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I. INTRODUCTION

The study of technical solutions to the problem of managing fluid waste discharges in an estuary involves complicated relationships, such as those between the waste load, the location of discharges, the degree of treatment, the geometry of the estuary, the flow in the estuary, and the temperature. Models can be built to predict the probable consequences of each of the alternative solutions chosen. Using the predictions obtained from these models and whatever other information or insight is relevant, alternatives can be compared and conclusions drawn concerning the most desirable course of action. Such a model for well-mixed estuaries and coastal seas is presented ~~here~~ and ~~is~~ described in more detail in Reference 1. The model is being used and evaluated in a study of the extent and control of pollution in Jamaica Bay, New York.(2) Because the fluid waste discharged into the estuary is transported in it, a model of the fluid must be made in order to establish the relationship between the transport of the pollution and the transporting medium. Also, it can be visualized that constituents are transformed during the transport by chemical or biochemical action, and for this process a reaction model must be designed.

The flow in the estuary is governed by the shape of the estuary and its boundaries at the sea and the inland ends, as well as by the tides and the river discharges. The latitude of the area under investigation determines the effect of the earth's rotation on the flow, which is of particular importance in large estuaries and coastal seas at higher latitudes.

In the flow model, velocities of the water in the region are computed together with the water levels. Because the tide stage influences the location of the water's edge on the tidal flats, which affects surface area and cross sections to a considerable extent, such functional relationships must also be included in the model.

The fluid waste loads in the system influence the pollution constituent transport, while air temperature, solar radiation, and exchange of gases at the water surface influence the reactions.

In engineering and in systems analysis relationships are customarily displayed in graphical form. Because of the complexity of the results obtained from the simulations, the graphical results produced by the model permit an insight into the movements of the pollutants in the estuary in relation to the flow. The procedures used to derive these graphs are an integral part of the simulation model.

II. MOMENTUM AND MASS TRANSPORT

The fluid flow in an estuary is three-dimensional and time-dependent. Because the flow is mainly horizontal, the classic approach is to assume that the pressures are hydrostatic and that only shear stresses from horizontal velocity components are important.

In this classical approach, it is further assumed that gross motions only are considered and the effects of small-scale velocity fluctuation are aggregated into shear stress terms.

At present (1969), computational techniques are inadequate to deal with three-dimensional computations of fluid flow, and problems with the complicated boundaries of estuaries are beyond the capabilities of present computers. In the approach presented here, vertical integration of the equation of motion and continuity is used to reduce the problem to a two-dimensional one.

Such integration yields[1,3]

$$\frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial x} + V\,\frac{\partial U}{\partial y} - fV + g\,\frac{\partial \zeta}{\partial x} + g\,\frac{U\left(U^2 + V^2\right)^{\frac{1}{2}}}{C^2 H} - \frac{\tau^x}{\rho H} = 0 \qquad (1)$$

$$\frac{\partial V}{\partial t} + U\,\frac{\partial V}{\partial x} + V\,\frac{\partial V}{\partial y} + fU + g\,\frac{\partial \zeta}{\partial y} + g\,\frac{V\left(U^2 + V^2\right)^{\frac{1}{2}}}{C^2 H} - \frac{\tau^y}{\rho H} = 0 \qquad (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (HU)}{\partial x} + \frac{\partial (HV)}{\partial y} = 0 \qquad (3)$$

where U,V are the over-the-vertical averaged velocities along the eastward and northward directed axes, respectively, ρ is the density of water, g the gravity acceleration, f the Coriolis parameter, t the time, H the temporal depth, ζ the waterlevel elevation relative to the reference plane, C the Chezy term and τ^x and τ^y the windstress components on a finite surface element.

Vertical integration of the mass balance equations of dilute pollution constituents in well-mixed estuaries results in the expressions for the transport model

$$\frac{\partial(HP)}{\partial t} + \frac{\partial(HUP)}{\partial x} + \frac{\partial(HVP)}{\partial y} - \frac{\partial\left(HD_x \frac{\partial P}{\partial x}\right)}{\partial x} - \frac{\partial\left(HD_y \frac{\partial P}{\partial y}\right)}{\partial y} - R = 0 \tag{4}$$

where P is the vector of mass concentration of constituents averaged over the temperal depth, D_x,D_y are the dispersion coefficient and R is a vector. The elements of this vector include the local addition of the constituents in addition to the rate of production of the constituent in a water column with unit dimension.

For any point in the model, the following functional relation is used:

$$D_x = f_1(U,C,H) + D_w \tag{5}$$

$$D_y = f_2(V,C,H) + D_w \tag{6}$$

where D_w is a diffusion coefficient dependent on wave and wind conditions and the lateral diffusion.

