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FUEL ADDITIVE ATOMIZATION  
IN JET ENGINE TAILPIPES

by

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Task No. 867902

Work Unit No. 86790201

FINAL REPORT

August 1968 through November 1969  
January 30, 1970

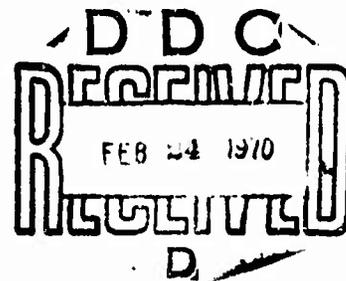
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Abstract

This report includes a review of the technical literature relating to the modelling of the atomization process, an analysis of the differences between conditions studied in the reports and those found in jet exhausts, and finally, recommendations for follow-up studies designed to modify early models as a consequence of those differences.

Participating Scientists

Philip Cole

and

Theodore Jungreis

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## I. INTRODUCTION AND SUMMARY

Investigations of the factors controlling the atomization of liquids by high-speed gas streams fall into two broad categories, distinguished by their approach to the problem of predicting droplet sizes, spray disintegration rates and areas, and spray or jet trajectories. One school of investigators, wary of the complexity of the process have attempted to empirically correlate free stream and injected liquid parameters with experimentally determined droplet size distributions or measures thereof (maximum droplet size, volume-median drop diameter, etc. ), spray areas, and spray or jet trajectories. The forms chosen for such correlation formulae have often been obtained by resorting to dimensional analysis, or the use of standard fluid mechanical non-dimensional numbers. Dimensional analysis can also be useful in planning the type of experimental data to be taken.

Very few investigators, until the last few years have attempted to model the actual disintegration or atomization process of a liquid jet injected cross-stream into a high speed gas flow. Two related papers which develop primarily theoretical models for this process will be discussed in detail. These papers contain predictions for those properties of the atomized liquid which are judged highly relevant to the current study.

On the basis of this search and evaluation of a small part of the literature, recommendations for further efforts in two related directions are proposed. First, the techniques of dimensional analysis should be

employed to assist in the formulation of any experimental program using the wind tunnel at Hanscom Field and in the development of an empirical correlation model. Secondly, a computer based modeling activity should be employed to obtain further improvement of existing theoretically derived models of the jet disintegration and atomization process. This will permit the elimination of some assumptions that have been employed in current models merely for the sake of convenience. The experimental data gathered from wind tunnel tests can then be used to verify both the empirical correlation model and the theoretical model. The results of these activities will permit a rational choice for those controllable model parameters which can best facilitate the degree of atomization or droplet size distribution desired.

## II. OVERALL STUDY OBJECTIVES AND THE LITERATURE SURVEY

The objective of this study is to determine those parameters which control the resulting droplet size distribution produced by the injection of a liquid into the tailpipe region of a turbojet engine. Parameter values must be selected which produce droplets having diameters within a certain range. These droplets will then serve as nuclei for the condensation of water vapor in the exhaust trail. It may be necessary to suggest a suitable mechanical design for the injection nozzles if simple orifices are not capable of producing the desired droplet sizes. However, mechanical design considerations may be of secondary importance if the droplet size produced by the primary atomization of simple liquid jets is sufficiently small.

The first phase of this effort consisted of a brief literature search using readily available sources. Our primary goal was to determine the current state of information relating to the mechanisms controlling the atomization of liquids injected in a cross or cocurrent direction into high speed gas streams. More specifically, most interest centered about the availability, relevance, and reliability of empirically and theoretically determined relations between droplet size distributions, spray or jet trajectories, and rate of disintegration of liquid jets as functions of liquid and gas stream parameters, and nozzle geometry.

Stimulus for the study of atomization phenomena arises in a variety of areas. Jet penetration studies have been made for many years to aid in the design of fire-hose nozzles, oil burner sprays, and agricultural spraying devices. The prediction of drop size distribution and spray

geometry is of fundamental interest in the study and design of diesel engine combustion chambers. Considerable work in this area has taken place in Germany and Russia. Chemical process designers have long studied atomization phenomena for such applications as spray driers and spray columns [4] [5]. Various types of spray nozzle designs have also been experimentally studied and reported on in the chemical engineering literature [6]. Unfortunately, in studies of this type surveyed, the ambient or free stream gas velocity is usually zero, and hence does not contribute to the atomization process directly.

The work of primary interest to this investigation appears in the aerospace related literature, and has arisen out of the need to study fuel injection phenomena in turbojet, ramjet, and rocket engines. In such cases the ambient or free stream gas velocity will usually be quite high and nearly always greater than the liquid injection velocity. Much of this work is directly applicable to the current study, and at worst will provide a sound basis for future efforts. Empirical correlations and theoretically obtained expressions for drop size distributions, mean, volume-median, and maximum drop sizes are available from several sources [7], [8], [9]. These predictive formulae usually are applicable to simple circular orifices. Both cross and cocurrent injector configurations have been employed. The results obtained in these studies may be entirely suitable for obtaining at least initial order of magnitude estimates for the present study, once the associated empirically derived parameters have been verified using data supplied by AFCRL.

As we have noted, investigations of atomization phenomena generally fall into one of two categories: empirical correlations or more theoretically based models of the actual mechanism of liquid jet disintegration. Within each category we shall discuss three separate articles, since they seem to represent the most carefully planned experimental programs from the empirical point of view and the best thinking from a theoretical point of view.

The fluid mechanical parameters that control the disintegration of liquid jets and sprays are most conveniently discussed if we first review some of the theoretically based models that have been proposed in the literature.

In 1961, Mayer [1] proposed a model for the shedding of droplets from a plane liquid-gas interface, where the gas velocity is sufficiently high to cause the unstable growth of capillary or surface tension dominated waves. Mayer acknowledged the lack of theoretical models for predicting drop-size distributions and then attempted to fill that need.

