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APPLICATION OF THE HERTZ CONTACT LAW TO PROBLEMS OF IMPACT IN PLATES

By
Jackson C. S. Yang
Do Sup Chun

5 SEPTEMBER 1969

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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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APPLICATION OF THE HERTZ CONTACT LAW TO PROBLEMS OF IMPACT IN PLATES

Prepared by:
Jackson C. S. Yang and Do Sup Chun*

ABSTRACT: The purpose of this paper is to determine: (1) the validity of the static treatment of pressure on thin plates for the dynamic cases of intermediate impact velocities; (2) the use of the Hertz law for the contact of an elastic projectile (steel) on elastic (steel), viscoelastic (Adiprene), and plastic (Lexan) bodies; (3) the use of viscoelastic and plastic materials as shock mitigators.

Equations for stress distribution in the simply supported square plate under transverse impact have been developed by using the static theory of thin plates combined with the Hertz contact law.

Experimental tests were performed using an air gun to impact steel spherical projectiles on steel plates with and without Adiprene and Lexan coverings. Two spherical projectiles of different radii were used. The radius of contact between the projectile and the covering was measured after each impact test. Strain measurements were also taken from strain gages mounted on the free side of the plate directly under the point of impact. Reasonable agreement between the results of the calculations based on the theoretical analysis and the experimental data was obtained.

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The work presented in this report is a part of the U. S. Naval Ordnance Laboratory's massive-glass hull development program. Experiments were performed in part at the University of Maryland. This report is intended for use in the design of plastic-buffer clads for the mitigation of damage to hulls during naval service.

GEORGE G. BALL
Captain, USN
Commander

A. E. Seigel
A. E. SEIGEL
By direction

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LIST OF SYMBOLS

a, b	lateral dimensions of the plate, in.
c	radius of contact, in.
D	flexural rigidity
E	Young's modulus of elasticity, psi
h	thickness of the plate, in.
n	constant dependent upon the materials and dimensions of two bodies
P	compressive force, lb
q	intensity of contact pressure, psi
v	velocity of impact, fps
W	deflection of the plate, in.
Mx, My	bending moments, lb-in.
Mxy	twisting moment, lb-in.
α	relative indentation, in.
ν	Poisson's ratio
σx, σy	bending stresses, psi
τxy	shearing stress, psi
J_1	Bessel function of order one
m_2	mass of projectile, lb sec^2/in.
q_o	maximum contact pressure, psi
R_1, R_2	radii of two contact bodies, in.
v_1, v_2	velocities of two bodies after contact, fps

INTRODUCTION

During the past decade, increasing attention has been focused on the problems attendant to the collision of a projectile and a target. Thus, the impact problems, as well as the fatigue problems, have been investigated and developed by many authors for various conditions. In the impact studies, the Hertz theory has been applied even though some questions have been raised concerning the validity of the theory under dynamic conditions, since the Hertz theory is based on the quasi-static contact of indenter against the infinitely large target. Introducing his law, Hertz published his original analysis of elastic contact stress under static loading in 1881 (Ref. 1), and his analysis revealed that the intensity of pressure q over the surface of contact between two general curved surfaces is represented by the ordinates of a semi-ellipsoid constructed on the surface of contact, as in Figure 1; thus

$$q = q_o \left(1 - \frac{x^2}{a'^2} - \frac{y^2}{b'^2}\right)^{1/2} \tag{1}$$

in which q_o is maximum pressure and expressed as

$$q_o = \frac{3P}{2\pi\, a'\, b'} \tag{2}$$

where P is total applied load, and his force-indentation law is

$$P = n\, \alpha^{3/2} \tag{3}$$

where n is determined with the properties and dimensions of two bodies, and α is the relative indentation.

His static analysis has also been used in investigating impact of elastic bodies.

As an example of the impact of two spheres (Fig. 2), if v_1 and v_2 are the velocities of two bodies after contact, then

$$\dot{\alpha} = v_1 + v_2 \tag{4}$$

With the above relation, using equations of motion of two bodies and the Hertz force-indentation law (Eq. (3)), we can find the maximum compressive force P acting between spheres during impact, and the corresponding radius of surface of contact.

Reference 2 indicates that for the case of impact of elastic bodies at moderate velocities, the problems of elastic contact and elastic impact are in essence identical (Love (1934)), and reference 3 indicates that the quasi-static treatment is found to approximate very closely the values of stress found in dynamic impacts.

C. V. Raman (Ref. 4) applied the Hertz theory (Eq. (3)) to the investigation of the coefficient of restitution, and found that for moderate thicknesses of plate (0.138" ~ 1"), the theoretically calculated and experimentally observed values agreed well.

In further investigations, Werner Goldsmith (Ref. 5) extended his experimental observations to intermediate velocities (30-300 fps) of impact, using the Hopkinson bar technique, and vindicated the use of the Hertz law for hard tool steel even when some permanent set was produced; however, he denied its application to aluminum.

For the purpose of determining the validity of applying the static treatment of plates and Hertz's law to thin plates under intermediate velocities of impact, the present report gives an analytical solution for the stress distribution of the plate under transverse impact. This is done by the combined application of the static theory of plates, together with Hertz's law.

A comparison is then made with the experimental data obtained using an air gun for intermediate velocities of impact. Two projectiles with different radii of curvature were used to impact against steel plates of various thicknesses with and without polyurethene (Adiprene) and polycarbonate (Lexan) coverings. The shock mitigation properties of the two claddings, Adiprene and Lexan, were also obtained.

