$$

with  $\rho_0 = 8.93 \text{ gm/cm}^3$  was chosen [6] for copper. All velocities are in units of km/sec. Equation (10) then yields parameters  $A = 279.7 \text{ kb}$  and  $n = 4.956$ . Available data [1] on polyethelene was used to determine the coefficients to be  $A = 9.64 \text{ kb}$  and  $n = 5.875$ . The initial density was taken to be  $.915 \text{ gm/cm}^3$ . The results of these calculations are shown in Figure 1 as pressure vs density relationships for various mass fractions of copper. A mass fraction of .9, for these materials, corresponds to a mixture of 48% copper, by volume.

Once the Hugoniot pressure vs. density relationship has been computed, the other shock properties can be determined from the Rankine-Hugoniot jump conditions, i.e.

$$D\rho_{\text{ave}} = (D-U)\rho_{f\text{ave}} \quad (12a)$$

$$P = \rho_{\text{ave}} DU \quad (12b)$$

where the composite is assumed to be macroscopically homogeneous. One particularly interesting result is given in Figure 2, where the shock speed at various pressure levels is plotted against the mass fraction of copper. The pronounced minimum at low pressures is particularly significant and suggests the possibility of a minimum in the sonic velocity. This subject will be treated in a later section.

c. Comparison with Theory for a Layered Composite

In Figure 3, shock speed vs. particle velocities as computed from this theory (the solid lines) are compared with results obtained previously for a layered composite having the same composition. Several mixtures

of aluminum with an assumed linear Hugoniot

$$D = 5.24 + 1.439U \text{ km/sec}$$

and polymethylmethacrylate, with an assumed linear Hugoniot of

$$D = 2.70 + 1.61U \text{ km/sec}$$

were considered. The results showed a surprisingly good agreement between the two computations, suggesting that the influence of the layering assumed in [2] is much less than might be expected.

#### d. Theory for Several Constituents

The theory may be readily extended to mixtures of three or more constituents. Denoting the  $i$ th constituent by a subscript  $i$ , the mass of that constituent in some volume element  $\Delta V$  is

$$M_i = \Delta V_i \rho_{e_i} = f_{v_i} \Delta V \rho_{e_i} \quad (13)$$

$$\text{Then } \rho_{0\text{ave}} = \frac{\sum M_i}{\sum \Delta V_i} = \frac{\sum_{i=1}^N f_{v_i} \rho_{e_i}}{\sum_{i=1}^N \frac{f_{m_i}}{\rho_{e_i}}} \quad (14)$$

and

$$\frac{f_{m_i}}{\rho_{e_i}} = \frac{f_{v_i}}{\rho_{0\text{ave}}}$$

Assuming each constituent may be described by Equation (6) or

$$\frac{\rho_{F_i}}{\rho_{0_i}} = \left\{ \frac{P}{A_i} + 1 \right\}^{\frac{1}{n_i}}, \quad (7c)$$

since

$$\rho_{F\text{ave}} = \sum_{i=1}^N \frac{f_{m_i}}{\rho_{F_i}}$$

we find

$$\frac{\rho_{0\text{ave}}}{\rho_{f\text{ave}}} = \frac{\sum_{i=1}^N \left\{ \frac{f_{m_i}}{\rho_{0_i}} \left( \frac{P}{A_i} + 1 \right) \right\}^{-\frac{1}{n_i}}}{\sum_{i=1}^N \frac{f_{m_i}}{\rho_{0_i}}} \quad (14a)$$

or

$$\frac{\rho_{0\text{ave}}}{\rho_{f\text{ave}}} = \sum_{i=1}^N f_{v_i} \left\{ \frac{P}{A_i} + 1 \right\}^{-\frac{1}{n_i}} \quad (14b)$$

### III. APPLICATION OF THEORY TO POROUS MEDIA

Of particular interest are composites where one of the constituents is air, such as would result from an imperfect packing. For air, however, Equation (6) is not appropriate. We assume air to be a perfect gas with constant ratio of specific heats  $\gamma$ , and

$$P = \rho RT \quad (15a)$$

$$E - E_0 = C_v(T - T_0) \quad (15b)$$

Using these and the Rankine Hugoniot jump condition for a one dimensional shock in a homogeneous material,

$$E - E_0 = \frac{(P + P_0)}{2} \left\{ \frac{1}{\rho_0} - \frac{1}{\rho_f} \right\} \quad (16)$$

to determine a shock pressure vs density relationship for air which is analogous to Equation (6), we find, for  $\gamma = 1.4$

$$\frac{P}{P_0} = \frac{6\rho/\rho_0 - 1}{6 - \rho/\rho_0} \quad \text{or} \quad \frac{\rho}{\rho_0} = \frac{6P/P_0 + 1}{6 + P/P_0} \quad (17)$$

Thus the average density of a composite containing  $N$  solid constituents and air is

$$\frac{\rho_{0\text{ave}}}{\rho_{f\text{ave}}} = \sum_{i=1}^N f_{v_i} \left\{ \frac{P}{A_i} + 1 \right\}^{-\frac{1}{n_i}} + f_{v_a} \left\{ \frac{6 + P/P_0}{6 + P/P_0 + 1} \right\} \quad (18)$$

where  $f_{v_a}$  is the initial volume fraction of air. For pressures of interest for solids,  $P/P_0 \gg 1$  and the last term may be written as  $f_{v_a}/6$ . Experimental data [7] on strong shocks in air indicates that the compression under shocks of strength greater than 200 bars is closer to ten than the value of six predicted by treating air as a perfect gas.

a. Comparison of Theory with Experimental Results on Materials of Low Porosity

Experimental data for a tungsten-copper mixture (Elkonite) was obtained from the literature [1] and compared with values predicted by this theory using

$$D = (4.029 + 1.237U) \text{ km/sec} \quad (19)$$

with  $\rho_0 = 19.22 \text{ gm/cm}^3$  for tungsten and

$$D = (3.940 + 1.489U) \text{ km/sec} \quad (20)$$

with  $\rho_0 = 8.93 \text{ gm/cm}^3$  for copper,

Constants  $A = .790 \text{ mb}$ , and  $n = 3.948$  were determined for tungsten and constants  $A = .280 \text{ mb}$  and  $n = 4.956$  were determined for copper.

The average compositions of the mixtures for which Hugoniot data was available are given in Table I. Shock pressures corresponding to the theoretical densities given in Table I were computed and compared with the experimental data. The agreement between the experimental data and the elementary theory was found to be remarkably good. It can be seen, however, from Table I that the samples apparently have some porosity; that is, the measured densities are not precisely the theoretical density of a mixture having the stated mass fraction of the two constituents, but are always somewhat lower. If the differences can be attributed to porosity, and the expression given by Equation (18) takes proper account of the pressure of air, then Equation (18) should yield values of final density which are in better agreement with the experimentally determined values than predictions as based on Equation (8). For each composition (nominal mass fraction of each) the quantity

prediction of the final density may be obtained by neglecting the porosity and using the initial density the density which the composite would have if there were no porosity. The three methods of calculation are compared in Figure 4. The solid lines indicate typical pressure-density relationships as might be computed from Equation (8) if the measured density  $\rho_0$  or the theoretical density (for no porosity) were used. The dashed curve shows the result to be expected if Equation (18), taking account of porosity, were to be used. Above the pressure at which the air is essentially compressed to its limiting density, the difference between the two curves is very small.