In Mayer's model the behavior of the gas-liquid interface region governs the atomization process. This is because the main body of liquid is not represented as a moving jet undergoing gross acceleration, but instead as a large deep body of fluid, fixed in space, with surface disturbances produced by the wind-like action of the gas stream. Surface tension and viscous forces are assumed to exist. In this case, the only way that wave motion on the surface of the liquid can persist and grow is through the action of surface forces, namely, surface wind generated

tractions, normal and tangential to the liquid surface.

Mayer's analysis represents an extension of the classical wave stability analyses, such as appears in Birkhoff [10], where viscous forces are assumed to be absent. The typical classical analysis is based on the Bernoulli equation for non-stationary motion of inviscid fluids in a gravitational field. In the classical inviscid model, relative surface velocity at the gas-liquid interface, surface tension forces, and acceleration normal to the gas-liquid interface are assumed to be present. With this type of model, shown that

- a) Relative surface velocity is always destabilizing, i. e., causes wave growth.
- b) Acceleration directed from a light toward a dense fluid is a destabilizing influence on the dense fluid.
- c) Surface tension stabilizes sufficiently short surface ripples.

As we have mentioned, Mayer's model includes viscous effects in the liquid jet. Superimposed normal acceleration is not considered. The growth and destabilization of surface ripples is, therefore, primarily attributable to surface forces produced by the direct action of a high-speed gas stream. The model is based on an analysis of wind induced surface waves presented in Lamb [11] and attributed to Jeffreys [12]. In this model a complete spectrum of small surface ripples of a periodic nature are assumed to exist. Surface wind action, therefore, produces a related normal pressure distribution. Tangential surface tractions can be shown to be negligible. An energy balance of the surface wave region yields the

following differential equation for wave amplitude.

$$\dot{A} = \frac{dA}{dt} = A \left[ \frac{\beta \rho_g (V_g - u)^2}{2 \rho_l u} k - \frac{2 \mu_l}{\rho_l} k^2 \right] \quad (1)$$

where

$A$  = wave amplitude

$\rho_g$  = gas density

$\rho_l$  = liquid density

$\mu_l$  = liquid viscosity

$V_g$  = gas velocity

$u$  = surface wave velocity

$k$  = wave number =  $\frac{2\pi}{\lambda}$ ;  $\lambda$  = wave length

$\beta$  = Jeffreys' sheltering parameter,  $\beta \approx 0.3$

The empirical constant  $\beta$  accounts for the fact that only part of a wave is significantly exposed to surface wind action. The wave velocity must be related to the particular type of forces which produce unstable wave growth. For capillary waves  $u = (\sigma k / \rho_l)^{1/2}$  where  $\sigma$  = surface tension. It follows that if  $V_g \gg u$

$$\dot{A} = A \left[ \frac{f}{\lambda^{1/2}} - \frac{\nu}{\lambda^2} \right] = A \phi(\lambda) \quad (2)$$

where  $f = \left(\frac{\pi}{2}\right)^{1/2} \frac{\beta \rho_g V_g^2}{(\sigma \rho_l)^{1/2}}$  = forcing parameter (3)

$$\nu = \frac{8\pi^2 \mu_l}{\rho_l} = \text{viscous damping parameter} \quad (4)$$

If  $\phi < 0$ , surface waves decay. The condition  $\phi = 0$  can be used to determine the minimum value of  $\lambda$  which is associated with wave growth.

As  $\lambda$  increases above this value,  $\phi$  reaches a maximum and then

decreases toward zero. Therefore, a range of  $\lambda$  exists in which wave growth is possible. However, waves of small  $\lambda$  decay due to viscosity and waves of large  $\lambda$  (long wavelength) grow very slowly because of inertial resistance. In any realistic situation, a complete spectrum of wavelengths will be present due to gas stream turbulence and pressure fluctuations. The character of this spectrum must, of course, be estimated.

Mayer postulates the following mechanism for the atomization process. When a wind induced wave of length  $\lambda$  has grown to an amplitude of order  $\lambda$  the wave crest is shed as a ligament, which rapidly collapses due to surface tension instability into droplets of a size proportional to  $\lambda$ . That is

$$D = F\lambda \quad (5)$$

where  $D$  = droplet diameter

$F$  = dimensionless scale factor, independent of  $\lambda$ , but possibly dependent on fluid properties. ( $F \approx 1$ )

Clearly, the rapidity with which waves grow to an amplitude of order  $\lambda$  is a function of  $\lambda$  or  $\phi(\lambda)$ . That is, the frequency of wave formation varies with  $\lambda$ . It is then postulated that  $\dot{m}(\lambda)$ , the mass shedding rate per unit surface area of liquid per unit wavelength, in the range  $\lambda$  to  $(\lambda + d\lambda)$  is proportional to  $\rho_L$  and  $\phi(\lambda)$ . A related expression for  $\dot{n}(\lambda)$  (droplet formation rate per unit area per unit wavelength) can then be obtained. There appears to be some discrepancy in the mass shedding rate relation, but it is not sufficient to invalidate the remainder of the analysis. Using the distribution relation  $\dot{n}(\lambda)$ , an expression for  $\bar{\lambda}$ , the mean

shedding wavelength, and  $\bar{D} = F\bar{\lambda}$ , the mean droplet diameter can be determined. It is readily shown that

$$\bar{D} = 9\pi (16)^{1/3} \frac{F}{\beta^{3/2}} \left[ \frac{\mu_2 (\sigma/\rho_2)^{1/2}}{\rho_1 V_1^2} \right]^{2/3} \quad (6)$$

If the apparent discrepancy in Mayer's analysis is removed, the value for  $\bar{D}$  is approximately halved.

The combined factor  $F/\beta^{3/2}$  must be obtained from an examination of experimental data. Mayer makes one such correlation with data obtained by Weiss and Worsham [13]. The use of their data yielded the values  $\beta \approx 0.3, F \approx 0.14$ , in reasonable agreement with the assumptions made previously.

Mayer does not claim that this model is directly applicable to liquid jets injected into a high-speed gas stream. For small diameter jets, only a small time interval will be available for wave growth, which places an upper limit on the range of wavelengths which can contribute to the shedding process. The important aspect of this preliminary model is that its concepts can be extended and developed to cover other atomization processes, namely, jets of small diameter and probably the secondary atomization of large droplets. In addition, with suitable modifications, the model can be extended to cover the growth of surface waves produced by acceleration induced destabilizing forces. Most of these extensions are embodied in two succeeding papers published by Adelberg [2], [3]. His results are directly applicable to the cross-stream injection of liquid jets in high velocity gas streams.

