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RASA REPORT 70-02

STUDY OF MODIFICATION OF
ROTOR TIP VORTEX
BY AERODYNAMIC MEANS

INTERIM TECHNICAL REPORT For the Period
1 February 69 Through 31 January 70

by

Stephen A. Rinehart



Prepared under Contract No. N00014-69-C-0169
Req. No. NR 212-194/12-5-68 (Code 461)

by

ROCHESTER APPLIED SCIENCE ASSOCIATES, INC.
100 Allens Creek Road, Rochester, New York 14618
716/271-3450

for

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
Washington, D.C. 20360

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ABSTRACT

Numerous research efforts have been conducted by different investigators to alter the characteristics of the tip vortex generated by a helicopter blade in order to alleviate the blade-vortex interaction problem as well as the noise problem associated with impulsive loading. Various approaches have been taken in these investigations including modifications of the loading distribution by taper and twist and altering the blade tip by utilizing porous sections. All of these approaches have not been universally successful for all flight conditions. The present analytical investigation shows that it should be possible to significantly alter the characteristics of the trailing tip vortex for all flight conditions in a beneficial manner by injecting an airstream directly into the forming tip vortex. Analytical expressions were developed for the initial and final states of the vortex in order to evaluate the effects of mass flow injection on the vortex strength, swirl velocity distribution, vortex core pressure, vortex core size and the induced drag on the blade. On the basis of the results that were obtained, it was shown that the required mass flow may be obtained from centrifugal pumping action by venting the blade and therefore the desired modification can be obtained apparently without significant performance penalties which would be unacceptable.

LIST OF SYMBOLS

A	area of nozzle or jet, ft ²
a_c	tip vortex core radius, ft
B	constant of integration occurring in solution for swirl velocity
b_θ, b_z	radii of the zones of mixing for the swirl and axial components, ft
C	$(2\pi)^{-1}$ times the circulation of the fluid, ft ² /sec
c	chord length of the wing, ft
D_c	total induced drag of the wing, lb
H	total head losses of the fluid flow, ft ² /sec ²
I_0	inertia dyadic about the rotation point, slugs-ft ²
k	constant strength of the tip vortex, (nondimensional)
l_θ, l_z	mixing lengths for the swirl and axial components, ft
M_z	specific torque acting on the fluid, ft-lb
m, n	integers occurring in the similarity solution for the swirling flow
\bar{p}_c	tip vortex core pressure, psf
p_∞	free-stream pressure, psf
Q	flow rate of fluid, ft ³ /sec
$\bar{R}, \bar{\theta}$	rotating unit vectors
R	radius of the rotor blade, ft
r, θ, z	cylindrical coordinates

s'	wing semispan; ft
T	thrust of the jet, lb
v_r, v_θ, v_z	radial, tangential, and axial velocity components of the fluid flow
V_∞	free-stream velocity, fps
x, y, z	Cartesian coordinates
Γ	circulation of the fluid, ft ² /sec
ν	the coefficient of viscosity
ρ	density of air, slugs/ft ³
σ_{max}	scale of velocity defect
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$	turbulent normal stress components of the fluid tensor
$\tau_{r\theta}, \tau_{\theta z}, \tau_{zr}$	turbulent shear stress components of the fluid tensor
σ	swirl velocity component, fps
Ω, ω	angular velocity of the rotor blade, rad/sec
ω_c	angular velocity of the vortex core, rad/sec
(—)	bars over variables are time averages
(')	prime denotes the variation of a quantity about the mean due to turbulence

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INTRODUCTION

While any airfoil develops a trailing tip vortex of significant strength, the helicopter blade, because of its loading distribution, tends to develop a relatively stronger tip vortex than that developed by a fixed wing of a conventional aircraft. Another important difference between the vortex-lifting surface relationship of the fixed wing and of a rotating lifting surface is that with the fixed wing the tip vortex streams away from the lifting surface after it is formed while with the helicopter, the vortex streaming from a given blade tends to interact with one or more of the following blades. This blade-vortex interaction has been of considerable concern as regards blade loading and dynamic response as well as regards noise associated with impulsive loading such as "blade slap." These characteristic noise signatures are of particular importance in detection problems associated with military operations and in annoyance problems associated with civil operations.

Numerous research efforts have been conducted by various investigators to alter the characteristics of the tip vortex and thus alleviate the blade response and noise problem. Several approaches have been employed including modification of the loading distribution by taper and twist and by altering the blade tip geometry using porous tips. All of these approaches, while sometimes achieving a moderate degree of success at a given flight condition have not been universally successful for all flight conditions. It was believed, however, that direct injection of an airstream into the forming tip vortex, would significantly alter the characteristics of the trailing vortex at all flight conditions in a manner that would be beneficial to both the blade dynamic response problem and impulsive noise.

The study was conducted in three steps. The initial work was directed at studying the characteristics of tip vortices. The second phase on which the majority of effort was expended, was the

determination of the effects of mass flow injection on the vortex strength, swirl velocity distributions, vortex core pressure changes, vortex core sizes and the induced drag on the blade. The final step was to analyze various means of injecting an airstream into the forming tip vortex and evaluating its possible effects on helicopter performance characteristics.

TECHNICAL DISCUSSION

Discussion of the Relevant Characteristics of Tip Vortices

A considerable number of studies have been conducted concerning the structure and formation of wing and rotor-blade tip vortices. Surveys of work carried out in this area are given in References 1 and 2. Reference 1 is concerned with experiments, while Reference 2 deals primarily with theoretical developments.

The results of the studies indicate that a complete understanding of the problem is still to be obtained. Theoretical treatments, such as the one in Reference 3, are based on the assumption that the flow is essentially two-dimensional, with perturbations in the direction of flight playing only a minor role. The analysis of Reference 3, which is representative, predicts a tip-vortex core diameter of 15% of span, with vortices spaced about 3/4 of the span apart, for a wing with elliptic loading. A parallel study in Reference 3 leads to an estimate of from 20 to 30 chord lengths for the distance required for roll-up of the wake vorticity into concentrated tip vortices. On the other hand, measurements taken behind model wings and full-scale aircraft wings, as reported in Reference 4, indicate that tip-vortex core diameters are only about 3% of span, and the vortices are located almost at the wing tips, so they are separated by a distance about equal to the span. Furthermore, the roll-up process is 90% complete within one chord length for a moderately loaded wing of high aspect ratio.

This large discrepancy between theory and experiment could be due to three-dimensional effects, as indicated by the approximate analysis outlined in Reference 5. By ordering terms in a manner similar to that of boundary-layer theory and making certain assumptions (the validity of which is difficult to assess) regarding energy losses, it was shown in Reference 5 that a strong axial flow

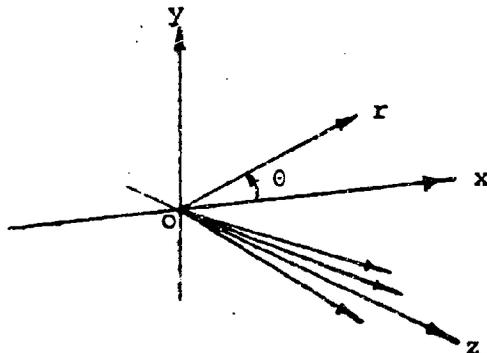
could exist in the core of a tip vortex. This flow would be in the same direction as the free stream, leading to a smaller core size than would be the case with no axial flow. Presumably, three-dimensional effects could also account for the difficulty in predicting the separation and rate of roll-up of the tip vortices.

It was felt that it would be beyond the scope of this study to develop a method for more accurately predicting tip-vortex characteristics. Thus, in analyzing the effects of injecting an airstream into a tip vortex, a range of values for core size and circulation representative of the measured values were used.

Theoretical Formulation

Problem Definition

The flow into which an airstream is to be injected is defined in cylindrical coordinates (r, θ, z) as sketched below.



The nominal flow is assumed to be incompressible and axisymmetric, with components $V_\theta(r)$ and $V_z(r)$ in the tangential and axial directions, respectively. The radial component of the nominal flow is taken to be zero, giving a pure swirling flow. A jet of incompressible fluid emanates from the origin into this flow, directed along the positive z-axis.

Since the tip vortex may already possess a strong axial flow component, a pure axial jet injected in the same direction as that axial flow may cause only a limited reduction in the swirl component of the nominal flow. Thus, it is assumed that the jet also has a swirl component opposing that of the nominal flow. Since variation of the tangential flow component due to mixing is to be analyzed in any case, consideration of a swirling jet rather than a simple axial jet does not appreciably complicate the problem.

In light of the objective of the present study, and since the jet would undoubtedly be turbulent for any practical application, it was decided to treat the case of turbulent mixing in order to insure that realistic results would be obtained. It is noted that the analysis of turbulent flows in general is somewhat less satisfactory than treatment of laminar flows, because a certain degree of empiricism is always necessary in obtaining a solution. However, the theory for turbulent free jets has met with considerable success, particularly when compared with analyses of other types of turbulent flows, such as in boundary layers, apparently because the assumptions applied to turbulent free jets are fairly realistic. For the present study, therefore, it was decided to develop the equations for a steady, axisymmetric, incompressible, turbulent flow created by injection of a swirling jet into a swirling stream. The general equations governing such a flow were derived by following a procedure analogous to the one used in Ref. 6 for formulating the equations for a turbulent flow referred to Cartesian coordinates. Thus, the radial, tangential, and axial flow components and the pressure can be written in the form

$$v_r(r, \theta, z, t) = \bar{v}_r(r, z) + v'_r(r, \theta, z, t)$$

$$v_\theta(r, \theta, z, t) = \bar{v}_\theta(r, z) + v'_\theta(r, \theta, z, t)$$

$$v_z(r, \theta, z, t) = \bar{v}_z(r, z) + v'_z(r, \theta, z, t)$$

$$p(r, \theta, z, t) = \bar{p}(r, z) + p'(r, \theta, z, t)$$

The quantities with bars over them are time averages and the primed quantities are variations about the mean due to turbulence. These expressions were substituted into the Navier-Stokes equations for incompressible flow referred to cylindrical coordinates (Ref. 6), the time average taken and suitable manipulations performed using the continuity equation. Assuming the flow is axisymmetric, the governing equations were found to be

$$\bar{v}_r \frac{\partial \bar{v}_r}{\partial r} - \frac{\bar{v}_\theta^2}{r} + \bar{v}_z \frac{\partial \bar{v}_r}{\partial z} = \frac{1}{\rho} \left[\frac{\partial \bar{\sigma}_{rr}}{\partial r} + \frac{\partial \bar{\tau}_{rz}}{\partial z} + \frac{1}{r} (\bar{\sigma}_{rr} - \bar{\sigma}_{\theta\theta}) \right]$$

$$\bar{v}_r \frac{\partial \bar{v}_\theta}{\partial r} + \frac{\bar{v}_\theta \bar{v}_r}{r} + \bar{v}_z \frac{\partial \bar{v}_\theta}{\partial z} = \frac{1}{\rho} \left[\frac{\partial \bar{\tau}_{r\theta}}{\partial r} + \frac{\partial \bar{\tau}_{\theta z}}{\partial z} + \frac{2}{r} \bar{\tau}_{r\theta} \right] \quad (1)$$