The comparison of theory using Equation (18) and experiment is shown for each data point below 2mb in Figure 5. The predicted final density was computed from Equation (18), with the volume fraction of air being determined for each specimen from the stated nominal composition and the measured density of that specimen. Aside from the systematic deviation in the 45/55 samples, the agreement is remarkably good, considering the drastic assumptions included in the theory.

As was indicated earlier, once the Hugoniot pressure density relationship is established, the other shock parameters may be determined. The copper-tungsten was found to have a minimum in the isobars of the shock speed vs mass fraction relationship as was found in the copper-polyethylene mixture.

b. Comparison of Theory with Results on Materials of High Porosity

Much data has been obtained from shock experiments on porous materials. Such data are of interest, for the use of porous samples permits points on the equation-of-state surface for the solid to be reached in a shock experiment which are not attainable in a solid specimen. The porous sample may also be viewed as a rather extreme composite material of the type being considered herein. Shock data on porous samples of copper, aluminum, beryllium, magnesium, and uranium were obtained from the literature and compared with the predictions of Equation (18). Shock pressures are plotted against particle velocity, where the particle velocity for the theoretical curve is determined through the use of Equations (12a) and (12b). The results of this comparison are shown in Figures 6, 7, and 8. In all of these comparisons,  $f_v$  is the volume fraction of the solid. The comparison of theory and experiment for copper, using data from McQueen, et al [1] shows a rather good agreement, particularly below 250 kb. Experimental data for pure copper ( $f_v = 1.0$ ) have been included for purposes of comparison. The theory generally underestimates the shock pressure in a porous material.

In Figure 7, data from McQueen, et al, has been combined with the data from the Shock Wave Compendium [8]. The agreement here is not as good as in the case of copper, the theory again leading to a systematic underestimation of the shock pressure. The data for pure aluminum is presented, for comparison, as the solid line. Limited data for beryllium, magnesium, and uranium, all from the Compendium,

are compared with the theory in Figure 8. The agreement here is again quite good for all three materials. Here the dashed lines represent data for the solid material, while the solid line represents the predictions of the theory, and the indicated points are data from the literature.

All of these calculations were made using  $\gamma = 1.4$  (limiting compression of six) for air. Repeating the calculations using a limiting compression of ten for air gave an insignificant change in the results when plotted as pressure vs particle velocity or pressure vs shock velocity. The final density, however, is affected.

### III. THE SPEED OF SOUND IN MIXTURES

It has been reported that the speed of sound in certain mixtures has been found to be less than that for either constituent. In particular, the speed of sound in "bubbly" water has been found to be as low as 65 ft/sec. [10]

It will be shown in what follows that an elementary theory for the speed of sound in mixtures can be developed along the lines of the development in the preceding section which does predict that certain mixtures will have sonic velocities below those of either constituent. Moreover, it will be shown to be unnecessary to assume any type of deformation not present in the acoustic disturbance of the constituents alone.

We will restrict our attention to homogeneous mixtures of two constituents; that is, mixtures in which the particles of the constituents are substantially smaller than any other physical dimension present in the problem. In an unbounded medium, the only dimension present is the wavelength of the acoustic disturbance; hence the theory is valid only for moderate and low frequencies. Further, we assume the relative amounts of the two constituents to be the same at all points and, there being no structure, the mixture to be isotropic.

Let two constituents of density  $\rho_{01}$  and adiabatic bulk modulus  $B_1$  at some reference pressure and temperature be mixed so as to create a mixture at the same pressure and temperature having volume fraction  $f_1$  of each constituent. Since the mixture is assumed to be homogeneous, any elemental volume,  $\Delta V_0$ , of the mixture contains a volume

$$\Delta V_{01} = f_1 \Delta V_0 \quad (22)$$

of each constituent. Since the mass of each constituent contained in  $\Delta V_0$  is

$$\Delta m_1 = \rho_{01} f_1 \Delta V_0 \quad (23)$$

the density of the mixture is given by

$$\rho_{0\text{ave}} = \frac{\Delta m_1 + \Delta m_2}{\Delta V_0} = \rho_{01} f_1 + \rho_{02} f_2 \quad (24)$$

i.e., the rule of mixtures.

Under a pressure change,  $\Delta P = P - P_0$ , the volume of each constituent will change, but not necessarily in the same amount. By definition,

$$\left. \frac{dP}{d\rho} \right|_i = \frac{B_i}{\rho_{0i}} \quad (25)$$

Hence for such an infinitesimal pressure change,

$$P - P_0 = \frac{B_i}{\rho_{0i}} (\rho_i - \rho_{0i}) \quad (26)$$

The volume of each constituent is then

$$\Delta V_i = \frac{\Delta m_i}{\rho_i} = \frac{f_i \Delta V_0}{\left\{ 1 + \frac{P - P_0}{B_i} \right\}} \quad (27)$$

and the average density at pressure  $P$  is

$$\rho_{\text{ave}} = \frac{\Delta m_1 + \Delta m_2}{\Delta V_1 + \Delta V_2} = \frac{\rho_{0\text{ave}}}{\frac{f_1}{1 + \frac{P - P_0}{B_1}} + \frac{f_2}{1 + \frac{P - P_0}{B_2}}} \quad (28)$$

Defining an effective bulk modulus for the composite through

$$B_{\text{ave}} = \rho_{0\text{ave}} \left. \frac{dP}{d\rho_{\text{ave}}} \right|_{\substack{P=P_0 \\ S=\text{const}}} \quad (29)$$

We find

$$B_{\text{ave}} = \frac{1}{\frac{f_1}{B_1} + \frac{f_2}{B_2}} \quad (30)$$

from which it is evident that the inverse quantity, the compressibility,

satisfies a rule of mixtures.

The sonic velocity for each constituent is defined to be

$$C_1^2 = \left. \frac{dF}{d\rho} \right|_s = \frac{B_1}{\rho_{01}} \quad (31)$$

Assuming the propagation through the mixture to be adiabatic, a sonic velocity can be defined

$$C_{ave}^2 = \left. \frac{dP}{d\rho} \right|_s = \frac{B_{ave}}{\rho_{0ave}} \quad (32)$$

Substituting the expression for the average bulk modulus and density, we find

$$C_{ave}^2 = \frac{1}{(f_1/B_1 + f_2/B_2) (\rho_{01}f_1 + \rho_{02}f_2)} \quad (33)$$

In terms of mass fractions,  $f_{m1}$  and  $f_{m2}$  rather than volume fractions, the above results become

$$\frac{1}{\rho_{0ave}} = \frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}} \quad (34)$$

$$B_{ave} = \frac{1}{\rho_{0ave}} \left\{ \frac{f_{m1}}{\rho_{01}} \frac{1}{B_1} + \frac{f_{m2}}{\rho_{02}} \frac{1}{B_2} \right\}^{-1} \quad (35)$$

$$C_{ave}^2 = \frac{\left\{ \frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}} \right\}^2}{\frac{f_{m1}}{\rho_{01}} \frac{1}{B_1} + \frac{f_{m2}}{\rho_{02}} \frac{1}{B_2}} \quad (36)$$

replacing the modulus with the sonic velocities of the constituents through Equation (32).

$$C_{ave}^2 = C_1^2 \frac{\left\{ f_{m1} + f_{m2} \frac{\rho_{01}}{\rho_{02}} \right\}^2}{f_{m1} + f_{m2} \left\{ \frac{\rho_{01} C_1}{\rho_{02} C_2} \right\}^2} \quad (37)$$

The quantity  $Z_1 = \rho_1 c_1$  appearing in this expression is known as the acoustic impedance. The minimum values in the sonic velocity in such cases as aluminum and polyethelene, and copper and polyethelene are below the sonic velocity of either constituent. However, other combinations, such as copper and lead, do not display this phenomenon.