THEORETICAL ANALYSIS

For the case of impact between two spheres of mass m_1 and m_2, investigations show that the duration of impact, i.e., the time during which the spheres remain in contact, is very long in comparison with the period of lowest mode of vibration of the spheres (Ref. 6). Vibrations can therefore be neglected, and it can be assumed that the force-displacement relation established for static conditions holds during impact. The force is basically a power law given by

$$P = n \alpha^{3/2} \tag{5}$$

where P is the compressive force, and the constant n is given by

$$n = \frac{4}{3\pi\ (K_1 + K_2)} \left(\frac{R_1\ R_2}{R_1 + R_2}\right)^{1/2} \qquad (6)$$

R_1, R_2 - Radii of the spherical surfaces of the two bodies at the point of contact

$$K_1 = \frac{1 - v_1^{\,2}}{\pi\ E_1}$$

$$K_2 = \frac{1 - v_2^{\,2}}{\pi\ E_2}$$

in which E and v are Young's modulus and Poisson's ratio, respectively.

This is similar to examining the material as a nonlinear spring. In our problem, a spherical-nosed projectile is impacted against a rigid flat surface. Then equation (6) may be simplified to

$$n = \frac{4\ R_2^{\ 1/2}}{3\pi(K_1 + K_2)} \qquad (7)$$

in which subscripts 1 and 2 represent plate and projectile, respectively.

During the impact of such a system, the following differential equation may be written:

$$m_2\ \ddot{\alpha} + n\ \alpha^{3/2} = 0 \qquad (8)$$

The solution to the system differential equation gives the displacement, velocity, and acceleration of the spherical-nosed projectile with respect to time. With the displacement of the spherical projectile known, the force acting on the plate at any time can be obtained from equation (5).

<u>Plate of Infinite Thickness</u>

The maximum compression of the cladding and the duration of impact can be determined directly by requiring the work done by the cladding to equal the kinetic energy of the system. The deflection of the plate is neglected, since it is very small when compared to the deflection of the cladding. In this case (Ref. 6)

$$\alpha_{max} = \left(\frac{5\ m_2 v_o^{\ 2}}{4\ n}\right)^{2/5} \tag{9}$$

$$t_d = \frac{2\ \alpha_{max}}{v_o} \int_0^1 \frac{d\alpha}{\sqrt{1 - (\alpha)^{5/2}}} \approx 2.94\ \frac{\alpha_{max}}{v_o} \tag{10}$$

Hertz's contact law is also used to estimate the maximum bearing stress which the system must sustain. The stress distribution is hemispherical in shape over a circle of contact of radius c (Ref. 6) given by

$$c = \left[\frac{3\ \pi\ P\ (K_1 + K_2)\ R_1 R_2}{4\ (R_1 + R_2)}\right]^{1/3} \tag{11}$$

For $R_1 = \infty$

$$c = \left[3/4\ \pi\ P\ (K_1 + K_2)\ R_2\right]^{1/3} \tag{12}$$

We have the maximum compressive force as

$$P_{max} = n\ \alpha_{max}^{\ 3/2} = \frac{4(R_2)^{1/5}\left(\frac{15}{16}\ \pi\ m_2\ v^2\right)^{3/5}}{3\ \pi\ (K_1 + K_2)^{\ 2/5}} \tag{13}$$

and the corresponding radius of the surface of contact can be obtained by substituting equation (13) into equation (11),

$$c_{max} = (R_2)^{2/5}\left(\frac{15\pi\ (K_1 + K_2)\ m_2\ v^2}{16}\right)^{1/5} \tag{14}$$

If the two bodies are of the same material,

$$P_{max} = \frac{4(R_2)^{1/5}\left(\frac{15}{16}\pi m_2 v^2\right)^{3/5}}{3\pi\left[\frac{2(1-\nu^2)}{\pi E}\right]^{2/5}} \tag{15}$$

$$c_{max} = (R_2)^{2/5}\left[\frac{15\ (1-\nu^2)\ m_2\ v^2}{8\ E}\right]^{1/5} \tag{16}$$

The peak bearing stress is obtained by equating the sum of the pressures over the contact area to the compressive force P. For the hemispherical pressure distribution this gives

$$q_o = \frac{3\ P_{max}}{2\ \pi\ c^2_{max}} \tag{17}$$

This stress can be written as a function of displacement

$$q_o = \frac{E}{\pi(1-\nu^2)}\left(\frac{\alpha_{max}}{R_2}\right)^{1/2} \tag{18}$$

With the maximum compression force, P_{max}, and the maximum radius of the surface contact, c_{max}, as equations (13) and (14) for different materials, and equations (15) and (16) for the same materials known, the stress can be calculated for the outer surface of the plate at the pole by the thin plate theory (Ref. 7).

Plate of Finite Thickness

In the previous analysis, due to the cladding the deflection of the plate is neglected since it is very small when compared to the deflection of the cladding. However, for impact tests without cladding, the bending of the plate should be taken into consideration.

From reference 7 by Goldsmith, a differential equation of motion was written for an approximate solution to the case of the central transverse impact of a sphere on a rectangular plate simply supported along all edges.

$$\frac{d^2\alpha}{dt^2} + \frac{1}{m} F(\alpha) + \frac{1}{16\, b^2}\left[\frac{3(1 - \nu^2)}{\rho\, E}\right]^{1/2} \frac{d}{dt} F\,(\alpha) = 0 \tag{19}$$

The elastic force-indentation law, equation (5), is now employed to eliminate the term $F(\alpha)$ from the equation. Introducing dimensionless variable defined by

$$\bar{\alpha} = \frac{\alpha}{\bar{T}\, v_o}\,, \quad \bar{t} = \frac{t}{\bar{T}} \tag{20}$$

equation (9) may be written as

$$\frac{d^2\bar{\alpha}}{d\bar{t}^2} + \left[1 + \lambda \frac{d}{d\bar{t}}\right] \bar{\alpha}^{\,3/2} = 0 \tag{21}$$

with

$$\bar{\alpha} = 0\,, \quad \frac{d\bar{\alpha}}{d\bar{t}} = 1 \quad \text{for } \bar{t} = 0$$

where $\bar{T}$, a constant with dimensions of time, is so chosen that the first two coefficients in the transformed equations are unity. For a striking sphere of radius R, the parameter λ is given by

$$\lambda = \frac{\pi^{3/5}}{3^{1/2}}\left(\frac{R}{2b}\right)^2 \left(\frac{\rho_2}{\rho_1}\right)^{3/5} \left[\frac{v_o}{\left[\dfrac{E_1}{\rho_1(1 - \nu_1^{\,2})}\right]^{1/2}}\right]^{1/5} \left[\frac{\dfrac{E_2}{1 - \nu_2^{\,2}}}{\dfrac{E_1}{1 - \nu_1^{\,2}} + \dfrac{E_2}{1 - \nu_2^{\,2}}}\right] \tag{22}$$

where subscripts 1 and 2 denote the plate and sphere, respectively.

