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Department of Naval Architecture and Marine Engineering

Cambridge, Massachusetts 02139

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WORKSHOP ON LIFTING-SURFACE THEORY IN SHIP HYDRODYNAMICS

Edited by

Bruce D. Cox

March, 1970

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13. ABSTRACT

The proceedings of a three day workshop on lifting-surface theory in ship hydrodynamics are presented in summary. Prepared papers and informal contributions are given on the subjects of steady and unsteady propellers, interaction effects, and cavitating flows. This is intended to document the essential ideas discussed and provide a current reference listing for researchers in this field.

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PREFACE

The idea of a workshop on lifting-surface theory in ship hydrodynamics came about as a result of an informal discussion among several of the participants at the Seventh Office of Naval Research Symposium on Naval Hydrodynamics held in Rome in August, 1968. The aim was to stimulate free transfer of ideas among those actively engaged in research in this field, particularly for the purpose of setting directions for future work.

The meeting was sponsored jointly by the Royal Netherlands Navy and the U.S. Office of Naval Research, and the great interest and effort of Ir. J.M. Dirkzwager and Mr. Ralph D. Cooper of these two organizations is greatfully acknowledged.

The three-day workshop was held at the Massachusetts Institute of Technology on April 30 to May 2, 1969, and the report which follows is a compendium of the workshop proceedings. It is intended to document the essential ideas presented and to provide a current reference listing. A complete record of the sessions was not attempted in view of the informal atmosphere, and the number of pages devoted to each topic does not necessarily correspond to the amount of time spent during the workshop. In particular, we have not attempted to reproduce in detail material which appears in recent publications, but have given appropriate references to these instances. Finally, in the interest of early publication of these proceedings, the participants have not been given the opportunity to edit the summaries of their contribution to this workshop. We would, therefore, suggest that subsequent references to the work of any of the participants be based on their published work rather than on the informal summaries contained herein.

B. D. Cox
J. E. Kerwin
J. N. Newman

Cambridge, Massachusetts
March 1970

LIST OF PARTICIPANTS

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
WASHINGTON, D.C.

Dr. W.B. Morgan
Dr. P. Pien
Dr. G. Chertock
Dr. M. Strasberg
Dr. Keith Kierney
Mr. J.B. Hadler
Mr. J. Cox
Mr. H. Cheng

DAVIDSON LABORATORY
HOBOKEN, NEW JERSEY

Dr. J.P. Breslin
Dr. S. Tsakonas

HYDRONAUTICS, INC.
LAUREL, MARYLAND

Mr. Marshall Tulin
Mr. C.C. Hsu

McDONNELL DOUGLAS AIRCRAFT CO.
LONG BEACH, CALIFORNIA

Dr. John Hess

CORNELL AERO LABORATORY
BUFFALO, NEW YORK

Mr. J.C. Erickson, Jr.

SAGE ACTION, INC.
ITHACA, NEW YORK

Dr. D.E. Ordway

U.S. NAVAL ORDNANCE TEST STATION
PASADENA, CALIFORNIA

Dr. David M. Nelson

List of Participants

BOLT, BERANEK AND NEWMAN, INC.
CAMBRIDGE, MASS.

Dr. N. Brown

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS.

Prof. J.E. Kerwin
Prof. J.N. Newman
Prof. J.H. Milgram
Prof. P. Leehey
Prof. S. Widnall
Prof. D. Cummings
Mr. B.D. Cox

CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

Prof. T.Y. Wu

PENNSYLVANIA STATE UNIVERSITY
STATE COLLEGE, PENNSYLVANIA

Prof. Barnes McCormick
Prof. Robert E. Henderson

UNIVERSITY OF MICHIGAN
ANN ARBOR, MICHIGAN

Prof. Finn C. Michelsen
Prof. William Grable

THE CATHOLIC UNIVERSITY OF AMERICA
WASHINGTON, D.C.

Prof. D.P. Wang

THE NETHERLANDS SHIP MODEL BASIN
WAGENINGEN, THE NETHERLANDS

Prof. Dr. J.D. Van Manen
Dr. G. Kuiper
Mr. M.W.C. Oosterveld

MATHEMATISCH INSTITUUT
GRONINGEN, THE NETHERLANDS

Prof. Dr. J.A. Sparenberg

List of Participants

TECHNISCHE HOGESCHOOL DELFT
DELFT, THE NETHERLANDS

Prof. R. Timman

INSTITUT FÜR SCHIFFBAU DER UNIVERSITÄT HAMBURG
HAMBURG, GERMANY

Mr. H. Thieme

INST. MEKANIK I., ROYAL INSTITUTE OF TECHNOLOGY
STOCKHOLM, SWEDEN

Prof. M. Landahl

STATENS SKEPPSPROVNINGSANSTALT
GÖTEBORG, SWEDEN

Mr. C.A. Johnsson

CAMPAGNE DE RESEARCHES ET D'ÉTUDES AERONAUTIQUE
PARIS, FRANCE

Dr. Daniel Fruman

NATIONAL PHYSICAL LABORATORY
FELTHAM, MIDDLESEX, ENGLAND

Dr. John English

I. STEADY FOILS AND PROPELLERS

The study of lifting surfaces in steady motion has a most fascinating history. One can trace a logical development from the earliest notions of two-dimensional airfoil flow to the vortex assembly in lifting flow as put forth by Lanchester, and subsequently, Prandtl's lifting line approximation for high aspect ratio wings. At the same time experimental results and practical requirements have guided the theoretical advances and pointed up new directions. Consequently, within perhaps the last twenty-five years, an intensive effort has been devoted to the more general lifting surface and associated effects such as cavitation, a free surface, and mutual interference. In ship hydrodynamics this is dramatically illustrated, for example, in propeller theory.

The last decade has been marked by the advent of two powerful tools--numerical analysis on high speed computers and formal perturbation expansion methods. These have provided a systematic means of studying a host of otherwise intractable problems. For instance, in an iteration scheme, the precision of the approximations is often a matter of choice. The widespread use of computers and expansion techniques is reflected in many of the contributions at this workshop.

The present session concerns the application of steady state lifting surface methods to different propeller problems:

1. Conventional Propeller Design

The "design problem" in lifting surface theory consists of determining the foil geometry for a given set of forces and load distributions. In moderately loaded propeller design, this is performed in two steps. First a lifting line calculation, usually according to the induction factor method, establishes the spanwise load distribution and approximate wake geometry. Using this information, a lifting surface calculation, based on a chosen chordwise load distribution (e.g. NACA a=.8 mean line), defines the required section shapes and planforms. Fig. I-1 illustrates how the requirement of propeller revolutions is satisfied by such a procedure (Johnsson 1965, Cheng 1964, 1965, and Morgan, et al., 1968). Blade thickness has been accounted for by source distributions, but no allowance for the influence of the hub is included.

Dr. Morgan raised the question of numerical methods, offering Fig. I-2 (Morgan, et al., 1968) as an example of the discrepancy between the discrete vortex element model and continuous modes. In the latter, a careful treatment of the singular integrals is necessary, while the discrete element method includes this implicitly. Professor Milgram noted the rapid computing time for the design problem (one minute on IBM 360) using lattice schemes and cited the remarkably close agreement between his yacht sail lifting surface program and Professor Kerwin's propeller calculation (for details see Milgram, 1968). Further appeal in this method lies in working directly with the vorticity strength instead of mode series coefficients. A detailed account of recent experience with several numerical methods has been given by Landahl, 1968.

The points far off in Fig. I-2 indicate the need for a heavily loaded propeller theory. It is generally recognized that the propeller race under these conditions can no longer be modeled as helical surfaces of free vorticity. Instead the trailing sheets roll up, drastically altering the downwash, for example on nearby blades. Recently, a vortex lattice method has been developed (Cummings 1968, Kerwin and Cummings 1968) in which the deformed wake geometry is sought by iteration. An alternative concept, based on the acceleration potential, has been put forth by Pien (1968) as a second-order theory. The question of the optimum circulation distribution remains open, since the classical Betz and Goldstein solutions must now be abandoned.

2. Contra-Rotating and Ducted Propeller Design

As examples of lifting surface theory applied to unconventional propeller configuration, results for contra-rotating and ducted propeller designs are presented in Fig. I-3 (Lindgren, et.al. 1968). For the former, two different lifting line approaches have been used. Suprisingly, the effective pitch obtained is almost as accurate as in the single propeller case, despite the approximation for mutual interference. Usually a time average value is taken for the induced velocity evaluated at both propellers. Hence, the problem is treated using steady flow methods.

In the case of ducted propellers, Mr. Johnsson noted the success of effective pitch prediction when duct thrust is moderate and separation is avoided. In order to properly calculate blade section camber, it is necessary to include the duct induced flow curvature as shown in Fig. I-4. (Johnsson, et al, 1968). This has been accomplished by modelling the duct as a thin ring wing when linearized theory is applicable (Morgan 1961).

An interesting example of cavitating ducted propellers with wide blades was presented by Mr. J. English. In this case, the cavity blockage tends to reduce the axial velocity into the blades in opposition to the influence of the accelerating duct.

3. Analysis of a Given Propeller

There are at least two principal objectives in formulating a procedure for analyzing a given lifting surface. If the procedure is fundamentally similar to the inverse, then it provides a powerful check on design theory without extensive model testing. Furthermore, it offers the means to predict the point of cavitation inception.

Existing methods for treating this problem are based on the Goldstein function to relate the sectional lifting coefficients to the geometrical and induced flow angles. Results for two propellers are shown in Fig. I-5 (Johnsson 1968). Here the assumption was made that the lift slope is proportional to a curved-flow camber connection from the lifting surface calculation. A similar approach was applied to the computation of the Troost B-Series Propellers (Kerwin 1959).

4. Prediction of Propeller Cavitation

It is well recognized that conventional propellers designed for fully wetted operation perform inadequately upon cavitation inception. One may, in principle, predict the onset of bubble cavitation, as noted earlier, by solving the direct problem for the blade chordwise pressure distribution and angle of attack. In addition, the cavitation pressure must be estimated. Results of calculations of this kind and comparison with experiment are shown in Fig. I-6 (Johnsson 1968).

A satisfactory potential-flow theory for predicting the inception and extent of sheet cavitation has not been developed. Indeed, there is some doubt as to the validity of representing a partially cavitating blade, for example, by a vortex system. The results of an approximate calculation using thick two-dimensional airfoil theory to model the cavity sheet at the leading edge are shown in Fig. I-7 (Johnsson 1968). Predictions of the cavitation patterns are given in Fig. I-8. Further theoretical work in this connection might be directed toward more acceptable models of the free vortex core.

5. Free-space Propeller Pressure Field

Steady lifting surface theory has been readily extended to map the fluctuating pressure and velocity field near a propeller in a uniform free stream. This will prove to be an invaluable tool for solving interaction problems.

A complete calculation based on the work of Kerwin and Leopold (1964) includes loading and thickness effects.

Denny (1967) has compared this and the results of Breslin (1964) with experiment as shown in Fig. I-9, 10. The S.S.P.A. calculations shown do not include the influence of the free trailing vortex which is seen to be small.

II AND III. UNSTEADY FOILS AND PROPELLERS

FOILS

In many current applications of lifting surfaces in ship hydrodynamics, the fundamental phenomena are associated with flow nonuniformity, arbitrary foil motion, or both. A knowledge of the fluid mechanics has provided, for example, the basis for related studies in body vibration and acoustics. In the cases of unconventional propulsion systems, this may be particularly important.

Flows which are non-stationary in an inertial reference frame are often approximated in some time-average sense as a first estimate. In certain circumstances, however, this can be misleading and even disastrous. A classic example is wing flutter response to small free stream disturbances. Early investigators attempted to account for mild unsteadiness by including small harmonic perturbations in the unsteady linearized theory. This approach, dating back to the 1930's, is the basis of almost all modern theories.

Professor Landahl briefly reviewed the state of the art of numerical analysis in unsteady lifting surface theory. For a comprehensive survey of this subject and an extensive bibliography the reader is referred to Landahl (1968). There are several mathematical formulations of the linearized planar oscillating wing problem. On the mean surface, the normal velocity is specified. Off the wing surface and at the trailing edges, the loading is zero. This is a mixed boundary value problem leading to an integral equation for the velocity potential, pressure, or other related function. The principal task lies in selecting the load distribution functions whose coefficients are unknown, and devising an efficient numerical scheme for evaluating the resulting integrals. The choice of functions has been guided by the known results of steady two-dimensional and lifting line theories. The unknown coefficients are then determined by satisfying the tangency condition in a suitably approximate fashion (collocation, least squares, or variational methods, for example). The chordwise integration is usually performed first in which the square root singularity at the leading edge can be properly accounted for. The more difficult spanwise integration is discussed in the cited references.

For the subsonic regime, numerical calculations of these kinds and comparison with experimental and analytical results (van Spiegel, 1959) are quite good. Although numerical methods are capable of high accuracy, computational time grows if many modes of loading are needed, as for instance, in non-planar configurations.

PROPELLERS

The working of a propeller in a ship's wake is a spectacular example of unsteady lifting surface flow with non-planar geometry. Early quasi-steady and empirical calculations have been surpassed in recent years with comprehensive mathematical models based on the fundamental theory referred to earlier. At least for conventional propellers, there now exist several computer programs capable of computing vibratory forces for a given wake pattern. A complete potential flow model of the hull-propeller unsteady interaction has not been successfully developed to date.

Professor Sparenberg opened the workshop discussion with some remarks concerning vertical axis propeller, which is a non-stationary problem regardless of inflow. This has been treated in an approximate fashion by considering the path traversed by one of the oscillating blades. An optimization is possible by minimizing the kinetic energy of the trailing free vortex sheet. The reader is referred to Sparenberg (1960, 1967, 1968) for specific details.

Two recently developed approaches to unsteady propellers were briefly mentioned by Doctors P. Pien and Neal Brown. Pien's method rests on the acceleration-potential formulation, in which the unknown slip-stream geometry is not required. This theory is put forth as valid to second order in induced velocity, although the intensive workshop discussion on this point implied that further justification is necessary. In Dr. Brown's work the propeller is considered to be operating in a homogeneous turbulent stream. The calculation of such quantities as the frequency components of the thrust may then be performed by treating the process as stochastic. For a selected distribution of say flow wave number, this can be computed directly in terms of propeller characteristics.

The following discussions presented at the Workshop are concerned with the numerical evaluation of integrals encountered in the usual formulation of unsteady propeller calculations.

PROPELLER LOADING DISTRIBUTIONS

Dr. S. Tsakonas

It is gratifying to find that Netherlands Ship Model Basin has developed its own program to deal with the propeller loading problem and that their results at this stage show such good agreement with the results of Davidson Laboratory of Stevens Institute of Technology. I am sure that if the number of chordwise modes are increased, the agreement will be even better.

The latest activities at DL in this particular field can be summarized by saying that the previously developed program for evaluating the steady and time-dependent loading distributions and resultant forces and moments has been extended and modified and the numerical procedure has been closely scrutinized with a view to achieving greater accuracy.

The problem of the unsteady chordwise loading distribution, which had shown no sign of convergence even though a large number of Birnbaum modes were taken for the assumed distribution, has been tackled. This instability has been found to be attributable to the linear dependence of the Birnbaum modes and has been corrected by modifying the kernel function so as to secure linear independence of the chordwise modes.

The numerical instability observed in the systematic calculations at NSRDC for propellers of EAR > 0.9 has been attributed to overlapping of the blade wakes which had not been taken into account in the original mathematical model. A correction for wide blades which eliminates the instability is now incorporated in the development.

Originally, greater emphasis had been placed on the unsteady problem than on the steady-state case. The latter part of the numerical solution has since been studied more carefully. The discrepancies which appeared between experiment and theory have been found to be the result of numerical inaccuracies introduced in the steady-state case by the approximation in the evaluation of blade-camber effects and by truncation of the infinite series resulting from the expansion of the kernel function. The program has been modified to correct these defects.

The improvement in stability of the chordwise distribution is evident in comparing Figs. III-1a and 1b for the earlier results with Figs. III-2a and 2b. Fairly extensive calculations had shown that, in the case of the marine propeller, the coefficients $L^{(\bar{n})}$ of the Birnbaum chordwise modes remain of the same order of magnitude and also tend to a constant value C which depends on EAR, on frequency and on spatially varying inflow field. In an attempt to sum up the slowly converging series, use was made of the Cesaro summability method, which is a proper procedure for obtaining the sum of slowly convergent or even divergent series. As seen in Figs. III-1a and 1b, the resulting chordwise distribution is fairly stable after six Birnbaum modes, over the major portion of the chord, but not near the edges. The assumed distribution is here of the form

$$L^{(o)} \cot\frac{\theta}{2} + \sum_{\bar{n}=1}^{\infty} (L^{(\bar{n})} - C) \sin \bar{n}\theta + C \sum_{\bar{n}=1}^{\infty} \sin \bar{n}\theta \qquad (1)$$

where $L^{(\bar{n})} - C \to 0$ as $\bar{n} \to \infty$

The last term is a divergent series whose value exists in the Cesaro sense and is given by

$$C \sum_{\bar{n}=1}^{\infty} \sin \bar{n}\theta = \frac{C}{2} \cot \frac{\theta}{2} \qquad (2)$$

Thus, the terms of the Birnbaum series are not linearly independent, and curtailment of this infinite series leads to error.

This error has been corrected by rewriting a curtailed series

$$S_{\bar{n}} = L^{(o)} \cot \frac{\theta}{2} + \sum_{\bar{n}=1}^{M} L^{(\bar{n})} \sin \bar{n}\theta$$

using (2), as

$$S_{\bar{n}} = (L^{(o)} + \frac{C}{2}) \cot \frac{\theta}{2} + \sum_{\bar{n}=1}^{M-1} (L^{(\bar{n})} - C) \sin \bar{n}\theta$$

$$- C\left[\sum_{\bar{n}=1}^{\infty} \sin \bar{n} - \sum_{\bar{n}=1}^{M} \sin \bar{n}\theta\right] \tag{3}$$

Then, after application of the generalized lift operators to the surface integral equation which relates the lift and downwash distributions on the lifting surface, the chordwise integrations are done analytically and the surface integral can be written, for each strip and order $\bar{m}$ of lift operator mode, as products of loading coefficients and kernel functions:

$$\ell^{(o)}k^{(\bar{m},o)} + \ell^{(1)}k^{(\bar{m},1)} + \ldots + \ell^{(n)}k^{(\bar{m},n)}$$
$$- C\left[\frac{1}{2} k^{(\bar{m},o)} - \sum_{\bar{n}=1}^{M} k^{(\bar{m},\bar{n})}\right] \tag{4}$$

where $\ell^{(o)} = L^{(o)} + \frac{C}{2}$

$\ell^{(n)} = L^{(n)} - C \quad , n > 0$

and $M \geq (n + 1)$ for a solution to be possible. The last kernel element, the coefficient of C in (4), can be evaluated in one step.

The results of using (3) and (4) in evaluating the unsteady chordwise distribution for the same 3-blade propeller of EAR = 0.6 are presented in Figs. 2a and 2b, for two combinations of M and maximum n, M= 5, n max 2 and M = 10, n max = 4. Thus, this method secures convergence of the chordwise distribution.

That the corrections for "overlapping" of blade wakes in the case of wide blades has secured smooth stable values is attested by Figs. III-3a, 3b, 4 and 5 which present the calculated results for a range of EAR. Fig. III-3a for mean thrust coefficients also feflects the improvement in evaluation of camber effects and the modification of the program to eliminate inaccuracies due to truncation of series.

Figs. III-6 and 7 of the vibratory thrust and torque coefficients show the improvement obtained by the further refinement of dividing rhe propeller blade span into 16 strips rather than 8.

AN EXACT TREATMENT OF THE LINEARIZED LIFTING SURFACE INTEGRAL EQUATION -- Ir. G. Kuiper

We will pay some attention to the numerical solution of the lifting surface theory for a screw behind a ship.

There is a rapidly growing amount of literature on this subject. A detailed theoretical approach of the lifting surface theory was given by Yamazaki (1968). He used a linearized theory in which the fixed boundaries of hull and rudder were taken into account. In the calculations, however, many simplifications were made, so that the influence of fixed boundaries near the screw was not calculated.

At the same Symposium in Rome, Pien (1968) introduced a general theory in which the influence of the hull and rudder was neglected, but where the equations of motion were not linearized. Only a few computional results are available yet.

The greatest amount of numerical results was published by Jacobs and Tsakonas in a series of publications (1966, 1967, 1968). They started from the linearized equations of motion and used the formulation of the acceleration potential as was also formulated by Sparenberg, Hanaoka, and others. No fixed boundaries are taken into account, except at the screw blade itself.

But also the solution of the integral equation of this simplified model without friction, hull, rudder, or blade thickness and with the assumption that the induced velocities are small gives a lot of problems. Numerous assumptions have been tried, especially as to the chordwise pressure distribution and the helicoidal wake. As to the chordwise pressure distribution the most usual methods are:

1. The loading concentrated at 1/4 chord line (Weissinger model).
2. The flat plate pressure distribution (first term of the Birnbaum distribution).
3. An arbitrary pressure distribution (mostly evaluated in a Birnbaum series with a finite number of terms).

As to the solution of the kernel of the downwash integral, some methods from the unsteady wing theory can be used. Jacobs and Tsakonas use in their already mentioned results a staircase approximation of the helicoidal wake, together with an arbitrary chordwise pressure distribution. Earlier they had tested the exact treatment of the integral equation assuming a flat plate chordwise pressure distribution. The conclusion was that the exact treatment gave 2 percent higher values for thrust in the stationary case and 15 percent higher values in the unsteady case, compared with the staircase approximation.

At the NSMB a solution of the integral equation has been developed by Verbrugh (1968) in which an arbitrary chordwise pressure distribution is combined with an exact treatment of the wake, which is in fact the solution of the used model. Very recently some preliminary results of this approach have become available and a very rough comparison of some results is possible now.

