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PROPAGATION OF FIRST-ORDER ELECTROMAGNETIC DISCONTINUITIES IN AN ISOTROPIC MEDIUM

Prepared by

Lehigh University
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Abstract

The propagation of a first-order electromagnetic discontinuity is discussed. Expressions are obtained for the possible velocities of propagation as functions of the field strengths ahead of the surface of discontinuity. Expressions are also obtained for the growth in the magnitude of the discontinuity as the wave progresses.

1. Introduction.

In this paper we shall discuss the propagation of electromagnetic waves in which the fields are continuous with respect to space and time, but where spatial and time deriatives may be discontinuous on some surface. Such discontinuities are called first-order discontinuities. Until recently studies of the propagation of discontinuities of this type have been confined to fluid dynamics (see, for example, Courant and Friedrichs[1]). More recently, using a technique based on the compatibility conditions at a moving surface of discontinuity, Thomas[2] has studied the growth of elastic waves of this type in linear materials. Varley and Cumberbatch[3] have considered the more general problem of a quasilinear, first order, system of hyperbolic equations using a variant of the method for a linear system described in Courant and Hilbert[4]. In this paper we apply the procedure of Varley and Cumberbatch to the study of the propagation of first-order electromagnetic discontinuities in a non-linear centrosymmetric isotropic material, for which the electric displacement field depends only on the electric field and the magnetic induction field depends only on the magnetic intensity field.

We obtain first the secular equation for the propagation of a first-order electromagnetic discontinuity surface. We find that there are, in general, two possible propagation velocities in a specified forward direction and correspondingly two possible polarization directions. Both the velocities and directions of polarization depend on the magnetic induction field and electric displacement field immediately ahead of the discontinuity.

In §4, we obtain an expression for the growth of the magnitude of the discontinuity as a surface of discontinuity propagates. In §§5 and 6 this result is specialized to two cases in which the fields ahead of the surface of discontinuity are uniform and constant. In one of these the magnetic induction field is taken normal to the surface of discontinuity and in the other, the electric displacement field is taken to be normal to this surface.

2. Propagating Surfaces.

In this section, we consider the propagation of an electromagnetic wave for which the magnetic induction field $\underset{\sim}{B}$ and elastic displacement field $\underset{\sim}{D}$ are continuous, while their first derivatives, both with respect to space and time, may be discontinuous across some surface. This singular surface is called a first-order singular surface.

Let $\underset{\sim}{x}$ be the vector position of a generic point on this singular surface with respect to a fixed origin. Let

$$\phi(\underset{\sim}{x},t) = 0 \ , \tag{2.1}$$

for fixed t, be the equation of the singular surface at time t. We define α by

$$\alpha = \phi(\underset{\sim}{x},t) \ , \tag{2.2}$$

and assume that ϕ is a continuous function of $\underset{\sim}{x}$ and of t, which has continuous first derivatives with respect to space and time. We also assume that $\partial\phi/\partial t \neq 0$ in some neighborhood of the propagating singular surface, i.e. $\alpha = 0$.

In view of the definition (2.2) of α, we see that α = constant describes a propagating surface, which is the singular surface if the constant is zero. Also, the variables $\underset{\sim}{x},\alpha$ define a point on such a surface. Let $\underset{\sim}{n}(\underset{\sim}{x},\alpha)$ denote the unit normal at the point $\underset{\sim}{x}$ to the surface α = constant, drawn in the direction of propagation. Let $V(>0)$ denote the speed of propagation of the surface. From the definition of the speed V of a moving surface at a point as the speed with which the surface moves normal to itself at that point, we have

$$V = \underset{\sim}{n} \cdot \frac{d\underset{\sim}{x}}{dt} \ , \tag{2.3}$$

where

$$\underset{\sim}{n} = \underset{\sim}{\nabla}\phi/(\underset{\sim}{\nabla}\phi\cdot\underset{\sim}{\nabla}\phi)^{1/2} \quad . \tag{2.4}$$

Now, from (2.2), we have for the moving surface α = constant,

$$\underset{\sim}{\nabla}\phi \cdot \frac{d\underset{\sim}{x}}{dt} + \frac{\partial\phi}{\partial t} = 0 \quad . \tag{2.5}$$

Multiplying (2.5) throughout by $\underset{\sim}{n}$ and using (2.3) and (2.4), we obtain

$$\frac{\underset{\sim}{n}}{V} = - \frac{\underset{\sim}{\nabla}\phi}{\partial\phi/\partial t} \quad . \tag{2.6}$$

Now, any function f, say, of $\underset{\sim}{x}$ and t may also be regarded as a function $\bar{f}$, say, of $\underset{\sim}{x}$ and α, where α is defined by (2.2). We then have

$$f(\underset{\sim}{x},t) = \bar{f}(\underset{\sim}{x},\alpha) \quad . \tag{2.7}$$