The results of numerical integrations of equation (21) for several values of λ are shown in Figure 3 for dimensionless displacement histories $\dfrac{\alpha}{\bar{T}\, v_o}$ as a function of dimensionless time $\left(\dfrac{t}{\bar{T}}\right)$. From Hertz's contact force law $\dfrac{F}{P} = \left(\dfrac{\alpha}{\bar{T}\, v_o}\right)^{3/2}$ this gives us the force F acting on the plate at anytime, having taken into consideration the bending of the plate.

Derivation of Stress

1. Case of hemispherically distributed contact pressure.

According to Hertz, the intensity of pressure q over the surface of contact is represented by the ordinates of a hemisphere of radius c constructed on the surface of contact as shown in Figure 4 (special case of Fig. 1)(Refs. 6, 8).

Taking a coordinate system as shown in Figure 4, the deflection of the plate must satisfy the differential equation,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q(x,y)}{D} \tag{23}$$

where $D = \dfrac{Eh^3}{12\,(1 - \nu^2)}$ (flexural rigidity of the plate)

$$q\,(x,y) = \frac{3\,F}{2\pi\,c^2}\sqrt{1 - \frac{(x-\xi)^2}{c^2} - \frac{(y-\eta)^2}{c^2}}$$

Solving the differential equation with the appropriate boundary conditions, we have the general form of the deflection of a simply supported thin plate subjected to the transverse impact load as follows (Ref. 7):

$$W = \frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{A_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{24}$$

where $A_{mn} =$

$$\frac{GF}{ab\pi c^3}\int_0^c\int_0^{2\pi}\sqrt{c^2 - r^2}\,\sin\frac{m\pi(\xi+y\cos\theta)}{a}\,\sin\frac{n\pi(\eta+y\sin\theta)}{b}\,r\,dr\,d\theta \tag{25}$$

By the assumption of a thin plate, $\sigma_z = 0$, we obtained the general forms of stresses as follows:

$$\sigma_x = \frac{12z}{h^3\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn\left(\frac{m^2}{a^2} + \frac{vn^2}{b^2}\right)}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{26}$$

$$\sigma_y = \frac{12z}{h^3\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn\left(v\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{27}$$

$$\tau_{xy} = -\frac{12z(1 - v)}{h^3\pi^2 ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn\,(m)(n)}{\left(\frac{m^2}{a^2} + \frac{n^2}{h^2}\right)^2} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \tag{28}$$

For a square plate ($a = b$ and $\xi = \eta = a/2$),

$$\sigma_x = \frac{12a^2z}{h^3\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn(m^2+vn^2)}{(m^2 + n^2)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} \tag{29}$$

$$\sigma_y = \frac{12a^2z}{h^3\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn(vm^2 + n^2)}{(m^2 + n^2)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a} \tag{30}$$

$$\tau_{xy} = -\frac{12a^2z(1 - v)}{h^3\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{Amn\,(m)\,(n)}{(m^2 + n^2)^2} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{a} \tag{31}$$

Since maximum bending moments occur at the center of the plate ($x = y = \frac{a}{2}$) and the maximum bending stresses at $Z = \pm \frac{h}{2}$, we have

$$\sigma_x)_{max} = \frac{6a^2}{h^2\pi^2} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{Amn(m^2 + \nu n^2)}{(m^2 + n^2)^2} (-1)^{\left(\frac{m+n}{2} - 1\right)} \tag{32}$$

$$\sigma_y)_{max} = \frac{6a^2}{h^2\pi^2} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{Amn(\nu m^2 + n^2)}{(m^2 + n^2)^2} (-1)^{\left(\frac{m+n}{2} - 1\right)} \tag{33}$$

$$\tau_{xy} = 0 \tag{34}$$

Notice that $\sigma_x)_{max} = \sigma_y)_{max}$.

2. Case of uniformly distributed contact pressure.

Y. M. Tsai and H. Kolsky (Ref. 3) found in the study of wave propagation that the theoretical analysis gave close agreement with the experimental pulse shapes, by assuming that the area of contact has a finite radius and stress is distributed uniformly over this circular area. Therefore, for the simplification of the problem, we can assume that the compressive force is uniformly distributed over the contact area as shown in Figure 5. Since the compressive force P is uniformly distributed over the contact area

$$Amn = \frac{4F}{ab\ \pi\ c^2} \int_0^c \int_0^{2\pi} \sin \frac{m\pi(\xi + r\cos\theta)}{a} \sin \frac{n\pi(\eta + r\sin\theta)}{b}\, dr d\theta \tag{35}$$