As an illustration of the way in which this problem can be handled, I will give a very short review of this solution.

Using cylincrical coordinates as shown in Fig. III-8, we can formulate the helicoidal surface F (eq. 1).

The acceleration potential of a rotating pressure dipole distribution $\mu(\xi,\zeta,\theta)$, lying on F with its axis normal to F, can be given by eq. 2.

We now need the velocities, induced by this dipole layer, to fulfill the boundary conditions: tangential velocities at the screw blade. The velocity potential can be found by substituting the acceleration potential in a solution of the linearized equations of motion (eq. 3).

In the unsteady linearized model we can assume that the pressure dipoles are lying on the helicoidal surface F. The pitch of this surface is chosen to give no disturbances in the mean flow. The boundary conditions are now that the wake velocities must be cancelled by the induced velocities, which can be found by differentiating the velocity potential. This finally results in eq. 4.

All assumptions are now in this equation. We have neglected the thickness of the screw blade by using a pressure dipole layer. We have assumed that the induced velocities are small by using the linearized equations of motion, which also gives the possibility of using the helicoidal surface as the screw blade. In the solution of the linearized equations of motion, it is assumed that far before the screw the flow is undisturbed. At the same time we have restricted the direction of the flow at the screw blade only, which implies the absence of other boundaries. We also have summed up the influence of more blades.

The unknown pressure dipole distribution must be solved now. It is not my intention to treat the solution from the beginning to the end, but I will indicate the different steps that are taken.

The intention is to solve this integral numerically, which means that we have to separate the singularities in it. The kernel of eq. 4 has strong singularities of the Hadamard type, which can be separated by approaching the singular points from a surface slightly different from the helicoidal surface and by integrating over a small strip of the screw blade, in which the singularity occurs. The integral equation can be written as eq. 5, Fig. III-9.

The kernel cannot be integrated numerically yet because of the limit, while it is also not suited for an analytical treatment. Many of the simplifications or approximations are made to be able to treat the integral analytically, to find a formulation which cancels the separated mean value and makes further numerical treatment possible.

At the NSMB the character of the singularities in the kernel for $r = \rho$ or $\beta = 0$ were analysed, mainly by evaluation into power series. When we know the character of the singularities, we can subtract a simplified function with the same singularity from the integrand of the kernel. This leaves an integrand under the integration sign which even has more terms than the original integrand, but which is regular for $\beta = 0$.

The subtracted terms have to be added separately. They are, however, more simple than the original terms and can be integrated analytically. In this way we can separate the kernel in two parts, a complicated, but regular part, which can be integrated numerically, and a part which is singular, but which is integrated analytically.

As to the following calculations, we will be short. The evaluation of the chordwise pressure distribution in a Birnbaum series (eq. 6), makes it possible to separate the chordwise and the spanwise integration. This is expressed in eq. 7. The integration of the regular part of the kernel is included in the function $F_{3,p}$.

Spanwise integration is now done by the collocation method. Integration of the first term of eq. 7 from $-\beta$ to β just cancels the separated mean value.

In this way the integral equation can be reduced to a set of linear equations. When we write the wake ASA Fourier series we can consider each harmonic separately.

The summation over p in eq. 8 is introduced by the chordwise integration, the summation over l by the spanwise integration. When we realize the complicated nature of the function F_p, it is clear that the computation of the equation for an increasing number of points over the screw blade very rapidly increases the computing time.

NUMERICAL RESULTS

The results that have been obtained are very preliminary. The program is being tested at the moment and due to the complexity of the problem this takes much time.

The first thing that should be tested is the convergence of the chordwise and spanwise pressure distribution. We will look at the third harmonic of a three blade screw.

Fig. III-10 A criterion for the chordwise convergence is the convergence of the Birnbaum coefficients. As is shown, a nice convergence is found, so a restricted number of chordwise modes gives a satisying result.

Fig. III-11 In this figure, two spanwise lift distributions are drawn with resp. 6 and 8 spanwise control points. The difference can be neglected.

Fig. III-12 does the same for the chordwise pressure distribution for two radii. As could be expected after what was seen in Fig. III-10, the difference after more than 4 chordwise control points can be neglected.

Fig. III-13 shows the amplitude over the radius of the third harmonic of the wake which was used. This wake was generated in a tank by a three bladed "cross plate" wake generator. Measurements on screws in thus generated wakes were done by Wereldsma (1966) at the NSMB. The maximum amplitude of this harmonic is about 10 percent of the mean velocity.

Fig. III-14 shows the amplitude of the unsteady pressure distribution over the screw blade. The already mentioned measurements of Wereldsma are a nice testcase for comparison because at Davidson Laboratory, Jacobs and Tsakonas have compared their computational results with these measurements. In the next figures, for the unsteady cases, the results of the NSMB are given with only one chordwise control point, the flat plate distribution. Due to the lack of time they could not be run for more chordwise modes any more. Still, a rough comparison is possible.

Fig. III-15 gives a comparison of the thrust coefficients measured by Wereldsma, computed by Davidson Lab. and by the NSMB.

Fig. III-16 gives the torque coefficients for these cases. The trends between the measured and computed results are the same. The values are however rather different between measurements and calculations. As to the comparison of the two calculations, no other conclusion should be drawn yet than that the results are in good accordance with each other. The necessary accuracy of different integrals and evaluations is still tested at the NSMB and this might still change the results for a few percents. Besides, the results are for a flat plate pressure distribution only.

Fig. III-17 and III-18 give the results of a comparison at a lower advance ratio to test the influence of that parameter.

Some computations are shown for the steady case on a screw of the B-series with 3 blades and a blade-area ratio of 0.50.

Fig. III-19 shows the computed and measured thrust coefficient for different advance ratios. In these computations four terms of the Birnbaum series are taken into account, which is enough for a good convergence.

It can be seen that the influence of the linearization appears at $J = 0.8$.

Fig. III-20 gives the torque coefficients of the same B 3-50 screw. As could be expected the torque is much more sensible to linearization. Without a friction and induced velocities, the efficiency is always one.

Fig. III-21 shows the pressure distribution in the steady case. It can be seen that the entrance is nearly shockfree at the root, which becomes less at the tip of the blade.

For the unsteady case, Jacobs and Tsakonas (1967) also compared their results with experiments in a screen generated wake in a tunnel, carried out at the NSRDC. The agreement between tests and computations was better than in the case of the NSMB measurements. We, therefore, make a comparison for these cases, too.

Fig. III-22 gives the thrust coefficients for the third harmonic.

Fig. III-23 gives the torque coefficients of those cases. Again the computational results of the NSMB fairly good agree with the results of Davidson Lab. This reveals that, to be able to test the reliability of the theory and to use the programs for prediction of vibratory forces, reliable and accurate measurements of the wake behind a ship are necessary.

IV. INTERACTION EFFECTS

Lifting surfaces rarely operate in an environment free from neighboring interferences. This is particularly true in ship hydrodynamics where a surface of constant pressure is in close proximity. The classical propeller-hull interaction problem has plagued naval architects probably since the first propeller vibrated. Lately, the trend toward higher speed and power and more flexible maneuvering has amplified the interaction problem more than ever.

The interaction of a wing and tip vortex is being studied experimentally by Professor McCormick. Full scale observations on an airplane have been recorded, including vortex velocity measurements (shown in motion picture). This has been supplemented with tests of model rotating blades (1500 rpm) passing through a vortex and recording the pressure variation with distance to the core.

As an example of the widespread existence of interference effects in ship hydrodynamics, Professor Newman described his recent work on the lateral flow past a slender ship hull in shallow water. Using the simple image approximation for the free surface (valid at low Froude numbers), the problem is equivalent to the interference effects between a low-aspect ratio foil and closely-spaced walls which are perpendicular to the spanwise coordinate. The immediate application of this work is to maneuvering characteristics in shallow water, but if the slender-body restriction is removed, it is relevant also to the gap effects in a ducted propeller. A brief outline of the analysis is given below, and details may be obtained from Newman, 1969.

Due to the proximity of the ship's bottom to the bottom of the water, the usual strip-theory approximation to the cross-flow is not valid. Part of the flow is diverted around the ends, so that the flow is three-dimensional, but a complete solution can be obtained for slender bodies (or small values of the draft-length ratio) from a matching procedure. The inner solution is the cross-flow problem, with a lateral streaming flow past the two-dimensional section of the hull, and its image above the free surface, and constrained by the bottom and its image, but the stream velocity in this inner region is unknown because the flux of fluid around the body ends is not determined at this point. In the outer problem the body reduces to a line of singularities and the flow is two-dimensional in the horizontal plane, constrained by the bottom of the fluid and its image above the free surface. Part of this outer flow passes around the ends of the body and part passes "through" the body (the latter portion being the flux which passes through the inner region). Matching of the inner and outer solutions yields an integral equation identical to the Prandtl lifting-line equation of aerodynamics, the solution of which determines the unknown stream velocity in the inner region. In this manner expressions have been found for the added mass, added moment of inertia, side force due to yaw angle, and yaw moment due to yaw angle. The side force due to yaw angle is in reasonable agreement with recent experiments conducted by Norrbin (at the Swedish State Towing Tank) on a tanker model. Extensions of the same technique to wave problems (exciting forces in oblique waves in shallow water) are currently being developed by Tuck.

Professor Timman outlined an optimization problem concerning the hull-propeller combination. The shaft horsepower is minimized under the constraint that propeller thrust is the sum of bare hull resistance and propeller induced resistance (i.e., "thrust deduction").

The matter of vibrating forces in a propeller shaft was mentioned by Professor F. M. Lewis. Some experimental results illustrating the effects of rudder and propeller clearance have been published (Lewis, 1969).

A theoretical study of this problem, based on the linearized approximation is given in the following discussion by Dr. Tsakonas. A limited number of numerical results have been obtained.

PROPELLER-RUDDER INTERACTION

Dr. S. Tsakonas

The interaction between propeller and rudder when both lifting surfaces are immersed in spatially nonuniform flow (hull wake) has been studied on the basis of the linearized unsteady lifting surface theory. The kinematic boundary on both propeller and rudder yield a pair of surface integral equations:

$$W_p(x_p,r_p,\varphi_p;t) = \iint_{S_p} L'_p(\xi_p,\rho_p,\theta_p;t)\, K_{pp}(x_p,r_p,\varphi_p;\xi_p,\rho_p,\theta_p;t)\, d\, S_p$$

$$+ \iint_{S_R} L_R \; L'_R(\xi_R,0,\zeta_R;t)\, K_{RP}(x_p,r_p,\varphi_p;\xi_R,0,\zeta_R;t)\, d\, S_R \qquad (1)$$

$$W_R(x_R,0,z_R;t) = \iint_{S_p} L'_p(\xi_p,\rho_p,\theta_p;t)\, K_{PR}(x_R,0,z_R;\xi_p,\rho_p,\theta_p;t)\, d\, S_p$$

$$+ \iint_{S_R} L'_R(\xi_R,0,\zeta_R;t)\, K_{RR}(x_R,0,z_R;\xi_R,0,\zeta_R)\, d\, S_R \quad . \qquad (2)$$

where x_p, r_p, φ_p and ξ_p, ρ_p, θ_p: cylindrical coordinates of control and loading points, respectively. associated with the propeller

x_R, y_R, z_R and ξ_R, η_R, ζ_R: Cartesian coordinates of the corresponding points associated with the rudder

t: time, sec.

S_p, S_R: propeller and rudder surfaces, respectively, ft^2

W_p, W_R: downwash velocity distributions normal to propeller and rudder, respectively, ft/sec

L'_p, L'_R: unknown loading on propeller blade and rudder, respectively, lb/ft^2

K_{ji}: induced velocity on element i due to oscillatory load of unit amplitude located at element j, ft/lb sec

The second term on the right-hand side of Eq. (1) and the first term on the right-hand side of Eq. (2) represent the interference effect of the rudder on the propeller and of the propeller on the rudder, respectively. The remaining terms of Eq. (1) and (2) represent the contribution to downwash from the individual lifting surface. The kernel functions K_{PP}, K_{PR} and K_{RR} have high-order singularities with Hadamard finite contributions, whereas the cross-coupling term K_{RP} is not singular.

With oscillatory loading and prescribed downwash velocity expressed as

$$L_j'(\xi,\rho,\theta;t) = \sum_{\lambda_K=0}^{\infty} L_j'^{(\lambda_k)}(\xi,\rho,\theta)e^{i\lambda_k\Omega t}$$

$$W_j(x,r,\varphi;t) = \sum_{q=0}^{\infty} W_j^{(q)}(x,r,\varphi)e^{iq\Omega t}$$

it can be shown that Eqs. (1) and (2) are frequency coupled. For each harmonic q of the wake, Eq. (1) can be written, after the chordwise integrations, as

$$W_p^{(q)} = \int_{\rho_p} L_p^{(q)} K_{pp}^{(m_1)}\, d\rho_p + \sum_{\lambda_2=0,N}^{\infty} \int_{\zeta_R} L_R^{(\lambda_2)} K_{RP}^{(m_2)}\, d\zeta_R \qquad (3)$$

where $m_1 = q + \ell_1 N, \quad \ell_1 = 0, \pm 1, \pm 2, \ldots.$

$m_2 = q - \lambda_2$

and Eq. (2) can be written as

$$W_R^{(q)} = \sum_{\lambda_3=0}^{\infty} \int_{\rho_p} L_p^{(\lambda_3)} K_{PR}^{(m_3)}\, d\rho_p + \int_{\zeta_R} L_R^{(q)} K_{RR}\, d\zeta_R \qquad (4)$$

where $m_3 = \lambda_3 - q$ and $q = \ell_3 N$

These equations exhibit the mechanism of the filtering effect of the propeller on the harmonic constituents of the wake which allows the rudder to be exposed only to the blade harmonic and multiples thereof.

It is seen from Eqs. (3) and (4) that a large number of harmonic constituents of the loadings participate in the interaction phenomenon, a fact which makes a direct solution by the collocation method rather impractical. An iterative procedure has therefore been developed which is not only physically plausible but advantageous computationally.

It is assumed initially that the effect of the rudder on the propeller is negligible and hence the propeller loading $L_P^{(q)}$ can be evaluated at any frequency q by the known numerical method. Substitution of these loadings into Eq. (4) reduces the latter to an integral equation with unknown $L_R^{(\ell N)}$ which can be solved for each $\ell = 0,1,2....$ Upon substituting $L_R^{(O)}$, $L_R^{(N)}$, $L_R^{(2N)}$, etc., into Eq. (3), new propeller loadings $L_P^{(q)}$ can be obtained. If these new values differ from the previous values, a new iteration is started with the new $L_P^{(q)}$. This process will continue until stable values are achieved for all the loadings.

The numerical procedure has been programmed for a high-speed digital computer and evaluates not only the steady and time-dependent pressure distributions on both lifting surfaces but the corresponding hydrodynamic forces and moments generated by the propeller and the rudder as well.

From a limited number of calculations it is found that axial clearance between propeller and rudder is a very important parameter. In fact, theory shows the oscillatory variation of blade-frequency rudder force and rudder stock moment with axial clearance which has been found in experiments. Furthermore, the inadequacy of the currently used method, based on a modification of the low aspect ratio theory, for the calculation of the steady-state rudder lateral force and rudder stock moment, is demonstrated. The failure of any semi-empirical method to take cognizance of the vibratory components of rudder force and stock moment can easily lead to erroneous conclusions with dangerous consequences.

V. LIFTING CAVITATING SURFACES--STEADY

The remarkable phenomenon of a hydrofoil operating with a fully developed trailing cavity has no counterpart in aerodynamics. Certainly the early investigators in this field could not have foreseen the extent of theoretical and experimental study which followed. Today, as a result, the "design" of supercavitating hydrofoils is possible. In essence, a new dimension in lifting surface technology has been opened.

The rich literature on cavity flows is predominantly a two-dimensional theoretical treatment and comparison with experiment. This has provided, for example, a basis for the design of propeller blades in a strip theory sense. Some considerations on the three-dimensional aspects of cavity flows are discussed below.

"Cavitating Hydrofoils of Finite Span"

Professor Patrick Leehey

Although considerable theoretical and experimental work has been done on cavitating hydrofoils of finite span, neither theory nor experiment has led us to a fully satisfactory understanding of the flow problem involved. The significant parameters affecting the force coefficients have not as yet been clearly identified.

The theoretical work falls into two basic categories. First, for foils of moderate to large aspect ratio, we have the theories of Widnall (1966), Johnson (1958), and Cumberbatch (1960). Widnall derived coupled integral equations for the velocity potential which she solved numerically under apriori assumptions as to cavity length and shape. Johnson applied well-known results from aerodynamic theory to correct for downwash and cross-flow lift. His analysis is restricted to zero cavitation numbers. Cumberbatch's work is the first attempt at matching three- and two-dimensional problems. He treats the very difficult case of heavy tip loading with separate tip and central body cavities. He was forced to make the severe assumption that the center line cavity length is independent of aspect ratio. This assumption is in conflict with experimental results.

At the other extreme, Tulin (1959), Cumberbatch and Wu (1961), and Kaplan, Goodman, and Chen (1966) treat the problem of a slender supercavitating delta wing. For this problem, the expansion is essentially directly in aspect ratio and slender body theory is used.

The available experimental work related to the former problem is typified by the work of Kermeen (1961), Schiebe and Wetzel (1961), and Scherer and Auslaender (1964). In all cases, foils of rectangular plan form and constant section were used. Aspect ratios from 0.5 to 6, with either natural cavitation or ventilation were studied. Although these experimental results are very useful from the viewpoint of practical application, they do not add greatly in establishing correspondences between theory and experiment. Many of the tests were run at large angles of attack and heavy loading. This, together with the plan forms used, led to severe tip cavitation, representing the most difficult of the theoretical problems.

Friedrichs (1966) first applied the methods of matched asymptotic expansion in inverse aspect ratio. Van Dyke (1964) showed that a solution of Prandtl's integral equation is unnecessary and such solutions are inconsistent with systematic expansions to higher order in inverse aspect ratio. His third order solution is superior to the integral equation solution for aspect ratios greater than one.

The theory of unsteady two-dimensional cavitating hydrofoils met with an earlier logical dilemma. Within the framework of two-dimensional hydrodynamics, time dependent change in cavity area led to infinite pressure at infinity. Various attempts at resolving this difficulty appear in the literature. Some, e.g., Geurst (1961), require the cavity area to be fixed. Timman (1958) proposed compensating wake sources and sinks. Leehey (1962) showed that the problem could be resolved by considering compressibility of the fluid. Benjamin

(1964), however, pointed out that the practical range of oscillation frequencies compared the practical range of spans indicated that three-dimensional effects would come into play before compressibility effects, for spans would be very much less than typical acoustic wave lengths. He further outlined an approach to the problem based upon matched asymptotic expansions. Leone (1968) followed up on Benjamin's ideas and used matching with an outer flow to set the proper boundary conditions for the unsteady two-dimensional flow. His interest, however, was particularly in this question rather than in effecting an adequate solution to the three-dimensional problem.

Except for the work of Cumberbatch, it was apparently not clearly recognized that matching with the outer three-dimensional flow applied equally well to steady flows. Moreover, neither Benjamin nor Leone recognized that logarithmic terms in inverse aspect ratio should appear in the inner flow expansion for essentially the same reasons that they appear in Van Dyke's airfoil theory. The stage appears to be set now for bringing together the various aspects of the steady flow three-dimensional problem for supercavitating hydrofoils. It is necessary, however, that suitable additional experiments be performed in order that rational comparisons of theory with experiment can be made.

A systematic theory for lightly loaded partial and fully cavitated hydrofoils of finite span in steady flow is being developed. The problem is one involving three small parameters ϵ, the inverse aspect ratio, σ, the cavitation number, and α, the angle of attack. Geurst (1960) and others have shown that $J=\alpha/\sigma$ is a similarity parameter of the two-dimensional problem. The dimensionless cavity length is a unique function of J, but the solution is not uniform in J, becoming singular at J=0.1 where unsteady jumps between partial and fully cavitating flow occur as confirmed experimentally by Meijer (1959). It has been shown in second order aerodynamic theory that the wake downwash velocity at infinity is proportional to:

$$\epsilon\alpha \int_{-b}^{b} \frac{c'(\zeta)d}{z-\zeta}$$

where b is the semi-span and c(z) is the sectional chord. This must be small if the second order lifting line potential is to be determined by a planar wake vortex sheet. This we define as the lightly loaded case. For sufficient regularity of plan form we may consider $K = \epsilon\alpha$ as a second similarity parameter of the problem, recognizing that this appears not for physical results, but to make the problem tractable. For cavitating flow, we find that the expansion to third order in ϵ of the outer problem has coefficients in the orders:

$$1:K:\epsilon K \text{ and } \epsilon KJ^{-3}$$

Matching of the third inner to the third outer problem introduces log ε coefficients in the inner problem. These come from three sources. Two are a correction to the lifting line problem and the lateral dipole associated with the sectional moment coefficient as in Van Dyke's theory. The third, corresponding to the εKJ^{-3} term in the outer expansion relates to the sectional cavity area. The necessity for carrying the problem for the cavitating hydrofoil to third order is now apparent; for J is typically very small, especially in supercavitating flows. Hence, the influence of cavity area may be very important in the three-dimensional problem. We remark that the inner thickness and lifting problems are not separable for cavity flows in contrast to the airfoil problem. We also remark that the cavity length and area are derived results at any stage of the problem and are themselves not subject to direct expansion in ε in the inner problem.

Considerable analytical work remains to be done. Certain third order terms of the inner problem must be solved, for example, the camber effect from flow curvature. Further, matching will be required when $c'(z)$ is not small near the tips.

....

