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Technical Report 475

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B. Loesch
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Prepared for the Office of the Chief of Research and Development,
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LINCOLN LABORATORY

A TECHNIQUE FOR SYNTHESIZING SIGNALS AND THEIR MATCHED FILTERS

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ABSTRACT

A nonuniformly tapped delay line is the key component in a device that can be used both to generate an arbitrary waveform and to act as a matched filter for this same waveform. Formulas for calculating the tap delays and weights needed to generate a given waveform are derived. The effects of delay-line attenuation and a technique for compensating for these effects are discussed.

An experimental system embodying these design techniques is described. This system generates and receives a linear FM waveform of 0.5-μsec duration and either 200- or 400-MHz bandwidth. Construction details and the results of many tests run on the system are given.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

CONTENTS

A TECHNIQUE FOR SYNTHESIZING SIGNALS AND THEIR MATCHED FILTERS

I. INTRODUCTION

Two basic operations performed by many radars are the generation of a transmitter waveform and its subsequent processing at the receiver by means of a matched filter. A great many techniques for realizing one or both of these objectives have been analyzed and built in the past. The purpose of this report is to describe a new approach to this problem that has been developed over the last few years. Since the operations of signal generation and matched filtering also arise in fields such as communications and sonar, the results of this report have possible applications outside the domain of radar. Radar terminology is used in the sequel simply because that is the field with which the authors are most familiar.

The system to be discussed is illustrated in simplified form in Fig. 1. It consists of a nonuniformly tapped delay line, whose taps are weighted and then summed, followed by a bandpass zonal filter. In the waveform generation mode, an impulse is applied at terminal 1 and the desired waveform obtained at terminal 3. This same device also can be used as a matched filter for the waveform that it generates by applying the latter to terminal 2 and taking the output at terminal 4. (Terminals 3 and 4 are identical if the summing bus is lossless.) This is one of the most important features of this device because it eliminates the need for separate signal generation and reception equipment. The class of signals that can be generated and received in this manner is limited only by equipment problems such as the limited time-bandwidth product of existing delay lines and the density with which taps can be placed on these lines.

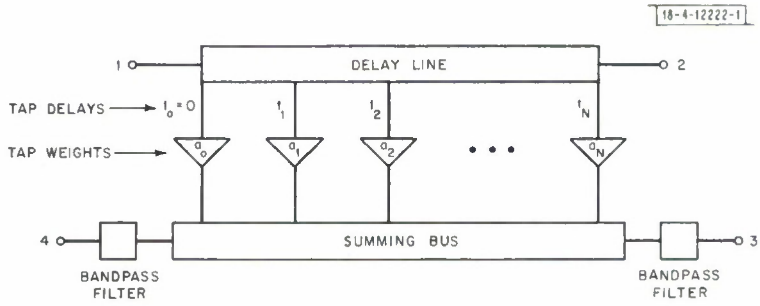

Fig. 1. Simplified diagram of waveform generation/matched filter system.

One frequently suggested tapped-delay-line technique for realizing signals and their matched filters is based on the bandpass sampling theorem.[1] This method uses a uniformly tapped delay line with amplitude weights and phase shifts on each tap. The disadvantages of such a system are that good phase shifters are difficult to build and that the phase shifts must be changed by 180° between transmission and reception. The latter procedure is required to make the receive filter matched to the waveform that was generated on transmission. The synthesis technique described in this report avoids both of these drawbacks.

The body of the report consists of a theoretical section and an experimental section. The theoretical section is devoted to the derivation of formulas from which the tap spacings and tap weights needed to generate a given narrowband waveform can be calculated. In addition, the effect of such factors as delay-line attenuation and the shape of the delay-line exciting pulse are discussed.

The experimental section of the report describes an application of these thcorctically derived waveform generation and reception techniques to the design and fabrication of a linear FM radar pulse-compression system. The system that was built generates and comprcsses a linear FM waveform of 0.5-μsec duration and 400-MHz bandwidth. The details of its construction and performance arc given, as well as a discussion of some of the problems encountered with this type of system.

Much of the work described in this report has been previously documented in internal memoranda, and a patent* has bcen granted on this network.

II. WAVEFORM SYNTHESIS THEORY

The procedure for selecting the tap delays and weights to synthcsize a given waveform is quite straightforward. Suppose that the waveform to be generatcd is given in the form

$$s(t) = E(t) \cos 2\pi \left[f_o t + \varphi(t)\right] \quad , \tag{1}$$

where $E(t)$ denotes the desired envelope modulation, $\varphi(t)$ the desired phase modulation, and f_o the carrier frequency. Then, as will be demonstrated, the proper tap delays to use are given by the solutions to the equation

$$f_o t_k + \varphi(t_k) = k \quad , \quad k = 0, \pm 1, \pm 2, \ldots \quad , \tag{2}$$

and the corresponding tap weights are given by

$$a_k = \frac{1}{2} E(t_k) \left[f_o + \varphi'(t_k)\right]^{-1} \quad , \tag{3}$$

where φ' denotes the derivative of φ. The bandpass filter to use in connection with this tapped delay line should be centered at f_o and have a bandwidth just large enough to pass $s(t)$ without objectionable distortion. Note that, although Eq. (2) will have an infinite number of solutions in general, only a finite number of them will be needed as long as $s(t)$ [and hence $E(t)$] is time limited because then all but a finite number of the a_k's will be zero.

The synthesis procedure just described will yield an accurate replica of the desired waveform $s(t)$ only if the bandwidth of $s(t)$ covers considerably less than an octave of frequency. The reason for this will become apparent when the derivation of the procedure is given. For signals whose bandwidths violate this condition, the synthesis procedure must be modified in order to yield accurate results.

The mathematical justification for the synthesis procedure just described is based on an intriguing expansion formula. The derivation of this formula begins with the well known Fourier series representation of a uniform impulse train,[2]

$$\sum_{k=-\infty}^{\infty} \delta(x - k) = \sum_{k=0}^{\infty} \epsilon_k \cos 2\pi kx \quad , \tag{4}$$

* Potent No. 3,430,241, 25 Februory 1969, Frequency Modulated Wave Pulse Tronsmission ond Reception.

where

$$\epsilon_k = \begin{cases} 1 & , \quad k = 0 \\ 2 & , \quad k \neq 0 \end{cases}$$

and $\delta(x)$ denotes a unit impulse located at $x = 0$. Equation (4) is valid for all x; therefore, the substitution $x = f_o t + \varphi(t)$ is permissible and yields the result,

$$\sum_{k=-\infty}^{\infty} \delta\,[f_o t + \varphi(t) - k] = \sum_{k=0}^{\infty} \epsilon_k \cos 2\pi k\,[f_o t + \varphi(t)] \quad . \tag{5}$$

The function $\delta\,[f_o t + \varphi(t) - k]$ represents a series of impulses located at those values of t for which $f_o t + \varphi(t) = k$. For convenience, assume that $f_o t + \varphi(t)$ is monotonic so that this equation has only one solution, i.e., for each k there exists a unique t_k such that

$$f_o t_k + \varphi(t_k) = k \quad . \tag{6}$$

If $f_o t + \varphi(t)$ is not monotonic, the synthesis procedure must be modified in an obvious way. The value (area) of the impulse located at t_k is given by

$$\int_{-\infty}^{\infty} \delta\,[f_o t + \varphi(t) - k]\, dt = \int_{-\infty}^{\infty} \frac{\delta(y)}{f_o + \varphi'[t(y)]}\, dy = \frac{1}{f_o + \varphi'[t(o)]} = \frac{1}{f_o + \varphi'(t_k)} \quad ,$$

where the first equality is a result of the change of variable $y = f_o t + \varphi(t) - k$ and the assumption that $f_o + \varphi'(t_k) \neq 0$. Since the instantaneous frequency $f_o + \varphi'(t)$ of any practical signal will never be equal to zero, this assumption will always be met in practice. Thus, it can be concluded that

$$\delta\,[f_o t + \varphi(t) - k] = \frac{\delta(t - t_k)}{f_o + \varphi'(t_k)}$$

and, therefore, that

$$\sum_{k=0}^{\infty} \epsilon_k \cos 2\pi k\,[f_o t + \varphi(t)] = \sum_{k=-\infty}^{\infty} \frac{\delta(t - t_k)}{f_o + \varphi'(t_k)} \quad . \tag{7}$$

Multiplication of both sides of Eq. (7) by $\frac{1}{2} E(t)$ now leads to the final expansion formula,

$$\frac{1}{2} \sum_{k=0}^{\infty} \epsilon_k E(t) \cos 2\pi k\,[f_o t + \varphi(t)] = \sum_{k=-\infty}^{\infty} \frac{\frac{1}{2} E(t_k)}{f_o + \varphi'(t_k)}\, \delta(t - t_k) \quad . \tag{8}$$

The last step in this derivation has made use of the identity $E(t)\,\delta(t - t_k) = E(t_k)\,\delta(t - t_k)$ which is valid as long as $E(t)$ is continuous at $t = t_k$. Should $E(t)$ have a jump discontinuity at $t = t_k$, then the identity is again valid as long as the value of $E(t)$ at t_k is defined to be $\frac{1}{2}\,[E(t_k^+) + E(t_k^-)]$. This will be assumed to be the case throughout this report.