Provided that the circle r = c remains entirely inside the boundary of the plate, the evaluation of the integral equation (31) gives the expression

$$Amn = \frac{8F}{abc\ \gamma_{mn}} J_1\ (\gamma_{mn}C) \sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \tag{36}$$

in which $\gamma_{mn} = \pi \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ and $J_1\ (\gamma_{mn}C)$ is the Bessel function of order one, with the argument $\gamma_{mn}C$ (Ref. 11). Therefore, the expression for the deflection of the plate becomes, by substituting equation (36) into equation (24)

$$W = \frac{8F}{\pi^5 Dabc} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{J_1(\gamma_{mn}C) \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b}}{\left[\left(\frac{m^2}{a^2}\right) + \left(\frac{n^2}{b^2}\right)\right]^{5/2}}$$

$$\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (37)$$

For a square plate a = b and $\eta = \xi = a/2$

$$W = \frac{8Fa^3}{\pi^5 Dc} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{J_1(\gamma_{mn}C) (-1)^{\left(\frac{m+n}{2}-1\right)}}{(m^2 + n^2)^{5/2}}$$

$$\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (38)$$

As in the previous section, the stress at the center of the surface of the plates

$$\sigma_x)_{max} = \sigma_y)_{max} = \frac{48Fa}{\pi^3 h^2 c} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{J_1(\gamma_{mn}C) (-1)^{m+n} (m^2+\nu n^2)}{(m^2 + n^2)^{5/2}} \qquad (39)$$

$$\tau_{xy} = 0$$

EXPERIMENTAL TESTS

Test Apparatus

An air gun and two steel projectiles having 0.5-inch and 1.0-inch radii spherical heads were used. Air pressures up to 60 psi, velocities up to 85 fps were obtained.

Test Specimens and Coverings

Steel square plates (6 in. x 6 in.) in various thicknesses, and Adiprene (h - 0.29 in.) and Lexan (h - 0.19 in.) coverings were used. Rockwell hardness tests were carried out to obtain the mechanical properties of steel, and the results were interpreted using the ASM Handbook (Ref. 10).

The mechanical properties of the specimens and coverings are as in Table 1.

Comparison of the Test Results and the Computed Values

The comparison of experimental results with theoretical values for velocity of impact versus radius of contact for Lexan and Adiprene coverings is shown in Figures 8 and 9, respectively. Experimental and theoretical values for strain versus velocity of impact on the steel plate with and without coverings are shown in Figure 10 for 1-inch radius projectiles, and Figure 11 for 1/2-inch radius projectiles.

DISCUSSION AND CONCLUSION

The radius of contact between the projectile and the covering was measured after each impact test. These values were compared with the theoretically calculated values using the Hertz law for the contact of elastic (projectile) and viscoelastic (Adiprene) bodies, and elastic (projectile) and plastic (Lexan) bodies. It was found that there was a reasonable agreement between experimental and theoretical values. This confirms the validity of using the Hertz contact law in problems of impact, and it also supports the conclusion that the application of the Hertz law can be extended to the contact of viscoelastic bodies (Refs. 5 and 9).

In order to compare the experimental results with the theoretical values of the strain for the plate with coverings, compressive forces and radii of contact between projectiles and coverings were first calculated. Then the plate was considered to be subjected to these forces without coverings. The values of stress and strain, using the biaxial stress-strain relations, were calculated by the computer.

The experimental data were obtained from the strain gages, mounted on the free side of the plate directly under the point of impact and connected to an oscilloscope.

From the comparison, it was found that the experimental results were in good agreement with the theoretical values for the steel plate, considering the fact that there existed a certain depth of layer of covering between projectile and plate, and a possible friction loss between the covering and the plate.

For the different radii of projectiles, it was found that, even with the same mass and at the same velocity of impact, the projectile of larger radius produced a greater compressive force, as well as a larger radius of contact between bodies, thus producing a larger strain, both theoretically and experimentally.

From an evaluation of the theoretical results for both the hemispherically and uniformly distributed contact pressure, the latter was found to be closer to the experimental results. However, since all of the results are reasonably close, both cases can be used to predict the approximate impact characteristics of thin plates.

It was noticed that under the above conditions all plates tested remained with the elastic range.

From the comparison of steel plate with and without coverings, it was found that Adiprene served as the best mitigator of shock. It reduced the magnitude by a factor of 10 on the average. Lexan also served as a good mitigator, where a factor of 5 was obtained.

In addition, experimental tests were also carried out on the plates without coverings where plastic yieldings (a slight permanent set) occurred at the point of contact. Observing the results, the same conclusion can be obtained that the Hertz law is applicable for steel plates. This also verifies the conclusion reached by Werner Goldsmith (Ref. 5) that the Hertz law is valid under proper circumstances even when some permanent set is produced.

It is further suggested that the problem of large plastic deformation be studied, combining the theory of plasticity with some modification of the Hertz law.

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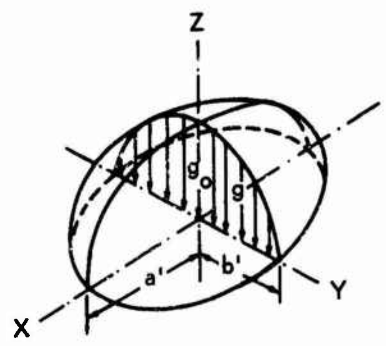