$$\bar{v}_r \frac{\partial \bar{v}_z}{\partial r} + \bar{v}_z \frac{\partial \bar{v}_z}{\partial z} = \frac{1}{\rho} \left[\frac{\partial \bar{\tau}_{rz}}{\partial r} + \frac{\partial \bar{\sigma}_{zz}}{\partial z} + \frac{1}{r} \bar{\tau}_{rz} \right]$$

$$\frac{\partial}{\partial r} (r \bar{v}_r) + \frac{\partial}{\partial z} (r \bar{v}_z) = 0$$

where

$$\bar{\sigma}_{rr} = -\bar{p} - \rho \bar{v}_r'^2 + 2\nu \frac{\partial \bar{v}_r}{\partial r}$$

$$\bar{\sigma}_{\theta\theta} = -\bar{p} - \rho \bar{v}_\theta'^2 + 2\nu \frac{\bar{v}_r}{r}$$

$$\bar{\sigma}_{zz} = -\bar{p} - \rho \bar{v}_z'^2 + 2\nu \frac{\partial \bar{v}_z}{\partial z}$$

$$\bar{\tau}_{r\theta} = -\rho \bar{v}_r' v_\theta' + \nu r \frac{\partial}{\partial r} \left(\frac{\bar{v}_\theta}{r} \right)$$

$$\bar{\tau}_{rz} = -\rho \overline{v'_r v'_z} + \nu \left(\frac{\partial \bar{v}_r}{\partial z} + \frac{\partial \bar{v}_z}{\partial r} \right)$$

$$\bar{\tau}_{\theta z} = -\rho \overline{v'_\theta v'_z} + \nu \frac{\partial \bar{v}_\theta}{\partial z}$$

in which ν is the coefficient of viscosity. The averages of terms involving turbulent fluctuations represent the turbulent shear. The ordinary viscous terms, i.e. those multiplying ν , can generally be dropped, as they are usually much smaller than the turbulent shears. The general problem formulation is completed by specification of the boundary conditions:

$$\left. \begin{array}{l} \bar{v}_r \rightarrow 0 \\ \bar{v}_\theta \rightarrow V_\theta(r) \\ \bar{v}_z \rightarrow V_z(r) \end{array} \right\} \begin{array}{l} \text{as } r \rightarrow \infty \\ \text{and/or } z \rightarrow -\infty. \end{array}$$

Equations for Solution - The Boundary Layer Approximation

The analysis of turbulent free jets, either two-dimensional or axisymmetric, generally proceeds by simplifying the governing equations according to assumptions analogous to those employed in boundary layer theory (Ref. 6). Specifically, it is assumed that dependent variables change much more rapidly in the transverse (i.e., radial) direction than in the downstream, or axial direction. In addition, the transverse, or radial, flow component is assumed to be small compared with the axial component.

The boundary-layer type of approximation was assumed to be valid for the solution of Eqs. (1), with the additional assumption that \bar{v}_θ and \bar{v}_z are of the same order and that ordinary viscous

terms can be dropped in favor of turbulent shears. The resulting equations governing injection of a swirling jet into a swirling stream are:

$$\frac{1}{\rho} \frac{\partial \bar{p}}{\partial r} = \frac{\bar{v}_\theta^2}{r} \quad (2.1)$$

$$\frac{\bar{v}_r}{r} \frac{\partial}{\partial r} (r \bar{v}_\theta) + \bar{v}_z \frac{\partial \bar{v}_\theta}{\partial z} = \frac{1}{\rho} \left(\frac{\partial \bar{\tau}_{r\theta}}{\partial r} + \frac{2}{r} \bar{\tau}_{r\theta} \right) \quad (2.2)$$

$$\bar{v}_r \frac{\partial \bar{v}_z}{\partial r} + \bar{v}_z \frac{\partial \bar{v}_z}{\partial z} = - \frac{\partial \bar{p}}{\partial z} + \frac{1}{\rho} \left(\frac{\partial \bar{\tau}_{rz}}{\partial r} + \frac{\bar{\tau}_{rz}}{r} \right) \quad (2.3)$$

$$\frac{\partial}{\partial r} (r \bar{v}_r) + \frac{\partial}{\partial z} (r \bar{v}_z) = 0 \quad (2.4)$$

where, now,

$$\frac{\bar{\tau}_{r\theta}}{\rho} = - \overline{v'_r v'_\theta} \quad (2.5)$$

$$\frac{\bar{\tau}_{rz}}{\rho} = - \overline{v'_r v'_z} \quad (2.6)$$

The solution of Eqs. (2) is aided by the following considerations. First, there are two quantities which are invariant with axial distance from the origin, involving the unknown flow components. These quantities are calculated by momentum considerations in the following way.

Let T denote the thrust acting on the nozzle injecting the fluid jet. Integrating over a cylindrical control surface containing the origin, it is found that

$$\frac{T}{\rho} = \int_0^\infty \int_0^{2\pi} \left[\frac{\bar{p}}{\rho} (r, z) - \frac{\bar{p}}{\rho} (r, -\infty) + \bar{v}_z^2 (r, z) - \bar{v}_z^2 (r, -\infty) \right] r \, d\theta \, dr .$$

But, from the first of Eqs. (2),

$$\frac{\bar{p}_\infty - \bar{p}(r, z)}{\rho} = \int_r^\infty \bar{v}_\theta^2(\xi, z) \frac{d\xi}{\xi}$$

$$\frac{p_\infty - \bar{p}(r, -\infty)}{\rho} = \int_r^\infty v_\theta^2(\xi) \frac{d\xi}{\xi}$$

where, of course, $p_\infty = \lim_{r \rightarrow \infty} p(r, z)$, a constant. Also,

$$\bar{v}_z(r, -\infty) = v_z(r) .$$

Substituting these relations back in the expression for T, and integrating the pressure terms by parts, it is found that

$$\frac{T}{2\pi\rho} = \int_0^\infty \left\{ \bar{v}_z^2(r, z) - v_z^2(r) + \frac{1}{2} \left[\bar{v}_\theta^2(r, z) - v_\theta^2(r) \right] \right\} r dr \quad (3)$$

In an analogous manner, the torque acting on the nozzle due to the swirl which it imparts to the jet is calculated from consideration of the flux of angular momentum from a cylindrical control volume. If the shear acting on the control surface is discarded in favor of the remaining terms, consistent with the ordering approximations imposed on the differential equations, the torque, denoted M_z , is found to be given by

$$\frac{M_z}{2\pi\rho} = \int_0^\infty \left[\bar{v}_z(r, z) \bar{v}_\theta(r, z) - v_z(r) v_\theta(r) \right] r^2 dr \quad (4)$$

The expressions for both T and M_z , i.e., Eqs. (3) and (4), must be invariant with z.

The other consideration aiding the solution of Eqs. (2) involves calculation of the turbulent shears $\tau_{r\theta}$ and τ_{rz} . In the case of free turbulent flows, Prandtl's hypothesis is generally employed for this purpose (Ref. 6). For the problem at hand, this hypothesis is formulated according to

$$\left. \begin{aligned} \frac{\bar{\tau}_{r\theta}}{\rho} &= \nu_{\theta}(z) \left[\frac{\partial \bar{v}_{\theta}}{\partial r} - \frac{\bar{v}_{\theta}}{r} \right] \\ \frac{\bar{\tau}_{rz}}{\rho} &= \nu_z(z) \frac{\partial \bar{v}_z}{\partial r} \end{aligned} \right\} \quad (5)$$

while

$$\begin{aligned} \nu_{\theta}(z) &= \kappa_{\theta} b_{\theta}(z) \left[\bar{v}_{\theta\max} - v_{\theta}(b_{\theta}) \right] \\ \nu_z(z) &= \kappa_z b_z(z) \left[\bar{v}_{z\max} - v_z(b_z) \right] \end{aligned} \quad (6)$$

where κ_{θ} and κ_z are constants, b_{θ} and b_z are the radii of the zones of mixing for the swirl and axial components, respectively, and $\bar{v}_{\theta\max}$ and $\bar{v}_{z\max}$ are the maximum values of \bar{v}_{θ} and \bar{v}_z , respectively, in the mixing zone.

As can be seen from Eqs. (5) and (6), Prandtl's hypothesis asserts that the shear is proportional to rate of change of strain, just as for laminar shear, and that the constant of proportionality, or apparent kinematic viscosity, is invariant over the radial extent of the mixing zone, but that it does vary axially in the manner of Eqs. (6). The apparent kinematic viscosity is assumed here to be different for mixing of the axial and swirl components, but each is taken to obey Prandtl's further assertion (Eqs. (6)) that they vary in proportion to the size of the mixing zone and the maximum differences in the mean flow.

The relations given in Eqs. (2) through (6) constitute the formulation of the problem of a steady, axisymmetric, incompressible, free turbulent flow as created by the injection of a fluid jet into a swirling stream. It was believed unlikely that an exact solution could be obtained, either analytically or numerically to this set of nonlinear partial differential equations within the constraints of this study. Therefore, it was necessary to determine and apply rational approximations in these relations to simplify the problem.

Limiting Cases (Similarity Solutions)

Problems involving turbulent flows are generally very complicated in nature and to examine such problems from an analytical viewpoint requires the aid of semi-empirical hypothesis. In the present analysis, Prandtl's hypothesis is employed for this purpose as is often done with good results in analyzing free turbulent flows. This hypothesis states that the shearing stress is proportional to the shearing strain (velocity gradients) but the proportionality constant must be determined experimentally.

In analyzing the injection problem of the free turbulent jet, two limiting cases have been considered which enable explicit results to be obtained. Besides the governing differential equations of motion, the flow of a swirling jet is further specified by two invariant quantities; the linear and angular momentum.

It is possible to introduce two similarity scales depending on whether one controls the flow by linear or angular momentum, that is, depending on if one has a strong axial jet with weak swirl or a weak axial jet with strong swirl. Chervinsky [7] has shown recently that in a study of the flow field in a turbulent swirling jet far from the orifice that similar solutions of the components of the velocity exist for these two limiting cases. Also, Chervinsky [7] obtained good similarity for the tangential and axial velocity profiles in a series of experiments on swirling

jets issuing from a round orifice. The similarity scale for the swirl component is assumed to be given by

$$k_{\theta} v_z^{1/4}, \quad (7)$$

and for the axial component, one has

$$k_z v_z^{1/3}. \quad (8)$$

It is also assumed that the initial flow field for both limiting cases before injection is specified by the following boundary conditions:

$$z = 0, \quad v_{\theta}(r) = \frac{\Gamma_0}{2\pi r}, \quad v_z = v_{\infty} = \text{constant} \quad (9)$$

which assumes the generation of a potential vortex at the origin where Γ_0 is the circulation in the initial vortex.