Professor Wu made note of the cavity arrangement which has been observed in finite span cavity flows. For example, it is possible to have the tip votex cavities separated from the centerline cavity by a fully wetted region. Thus the "one cavity" model will not, in general, be valid for all values of the flow parameters.

The two dimensional theory has been extended by Mr. Hsu to treat a cascade of cavitating lifting surfaces. This may have an important application in supercavitating pump design.

One aspect of cavity flows which has always plagued theorists is the initially unknown location of the free streamline. In partially cavitating flow this is particularly difficult to determine with an analytical model. Professor Tumman offered an alternative approach. Consider the quantity:

$$\left(v-\frac{\partial f}{\partial x}\right) \quad (u)$$

where u, v=perturbation velocities

f(x)=foil geometry

On the wetted portion of the foil, the first factor is zero, while on the cavity wall the second is zero. Thus for this streamline, the product is zero everywhere. This provides the governing constraint in a numerical search process for this streamline.

VI. LIFTING CAVITATING SURFACES--UNSTEADY

In nearly all applications of cavitating lifting surfaces in a ship environment the cavity structure and associated body loading depend in a non-trivial way on the flow unsteadiness. A classic example is a supercavitating hydrofoil running under a train of waves. This situation is fundamentally the same as a supercavitating propeller operating in a ship wake. Consequently, as in fully wetted lifting surface technology, cavity research is being, in part, directed to the question of time dependent flows. The principal goal is to provide a design theory which realistically accounts for external influences on the foil.

In linearized, two-dimensional steady flow theory, the free surface forming the cavity wall is taken to be a streamline of constant pressure. At the cavity trailing edge this is not a valid approximation. Fortunately, the various models for the rear of the cavity yield essentially the same results for such quantities as lift coefficient. However, in the unsteady formulation, a more careful treatment of the wake is required. Professor Wu mentioned some general considerations for these problems.

(1) Choice of reference frame

<u>accelerating flow</u>

$$\frac{p}{\rho} + \phi_t + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi = C(t)$$

$$p \rightarrow \rho\dot{U}x \ , \ \vec{x} \rightarrow \infty$$

$$\nabla\phi \rightarrow U(t) \ , \ \vec{x} \rightarrow \infty$$

<u>accelerating body</u>

$$\frac{p}{\rho} + \phi_t - ax + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi = C(t)$$

$$p \rightarrow p_\infty \ , \ \vec{x} \rightarrow \infty$$

$$\nabla\phi \rightarrow 0 \ , \ \vec{x} \rightarrow \infty$$

For example, Von Karman (1949) considered the case of a flat plate placed broadwise in an <u>accelerating stream</u>. This was solved by assuming:

$$\Phi(\vec{x},t) = U(t)\ \phi(x,y)$$

(2) Role of time in free surface problems

In problems with <u>no free surface</u>, one may write the body boundary condition as:

$$\vec{n}(t) \cdot \left\{ \phi(\vec{x},t) - \vec{V}_b(\vec{x},t) \right\} = 0 \qquad \text{on} \quad \vec{x} = \vec{x}_b$$

Here time enters only as a parameter. By introducing a free surface, where the boundary condition contains time differentially, the past history of the flow at the boundary is required.

(3) Area and volume time derivatives

In three-dimensional flow, volumes may change with time without violating energy conservation.

$$\frac{\partial(\text{vol})}{\partial t} = \frac{\partial\phi}{\partial R} \cdot 4\pi R^2 \rightarrow \phi \sim \frac{\partial(\text{vol})}{\partial t} \frac{1}{R} \quad , \quad R \rightarrow \infty$$

In two-dimensions, this is not the case for incompressible flow since:

$$\frac{\partial(\text{area})}{\partial t} = \frac{\partial\phi}{\partial R} \cdot 2\pi R \rightarrow \phi \sim \frac{\partial(\text{area})}{\partial t} \log R \ , \ R \rightarrow \infty$$

To avoid infinite energy transmission, there have been several models proposed to "remove liquid" (Timman, 1958; Leehey, 1962; Hsu & Chen, 1962).

As an example of recent work, Professor Wu outlined an approach to the problem of a flat plate moving normal to itself with cavity pressure a function of time. By conformal mapping this problem can be transformed into a Riemann-Hilbert problem in which the real and imaginary parts of an analytic function are prescribed on segments of the real axis of the transform plane.

The importance of considering energy coupling between the time dependent cavity pressures and waves on the cavity wall was cited by Tulin (Hsu and Chen, 1962).

Professor Widnall presented a theory for unsteady, finite span supercavitating hydrofoils based on a computer aided numerical solution similar to the fully wetted case. The problem for the perturbation pressure is formulated using linear approximations, but with the cavity plan form necessarily specified apriori (the results for, say, lift coefficient are found to be insensitive to cavity length if it is greater than two chord lengths). This results in coupled integral equations for the unknown loading distribution (pressure doublets) and cavity-foil thickness distribution (pressure sources).

These equations have been solved by choosing a set of loading distribution modes with unknown coefficients, which are found by satisfying the equation at a selected number of control points. The pressure source distribution functions are chosen so as to permit direct integration. Numerical results and comparison with experiments may be found in Widnall (1966).

In answer to the question of whether the cavity pressure remains constant, Mr. Tulin suggested that a two-dimensional ventilated unsteady model might be explored. He also noted that for low aspect ratios, non-linear effects became important because a large angle of attack is necessary for the cavity to break free.

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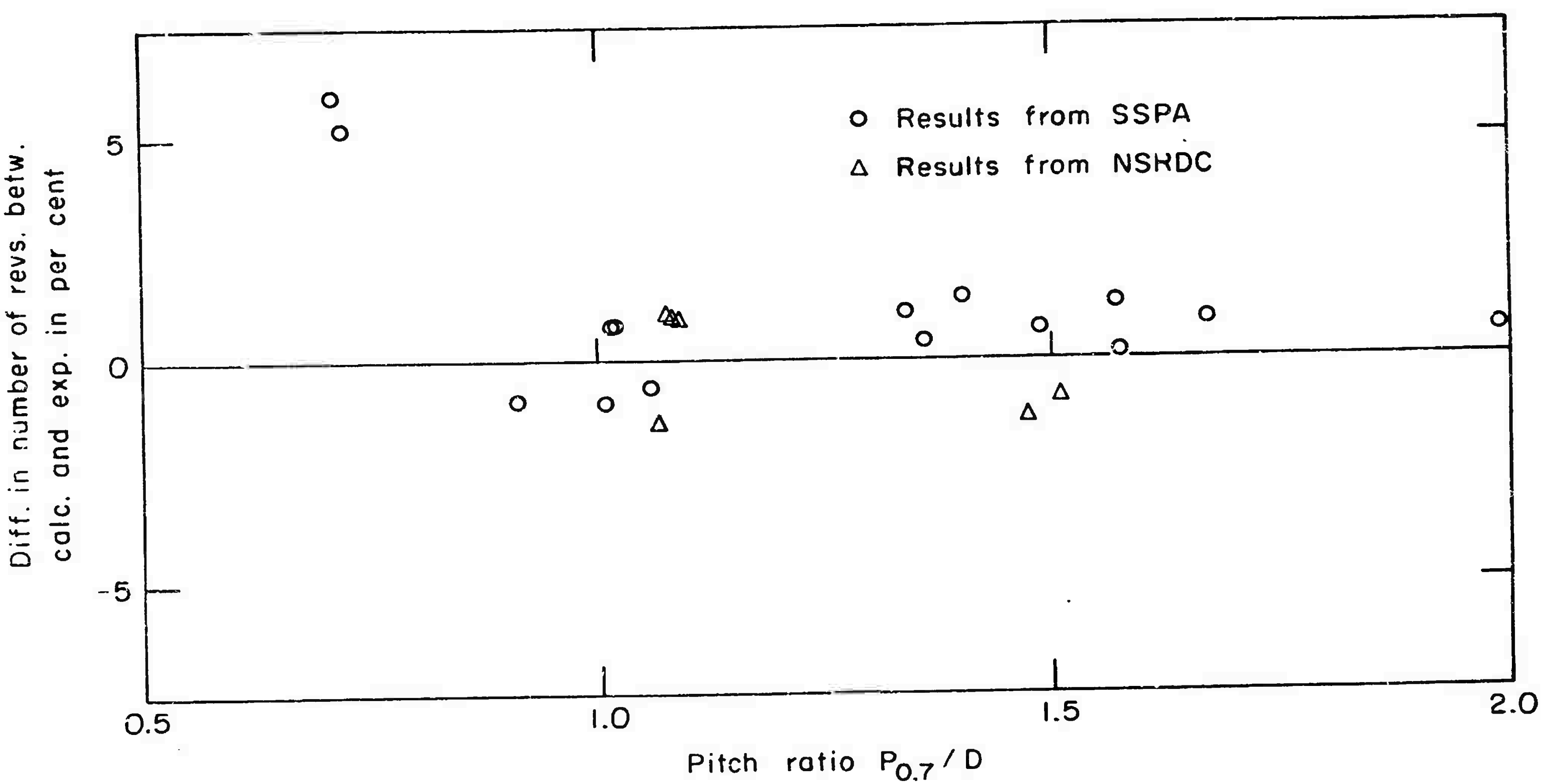


Results from SSPA
Results from NSRDC
Diff. in number of revs. betw.
calc. and exp. in per cent
5
0
-5
0.5
1.0
1.5
2.0
Pitch ratio $P_{0.7}/D$

Fig. I-1

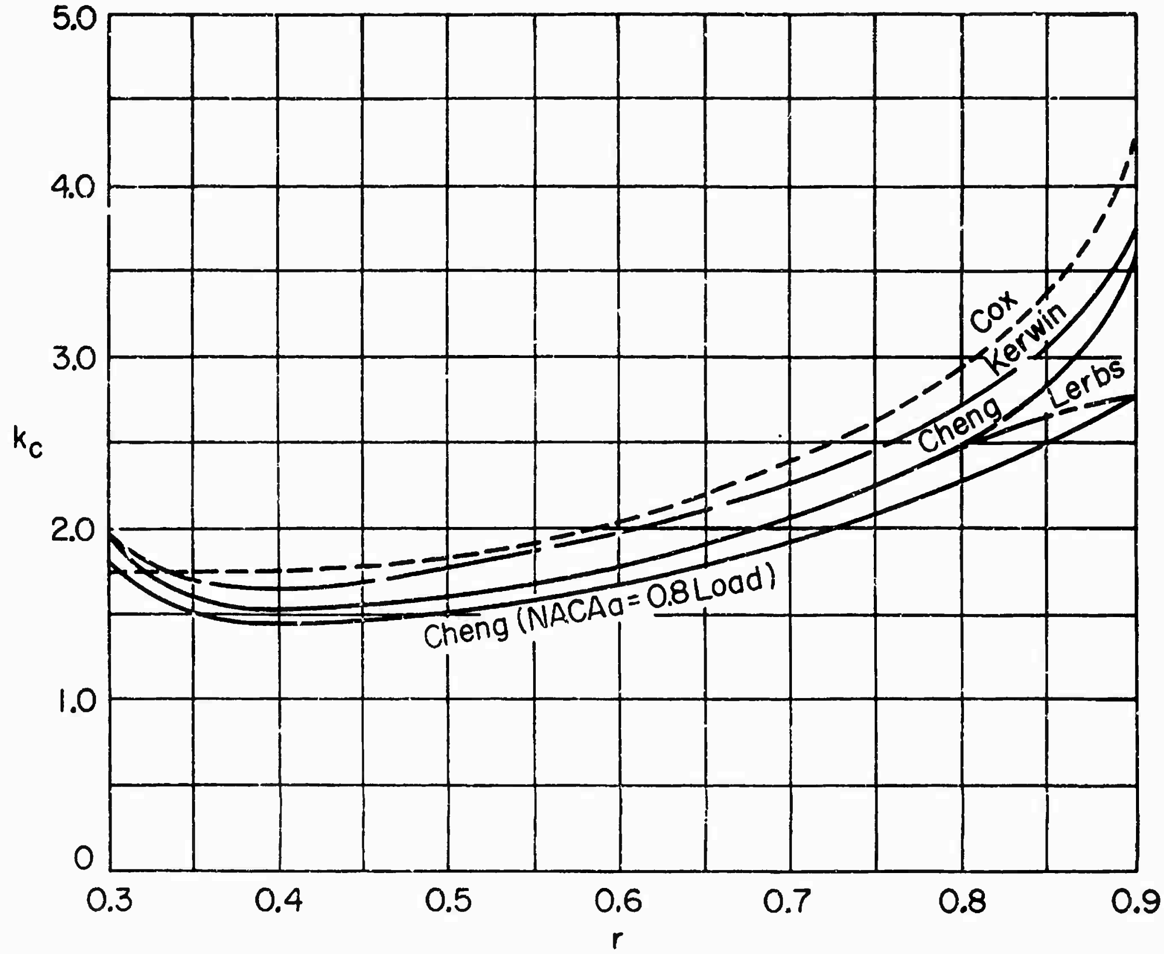


Fig. I-2 Comparison of camber correction factors for three blades, $\pi\lambda_i = 1.0472$, and $A_E / A_O = 0.75$

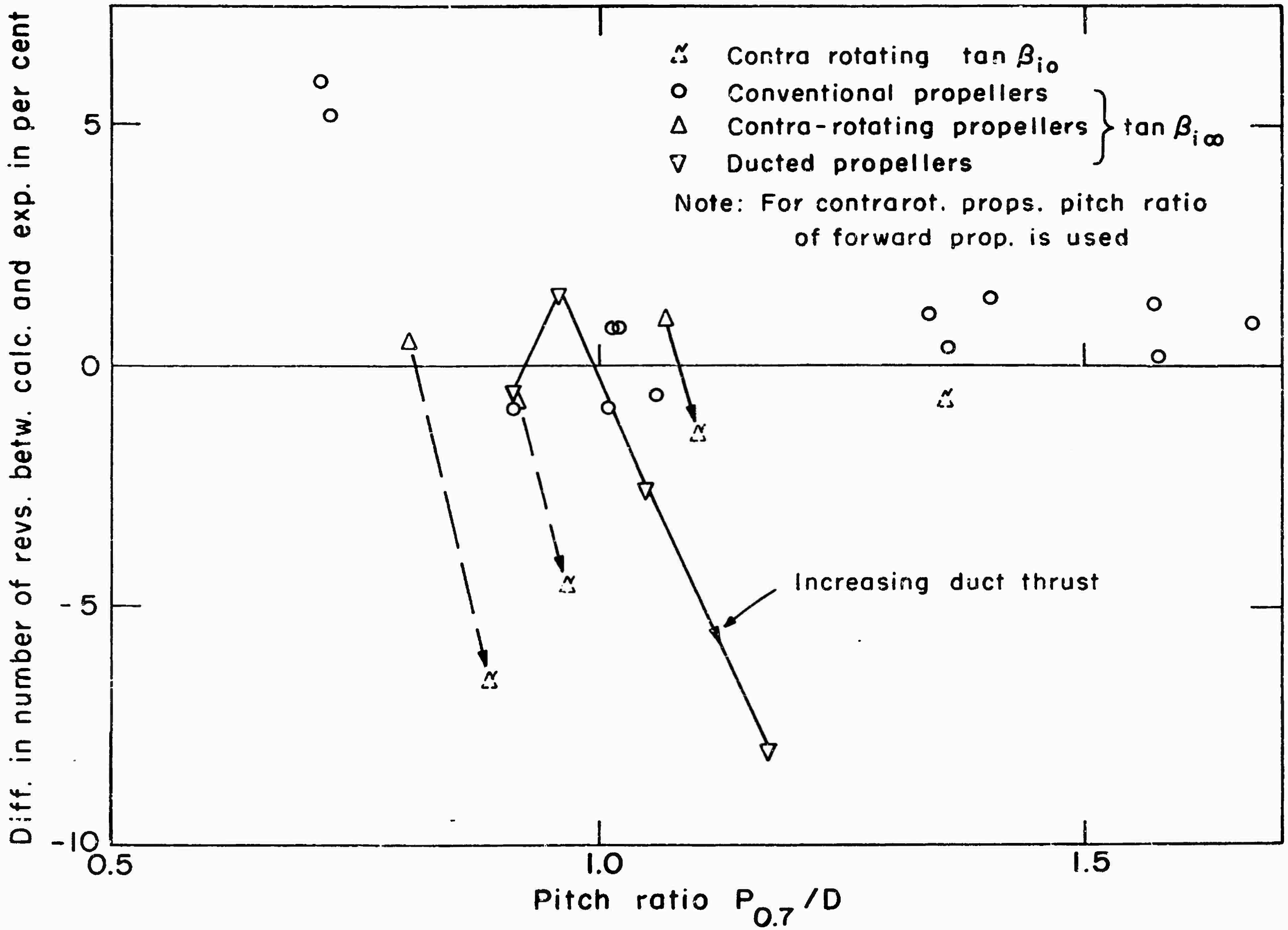

Contra rotating tan β_{io}
Conventional propellers
Contra-rotating propellers
Ducted propellers
tan $\beta_{i\infty}$
Note: For contrarot. props. pitch ratio of forward prop. is used
Increasing duct thrust
5
0
-5
-10
0.5
1.0
1.5
Diff. in number of revs. betw. calc. and exp. in per cent
Pitch ratio $P_{0.7}/D$

Fig. I-3

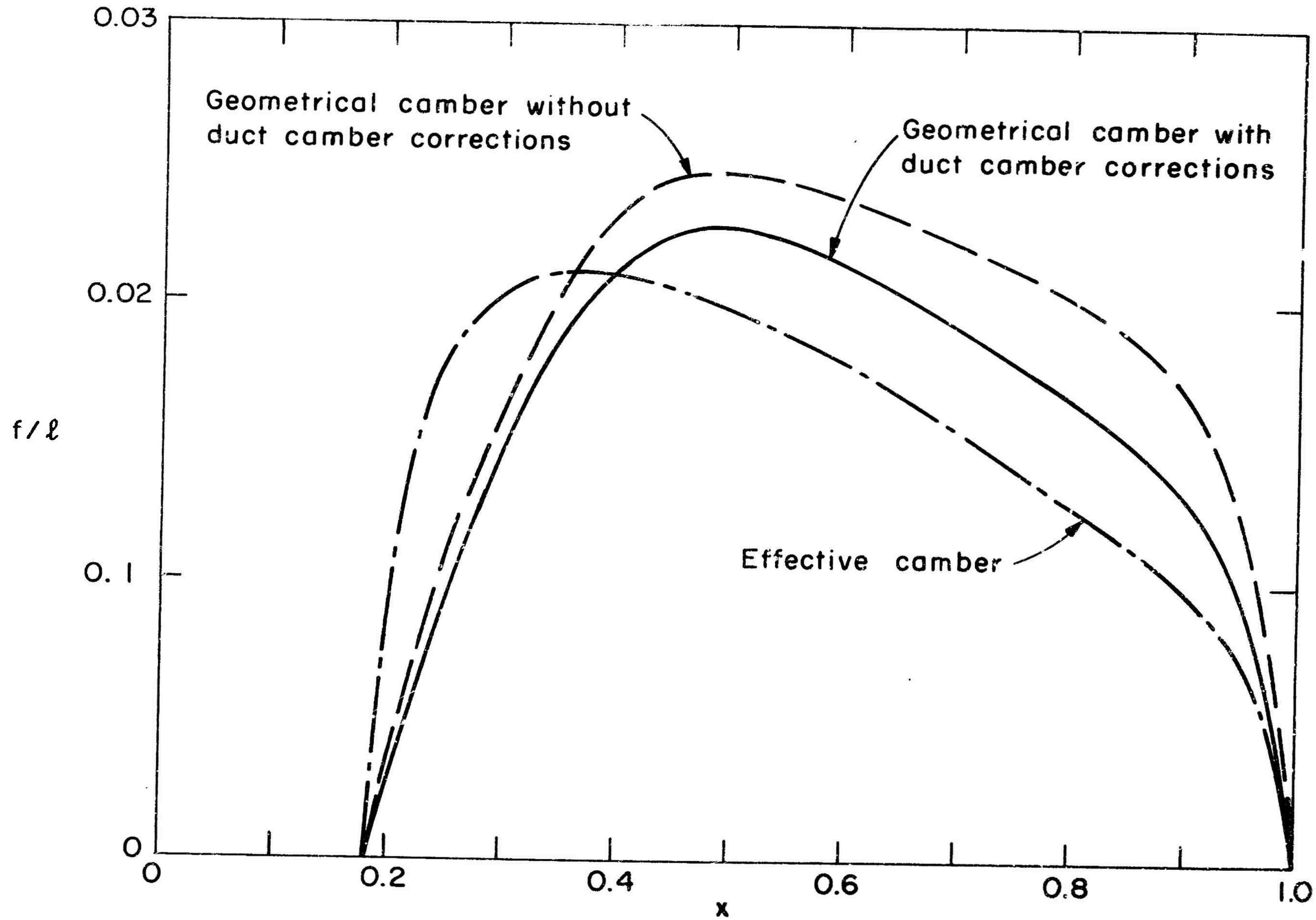

0.03
0.02
0.1
0
f/ℓ
Geometrical camber without
duct camber corrections
Geometrical camber with
duct camber corrections
Effective camber
0
0.2
0.4
0.6
0.8
1.0
x