Adelberg has attempted to eliminate some of the limitations of Mayer's model when it is applied to the atomization of liquid jets of small diameter, injected cross-stream into a gas stream. Expressions are also developed to describe the trajectory of the jet and its penetration into the flowing gas stream. The shedding mechanism proposed by Mayer is adopted essentially in tact. The main difference in Adelberg's model is attributable to three facts:

- a) The gas-liquid interface is no longer planar, and the mass of liquid is finite. The center of mass of any section of the jet can be subjected to severe accelerations, provided the free stream dynamic pressure is sufficiently high.
- b) Because the jet is of finite size and rapidly disintegrates, surface waves must grow to shedding amplitude within a short time, so that there is an upper limit on wavelengths which can contribute to shedding process.
- c) Mayer's capillary wave growth model is inadequate when the free stream dynamic pressure is very high, i. e. , when  $\frac{1}{2} \rho_g V_g^2 > 300 \text{ lb/ft}^2$  for jets having an initial diameter of order 0.02 to 0.1 in. In such cases, surface waves grow as "gravity" or acceleration waves, the acceleration being roughly normal to the trajectory of the jet.

Adelberg develops two sets of predictive formulae for mean droplet diameter, mass shedding rate, etc. One set is applicable to capillary wave growth (low free stream dynamic pressures), and one is

applicable to acceleration wave growth (high free stream pressure). All such formulae contain a single parameter which can only be estimated and must be evaluated using experimental data. This parameter is related to the scale factor  $F$  chosen by Mayer.

The actual process of wave growth is perhaps a bit more complex than has been stated above. Initial jet surface roughness is usually present due to internal turbulence and external free stream pressure fluctuations. In cases where the free stream dynamic pressure is high, initial wave growth, for waves larger than some minimum size, is probably due to capillary forces. Growth then takes place in a region where both capillary and dynamic pressure (acceleration) forces are important. Most growth is sustained in a much larger region where acceleration forces predominate. In cases where the free stream pressure is low, the acceleration dominated region is never encountered. In all the situations for which these models are applicable, viscous forces, while present, have negligible effect. They merely serve to define the minimum wavelength beyond which capillary or acceleration waves will grow.

We will now review the development of the models described by Adelberg. Equation (1) is still used as a starting point. An expression for  $u$ , the wave velocity, must be determined. For capillary waves

$$u = (\sigma k / \rho_L)^{1/2} \quad (7)$$

while for acceleration waves

$$u = (a/k)^{1/2} \quad (8)$$

where  $a$  = acceleration normal to the surface of the jet and directed along the radius of curvature of the jet trajectory. Equations (7) and (8) are applicable if the wavelengths present on the jet surface are small in comparison with the jet diameter. If dynamic pressure forces dominate the wave growth process, then  $a/k > \sigma k / \rho_L$ , or  $\ell = 2\pi/k > (4\pi^2 \sigma / a \rho_L)^{1/2}$ . If Equation (8) is to be used, an estimate for  $a$ , the acceleration must be made. It is assumed that the acceleration of any element of the fluid jet is constant, and that the dynamic pressure induced surface force can be approximated by employing the force relationship for an inclined cylinder in cross stream flow. Finally, one obtains

$$\dot{A} = A \left[ \frac{G}{\lambda^{3/2}} - \frac{v}{\lambda^2} \right] = A \phi(\lambda) \quad (9)$$

$$G = \frac{\pi^2 \beta D_0^{1/2} V_L}{C_{D_0}^{1/2} \sin \theta} \left[ \frac{\rho_g V_g^2}{\rho_L V_L^2} \right]^{1/2} \quad (10)$$

$D_0$  = initial jet diameter

$C_{D_0}$  = drag coefficient for a cylinder in cross flow

$\theta$  = angle between jet and wall

For the case of capillary wave growth, Equations (2) and (3) are still applicable. The mass shedding rate is again assumed proportional to  $\phi(\lambda)$  and an undetermined constant of proportionality  $K$ , ( $K \approx 1$ ), which is analagous to Mayer's  $F$ . Minimum values for  $\lambda$  can be determined by setting  $\phi(\lambda) = 0$ , where the appropriate  $\phi$  is chosen for the capillary or acceleration regimes. It is assumed that the maximum wavelength which contributes to the shedding process is

proportional to the jet diameter, that is

$$\lambda_{\max} = ed \quad (11)$$

where  $e < 1$ , and  $d$  = jet diameter at any point along its trajectory. The mean mass loss rate per unit length of the jet can then be given by

$$\dot{m} = \int_{\lambda_{\min}}^{ed} K \rho_L \lambda^2 \phi(\lambda) P(\lambda) d\lambda \quad (12)$$

where

$$P(\lambda) = \text{probability that a wave occurs in the} \\ \text{wavelength range} \\ \lambda \text{ to } (\lambda + d\lambda)$$

The mass loss rate for the entire jet can then be given as

$$\dot{M} = \int_0^{S_0} \dot{m} ds \quad (13)$$

where  $ds$  = differential element of arc measured along the jet axis. The mass loss rate expressions can be combined with expressions for the acceleration and surface force acting on a differential element of the jet to generate expressions for the trajectory of the jet and its diameter at any point along the trajectory. We shall not deal with these aspects of the jet in detail here, but instead turn to a discussion of how estimates for the mean droplet diameter can be developed.

Adelberg relates the droplet formation rate per unit length of the jet to the mass loss rate per unit length by following the argument proposed by Mayer. He also accepts Mayer's hypothesis that the mean droplet diameter is directly related to the mean shedding wavelength.