The speed of sound in a homogeneous mixture has been previously found by Wood [11] to be

$$1/c^2 = [\alpha\rho_2 + (1-\alpha)\rho_1] \left\{ \frac{\alpha}{\rho_2 c_2^2} + \frac{1-\alpha}{\rho_1 c_1^2} \right\} \quad (38)$$

where

$c$  is the speed of sound in the mixture

$\alpha$  is the volume fraction of phase 2

$\rho_1, \rho_2$  are the densities of phases 1 and 2, respectively, and

$c_1, c_2$  are the sonic speeds in each phase, defined by

$c_1^2 = \frac{dP}{d\rho_1}$  where the derivative is evaluated along an appropriate thermodynamic path.

If a minimum in the sonic speed exists, the above expression for the speed,  $c$ , as a function of volume fraction  $\alpha$ , must have an extremal value in the range  $0 < \alpha < 1$ . The sonic speed of such a mixture can also be written as a function of the mass fraction,  $x$ , of phase 2

$$x = \frac{\alpha\rho_2}{\rho_a} \quad (39)$$

where the average density,  $\rho_a$ , of the mixture is given by

$$\rho_a = \alpha\rho_2 + (1-\alpha)\rho_1 = [x/\rho_2 + (1-x)/\rho_1]^{-1} \quad (40)$$

The sonic speed, in terms of  $x$ , is

$$c^2 = \left[ \frac{x}{\rho_2} + \frac{(1-x)}{\rho_1} \right] \left/ \left[ \frac{x}{Z_2^2} + \frac{(1-x)}{Z_1^2} \right] \right. \quad (41)$$

where  $Z_1$  are the impedances,  $Z_1 = \rho_1 c_1$ . This expression is equivalent to Equation (37). Differentiating Equation (41) with respect to  $x$ , setting to zero, and solving for the mass fraction at which the extremal value occurs, yields

$$x = (B-2A)/BA \quad (42)$$

where

$$A = \rho_1/\rho_2 - 1 \quad (43a)$$

$$B = (Z_1/Z_2)^2 - 1 \quad (43b)$$

But the mass fraction must lie in the range  $0 \leq x \leq 1$ . Hence, for a minimum in the sonic speed to occur, the properties of the two materials must satisfy certain inequalities. These are:

If  $A$  and  $B$  are both negative, then

$$2A \leq B \leq 2A/(1-A) \quad (44)$$

If  $A$  and  $B$  are both positive, then

$$2A \leq B \leq 2A/(1-A) \text{ for } A < 1 \text{ and} \quad (45a)$$

$$B \geq 2A \text{ for } A > 1 \quad (45b)$$

If  $A$  and  $B$  are of unlike sign, no extremal value can occur. Testing the sign of the second derivative of the expression given by Equation (41) we find that the extremal values which occur when the above inequalities are satisfied are minimal. The shaded area in Figure (9) indicates the range of values of density ratios and impedance ratios of the two materials for which a minimum in the sonic speed will occur.

This analysis can be applied to two perfect gasses, each satisfying the adiabatic pressure volume relationship

$$\frac{P_1}{P_{01}} = \left( \frac{\rho_1}{\rho_{01}} \right)^{\gamma_1} \quad (46)$$

which leads to sonic velocities given by

$$C_1^2 = \frac{P_{01}}{\rho_{01}} \gamma_1 \quad (47)$$

For two such gasses, the ratio of impedances is

$$\frac{\rho_{01} C_1}{\rho_{02} C_2} = \sqrt{\frac{\gamma_1 \rho_{01}}{\gamma_2 \rho_{02}}} \quad (48)$$

Thus, for two gasses having the same  $\gamma$ , the parameters A and B defined by Equations (43a) and (43b) are identical and the sonic velocity of the mixture becomes

$$\frac{C_{ave}^2}{C_1} = f_{m1} + \frac{\rho_{01}}{\rho_{02}} f_{m2} \quad (49a)$$

or

$$C_{ave}^2 = \gamma P_0 \left\{ \frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}} \right\} = \frac{\gamma P_0}{\rho_{0,ave}} \quad (49b)$$

i.e., a rule of mixtures. It should be noted, however, that the rule of mixtures applies to the squares of the sonic velocities. If, however, the two gasses do not have the same  $\gamma$ , then

$$B = \frac{\gamma_1 \rho_{01}}{\gamma_2 \rho_{02}} - 1, \quad A = \frac{\rho_{01}}{\rho_{02}} - 1 \quad (50)$$

Thus, if the denser material has a larger  $\gamma$ , a minimum in the sonic velocity of the mixture is possible and will occur if inequality (45b) is satisfied.

One other special case is of interest. If any two materials have the same impedance, Z, then  $B = 0$  and

$$C_{ave} = Z \left( \frac{f_{m1}}{\rho_{01}} + \frac{f_{m2}}{\rho_{02}} \right) = f_{m1} C_1 + f_{m2} C_2 \quad (51)$$

i.e., a rule of mixtures for the velocities.

#### IV SUMMARY

A simple theory for predicting the pressure density relationship for a homogeneous composite under shock loading was developed and compared with some available experimental data. The good agreement between theory and experiment suggests that the simple theory may have application to composites of this type. Since this theory which assumes the composite to have no geometrical structure leads, in the one case considered, to results surprisingly similar to prediction from a theory for laminated composites, it may be that the influence of structure is not great and a simple theory can be applied with at least fair accuracy to composites which do display a geometrical structure. This possibility should be considered further.

The new theory was compared with experimental data for highly porous materials by treating the porous material as a homogeneous composite of metal and air. The comparison shows agreement which is qualitatively good, but only in some cases are the predictions entirely satisfactory.

Conditions under which the sonic speed of a mixture can be less than that of either constituent are developed from the well-known expression for the sonic speed. For a non-reacting mixture of two phases, such a minimum can occur if certain inequalities relating the ratio of densities and the ratio of impedances are satisfied. It is necessary, but by no means sufficient, that the denser material have the greater impedance.

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The author is indebted to Mr. Alan Hopkins, a graduate student in the School of Engineering, Air Force Institute of Technology, who performed the computations for this study and is presently engaged in an experimental evaluation of this theory.