Fig. I-4

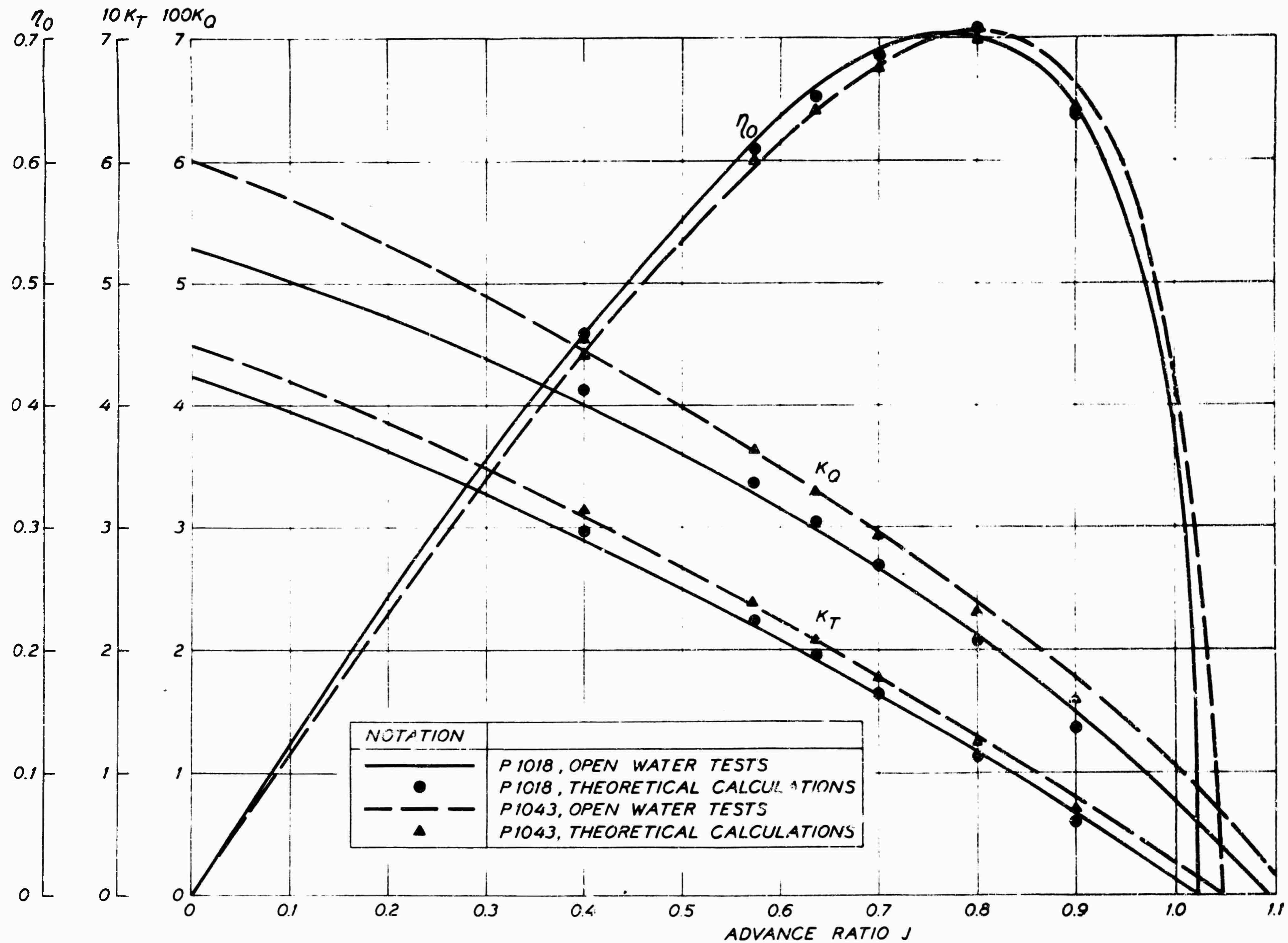
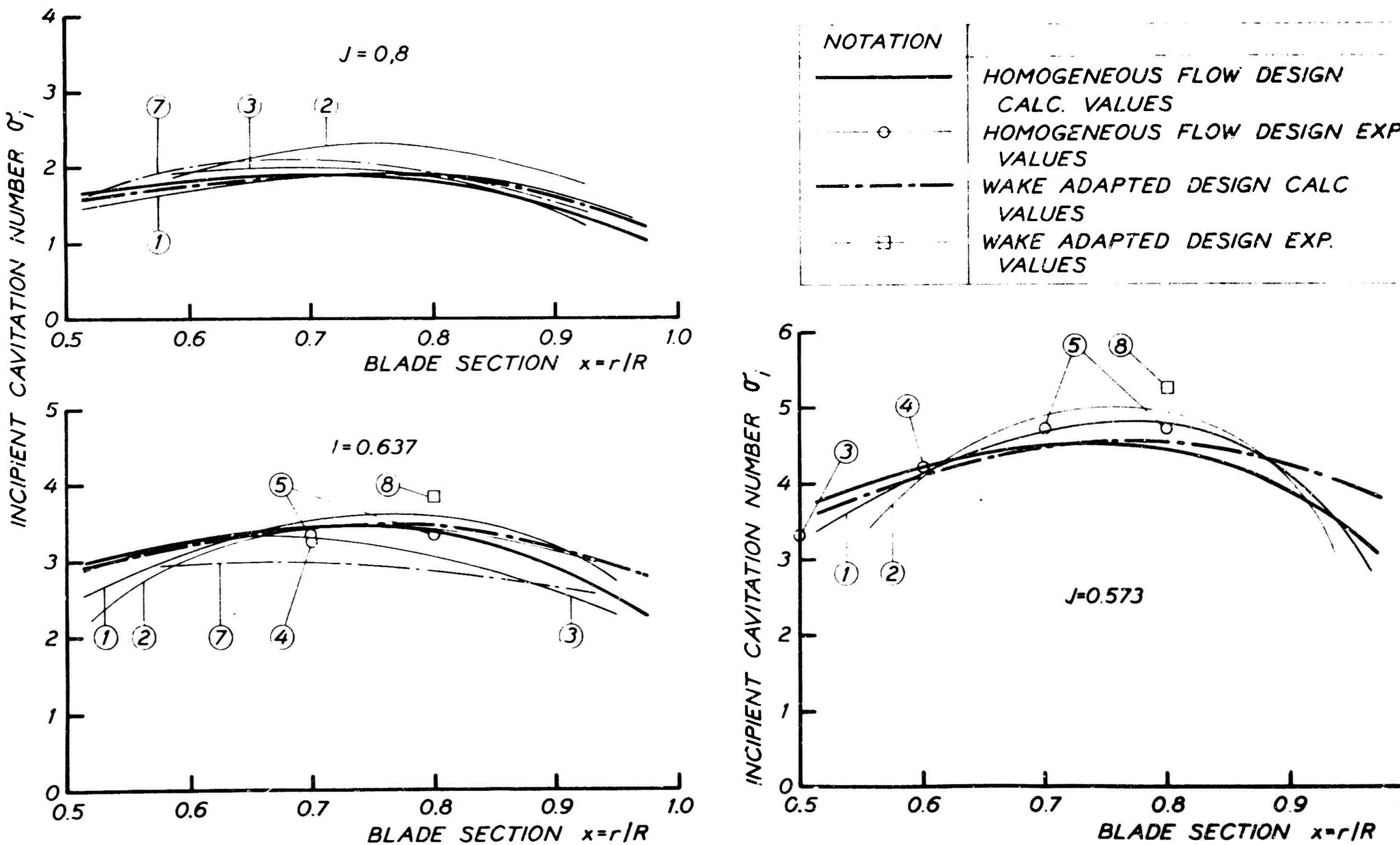
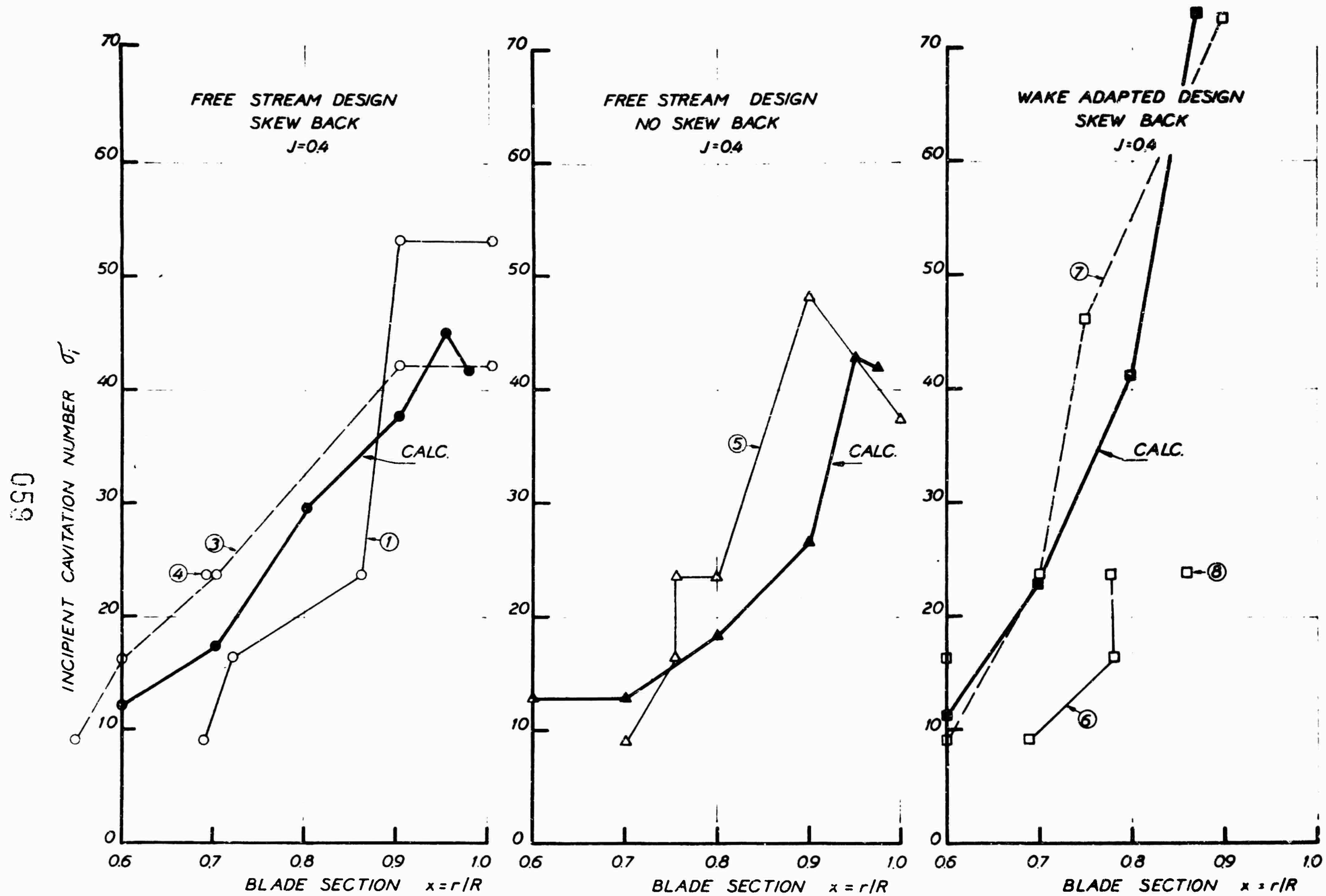

η_0
$10K_T$
$100K_Q$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
ADVANCE RATIO J
η_0
K_Q
K_T
NOTATION
P 1018, OPEN WATER TESTS
P 1018, THEORETICAL CALCULATIONS
P1043, OPEN WATER TESTS
P1043, THEORETICAL CALCULATIONS

Fig. 5



NOTATION
HOMOGENEOUS FLOW DESIGN CALC. VALUES
HOMOGENEOUS FLOW DESIGN EXP VALUES
WAKE ADAPTED DESIGN CALC VALUES
WAKE ADAPTED DESIGN EXP. VALUES
J = 0,8
I = 0.637
J=0.573
INCIPIENT CAVITATION NUMBER σ_i
BLADE SECTION x=r/R

Fig. I-6



FREE STREAM DESIGN
SKEW BACK
J=0.4
FREE STREAM DESIGN
NO SKEW BACK
J=0.4
WAKE ADAPTED DESIGN
SKEW BACK
J=0.4
CALC.
INCIPIENT CAVITATION NUMBER σ_i
BLADE SECTION x = r/R
0
10
20
30
40
50
60
70
0.6
0.7
0.8
0.9
1.0
1
3
4
5
6
7
8

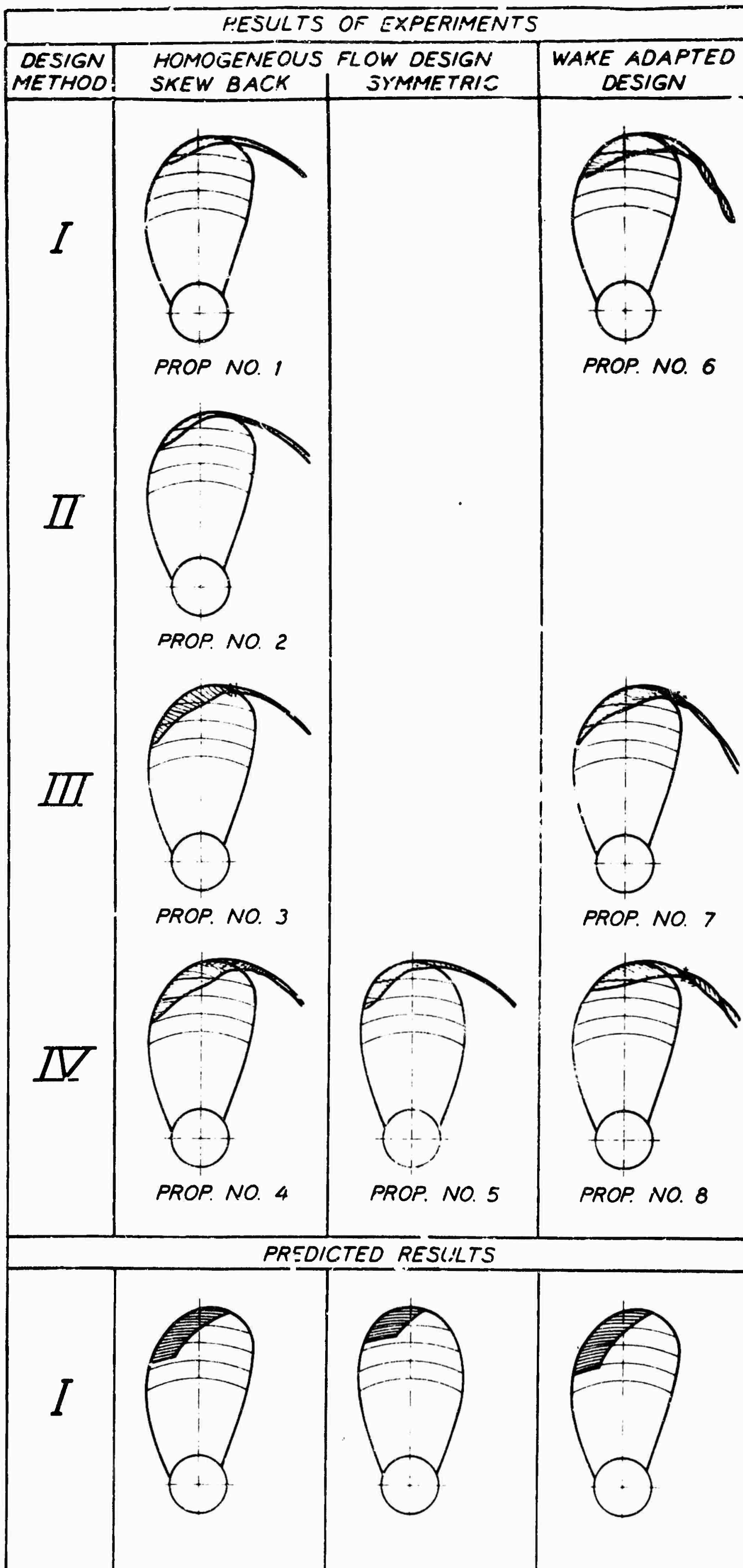

RESULTS OF EXPERIMENTS
DESIGN METHOD
HOMOGENEOUS FLOW DESIGN
SKEW BACK
SYMMETRIC
WAKE ADAPTED DESIGN
I
PROP NO. 1
PROP. NO. 6
II
PROP. NO. 2
III
PROP. NO. 3
PROP. NO. 7
IV
PROP. NO. 4
PROP. NO. 5
PROP. NO. 8
PREDICTED RESULTS
I

Fig. I-8

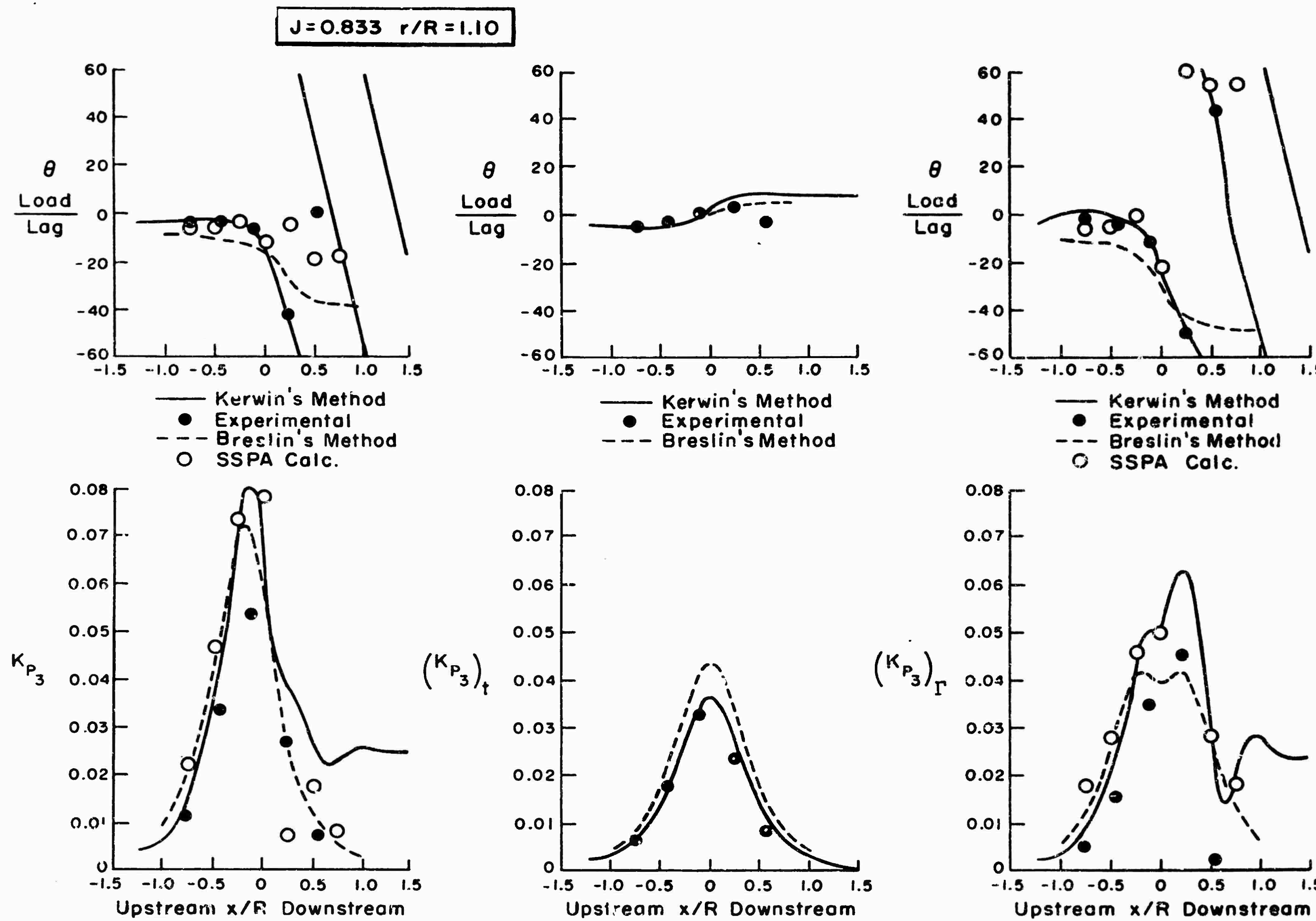

J=0.833 r/R=1.10
θ Load/Lag
60
40
20
0
-20
-40
-60
-1.5
-1.0
-0.5
0
0.5
1.0
1.5
Kerwin's Method
Experimental
Breslin's Method
SSPA Calc.
K_{P_3}
$(K_{P_3})_t$
$(K_{P_3})_\Gamma$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Upstream x/R Downstream

