ERRORS IN THE TRAIL RESULTING FROM IGNORING EITHER THE MEASURED TIME OF FLIGHT OR THE MEASURED RANGE

R. H. Kent, et al

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

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by

R. H. Kent
F. V. Reno

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U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
BALLISTIC RESEARCH LABORATORY
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MEASURED TIME OF FLIGHT OR THE MEASURED RANGE

Abstract

Expressions for the errors in the trail, range, deflection and time of flight resulting from ignoring either the measured time of flight or the measured range are deduced. Results of calculations of the resulting errors in trail, time of flight, range and deflection for the bomb M38A2 are presented. The desirability of the introduction of a ballistic coefficient $C$ based on trail is mentioned. The reality and the sources of the discrepancy between the ballistic coefficients deduced from range and time of flight respectively are discussed.

Two different procedures have been used to determine the trail of a bomb from observations on certain elements of its trajectory. One of these procedures, designated by Procedure A, uses measurements of the range and ignores the measured time of flight. The other procedure, designated by B, uses measurements of the time of flight and ignores the range. In the following we shall not attempt to describe the complicated methods actually used but for the sake of clarity and brevity shall describe other procedures theoretically simpler which might be used for the determination of the trail from observations on the range or the time of flight.

* Although the methods to be described are simpler in theory than those actually used, they are much more laborious in practice.
We adopt a coordinate system fixed with respect to the earth with origin at the vertical projection of the airplane on the ground and with the vector ground speed as the X axis. (See Fig. 1).

![Airplane at Instant of Release](image)

Fig. 1

We adopt symbols as follows.

- $C_x =$ ballistic coefficient determined from $x$.
- $C_t =$ ballistic coefficient determined from $t$.
- $t_w =$ time of flight under ballistic table conditions.
- $u =$ resultant air speed.
- $u_x =$ range component of air speed.
$u_z =$ deflection component of air speed.
$\nu =$ ground speed.
$x_\omega =$ range under ballistic table conditions.
$x_{\omega b} =$ range under bombing table conditions.
$x_{\omega A} =$ range under bombing table conditions using data obtained by Procedure A.
$x_{\omega B} =$ range under bombing table conditions using data obtained by Procedure B.
$y_\omega =$ altitude of airplane at release.
$z_{\omega b} =$ deflection under bombing table conditions.
$z_{\omega A} =$ deflection under bombing table conditions using data obtained by Procedure A.
$z_{\omega B} =$ deflection under bombing table conditions using data obtained by Procedure B.
$\Delta t =$ error in $t_\omega$.
$\Delta x =$ error in $x_\omega$.
$\Delta z =$ error in $z_{\omega b}$.
$\Delta \lambda =$ error in $\lambda$.
$\lambda =$ trail under bombing table conditions.
$\lambda_A =$ trail under bombing table conditions obtained from data of Procedure A.
$\lambda_B =$ trail under bombing table conditions obtained from data of Procedure B.
$\lambda_x =$ $x$ component or range component of $\lambda$.
$\lambda_z =$ $z$ component or deflection component of $\lambda$.
$\lambda_{xA} =$ $x$ component of $\lambda_A$.
$\lambda_{ZA} =$ $z$ component of $\lambda_A$.
$\lambda_{xB} =$ $x$ component of $\lambda_B$.
$\lambda_{ZB} =$ $z$ component of $\lambda_B$. 

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Procedure A

The procedure is described in the following. The range is obtained from direct measurements of the position of the airplane at the instant of release and the coordinates of the point of fall taken with respect to the earth. The range obtained in this way is then corrected to ballistic table conditions. These latter assume no wind, standard density and temperature structure and no rotation of the earth. The range corrected to these ballistic table conditions is designated by $x_w$. The ballistic coefficient is determined from $x_w$ and is designated by $C_x$. The time of flight is now computed from $C_x$ and is designated by $t_w(C_x)$. The trail $\lambda_A$ is then computed by the relation

$$\lambda_A = v_g t_w(C_x) - x_w = ut_w(C_x) - x_w. \quad (1)$$

Since there is no wind, $u = v_g$ and $\lambda_A = \lambda_{xA}$. In general, however, $t_w \neq t_w(C_x)$ since the ballistic coefficient determined from range $C_x$ differs from that determined by the time of flight, $C_t$. Let $\Delta t = t_w(C_x) - t_w$ and let $\Delta \lambda$ = the error in the trail.