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Table I

## COMPOSITIONS AND DENSITIES FOR COPPER-TUNGSTEN MIXTURES (Elkonite)

<u>Designation</u>	<u>% Cu (By wt)</u>	<u>% W (By wt)</u>	<u>Theoretical Density <math>\rho_0</math></u>	<u>Measured Density <math>\rho_e</math></u>	<u><math>\rho_e / \rho_0</math></u>
2128C	74.5	25.5	10.35	9.69	.935
1W3	45	55	12.68	12.40	.978
3W3	31	69	14.05	13.80	.980E
10W3	24	76	15.09	14.85	.983

TABLE II COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL RESULTS FOR ELKONITE

Nominal Composition Cu/W - Mass Fraction	N Number of Samples	$Q_1$	$Q_2$	$Q_3$	$Q_4$
74.5/ 25.5	18	.0138	.0563	.0564	.00851
45/ 55	13	.0070	.0286	.0284	.01151
32/ 68	5	.044	.0361	.0105	.00624
24/ 76	16	.0040	.0186	.0170	.00573

## LIST OF ILLUSTRATIONS

- Figure 1 Pressure Density Relationship for a Copper-Polyethelene Mixture
- Figure 2 Shock Velocity as a Function of Composition for a Copper-Polyethelene Mixture
- Figure 3 Shock Speed vs Particle Velocity for Aluminum-Polymethylmethacrylate by Two Theories
- Figure 4 Comparison of Three Theoretical Pressure Density Relationships for a Porous Material
- Figure 5 Experimental and Theoretical Values of Shock Pressure vs Density for Elkonite
- Figure 6 Comparison of Theory with Data for Porous Copper from McQueen, et al
- Figure 7 Comparison of Theory with Data for Porous Aluminum
- Figure 8 Comparison of Theory with Data for Several Porous Materials
- Figure 9 Density and Impedance Ratios for which Minima in Sonic Speeds Can Occur

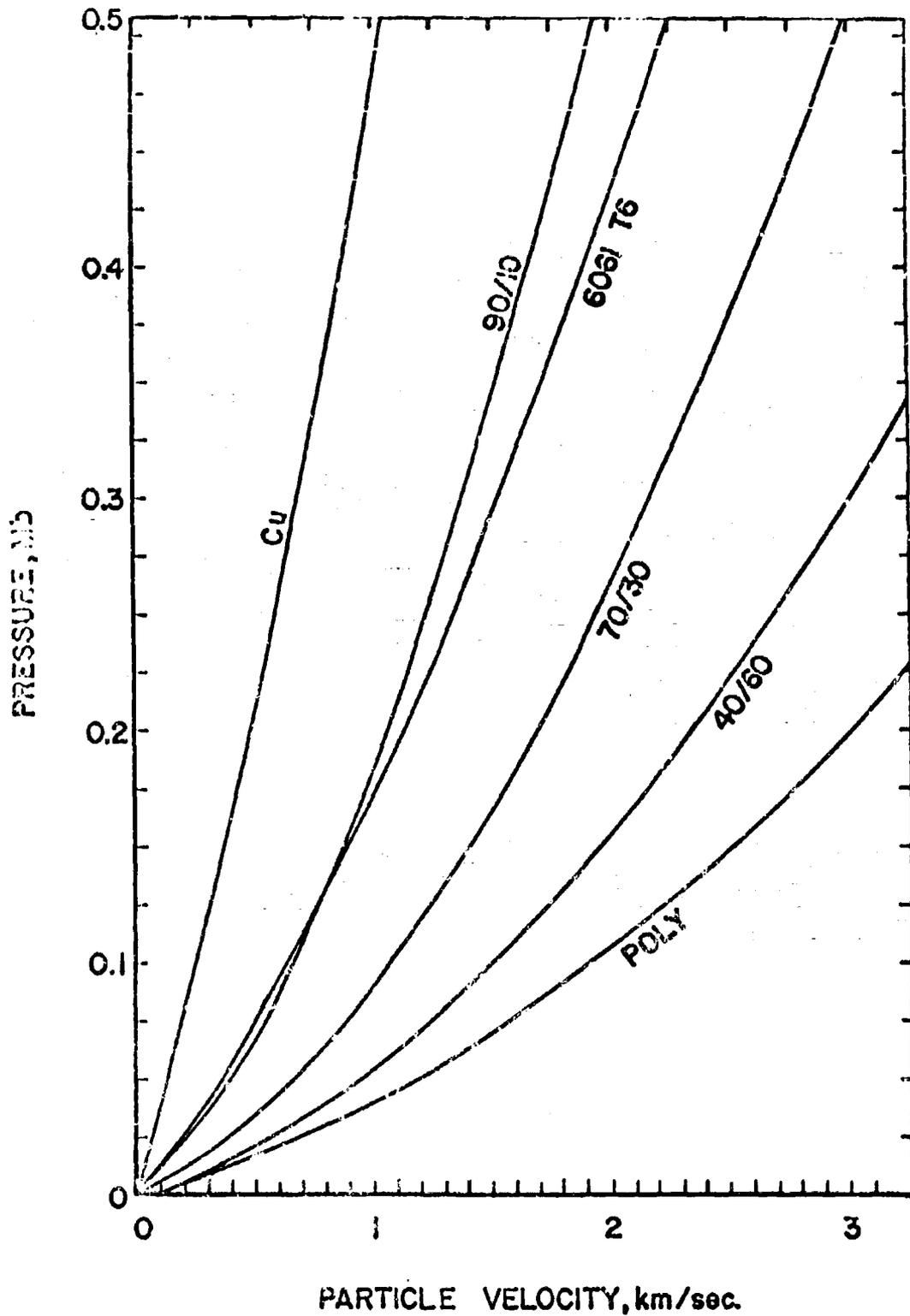
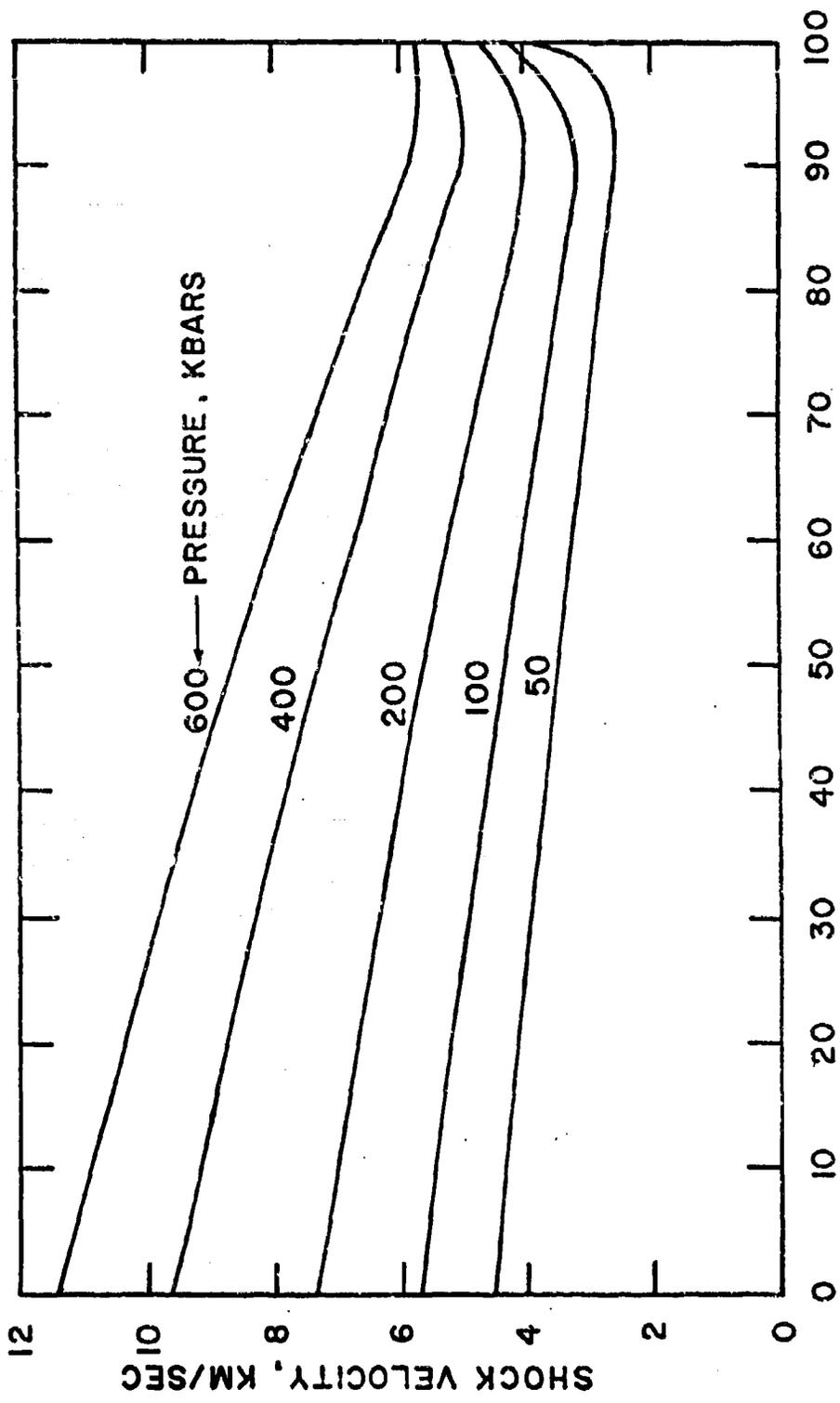