That is,

$$\dot{n}(\lambda) = \frac{\dot{m}(\lambda)}{\frac{\pi}{6} K_2 \lambda^3} = \frac{6 K \phi(\lambda)}{K_2 \pi \lambda} \quad (14)$$

where  $\dot{n}$  = droplet formation rate per unit jet length and  $K_2$  is a proportionality constant of order unity. The mean shedding wavelength  $\bar{\lambda}$  can then be defined as

$$\bar{\lambda} = \left[ \int_{\lambda_{min}}^{ed} \int_0^{S_0} \lambda \dot{n}(\lambda) d\lambda ds \right] / \left[ \int_{\lambda_{min}}^{ed} \int_0^{S_0} \dot{n}(\lambda) d\lambda ds \right] \quad (15)$$

where  $S_0$  = maximum jet length.

Appropriate expressions for  $\phi(\lambda)$  for the capillary and acceleration wave regimes can be employed in Equations (14) and (15) to obtain estimates for  $\bar{\lambda}$ . Finally,

$$\bar{D} = K_3 \lambda \quad (16)$$

where  $K_3$  is a proportionality constant of order unity. The mean droplet size can then be given as

$$\bar{D} = 12\pi K_3 (16)^{1/3} \left[ \frac{\mu_L (\sigma/\rho_L)^{1/2}}{\beta \rho_g V_g^2} \right]^{2/3} \quad (17)$$

which is valid for the acceleration regime, and

$$\bar{D} = K_3 (16)^{2/3} \left( \frac{2\pi e D_0}{g} \right)^{1/2} \left[ \frac{\mu_L (\sigma/\rho_L)^{1/2}}{\beta \rho_g V_g^2} \right]^{1/3} \times \left[ 1 - \frac{\kappa_1 \beta (\pi/2)^{1/2} e^{3/2}}{5 \lambda_0} \right] \quad (18)$$

for the capillary regime, where

$$\lambda_0 = \frac{2}{5} K \beta (\pi/2)^{1/2} e^{3/2} \quad (19)$$

The most significant difference between these two results is that the mean droplet diameter depends upon  $D_0$ , the initial jet diameter only in the capillary wave regime. In the acceleration regime, the mean droplet size is independent of  $D_0$ .

Adelberg has correlated the droplet size predictions yielded by these models with several sources of data, primarily for cases where capillary forces dominate. On the basis of these correlations, it appears that the following values should be used for the constants appearing in Equations (17) - (19):

$$\beta = 1$$

$$e = 0.06$$

$$K = 1$$

$$K_3 = \begin{cases} 1.4 & \text{(capillary regime)} \\ 0.7 & \text{(acceleration regime)} \end{cases}$$

In general, reasonable agreement was obtained when comparing Equations (17) and (18) with existing empirical correlations appearing in the literature. The predicted exponents for  $\sigma$ ,  $\mu_l$ ,  $\rho_l$ ,  $\rho_g$ ,  $V_g$ , and  $D_0$  generally agree both with respect to sign and approximate magnitude.

An attempt was made to employ Equation (18) to predict  $\bar{D}$  under conditions approximating a typical jet engine tailpipe. Liquid jet disintegration is clearly a result of capillary instability because of the low free stream dynamic pressure (approximately 100 lb/ft<sup>2</sup>). The following values were employed in trial calculations:

$$K = 1$$

$$K_3 = 1.4$$

$$e = .06$$

$$\beta = 1$$

$$\rho_l = 1.5 \text{ gm/cm}^3$$

$$\rho_g = 24.54 \text{ gm/cm}^3$$

$$V_g = 35 \times 10^3 \text{ cm/sec.}$$

It follows that

$$f_{\sigma} = 7.35 \times 10^{-3}$$

and

$$\bar{D} = 0.0589 \mu_L^{1/3} \sigma^{1/6} D_o^{1/2}$$

This equation was used to produce the following table.

$D_o$	$\sigma$ (Dynes/cm) $\mu_L$ (Dyne-sec/cm <sup>2</sup> )	$\bar{D}$ (Microns)							
		25				50			
		.001	.005	.010	.050	.001	.005	.010	.050
.025 cm		15.9	27.2	34.3	58.6	17.8	30.5	38.5	65.8
.050 cm		22.5	38.5	48.5	82.9	25.3	43.2	54.2	93.1
.100 cm		31.8	54.4	68.5	117.2	35.7	61.1	76.9	131.6

A plot of this table is shown in Figure 1.

Cross-stream injection of liquid jets into high-velocity air streams have been studied experimentally by Ingebo and Foster [7]. By employing dimensional analysis techniques, an expression correlating the ratio of the volume-median drop diameter to the jet orifice diameter,  $D_{30}/D_o$  \*, with a modified Weber-Reynolds number ratio was obtained. A similar relation for the ratio  $D_m/D_o$  was obtained, where  $D_m$  is the maximum drop diameter observed. Several distribution expressions relating R, the volume fraction of drops having diameters greater than D with respect to variations in D were employed.

This work was performed to simulate the injection and breakup of fuel jets under conditions similar to those encountered in ramjet engines

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\*Note:  $D_{30} = (\sum n D^3 / \sum n)^{1/3}$

D = droplet diameter

n = number of drops in a given size range

and afterburners. Air was employed as the gas, and in some cases it was heated to 900°F. Test liquids employed for injection were iso-octane, JP-5, water, benzene, and carbon tetrachloride. Air stream velocities ranged from 100 to 700 ft/sec.