Since the present analysis is based on the existence of similarity solutions, then the interest here is focused on steady, time-averaged, axisymmetric flow fields which admit the customary boundary-layer-type approximations in which the axial gradients are of small magnitude compared with the radial gradients, that is,

$$\frac{\partial}{\partial z} \ll \frac{\partial}{\partial r}, \quad \bar{v}_r \ll \bar{v}_z, \quad (10)$$

where \bar{v}_r is the radial component of the velocity and \bar{v}_z is the axial component.

Similarity Solutions for Direct Linear Injection

For the case of a strong axial jet with a weak swirl component, the control of the flow field will be by linear momentum. However, due to turbulent mixing, the injected mass of air will develop a

swirl component far downstream, where it is further assumed that the usual boundary-layer-type approximations are supplemented by the condition that,

$$|\bar{v}_z - V_\infty| \ll V_\infty \quad (11)$$

A characteristic feature of a steady trailing line vortex from one side of a wing or rotor blade is the existence of a strong axial flow near the axis of symmetry. The coupling of the axial and swirl velocity components occurs through the pressure term for a steady line vortex and any change in the tangential velocity component with distance z downstream produces an axial pressure gradient and consequently an axial acceleration or deceleration. The process of contraction or expansion of a vortex core depends upon the axial acceleration of elements of fluid entering the core as pointed out by Batchelor [5] who showed the influence of a positive axial pressure gradient arising from the centrifugal force on the decay of the axial component of the velocity. Of course, the change in diameter of a vortex core would also depend on the wing geometry.

In light of these considerations, a similarity solution for the flow in a trailing line vortex far downstream from the point of injection was developed analogous to Batchelor's solution but of course retaining the assumption that the flow is turbulent. The resulting asymptotic solution for the axial velocity field is given as follows:

$$\bar{v}_z = V_\infty - \frac{C_0^2}{8\nu_\theta z} \log \frac{zV_\infty}{\nu_\theta} Q_1(\eta) + \frac{C_0^2}{8\nu_\theta z} Q_2(\eta) - \frac{BV_\infty^2}{8\nu_\theta z} e^{-\eta} \quad (12)$$

where

$$\eta = \frac{V_\infty r^2}{4\nu_\theta z} \quad (13)$$

and $Q_1(\eta)$ and $Q_2(\eta)$ are known functions. The apparent kinematic viscosity ν_θ was assumed constant as a first approximation. This approximation has been employed by other investigators for free turbulent flows with good success (i.e., see Newman [8]). In addition, this assumption was employed by Eskinazi in investigating the viscous decay of a steady, two-dimensional viscous vortex. It was found that the various experimental velocity distributions in the vortex were in good agreement with a linearized, axisymmetric, laminar, incompressible, viscous vortex theory due to Newman when an apparent kinematic viscosity of eight to ten times the laminar kinematic viscosity was used. This result illustrates the fact that ordinary vorticity diffusion is stronger than that based on a purely laminar case and therefore the flow should be characterized as turbulent.

Velocity Field

Far downstream, where the boundary-layer-type approximations $\partial/\partial z \ll \partial/\partial r$ and $\bar{v}_r \ll \bar{v}_z$ are supplemented by the approximation

$$|\bar{v}_z - V_\infty| \ll V_\infty,$$

the equation of motion (2.3) reduces to

$$V_\infty \frac{\partial \bar{v}_z}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \nu \left(\frac{\partial^2 \bar{v}_z}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_z}{\partial r} \right) \quad (14)$$

and the equation for the swirl component becomes

$$V_\infty \frac{\partial \bar{v}_\theta}{\partial z} = \nu \left(\frac{\partial^2 \bar{v}_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_\theta}{\partial r} - \frac{\bar{v}_\theta}{r^2} \right).$$

This latter equation may also be written as

$$V_\infty \frac{\partial C}{\partial z} = \nu_\theta r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial C}{\partial r} \right). \quad (15)$$

The solution of this equation for C or \bar{v}_θ as a function of r at any value of z may be written down explicitly. As is well-known, the asymptotic form of the solution as $z \rightarrow \infty$ is

$$C = r\bar{v}_\theta = C_0 \{1 - B \exp(-\eta)\}$$

where

$$\eta = \frac{V_\infty r^2}{4v_\theta z} \quad \text{and}$$

C_0 is the (non-zero) value of C at large r . Hence the resulting velocity field after injection for the azimuthal component of the flow field becomes

$$\bar{v}_\theta = \frac{C_0}{r} \{1 - B \exp(-\eta)\} \quad (16)$$

The arbitrary constant of integration B which occurs in the above solution for the azimuthal velocity must be evaluated. For this purpose, the momentum integral theorem is employed, with a control surface enclosing the nozzle at the origin in the form of a right circular cylinder with generators parallel to the z -axis and of which A is the area of each end face. The upstream end face and the curved surface of the cylinder are both at a sufficiently large distance from the jet, so that conditions there are approximately as in the free stream. Therefore, it follows that the thrust produced by the jet is given in the usual way as

$$\frac{T}{\rho} = \int \left\{ \bar{v}_z (V_\infty - \bar{v}_z) + \frac{p_\infty - \bar{p}}{\rho} \right\} dA \quad (17)$$

where

$$\int_A (p_\infty - \bar{p}) dA = \text{resultant normal force on the end faces and } T$$

denotes the thrust of the jet. For steady flow the magnitude of the thrust produced by the jet is given by

$$T = \left(- \rho A_j v_j^2 \right) \quad (18)$$

where v_j is the exit velocity of the injected airstream leaving the nozzle and A_j is the area of the nozzle. It has been shown previously that the pressure term is given by

$$\frac{p_\infty - \bar{p}}{\rho} = \int_r^\infty \frac{C^2}{r^3} dr \quad (19)$$

where

$$C = r \bar{v}_\theta .$$

Far downstream, the trailing vortex is approximately cylindrical, and the pressure in the core is determined by the balance with centrifugal force, where the pressure is given by equation (19). Solving (19) for p , and inserting it into (17) and performing integrations by parts yields

$$\frac{T}{2\pi\rho} = \frac{C_0^2}{4} + \int_0^\infty \left\{ v_\infty (v_\infty - \bar{v}_z) r - \frac{1}{2} \frac{\partial C^2}{\partial r} \log \frac{r v_\infty}{v_\theta} \right\} dr \quad (20)$$

in which, consistent with Batchelor's approximations, $v_z (v_\infty - v_z)$ has been approximated by $v_\infty (v_\infty - v_z)$ and therefore the integrand can be taken zero outside the vortex core. Also, it is noted that

$$\left(\frac{C}{C_0} \right)^2 = (1 - Be^{-\eta})^2 \quad (21)$$

which follows from equation (16). Introducing the change of variable

$$\eta = \frac{v_\infty r}{r v_\theta z} \quad (22)$$

into (20) and performing the quadratures yields the following

quadratic equation for the constant of integration B in terms of the thrust T,

$$B^2 \left[(.5) \log_e \left(\frac{4zV_\infty}{v_\theta} \right) - .635 \right] + B \left[.577 - \log_e \left(\frac{4zV_\infty}{v_\theta} \right) \right] + \left[A - \frac{T}{2\pi\rho} \right] = 0 \quad (23)$$

where

$$A = \int_0^{\infty} V_\infty (V_\infty - \bar{v}_z) r dr = \int_0^a V_\infty (V_\infty - \bar{v}_z) r dr .$$

The value of the integral A can now be estimated via energy arguments, and B can then be calculated at a given downstream location for a given thrust injection. The velocity v_z is usually greater than V_∞ in the core and approaches V_∞ with increasing downstream locations. Outside of the vortex core $v_z = V_\infty$ (free stream velocity).

It is also possible to examine the effect of an injected air-stream on the axial velocity assuming that the injection process is of sufficient magnitude to induce turbulence and the swirl component decays independently of the axial velocity by again using the momentum integral theorem in conjunction with Batchelor's form of solution for the axial velocity.

In the case of flow in a tip vortex shed from a rotor blade or wing in an infinite body of fluid, all streamlines originate in a region where the pressure is uniform and equal to p_∞ and the fluid velocity is uniform with components $(V_\infty, 0, 0)$. Some streamlines will pass through the boundary layer at the wing where viscous forces are appreciable, and the Bernoulli function at any point in the vortex may therefore be written as

$$\frac{\bar{p}}{\rho} + \frac{1}{2} (\bar{v}_r^2 + \bar{v}_\theta^2 + \bar{v}_z^2) = \frac{p_\infty}{\rho} + \frac{1}{2} V_\infty^2 - \Delta H .$$

Batchelor has shown that the magnitude of the axial velocity (in the absence of total head losses ($H=0$)), in the vortex core is given by (for a vortex core of radius a_c which rotates rigidly with angular velocity ω_c):

$$\bar{v}_z = V_\infty, \text{ for } r \geq a_c \quad (24)$$

$$\bar{v}_z = \{V_\infty^2 + 2\omega_c^2 (a_c^2 - r^2)\}^{1/2} \text{ for } r \leq a_c.$$

The interesting feature of this equation is that it shows there is an excess axial velocity in the core of the vortex. Outside the core, $v_z = V_\infty$ (free stream velocity); but inside the core $v_z > V_\infty$. This result, however, is in disagreement with the linearized analysis of Newman which predicts an axial velocity deficiency ($v_z < V_\infty$) in the vortex core. At present, the question of whether there is an axial velocity defect in the vortex core has not been resolved either experimentally or theoretically.

Circulation

At a given location downstream, the tangential velocity is a function of r only, and the circulation in a circular region of radius r is

$$\Gamma = 2\pi(r - r_0)\bar{v}_\theta(r, z). \quad (25)$$

Drag

Spreiter and Sacks [3] have shown that the induced drag of the wing in terms of the vortex core radius for the rolled up vortices far behind the wing, assuming that the vortex cores are circular in shape and rotating as solid bodies has the form

$$D_c = \frac{\rho_0 \Gamma_0^2}{8\pi} \left\{ 1 + 4 \log_e \frac{2s' - a_c}{a_c} \right\} \quad (26)$$

where a_c denotes the tip vortex core radius and s' denotes the semispan of the vortices. Therefore, knowing the vortex core size which can be obtained from the graph of the tangential velocity distribution in the radial direction, the vortex core drag can then be computed from equation (26).