The left-hand side of Eq. (8) consists of the desired signal plus all of its harmonics including the zeroth. It follows that, as long as the frequency spread of $s(t)$ is considerably less than an octave, these harmonics will be well separated in frequency and $s(t)$ can be filtered out from this combination by means of a bandpass filter centered at f_o and having a bandwidth just large enough to pass $s(t)$ without undue distortion. Since the right-hand side of Eq. (8) is exactly the output of a delay line tapped and weighted according to the synthesis procedure described above, it follows that this procedure will yield an accurate replica of any sufficiently narrowband signal.

The foregoing derivation has assumed that the tapped delay line was excited with a perfect impulse. It should be obvious now that essentially the same results would be obtained by exciting the delay line with a pulse short enough so that its spectrum is more or less flat in the frequency interval occupied by the desired signal s(t). Still another approach is to excite the line with an RF pulse having the same carrier frequency as the desired signal. The duration of such an RF pulse must be smaller than the reciprocal bandwidth of s(t) so that its spectrum will be essentially flat over the frequency interval occupied by s(t). This is the approach that was used in the experimental system to be described later.

The bandwidth restriction inherent in the synthesis technique just described can be removed by the following modification of the tapping and weighting procedure. This new procedure is based on an expansion formula similar in character to Eq. (8). The derivation of the formula begins by multiplying both sides of Eq. (4) by $\cos(2\pi x/n)$ with the result,

$$\cos\frac{2\pi}{n}x \sum_{k=0}^{\infty} \epsilon_k \cos 2\pi kx = \sum_{k=-\infty}^{\infty} \cos\left(\frac{2\pi k}{n}\right) \delta(x-k) \quad . \tag{9}$$

The substitution $x = n\,[f_o t + \varphi(t)]$ now transforms Eq. (9) into the form

$$\begin{aligned} \cos 2\pi\,[f_o t + \varphi(t)] \sum_{k=0}^{\infty} \epsilon_k \cos 2\pi nk\,[f_o t + \varphi(t)] &= \sum_{k=-\infty}^{\infty} \cos\frac{2\pi k}{n}\, \delta\left\{n\,[f_o t + \varphi(t)] - k\right\} \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{n} \cos\frac{2\pi k}{n} \frac{\delta(t-t_k)}{f_o + \varphi'(t_k)} \quad , \end{aligned} \tag{10}$$

where the t_k's are the solutions to the equation

$$f_o t_k + \varphi(t_k) = \frac{k}{n} \quad , \qquad k = 0, \pm 1, \ldots \quad , \tag{11}$$

which implies that n taps are to be provided for each cycle of the desired signal.

Multiplication of Eq. (10) by E(t) yields the desired expansion formula,

$$\begin{aligned} E(t) \cos 2\pi\,[f_o t + \varphi(t)] \sum_{k=0}^{\infty} \epsilon_k \cos 2\pi nk\,[f_o t + \varphi(t)] &= \sum_{k=-\infty}^{\infty} \frac{1}{n} \cos\frac{2\pi k}{n} E(t_k) \\ &\times \frac{\delta(t-t_k)}{f_o t + \varphi'(t_k)} \quad . \end{aligned} \tag{12}$$

The left-hand side of Eq. (12) consists of the desired signal (actually twice the desired signal for n = 1, 2) plus its $n-1^{st}$, $n+1^{st}$, $2n-1^{st}$, $2n+1^{st}$, $3n-1^{st}$, ..., harmonics. The right-hand side of Eq. (12) is the output of a tapped delay line that has been excited by an impulse and has tap delays given by Eq. (11) and tap weights given by

$$a_k = \frac{1}{n} E(t_k) \cos\left(\frac{2\pi k}{n}\right) [f_o + \varphi'(t_k)]^{-1} \quad . \tag{13}$$

Since, by choosing n sufficiently large, the frequency separation between the desired signal and its nearest harmonic (the $n - 1^{st}$ for $n > 2$) can be made as large as desired, it follows that the tapped-delay-line system depicted in Fig. 1 can be made to generate an accurate replica of any waveform, regardless of its bandwidth, by choosing n large enough and then using the tap delays and weights given by Eqs. (11) and (13). Care should be taken, however, not to pick n any larger than is necessary to enable the desired signal to be adequately filtered from its harmonics because the density of taps on the line, and hence the difficulty of implementing the system, increases with increasing n.

It should be noted that, although the preceding modification of the synthesis procedure was presented as a means for overcoming the bandwidth restriction of the earlier technique, the modified procedure also can be useful for simplifying the filtering of the desired signal from its harmonics even when it is used to synthesize a signal whose bandwidth is less than an octave. Another way of simplifying the filtering operation is to raise the carrier frequency of the desired signal, thus reducing its fractional bandwidth. Because this artifice may be inconvenient in some applications, it is useful to have an alternate technique available in the form of the modified tapping procedure just described.

As a concrete example of the above technique, consider the design of a tapped-delay-line system to generate the linear FM waveform given by

$$s(t) = \begin{cases} \cos 2\pi\, [f_o t + \frac{1}{2}\mu t^2] \quad , & |t| < \frac{T}{2} \quad , \\ 0 \quad , & |t| > \frac{T}{2} \quad , \\ \frac{1}{2} \cos 2\pi\, [\pm f_o \frac{T}{2} + \frac{1}{8}\mu T^2] \quad , & t = \pm \frac{T}{2} \quad . \end{cases} \tag{14}$$

This waveform has been centered at $t = 0$ for mathematical convenience. The tap delays are the solutions to Eq. (11) which in this case assumes the form

$$f_o t_k + \frac{1}{2}\mu t_k^2 = \frac{k}{n} \quad . \tag{15}$$

The solutions of this equation lying in the interval $[-\frac{T}{2}, \frac{T}{2}]$ are given by*

$$t_k = -\frac{f_o}{\mu} + \left[\left(\frac{f_o}{\mu}\right)^2 + 2\frac{k}{n\mu}\right]^{1/2} \quad , \tag{16}$$

where

$$n\,(-f_o \frac{T}{2} + \frac{1}{8}\mu T^2) \leqslant k \leqslant n\,(f_o \frac{T}{2} + \frac{1}{8}\mu T^2) \quad , \tag{17}$$

and the corresponding tap weights are given by

$$a_k = \begin{cases} \frac{1}{n}\,[f_o + \mu t_k]^{-1} \cos \frac{2\pi k}{n} \quad , & |t_k| \leqslant \frac{T}{2} \quad , \\ 0 \quad , & |t_k| > \frac{T}{2} \quad , \\ \frac{1}{2n}\left[f_o \pm \mu \frac{T}{2}\right]^{-1} \cos \frac{2\pi k}{n} \quad , & t_k = \pm \frac{T}{2} \quad . \end{cases} \tag{18}$$

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* For an intuitive derivation of this formula in a context different from that of this paper see Ref. 3. Reference 4 is also pertinent in this connection.

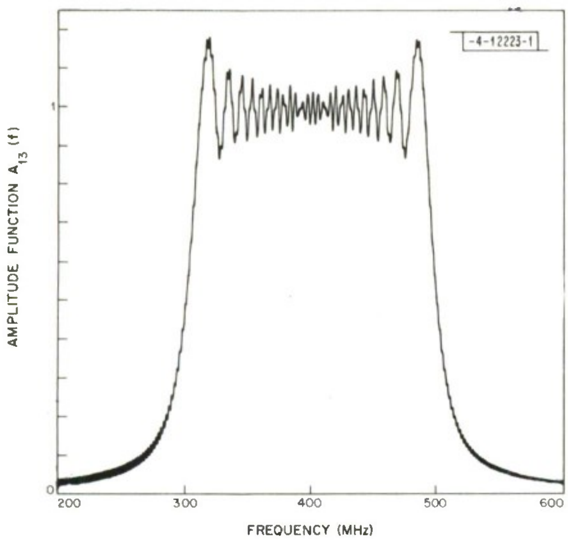


Fig. 2. Amplitude spectrum of linear FM waveform generated by tapped delay line.