Preliminary tests were conducted to determine the effect that injection conditions, namely, liquid jet velocity  $V_l$ , orifice discharge coefficient  $C_o$ , and the length diameter ratio for the orifice had on volume-median drop diameter  $D_{30}$ . These tests indicated that these injection conditions had little if any effect on  $D_{30}$ . This may be explained by noting that the air stream is initially normal to the jet. The fact that  $V_l$  does not enter into the expression for  $D_{30}$  agrees with the results of Mayer and Adelberg. (This is not true, however, for the relations obtained by Ingebo [8] in a study of injection where the liquid jet was injected cocurrent to the gas stream.) The only injection parameter which must be included in the correlation is the orifice diameter  $D_o$ . The following functional relationship was assumed.

$$D_{30} = f(D_o, \rho_l, \rho_g, \mu_l, \mu_g, V_g, \sigma) \quad (20)$$

$$= C_1 D_o^a \rho_l^b V_g^c \sigma^d \mu_l^e \rho_g^f \mu_g^g$$

where  $C_1$ ,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ ,  $f$ , and  $g$  are to be determined. By simple dimensional analysis, three of the exponents can be related to the other four, and the following relationship obtained.

$$\frac{D_{30}}{D_o} = C_1 \left( \frac{\sigma}{D_o \rho_l V_g^2} \right)^d \left( \frac{\mu_l}{D_o \rho_l V_g} \right)^{g+e} \left( \frac{\rho_g}{\rho_l} \right)^f \left( \frac{\mu_g}{\mu_l} \right)^g \quad (21)$$

which includes four dimensionless groups. No appreciable effect could be attributed to the group  $\left( \frac{\mu_g}{\mu_l} \right)$  i. e.,  $g \approx 0$ . Let

$$We = \frac{D_o \rho_l V_g^2}{\sigma} = \text{Liquid jet Weber number} \quad (22)$$

$$Re = \frac{D_o \rho_l V_g}{\mu_l} = \text{Liquid jet Reynolds number} \quad (23)$$

Note that neither of these definitions is strictly correct since both contain both gas and liquid fluid parameters. By utilizing simple correlation techniques it was found that

$$\frac{D_{30}}{D_o} = 3.9 Re^{-0.25} We^{-0.25} \quad (24)$$

A similar expression, obtained in the same manner, is given for the ratio

$$\frac{D_{10}}{D_o} = 22.3 Re^{-0.24} We^{-0.24} \quad (25)$$

Each correlation equation contains six parameters, and excludes gas stream viscosity, liquid injection velocity, and all orifice parameters except  $D_o$ .

The values for  $D_{30}$  employed in determining the correlations described here were obtained from the raw test data by applying the Nukiyama-Tanasawa expression for drop size distribution.

$$\frac{dR}{dD} = \frac{b^{6/\beta}}{\Gamma(6/\beta)} D^5 \exp(-bD^\beta) \quad (26)$$

Here,  $R$  = volume fraction of drops having diameters greater than  $D$

$b$  = constant determined from the data

$\beta$  = constant = 1 here

$\Gamma$  = Gamma function.

It is then possible to show that

$$D_{30} = \left( \frac{\Gamma(6)}{\Gamma(3)} b^{-3} \right)^{1/3} = 3.915/b \quad (27)$$

Finally, the Nukiyama-Tanasawa expression for drop size distribution may be modified by employing the relations for  $D_{30}/D_0$ ,  $D_m/D_0$ , and  $D_{30}$ , to eliminate  $b$ . The result is

$$\frac{dR}{dD} = 10^6 \frac{D^5}{D_m^6} R_c^{-0.24} W_e^{-0.24} \exp(-22.3 \times R_c^{-0.04} W_e^{-0.04} D/D_m) \quad (28)$$

a result which shows the effect of maximum drop diameter, Weber number, and Reynolds number on the complete drop size distribution function. It should be noted that for this expression  $D$  must always be less than or equal to  $D_m$ , a restriction which does not appear in the original Nukiyama-Tanasawa distribution function.

Equation (24) was used to obtain an estimate for  $D_{30}$ , the volume median drop diameter. The same data that was employed in the result given by Adelberg was used here:

$$D_{30} = 0.0348 \mu_l^{1/4} \sigma^{1/4} D_0^{1/2}$$

This equation was used to produce the following table:

	D <sub>30</sub> (MICRONS)								
	(Dynes/CM)	25				50			
	(Dyne-Sec/CM <sup>2</sup> )	.001	.005	.010	.050	.001	.005	.010	.050
.025 CM		21.9	32.7	38.9	58.2	26.0	38.9	46.3	69.2
.050 CM		30.9	46.2	54.9	82.1	36.7	54.9	65.3	97.6
.100 CM		43.7	65.3	77.7	116.2	52.0	77.7	92.4	138.2

A plot of this table is shown in Figure 2.

Clark [9] has also studied the breakup of a water jet injected cross-stream into a nitrogen gas stream. In this investigation, interest centered around obtaining an expression for the relative cross-sectional area of the jet at any given point along the trajectory of the jet. An empirical expression

for this area was derived. Drop size measurements and correlation relations were not developed. However, Clark proposed a model for the breakup of the jet. The model attempts to relate the breakup rate to such parameters as free stream and liquid densities, velocities, orifice diameter, and the distance over which the gas stream acts on an element of the jet. A single non-dimensional correlating parameter is developed which includes the physical parameters noted above. This parameter is not directly related to the usual non-dimensional force ratio numbers employed by most investigators.

The model of the jet disintegration process proposed by Clark will now be described in detail. The jet cross section is initially roughly circular with superimposed surface roughness attributable to turbulence generated at the entry region of the injector orifice. Normal and tangential components of the dynamic pressure of the free stream gas tend to distort the cross-section of the jet and tear off liquid ligaments from its periphery. For small surface disturbances, surface tension acts as a stabilizing mechanism, but for larger disturbances it tends to promote jet disintegration. Internal liquid viscosity acts as a stabilizing force, retarding the effects of external, surface and internal turbulence generated shearing forces. It is shown that the parameters

$$V = (V_g^2 + V_l^2)^{1/2} = \text{relative gas velocity}$$

$$\rho_l = \text{liquid density}$$

$$\rho_g = \text{gas density}$$

$$D_o = \text{orifice diameter}$$

$$t = \text{action time during which breakup occurs.}$$

can be combined into a dimensionless number

$$\epsilon = \frac{\delta}{D_0} = \frac{f_g}{\rho_l} \left( \frac{Vt}{D_0} \right)^2 \quad (29)$$

where  $\delta$  is the maximum spreading of the cross-section of the jet from its initial circular configuration. The rate of breakup is assumed to be related to  $\epsilon$  alone. This nondimensional number  $\epsilon$  is obtained by neglecting gravitational and viscous effects, and is independent of surface tension. These assumptions are, therefore, valid only for situations where the Weber number is quite high. This clearly makes this model of the shedding process applicable to what Adelberg calls the acceleration regime.