Fig. I-9

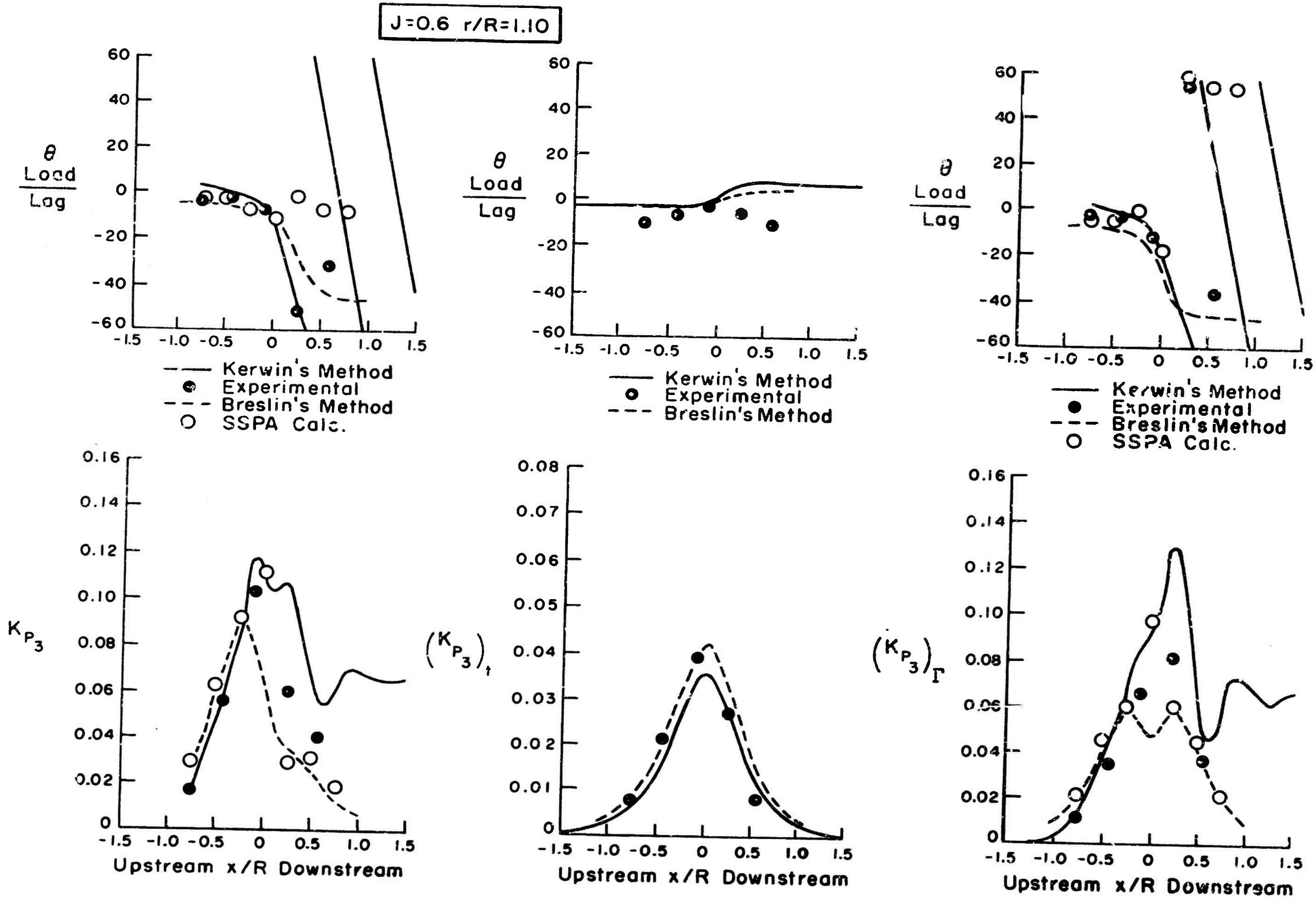

J=0.6 r/R=1.10
θ
Load
Lag
Kerwin's Method
Experimental
Breslin's Method
SSPA Calc.
K_{P_3}
$(K_{P_3})_t$
$(K_{P_3})_\Gamma$
Upstream x/R Downstream

Fig. I-10

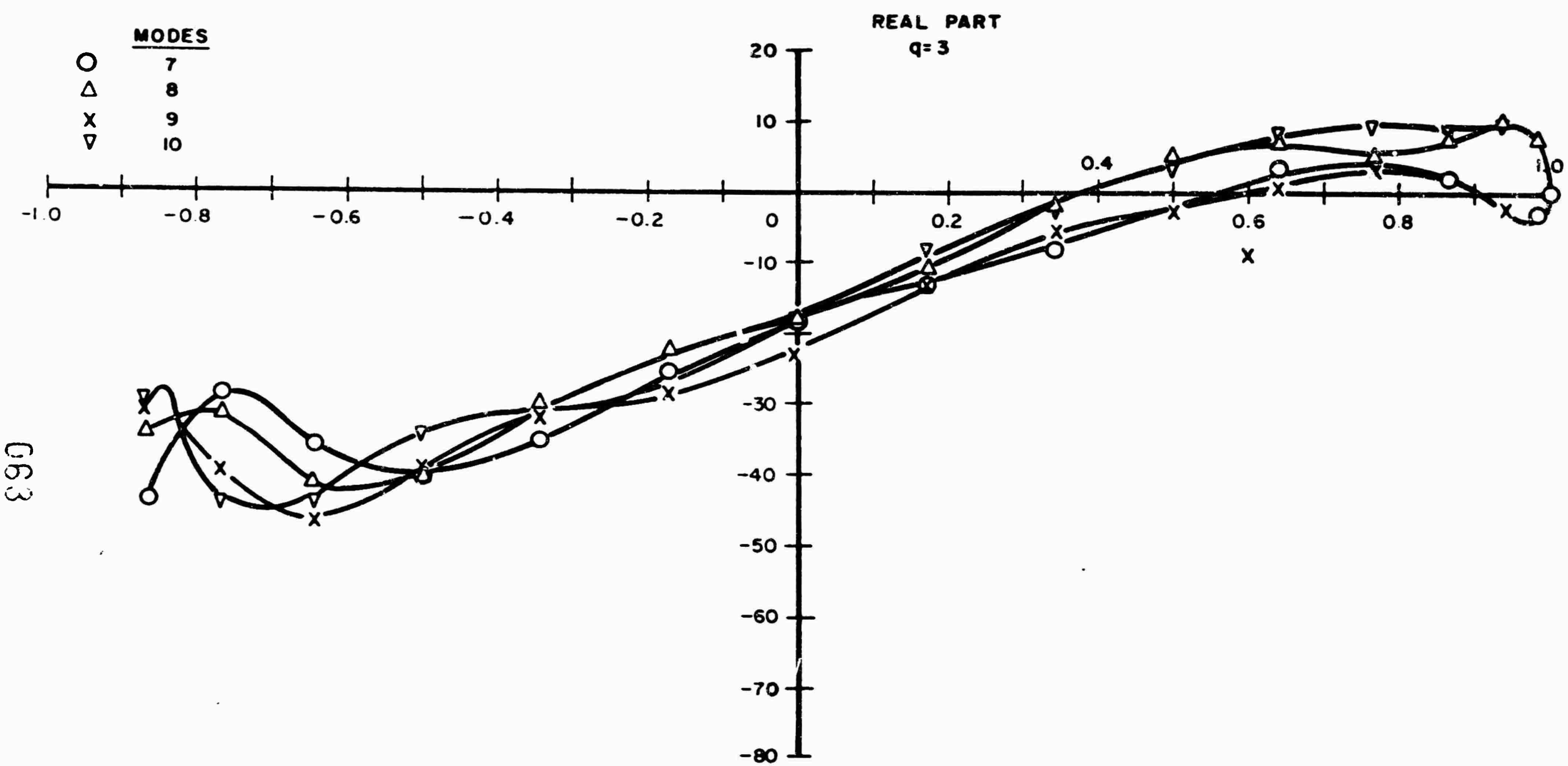


Fig. III-1b UNSTEADY CHORDWISE DISTRIBUTION AT 0.75 RADIUS FOR PROPELLER OF EAR = 0.6 AT DESIGN J (CESARO-SUMMABILITY OF SINE TERMS)

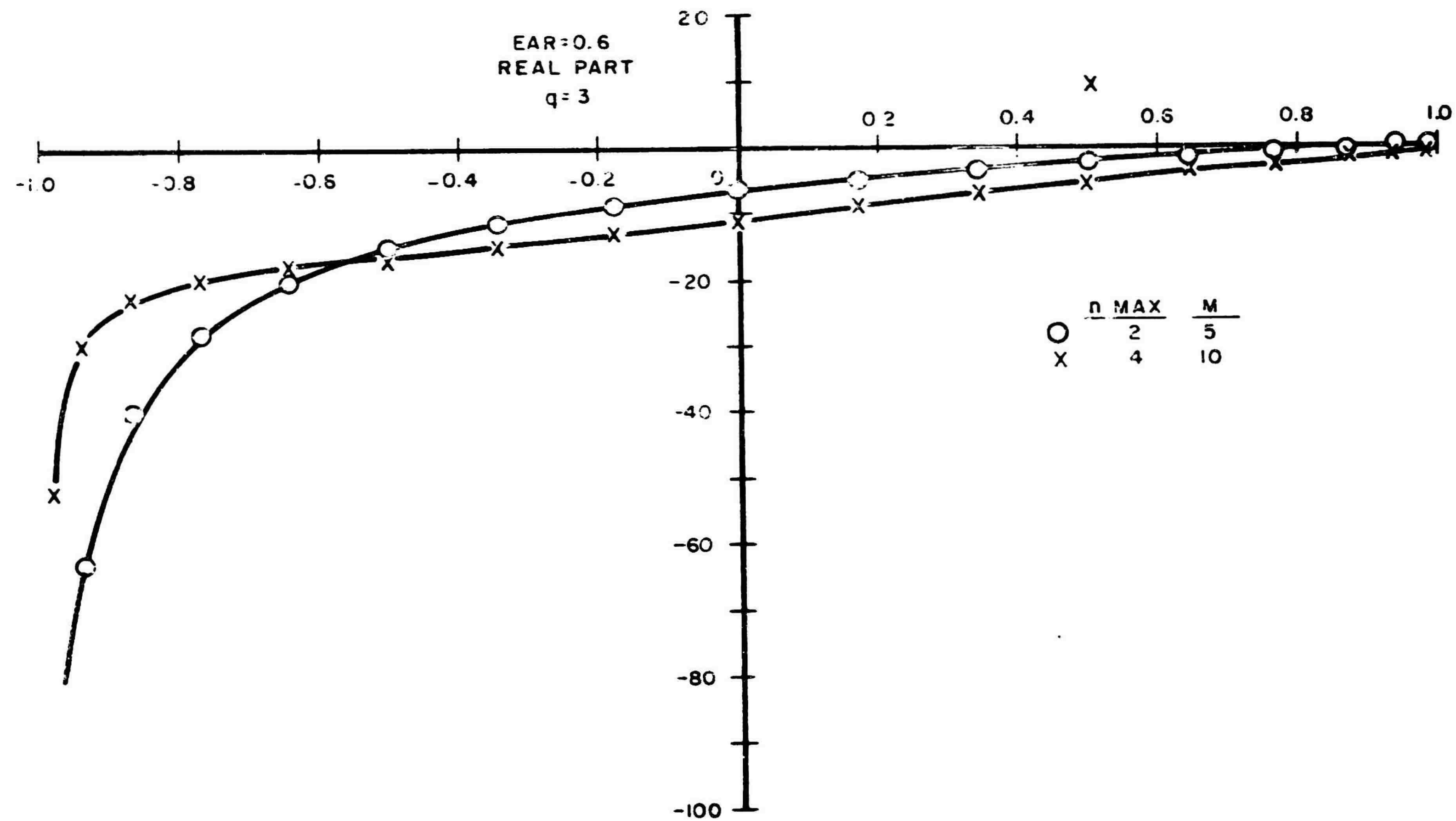


Fig. III-2 UNSTEADY CHORDWISE DISTRIBUTION AT 0.65 RADIUS FOR PROPELLER OF EAR = 0.6 (CESARO-SUMMABILITY OF SINE TERMS)

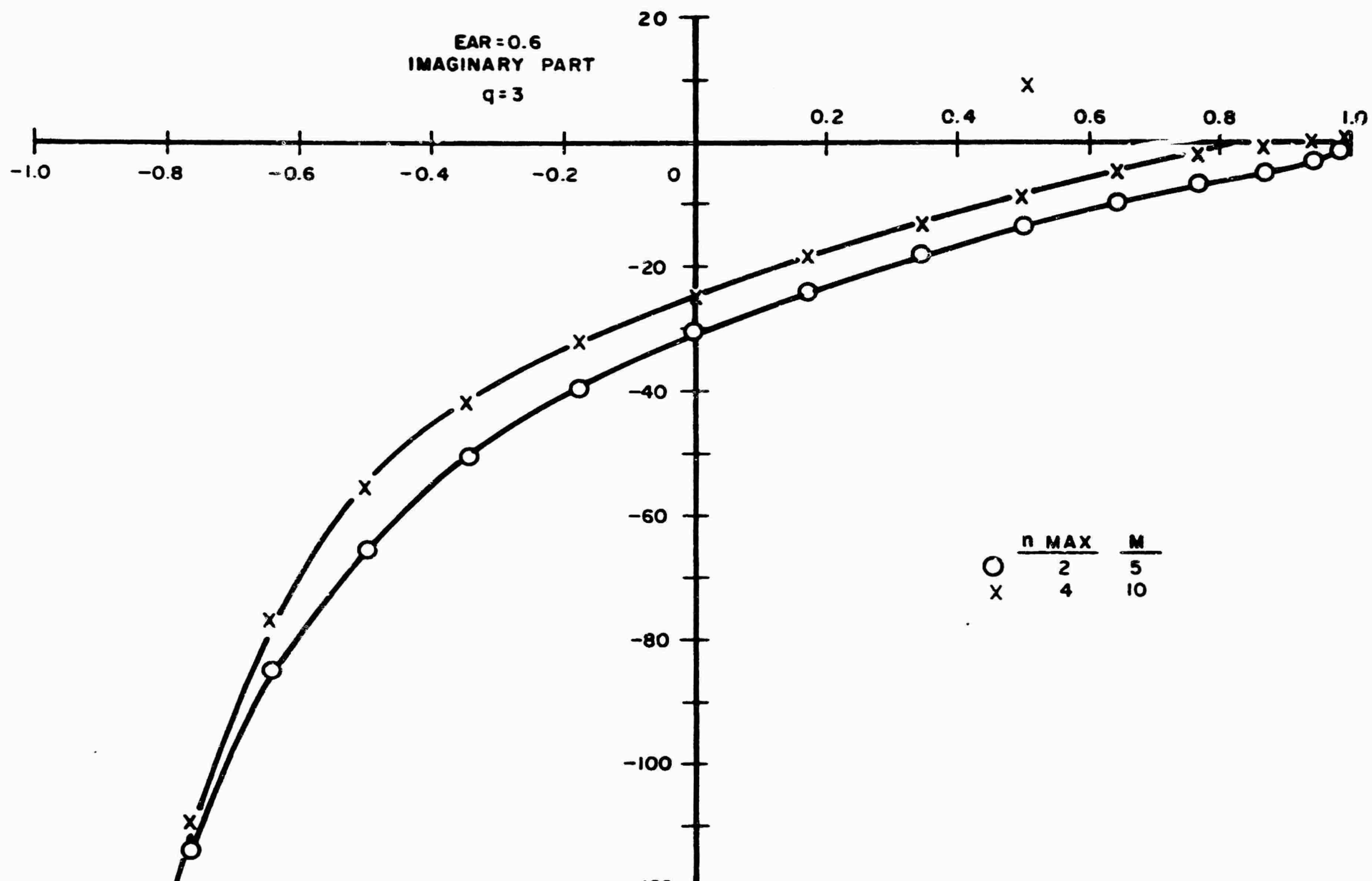


Fig. III-2b UNSTEADY CHORDWISE DISTRIBUTION AT 0.65 RADIUS FOR PROPELLER OF EAR = 0.6 (Cesaro-Summability of Sine Terms)