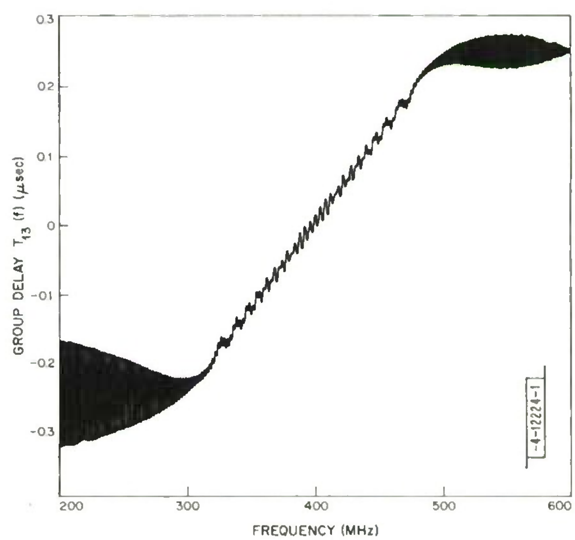


Fig. 3. Group delay of linear FM waveform generated by tapped delay line.

A fixed delay must be added to the tap delays given by Eq. (16) to make them all non-negative and thus realizable. It is easy to use Eq. (17) to verify that the number of taps needed to generate this waveform is the greatest integer contained in $nf_o T + 1$. The bandpass filter should be centered at f_o Hz and have a bandwidth roughly equal to the signal bandwidth μT Hz.

Using the formulas derived here, an ideal tapped-delay-line system for generating a 0.5-μsec linear FM waveform sweeping from 300 MHz to 500 MHz was simulated on a computer. The second-order mode (n = 2) was used in this simulation, and the bandpass filter was assumed to be flat over all frequencies of interest. This design corresponds to the experimental unit to be described in the next section of this report.

Figures 2 and 3 depict the normalized amplitude response $A_{13}(f)$ and group delay $T_{13}(f)$ of this linear FM system. These functions are derived from the terminal 1 to terminal 3 transfer function $H_{13}(f)$ of the tapped delay line by means of the equations

$$\left.\begin{aligned} A_{13}(f) &= \mu \, |H_{13}(f)| \quad , \\ T_{13}(f) &= -\frac{1}{2\pi}\frac{d}{df}\,[\arg H_{13}(f)] \quad , \end{aligned}\right\} \tag{19}$$

where

$$H_{13}(f) = \sum_{k=-\infty}^{\infty} a_k \, e^{-j2\pi f t_k} \quad , \tag{20}$$

and the a_k's and t_k's are to be determined from Eqs. (16) and (18). The resulting expression for $H_{13}(f)$ is

$$H_{13}(f) = \frac{1}{400} \sum_{k=-174}^{224} (-1)^k \left(1 + \frac{k}{400}\right)^{-1/2} e^{-j2\pi f\{-1+[1+(k/400)]^{1/2}\}} - \frac{1}{200}$$
$$\times \left[\frac{1}{3}\, e^{j(\pi f/2)} + \frac{1}{5}\, e^{-j(\pi f/2)}\right] \quad , \tag{21}$$

where frequency is measured in MHz. The agreement between these curves and the ones corresponding to a system that generates the same linear FM waveform exactly is very good. In this connection, it is important to note that the ripple present on the group delay curve is, for the most part, the Fresnel ripple associated with ideal linear FM which is sharply truncated in time and not the result of spectral overlap from the higher-order harmonics generated by the tapped delay line. The ripples in the time-delay function, $T_{13}(f)$, shown in Fig. 3 are exactly equal and opposite to those of the time-delay function, $T_{24}(f)$, of the tapped delay line. This two-way matched characteristic of the network is inherent in its design.

The preceding discussion has been couched in the language of waveform synthesis, but, as pointed out in the introduction, the same device used to generate a given waveform can be used as a matched filter for that waveform. This fact can be established by comparison of the two transfer functions, $H_{13}(f)$ and $H_{24}(f)$. For a delay line having N taps and a total delay of $T = t_{N-1}$ seconds (t_o has been set equal to zero for convenience), these functions are given by

$$H_{13}(f) = H(f) \sum_{k=0}^{N-1} a_k \, e^{-j2\pi f t_k} \quad , \tag{22}$$

$$H_{24}(f) = H(f) \sum_{k=0}^{N-1} a_k \, e^{-j2\pi f(T-t_k)} \quad , \tag{23}$$

where the a_k's and t_k's depend on the desired waveform in the now familiar way and H(f) denotes the transfer function of the bandpass filter. The functions $H_{13}(f)$ and $H_{24}(f)$ will be matched (except, perhaps, for an inconsequential time delay) if $H_{13}(f) = H_{24}^*(f)\, e^{j2\pi f\delta}$ for some constant δ. Comparison of Eqs. (22) and (23) shows that this will be the case provided that the phase of H(f) is linear over the pass band of interest. This condition will be met, at least approximately, by most bandpass filters used in this application.

The preceding discussion has assumed that both the delay line and the summing bus are ideal elements. This is far from the case in actual practice. The delay line, in particular, will usually exhibit an attenuation characteristic that rises significantly with increasing frequency because of increasing losses due to skin effect. One consequence of such an attenuation characteristic is that the device will not generate the waveform for which the tap delays and weights were designed. Another, usually more serious, consequence is that the device, when used as a receiver filter, is no longer necessarily a matched filter for the waveform that it generates. This fact can be seen from the expressions for $H_{13}(f)$ and $H_{24}(f)$ modified to take into account the effects of delay line and summing bus attenuation. These expressions are

$$H_{13}(f) = H(f) \sum_{k=0}^{N-1} a_k \, [A(f)]^{t_k} \left[\prod_{i=k+1}^{N-1} B_i(f) \right] e^{-j2\pi f t_k} \quad , \tag{24}$$

$$H_{24}(f) = H(f) \sum_{k=0}^{N-1} a_k \, [A(f)]^{T-t_k} \left[\prod_{i=1}^{k} B_i(f) \right] e^{-j2\pi(T-t_k)} \quad , \tag{25}$$

where A(f) denotes the attenuation per unit delay of the delay line and $B_k(f)$ denotes the attenuation of that portion of the summing bus connecting the $(k-1)^{st}$ tap to the k^{th} tap. Assuming that the phase of H(f) is linear, it is seen from Eqs. (24) and (25) that a sufficient condition for $H_{13}(f)$ to be matched to $H_{24}(f)$ is that

$$[A(f)]^{t_k} \prod_{i=k+1}^{N-1} B_i(f) = [A(f)]^{T-t_k} \prod_{i=1}^{k} B_i(f) \quad , \quad k = 0, \ldots, N-1 \quad . \tag{26}$$

It now follows easily that Eq. (26) can be satisfied for $k = 0, \ldots, N-1$ if, and only if,

$$[A(f)]^{t_{k+1}-t_k} = B_{k+1}(f) \quad , \quad k = 0, \ldots, N-1 \quad . \tag{27}$$

Stated in physical terms, Eq. (27) says that H_{13} will be matched to H_{24} if the summing bus attenuation between any two adjacent taps is equal to the delay-line attenuation between the same two taps. When Eq. (27) is satisfied, $H_{13}(f)$ assumes the form

$$H_{13}(f) = H(f)\,[A(f)]^T \sum_{k=0}^{N-1} a_k\, e^{-j2\pi f t_k} \quad , \tag{28}$$

and $H_{24}(f) = H^*_{13}(f)$ except perhaps for a time delay. It will be noted from Eq. (28) that, even when the summing bus attenuation has been designed so as to "match" the delay-line attenuation as required by Eq. (27), the signal generated by the system differs from the one for which the a_k's and t_k's were designed by the factor $[A(f)]^T$. This can be taken into account in the design of the zonal filter H(f) or otherwise equalized externally to the tapped-delay-line network. Finally, it should be noted that the condition that H(f) have a linear phase characteristic is not really necessary to assure that H_{13} and H_{24} are matched. All that is really necessary to insure this system match is that the bandpass filter used on reception be matched to the bandpass filter that was used on transmit, i. e., that the sum of their phase characteristics be linear.