The actual sequence of events included in the model of the breakup process can be described as follows. The injected jet, which has an initially circular cross section, is exposed to a pressure distribution imposed by the free stream. This distribution can be approximated by the pressure distribution associated with a long circular cylinder in a uniform flow field. Normal components of this pressure distribution create a pressure gradient within the jet and flatten its cross-section in directions transverse to the flow direction. At the edges of this now flattened sheet of fluid, the combined action of tangential components of the free stream dynamic pressure and surface tension forces tear off ligaments which quickly collapse into drops due to further surface tension produced instabilities. (The wavelength of the flattened jet protrusions is, therefore, approximately  $\delta$  where  $\delta$  is the order of  $D_0$ . This appears to be much higher than the wavelengths considered

important by Mayer and Adelberg.) The rate of jet breakup is, therefore, assumed to be controlled by the distortion rate of the liquid cross-section. Neglecting surface tension forces in predicting  $\delta$  makes the model inapplicable for cases where the Weber number is relatively low.

Examination of experimental data indicates that the following factors tend to increase the rate of distortion and subsequent breakup; increasing gas velocity  $V_g$ , gas density  $\rho_g$ , and action time  $t$ . Furthermore, increasing liquid velocity  $V_l$  and jet diameter tend to decrease the rate of breakup. However, Clark makes no statement as to how these factors control the resultant drop size distribution. It should also be clear that initial jet turbulence can materially effect the rapidity of breakup. The experimental results also seem to indicate that the action time  $t$  is a function of the dynamic pressure associated with the free stream and that the vector sum  $V = (V_g^2 + V_l^2)^{1/2}$  is better correlated to jet breakup than is the gas velocity  $V_g$  alone. Within the assumptions made about the forces controlling breakup, the most important conclusion is that rate of breakup is inversely proportional to jet diameter, i. e.,  $\epsilon \propto D_c^{-2}$ . The model and experiments are not to be assumed representative of breakup phenomena where surface tension forces are important. The data also show that no jet breakup occurs for values of  $\epsilon < 1$ , thus indicating that shedding of liquid will not occur until the jet cross-section is considerably flattened. Furthermore, jet breakup is essentially completed when  $\epsilon$  has increased to a value of 10 or 15.

Clark claims that the breakup data for liquid jets are well correlated with the suggested breakup criteria  $\epsilon$ . He further contends

that the same criteria is applicable to the breakup of liquid drops, and to fluids having lower surface tensions and densities than water. He has attempted to correlate his model with some data presented by Ingebo and Foster. The best correlation is obtained at higher Weber numbers.

The notion that the flattening of the jet cross-section is related to jet breakup seems plausible. Such flattening exposes more fluid to the action of the gas stream. However, the distortion or spreading  $\delta$  would appear to be much larger in magnitude than the wavelengths and resultant drop sizes predicted by Mayer and Adelberg. This indicates that it would be difficult to relate  $\delta$  to mean droplet size, or some other appropriate measure of atomization.

Ingebo [8] has correlated maximum drop diameters observed for water and ethanol injected into cocurrently flowing gas streams of nitrogen or helium. Four separate injection regimes were studied, viz:

- a) Pendant drops of liquid in still air where liquid velocity approaches zero.
- b) Injection of liquid into a moving gas stream where liquid velocity is equal to gas velocity. Velocities ranged from 610 to 6250 cm/sec.
- c) Injection of liquid into a moving gas stream where stream velocity exceeded liquid velocity over a range of 1,525 to 12,078 cm/sec.
- d) Same as (c), but with gas stream accelerations of  $8.33 \times 10^5$  to  $192.15 \times 10^5$  cm/sec<sup>2</sup> present and gas stream velocities over a range of 5,399 to 13,725 cm/sec. A

few trials were also made employing gas stream deceleration.

For case (b) no acceleration is imposed on the liquid jet by the free stream, whereas in case (c) relative velocity differences impose accelerations on the liquid jet. In case (d) both the relative velocity differences and free stream acceleration produce jet acceleration. All velocities and velocity differences were below sonic velocity.

Ingebo's measure of the fineness of the atomized liquid is the ratio of orifice diameter to maximum drop diameter. This ratio was correlated to six dimensionless numbers which characterize the ratios of various forces which either produce or retard liquid jet instability. These forces are:

- a) Hydrostatic or gravity forces on the liquid jet.
- b) Internal liquid jet hydrodynamic forces.
- c) External gas stream hydrodynamic forces, i. e., dynamic pressure forces.
- d) Gas stream acceleration forces.
- e) Internal jet viscous forces.
- f) Liquid jet surface tension forces.
- g) External gas stream viscous forces.

It is possible to combine these forces in various ratios to form non-dimensional numbers. These numbers can then be used as parameters in a correlation expression for maximum drop diameter, namely:

$$\begin{aligned} \frac{D_o}{D_m} &= f(B_o, Re_l, Re_g, We_l, We_g, A_c) \\ &= C_1 + C_2 B_o^a Re_l^b Re_g^c We_l^d We_g^f A_c^h \end{aligned} \quad (30)$$

where

$D_o$  = orifice diameter

$D_m$  = maximum drop diameter

$B_o = \rho_l D_o^2 g / \sigma$  = Bond gravitational acceleration number

$Re_l = \rho_l D_o v_l / \mu_l$  = Liquid jet Reynolds number

$Re_g = \rho_g D_o v_g / \mu_g$  = Gas stream Reynolds number

$We_l = \rho_l D_o (v_g - v_l)^2 / \sigma$  = Liquid jet Weber number

$We_g = \rho_g D_o (v_g - v_l)^2 / \sigma$  = Gas stream Weber number

$A_c = \rho_g D_o^2 a / \sigma$  = Aerodynamic acceleration number

and

$\rho_l$  = liquid density

$\rho_g$  = gas density

$\mu_l$  = liquid viscosity

$\mu_g$  = gas viscosity

$v_l$  = liquid jet velocity

$v_g$  = gas stream velocity

$\sigma$  = surface tension of liquid

$g$  = gravitational acceleration

$a$  = gas stream acceleration

Separate correlations were obtained for each of the four separate types of injection described above. The correlation which includes free stream acceleration also fits with good accuracy the three other less general cases studied. Therefore, it may be inferred that each new set of forces imposed on the liquid jet is additive and extend the ratio  $D_o/D_m$  to higher values, i. e., relative velocity and stream acceleration each

tend to reduce the maximum drop diameter.