Similarity Solution for Swirl Injection

The problem to be solved for this case is formulated as follows (bars over the time-averaged quantities have been omitted):

$$\frac{p_\infty - \bar{p}}{\rho} = \int_r^\infty \frac{\bar{v}_\theta^2(\xi, z) d\xi}{\xi} \quad (27)$$

$$\frac{\bar{v}_r}{r} \frac{\partial}{\partial r} (r\bar{v}_\theta) + \bar{v}_z \frac{\partial \bar{v}_\theta}{\partial z} = v_\theta(z) \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r\bar{v}_\theta) \right] \quad (28)$$

$$\bar{v}_r \frac{\partial \bar{v}_z}{\partial r} + \bar{v}_z \frac{\partial \bar{v}_z}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \frac{v_z(z)}{r} \frac{\partial}{\partial r} \left[r \frac{\partial \bar{v}_z}{\partial r} \right] \quad (29)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r\bar{v}_r) + \frac{\partial \bar{v}_z}{\partial z} = 0 \quad (30)$$

where by Prandtl's hypothesis

$$v_\theta(z) = \kappa_\theta b_\theta(z) \left| \bar{v}_{\theta \max} - v_\theta(b_\theta) \right| \quad (31)$$

$$v_z(z) = \kappa_z b_z(z) \left| \bar{v}_{z \max} - v_z(b_z) \right| \quad (32)$$

in which b_θ and b_z are the radii of the mixing zones and κ_θ and κ_z are the constants. In order to deduce similarity and satisfy the boundary conditions a solution of the form

$$\bar{v}_\theta(r, z) = V_\theta(r) + \bar{\sigma}(r, z) \quad (33)$$

$$\bar{v}_z(r, z) = V_z(r) + \bar{u}(r, z) \quad (34)$$

is assumed. For the case of a weak axial jet with strong swirl, it is assumed that

$$v_z \ll V_z \quad (\text{i.e., weak axial jet})$$

while $\bar{\sigma}$ is of order V_z and V_θ . The governing equations for the swirl case then reduces to

$$\frac{p_\infty - \bar{p}}{\rho} = \int_r^\infty [V_\theta(\xi) + \bar{\sigma}(\xi, z)]^2 \frac{d\xi}{\xi} \quad (35)$$

$$V_z(r) \frac{\partial \bar{\sigma}}{\partial z} = v_\theta(z) \frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial}{\partial r} [r(V_\theta + \bar{\sigma})] \right\} \quad (36)$$

$$\bar{v}_r \frac{\partial V_z}{\partial r} + V_z(r) \frac{\partial \bar{v}_z}{\partial r} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \frac{v_z(z)}{r} \frac{\partial}{\partial r} \left[r \frac{\partial}{\partial r} (V_z + \bar{v}_z) \right] \quad (37)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r \bar{v}_r) + \frac{\partial \bar{v}_z}{\partial z} = 0 \quad (38)$$

$$v_\theta(z) = \kappa_\theta b_\theta(z) |\sigma_{\max}| \quad (39)$$

$$v_z(z) = \kappa_z b_z(z) |v_{z\max}| \quad (40)$$

and from (3) and (4)

$$\frac{T}{2\pi\rho} = \int_0^{\infty} (2V_z \bar{v}_z + \frac{1}{2} (\bar{\sigma}^2 + 2V_\theta \bar{\sigma})) r dr \quad (41)$$

$$\frac{M_z}{2\pi\rho} = \int_0^{\infty} r^2 V_z(r) \bar{\sigma}(r, z) dr. \quad (42)$$

In problems involving free turbulent jets it is usually assumed that the mixing length is proportional to the width of the jet, that is,

$$\frac{l_\theta}{b_\theta} = \beta = \text{constant.}$$

It may be shown that the mixing length l_θ associated with the swirl component is

$$l_\theta \sim z^{1/4} \quad (43)$$

and the scale of velocity defect

$$\sigma_{\max} \sim z^{-3/4} \quad (44)$$

and therefore the apparent kinematic viscosity may be seen to be

$$\nu_\theta(z) \sim l_\theta \sigma_{\max} \sim z^{-1/2} \quad (45)$$

Equation (36) characterizes the fact that via the assumptions employed, the swirling motion of the fluid has been uncoupled from the axial and radial motion and may therefore be determined independently. In order to solve this equation for σ , it will now be assumed that the free stream axial velocity component is constant, that is,

$$V_z(r) = \text{constant} = V_z. \quad (46)$$

Thus, we are required to solve Equation (36) under the stipulation that

$$\frac{M_z}{2 \rho V_z} = \text{constant} = \int_0^{\infty} r^2 \bar{\sigma}(r, z) dr, \quad (47)$$

where V_z is given in (46). It is now assumed that a similarity solution exists. Introducing the notation

$$\xi(r, z) = z \quad (48)$$

and

$$\eta(r, z) = rz^{-p} \quad (49)$$

then it follows that

$$\frac{\partial \bar{\sigma}}{\partial z} = \frac{\partial \bar{\sigma}}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial \bar{\sigma}}{\partial \eta} \frac{\partial \eta}{\partial z},$$

$$= \frac{\partial \bar{\sigma}}{\partial \xi} - \frac{p\eta}{\xi} \frac{\partial \bar{\sigma}}{\partial \eta},$$

$$\frac{\partial \bar{\sigma}}{\partial r} = \frac{\partial \bar{\sigma}}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial \bar{\sigma}}{\partial \eta} \frac{\partial \eta}{\partial r} = \frac{1}{\xi^p} \frac{\partial \bar{\sigma}}{\partial \eta},$$

and

$$\frac{\partial^2 \bar{\sigma}}{\partial r^2} = \frac{1}{\xi^{2p}} \frac{\partial^2 \bar{\sigma}}{\partial \eta^2}.$$

It is further assumed that the V_θ will be such that

$$\frac{\partial}{\partial r} \left\{ \frac{1}{r} \frac{\partial}{\partial r} (rV_\theta) \right\} = 0 \quad (50)$$

for every such r and therefore the differential equation to be solved becomes:

$$\sqrt{z} \frac{\partial \bar{\sigma}}{\partial z} = \frac{k}{V_z} \left\{ \frac{\partial^2 \bar{\sigma}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\sigma}}{\partial r} - \frac{\bar{\sigma}}{r^2} \right\} . \quad (51)$$

It may be noted that Equation (50) does not place any restrictions of particular note on the problem since one often encounters the boundary condition that

$$V_\theta = \frac{\Gamma_0}{2\pi r} \text{ at } z = 0 \quad (52)$$

where Γ_0 is the circulation in the initial vortex (assumed potential) generated at $z=0$, and therefore for this important case, e.g. (50) is identically satisfied.

From (49), it follows that

$$r = \xi^p \eta$$

and inserting this change of variable into the differential equation yields

$$\frac{\partial \bar{\sigma}}{\partial \xi} - \frac{p\eta}{\xi} \frac{\partial \bar{\sigma}}{\partial \eta} = \frac{k}{V_z \xi^{2p+1/2}} \left\{ \frac{\partial^2 \bar{\sigma}}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \bar{\sigma}}{\partial \eta} - \frac{\bar{\sigma}}{\eta^2} \right\} . \quad (53)$$

In accordance with the assumption that a similarity solution exists, it is further assumed that the swirl component of the velocity is given by the following expression

$$\bar{\sigma}(\eta, \xi) = \xi^m f(\eta) . \quad (54)$$

From the requirement that the net angular momentum must be constant, i.e. Equation (47), we have the result

$$\begin{aligned} \frac{M_z}{2\pi\rho V_z} &= \int_0^\infty r^2 \bar{\sigma}(r, z) dr = z^m \int_0^\infty r^2 f\left(\frac{r}{z^p}\right) dr \\ &= z^{m+3p} \int_0^\infty n^2 f(n) dn . \end{aligned} \quad (55)$$

Since it is required that this quantity be invariant with respect to z , one then obtains the following equation for m and p :

$$m + 3p = 0 \quad (56)$$

Inserting (53) into (54), one finds the equation of motion can be written as

$$mf - pnf' = \frac{k}{V_z} \xi^{1/2-2p} \left\{ f'' + \frac{f'}{n} - \frac{f}{n^2} \right\} , \quad (57)$$

where

$$f' = \frac{df}{dn} ,$$

and upon inspection of (57) it can be seen that

$$p = \frac{1}{4} \quad (58)$$

and hence from (56)

$$m = -\frac{3}{4} . \quad (59)$$

Finally, the differential equation for which f must be a solution is

$$f'' + \left(\frac{1}{n} + \frac{V_z}{4k} n \right) f' + \left(\frac{3V_z}{4k} - \frac{1}{n^2} \right) f = 0 \quad (60)$$

and if the notation $\lambda = \eta \left(\frac{V_z}{4k} \right)^{1/2}$ is introduced in (60) then

$$\frac{d^2 f}{d\lambda^2} + \left(\frac{1}{\lambda} + \lambda \right) \frac{df}{d\lambda} + \left(3 - \frac{1}{\lambda^2} \right) f = 0 \quad (61)$$

A solution of Equation (61) which may be verified by direct substitution is given by

$$f(\lambda) = A \lambda e^{-\lambda^2/2} \quad (62)$$

This solution satisfies the two boundary conditions:

- (i) f is finite at the origin,
and
(ii) f is zero at infinity (i.e., $\lim_{\lambda \rightarrow \infty} f(\lambda) = 0$).

The general solution for the swirl component of the velocity field is therefore given by:

$$\bar{v}(r, z) = C z^{-3/4} \left(r z^{-1/4} \right) e^{-\frac{r^2 V_z}{8k\sqrt{z}}}$$

$$\bar{v}(r, z) = \frac{Cr}{z} \exp \left\{ -\frac{r^2 V_z}{8k\sqrt{z}} \right\} \quad (63)$$

To determine the constant of integration, (63) is inserted into (42) which gives us

$$\frac{M_z}{2\pi\rho V_z} = \frac{C}{z} \int_0^\infty r^2 \exp \left(\frac{-r^2 V_z}{8k\sqrt{z}} \right) (r dr) = \frac{32k^2}{V_z^2} C \quad (64)$$

Thus, the constant of integration is then

$$C = \frac{M_z V_z}{64\pi\rho k^2} , \quad (65)$$

and the general solution has the form

$$\bar{\sigma}(r,z) = \left(\frac{M_z V_\infty}{64\pi\rho k^2} \right) \frac{r}{z} \exp\left(-\frac{V_\infty}{8k} \frac{r^2}{\sqrt{z}} \right) \quad (66)$$

where the free stream axial velocity V_z has been denoted by V_∞ . The solution for the tangential velocity is then given by the sum of Equation (66) and (52).

Circulation

At a given location downstream, the tangential velocity is a function of r only, and the circulation in a circular region of radius r is given as follows:

$$\int_0^{2\pi} \sigma(r) r d\theta = C , \quad r \leq a_c$$

where $\sigma(r)$ is obtained from Equation (66).

Drag

The induced drag created by the vortex is calculated by Equation (26) by substituting in the vortex core size obtained from the graph of the swirl velocity distribution.

DISCUSSION OF RESULTS

The equations for the azimuthal and axial velocity components, circulation, vortex core drag, and vortex core size for both the linear and swirl injection cases were developed to investigate the effects of mass injection on a tip vortex. The results of various representative cases are presented in Figures 1 - 6.

The theoretical values of the tangential velocity for one to fifteen chord lengths downstream for the linearized solution are depicted in Figure 1 for three different values of the thrust parameter.