Whence

$$\frac{\partial f}{\partial t} = \frac{\partial\bar{f}}{\partial\alpha}\frac{\partial\alpha}{\partial t} = \frac{\partial\bar{f}}{\partial\alpha}\frac{\partial\phi}{\partial t} \quad . \tag{2.8}$$

We define the operator $\underset{\sim}{\bar{\nabla}}$ as the spatial gradient, holding α constant. It follows, from (2.7), that

$$\underset{\sim}{\nabla}f = \underset{\sim}{\nabla}\bar{f} = \underset{\sim}{\bar{\nabla}}\bar{f} + \frac{\partial\bar{f}}{\partial\alpha}\underset{\sim}{\nabla}\phi \quad . \tag{2.9}$$

With (2.6) and (2.8), this yields

$$\underset{\sim}{\nabla} f = \overline{\underset{\sim}{\nabla}}\bar{f} - \frac{\underset{\sim}{n}}{V}\frac{\partial \bar{f}}{\partial \alpha}\frac{\partial \phi}{\partial t} = \overline{\underset{\sim}{\nabla}}\bar{f} - \frac{\underset{\sim}{n}}{V}\frac{\partial f}{\partial t} \quad . \tag{2.10}$$

Thus, we may take the operator $\overline{\underset{\sim}{\nabla}}$ as

$$\overline{\underset{\sim}{\nabla}} = \underset{\sim}{\nabla} + \frac{\underset{\sim}{n}}{V}\frac{\partial}{\partial t} \quad . \tag{2.11}$$

Here we may point out that the operator $\overline{\underset{\sim}{\nabla}}$ is a surface operator on the surface $\phi = 0$ since by the choice of the new set of coordinates, the variables involved in the differentiation correspond to a displacement on the surface[4].

3. The Secular Equation.

In the absence of free charges, Maxwell's equations may be written as

$$
\begin{aligned}
\boldsymbol{\nabla}\times\mathbf{E} + \partial\mathbf{B}/\partial t &= 0 \quad , \\
\boldsymbol{\nabla}\times\mathbf{H} - \partial\mathbf{D}/\partial t &= 0 \quad ,
\end{aligned}
\tag{3.1}
$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic intensity fields respectively.

We shall consider that the medium in which the wave is propagating is isotropic, centrosymmetric and non-dissipative and that $\mathbf{E}$ is determined only by $\mathbf{D}$ and $\mathbf{H}$ is determined only by $\mathbf{B}$. It follows that the constitutive equations take the forms

$$\mathbf{E} = \varepsilon(\mathbf{D}\cdot\mathbf{D})\mathbf{D} \qquad \text{and} \qquad \mathbf{H} = \mu(\mathbf{B}\cdot\mathbf{B})\mathbf{B} \quad , \tag{3.2}$$

where ε and μ are scalar functions of the indicated arguments.

Introducing (3.2) into (3.1), we obtain

$$
\begin{aligned}
&\boldsymbol{\nabla}\varepsilon\times\mathbf{D} + \varepsilon\boldsymbol{\nabla}\times\mathbf{D} + \frac{\partial\mathbf{B}}{\partial t} = \varepsilon'\boldsymbol{\nabla}(\mathbf{D}\cdot\mathbf{D})\times\mathbf{D} + \varepsilon\boldsymbol{\nabla}\times\mathbf{D} + \frac{\partial\mathbf{B}}{\partial t} = 0 \\
&\text{and} \\
&\boldsymbol{\nabla}\mu\times\mathbf{B} + \mu\boldsymbol{\nabla}\times\mathbf{B} - \frac{\partial\mathbf{D}}{\partial t} = \mu'\boldsymbol{\nabla}(\mathbf{B}\cdot\mathbf{B})\times\mathbf{B} + \mu\boldsymbol{\nabla}\times\mathbf{B} - \frac{\partial\mathbf{D}}{\partial t} = 0 \quad ,
\end{aligned}
\tag{3.3}
$$

where $\varepsilon' = \partial\varepsilon/\partial(\mathbf{D}\cdot\mathbf{D})$ and $\mu' = \partial\mu/\partial(\mathbf{B}\cdot\mathbf{B})$.

We bear in mind the known vector identity

$$\boldsymbol{\nabla}(\mathbf{D}\cdot\mathbf{D}) = 2\ [(\mathbf{D}\cdot\boldsymbol{\nabla})\mathbf{D} + \mathbf{D}\times(\boldsymbol{\nabla}\times\mathbf{D})] \quad . \tag{3.4}$$

Substituting from (3.4) and (2.11) in $(3.3)_1$, we obtain

$$\varepsilon \underset{\sim}{n} \times \frac{\partial \underset{\sim}{D}}{\partial t} + 2\varepsilon'(\underset{\sim}{D}\cdot\frac{\partial \underset{\sim}{D}}{\partial t})(\underset{\sim}{n}\times\underset{\sim}{D}) - V\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$= 2\varepsilon' V[(\underset{\sim}{D}\cdot\underset{\sim}{\overline{\nabla}})\underset{\sim}{D}\times\underset{\sim}{D} + \{\underset{\sim}{D}\times(\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{D})\}\times\underset{\sim}{D}] + \varepsilon V\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{D} \quad . \tag{3.5}$$