Since the correct trail under both bombing table and ballistic table conditions is given by

$$\lambda = u t_w - x_w$$

it follows that,

$$\Delta \lambda = \lambda_A - \lambda = u t_w(C_x) - ut_w = u \Delta t. \quad (2)$$

Suppose that bombs are subsequently dropped, under bombing table conditions with $\lambda_A$ and $t_w(C_x)$ as determined by Procedure A. The bombing table conditions assume a standard density and temperature structure and no rotation of the earth. The wind is constant in intensity and direction but not necessarily zero. The air speed, $u$, is assumed to be equal to that obtained when the trail $\lambda_A$ was determined and of course the altitude $y_w$ is the same. The air speed, $u$,
makes an angle, $\phi$, with the ground speed $v$ as shown in Fig. 2. The angle $\phi$ is the azimuth of the trail and under the stated conditions is equal to $\psi$ the drift angle. The correct range under standard bombing conditions, $x_{wb}$, is given by

$$x_{wb} = v_g t\omega - \lambda \cos \phi$$

while the range, $x_{wA}$, obtained by using $\lambda_A$ and $t\omega(C_x)$ will be

$$x_{wA} = v_g t\omega(C_x) - \lambda_A \cos \phi = v_g (t\omega + \Delta t) - (\lambda + u\Delta t) \cos \phi$$

from (2).
Hence the error in the range, \( \Delta x \), from using Procedure A is

\[
\Delta x = x_{w_A} - x_{w_B} = v_g(t_w + \Delta t - t_w) - (\lambda + u \Delta t - \lambda) \cos \varphi
\]

\[
= v_g \Delta t - u \Delta t \cos \varphi = \Delta t(v_g - u_x) = \Delta t w_x
\]

where \( u_x \) is the \( x \) component of \( u \) and \( w_x \) is the \( x \) component of the constant wind. If the ground speed is equal to the air speed component, \( u_x \),

\[
\Delta x = 0.
\]

In other words although Procedure A produces errors both in \( t_w \) and \( \lambda \), the resultant range will be correct unless \( v_g \neq u_x \).

The deflection under bombing table conditions, \( z_{\omega_B} \), is given by

\[
\lambda \sin \varphi
\]

while according to Procedure A the deflection would be

\[
z_{\omega_A} = \lambda_A \sin \varphi = (\lambda + u \Delta t) \sin \varphi.
\]

Hence the error in the deflection, \( \Delta z \), is

\[
\Delta z = (\lambda + u \Delta t) \sin \varphi - \lambda \sin \varphi = u \Delta t \sin \varphi = u_z \Delta t = - w_z \Delta t
\]

where \( u_z \) is the \( z \) component of \( u \) and \( w_z \) is the \( z \) component of the constant wind. (see Fig. 3).
The deflection obtained by Procedure A will be in error unless \( \Delta t = 0 \).

Procedure B

Under Procedure B, the only observationally measured element employed in obtaining a true trail is the time of flight, reduced to ballistic table conditions. The ballistic coefficient, \( C_t \), is calculated directly from the reduced time of flight. It is then assumed that the range is the tabular range \( x(C_t) \) corresponding to \( C_t \), and the trail, \( \lambda_B \), is obtained by the relation

\[
\lambda_B = v_g t_w - x_w (C_t)
\]

The correct trail under the ballistic table conditions* for which the range \( x(C_t) \) is computed, is given by

\[
\lambda = v_g t_w - x_w .
\]

Hence the error in the trail \( \Delta \lambda \) is given by

\[
\Delta \lambda = x_w - x_w (C_t) .
\]

* Of course the magnitude of the trail under standard ballistic conditions is equal to the magnitude under standard bombing conditions since the magnitude of the trail depends upon the air speed not the ground speed.
Under bomb ing table conditions, \( \Delta x \) the error in the range is given by

\[
\Delta x = \left[ x_\omega(C_t) - x_\omega \right] \cos \phi = \left[ x_\omega(C_t) - x_\omega \right] \frac{v - w_x}{u}
\]

The error in deflection obtained by Procedure B is given by

\[
\Delta z = - \left[ x_\omega(C_t) - x_\omega \right] \sin \phi = \left[ x_\omega(C_t) - x_\omega \right] \frac{w_y}{v}
\]

**A Correct Procedure**

A correct procedure is of course to measure both \( t_\omega \) and \( x_\omega \). The range and time of flight thus obtained can be reduced to the values which would have resulted under ballistic table conditions by removal of the effects on range and time of flight due to departures from standard ballistic table conditions. The trail under ballistic table conditions is accurately given by

\[
\lambda = v \frac{t_\omega}{G} - x_\omega
\]

where the values of \( x_\omega \) and \( t_\omega \) are the values of the measured range and time of flight reduced to ballistic table conditions.