It may be noted in Figure 1 that the tangential velocity distribution at one chord length downstream has not been significantly altered under mass flow injection rates considered whereas for eight chord lengths downstream, the swirl velocity component of the tip vortex has been considerably reduced from that of the classical (non-injected) case. Reducing the swirl component of the vortex core is important since this corresponds to a reduction in the strength (circulation) of the tip vortex. The reason the tangential velocity distribution has not changed to any appreciable extent at one chord length downstream is that the trailing line vortex has not had sufficient time to modify the injected airstream by imparting some of its rotational kinetic energy of the vortex core to the injected mass. While the theory developed is not adequate to accurately predict the near field flow effects (i.e., less than three or four chord lengths downstream), it does show that the mass injection is beginning to reduce the swirl. At 8 or 10 chord lengths downstream, the tip vortex has had sufficient time to entrain the injected airstream, thereby reducing its rotational velocity. It may also be noted that there is a definite growth in the vortex core as it progresses in the downstream direction. This is to be expected since it is in agreement with the experimentally observed fact that the core of a viscous vortex increases

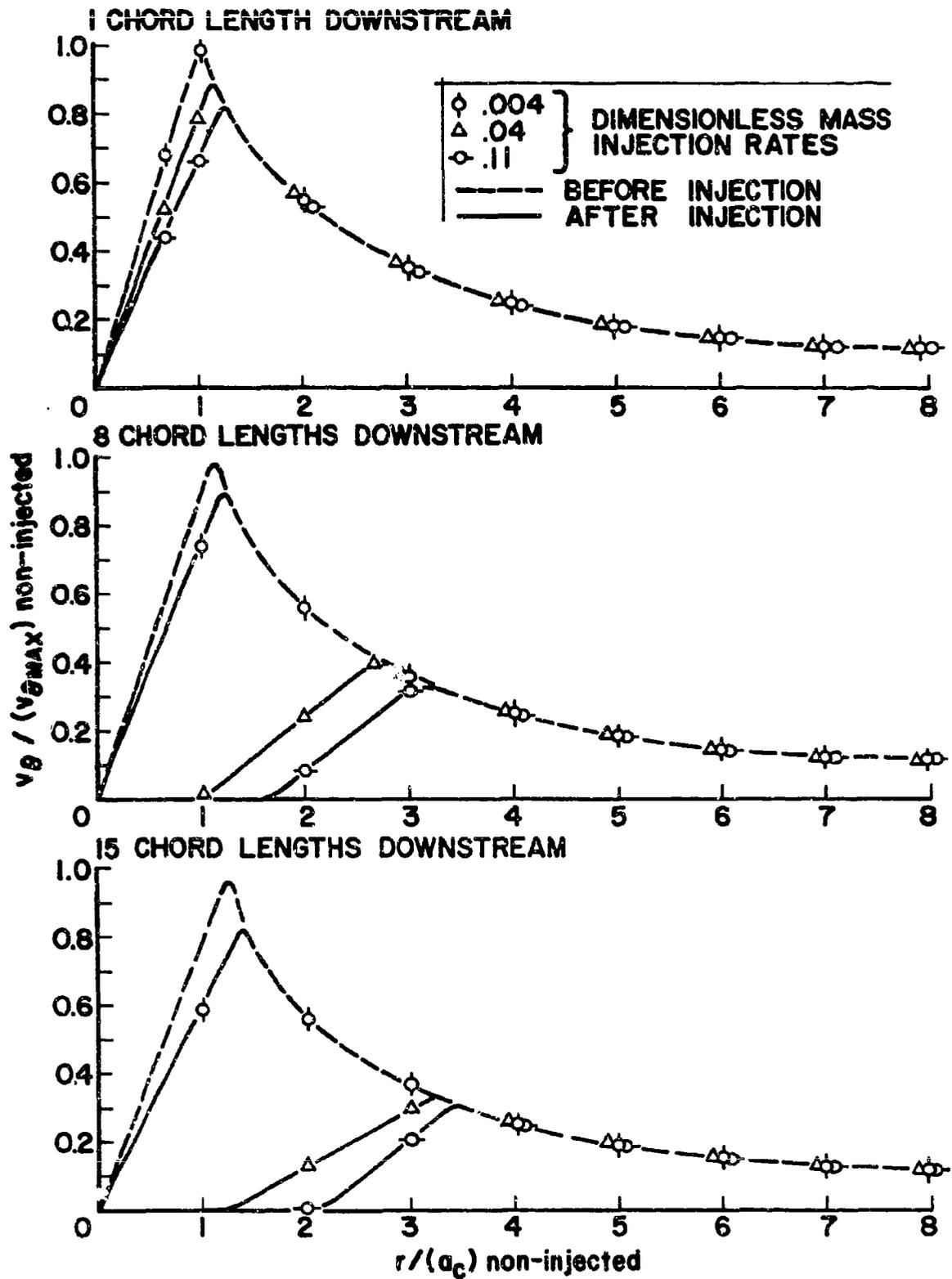


Figure 1. Dimensionless Tangential Velocity Through Vortex Center

in size with time (i.e., with downstream direction) and there is a corresponding decrease in the rotational velocity at a given radius inside the vortex core. Physically, the results observed in Figure 1. are quite plausible and in fact offer an explanation of the mechanism by which the noted changes of the vortex core can take place with mass injection.

It is known that the tip vortex consists of a finite central viscous core of fluid rotating like a solid body surrounded by a potential free vortex region. When an airstream is injected into the vortex core; the tip vortex will do work on the injected airstream by attempting to impart a rotational velocity component to the injected fluid. Furthermore, there will be a vigorous mixing action taking place (i.e., a free turbulent flow) between the tip vortex core and the injected mass which will cause the rotational velocity component of the tip vortex to be reduced through an exchange of momentum between the tip vortex core and the injected airstream. Thus, the normal viscous decay of a vortex has been greatly amplified by injecting the airstream into its core which creates a free turbulent flow and thereby greatly increases the viscous dissipation of the vortex core. As may be noted in Figure 1, the linear momentum of the injected fluid is of sufficient magnitude after a few chord lengths downstream to completely overcome the tangential velocity of the tip vortex near the axis of symmetry.

At a given location downstream, the tangential velocity v_{θ} is a function of only r , and the circulation in a circular region of radius r is

$$\Gamma = 2\pi(r - r_0)v_{\theta} .$$

As may be seen from this equation, the circulation will decrease (or increase) as the swirl component of velocity decreases (or increases). Since the theoretical values of the velocity

distribution v_{θ} are known, values of the circulation (strength) in the core of the tip vortex were calculated from the above equation and the results plotted in Figure 2 for both the injected and non-injected cases. The results shown indicate that injection of the airstream into the tip vortex core not only causes a decrease in the magnitude of the maximum circulation but also alters significantly the distribution of the circulation in the core because of the change in the swirl component of velocity.

A tip vortex usually remains fixed in size and strength for quite some time after its generation. It is precisely for this reason that the blade-vortex interaction problem has been of concern in helicopter dynamics where the helicopter blades must operate in each other's wakes. Figure 3 reveals that the direct injection of an airstream into the forming tip vortex can be of importance in helping to alleviate the blade-vortex interaction problem. While the ordinary vortex for the non-injected case will remain relatively fixed in size and strength, the injected vortex core will increase in size and thus lose strength.

It can be noted from Figure 3 for the case of 15 chord lengths downstream, the tip vortex for the injected case is roughly double that of the non-injected case for a thrust parameter of .11. However, as the thrust parameter increases from .11 to .20 there is very little additional increase in the growth of the tip vortex core. This indicated that it is not the velocity at which the airstream is injected but rather the amount of mass which is injected that is an important parameter. Furthermore, it illustrates the fact that there is an optimum amount of mass which can be injected for any given tip vortex strength.

It may also be observed from Figure 3 that direct injection is favored over reverse thrusting (i.e., suction) since direct injection will create a less concentrated tip vortex whereas suction can create a more concentrated vortex (i.e., small vortex core size)