Similarly, from $(3.3)_2$ we obtain

$$\mu \underset{\sim}{n} \times \frac{\partial \underset{\sim}{B}}{\partial t} + 2\mu'(\underset{\sim}{B}\cdot\frac{\partial \underset{\sim}{B}}{\partial t})(\underset{\sim}{n}\times\underset{\sim}{B}) + V\frac{\partial \underset{\sim}{D}}{\partial t}$$

$$= 2\mu' V[(\underset{\sim}{B}\cdot\underset{\sim}{\overline{\nabla}})\underset{\sim}{B}\times\underset{\sim}{B} + \{\underset{\sim}{B}\times(\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{B})\}\times\underset{\sim}{B}] + \mu V\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{B} \quad . \tag{3.6}$$

We now introduce the assumption that $\underset{\sim}{B}$ and $\underset{\sim}{D}$ are continuous at the moving surface $\phi = 0$. It follows that their tangential derivatives $\underset{\sim}{\overline{\nabla}}\underset{\sim}{B}$, $\underset{\sim}{\overline{\nabla}}\underset{\sim}{D}$, $\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{B}$ and $\underset{\sim}{\overline{\nabla}}\times\underset{\sim}{D}$ are also continuous across the surface.

We denote by $\underset{\sim}{b}$ and $\underset{\sim}{d}$ the jumps in $\partial\underset{\sim}{B}/\partial t$ and $\partial\underset{\sim}{D}/\partial t$ as we cross the surface $\phi = 0$ in the direction of $\underset{\sim}{n}$; thus

$$\underset{\sim}{b} = [\partial\underset{\sim}{B}/\partial t] \quad , \quad \underset{\sim}{d} = [\partial\underset{\sim}{D}/\partial t] \quad . \tag{3.7}$$

With (3.7) we obtain, from (3.5) and (3.6), on $\phi = 0$

$$\varepsilon\underset{\sim}{n}\times\underset{\sim}{d} + 2\varepsilon'(\underset{\sim}{D}\cdot\underset{\sim}{d})(\underset{\sim}{n}\times\underset{\sim}{D}) - V\underset{\sim}{b} = 0$$

and (3.8)

$$\mu\underset{\sim}{n}\times\underset{\sim}{b} + 2\mu'(\underset{\sim}{B}\cdot\underset{\sim}{b})(\underset{\sim}{n}\times\underset{\sim}{B}) + V\underset{\sim}{d} = 0 \quad .$$

From (3.8), we obtain

$$\underset{\sim}{n}\cdot\underset{\sim}{b} = 0 \quad \text{and} \quad \underset{\sim}{n}\cdot\underset{\sim}{d} = 0 \quad . \tag{3.9}$$

Thus, the jumps $\underset{\sim}{b}$ and $\underset{\sim}{d}$ are tangential to the surface $\phi = 0$.

Now, eliminating $\underset{\sim}{d}$ from (3.8), we obtain on $\phi = 0$

$$\begin{aligned}&\mu\varepsilon\underset{\sim}{n}\times(\underset{\sim}{n}\times\underset{\sim}{b}) + 2\mu'\varepsilon(\underset{\sim}{B}\cdot\underset{\sim}{b})\{\underset{\sim}{n}\times(\underset{\sim}{n}\times\underset{\sim}{B})\}\\ &\quad + 2\varepsilon'(\underset{\sim}{n}\times\underset{\sim}{D})[\mu\underset{\sim}{D}\cdot(\underset{\sim}{n}\times\underset{\sim}{b}) + 2\mu'(\underset{\sim}{B}\cdot\underset{\sim}{b})\{\underset{\sim}{D}\cdot(\underset{\sim}{n}\times\underset{\sim}{B})\}] + V^2\underset{\sim}{b} = 0 \quad .\end{aligned} \tag{3.10}$$

We introduce the notation

$$\underset{\sim}{B} = \underset{\sim}{B}_t + \underset{\sim}{B}_n \quad , \quad \underset{\sim}{D} = \underset{\sim}{D}_t + \underset{\sim}{D}_n \quad , \tag{3.11}$$

where $\underset{\sim}{B}_t$ and $\underset{\sim}{D}_t$ are tangential to the surface $\phi = 0$ and $\underset{\sim}{B}_n$ and $\underset{\sim}{D}_n$ are normal to it. We also write

$$\underset{\sim}{J} = \underset{\sim}{n}\times\underset{\sim}{D}_t \quad . \tag{3.12}$$