III. IMPLEMENTATION OF A TAPPED-DELAY-LINE NETWORK

Several tapped-delay-line networks have been constructed, using the synthesis technique described above, to generate and compress FM signals in radar application. The parameters of the linear FM network to be described are given below.

	Option 1	Option 2
Signal duration	0.5 μsec	0.5 μsec
Center frequency	400 MHz	800 MHz
Swept bandwidth	200 MHz	400 MHz
Taps/cycle	n = 2	n = 1

It may be noted that the calculated tap delays from Eq. (16), when using the parameters of Option 1, are exactly the same as the calculated tap delays when using the parameters of Option 2. Also, the total number of taps as determined by the range of values of k in Eq. (17) is the same for the two options. From Eq. (18) we see that the required magnitudes of the tap weights differ only by a normalization factor for the two options. However, under Option 1, alternate weights are of negative sign, whereas under Option 2, all weights are positive in sign.

The network was thus constructed with two summing buses, as shown in Fig. 4, so that either of the above options could be obtained. To obtain the parameters of Option 2, the two buses are added with 0° phase (all weights positive). To obtain the parameters of Option 1, each bus sums one set of alternately spaced taps, and these two sets are then added with 180° relative phase; this phase relationship is equivalent to the alternating negative signs in Eq. (18).

The coaxial cable used in this design is UT141-AA, which has a solid copper outer conductor of 0.141-inch o. d., solid teflon dielectric, and 50 ohms characteristic impedance. The tap spacings vary from 8.19 to 13.65 inches. The total number of taps is 401, corresponding to the signal duration times the (higher) center frequency.

Considerable experimental time was spent in determining a summing bus design which would match the attenuation characteristic of the coax line over the entire frequency range of interest. As shown above, this matched characteristic is necessary in order that all frequency components of an input signal to one end of the coax line will suffer the same attenuation to the opposite end of either bus for all connecting paths from the coax to the bus. This desired characteristic was closely approximated by using three parallel strands of 0.002-inch Midohm wire for the high-frequency half of each bus (shorter lengths of coax line), and two parallel strands of this same

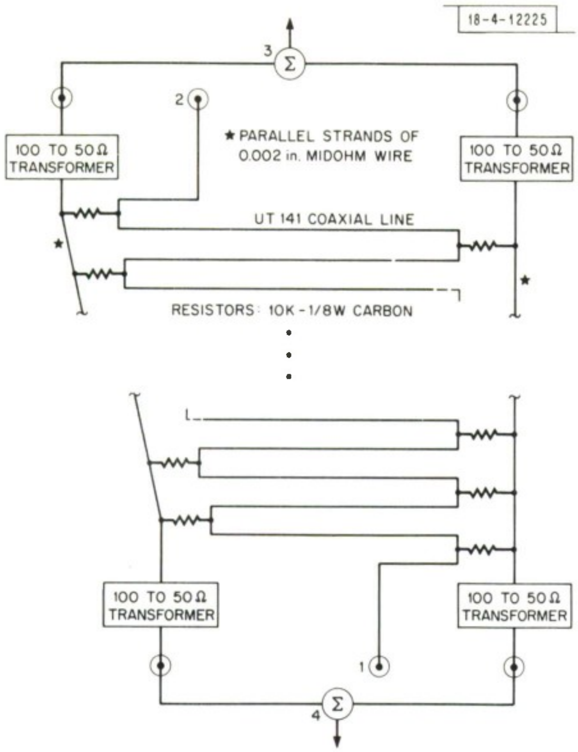


Fig. 4. Schematic diagram of network.

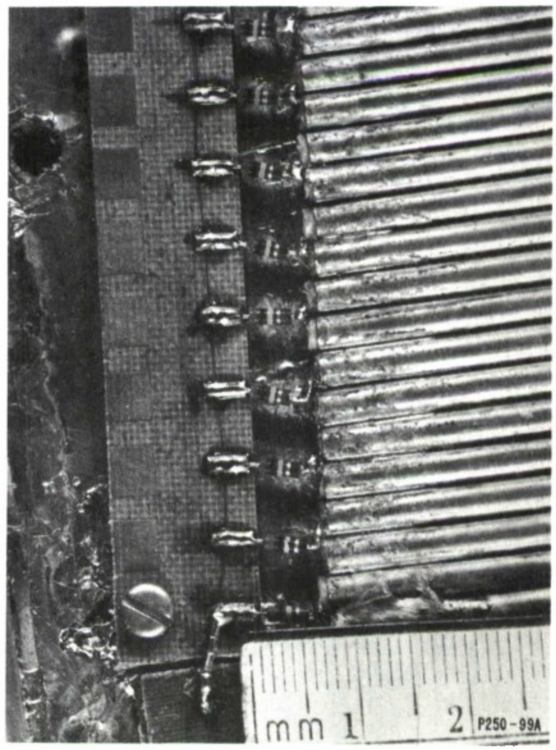


Fig. 5. Enlarged photograph of taps.

wire for the low-frequency half of each bus (longer lengths of coax line). With this design, the end-to-end summing bus loss increases from 37.0 to 46.0 dB across the 300-500 MHz band, while the end-to-end coax line loss increases from 35.5 to 44.0 dB across the same frequency band.

With the summing buses as described, it was found experimentally that the characteristic resistance of each bus was approximately 100 ohms. Tapered stripline matching transformers were therefore inserted as shown in Fig. 4 between the ends of each bus and the 50-ohm output connectors. With this design, the VSWR looking into any of the four bus connectors was better than 1.4 over the entire frequency range from 300 to 1000 MHz.

A photograph of a section of the network with the cover removed is shown in Fig. 5. The taps are formed by soldering together in pairs the sharply bent center conductors of adjacent sections and connecting these points to the summing bus with equal 10-kΩ, $\frac{1}{8}$-watt carbon resistors. This procedure, of course, violates the rule enunciated by the synthesis theory [Eq. (18)] for selecting the tap weights, which states that the weights at the "high-frequency" end of the tapped line (i. e., where the taps are close together) should be 0.80 and the weights at the "low-frequency" end should be 1.33, referred to unity weight at the center frequency. Moreover, the magnitude of the impedance of the 10-kΩ resistors varies across the frequency band in a way to make the approximation worse. For example, measurements (by the "width of VSWR minimum" method) show that their impedance varies from 5100 ohms at 300 MHz to 3700 ohms at 500 MHz. Resistors of this type having lower nominal values manifest less impedance variation with frequency (e. g., 1-kΩ resistors are flat across the band) but, because of the stronger coupling that they provide between the delay line and the summing bus, the impedance mismatch at the taps is increased, thereby increasing the magnitude of the intertap reflections. As will be seen below, the frequency dependence of the delay line is such as to compensate approximately for the improper tap weights, yielding an over-all spectrum amplitude flatter than might otherwise be expected.

The intertap reflections are, of course, of serious concern especially when the adjacent tap spacings are nearly equal and when the line attenuation is small. For this reason, the coaxial cable should be selected to have as large an attenuation as permitted by dynamic range considerations, so long as the resulting frequency dependence can be adequately matched by the summing bus design. The attenuation of UT141-AA at 400 MHz is 6.8 dB/100 ft and is a reasonable compromise for the parameters of the experimental system.

Taps formed in the way shown in Fig. 5 are accurately located in position on the coaxial line and are simple to make. This method of construction also places the taps in a convenient physical arrangement for attachment to the summing bus, which is fixed with respect to the large copper ground plane of the network. With the construction described, the individual tap reflections have been observed on a time domain reflectometer having 150-psec rise time; nearly all taps have a reflection coefficient of 0.01 or less, and the worst reflection observed was 0.02.