Figure 1 Pressure Density Relationship for a Copper-Polyethylene Mixture



WEIGHT PERCENT COPPER IN POLYETHELENE

Figure 2 Shock Velocity as a Function of Composition for a Copper-

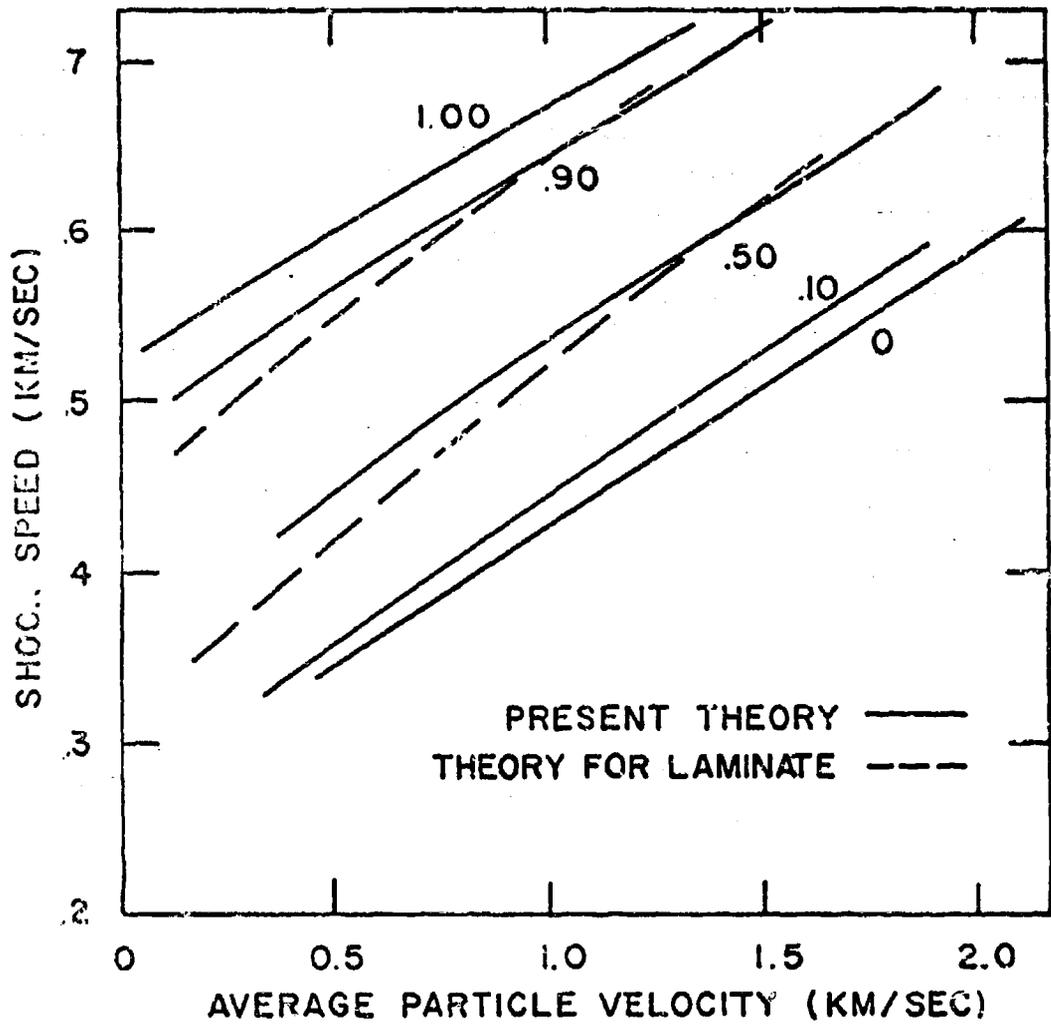


Figure 3 Shock Speed vs Particle Velocity for Aluminum-Polymethylmethacrylate by Two Theories