Using (3.11), (3.12) and $(3.9)_1$ in (3.10), we obtain

$$\begin{aligned}&(V^2-\mu\varepsilon)\underset{\sim}{b} - 2\mu'\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{b})\underset{\sim}{B}_t\\ &\quad -2\varepsilon'\underset{\sim}{J}[\mu(\underset{\sim}{b}\cdot\underset{\sim}{J}) + 2\mu'(\underset{\sim}{B}_t\cdot\underset{\sim}{b})(\underset{\sim}{B}_t\cdot\underset{\sim}{J})] = 0 \quad .\end{aligned} \tag{3.13}$$

Forming the dot product of (3.13) with $\underset{\sim}{B}_t$, we obtain

$$\begin{aligned}&[(V^2-\mu\varepsilon) - 2\mu'\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) - 4\mu'\varepsilon'(\underset{\sim}{B}_t\cdot\underset{\sim}{J})^2](\underset{\sim}{B}_t\cdot\underset{\sim}{b})\\ &\qquad = 2\mu\varepsilon'(\underset{\sim}{b}\cdot\underset{\sim}{J})(\underset{\sim}{B}_t\cdot\underset{\sim}{J}) \quad .\end{aligned} \tag{3.14}$$

Forming the dot product of (3.13) with $\underset{\sim}{J}$, we obtain

$$[-2\mu'\varepsilon(\underset{\sim}{J}\cdot\underset{\sim}{B}_t) - 4\mu'\varepsilon'(\underset{\sim}{B}_t\cdot\underset{\sim}{J})(\underset{\sim}{J}\cdot\underset{\sim}{J})](\underset{\sim}{B}_t\cdot\underset{\sim}{b})$$

$$= [(\mu\varepsilon - V^2) + 2\mu\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{J})](\underset{\sim}{b}\cdot\underset{\sim}{J}) \quad . \tag{3.15}$$

Eliminating $\underset{\sim}{b}$ from (3.14) and (3.15) and using the notation

$$x = V^2 - \mu\varepsilon \quad , \tag{3.16}$$

we obtain

$$x^2 - x\,[2\mu'\{\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) + 2\varepsilon'(\underset{\sim}{B}_t\cdot\underset{\sim}{J})^2\} + 2\mu\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{J})]$$

$$+ 4\mu\mu'\varepsilon\varepsilon'\{(\underset{\sim}{J}\cdot\underset{\sim}{J})(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) - (\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2\} = 0 \quad . \tag{3.17}$$

From (3.16) and (3.17), we obtain the two solutions for V^2:

$$V^2 = \mu\varepsilon + \mu'[\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) + 2\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2] + \mu\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{J})$$

$$\pm\{[\mu'\{\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) + 2\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2\} + \mu\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{J})]^2$$

$$- 4\mu\mu'\varepsilon\varepsilon'[(\underset{\sim}{J}\cdot\underset{\sim}{J})(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) - (\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2]^{1/2} \quad . \tag{3.18}$$

V^2 is real and positive if and only if

$$[\mu'\{\varepsilon(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) + 2\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2\} + \mu\varepsilon'(\underset{\sim}{J}\cdot\underset{\sim}{J})^2]^2$$

$$\geq 4\mu\mu'\varepsilon\varepsilon'[(\underset{\sim}{J}\cdot\underset{\sim}{J})(\underset{\sim}{B}_t\cdot\underset{\sim}{B}_t) - (\underset{\sim}{J}\cdot\underset{\sim}{B}_t)^2] \tag{3.19}$$

and $\mu\varepsilon > 0$. In this case we see that for given $\underset{\sim}{n}$, $\underset{\sim}{B}_t$ and $\underset{\sim}{D}_t$, there are two possible real speeds of propagation in the forward direction. Propagation in the opposite direction with equal speeds is also possible.

Provided that V^2 is a simple root of the equation (3.17), i.e. the greater than sign in (3.19) is valid, we can determine the direction of polarization of $\underset{\sim}{b}$ from equation (3.10). Then the direction of polarization of $\underset{\sim}{d}$ can be determined from equation $(3.8)_2$.

4. Growth Equation.

In this section we shall derive a differential equation governing the change in magnitude of the discontinuity. In order to do this it is more convenient to refer all equations to a rectangular cartesian reference system and to use indicial notation. With this notation, we obtain, by differentiating equations (3.3) with respect to t,

$$
\begin{aligned}
&e_{ijk}\{P_{k\ell}\frac{\partial}{\partial t}\left(\frac{\partial D_\ell}{\partial x_j}\right) + Q_{nk\ell}\frac{\partial D_n}{\partial t}\frac{\partial D_\ell}{\partial x_j}\} + \frac{\partial^2 B_i}{\partial t^2} = 0 \quad ,\\
&e_{ijk}\, R_{k\ell}\frac{\partial}{\partial t}\left(\frac{\partial B_\ell}{\partial x_j}\right) + S_{nk\ell}\frac{\partial B_n}{\partial t}\frac{\partial B_\ell}{\partial x_j}\} - \frac{\partial^2 D_i}{\partial t^2} = 0 \quad ,
\end{aligned}
\tag{4.1}
$$