Other methods of tapping have been considered briefly. If the delay line were to be implemented in stripline, couplers could be designed that satisfy the theoretical tap weighting formula and, moreover, might be more reproducible and amenable to mass production. Hybrid couplers for coaxial cable, however, would be costly in the numbers required. Another possibility would be to mill a small hole through the outer conductor and dielectric and partially into the center conductor. A coupling resistor could then be soldered to the center conductor or, alternatively, a capacitive probe coupler, supported in a holder affixed to the outer conductor, could be partially

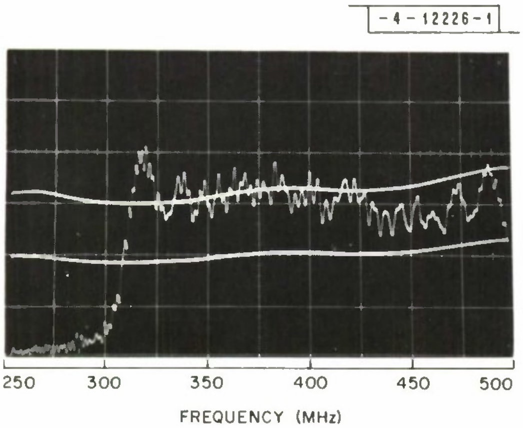


Fig. 6. Network omplitude response for frequency sweep low-to-high (−50 dB ond −53 dB reference levels olso shown).

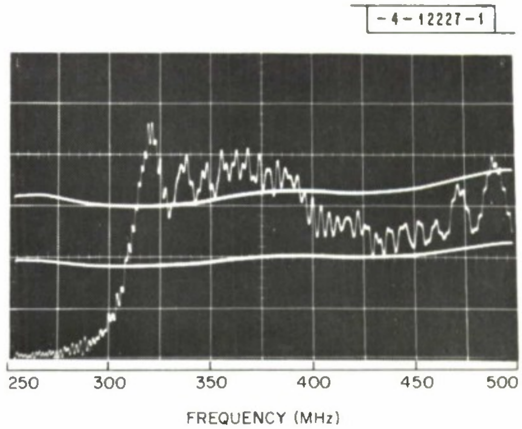


Fig. 7. Network omplitude response for frequency sweep high-to-low (−50 dB ond −53 dB reference levels olso shown).

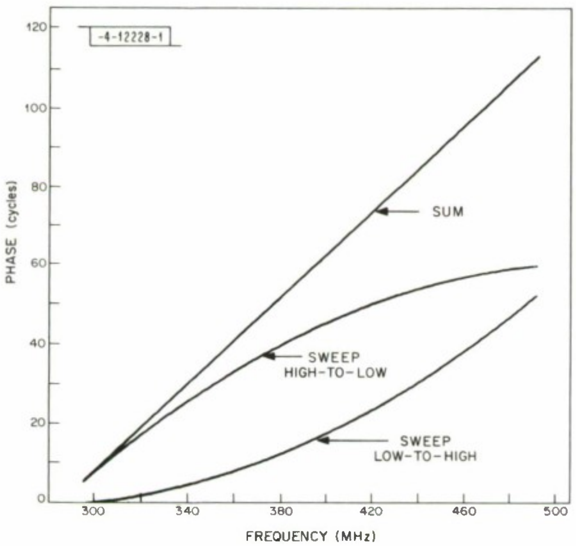


Fig. 8. Measured 300- to 500-MHz phose-frequency chorocteristics for both directions of frequency sweep, ond their sum.

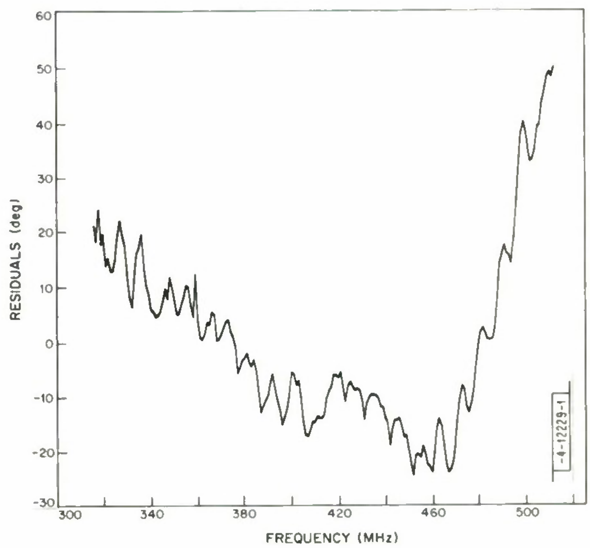


Fig. 9. Residuol 300- to 500-MHz phose deviotion of sum chorocteristic of Fig. 8 from lineor.

inserted into the hole. The reflection coefficients of single "taps" constructed using these alternative tapping methods have been measured. They do not appear to be superior to the taps formed as in Fig. 5.

A. Frequency Domain Measurements

The amplitude and phase characteristics of the tapped-delay-line network were measured by using a swept oscillator. First, in the 300- to 500-MHz band, the network amplitude response from terminal 2 to terminal 4 of Fig. 4 (frequency sweep low-to-high) is shown in Fig. 6, and the amplitude response from terminal 1 to terminal 3 (frequency sweep high-to-low) is shown in Fig. 7. Ideally, these two responses should be identical; they are actually quite closely matched, both in absolute magnitude and in over-all response, a direct indication that the summing bus attenuation closely corresponds to the coax delay-line attenuation.

In comparing these responses to the theoretical response of Fig. 2, it will be noted that these have nearly the ideal characteristic despite the "improper" tap weights and frequency-dependent tapping resistors that were used. The approximately flat response is the product of three factors as follows:

(1) The upward slope that would be obtained theoretically with equal weighting at all taps,

(2) The upward slope caused by the change of impedance of the 10-kΩ "resistors" with frequency,

(3) The downward slope caused by increased coax line (and bus) loss with frequency.

The measured phase-frequency characteristics for the two directions of frequency sweep are shown in Fig. 8. These measurements were made by recording the frequency of every π point as observed on a sampling scope. The two sets of measured data were fed into a computer to calculate the sum and plot all three curves. This sum characteristic should ideally be linear, corresponding to a constant time delay. The deviation in degrees of this sum phase curve from linear was determined by the computer and is shown in Fig. 9. This curve should indicate the residual phase error in the network. It is seen to consist of a large, single-cycle error componcnt, whose main effect will be to broaden the main lobe of the compressed pulse, and a more or less random error of rms value of about 3°, which will limit the sidelobe performance to the vicinity of -30 dB referred to the main lobe.

The measured phase data may also be used to compute the time-delay characteristics for the two directions of frequency sweep and their sum. These characteristics are shown in Fig. 10.

In the 600- to 1000-MHz frequency band (obtained by summing the two halves of the summing bus in phase, as described above), the same set of measurements was made. Figures 11 and 12 show the measured amplitude responses, again with reasonably closely matched characteristics, and Figs. 13 and 14 show the measured phase characteristics, their sum, and the residual phase error.

It will be noted in the amplitude response pictures that the network loss is considerably higher than in the lower frequency band and that there is considerably more variation in the reference levels. The reference level variation is primarily due to gain variations in the two amplifiers, and this variation indicates that they probably contribute partly to reduced network performance.

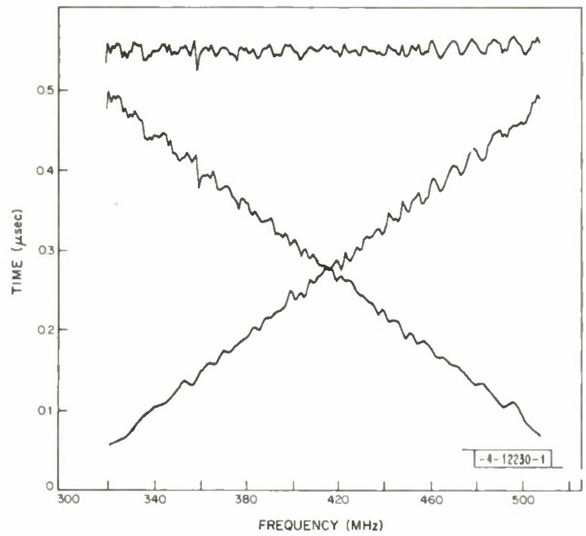


Fig. 10. Network time-deloy chorocteristics (300 to 500 MHz) computed from meosured phose doto, and their sum.

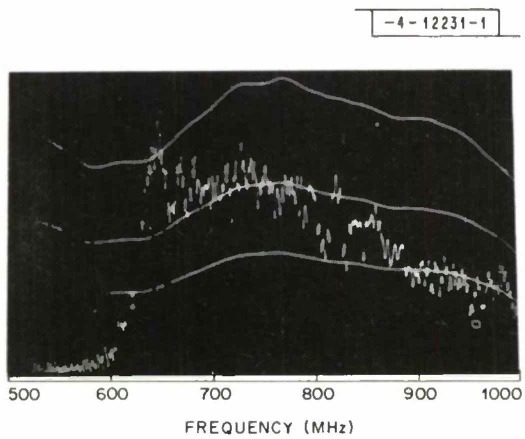


Fig. 11. Network amplitude response for frequency sweep low-to-high (−63 dB, −66 dB, and −69 dB reference levels also shown).