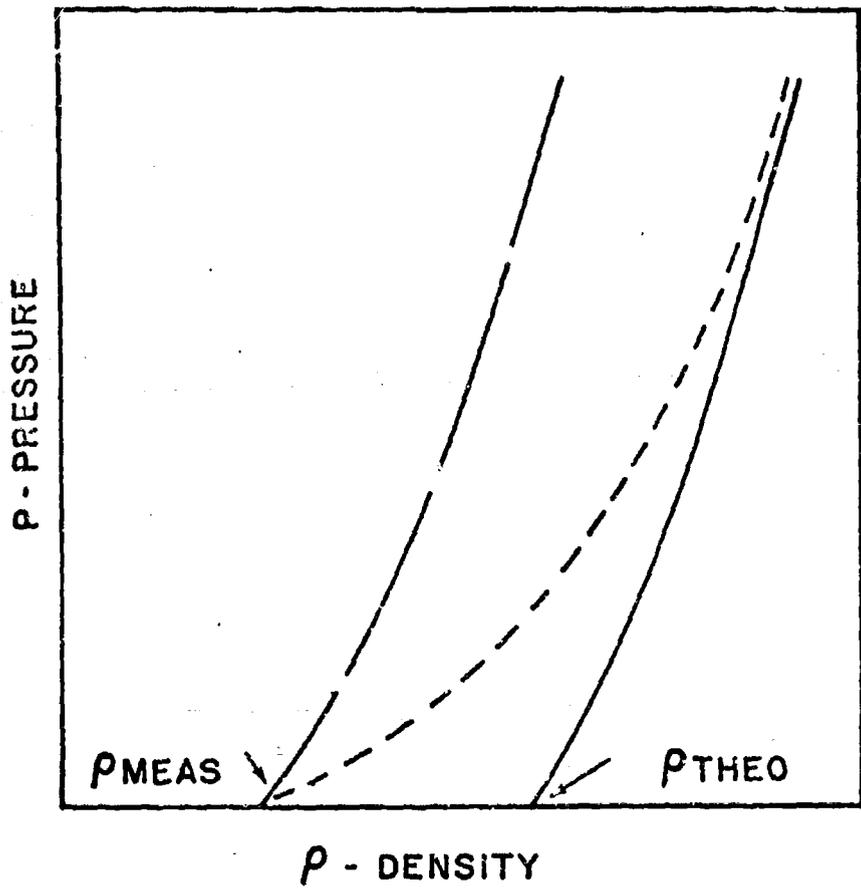


Figure 4 Comparison of Three Theoretical Pressure Density Relationships for a Porous Material

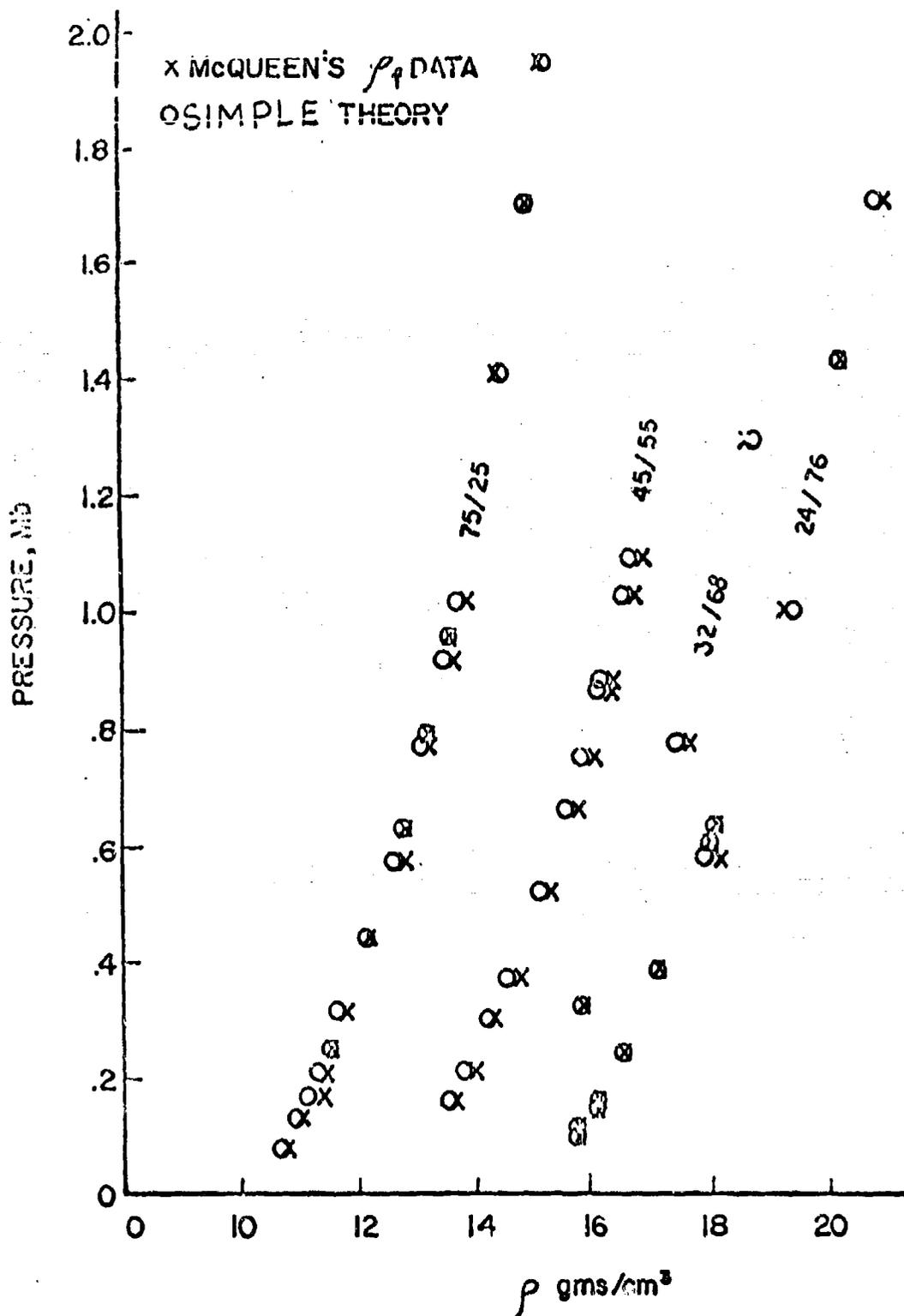


Figure 5

Experimental and Theoretical Values of Shock Pressure vs Density for Al<sub>2</sub>O<sub>3</sub>