where

$$
\begin{aligned}
P_{k\ell} &= \varepsilon\delta_{k\ell} + 2\varepsilon' D_k D_\ell \quad ,\\
R_{k\ell} &= \mu\delta_{k\ell} + 2\mu' B_k B_\ell \quad ,\\
Q_{nk\ell} &= \partial P_{k\ell}/\partial D_n = 2\varepsilon' D_n\delta_{k\ell} + 4\varepsilon'' D_n D_R D_\ell + 2\varepsilon' D_k\delta_{\ell n} + 2\varepsilon' D_\ell\delta_{kn} \quad ,\\
S_{nk\ell} &= \partial R_{k\ell}/\partial B_n = 2\mu' B_n\delta_{k\ell} + 4\mu'' B_n B_k B_\ell + 2\mu' B_k\delta_{\ell n} + 2\mu' B_\ell\delta_{kn} \quad ,
\end{aligned}
\tag{4.2}
$$

and e_{ijk} denotes the alternating symbol.

We now change the independent variables from x_i,t to x_i,α. In indicial notation, the relation (2.10) may be expressed as

$$\frac{\partial}{\partial x_j} = \left(\frac{\partial}{\partial x_j}\right)_\alpha - \frac{n_j}{V}\frac{\partial}{\partial t} \quad , \tag{4.3}$$

whence

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial t}\right) = \left(\frac{\partial}{\partial x_j}\right)_\alpha \frac{\partial}{\partial t} - \frac{n_j}{V}\frac{\partial^2}{\partial t^2} \quad . \tag{4.4}$$

Introducing (4.3) and (4.4) into (4.1), we obtain

$$\begin{aligned} e_{ijk}P_{k\ell}n_j \frac{\partial^2 D_\ell}{\partial t^2} - V\frac{\partial^2 B_i}{\partial t^2} &= Ve_{ijk}\{P_{k\ell}\left(\frac{\partial}{\partial x_j}\right)_\alpha \frac{\partial D_\ell}{\partial t} \\ &+ Q_{nk\ell}\frac{\partial D_n}{\partial t}\left(\left(\frac{\partial D_\ell}{\partial x_j}\right)_\alpha - \frac{n_j}{V}\frac{\partial D_\ell}{\partial t}\right)\} \end{aligned}$$

and

$$\begin{aligned} e_{ijk}R_{k\ell}n_j \frac{\partial^2 B_\ell}{\partial t^2} + V\frac{\partial^2 D_i}{\partial t^2} &= Ve_{ijk}\{R_{k\ell}\left(\frac{\partial}{\partial x_j}\right)_\alpha \frac{\partial B_\ell}{\partial t} \\ &+ S_{nk\ell}\frac{\partial B_n}{\partial t}\left(\left(\frac{\partial B_\ell}{\partial x_j}\right)_\alpha - \frac{n_j}{V}\frac{\partial B_\ell}{\partial t}\right)\} \quad . \end{aligned} \tag{4.5}$$

Eliminating $V\,\partial^2 D_i/\partial t^2$ from (4.5), we obtain

$$\begin{aligned} -(a_{is}+V^2\delta_{is})\frac{\partial^2 B_s}{\partial t^2} &= e_{ijk}\{V^2P_{k\ell}\left(\frac{\partial}{\partial x_j}\right)_\alpha \left(\frac{\partial D_\ell}{\partial t}\right) \\ &+ V^2Q_{nk\ell}\frac{\partial D_n}{\partial t}\left(\left(\frac{\partial D_\ell}{\partial x_j}\right)_\alpha - \frac{n_j}{V}\frac{\partial D_\ell}{\partial t}\right) \end{aligned} \tag{4.6}$$

$$-Ve_{\ell mn}P_{k\ell}R_{ns}n_j\left(\frac{\partial}{\partial x_m}\right)_\alpha\left(\frac{\partial B_s}{\partial t}\right)$$

$$-Ve_{\ell mn}P_{k\ell}S_{pns}n_j\left(\left(\frac{\partial B_s}{\partial x_m}\right)_\alpha \frac{\partial B_p}{\partial t} - \frac{n_m}{V}\frac{\partial B_s}{\partial t}\frac{\partial B_p}{\partial t}\right)\} \quad ,$$

where

$$a_{is} = e_{ijk}e_{\ell mn}P_{k\ell}R_{ns}n_jn_m \quad . \tag{4.7}$$

Using square brackets to denote the jump in a quantity across the surface of discontinuity, we have

$$\left[\frac{\partial D_n}{\partial t}\frac{\partial D_\ell}{\partial t}\right] = \left[\frac{\partial D_n}{\partial t}\right]\left[\frac{\partial D_\ell}{\partial t}\right] + \left[\frac{\partial D_n}{\partial t}\right]\left(\frac{\partial D_\ell}{\partial t}\right)_o + \left[\frac{\partial D_\ell}{\partial t}\right]\left(\frac{\partial D_n}{\partial t}\right)_o \quad , \tag{4.8}$$

where $(\partial D_n/\partial t)_o$ denotes the value of $\partial D_n/\partial t$ immediately in front of the wave. Also, we note that $P_{k\ell}$, $Q_{nk\ell}$, $R_{k\ell}$ and $S_{nk\ell}$ are continuous across the surface of discontinuity.