FIG. 1 CONTACT PRESSURE DISTRIBUTION, TWO GENERAL CURVED SURFACES

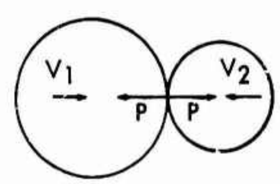

FIG. 2 IMPACT OF TWO SPHERES

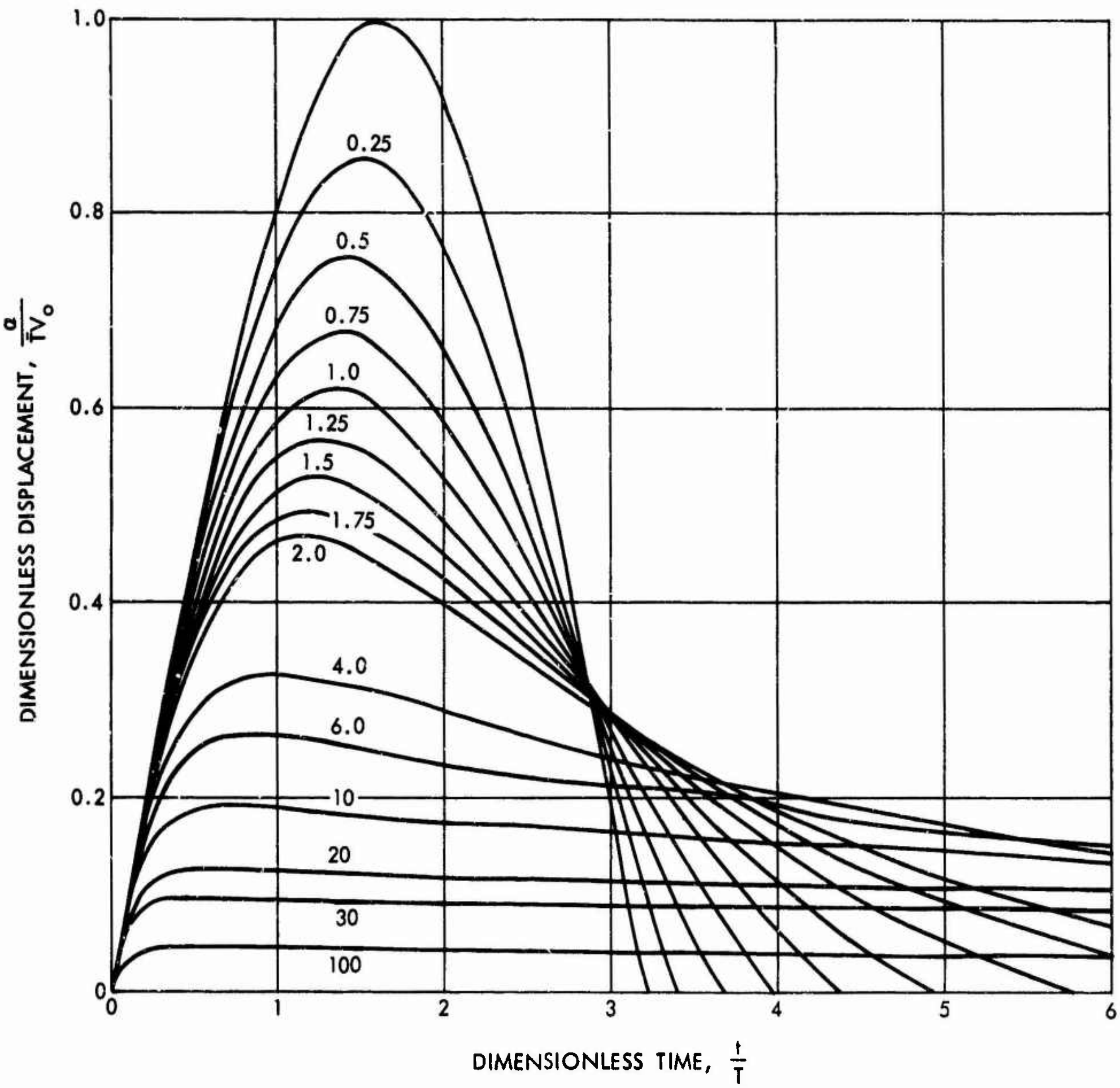

FIG. 3 DIMENSIONLESS DISPLACEMENT AS A FUNCTION OF DIMENSIONLESS TIME

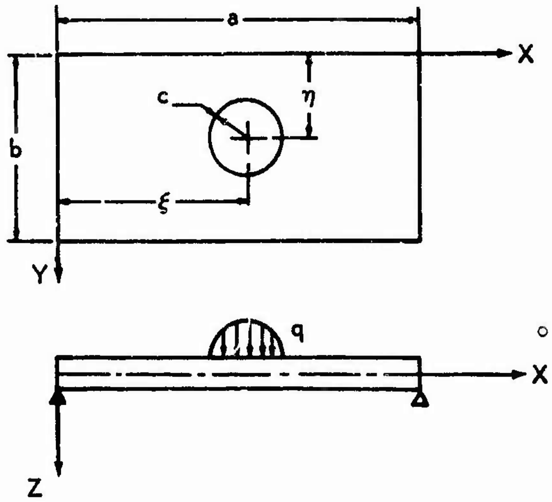

FIG. 4 HEMISPHERICALLY DISTRIBUTED CONTACT PRESSURE