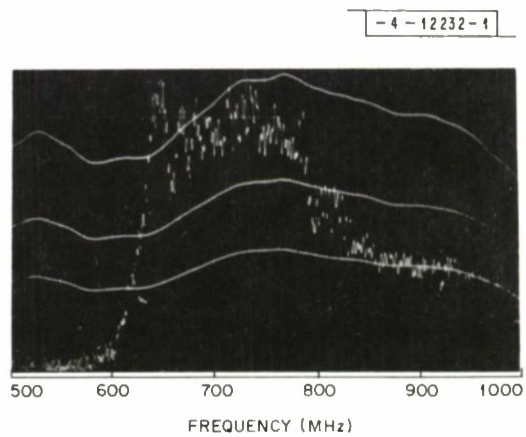
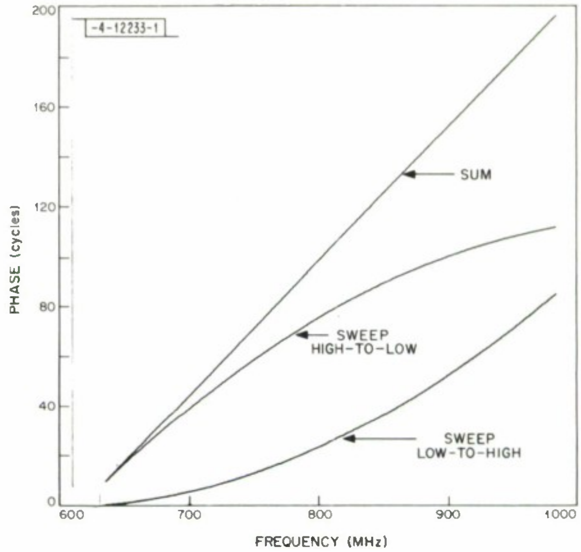


Fig. 12. Network omplitude response for frequency sweep high-to-low (−63 dB, −66 dB, ond −69 dB reference levels olso shown).

Fig. 13. Measured 600- to 1000-MHz phase-frequency characteristics for both directions of frequency sweep, and interpolated sum.



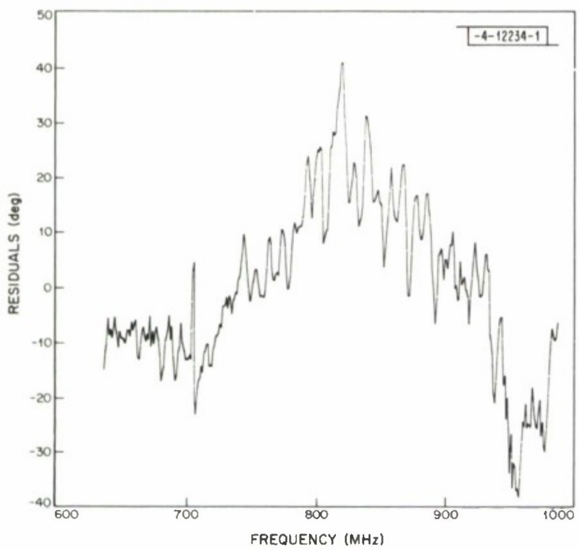


Fig. 14. Residual 600- to 1000-MHz phase deviation of sum characteristic of Fig. 13 from linear.

B. Block Diagram for System Tests

Before presenting the time domain measurements on the experimental tapped-delay-line pulse-compression system, it is desirable to discuss the test setup used for these measurements. A similar configuration would be required if the network were to be incorporated into a radar system, and some features of the test setup reflect this potential application. For example, if pulse-to-pulse phase coherence is required, it is necessary that the generating impulse be accurately phased with respect to the basic CW frequency source of the system. In the test setup, this is accomplished by deriving the impulse from a stable 10-MHz oscillator.

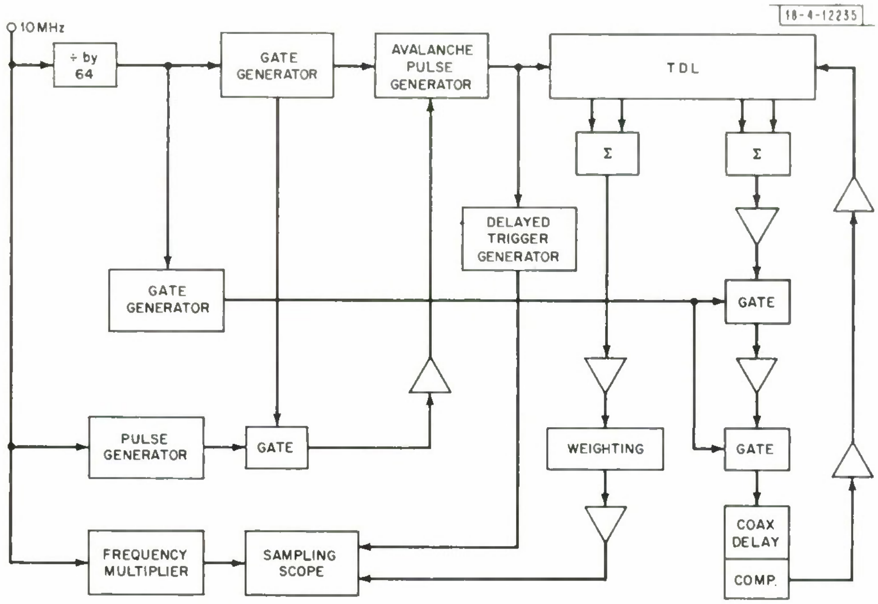


Fig. 15. System test block diagram.

The block diagram of the test setup is shown in Fig. 15. The 10-MHz CW source is connected to a digital clocked divider which produces output pulses spaced 6.4 μsec apart. A 0.2-μsec gate generator is triggered by these pulses, and this gate pulse is connected to an avalanche transistor pulse generator where it makes the avalanche circuit ready to "fire." The 10-MHz source is also connected to an HP10511 spectrum generator which produces a continuous sequence of very narrow sharp pulses with 0.1-μsec spacing. One of these sharp pulses will be passed through the HP10514 gate during the 0.2-μsec gate interval; this sharp pulse is amplified in an Avantek AP25 amplifier and then connected to the base of the avalanche transistor where it acts to "fire" that circuit and thus produce the input generating pulse to the tapped delay line. The process described produces accurate firing to within a few picoseconds, as was shown in subsequent tests.

In order to fully test the tapped delay line, the expanded pulse derived from the tapped delay line must be connected back into it in order to observe the compressed pulse output. Thus, to observe this compressed pulse without interference from the expanded pulse it is necessary to provide an external delay in the test system. This simulates the propagation delay experienced in a radar system. Since this loop includes amplifiers, it is necessary to gate these amplifiers "on" for a shorter time than the external delay in order to avoid loop oscillations.

In the right of Fig. 15 is shown the loop with the amplifiers, gate, and delay described above. The expanded pulse is obtained from an Anzac H81 transformer which, for operation at 400 MHz center frequency, sums the two buses with 180-degree phase. This expanded pulse is amplified in an Avantek AL25, gated through an HP10514, then amplified and gated again. (The gating rejection ratio of a single HP10514 is not sufficient to prevent loop oscillation.) It should be noted that the generator that drives these gates turns them "on" before the start of the expanded pulse and does not turn them "off" until well after the end of this pulse; the expanded pulse from the network is thus undistorted.

The required delay for system test is provided by a length of 0.5-inch alumispline coax cable. Over the frequency band from 300 to 500 MHz, the attenuation of this cable increases from 12.7 to 16.7 dB and, in order to flatten this slope in attenuation, a compensating circuit is added in series. The composite attenuation then varies from 20.7 to 19.3 dB over this same band.

Following the delay, two amplifiers – an Avantek AL25 and an AP25 – are used to drive the tapped-delay-line network with the delayed expanded pulse.

The compressed pulse is obtained from the other end of the buses, again summed with 180-degree phase. This compressed pulse is amplified, passed through a weighting filter, amplified again, and then observed on the sampling oscilloscope. The purpose of the weighting filter, details of which will be given later, is to reduce the time sidelobes of the compressed pulse to an acceptable level.* In order to accurately observe this delayed, compressed pulse, a variable-delay trigger (approximately 1.4-μsec delay) is provided for the oscilloscope. Finally, in order to compare the phase of the compressed pulse with that of the CW reference, a 40-times frequency multiplier was constructed to produce a 400-MHz CW reference.