Using these results, we obtain from (4.6), with the notation $b_i = [\partial B_i/\partial t]$, $d_i = [\partial D_i/\partial t]$,

$$-(a_{is}+V^2\delta_{is})\left[\frac{\partial^2 B_s}{\partial t^2}\right] = e_{ijk}\{V^2P_{k\ell}\left(\frac{\partial d_\ell}{\partial x_j}\right)_\alpha$$

$$+ V^2Q_{nk\ell}\{\left(\frac{\partial D_\ell}{\partial x_j}\right)_\alpha d_n - \frac{n_j}{V}\Big(d_nd_\ell$$

$$+ \left(\frac{\partial D_\ell}{\partial t}\right)_o d_n + \left(\frac{\partial D_n}{\partial t}\right)_o d_\ell\Big)\}$$

$$- Ve_{\ell mn}P_{k\ell}R_{ns}n_j\left(\frac{\partial b_s}{\partial x_m}\right)_\alpha$$

$$- Ve_{\ell mn}P_{k\ell}S_{pns}n_j\{(\frac{\partial B_s}{\partial x_m})_\alpha b_p - \frac{n_m}{V}(b_s b_p$$

$$+ (\frac{\partial B_p}{\partial t})_o b_s + (\frac{\partial B_s}{\partial t})_o b_p)\}\} \quad . \tag{4.9}$$

We now introduce the notation

$$b_s = \sigma r_s \quad , \tag{4.10}$$

where r_s is a unit vector in the direction of b_s. With (4.10) and (4.2), equation $(3.8)_2$ becomes

$$Vd_\ell = -R_{ns}e_{\ell mn}n_m b_s = -R_{ns}e_{\ell mn}n_m r_s \sigma \quad . \tag{4.11}$$

Using this result to substitute for Vd_ℓ in (4.9), we obtain,

$$-(a_{is}+V^2\delta_{is})[\frac{\partial^2 B_s}{\partial t^2}] = L_i + M_i\sigma + N_i\sigma^2 \quad , \tag{4.12}$$

where

$$L_i = -Ve_{ijk}e_{\ell mn}P_{k\ell}R_{ns}r_s\{n_m(\frac{\partial \sigma}{\partial x_j})_\alpha + n_i(\frac{\partial \sigma}{\partial x_m})_\alpha\} \quad ,$$

$$M_i = -V^2 e_{ijk}e_{\ell mn}P_{k\ell}(\frac{\partial}{\partial x_j})_\alpha (\frac{1}{V} R_{ns}n_m r_s)$$

$$+e_{ijk}Q_{nk\ell}R_{tp}n_s r_p\{-e_{nst}V(\frac{\partial D_\ell}{\partial x_j})_\alpha$$

$$+e_{nst}n_j(\frac{\partial D_\ell}{\partial t})_o + e_{\ell st}n_j(\frac{\partial D_n}{\partial t})_o\} \tag{4.13}$$

$$-Ve_{ijk}e_{\ell mn}P_{k\ell}n_j\{R_{ns}\left(\frac{\partial r_s}{\partial x_m}\right)_\alpha$$

$$+S_{pns}\left[\left(\frac{\partial B_s}{\partial x_m}\right)_\alpha r_p - \frac{n_m}{V} r_s\left(\frac{\partial B_p}{\partial t}\right)_o - \frac{n_m}{V} r_p\left(\frac{\partial B_s}{\partial t}\right)_o\right]\} ,$$

$$N_i = -V^{-1}e_{ijk}e_{nst}e_{\ell ab}Q_{nk\ell}R_{tp}R_{bc}n_jn_an_sr_pr_c$$

$$+e_{ijk}e_{\ell mn}P_{k\ell}S_{pns}n_jn_mr_sr_p \quad .$$

In our present notation $(3.8)_1$ may be written

$$Vb_i = e_{ijk}P_{k\ell}n_jd_\ell \quad . \tag{4.14}$$

With (4.11), we obtain

$$(a_{is}+V^2\delta_{is})b_s = 0 \quad , \tag{4.15}$$

where a_{is} is given by (4.7). We define a unit vector ℓ_i by

$$\ell_i(a_{is}+V^2\delta_{is}) = 0 \quad . \tag{4.16}$$

Multiplying (4.12) throughout by ℓ_i and using (4.16), we obtain

$$L + M\sigma + N\sigma^2 = 0 \quad , \tag{4.17}$$

where L, M and N are defined by

$$(L,M,N) = \ell_i(L_i,M_i,N_i) \quad . \tag{4.18}$$

Introducing (4.7) and (4.10) into (4.15) and differentiating with respect to n_p, we obtain

$$\{e_{ijk}e_{\ell mn}P_{k\ell}R_{ns}(\delta_{pj}n_m+\delta_{pm}n_j) + 2V\frac{\partial V}{\partial n_p}\delta_{is}\}r_s$$
$$+(e_{ijk}e_{\ell mn}P_{k\ell}R_{ns}n_jn_m+V^2\delta_{is})\frac{\partial r_s}{\partial n_p} = 0 \quad . \tag{4.19}$$