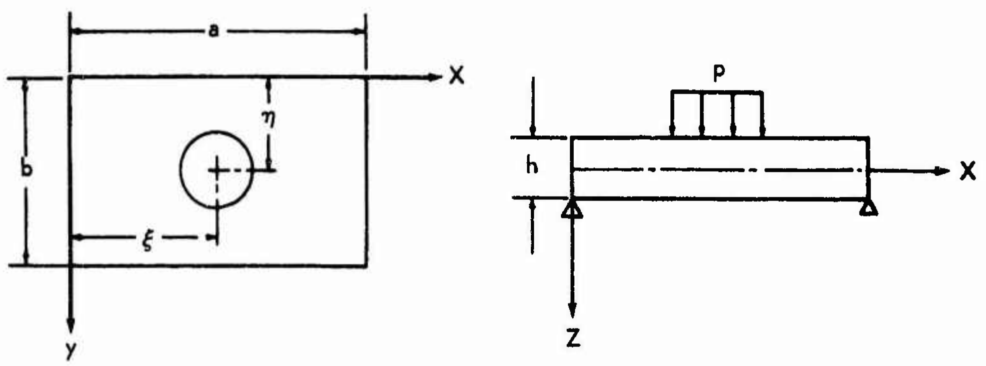

FIG. 5 UNIFORMLY DISTRIBUTED CONTACT PRESSURE

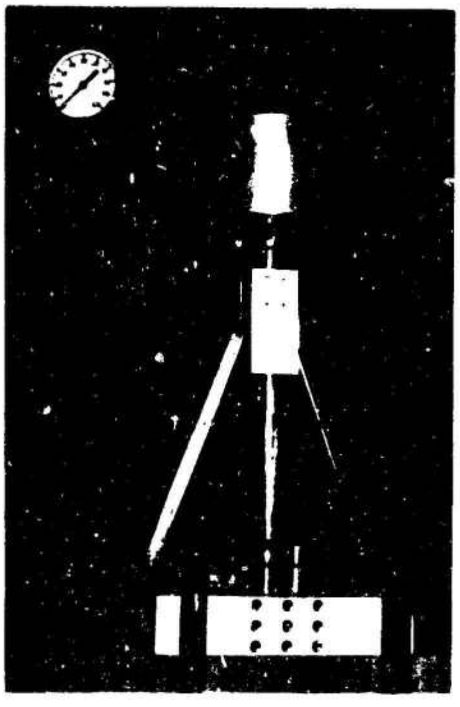

FIG. 6 AIR-GUN AND PROJECTILES

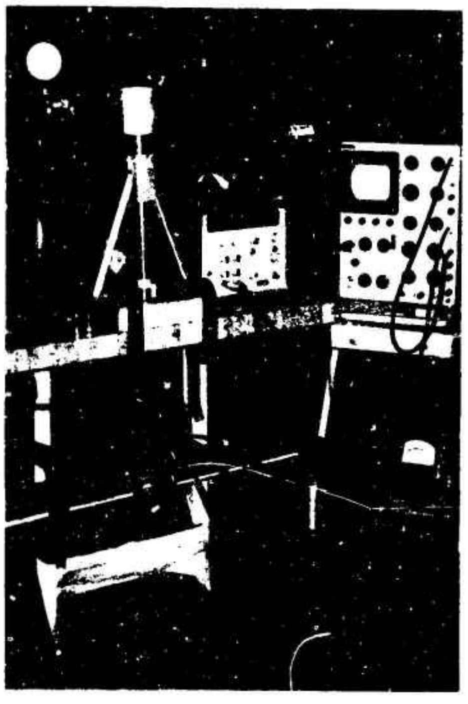

FIG. 7 ASSEMBLED TEST APPARATUS

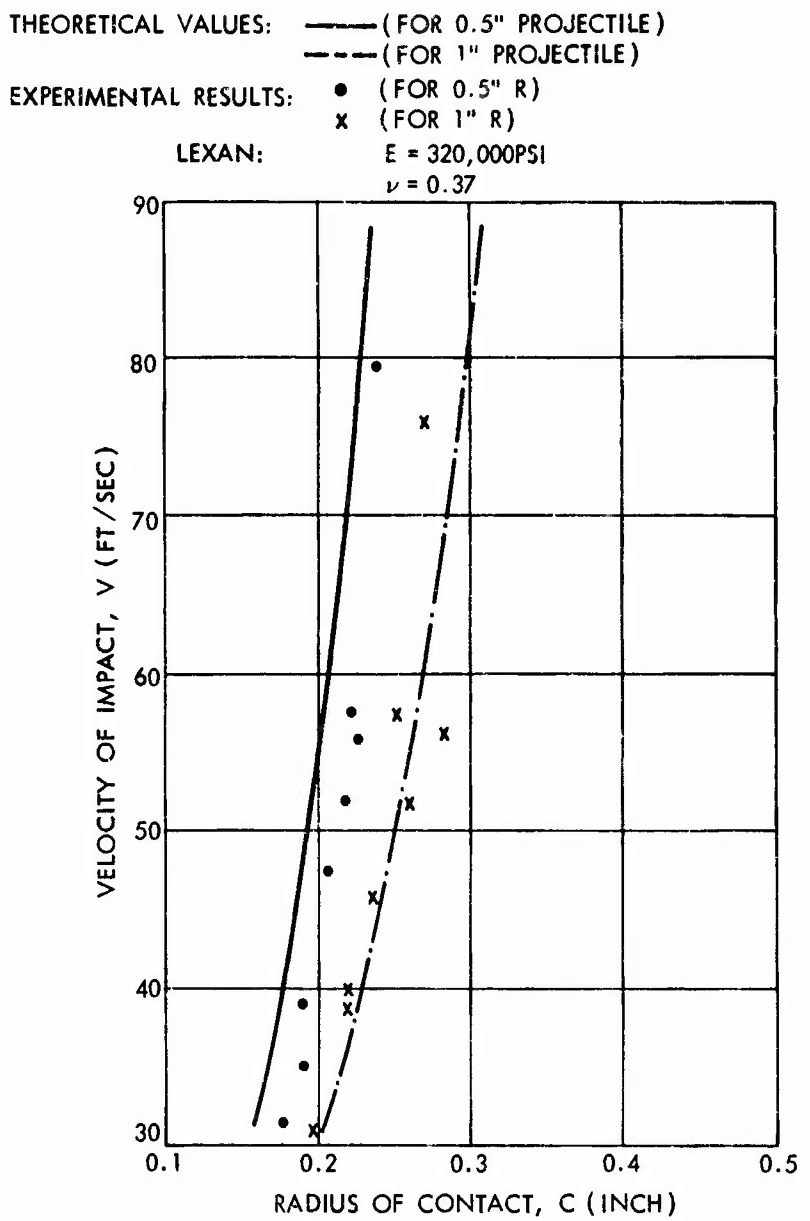

FIG. 8 VELOCITY OF IMPACT VS RADIUS OF CONTACT (LEXAN COVERING)