Ingebo's correlation model and fitting technique are quite straightforward and serve as a model for approaches of this type. His results clearly point out that increasing the relative velocity between the jet and stream and increasing the stream acceleration will both reduce  $D_m$ . Comparison is made with two other correlation relations obtained by other investigators. Where comparison is possible, agreement is fairly good.

Since cross-stream injection is probably of primary interest in this project, it is unfortunate that Ingebo's carefully detailed report considers only cocurrent injection. His results cannot be assumed to be applicable to cross-stream injection unless the major portion of the shedding process takes place after the jet trajectory has been shifted to a nearly co-stream direction, and if we consider only cases where

$V_g \gg V_c$  . Unfortunately, it appears that considerable shedding takes place in cross-stream injected jets before the jet trajectory has been shifted toward a co-stream direction.

### III. CONCLUSIONS AND RECOMMENDATIONS

There appears to be reasonable agreement between two predictive models for cross-stream injection discussed in this report, i. e., the theoretical model developed by Adelberg [2], [3], and the empirical correlation model developed by Ingebo and Foster [7]. Mean and volume-median drop sizes calculated using these models are in reasonable agreement but indicate that there may be some difficulty in producing drops in the 40 micron range. Clearly, every attempt must be made to minimize the surface tension and viscosity of the injected liquid and to minimize orifice diameter. Mechanical, design, and nozzle clogging considerations will fix a lower limit on the orifice diameter which can be employed. This limit has not been determined at this time. However, Ingebo [8] has employed orifice diameters as small as 0.0254 cm. for injecting ethanol, and Ingebo and Foster [7] have employed orifice diameters of 0.0254 to 0.01016 cm. for the injection of isooctane, JP-5, benzene, carbon tetrachloride and water. If we assume that an orifice diameter of 0.050 cm. is realizable, then liquid viscosity must be less than about 0.002 dyne-sec/cm<sup>2</sup> if the surface tension is about 50 dynes/cm or less. This suggests that every attempt should be made to reduce the viscosity and surface tension of the injected liquid by preheating. The surface tension and viscosity of liquids decrease rapidly in the temperature range of 0° - 80°C. This behavior should be exploited to reduce mean droplet sizes.

We recommend that further efforts by LS & R be directed toward the development of:

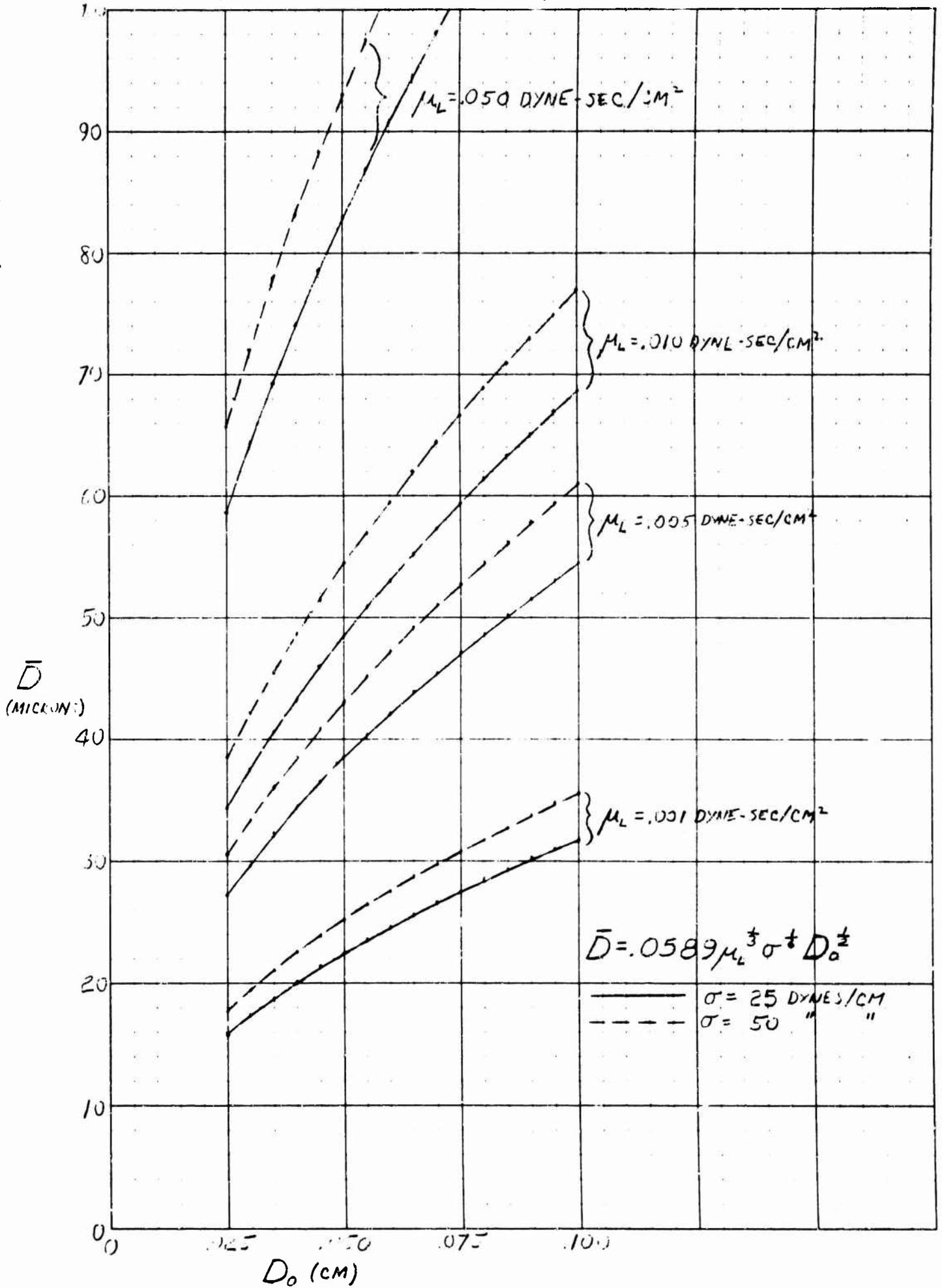
- a) An empirically oriented correlation model for predicting mean drop sizes. The primary utility of this model will be in structuring the experimental program to be undertaken at AFCRL. Dimensional analysis techniques will be employed to determine those particular groupings of physical parameters which are the most relevant and reliable indicators of mean drop size or drop size distribution, and how those physical parameters are to be varied in a series of experiments. These efforts should insure that sufficient data of the correct type is taken to permit the verification of the correlation model and a theoretically based model.
- b) A theoretically based prediction model. This model will use Adelberg's prior work as a basis. Improvement will be sought by removing some of the restrictive assumptions employed by Adelberg merely for the sake of convenience. These include:
- i) The assumption that the jet cross section is nearly circular.
  - ii) The assumption that the speed of the jet is constant.
  - iii) The assumption that the probability that a capillary wave occurs in a given wavelength range from  $\lambda$  to  $(\lambda+d\lambda)$  is uniform.
- The removal of these assumptions can only be accomplished