Multiplying throughout by ℓ_i and bearing in mind (4.16) and (4.7), we obtain

$$e_{ijk}e_{\ell mn}P_{k\ell}R_{ns}(\delta_{pj}n_m+\delta_{pm}n_j)\ell_i r_s = -2V\frac{\partial V}{\partial n_p}\ell_i r_i \quad . \tag{4.20}$$

Since a_{is}, defined by (4.7), is homogeneous of second degree in n_i and V is determined by the discriminant of equation (4.15), it follows that V is homogeneous of first degree in n_i. Therefore,

$$V = n_p\ \partial V/\partial n_p \quad . \tag{4.21}$$

Now, suppose a point x_p depends on t in such a manner that it always lies on the moving surface of discontinuity. Then its velocity is dx_p/dt. The velocity of the surface normal to itself is V, which is therefore given by

$$V = n_p\ dx_p/dt \quad . \tag{4.22}$$

From (4.21) and (4.22), we obtain

$$n_p\ \partial V/\partial n_p = n_p dx_p/dt \quad . \tag{4.23}$$

A possible choice for the dependence of x_p on t which satisfies the condition that x_p shall always be on the surface of discontinuity is that for which

$$dx_p/dt = \partial V/\partial n_p \quad . \tag{4.24}$$

We shall make this choice* and calculate the growth in the magnitude of the singularity as we move along the locus of x_p.

From $(4.13)_1$, (4.18), (4.20) and (4.24), we obtain

$$L = 2V^2 \frac{\partial V}{\partial n_p}\left(\frac{\partial \sigma}{\partial x_p}\right)_\alpha \ell_i r_i = 2V^2 \frac{d\sigma}{dt} \ell_i r_i \quad . \tag{4.25}$$

We introduce (4.25) into equation (4.17) and make the transformation

$$\sigma = 1/u \quad . \tag{4.26}$$

We obtain

$$\frac{du}{dt} = \overline{M}u + \overline{N} \quad , \tag{4.27}$$

where

$$\overline{M} = M/2V^2 \ell_i r_i \quad , \quad \overline{N} = N/2V^2 \ell_i r_i \quad . \tag{4.28}$$

This equation has the general solution

$$u = \frac{1}{\sigma} = e^{\int \overline{M} dt} \left\{ C + \int \overline{N}\, e^{-\int \overline{M} dt} dt \right\} \quad , \tag{4.29}$$

---

*In this case the locus of x_p is, in fact, a bicharacteristic of the secular equation $|a_{is} + V^2 \delta_{is}| = 0$.

where C is a constant of integration.

We see that if the fields $\underset{\sim}{B}$ and $\underset{\sim}{D}$ are known before passage of the surface of discontinuity, the velocity of such a surface, propagating in a given direction, can be determined from (3.18). $P_{k\ell}$ and $R_{k\ell}$ defined by (4.2) can be determined and hence a_{is} can be determined. r_i can then be calculated from (4.15) and (4.10), and ℓ_i from (4.16). Then, $\overline{M}$ and $\overline{N}$ can be determined from (4.28), (4.18) and (4.13).

5. Special case: magnetic induction field ahead of wave in direction of propagation.

We now assume that the fields ahead of the moving surface of discontinuity are uniform. This implies that the speeds of propagation V given by (3.18) are constant. It then follows, from (4.15), that $\underset{\sim}{b}$ has a constant orientation (i.e. $\underset{\sim}{r}$ is a constant) and, from (4.16), that the unit vector $\underset{\sim}{\ell}$ is constant. We see, from (4.13), that in this case $M_i = 0$ and N_i is constant. Then, from (4.18), $M = 0$ and N is constant. We then obtain from (4.27) or (4.29)

$$u = \frac{1}{\sigma} = \overline{N}t + \frac{1}{\sigma_o} \quad , \tag{5.1}$$

where σ_o is the value of σ when $t = 0$. The jump magnitude σ increases or decreases accordingly as $\overline{N}$ is negative or positive.

In the particular case when the direction of propagation is in the 1-direction and the applied magnetic induction field into which the wave propagates is also in the 1-direction, we have

$$n_i = \delta_{i1} \quad \text{and} \quad B_i = B\delta_{i1} \quad . \tag{5.2}$$

Introducing (5.1) into (3.18), we have since $\underset{\sim}{B}_t = 0$,

$$V^2 = \mu\varepsilon \quad \text{or} \quad V^2 = \mu\varepsilon + 2\mu\varepsilon'(D_2^2+D_3^2) \quad . \tag{5.3}$$

Corresponding to the first of these results, we obtain

$$\underset{\sim}{r} = (D_2^2+D_3^2)^{-1/2}\,(0, D_2, D_3) \tag{5.4}$$

and corresponding to the second, we have

$$\underset{\sim}{r} = (D_2^2+D_3^2)^{-1/2}(0,-D_3,D_2) \quad . \tag{5.5}$$

The directions of polarization of the $\underset{\sim}{b}$ vectors associated with the two waves are perpendicular to each other.