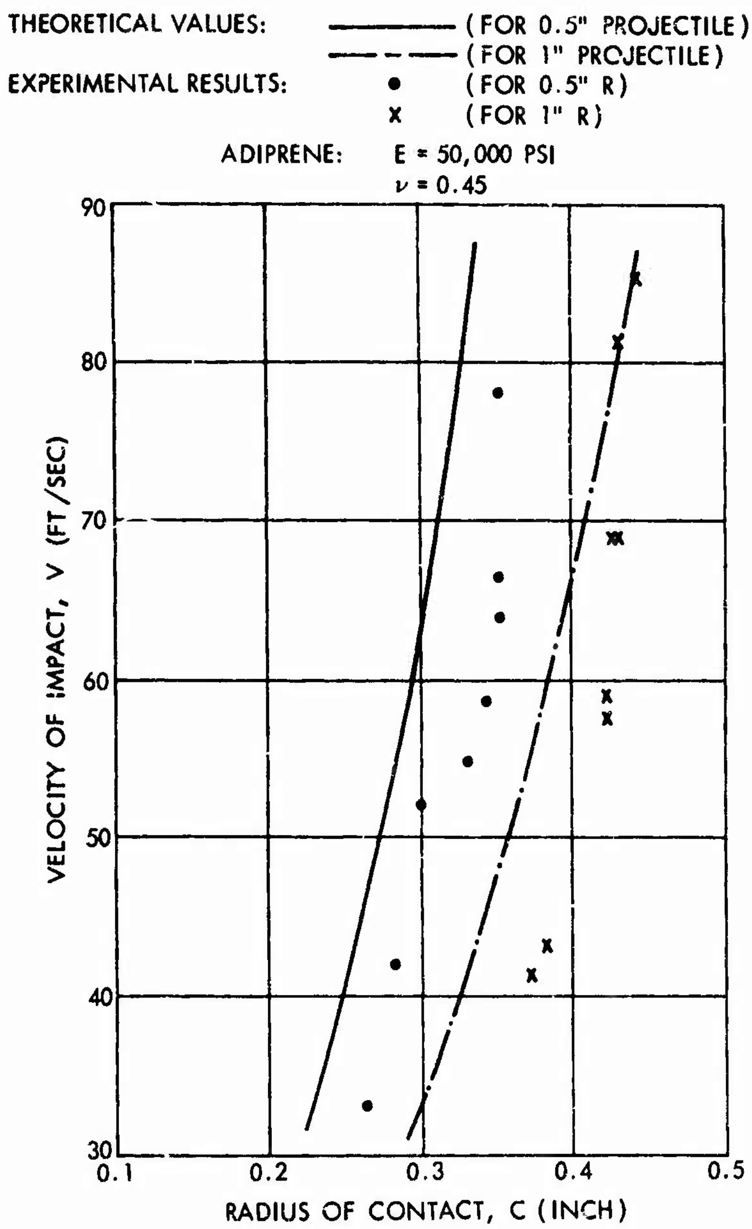

FIG. 9 VELOCITY OF IMPACT VS RADIUS OF CONTACT (ADIPRENE COVERING)

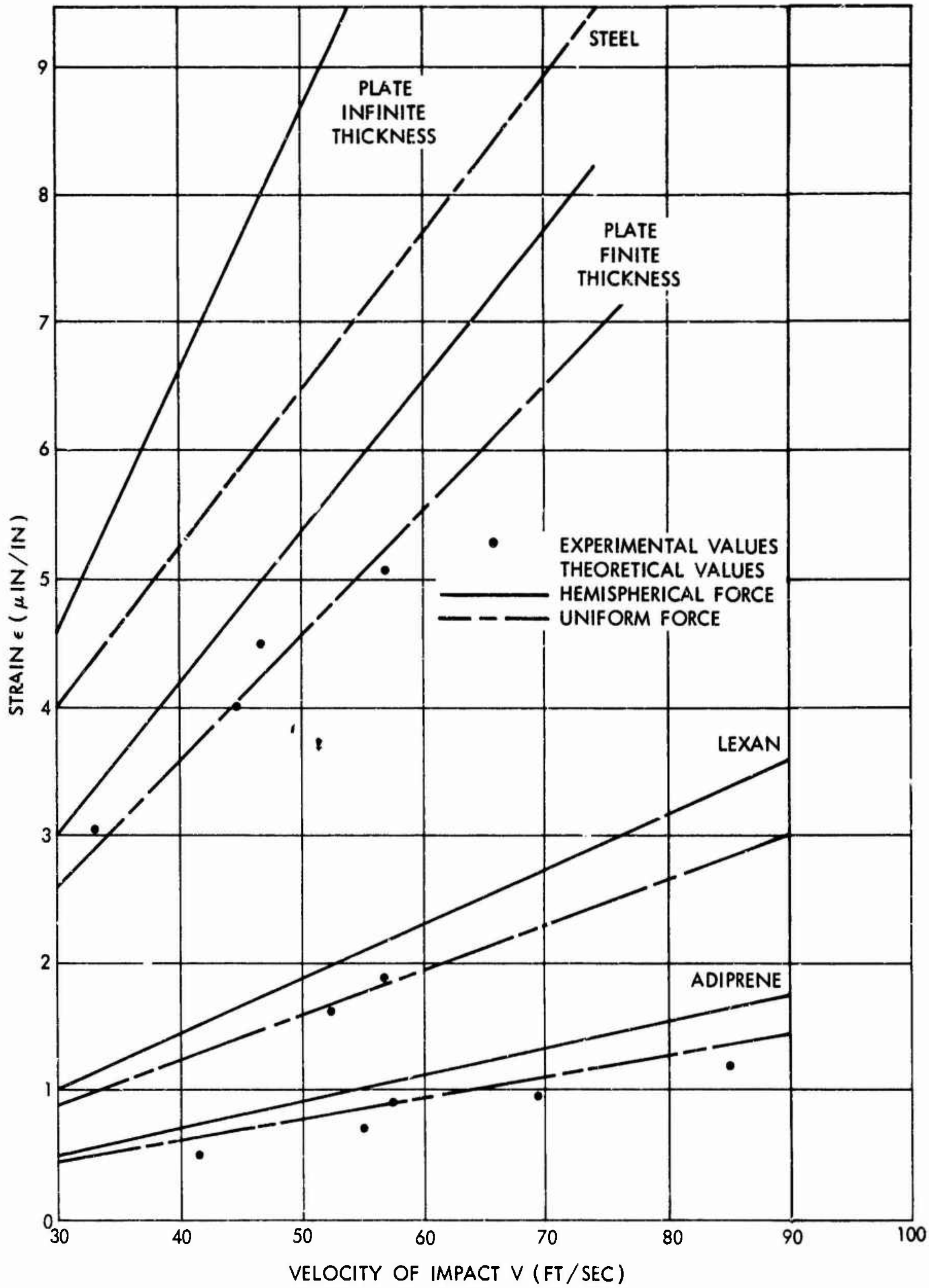

FIG. 10 STRAIN VS VELOCITY OF IMPACT (1"-R PROJECT)