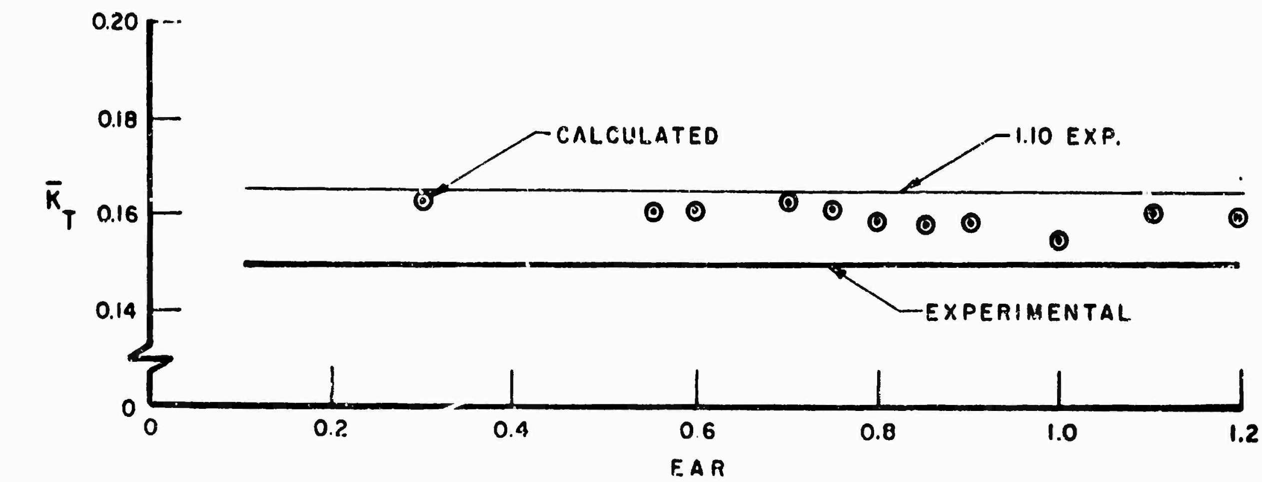


Fig. III-3a MEAN THRUST COEFFICIENTS FOR 3-BLADED PROPELLERS

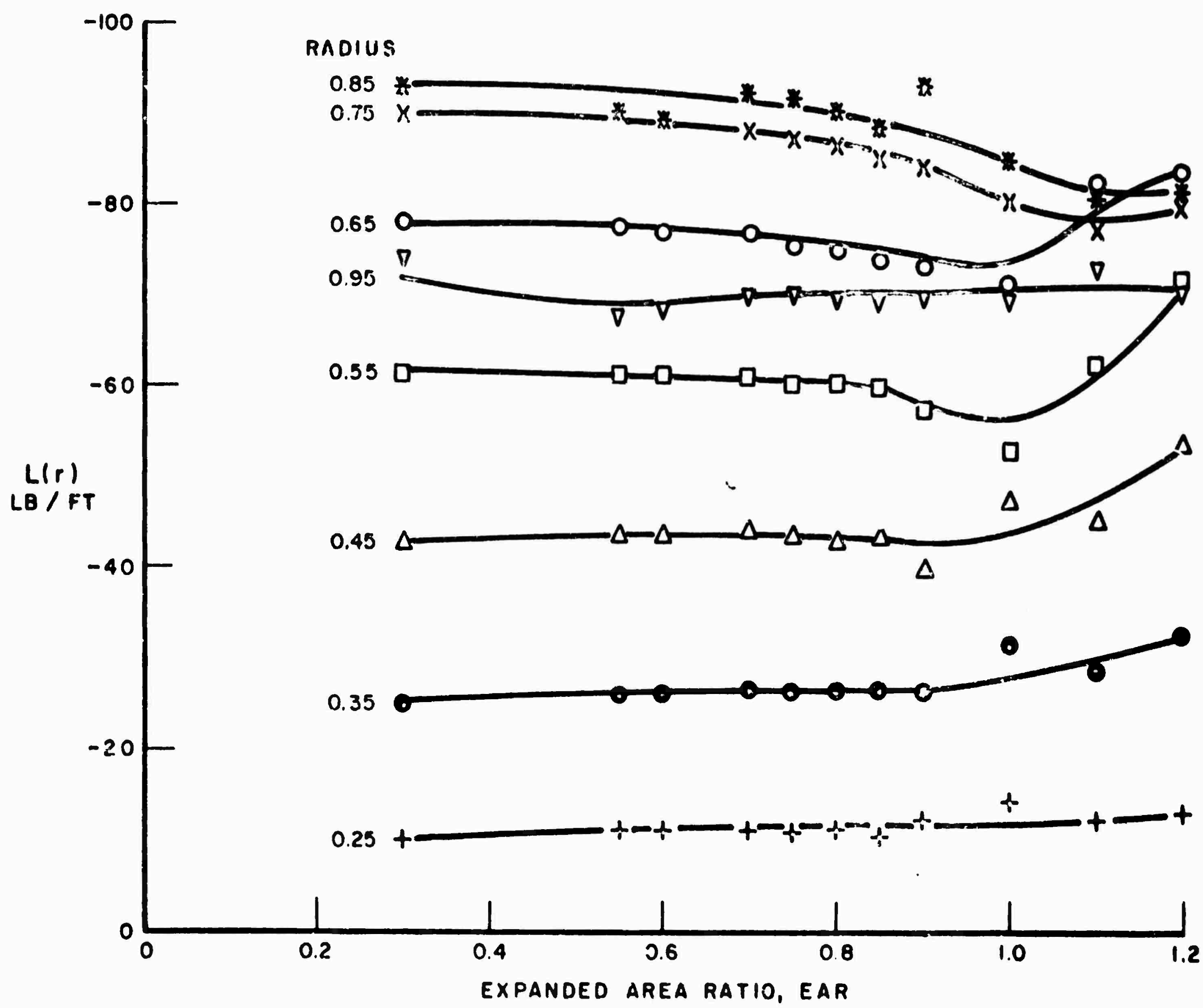


Fig. III-3b STEADY-STATE SPANWISE LOADINGS FOR 3-BLADED PROPELLERS

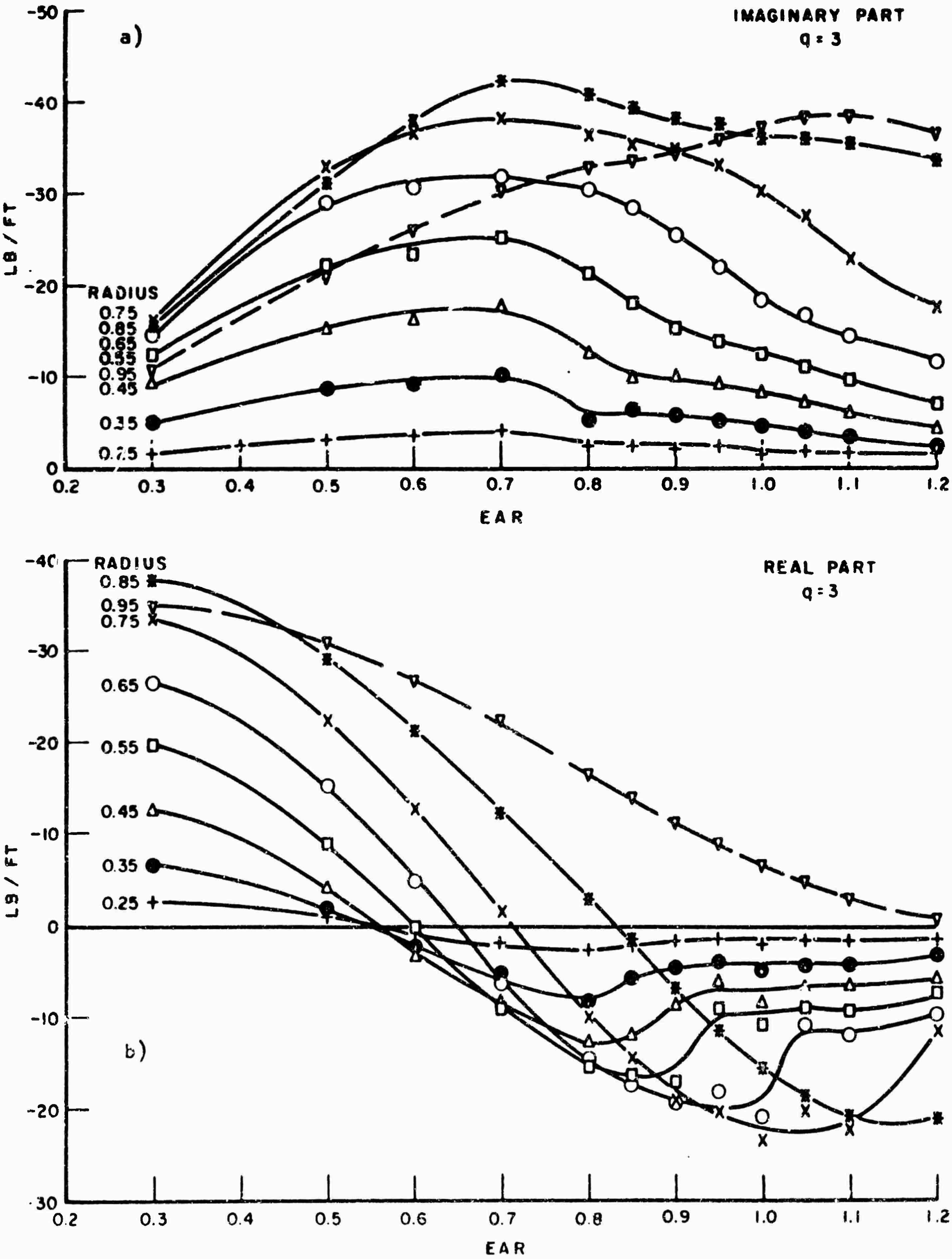


Fig. III-4 SPANWISE LOADING VERSUS EAR FOR 3-BLADED PROPELLERS AT q = 3

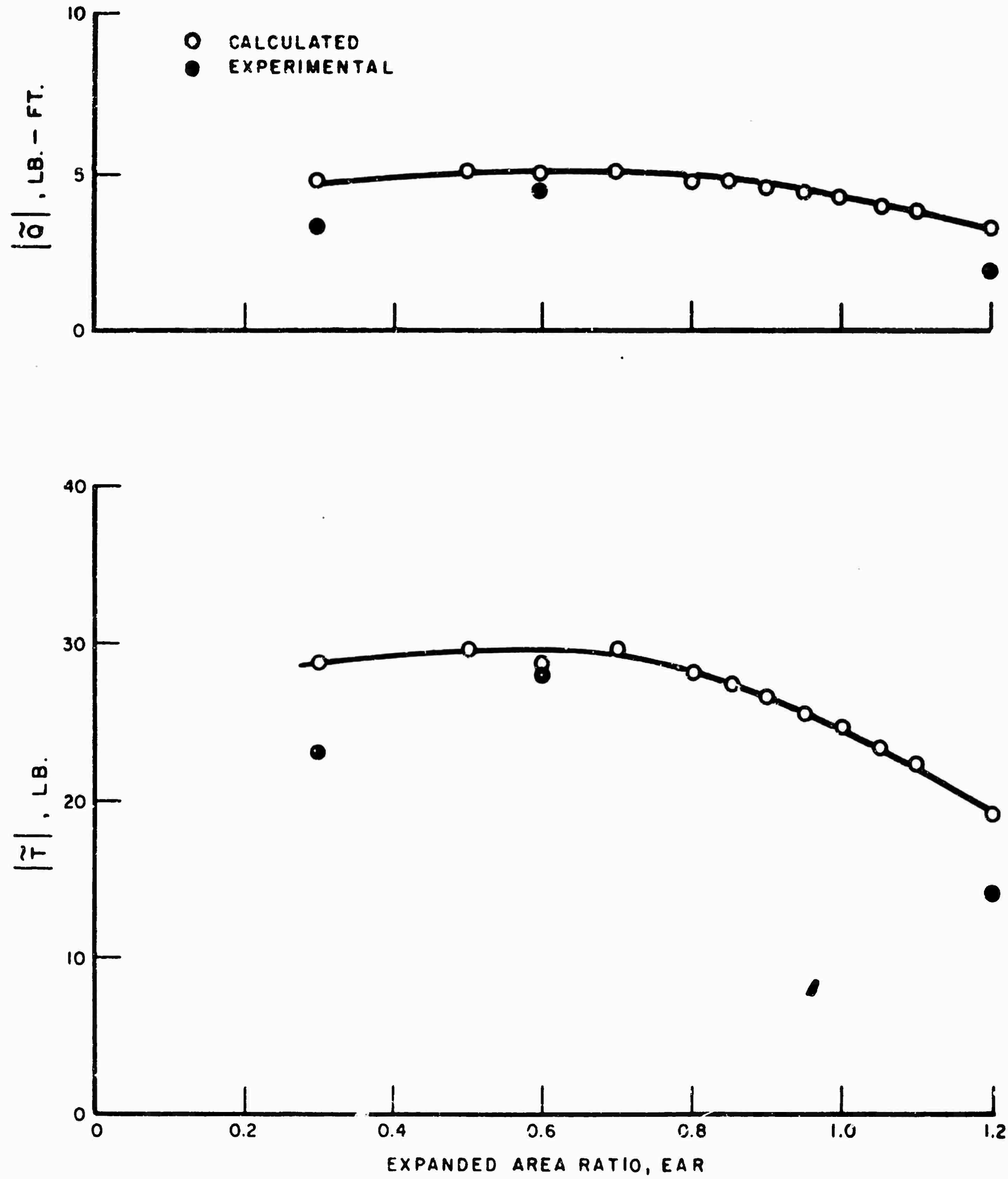


Fig. III-5 AMPLITUDES OF VIBRATORY THRUST AND TORQUE FOR 3-BLADED PROPELLERS

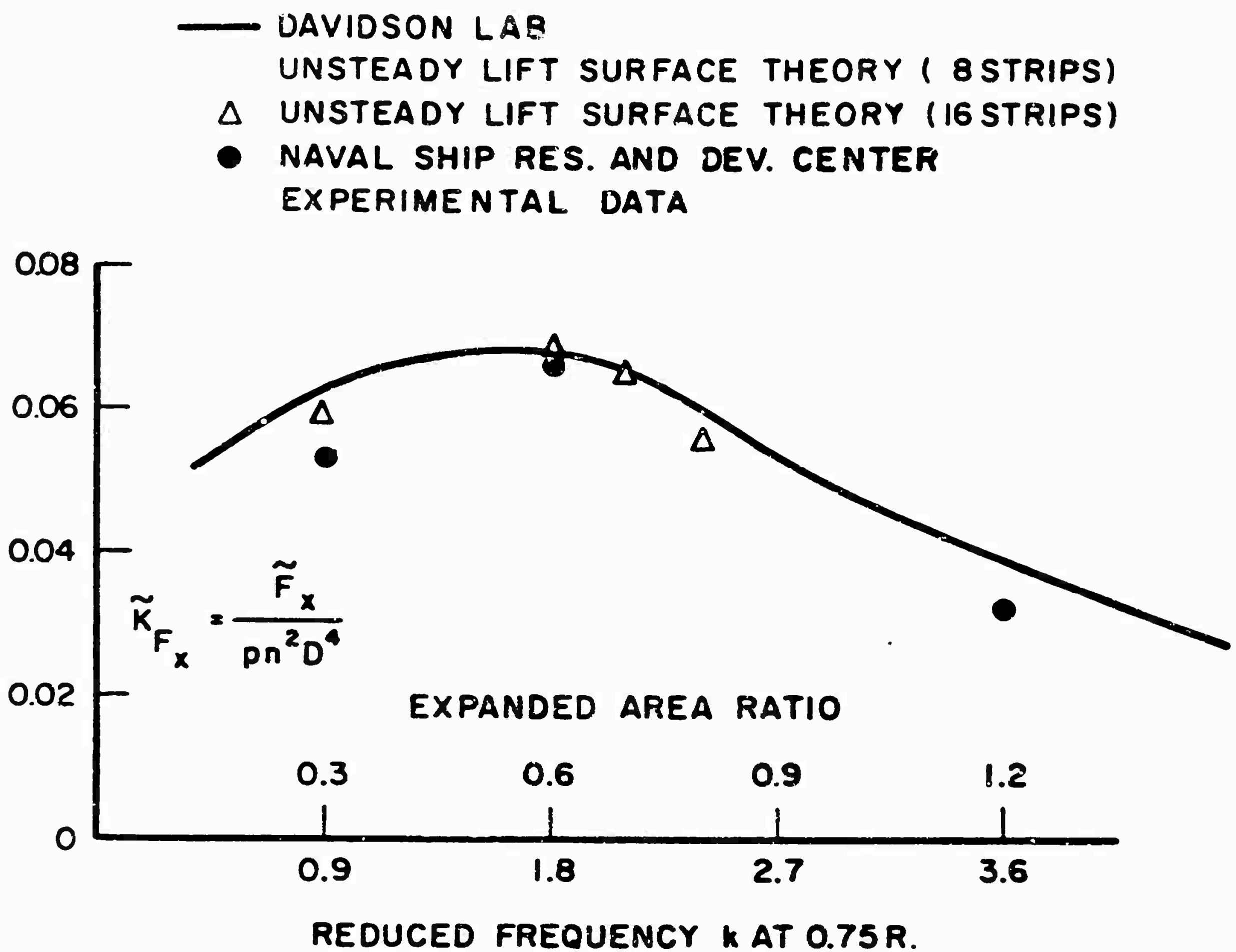


Fig. III-6 VIBRATORY THRUST COEFFICIENTS

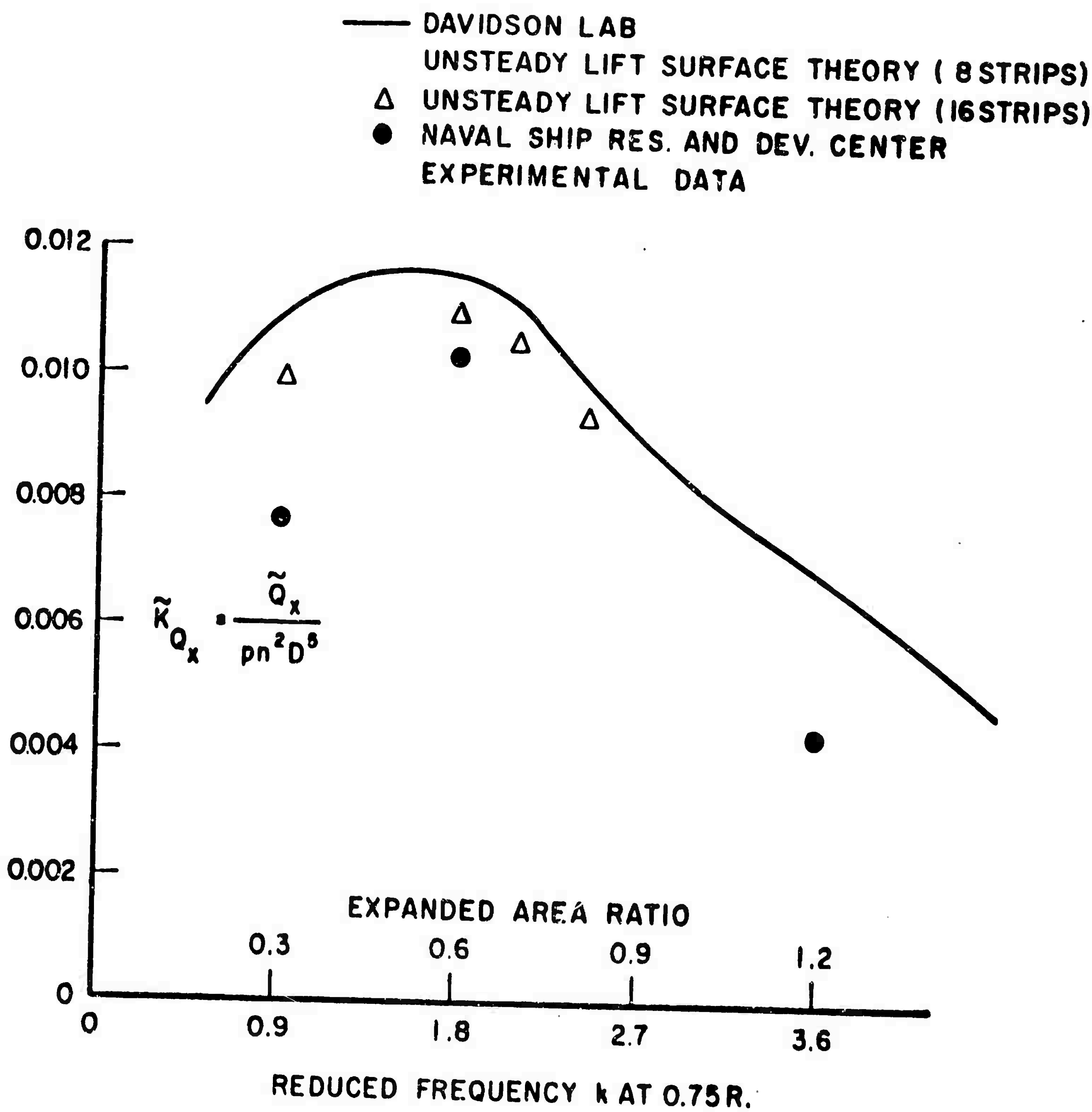
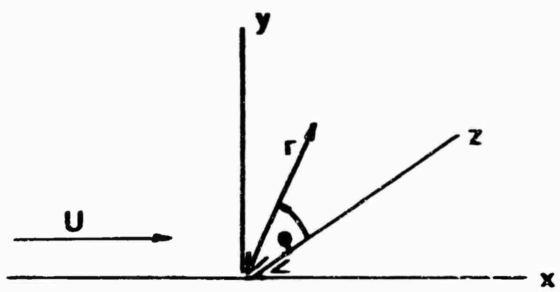


Fig. III-7 VIBRATORY TORQUE COEFFICIENTS



(1) $F: \quad \varphi - ax + \omega t = 0$

(2) $$\psi(x,r,\varphi,t) = -\frac{\mu(\xi,\rho,\theta)}{4\pi U(1+a^2\rho^2)^{1/2}} \frac{a\rho(x-\xi) + r\sin\{a(x-\tau)-\omega t-\varphi\}}{\{(x-\xi)^2+r^2+\rho^2-2r\rho\cos(a\xi-\omega t-\varphi)\}^{3/2}}$$

(3) $$\phi(x,r,\varphi,t) = \frac{1}{U}\int_{-\infty}^{x} \psi\left(x,r,\varphi,i-\frac{x-x_1}{U}\right) dx_1$$

(4) $$aru_0 - w_0 = \frac{1}{4\pi U}\sum_{m=0}^{\infty}\int_{r_i}^{r_0} d\rho \int_{x_l}^{x_t} \mu_m(\xi,\rho)\, e^{im(\omega t - a(x-\xi))}\, K(x,r,\xi,\rho)\, d\xi$$

$$K(x,r,\xi,\rho) = \sum_{k=0}^{n-1}\int_{-\infty}^{x-\xi} e^{im(a\tau+\frac{k}{n}2\pi)}\left[\frac{a^2r\rho+\cos(a\tau+\frac{k}{n}2\pi)}{R^3} - \frac{3\{ar\tau-\rho\sin(a\tau+\frac{k}{n}2\pi)\}\{a\rho\tau-r\sin(a\tau+\frac{k}{n}2\pi)\}}{R^5}\right] d\tau$$

m= harmonic.

n = number of blades.

U = mean velocity

u_0, w_0 = wake in x and φ direction.

μ = pressure dipole.

r_i, r_0 = inner and outer radius

x_l, x_t = leading and trailing edge.

Fig. III-8 FORMULATION OF THE LIFTING SURFACE INTEGRAL EQUATION.

$$(5) \quad a r u_o - w_o = \frac{1}{4\pi U} \sum_{m=0}^{\infty} e^{im(\omega t - ax)} \lim_{\beta \to 0} \left[\left\{ \int_{r_i}^{r-\beta} + \int_{r+\beta}^{r_o} \right\} \int_{x_l}^{x_t} \mu_m(\xi,\rho)\, e^{ima\xi}\, K(x,r,\xi,\rho)\, d\xi\, d\rho \; + \right.$$

$$- \frac{4(1+a^2r^2)^{1/2}}{\beta} \int_{x_l(r)}^{x} \mu(\xi,r)\, e^{ima\xi}\, d\xi$$

$$(6) \quad \mu(\xi,\rho) = \sum_{p=0}^{cp} C_{mp}(\rho) \cdot \frac{2}{\pi} \frac{\cos p\varphi + \cos(p+1)\varphi}{\sin\varphi} \qquad \frac{\xi - x_l(\rho)}{l(\rho)} = \frac{1}{2}(1-\cos\varphi).$$

$$(7) \quad \int_{x_l}^{x_t} \mu(\xi,\rho)\, e^{ima\xi} K(x,r,\xi,\rho)\, d\xi = \sum_{p=0}^{cp} C_{mp}(\rho) \left\{ \frac{(1+a^2 r \rho)^{1/2}}{(r-\rho)^2} F_{1,p}(x,r,\rho) + \ln|r-\rho|\, F_{2,p}(x,r,\rho) + F_{3,p}(x,r,\rho) \right\}.$$

$$(8) \quad a r u_o - w_o = \sum_{m=0}^{\infty} \frac{e^{m\omega t}}{4\pi U} \sum_{l=1}^{\bar{n}} \left[\sum_{p=0}^{cp} \left\{ b_1(r,l)\, F_{1,p}(\theta_l) + b_2(r,l)\, F_{2,p}(\theta_l) + F_{3,p}(\theta_l) \right\} \right] C_{m,p}(\rho).$$

Fig. III-9 SOLUTION OF THE LIFTING SURFACE INTEGRAL EQUATION.