Again, for the wave corresponding to the first of the solutions (5.3), we find that $\overline{N} = 0$, so that the discontinuity propagates unchanged.

We now consider the wave corresponding to the second of the solutions (5.3). Introducing (5.2) into $(4.2)_2$, we obtain

$$R_{ns} = \mu\delta_{ns} + 2\mu' B^2 \delta_{n1}\delta_{s1} \quad . \tag{5.6}$$

From $(4.2)_1$, (4.7), (5.2) and (5.6) we see that a_{is} is symmetric for interchange of i and s. Hence, with (5.5),

$$\ell_i = r_i = (D_2^2+D_3^2)^{-1/2}(-D_3\delta_{i2}+D_2\delta_{i3}) \quad . \tag{5.7}$$

From (4.18) and $(4.13)_3$, we obtain, using (5.2),

$$N = -V^{-1} e_{i1k} e_{n1t} e_{\ell 1b} Q_{nk\ell} R_{tp} R_{bc} r_p r_c \ell_i$$

$$+ e_{i1k} e_{\ell 1n} P_{k\ell} S_{pns} r_s r_p \ell_i \quad . \tag{5.8}$$

Introducing the expressions (5.7) for r_i and ℓ_i into (5.8) and using (5.6), (4.2) and (5.2), we obtain, with $(5.3)_2$ and $(4.28)_2$

$$\overline{N} = -\frac{\mu^2(3\varepsilon'+2\varepsilon'' D^2)D}{(\mu\varepsilon+2\mu\varepsilon' D^2)^{5/2}} \quad , \tag{5.9}$$

where $D = (D_2^2+D_3^2)^{1/2}$ is the magnitude of the component of the electric displacement field normal to the direction of propagation. (In carrying out the calculation, we find that the second term on the right hand side of (5.8) is zero.) In the case when $D = 0$, the two waves with velocities given by (5.3) have the same speeds and can be propagated with arbitrary direction of polarization in the 23-plane.

6. Special case: electric displacement field ahead of wave in direction of propagation.

We again assume that the fields ahead of the moving surface of discontinuity are uniform, so that (5.1) still applies. We again take the 1-direction as the direction of propagation, but now assume that the electric displacement field is in the 1-direction, so that

$$n_i = \delta_{i1} \quad \text{and} \quad D_i = D\delta_{i1} \ . \tag{6.1}$$

It follows from (3.12) and (6.1) that

$$\underset{\sim}{J} = 0 \ . \tag{6.2}$$

Introducing this result into (3.18) we obtain

$$V^2 = \mu\varepsilon \quad \text{or} \quad V^2 = \mu\varepsilon + 2\mu'\varepsilon \, \underset{\sim}{B}_t \cdot \underset{\sim}{B}_t \ . \tag{6.3}$$

Corresponding to the first of these results, we have

$$\underset{\sim}{r} = (B_2^2+B_3^2)^{-1/2}(0,-B_3,B_2) \tag{6.4}$$

and corresponding to the second of these results, we have

$$\underset{\sim}{r} = (B_2^2+B_3^2)^{-1/2}(0,B_2,B_3) \ . \tag{6.5}$$

From (6.4) and (6.5) it follows that the directions of polarization of the electric vectors for the two waves are perpendicular.

For the wave corresponding to the first of the solutions (6.3), $\overline{N} = 0$, so that, from (5.1), the discontinuity propagates unchanged.

We now consider the wave corresponding to the second of the solutions (6.3). As in §5, we again obtain the expression (5.8) for N. Introducing (6.1) into $(4.2)_1$, we obtain

$$P_{k\ell} = \varepsilon\delta_{k\ell} + 2\varepsilon' D^2 \delta_{k1}\delta_{\ell 1}. \tag{6.6}$$

Proceeding in a manner analogous to that used in the previous section, we find that

$$N = -\frac{B(3\mu' + 2\mu'' B^2)}{\mu\varepsilon + 2\mu'\varepsilon B^2}, \tag{6.7}$$

where $B = (B_2^2 + B_3^2)^{1/2}$ is the magnitude of the component of the magnetic induction field normal to the direction of propagation. (In carrying out the calculation, we find that the first term on the right hand side of (5.8) is zero.)

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13. ABSTRACT

The propagation of a first-order electromagnetic discontinuity is discussed. Expressions are obtained for the possible velocities of propagation as functions of the field strengths ahead of the surface of discontinuity. Expressions are also obtained for the growth in the magnitude of the discontinuity as the wave progresses.

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