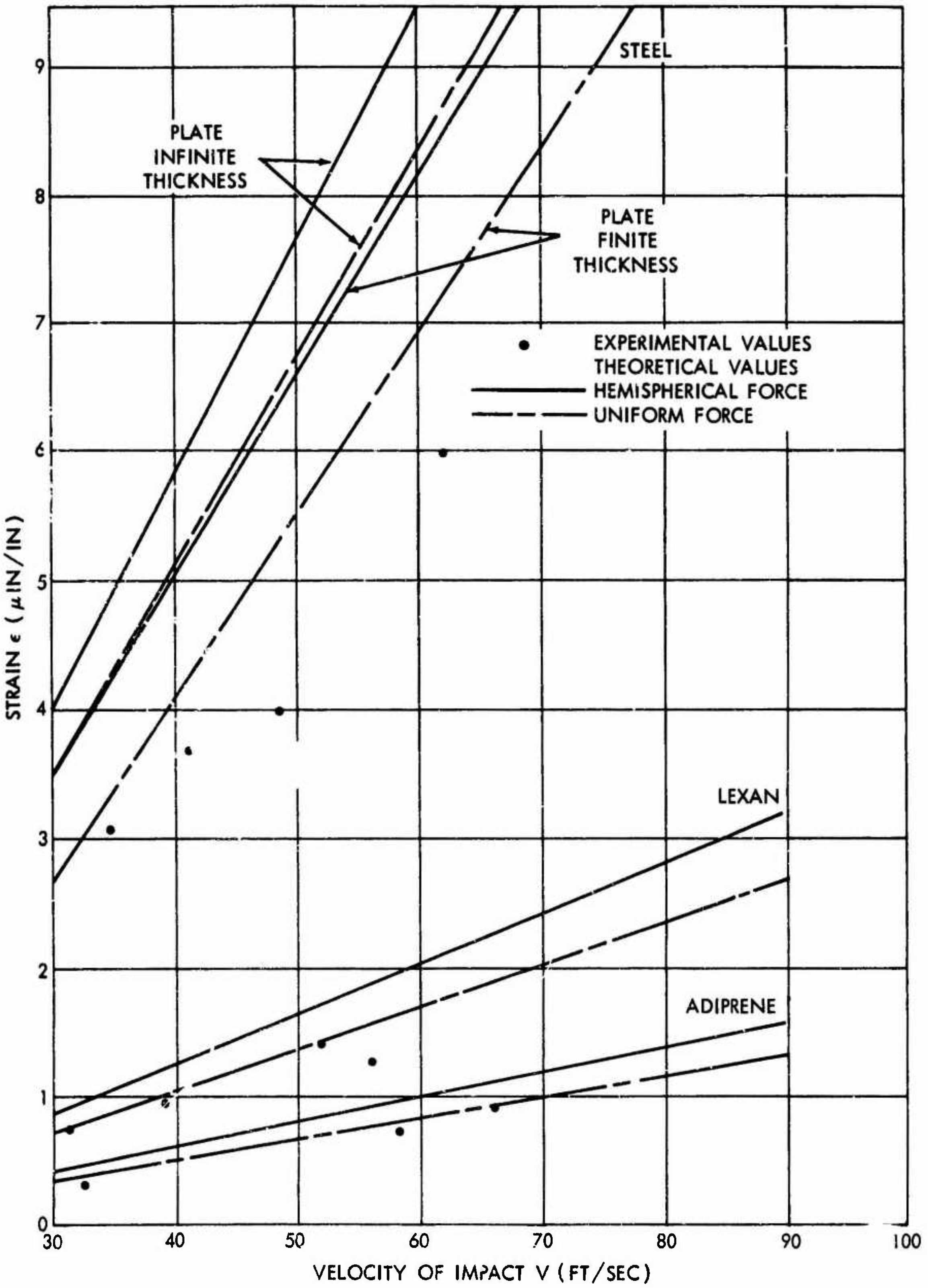

FIG. 11 STRAIN VS VELOCITY OF IMPACT (0.5"-R PROJECT)

Table 1

MECHANICAL PROPERTIES OF SPECIMENS AND COVERINGS

(Specimens: 6 in. x 6 in. x 0.375 in.)

Materials	Modulus of Elasticity, E, Psi	Poisson's Ratio	Hardness, R_B	Tensile Strength Ksi	Yield Strength Ksi
Steel, carbon and alloy	30×10^6	0.3	74	63	35
Adiprene	50×10^3	0.45			
Lexan	320×10^3	0.37			

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13. ABSTRACT

An experimental and theoretical investigation of transverse impact produced by the collision of strikers with steel square plates with and without covering was performed. The purpose of this report is to determine (1) the validity of the static treatment of pressure on thin plates for the dynamic cases of intermediate impact velocities, (2) the use of the Hertz law for the contact of an elastic projectile (steel) on elastic (steel), viscoelastic (Adiprene), and plastic (Lexan) bodies, and (3) the use of viscoelastic and plastic materials as shock mitigators.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Impact						
Hertz law						
Contact law						
Plates						
Mitigation						
Transverse impact						
Elastic						
Plastic						
Viscoelastic						

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