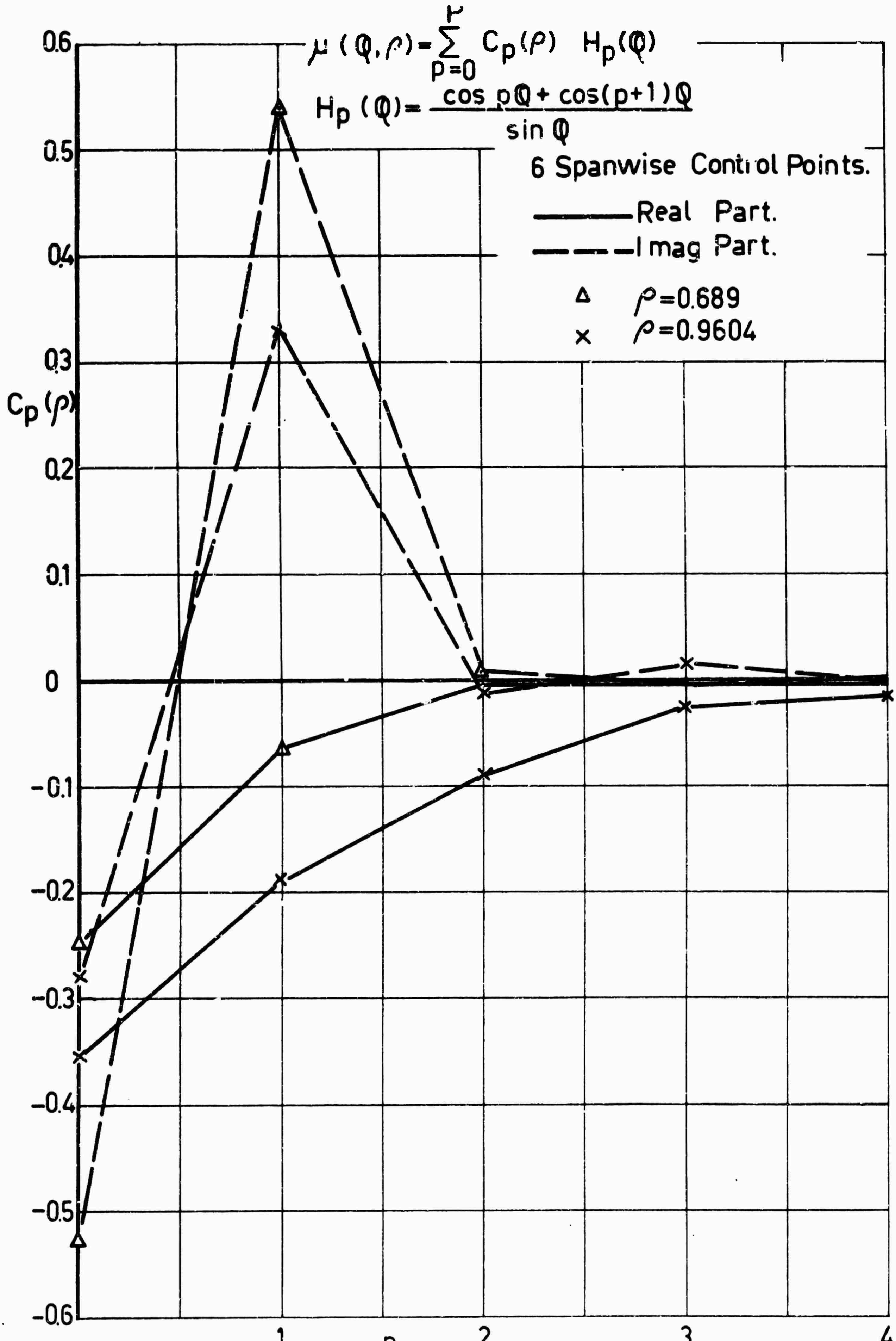


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Fig. III-10 CONVERGENCE OF THE COEFFICIENTS OF THE BIRNBAUM SERIES

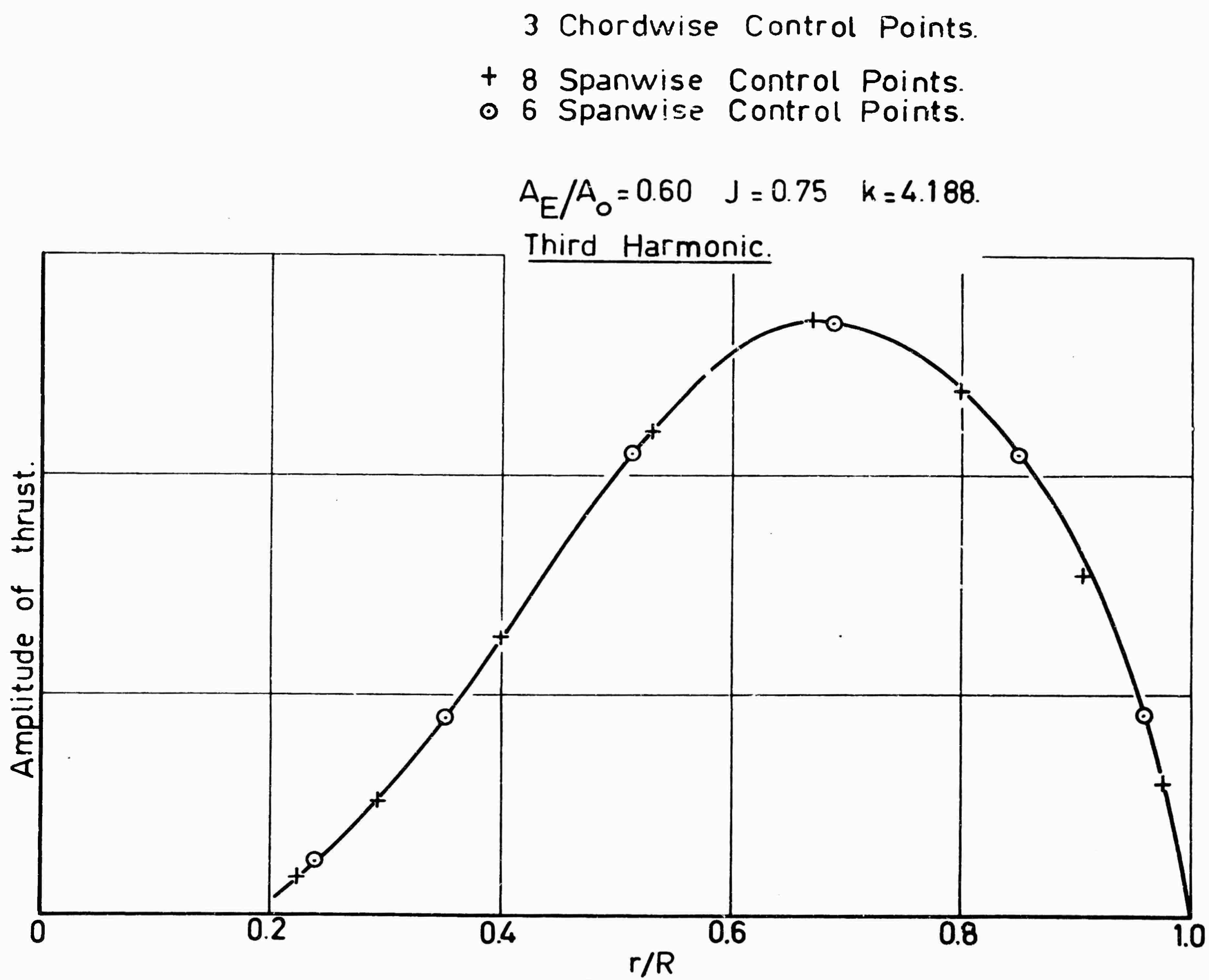

Fig. III-11 Unsteady spanwise thrust distribution.

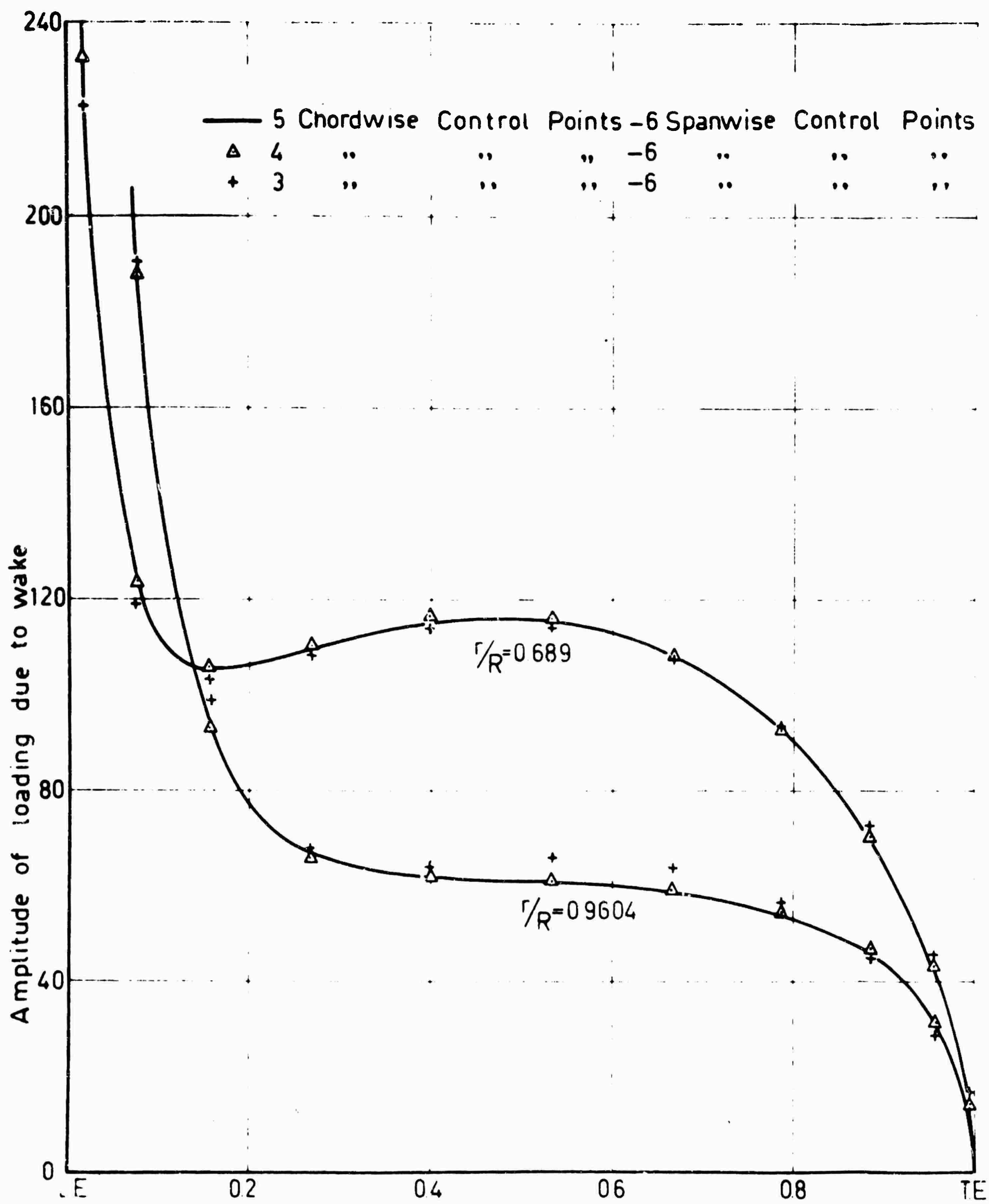

Fig. III-12 CONVERGENCE OF CHORDWISE UNSTEADY LOADING DISTRIBUTION

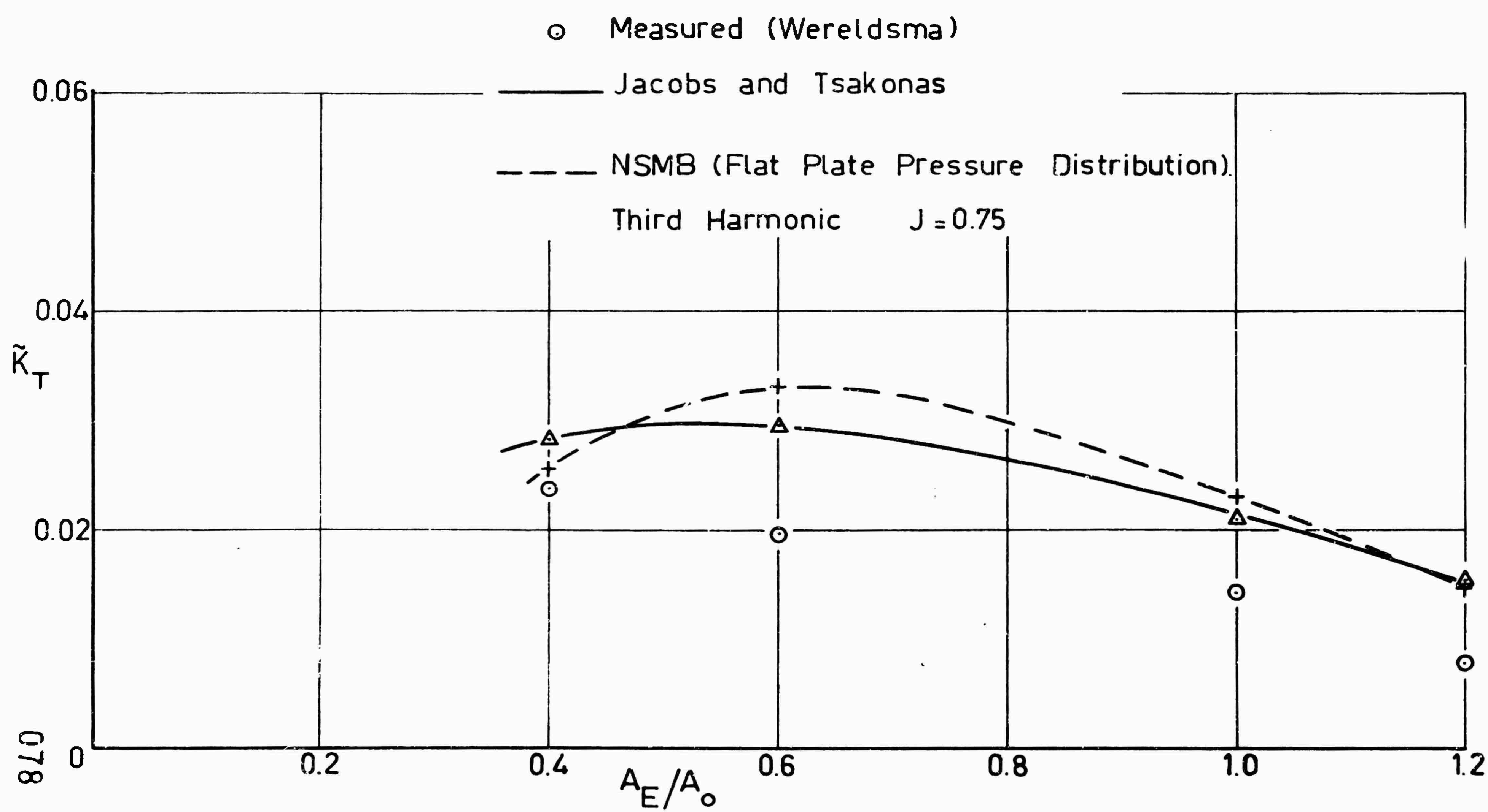

Fig. III-15 Comparison of preliminary results, (thrust coefficient).

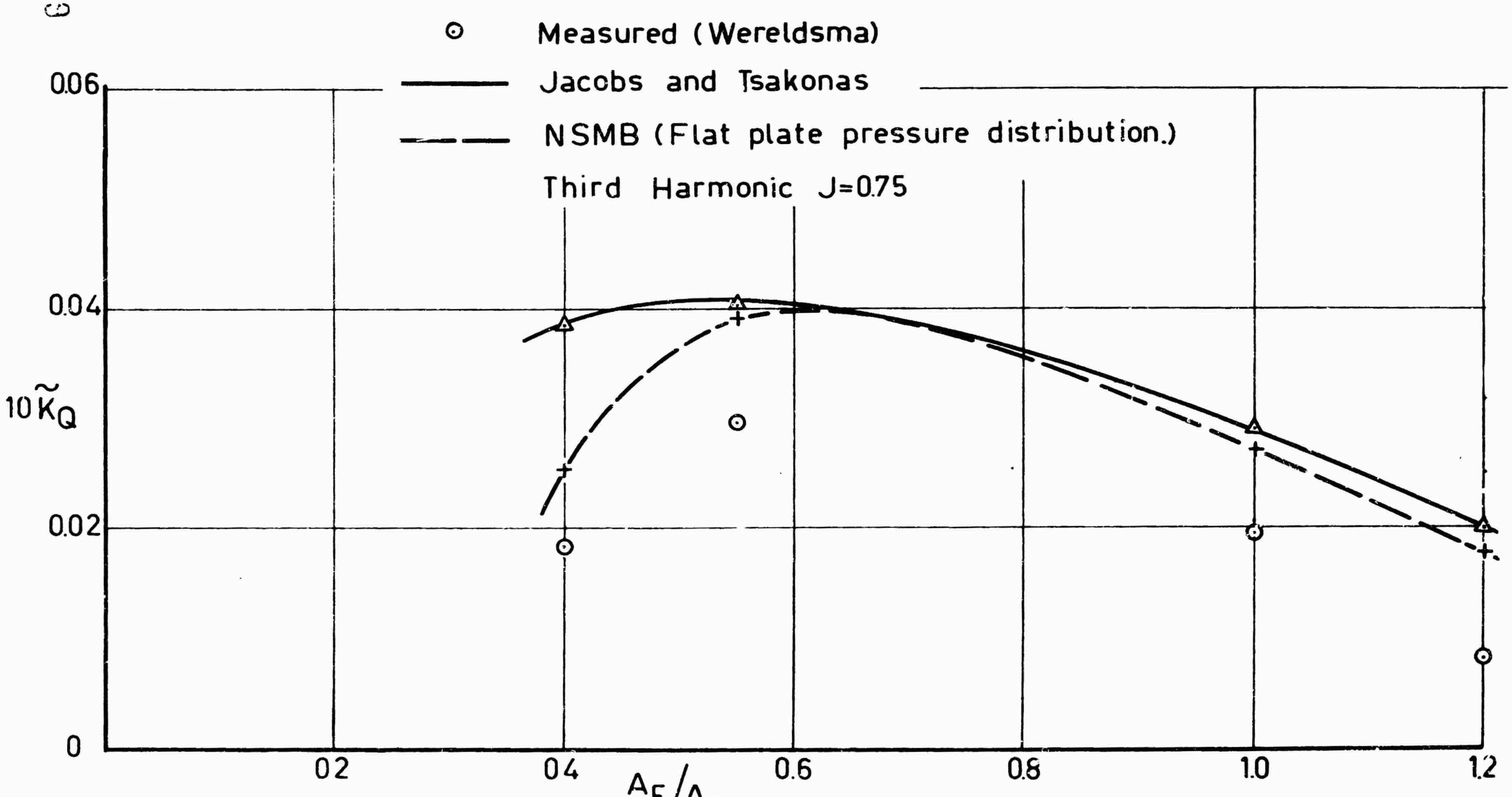

Fig. III-16 Comparison of preliminary results (torque coefficient).

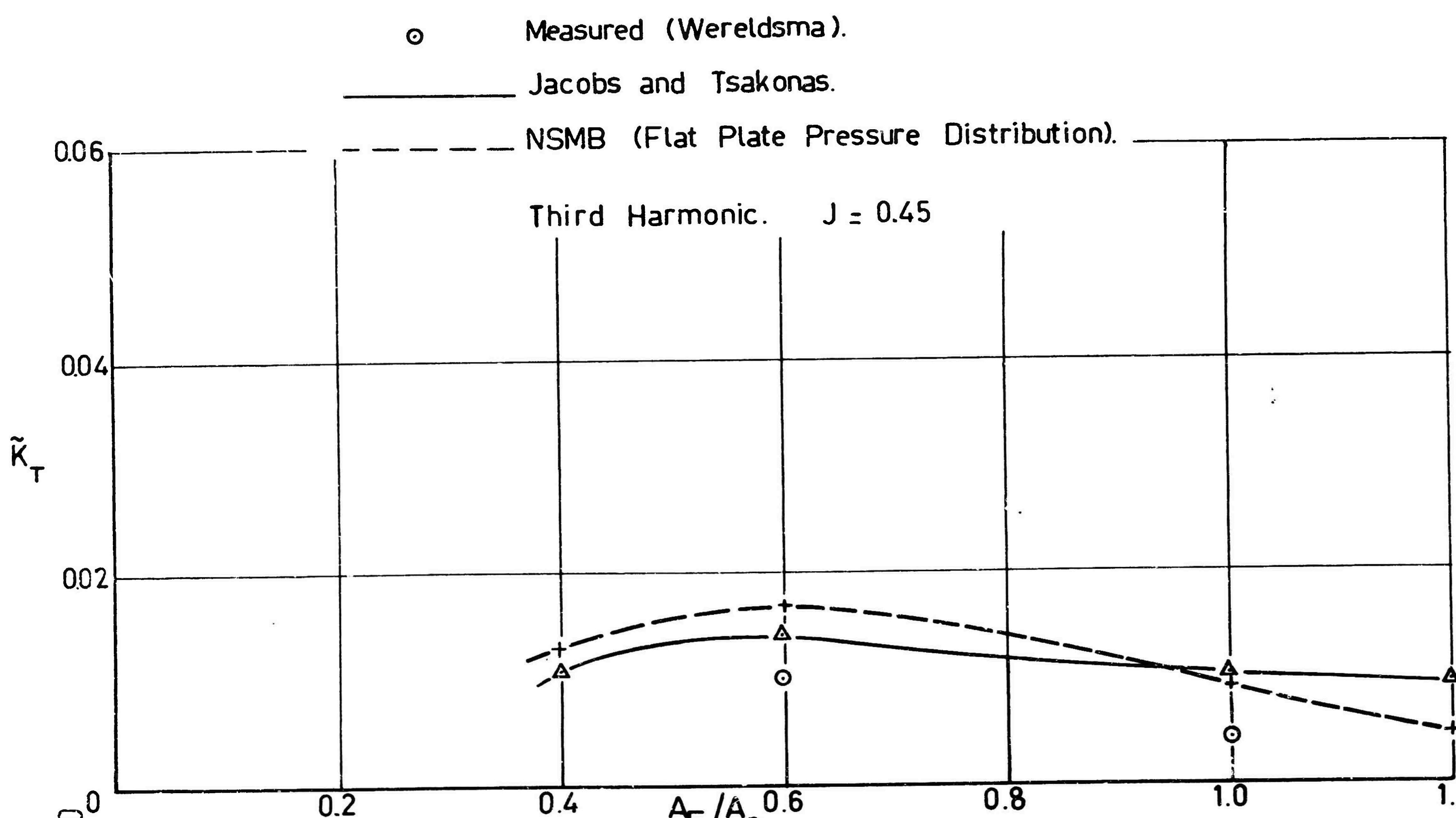

Fig. III-17 Comparison of preliminary results (thrust coefficient).