The test system as described thus far will show the performance of the tapped-delay-line network for expanded and compressed pulses centered at 400 MHz with 200-MHz sweep width. In order to observe performance at the 800-MHz center frequency with a 400-MHz sweep width, three system changes are made as follows:

(1) An Anzac D1 frequency doubler is used to obtain the wider band pulse. Since the frequency doubler is simply a full-wave rectifier, this is practically equivalent to simply adding all the delay-line taps in phase as discussed earlier, at least in the zone of interest centered at 800 MHz. This was done for practical reasons such as the lower available impulse energy and the increased line losses in the 800-MHz zone. In-phase addition of the taps is still used when compressing the expanded pulse so that the modified system still retains the feature of using the same delay line on transmit and receive.

(2) The Anzac transformer at the left in Fig. 15 is changed from an H81 to an H8 which sums the two buses with 0 degrees relative phase. The two AP25 amplifiers are changed to AL50 units, and the weighting network is changed to one centered at 800 MHz.

(3) The 40-times frequency multiplier is changed to 80-times to give an 800-MHz CW reference signal.

C. Time Domain Test Results

The time waveforms which are produced in the system described are shown in this section. First, the input generating pulse from the avalanche transistor circuit is shown in Fig. 16. This pulse is an approximate single cycle of about 600 MHz and is obtained by the use of a highly damped L-C circuit in the avalanche discharge path. The shape of this pulse is not critical, but it must be a large-amplitude pulse in order to overcome the large attenuations of the delay-line and summation resistors.

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* See Ref. 5 for o more complete discussion of weighting.

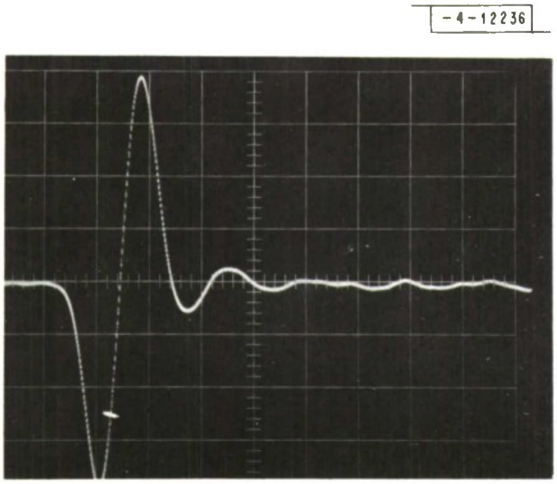

Fig. 16. Input generating pulse. Horizontal: 1.0 nsec/cm, vertical: 5.0 volts/cm.

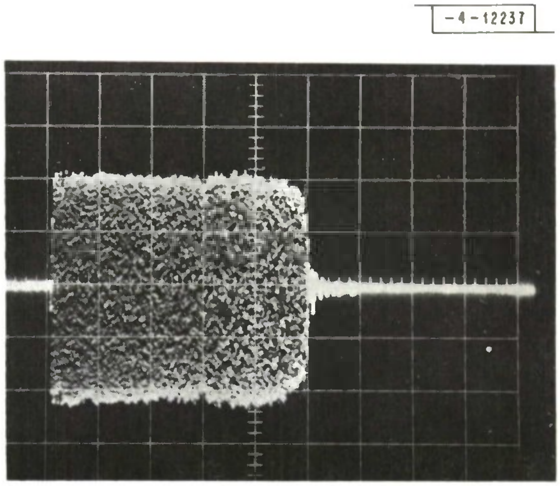

Fig. 17. Expanded pulse; frequency sweep low-to-high from 300 to 500 MHz; 0.1 μsec/cm sweep speed.

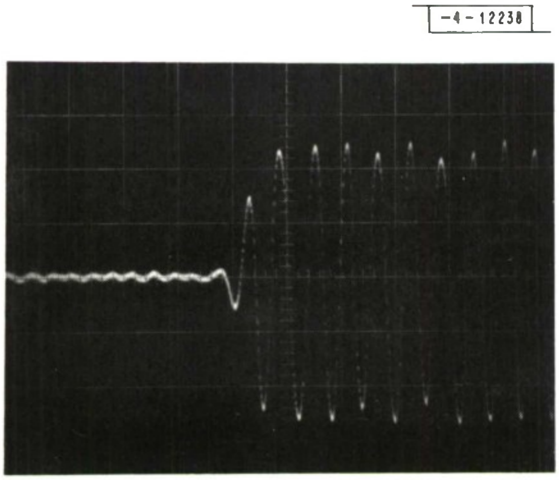

Fig. 18. Start of expanded pulse; frequency sweep low-to-high from 300 to 500 MHz; 5.0 nsec/cm sweep speed.

The expanded pulse is shown in Fig. 17, but the sampling rate of the scope is not nearly high enough to show the details of this waveform at this sweep speed. In Fig. 18 is shown the start of this same expanded pulse with a much faster sweep speed. Both pictures show that this pulse is nearly rectangular with sharply truncated leading and trailing edges; there has been no gate clipping or limiting on either of these waveforms.

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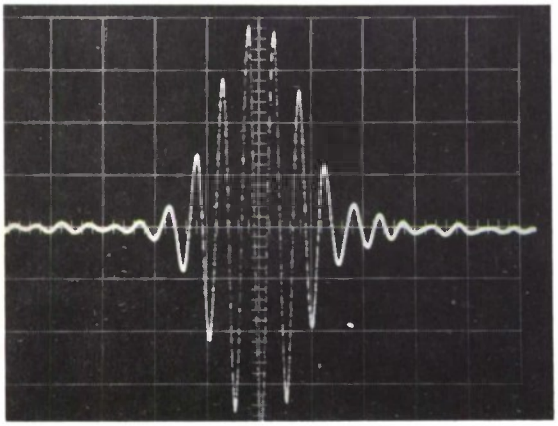

Fig. 19. Compressed pulse for Δf = 200 MHz; 5.0 nsec/cm sweep speed.

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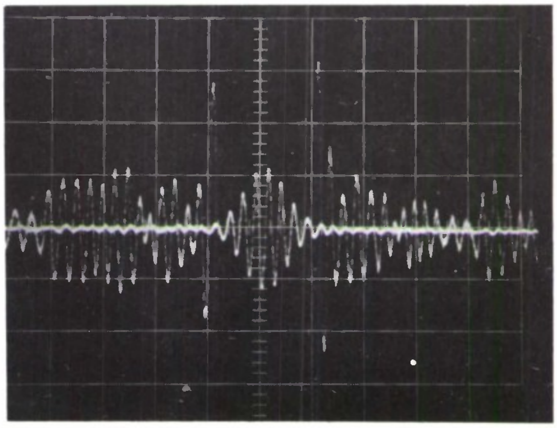

Fig. 20. Compressed pulse with Δf = 200 MHz at 0 dB and at −30 dB superimposed; 10 nsec/cm sweep speed.

The compressed pulse for Δf = 200 MHz is shown in Fig. 19. It will be noted that the pulse is nearly symmetrical and the −6.0 dB width is about 10 nsec. In Fig. 20, this same pulse is shown in double exposure at 0 dB and at −30 dB to show the sidelobe level; it will be noted that the worst sidelobe is 30 dB below peak response.

In Fig. 21 is shown a Lissajous pattern with the central portion of the compressed pulse of Fig. 19 as vertical deflection and with the 400-MHz CW signal as the horizontal deflection. It will be noted that there is no measurable phase jitter between these two signals, thereby proving that a tight phase lock of the generating pulse to the CW source has been obtained.

The compressed pulse for Δf = 400 MHz is shown in Fig. 22. It will be noted that this pulse is not quite symmetrical; the −6.0 dB width is about 5.5 nsec. In Fig. 23, this same pulse is shown in double exposure at 0 dB and at −29 dB, and it may be noted that the worst sidelobe is about 28 dB below peak response.

Finally, in Fig. 24 is shown a Lissajous pattern with the central portion of the compressed pulse of Fig. 22 used for the vertical deflection and with the 800-MHz CW signal as the horizontal deflection. In this Lissajous pattern, the width of the trace may be an indication of phase jitter, but it may instead be caused by a small time jitter in the triggering of the sampling scope; this has not yet been determined.