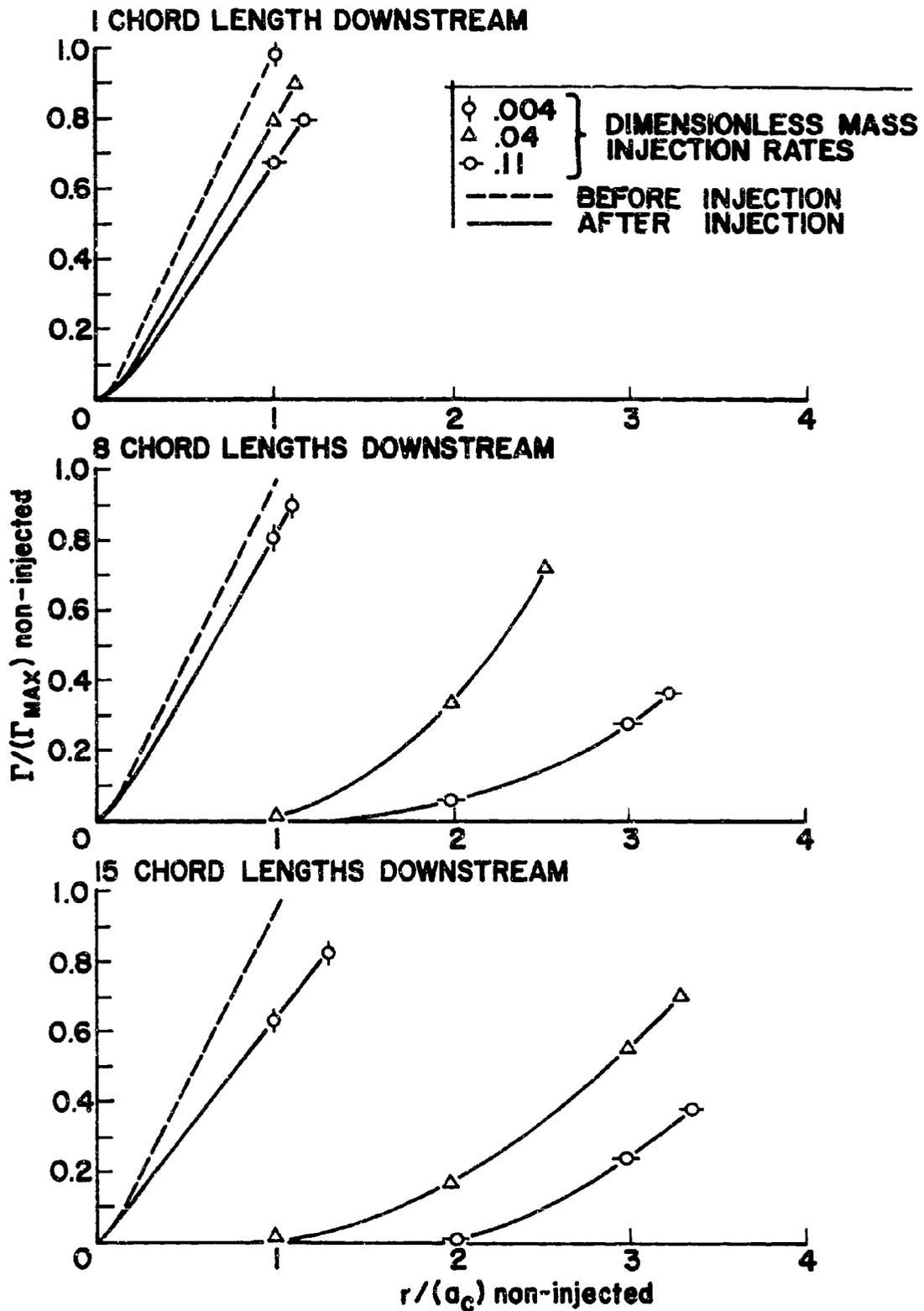


Figure 2. Dimensionless Circulation Through Vortex Center

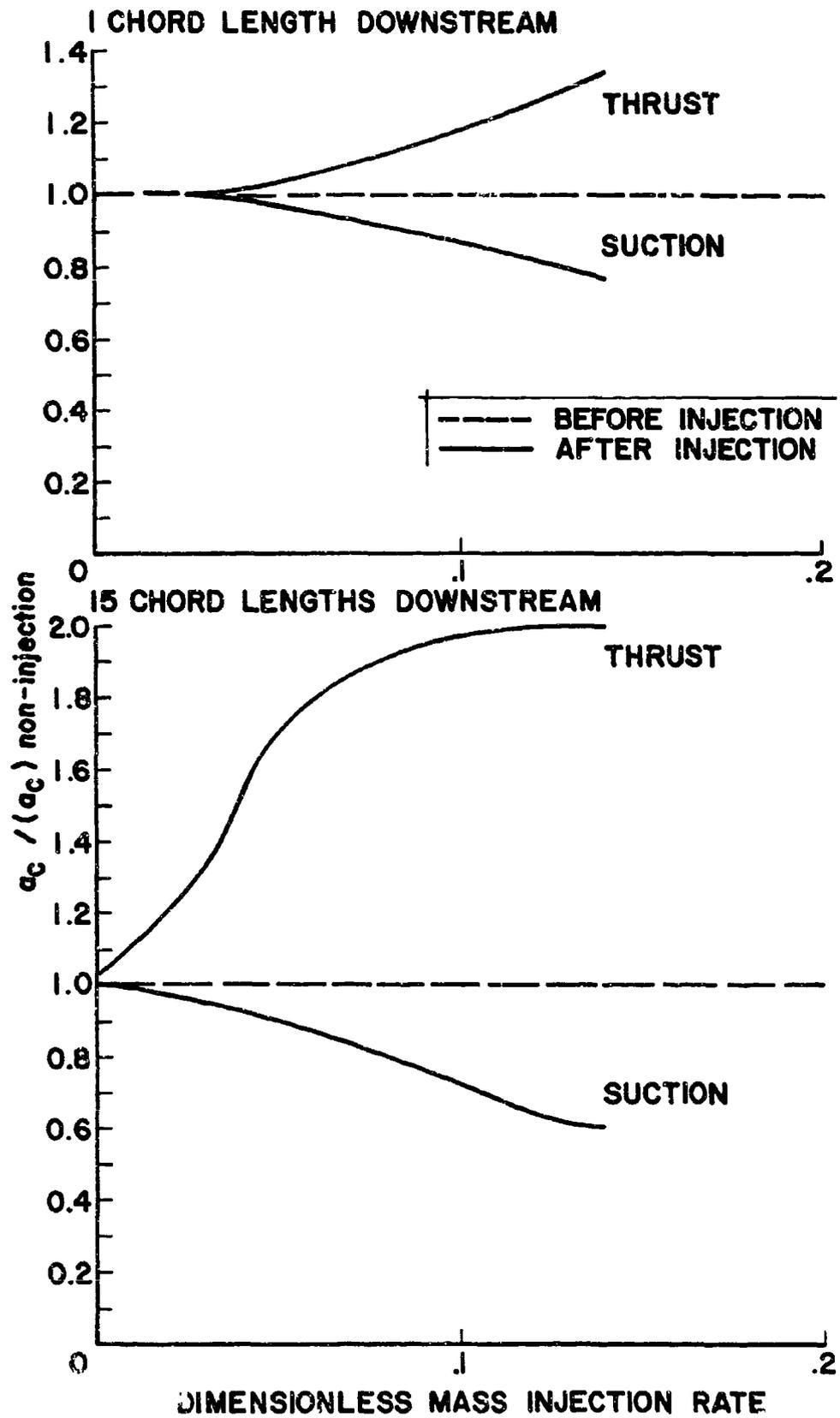


Figure 3. Dimensionless Vortex Core Size Versus Mass Injection Rates

by removing mass from the vortex core. Also, the reverse thrusting (suction) will cause a parasitic thrust on the rotor blade.

An interesting feature of steady axisymmetric flow fields is that strong axial flows occur near the axis of symmetry. As Batchelor [5] has shown, the coupling between the azimuthal (swirl) and axial (downstream) components of the motion in a steady line vortex is provided by the pressure. The radial pressure gradient will balance the centrifugal force field, and any change in the tangential (swirl) motion downstream creates an axial pressure gradient and consequently an axial acceleration or deceleration of the fluid.

The variation in the axial velocity across the vortex core for two different downstream locations (one chord length and ten chord lengths downstream) for the injected cases has been evaluated for McCormick's data [4] and the results are shown in Figure 4. The interesting feature of Figure 4 (top curve for one chord length downstream) shows that there is an axial velocity, and a large one, in the core of the tip vortex. Outside the core, where the circulation is constant, the axial velocity $v_z = V_\infty$, the free stream velocity; but inside the tip vortex core $v_z > V_\infty$ and increases toward the axis of symmetry up to a value of 2.72 times the free stream velocity (V_∞). The data used for computing these curves was taken from Reference [4] obtained by wind tunnel tests of a rectangular semi-span wing having an 18-inch span, a 5.85-inch chord, and a free stream velocity of 100 feet per second. Since such a model is similar to full-scale helicopter blades, the theoretical results presented in Figures 3, 4 and 5 will be equally applicable to full-scale rotor blades. Since mass injection into the forming tip vortex will create a continual slowing-down of the tangential motion by viscous mixing of the injected fluid in the downstream direction and consequently leads to a positive axial pressure gradient with continual loss of axial momentum, it might be expected that the axial velocity in the vortex core will be

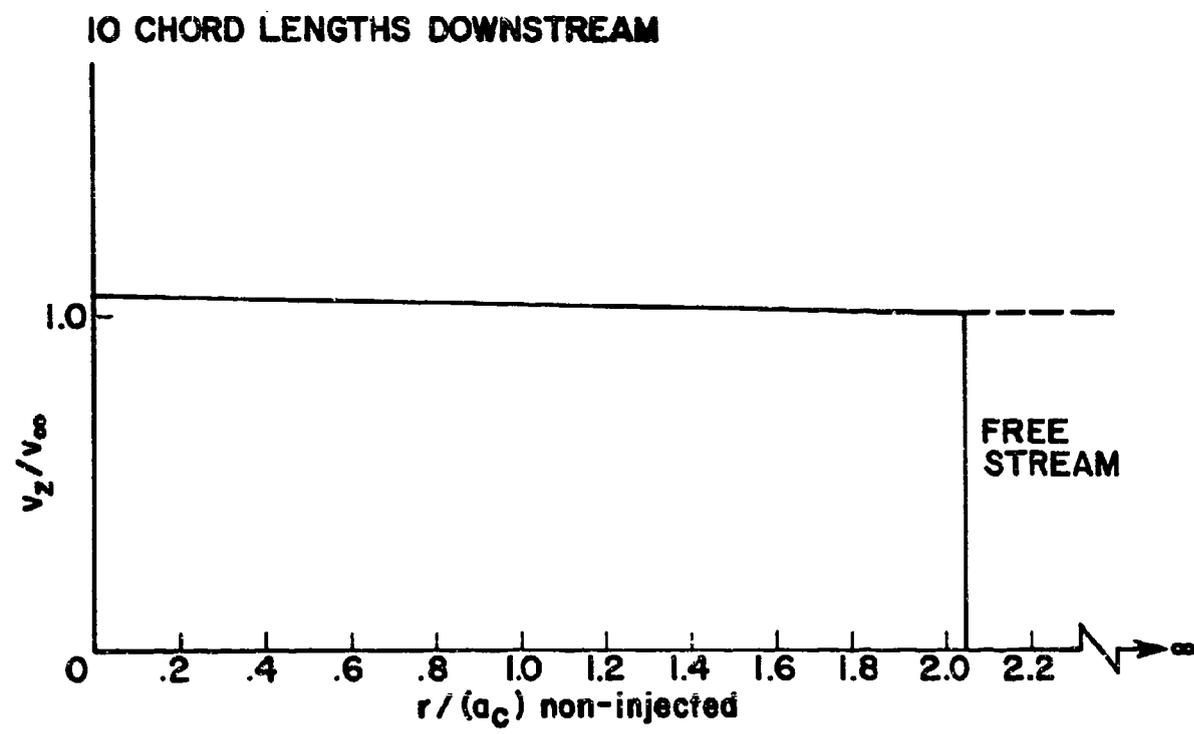
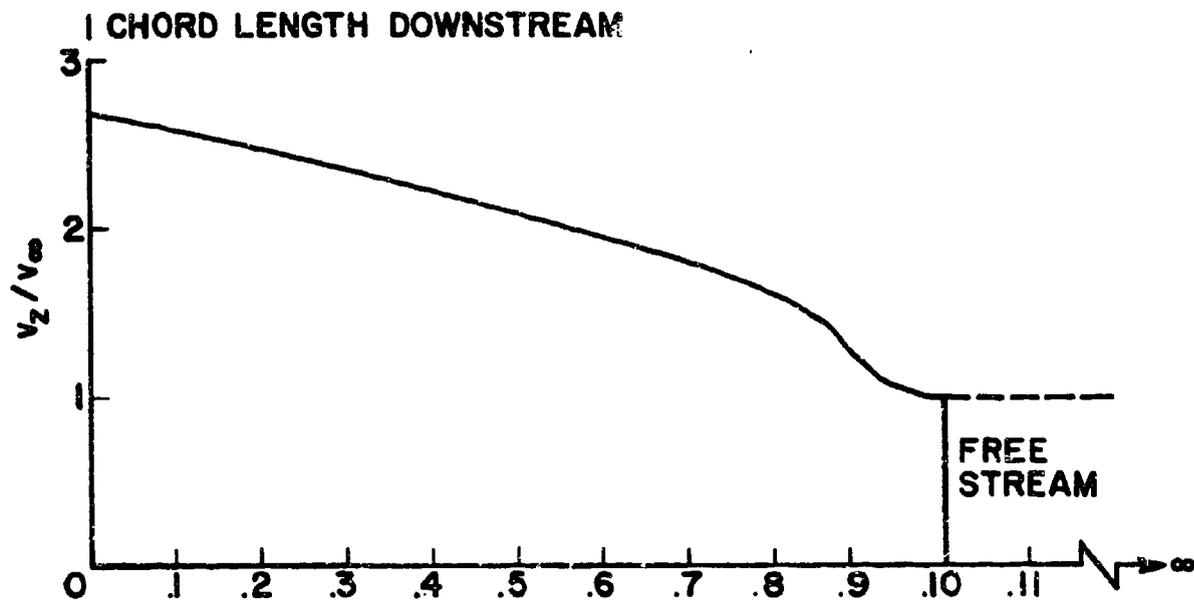