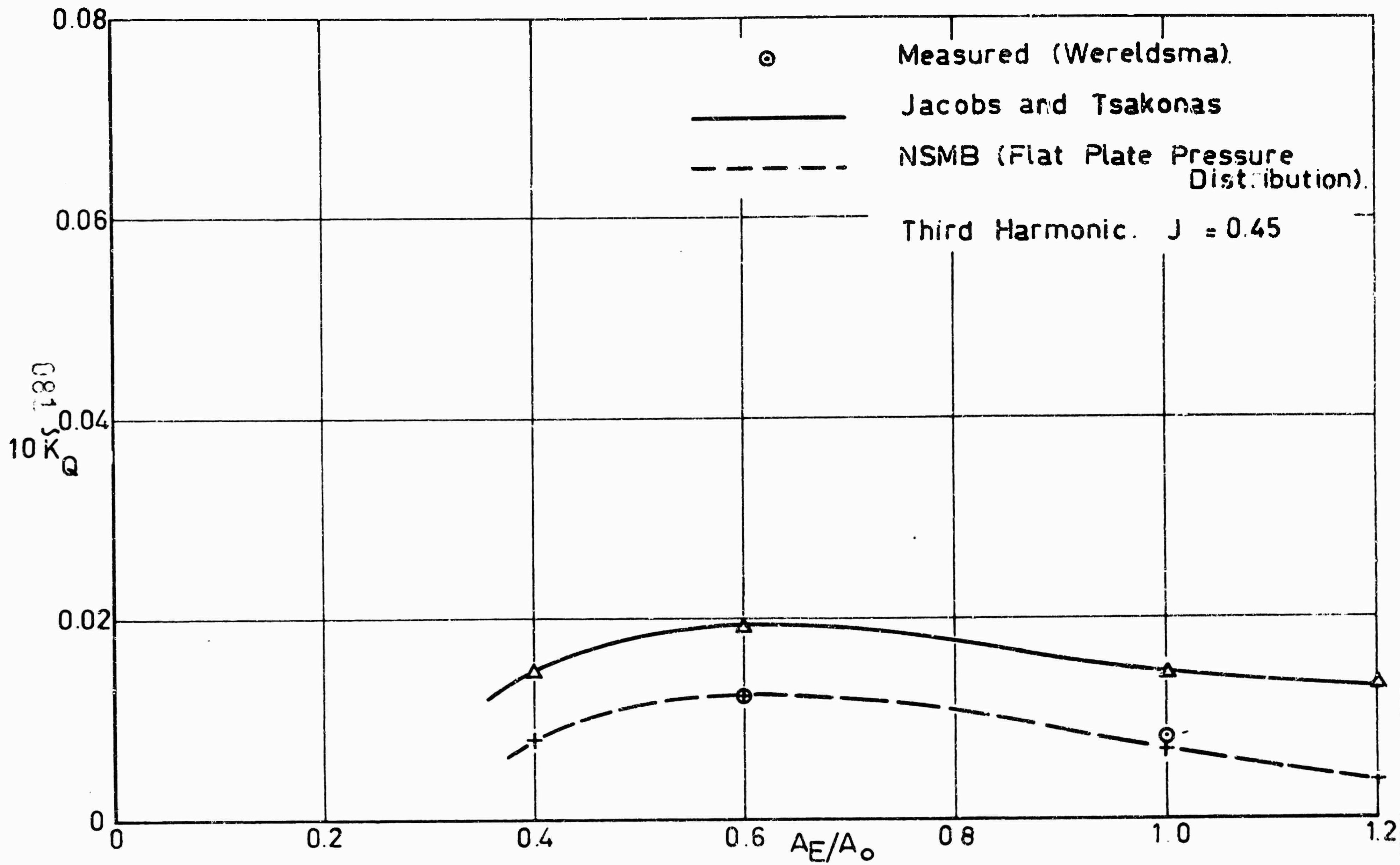

Fig. III-1 Comparison of preliminary results (torque coefficient)

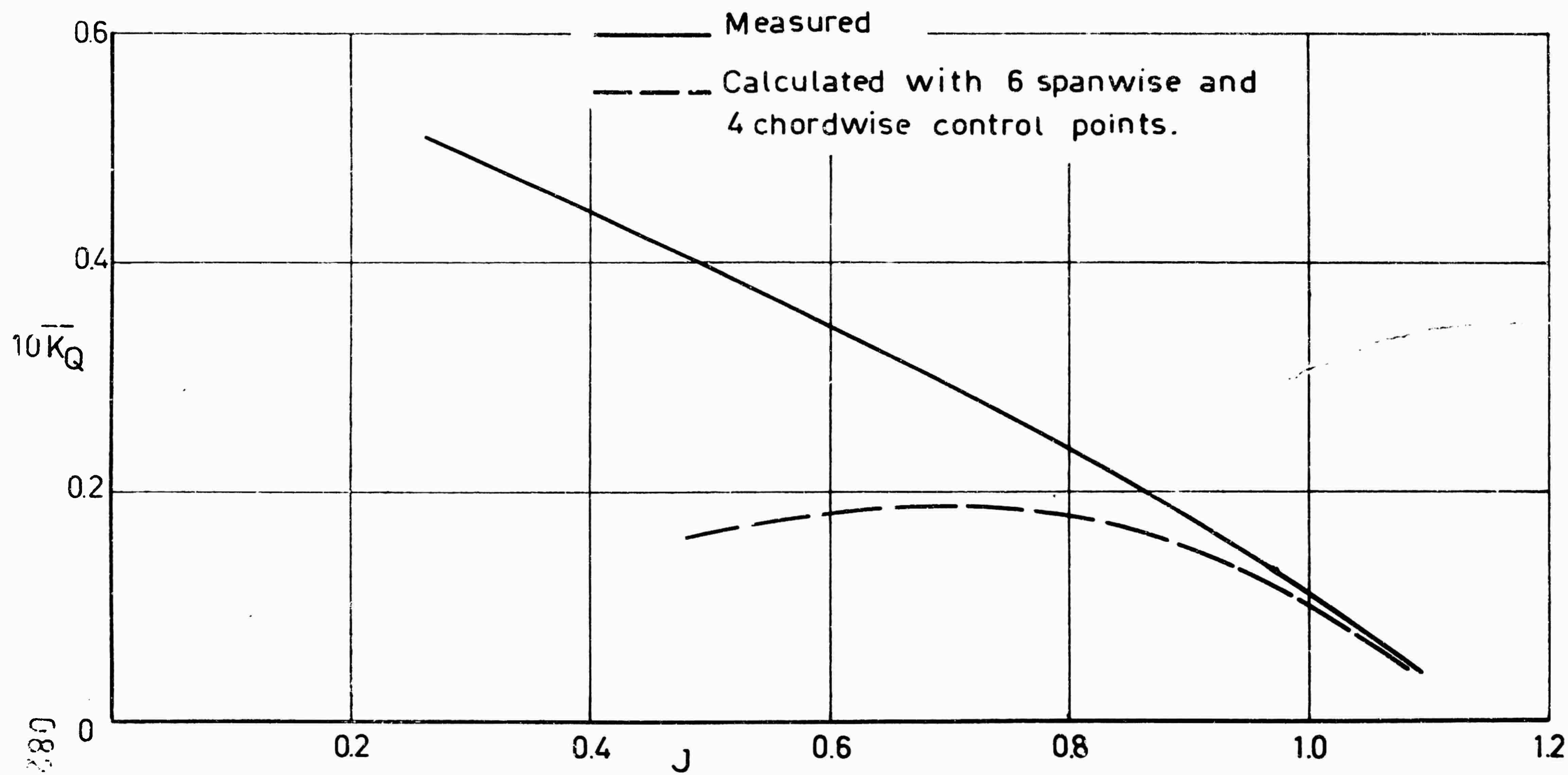

Fig. III-20 Mean torque coefficient of B 3-50 screw ($P/D = 1.0$).

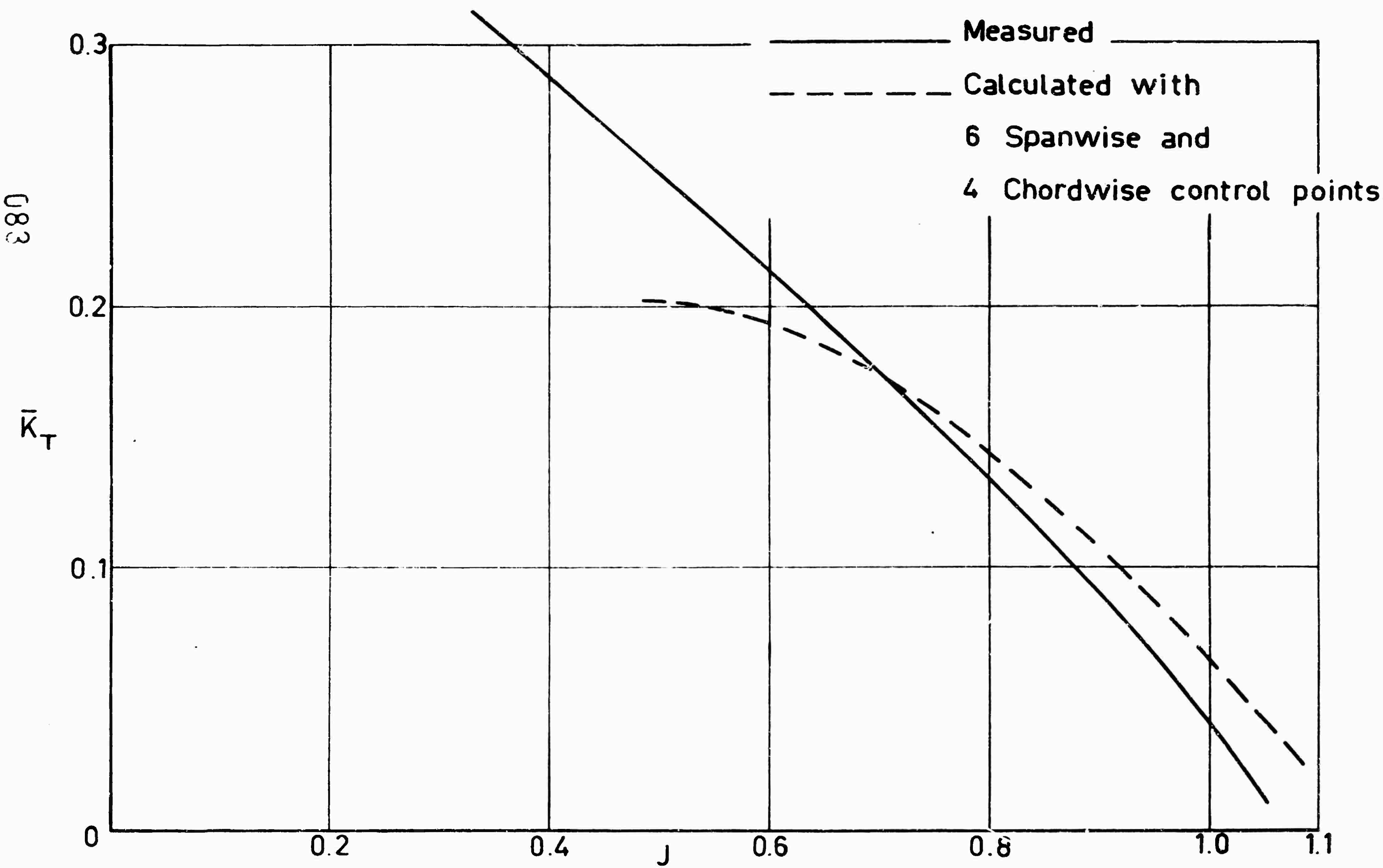

Fig. III-19 Mean thrust coefficient of B 3 - 50 screw (P/D = 1.0)

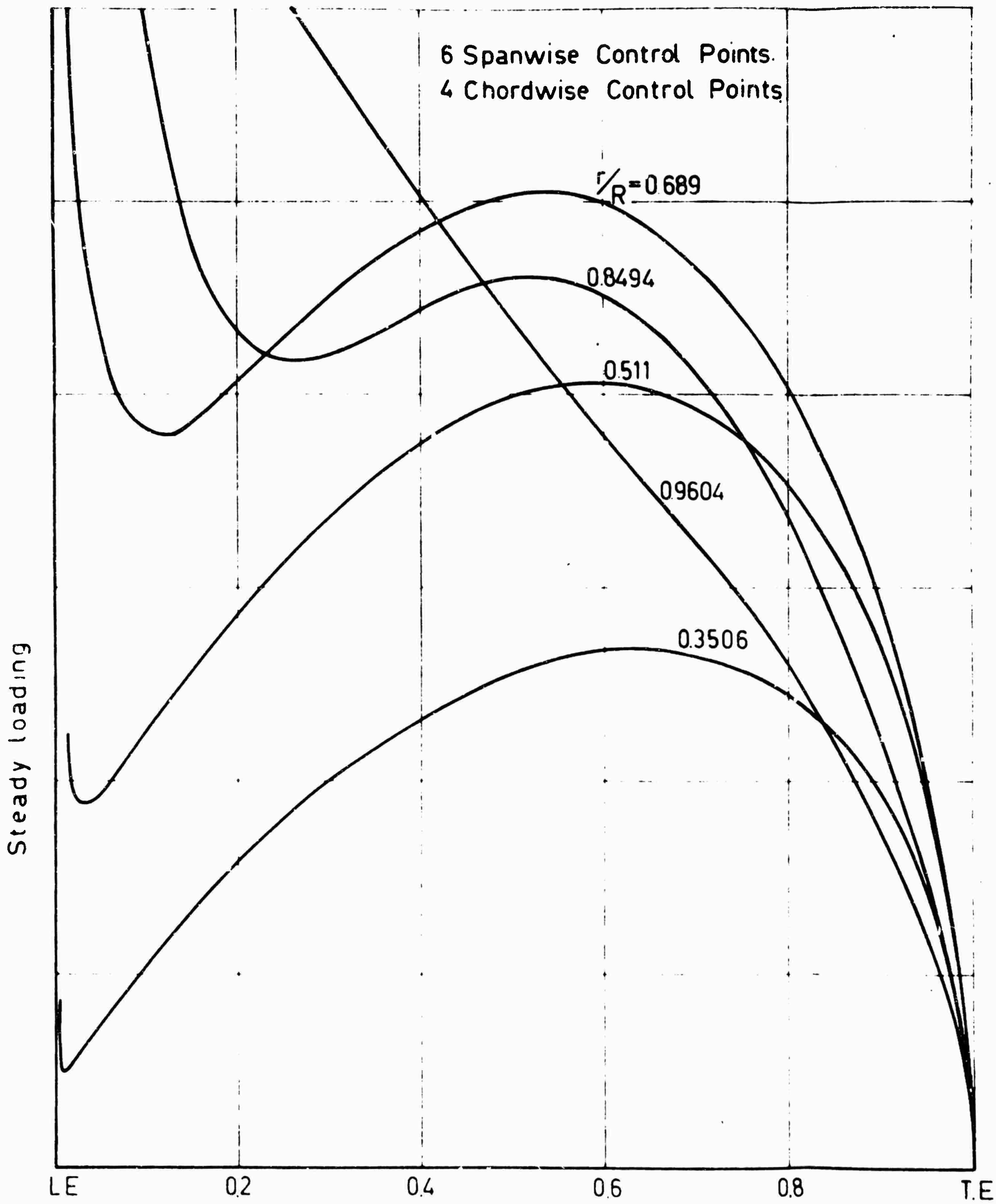

Fig. III-21 STEADY CHORDWISE PRESSURE DISTRIBUTION OF B 3-50 SCREW

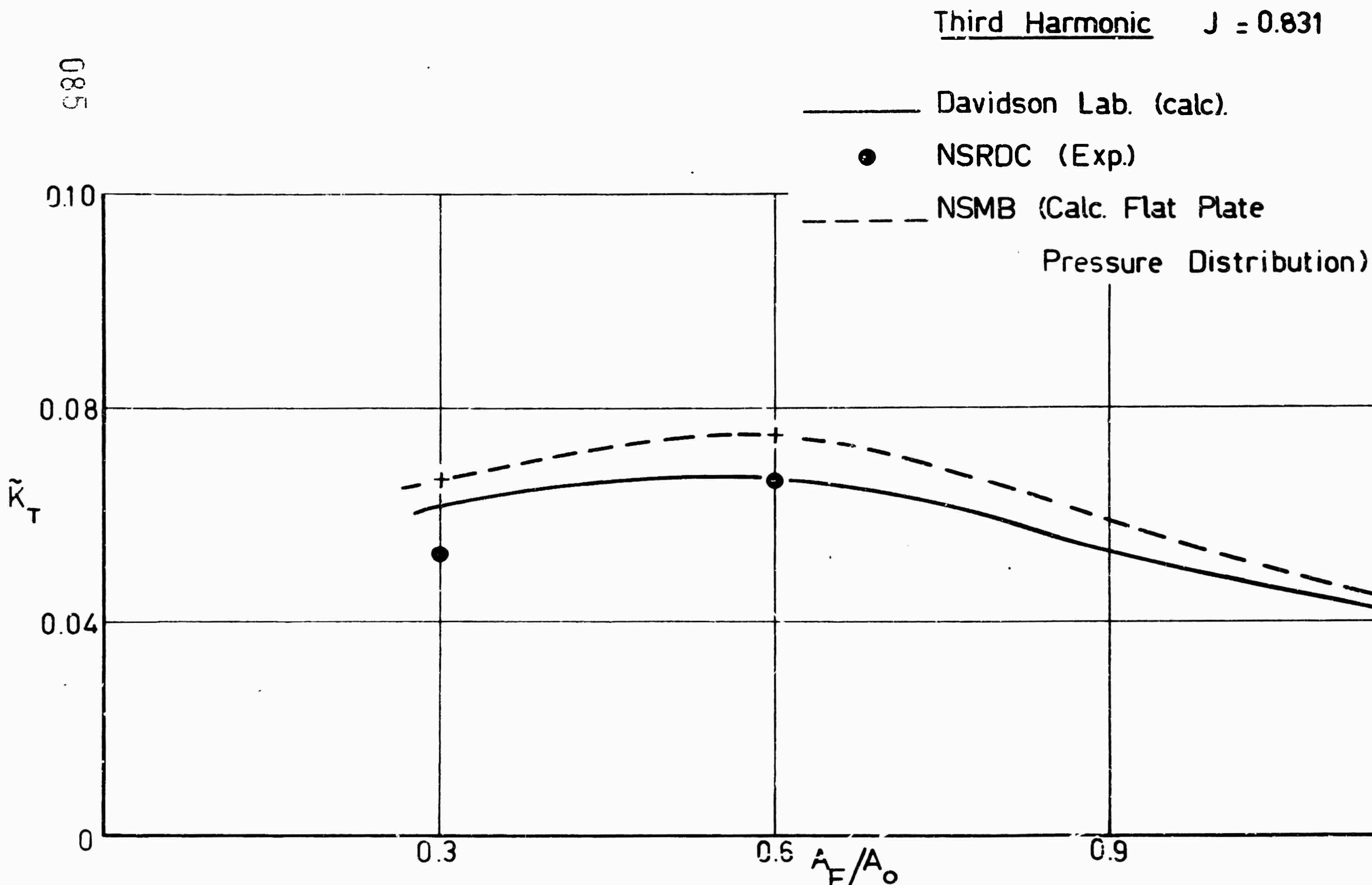

Fig. III-22 Comparison of vibratory thrust coefficients in screen generated wake.

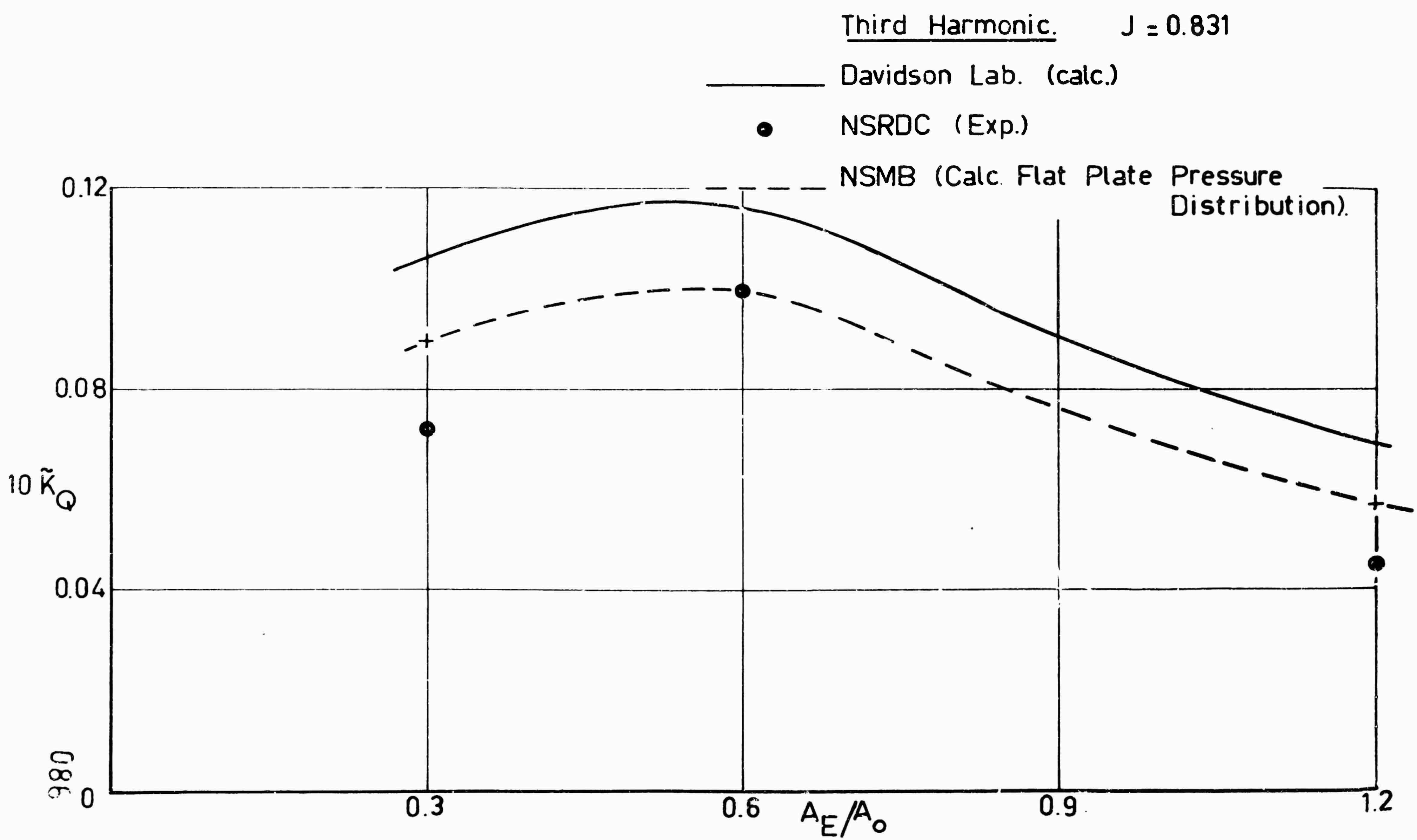

Fig. III-23 Comparison of vibratory torque coefficients in screen generated wake.

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