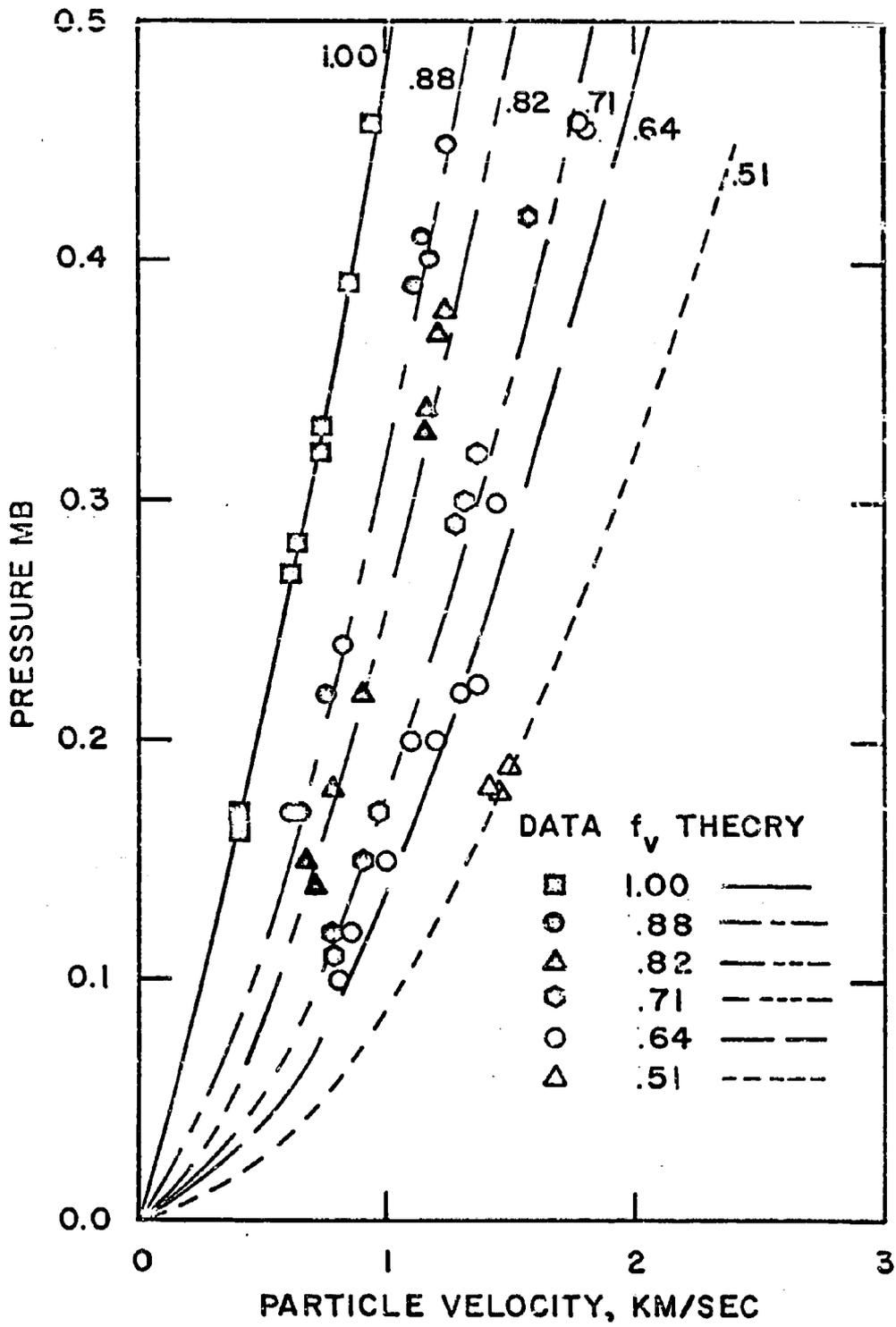


Figure 6

Comparison of Theory with Data for Porous Copper from McQueen, et al

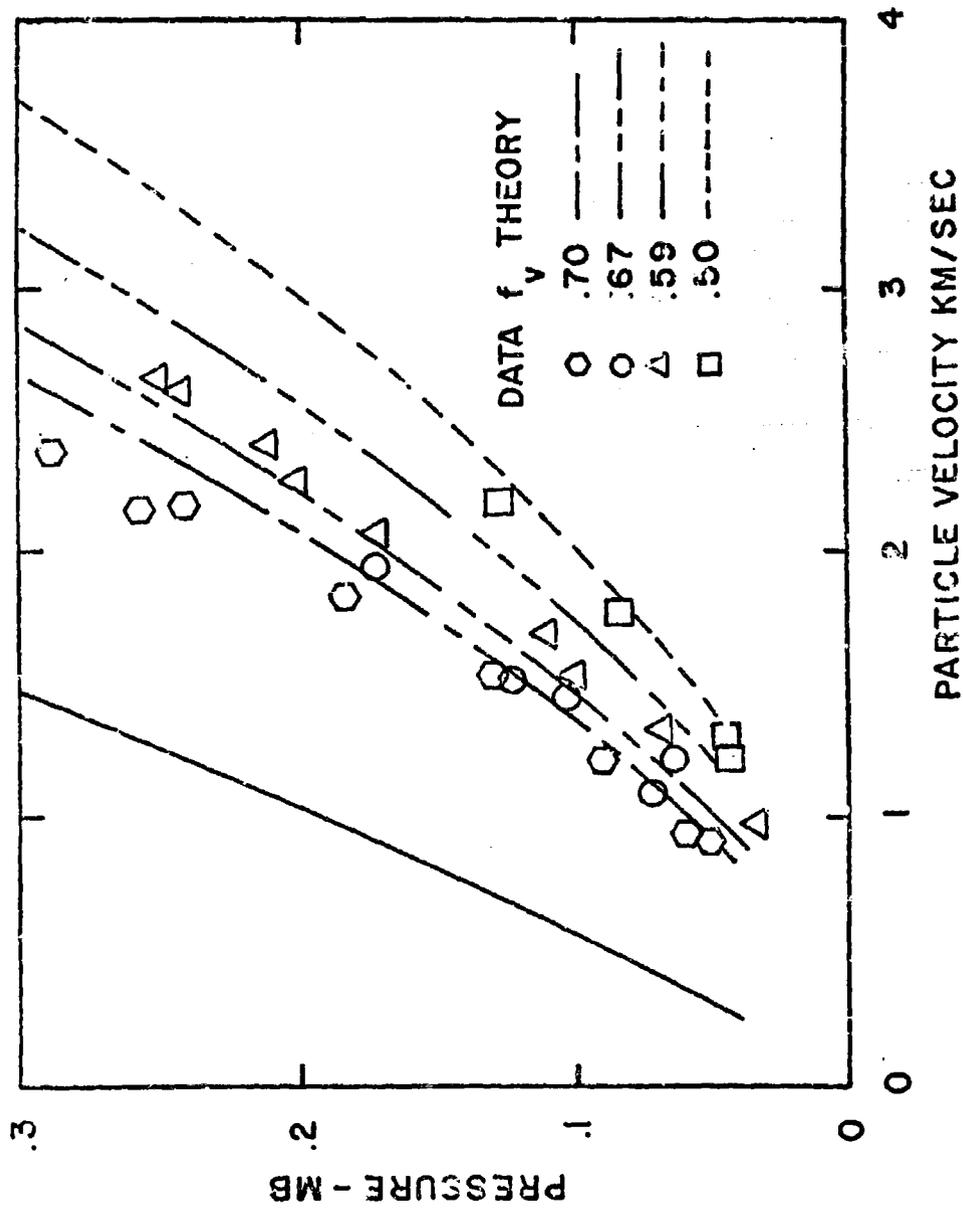


Figure 7 Comparison of Theory with Data for Porous Aluminum

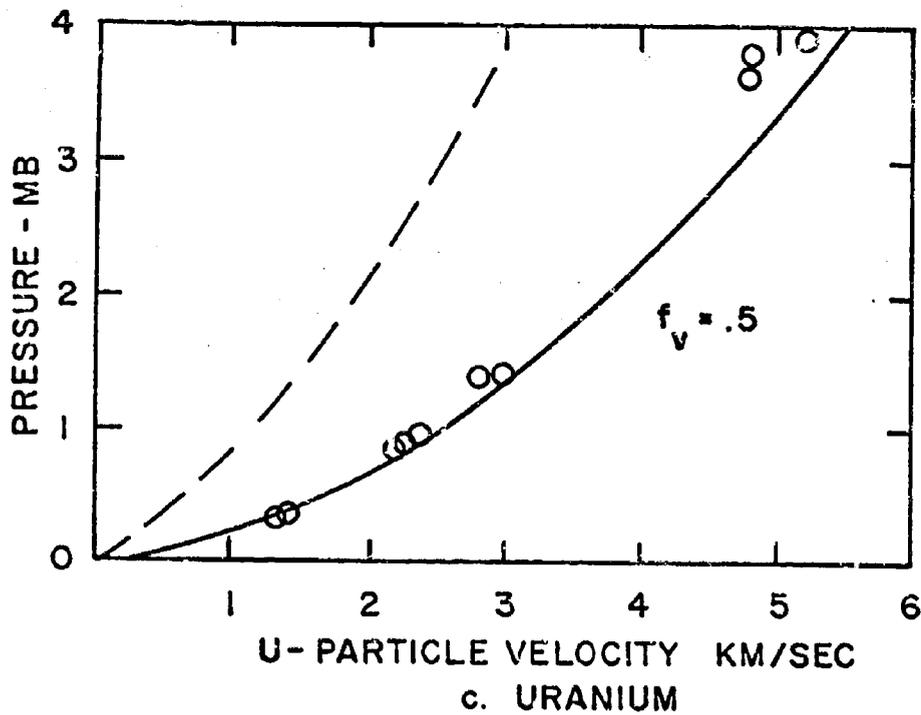
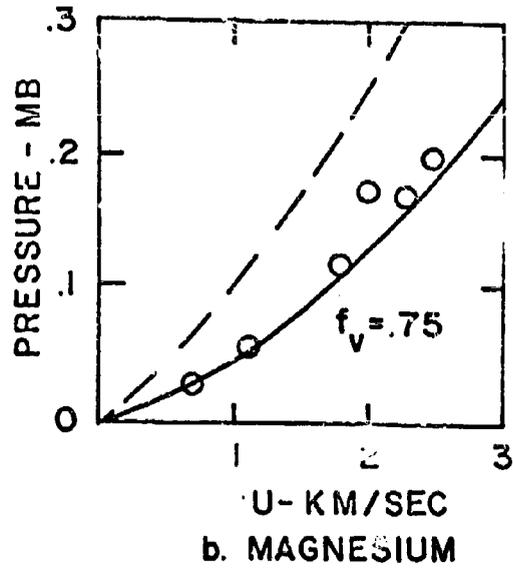
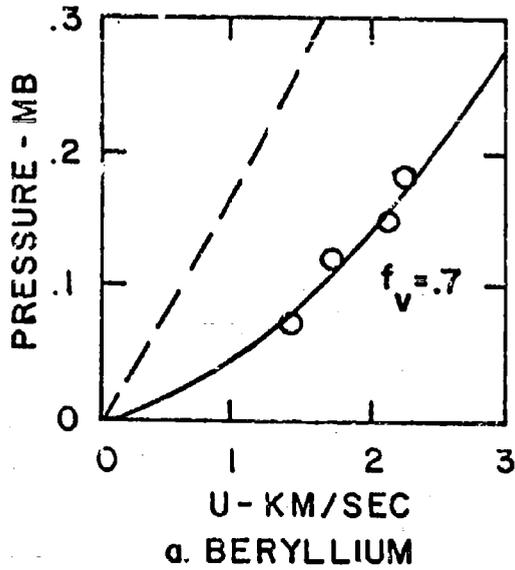


Figure 8 Comparison of Theory with Data for Several Porous Materials

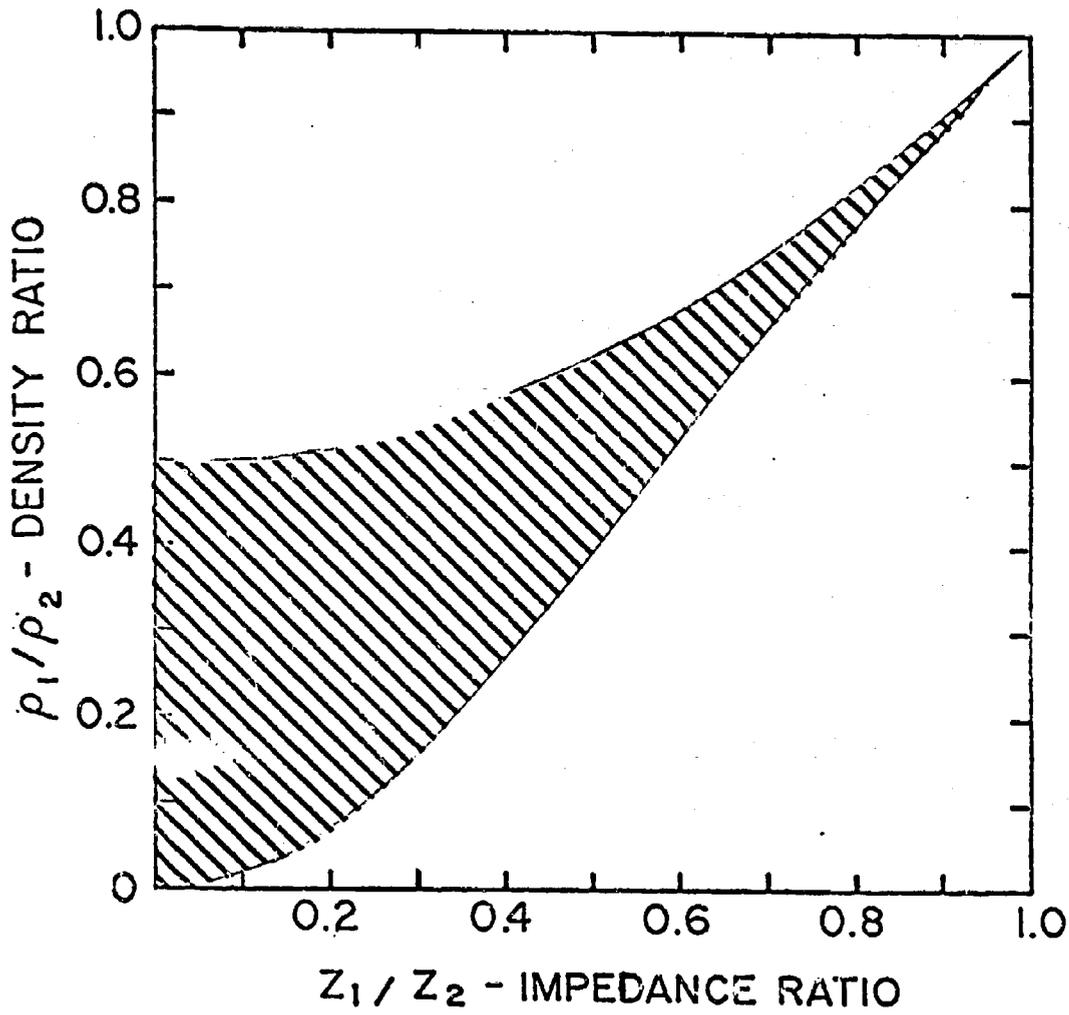


Figure 9

Density and Impedance Ratios for which Minima in Sonic Speeds Can Occur

TORVIR  
FIG. 9

**Unclassified**

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13. ABSTRACT

A Hugoniot pressure density relationship for homogeneous mixtures is calculated from the Hugoniot pressure density relationship of the constituents. The calculated values are found to be in reasonable agreement with available experimental data on mixtures of solids and porous materials.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
<b>Shock Propagation</b> <b>Shock Admitt</b> <b>Magmat</b> <b>Mixtures</b> <b>Composite Materials</b>						

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