Figure 4. Dimensionless Axial Velocity Through Vortex Center

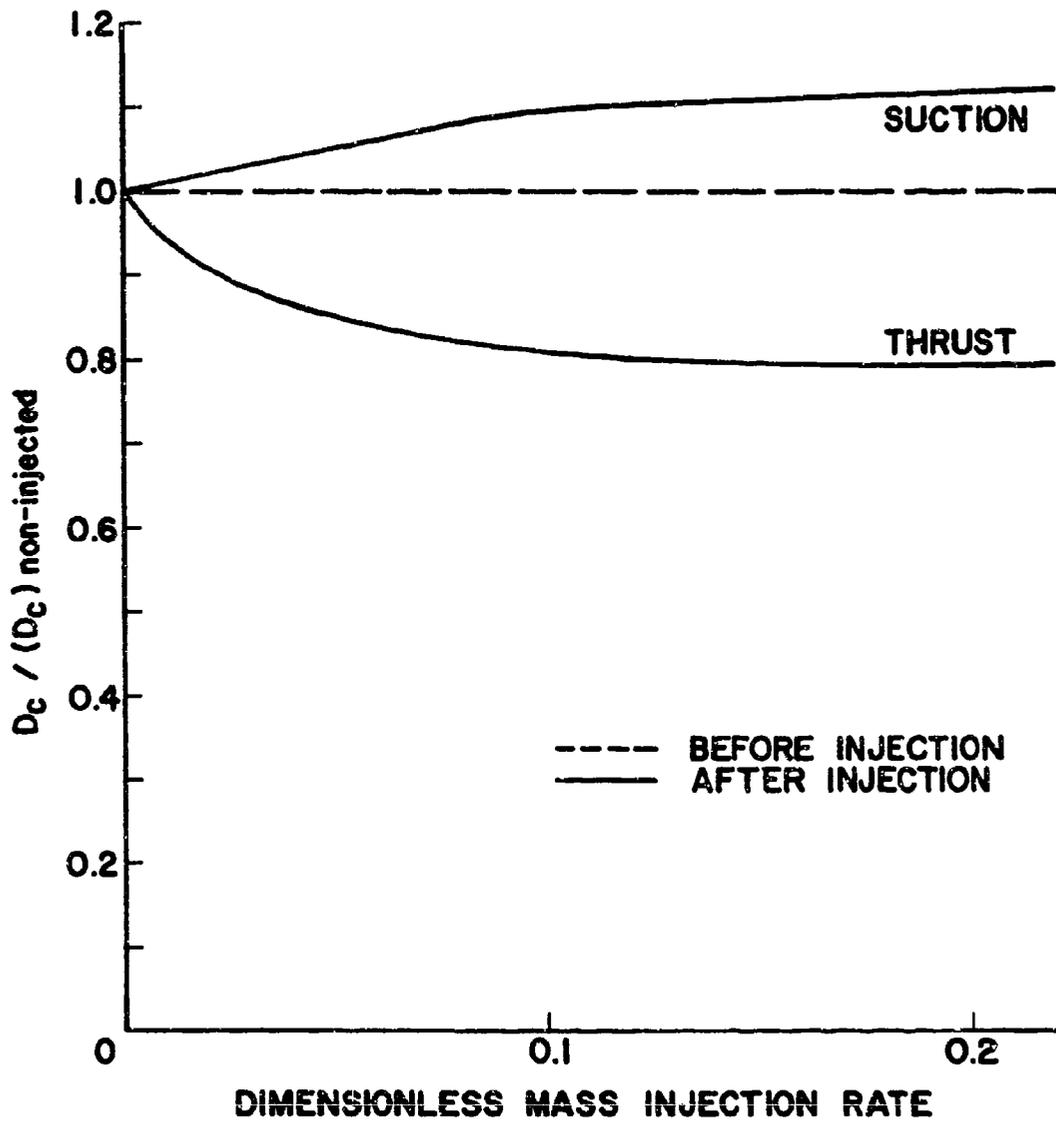


Figure 5. Dimensionless Vortex Core Drag Versus Mass Injection Rates

closer to the free stream velocity as the vortex progresses in a downstream direction. The bottom graph in Figure 4 depicts this situation with the maximum $v_z = 1.03$ times the free stream velocity at 10 chord lengths behind the wing. This result seems to be in agreement with the physics of the dissipation of a tip vortex and shows that direct injection of an airstream into the tip vortex core does create a faster dissipation of the trailing line vortex.

The induced drag associated with the tip vortex core has been considered by expressing the drag as an integral over a transverse plane which is independent of z , the downstream location. This drag is related to the vortex core size and thus depends on the thrust of the injected airstream. The effects of direct injection and suction on the induced drag of the wing have been computed for McCormick's wind tunnel test data and the results are presented in Figure 5. The theoretical values for the drag show a decrease of the induced drag for the injected case, while the drag remains constant for the non-injected case. A reduction in the induced drag on the wing should occur for the direct injection case since there is a corresponding reduction of the induced velocity on the wing. Since the present calculations neglect the three-dimensionality of the resulting flow field, the resulting percentage decrease as given in Figure 5 for the induced drag (i.e., approximately 20 per cent) are believed to be too high. A reduction of the induced drag of about three to five percent would seem more reasonable. The results presented in Figure 5 also show that suction will create a slight increase in the induced drag since suction generates a more concentrated tip vortex with a higher induced velocity field. Thus, suction is unfavorable in comparison with the direct injection case.

From Figure 6, the tangential velocity distribution across the vortex core is presented for various downstream locations for the reverse swirl case. The initial value of the circulation was chosen to be $350 \text{ ft}^2/\text{sec}$ (about twice as high as would be

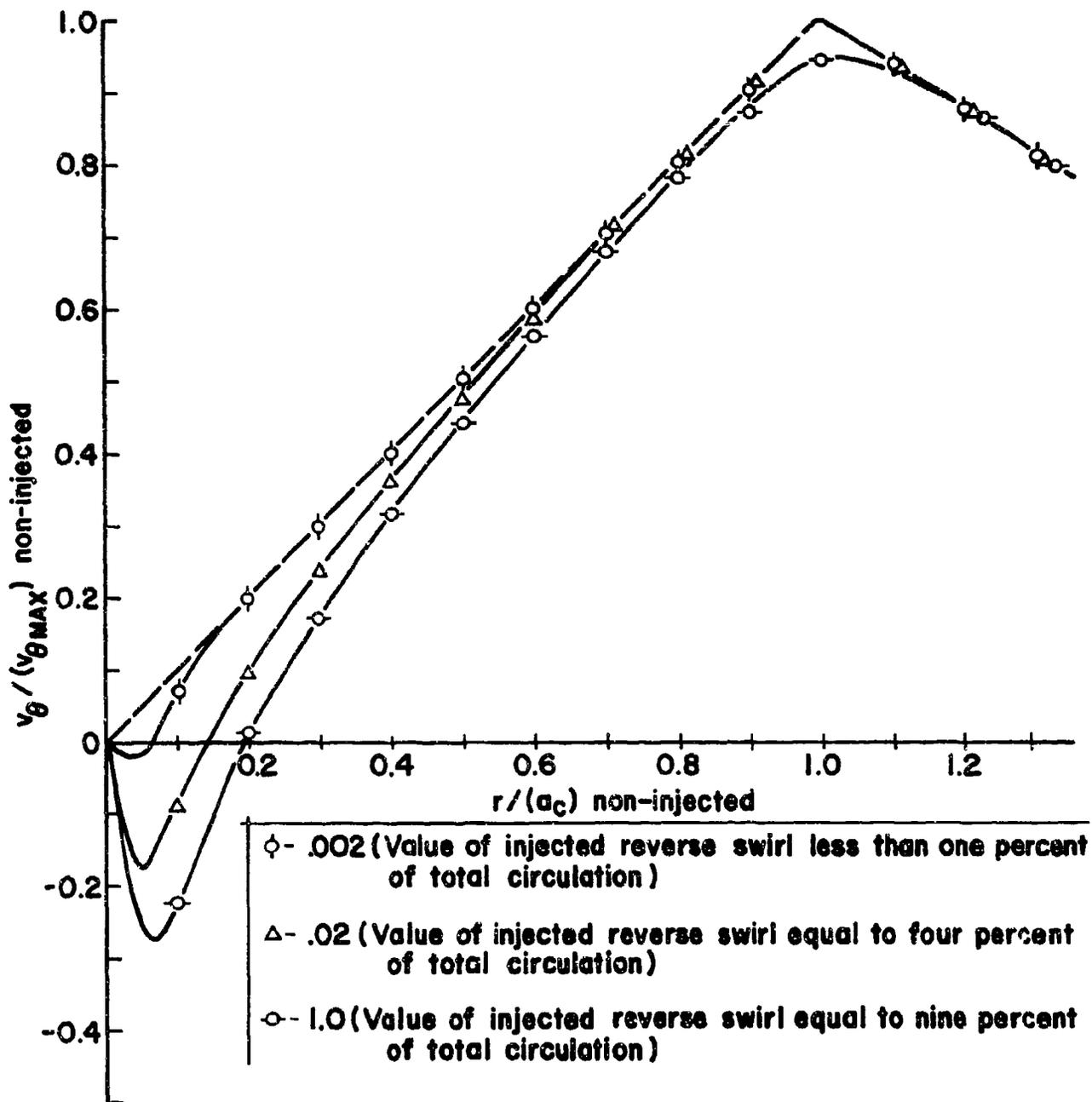


Figure 6. Dimensionless Tangential Velocity Through Vortex Center (Reverse Swirl Case)

encountered for a realistic rotor blade) and a free stream velocity of 600 feet per second (comparable to tip speeds encountered in actual helicopter flight regimes). The results shown are based on the assumption that reverse swirl is injected into the forming tip vortex core.

As may be noted from the results presented in Figure 6, the effect of injecting an airstream with reverse swirl is not favorable since it does not create a rapid dissipation of the tip vortex core even for relatively large amounts of injected fluid. It is noted that a reverse flow field in the tip vortex core is possible since the angular momentum of the injected fluid can be sufficient to overcome the angular velocity of vortex core in a neighborhood of the axis of symmetry. To investigate the effect of initial circulation strength on the results, a circulation strength of about 100 ft²/sec was used. The results obtained, however, were very similar to those of the higher circulation in that the injection of a reverse swirling flow into the tip vortex core did not significantly dissipate the vortex core.