III. THE COMPUTATIONAL MODEL

The discrete values of the variables are described on a grid. A space-staggered scheme is used, in which water levels and velocities are described at different grid points (Fig. 1). The mass densities are described at the same location as the water levels. This scheme has the advantage that in the formula operated upon in time there exists a centrally located spatial derivative for the linear term. The water

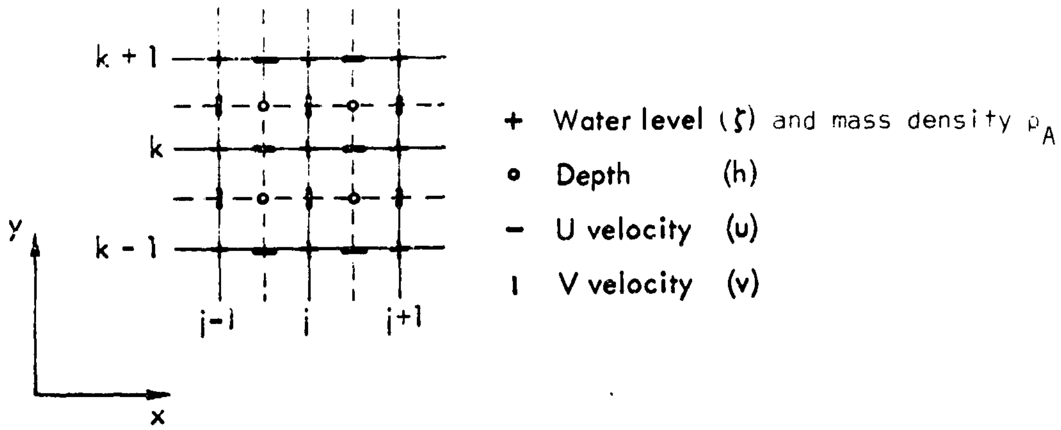

Fig. 1—Space-staggered scheme

level ζ and the mass densities ρ_A are described at integer values of j and k, the velocities u are described at half-integer values of j and integer values of k, and the velocity v is described at integer values of j and half-integer values of k. The water depth is described at the half-integer values of j and k.

Fractional and whole time steps are used: The fractional steps are used for the equations of continuity for the fluid mass and the mass densities, and the whole steps for the velocity components u and v. The velocity components are staggered in time.

The following notation is used for the approximation of the differential equations by a system of difference equations:

$$F \equiv F(j\Delta x,\ k\Delta y,\ n\Delta t)$$

where $(x,y,t) = (j\Delta x,\ k\Delta y,\ n\Delta t)$ and $j,k,t = 0,\ \pm\frac{1}{2},\ \pm 1,\ \pm 3/2,\ \ldots$

The following symbolic expressions are used for averages and differences. They are shown only for x, but are also used for y and t:

$$\bar{F}^x = \frac{1}{2}\left\{F[(j+\tfrac{1}{2})\ \Delta x,k\Delta y,n\Delta t] + F[(j-\tfrac{1}{2})\ \Delta x,k\Delta y,n\Delta t]\right\} \tag{7}$$

$$\delta_x F = \frac{1}{\Delta x}\left\{F[(j+\tfrac{1}{2})\ \Delta x,k\Delta y,n\Delta t] - F[(j-\tfrac{1}{2})\ \Delta x,k\Delta y,n\Delta t]\right\} \tag{8}$$

$$\bar{\bar{F}} = \frac{1}{4}\, F[(j+\tfrac{1}{2})\,\Delta x,\ (k+\tfrac{1}{2})\,\Delta y, n\Delta t] + F[(j+\tfrac{1}{2})\,\Delta x,\ (k-\tfrac{1}{2})\,\Delta y, n\Delta t]$$

$$+ F[(j-\tfrac{1}{2})\,\Delta x,\ (k+\tfrac{1}{2})\,\Delta y, n\Delta t] + F[(j-\tfrac{1}{2})\,\Delta x,\ (k-\tfrac{1}{2})\,\Delta y, n\Delta t] \qquad (9)$$

Special notations are used only for the indication of shifted time levels.

$$\delta_{+\frac{1}{2}t}\, F = \frac{2}{\Delta t}\left\{F[j\Delta x, k\Delta y,\ (n+\tfrac{1}{2})\,\Delta t] - F(j\Delta x, k\Delta y, n\Delta t)\right\} \qquad (10)$$

$$F_+ = F[j\Delta x, k\Delta y,\ (n+\tfrac{1}{2})\,\Delta t] \qquad (11)$$

$$F_- = F[j\Delta x, k\Delta y,\ (n-\tfrac{1}{2})\,\Delta t] \qquad (12)$$

The sets of finite-difference equations will now be presented in two sequential groups as they are used, and their use will be discussed subsequently. For the time level n, we use

