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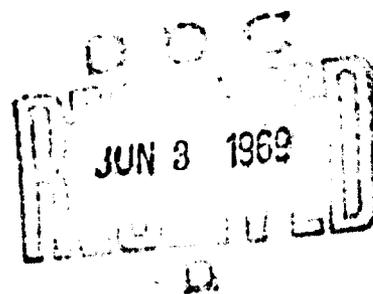
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TECHNICAL REPORT NO. S 1001

SIMULATION WITH MARKOV TRANSITION

MATRIX MODEL OF A

REQUISITION PROCESSING SYSTEM



OPERATIONS RESEARCH BRANCH  
OPERATIONS IMPROVEMENT DIVISION

May 1969

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by Irwin F. Goodman

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SIMULATION WITH MARKOV  
TRANSITION MATRIX MODEL OF  
A REQUISITION PROCESSING SYSTEM

IRWIN F. GOODMAN

ABSTRACT

A cursory review of the literature relating to the application of the Markov Transition Probability Matrix for the evaluation and analysis of problems was accomplished. A FORTRAN IV computer time sharing program, based upon the mathematics of Markov transition Matrices, has been developed and documented. The program was initially developed with data based upon a classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. The resulting Markov Model has been applied to a requisitioning system, an essentially equivalent problem. Some analysis results are presented following the application of the computer program to a requisitioning system. The computer program has been written generally enough for application to such other diverse problem areas as charge accounts, tank battles and reliability and maintainability.

SIMULATION WITH MARKOV  
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A REQUISITION PROCESSING SYSTEM

FOREWORD

The author wishes to acknowledge Mr. Larry Pyles,  
operations research analyst and Mr. Mike Spinelli, management  
analyst for their assistance during preparation of the  
graphs and the flow charts.

SIMULATION WITH MARKOV  
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## A. Introduction

A cursory review of the literature related to Markov transition probability matrix modeling was accomplished. A short bibliography of related books is included in a later section. Some of the more essential Markov techniques of analysis were organized together and then programmed in FORTRAN for solution on a computer time sharing terminal. The purpose of the model is to describe a requisition processing system. An equivalent problem involves the classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. Therefore, initially, the computer program and model were developed and checked out based upon the random walk problem. Then, the model was applied to a requisitioning system.

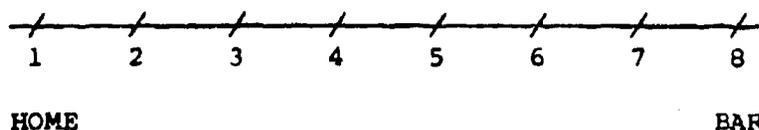
The data used in this report are presented only for the sake of illustrating the theory employed. The findings are consistent with the data presented but therefore not necessarily with the real-world situation.

## B. Markov Chains

A Markov Chain is defined as a probabilistic process in which the probability of moving from one state to another state may depend on the present state, but on no other past history. Classically, this process is usually exemplified in terms of the "wandering drunk" problem which is an example of a random walk.

### "Wandering Drunk" Problem

A long street has eight intersections. A drunk wanders along the street. His home is located at intersection 1 and his favorite bar at intersection number 8.



Intersections 2 - 7 Are Referred To As Street Corners

At each intersection other than his home or the bar, he moves in the direction of the bar with probability  $1/4$  and in the direction of his home with probability  $3/4$ . He never wanders down a side street. If he arrives at his home or the bar, he remains there. When he remains, we say that the process is "absorbed".

Some of the typical questions that an analysis of this problem would address itself to are as follows:

- a. What is the chance that, starting at a given corner, the drunk will end up at his home or at the bar?
- b. If the drunk starts at a particular corner, how many blocks, on the average, will the drunk walk before being "absorbed", that is, arrive at his home or the bar?

Stated in the Markov language of probability:

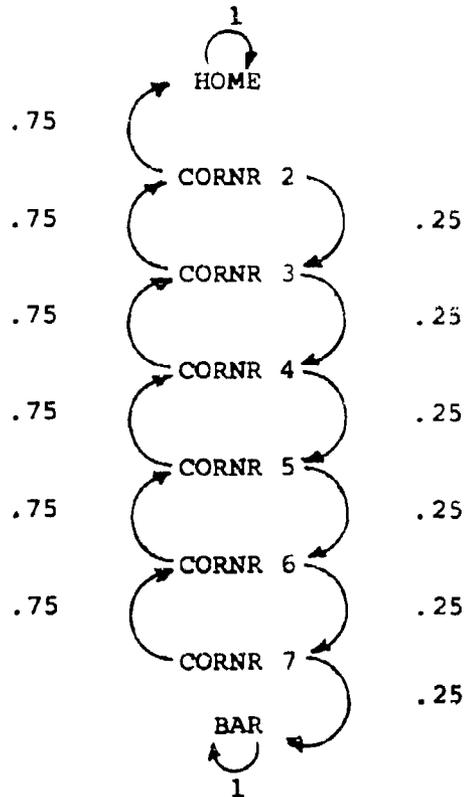
What are the absorption probabilities, and what is the mean time to absorption?