by resorting to computer based numerical techniques. That is, it will no longer be possible to employ a solely analytical approach and obtain explicit formulae for mean drop size, etc. It should be emphasized that the degree and validity of any improvements can only be evaluated by correlating this model with actual wind tunnel data. Clearly, if the improved model yields significantly better correlation with AFCRL obtained data than does Adelberg's model, this effort will have been successful.

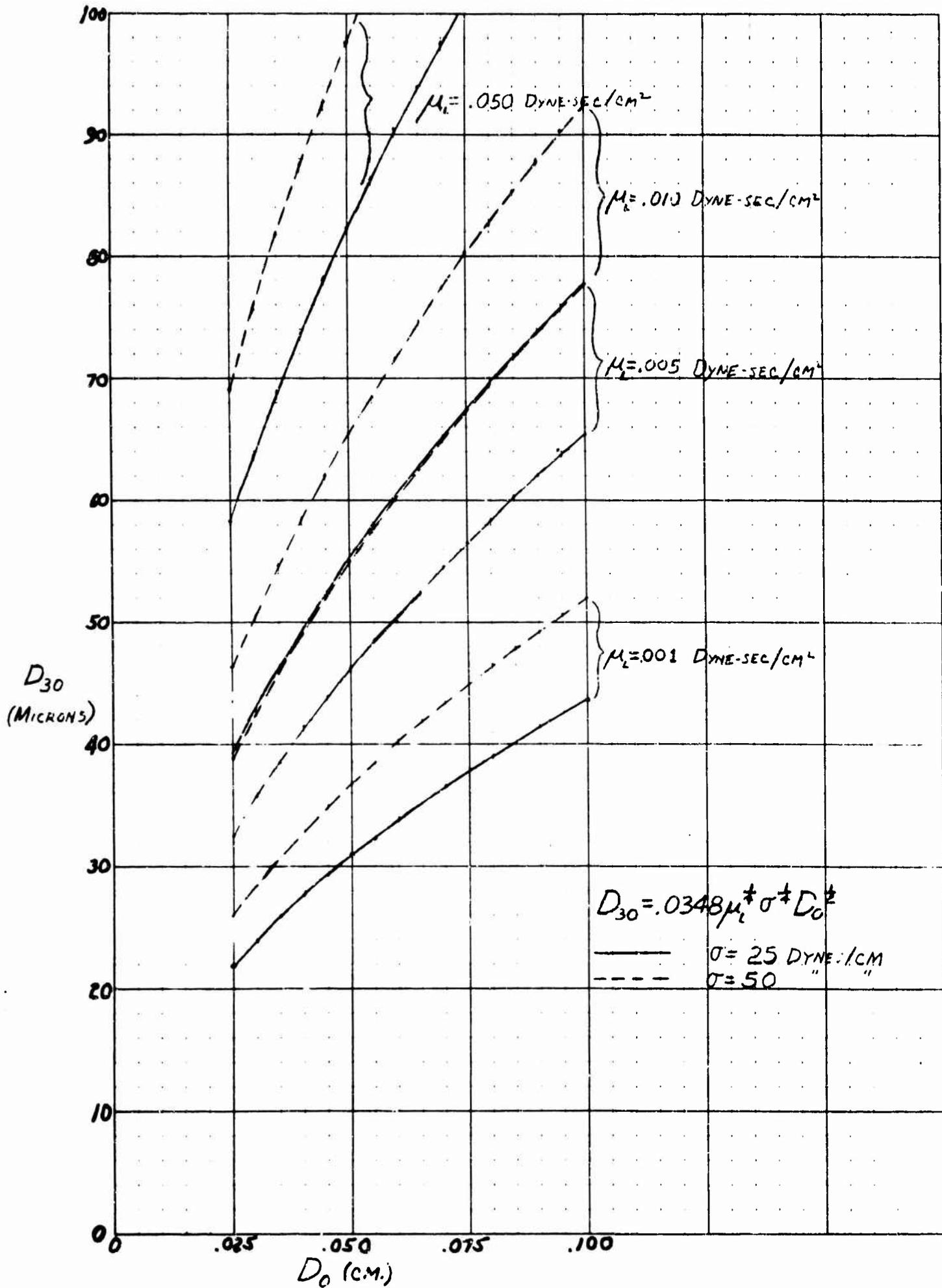
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FIGURE 1. MEAN DROPLET SIZE VS JET ORIFICE DIAMETER



32 FIGURE 2. VOLUME-MEDIAN DROP DIAMETER VS JET ORIFICE DIAMETER



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13. ABSTRACT <p>This report includes a review of the technical literature relating to the modelling of the atomization process, an analysis of the differences between conditions studied in the reports and those found in jet exhausts, and, finally, recommendations for follow-up studies designed to modify early models as a consequence of those differences.</p>		

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