$$\delta_t u - f\bar{\bar{v}} + u_+\,\overline{\delta_x u_-}^x + v\,\overline{\delta_y u}^y + g\,\overline{\delta_x \zeta}^t - g\,\frac{\overline{u}^t\left[(u_-)^2 + (\bar{\bar{v}})^2\right]^{\frac{1}{2}}}{\left(\overline{h}^y + \overline{\zeta}^x\right)\left(\overline{C}^x\right)^2} = 0 \quad \text{at } j+\tfrac{1}{2}, k, n \qquad (13)$$

$$\delta_{+\frac{1}{2}t}\zeta + \partial_x\left[\left(\overline{h}^y + \overline{\zeta}^x\right) u_+\right] + \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y\right) v\right] = 0 \quad \text{at } j,k,n \qquad (14)$$

$$\delta_{+\frac{1}{2}t}\left[P(\bar{\bar{h}} + \zeta)\right] + \delta_x\left[\left(\overline{h}^y + \overline{\zeta}^x\right) u_+\overline{P}^x_+\right] + \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y\right) v\overline{P}^y\right]$$

$$- \delta_x\left[\left(\overline{h}^y + \overline{\zeta_+}^x\right) D_x\delta_x P_+\right] - \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y\right) D_y\delta_y P\right] + R = 0 \quad \text{at } j,k,n \qquad (15)$$

and the finite difference approximations at time level n+½ are

$$\delta_t v + f\bar{\bar{u}} + u\,\overline{\delta_x v_-}^x + v_+\,\overline{\delta_y v_-}^y + g\,\overline{\delta_y \zeta}^t - g\,\frac{\overline{v}^t\left[(\bar{\bar{u}})^2 + (v_-)^2\right]^{\frac{1}{2}}}{\left(\overline{h}^x + \overline{\zeta}^y\right)\left(\overline{C}^y\right)^2} = 0 \quad \text{at } j, k+\tfrac{1}{2}, n+\tfrac{1}{2} \qquad (16)$$

$$\delta_{+\frac{1}{2}t}\zeta + \delta_x\left[\left(\overline{h}^y + \overline{\zeta}^x\right) u\right] + \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y\right) v_+\right] = 0 \qquad \text{at } j,k,n+\tfrac{1}{2} \tag{17}$$

$$\delta_{+\frac{1}{2}t}\left[P\left(\overline{\overline{h}} + \zeta\right)\right] + \partial_x\left[\left(\overline{h}^y + \overline{\zeta}^x\right) u\overline{P}^x\right] + \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y\right) v_+\overline{P}^y_+\right] - \partial_x\left[\left(\overline{h}^y + \overline{\zeta}^x\right) D_x\delta_x P\right] - \delta_y\left[\left(\overline{h}^x + \overline{\zeta}^y_+\right) D_y\delta_y P_+\right] + R = 0 \tag{18}$$

The sets of equations can be solved very rapidly as described in Ref. 1. The method consists of solving a set of linear equations, each having three unknown values.

IV. SIMULATIONS

Jamaica Bay is the area for which the model is applied. It has a very complicated bathymetry; many natural and dredged channels are separated by tidal flats. Thus, a finite-difference approximation of the bay bottom with a small grid size is required. The number of grid points upon which the computations are made, however, is limited by the size of the memory of presently available computers.

A grid size of 500 ft was selected. This grid size permitted several grid points to be present across the main channels in the bay and left space in the computer memory for program growth during development. The computational arrays were 78 × 61 points. Two arrays each are used for the two velocity components and three arrays for the water levels, since three time levels are involved. One array is used for depth data and one for the Chezy coefficients. If three pollution substances are computed, six arrays are necessary, i.e., one for each of the subsequent time levels. Thus, in total, 15 arrays are to be kept in memory, which requires approximately 75,000 storage locations.

Since the problem is time-dependent, large volumes of input data are generally required. These input data consist of the time histories of the following:

1. Water levels and mass densities at the mouth of the bay.
2. Discharges at the location of the outfalls of treatment plants and the combined sewer overflows.

3. Mass densities associated with these discharges.
4. Wind speed and direction.

The environment simulation program described in this paper generates large volumes of data in numerical form. Printed numerical data become unmanageable, and extracting important data from a large volume of material is a considerable task. Thus, programs were written for machine-made drawings to represent conditions at selected times.

Using the Integrated Graphics System, which supports an S-C 4060,[3] graphic outputs were made of the type shown in Fig. 2.

This graphic output displays the outline of the bay (approximate location of the high water line) and the grid points upon which the water levels and mass densities are computed. The total number of points varies from output to output as the locations of the water's edge in the marshy areas determine which point is included. The blank areas with no dots (.) are thus indications for land.

The U velocity component is computed at a location between the water level points in the x-direction, and the V velocity component is computed at a location between water level points in the y-direction (Fig. 1). In order to display the magnitude and direction of the velocity in the field of computation, velocity vectors were computed at the locations of the water levels from the average values of the adjacent components. To enhance the representation, vectors are plotted only at every second point in the x- and y-directions.