The compressed pulses shown in this section are, of course, affected by the weighting network which is simply a three-tap coax line with tap spacings of 1/Δf. The weights on these taps were adjusted experimentally for the best achievable reduction of the time sidelobes on the compressed pulse. The frequency responses of the resulting 400-MHz and 800-MHz weighting networks are shown in Figs. 25 and 26, which reveal the familiar "bell" shaped structure usually associated

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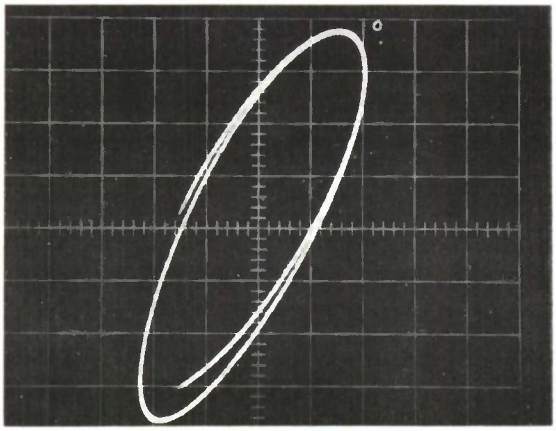

Fig. 21. Lissajous pattern with 400-MHz CW horizontol deflection ond 400-MHz compressed pulse verticol deflection.

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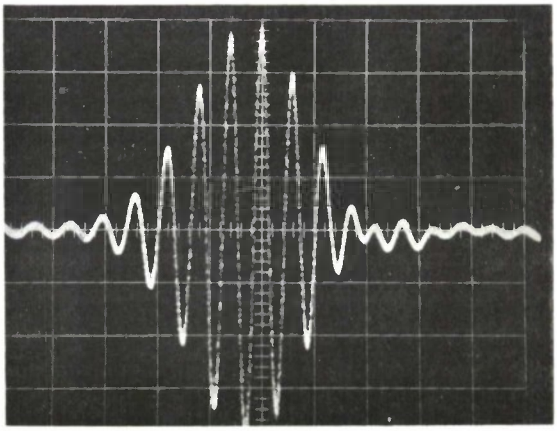

Fig. 22. Compressed pulse for Δf = 400 MHz; 2.0 nsec/cm sweep speed.

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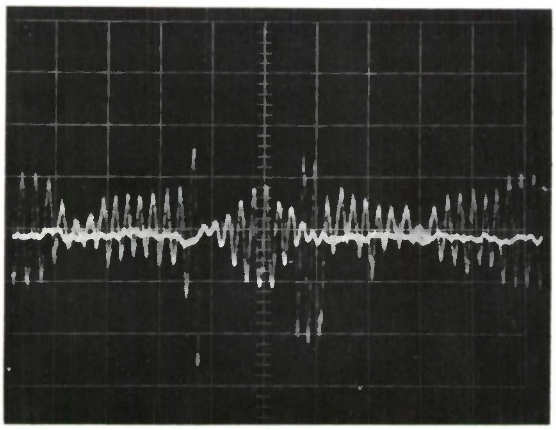

Fig. 23. Compressed pulse with Δf = 400 MHz at 0 dB ond ot -29 dB superimposed; 5.0 nsec/cm sweep speed.

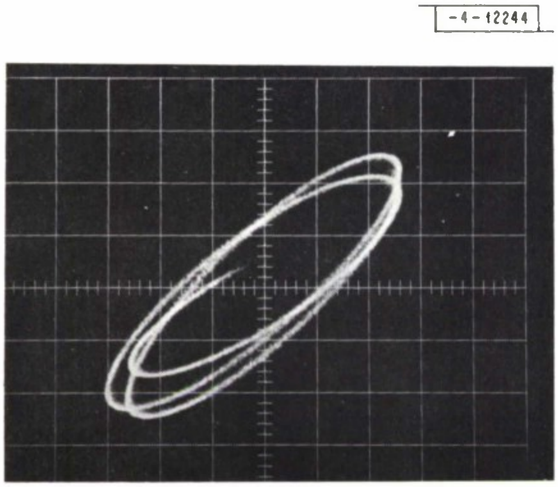

Fig. 24. Lissajous pattern with 800-MHz CW horizontal deflection and 800-MHz compressed pulse vertical deflection.

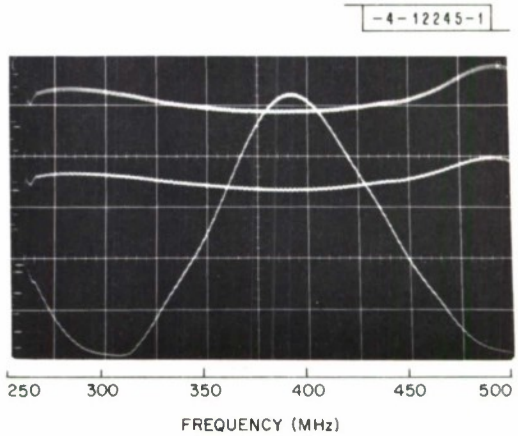

Fig. 25. Weighting network amplitude response for $\Delta f = 200$ MHz (-15 dB and -18 dB reference levels also shown).

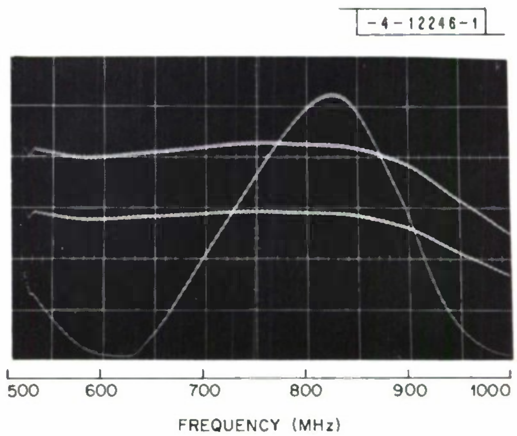

Fig. 26. Weighting network amplitude response for $\Delta f = 400$ MHz (-15 dB and -18 dB reference levels also shown).

with weighting networks. Several different tap spacings were also tried, but no improvement over the original network was obtained. Further sidelobe improvement would no doubt be obtained by using more than three taps in the network, but this has not been attempted to date.

IV. SUMMARY AND CONCLUSIONS

An arbitrary time-limited waveform may be synthesized by impulsing a nondispersive nonuniformly tapped delay line, weighting and summing the tap outputs, and passing the resulting signal through a bandpass zonal filter. Formulas for the location and weights of the taps were derived. It was shown that such a network can be used as a filter matched to the signal that it generates when the frequency dependence of the attenuation of the summing bus matches that of the delay line.

The construction details of an experimental network, using coaxial delay cable, for the generation and compression of a wideband, short-duration linear FM pulse were presented, together with frequency domain and time domain measurements of the performance that was obtained. Some consideration was given to the generation of the driving impulse and the achievement of the phase coherence frequently required in a system application.

In principle, any form of delay line that can be tapped may be used in the construction of networks of this type. Since the time-bandwidth product available with quartz delay lines is considerably greater than that of electrical lines, it is natural to consider tapping quartz lines. Since this can be done in several ways, it is possible that networks of much larger time-bandwidth product might be implemented by this means. An attempt to tap a superconducting niobium-teflon-lead coaxial cable so as to generate the same waveform described in this report has also been made, but the results to date have been poor.

The use of nonlinear FM waveforms for various applications has been discussed in the literature.[6,7] Such waveforms can be quite difficult to generate and receive by using conventional techniques; however, they present no special problems (at least in principle) to the tapped-delay-line technique.

As pointed out in this report, the waveform derived from a tapped-delay-line network may be sharply truncated in time, and a waveform with a rectangular time envelope may be obtained without any limiting or gating. Since most transmitters act as limiters, this network property is of importance, and one that is difficult to obtain with other network types.

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13. ABSTRACT

A nonuniformly tapped delay line is the key component in a device that can be used both to generate an arbitrary waveform and to act as a matched filter for this same waveform. Formulas for calculating the tap delays and weights needed to generate a given waveform are derived. The effects of delay-line attenuation and a technique for compensating for these effects are discussed.

An experimental system embodying these design techniques is described. This system generates and receives a linear FM waveform of 0.5-μsec duration and either 200- or 400-MHz bandwidth. Construction details and the results of many tests run on the system are given.

14. KEY WORDS

matched filter
pulse compression
signal synthesis
tapped delay line