The trail, in a perfectly general sense, is the perpendicular distance in the horizontal plane \( y = 0 \) from the point of impact to the vertical through the point vector \( v G w \). The definition arises from the phenomenon noted by the bombardier that the bomb in flight "trails" behind the vertical through the bomb rack.

In reducing the observations and preparing tables of the trail as a function of altitude it is sometimes convenient to introduce a coordinate system moving with the airplane as shown in Fig. 4. In this system, the trail is simply the distance from the \( y \) axis to the point of fall. The initial velocity of the bomb is zero. At the airplane and, if bombing table conditions hold, at other points of the trajectory there is a wind in this coordinate system equal in magnitude to the airspeed of the airplane but opposite in direction. If bombing table conditions do not hold, the wind varies along the trajectory.
Trajectories may be computed with such a coordinate system, the advantages of which will be discussed in a report to appear shortly. In such computations the ballistic coefficient which with the observed atmospheric structure produces a trail equal in magnitude and direction to the observed trail might be called the trail ballistic coefficient. However, in general it is impossible with a given C to reproduce exactly both the magnitude and direction of the trail. This results either from inaccurate measurements or an inadequate theory or a combination of the two. For practical purposes in the reduction of observations that ballistic coefficient which produces a trail under the observed meteorological structure and initial conditions equal in magnitude to the component of the observed trail in a direction determined by the wind structure is called the trail ballistic coefficient and is designated by $C_a$. 

Fig. 4
A direct inference of $C_\lambda$ can be made from a measured trail most conveniently from a table of $\lambda$ as a function of $u$, $y$, and $C$. Such a table is being prepared and will be employed during the bomb test firing program at Aberdeen Proving Ground during the current year. In lieu of such a table, several equispaced values of $\lambda$ as a function of $u$, $y$, and $C$ can be constructed from existing tables of $x$ and $t$ from which the value of $C_\lambda$ can be determined by inverse interpolation for any particular observed case. As a convenient and accurate approximation derivable from theory, accurate to within the individual probable error of one determination of $C_\lambda$ it has been found that

$$C_\lambda = \frac{2C_x C_t}{C_x + C_t}.$$

The trail ballistic coefficient, once determined from observations, can be employed as argument in entering a table of trail angles in the construction of bombing tables, thus obviating the laborious construction of separate ranges and times of flight as functions of the $C_x$ and $C_t$ inferred from observation. A table of this kind, as one of a series of "Bomb Ballistic Auxiliary Tables" is being prepared, and the method is being currently employed in the production of bombing tables at Aberdeen Proving Ground.

A plot of $C_\lambda$ vs $y$ for the bomb M38A2 is shown on plot 2.

**NUMERICAL EXAMPLES**

Table I shows the values of $\Delta \lambda$, $\Delta t$, $\Delta x$ and $\Delta z$ the errors under bombing table conditions in the trail, time of flight, range and deflection respectively for the bomb M38A2 for various conditions when the trail is determined by Procedure A. The values of $C_t$ and $C_x$ on which these results are based are given in plot 1.
Table I

Errors of Procedure A

Bomb M38A2, True Air Speed = 170 mi/hr

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Ground Speed</th>
<th>Range Wind</th>
<th>Cross Wind</th>
<th>$\Delta \lambda$</th>
<th>$\Delta t$</th>
<th>$\Delta x$</th>
<th>$\Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>42</td>
<td>-0.17</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>42</td>
<td>-0.17</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>15,000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>57</td>
<td>-0.23</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>57</td>
<td>-0.23</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>15,000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>67</td>
<td>-0.27</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>67</td>
<td>-0.27</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table II shows the errors under bombing table conditions in trail, $\Delta \lambda$, range, $\Delta x$, and deflection, $\Delta z$, resulting from the substitution of $x_w(C_t)$ for $x_w$ according to Procedure B.