Variables such as mass densities of constituents (or the water levels) are plotted by means of isocontour lines. Different line thickness or dashing is available for such presentation. The subroutines to draw such contours are very extensive, since a search must be made at each location if the contour under consideration passes through the adjacent grids. In addition, proper terminations of such contours had to be made if the adjacent grid points had become land and were thus not computed.

The value of each isocontour line can be found from extrapolation of numerical values which are also plotted in the field at selected locations.

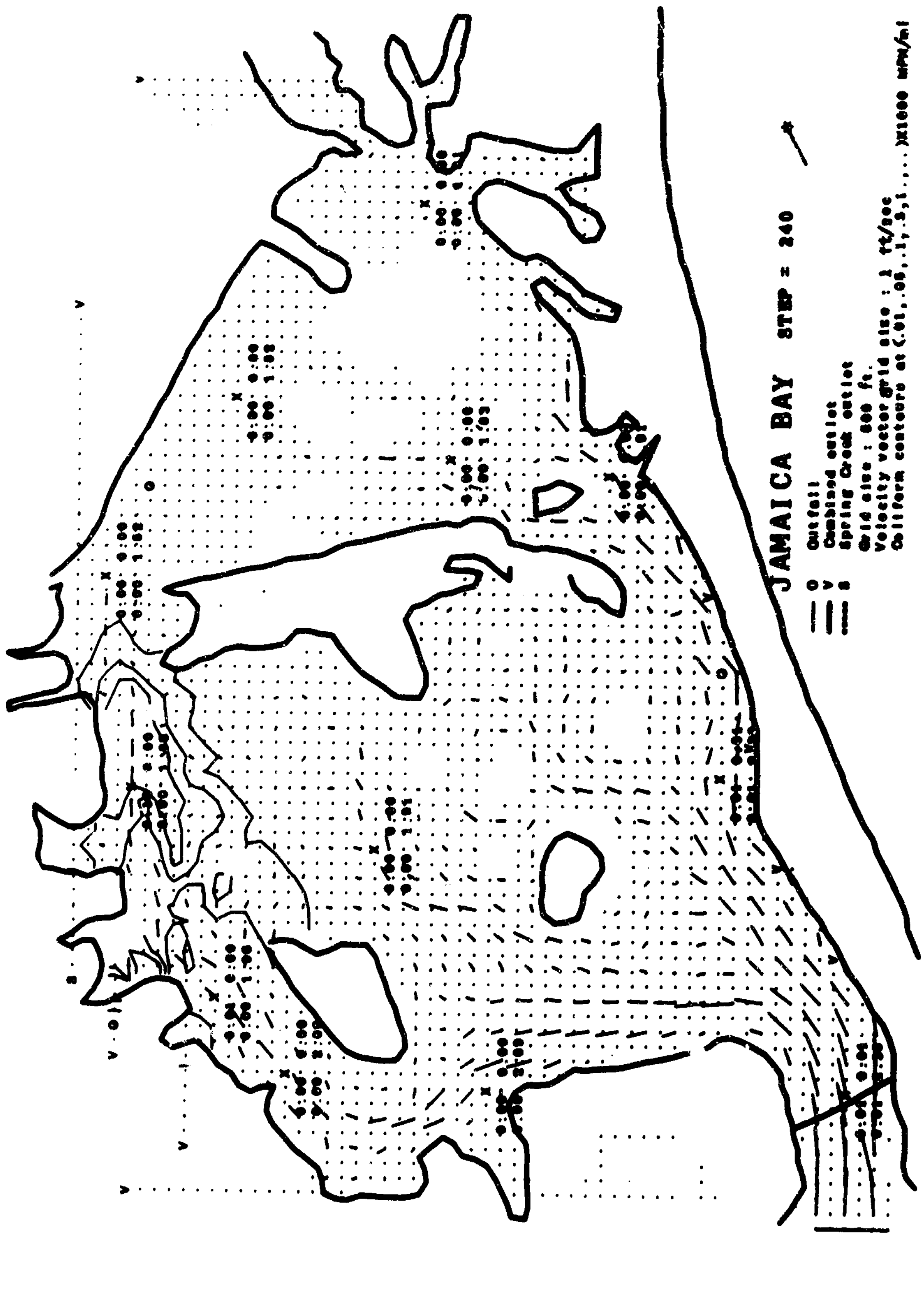

Fig. 2—Results after 8 hrs.

Figure 2 presents the pattern of the distribution of coliform bacteria from a source in the northwest section of the bay 8 hours after the start of the computation.

The model is now being verified for its hydraulic and water ality properties. The verification is based upon an extensive field investigation and analysis.

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