MEANS FOR OBTAINING THE REQUIRED MASS FLOW

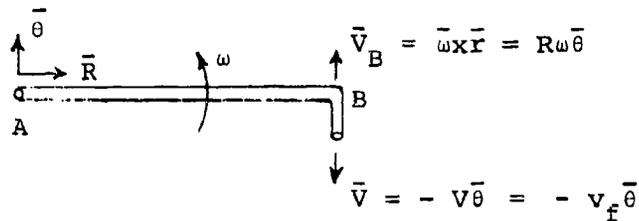
Since it has been determined that it was theoretically possible to achieve significant, beneficial modifications of the vortex core size, axial and swirl velocity distributions, vortex core drag and vortex strength in all flight regimes by injecting a given mass flow into the core of the tip vortex, it remains to be estimated how much of the required mass flow might be obtainable from centrifugal pumping and/or from utilization of the low pressure in the vortex core and thus how much must be supplied by other means. It is necessary to investigate these sources of obtaining the required mass flow, since they could very well result in performance penalties that might be unacceptable.

Basic Equations Governing Pumping Process

The following assumptions were made in the analysis:

1. The helicopter blade was replaced by a horizontal pipe AB of length R which is pivoted about a fixed support at A.
2. Suction and viscous drag forces associated with the pipe flow were neglected.

The blade was assumed to be always full of air and for dynamic purposes was regarded to be a thin rod of mass M.



Free Body of Blade

Let \bar{R} and $\bar{\theta}$ be rotating unit vectors along and perpendicular to the pipe AB and \bar{k} is the fixed unit vector forming a right-handed system with \bar{R} and $\bar{\theta}$. The angular velocity of the blade $\bar{\omega} = \omega\bar{k}$.

The absolute velocity of the air leaving the nozzle is given by

$$\bar{v}_f = -v_f \bar{e}.$$

The velocity of the air relative to the nozzle is given by

$$\bar{v}_f = (\bar{v}_f)_B + \bar{v}_B$$

and therefore

$$\bar{v}_f = (R\omega - (v_f)_B) \bar{e} \quad (69)$$

The redistribution of mass with respect to velocity for a rigid body rotating about a fixed axis about 0 is given by

$$\bar{r}_0 \times \bar{F} = \frac{d}{dt} (\bar{I}_0 \cdot \bar{\omega}) + \frac{dm_0}{dt} (\bar{r}_0 \times \bar{v}_0) - \frac{dm_1}{dt} (\bar{r}_1 \times \bar{v}_1) \quad (70)$$

where

\bar{I}_0 = inertia dyadic about the rotation point

$\bar{\omega}$ = angular velocity of the rigid body about 0

\bar{v}_0 = absolute velocity of the center of gravity of outgoing mass

\bar{v}_1 = absolute velocity of center of gravity of incoming mass

A = area of nozzle or jet

For the present case, $\bar{r}_1 = 0$, $\bar{r}_0 = R\bar{R}$ and

$$\frac{dm_0}{dt} = \rho Q = (\rho Q) \text{ out} = (\text{mass per unit time flowing}) \quad (71)$$

where Q is the flow rate or discharge rate out of the nozzle. Since ω is constant in magnitude and direction, and $\bar{r} \times \bar{F}$ (resultant torque on system) is equal to zero, then Equation (70) becomes

$$0 = \rho QR (R\omega - \frac{Q}{A}) \quad (72)$$

$$(v_f)_B = \frac{Q}{A}.$$

Therefore, from Equation (71) it follows that

$$Q = A(R\omega) = A(v_f)_B \quad (73)$$

and thus the discharge velocity at the exit (point B) is $R\omega$. As a check on this result, consider the sum of forces on a fluid element in the radial direction, that is,

$$m(\ddot{\bar{r}} - r\omega^2) = 0 \quad (74)$$

where

$$\ddot{\bar{r}} = \frac{d^2\bar{r}}{dt^2} = \frac{d}{dt}(\bar{v}) = \bar{v} \frac{d\bar{v}}{dr} \quad (75)$$

and \bar{r} = position vector from the origin at 0 to the fluid particle. Inserting (75) into (74) and performing the integrations yields

$$\frac{v^2}{2} = \frac{R^2\omega^2}{2}$$

or

$$(v_f)_B = \dot{\bar{r}} = R\omega \quad (76)$$

which is in agreement with the result Equation (73) found from the moment of momentum equation. If a helicopter blade has the following parameters:

$$R = 30 \text{ feet}$$

$$\omega = 30 \text{ rad/sec (cruise condition)}$$

$$A = 5.5 \times 10^{-3} \text{ ft}^2 \text{ (1" diameter nozzle)}$$

then from Equation (73) the discharge rate will be

$$Q = A(R\omega) = 4.95 \text{ ft}^3/\text{sec}$$

which would be sufficient to cause the beneficial modifications of the tip vortex core as previously noted (see Figure 1) since this corresponds to a thrust parameter of .11.

In addition to centrifugal pumping, it may also be possible to utilize the low pressure in the vortex core by means of a "pressure pump" effect. By applying the Bernoulli energy equation from the entrance to the exit of the pipe and neglecting head losses,

$$\frac{v_A^2}{2g} + \frac{p_A}{\gamma} = \frac{v_B^2}{2g} + \frac{p_B(r)}{\gamma} \quad (77)$$

and, therefore

$$p_0 - \bar{p}_C = \frac{\rho}{2} \left[(v_f)_B^2 - (R\omega)^2 \right] \quad (78)$$

where

ω = angular velocity of the blade

p_0 = stagnation pressure at entrance to pipe

\bar{p}_C = tip vortex core pressure = pressure at pipe exit.

Solving for $(v_f)_B$ yields

$$(v_f)_B = (R\omega) \left[1 + \frac{2(p_0 - \bar{p}_C)}{\rho (R\omega)^2} \right]^{1/2} \quad (79)$$

For a circular vortex, the pressure in the tip vortex core as given by Milne-Thomson [10] is

$$p_C = p_0 - \frac{k^2 \rho}{a_c^2} \left[1 - \frac{r^2}{2a_c^2} \right] \quad (80)$$

where $k = \frac{\omega a_c^2}{2}$ = strength of circulation. Defining the average pressure in the vortex core as

$$\bar{p}_c = \frac{\int_0^{2\pi} \int_0^{a_c} p_c r dr d\theta}{\pi a_c^2} \quad (81)$$

and inserting (80) into (81) yields

$$\bar{p}_c = \left[p_0 - \frac{k^2 \rho}{a_c^2} + \frac{k^2 \rho}{4a_c^4} \right] \quad (82)$$

If $\Delta p = p_0 - \bar{p}_c$, then it follows that

$$\Delta p = \frac{3k^2 \rho}{4a_c^2}$$

Substituting the expression for k^2 yields

$$\Delta p = \frac{3\rho \omega_c^2 a_c^2}{16}$$

and, thus

$$\frac{2\Delta p}{\rho R^2 \omega^2} = \frac{3}{8} \left(\frac{a_c}{R} \right)^2 \left(\frac{\omega_c}{\omega} \right)^2 \quad (83)$$

where ω_c = angular velocity of the tip vortex core.

Inserting (83) into (79) yields

$$(v_f)_B = (R\omega) \left\{ 1 + \frac{3}{8} \left(\frac{a_c}{R} \right)^2 \left(\frac{\omega_c}{\omega} \right)^2 \right\}^{1/2} \quad (84)$$

The second term in the parenthesis of Equation (84) represents the increase in the discharge velocity due to the low pressure in the vortex core. Note that if $p_c = p_0$ (stagnation pressure) in

Equation (79) then

$$(v_f)_B = R\omega$$

which is in agreement with Equation (76).

To estimate the order of magnitude of this effect, consider the experimental data of McCormick [4]:

$$V_\infty = 100 \text{ feet per sec}$$

$$R = 1.5 \text{ feet} = 18 \text{ inches}$$

$$R\omega = 100 \text{ feet per sec}$$

$$\omega = 66.6 \text{ rad/sec}$$

$$\text{chord} = 5.74 \text{ inches}$$

$$a_c = \text{vortex core radius} = .72 \text{ inches at one chord length}$$

$$\frac{\omega_c}{\omega} = \frac{2400}{66.6} = 37.0,$$

and $(v_f)_B = 133.5$ feet per second, the percent increase in the discharge velocity would be

$$\frac{v_{\text{new}} - v_{\text{old}}}{v_{\text{old}}} = \frac{133.5 - 100}{100} \quad 33\% \text{ increase}$$

assuming that all of the low pressure core could be effectively utilized.

CONCLUSIONS

The following is a summary of the effects which may be expected from the modification of a tip vortex by the injection of an airstream into the tip vortex core generated by a lifting surface:

1. Significant beneficial modifications of a tip vortex core may be achieved by direct injection of an airstream into the tip vortex core shed by a rotor blade.
2. Appreciable reduction of the swirl (tangential) velocity component of the tip vortex is possible under quite reasonable mass flow injection rates.
3. The beneficial modifications of a tip vortex by direct injection were not dependent on the free stream velocity and therefore should be obtainable over the complete flight regime of the helicopter. This represents an important improvement over other methods of circulation control.
4. Modifying a tip vortex by direct injection shows that it is the amount of mass which is injected into the tip vortex core that is important and not the velocity at which it is injected. Furthermore, there is an optimum mass injection rate above which very little additional benefits can be obtained.
5. The effect of direct injection is beneficial as regards rotor performance since it slightly reduces the induced drag on the helicopter blade which generates the tip vortex.
6. The advantages resulting from the injected airstream will be greater for those helicopters which have the smaller values of circulation per blade. Thus, for a given mass flow rate and a particular type of helicopter blade, the advantages of injection may be increased by adding more blades per rotor or by otherwise reducing the total circulation per blade.

7. The effects of injecting a reverse swirl into the forming tip vortex core and also withdrawing fluid from the tip vortex core by suction did not show significant beneficial modifications of the tip vortex core over a wide range of mass flow rates and therefore cannot be recommended as a possible means of modifying a tip vortex.
8. It has been estimated that the required mass flow for aerodynamic injection could be obtained from centrifugal pumping action of the blade.

RECOMMENDATIONS

Since it has been shown, theoretically, that significant beneficial modifications of a tip vortex are possible by means of direct injection of an airstream into the forming tip vortex, it remains to be seen if these changes may actually be obtained on a full-scale rotor system. It is therefore recommended that an experimental wind tunnel program be implemented using a scale model to verify that the theoretically predicted changes are actually possible.

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13. ABSTRACT Numerous research efforts have been conducted by different investigators to alter the characteristics of the tip vortex generated by a helicopter blade in order to alleviate the blade-vortex interaction problem as well as the noise problem associated with impulsive loading. Various approaches have been taken in these investigations including modifications of the loading distribution by taper and twist and altering the blade tip by utilizing porous sections. All of these approaches have not been universally successful for all flight conditions. The present analytical investigation shows that it should be possible to significantly alter the characteristics of the trailing tip vortex for all flight conditions in a beneficial manner by injecting an airstream directly into the forming tip vortex. Analytical expressions were developed for the initial and final states of the vortex in order to evaluate the effects of mass flow injection on the vortex strength, swirl velocity distribution, vortex core pressure, vortex core size and the induced drag on the blade. On the basis of the results that were obtained, it was shown that the required mass flow may be obtained from centrifugal pumping action by venting the blade and therefore the desired modification can be obtained apparently without significant performance penalties which would be unacceptable. (U)		

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