Table II

Errors of Procedure B

Bomb M38 A2, True Airspeed = 170 mi/hr

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Ground Speed</th>
<th>Range Wind</th>
<th>Cross Wind</th>
<th>$\Delta \lambda$</th>
<th>$\Delta x$</th>
<th>$\Delta z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>62</td>
<td>-62</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>62</td>
<td>-61</td>
<td>9</td>
</tr>
<tr>
<td>10,000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>72</td>
<td>-72</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>72</td>
<td>-71</td>
<td>11</td>
</tr>
<tr>
<td>15,000</td>
<td>170</td>
<td>0</td>
<td>0</td>
<td>76</td>
<td>-76</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>193.15</td>
<td>25</td>
<td>25</td>
<td>76</td>
<td>-75</td>
<td>11</td>
</tr>
</tbody>
</table>

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Discrepancy between $C_t$ and $C_x$: Sources of Discrepancy

It has been found by comparison of the experimental results for the firing programs of three bombs at Aberdeen Proving Ground, that the difference between $C_t$ and $C_x$ is in general several times the experimental error.

In the reduction of the observations care has been taken to eliminate the important systematic errors. While it is realized that some systematic errors remain, it appears that their magnitude should be much too small to account for the observed discrepancy between $C_t$ and $C_x$.**

The problem of hitting a target can evidently be solved only if the currently tabulated trail angle, time of flight and dropping angle correspond to the considerably differing ballistic coefficients derived for these elements from experiment.

It appears that there are two possible sources of the discrepancy between $C_t$ and $C_x$. In computing bomb trajectories, the bomb is currently assumed to be a particle and the drag function of that particle is assumed to be the Gavre function, $G_1$. The shapes of bombs differ appreciably from the flat based projectile with an ogive of small radius on which the Gavre function is based. Furthermore, the experimental difficulties in determining the drag at velocities below 900 ft/sec are considerable. For these reasons, it is likely that the drag function of the ordinary bomb differs appreciably from $G_1$. The use of an incorrect drag function would of course cause $C_t$ to differ in general from $C_x$.

The bomb is not a particle. It has not only drag but also a cross wind force. These depend upon the angle of yaw. The neglect of this dependence will cause the calculated trajectories to differ from the observed ones and will produce in general a $C_t$ different from $C_x$. In this connection mention is made of the following Ballistic Research Laboratory reports:

* Bomb, Practice, 100 lb. M38A2.
  Bomb, Demolition, 1100 lb. M33.
  Bomb, Demolition, 2000 lb. M34.

** The suggestion has been made that there may be a systematic plotting error. While there doubtless is one, it should not affect the measurement of the ground speed. The error in the position of the airplane caused by it is almost certainly far too small to account for the discrepancy between $C_t$ and $C_x$. **
In the reports listed no account is taken of the initial angular velocity of yaw which results from the considerable angular velocity of the tangent to the trajectory.* Report No. 82 is now being revised by H. P. Hitchcock to include this effect.

Acknowledgments

The inaccuracies inherent in Procedures A and B for obtaining the trail were pointed out by Lt. Col. H. H. Zornig. This paper was written at his suggestion. Hessra: F. S., Martin and R. P. Cronin assisted in the preparation of the tables and plots.

R. H. Kent
R. H. Kent

F. V. Reno
F. V. Reno

* See Darpas, Mem. d. l'Artilleries Franc. Tome. XVI, 4, p. 841 (1937). However, Darpas neglects the cross wind force entirely.
### Full Lift Coefficients as a Function of Altitude, $C_x$ AND $C_d$

<table>
<thead>
<tr>
<th>Degree of Flight</th>
<th>$C_x$</th>
<th>Purpose</th>
<th>Effect of Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>5° 25° 38°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1° 25° 31°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5° 25° 34°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7° 25° 37°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1° 25° 39°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7° 25° 37°</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observer: Brown, Mitchell, Miller
Computer: Bogan, Martin

Object: Fort, Practice, 14, 12, 236, 2
Instructor: Brooks, Otis, et al.

**NOTE**

- $C_x$  
- $C_d$

**PLOT 1**

- $F_{ECx}(y, w)$
- $F_{ECd}(y, w)$
Expressions for the errors in trail, range, deflection and time of flight resulting from ignoring either the measured time of flight or the measured range are deduced. Results of calculations of the resulting errors in trail, time of flight, range and deflection for the bomb M38A2 are given. The desirability of the introduction of a ballistic coefficient, $C$, based on trail is mentioned. The reality and the sources of the discrepancy between the ballistic coefficients deduced from range and time of flight respectively are discussed.