The transition probability matrix and diagram for this problem are as follows:

#### TRANSITION PROBABILITY MATRIX

STATE	HOME	BAR	CORNR 2	CORNR 3	CORNR 4	CORNR 5	CORNR 6	CORNR 7
Home	1	0	0	0	0	0	0	0
Bar	0	1	0	0	0	0	0	0
CORNR 2	.75	0	0	.25	0	0	0	0
CORNR 3	0	0	.75	0	.25	0	0	0
CORNR 4	0	0	0	.75	0	.25	0	0
CORNR 5	0	0	0	0	.75	0	.25	0
CORNR 6	0	0	0	0	0	.75	0	.25
CORNR 7	0	.25	0	0	0	0	.75	0

TRANSITION PROBABILITY DIAGRAM



In summary, the home and bar states are referred to as absorbing states and the other states, CORNR 2 thru CORNR 7, are referred to as transient states. In the case of absorption states, once there, the probability of remaining there is one (certainty). On the other hand with regard to the transition states, it is possible to leave as well as enter them.

The transition probability matrix for the "wandering drunk" problem was processed by the MARK1 computer program. The computer print - out is included in a later section. The results are briefly discussed as follows: Assuming the drunk initially can be at any one of the corners 2 thru 7 (away from the home or bar) with equal probability 1/6, then in the long run he will end up at his home with probability .92 and end up at the bar with probability .08. With regard to the typical questions discussed earlier, the results are as follows:

The probability that starting at a given corner, the drunk will end up at his home or the bar are as follows:

STATE	HOME	BAR
CORNR 2	.999	.001
CORNR 3	.996	.004
CORNR 4	.988	.012
CORNR 5	.963	.037
CORNR 6	.889	.111
CORNR 7	.667	.333

If the drunk starts at a given corner, the quantity of blocks, on the average, that he will walk before arriving at his home or the bar are as follows:

<u>INITIAL</u> <u>STATE</u>	<u>AVERAGE QUANTITY</u> <u>OF BLOCKS TO</u> <u>HOME OR BAR</u>
CORNR 2	2.0
CORNR 3	3.9
CORNR 4	5.8
CORNR 5	7.5
CORNR 6	8.5
CORNR 7	7.3

C. "A Requisition Processing System" Problem

Markov Transition Probability Matrix reasoning and techniques of analysis which were briefly discussed above in terms of a classical random walk problem have been applied to a requisition processing system. The system has been greatly simplified here for the sake of presentation of ideas and the mathematical modeling. The system is primarily automatic, consisting of a high speed automatic data processing computer system and the logic of a highly sophisticated inventory control system which together process the transactions. A requisition represents a demand on the system by a customer for materiel. This type of transaction (requisition) and the many other transactions necessary to keep the records current and decisions reliable are processed by the computer inventory control system.

A successful attempt has been made to mathematically model some of the aspects of such a system. To begin with, only the requisition type transactions were studied. Such a transaction was considered to be in the following states:

Actions completed:

MRO - Material Release Order

PASORD - Passing Order

REJCUS - Reject to Customer

BACKOR - Back-Order

Actions to be completed:

FRSPAS - First-Pass

SUBPAS - Subsequent - Pass

COMPLT - Completed action is defined as any one  
of the above four states (MRO, PASORD,  
REJCUS, BACKOR)

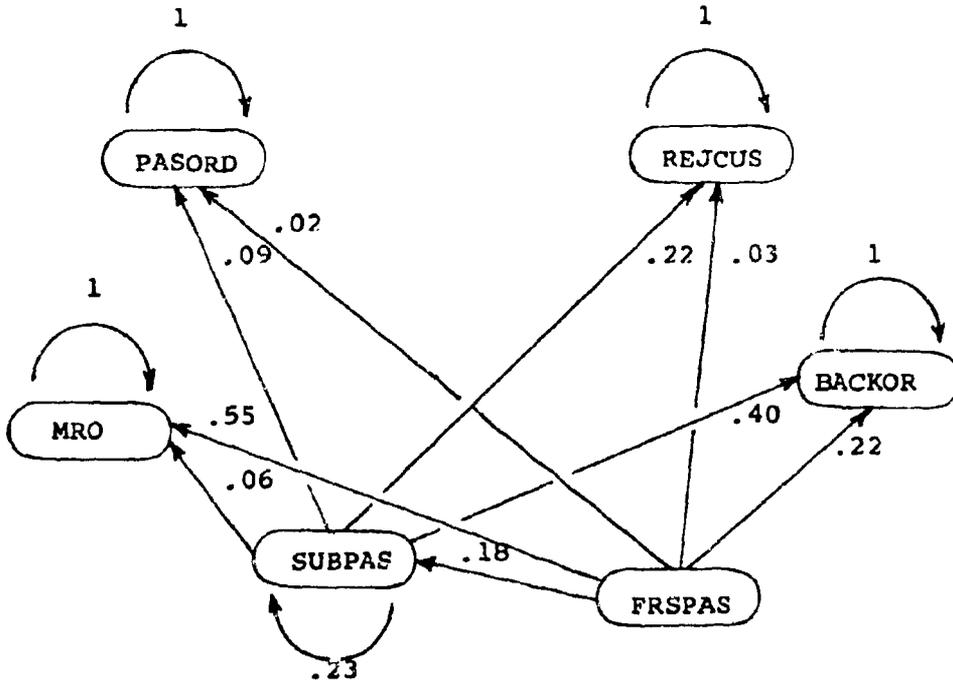
Two versions were modeled, one for all the above states, four absorbing and two transient, and the other one containing only one absorbing state (in place of the four absorbing states) and two transient states (same two transient states). The models are referred to as Model A and Model B. The states considered for each model are shown below:

MODEL A	MODEL B
<u>Absorption States:</u>	<u>Absorption States:</u>
MRO	COMPLT
PASORD	
REJCUS	
BACKOR	
<u>Transient States:</u>	<u>Transient States:</u>
FRSPAS	FRSPAS
SUBPAS	SUBPAS

The Model B version was defined with less states to facilitate a more in-depth variation of parameter study. The results of the variation of parameter study are presented in a later section. The transition probability matrices and diagrams for the Model A and Model B versions of the requisition processing system are as follows:

MODEL A

TRANSITION PROBABILITY DIAGRAM

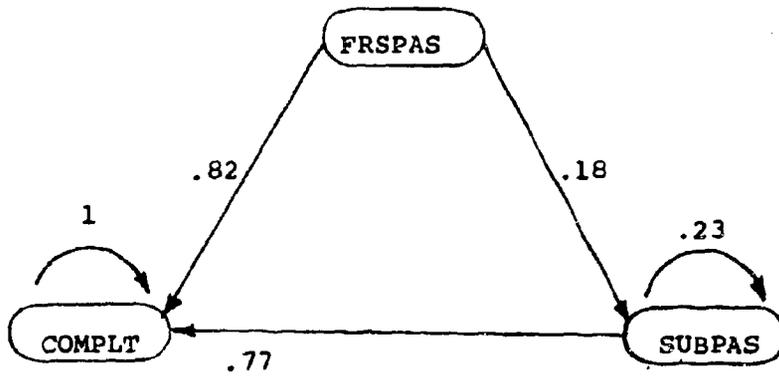


TRANSITION PROBABILITY MATRIX

STATE:	MRO	PASORD	REJCUS	BACKOR	FRSPAS	SUBPAS
MRO	1	0	0	0	0	0
PASORD	0	1	0	0	0	0
REJCUS	0	0	1	0	0	0
BACKOR	0	0	0	1	0	0
FRSPAS	.55	.02	.03	.22	0	.18
SUBPAS	.06	.09	.22	.40	0	.23

MODEL B

TRANSITION PROBABILITY DIAGRAM



TRANSITION PROBABILITY MATRIX

	COMPLT	FRSPAS	SUBPAS
COMPLT	1	0	0
FRSPAS	.82	0	.18
SUBPAS	.77	0	.23

The earlier questions regarding the "drunk" in the classical random walk problem can now be restated in the context of the requisition processing system as follows:

a. Given the transaction is in one of the transient states (first pass or subsequent pass), what is the probability the transaction will end up as a "MRO"?; as a passing order?; as a reject to customer?; as a backorder?

b. Given the transaction is in one of the transient states (first pass or subsequent pass), how many times, on the average, will the transaction be in the subsequent pass state prior to ending up as a MRO?; as a passing order?; as a reject to customer?; as a backorder?

D. Some Results with Model A

1. The Model A of the requisition processing system was processed by the MARK1 computer program. The computer print-out is included in a later section. Some of the results are as follows:

2. The expected number of times of remaining in a transient state prior to being absorbed are obtained from the fundamental matrix. The results are as follows:

a. A first pass (FRSPAS) transaction can be expected to be in a first pass state only once prior to being absorbed. This, agrees as it should with the "common sense" of the system.

b. A subsequent pass (SUBPAS) transaction can be expected to be in a first pass state zero times prior to being absorbed. This also agrees as it should with the "common sense" of the system by implying that once a transaction is a subsequent pass transaction it cannot pass through the first pass state.

c. A first pass (FRSPAS) transaction can be expected to be in a subsequent pass state about .23 times before being absorbed.

d. A subsequent pass transaction can be expected to be in a subsequent pass state 1.30 times prior to being absorbed.

3. The mean number of cycles until absorption is obtained from the T Matrix.

a. The mean number of cycles until completion for a first pass transaction is 1.23.

b. The mean number of cycles until completion for a subsequent pass transaction is 1.30.

4. The probability that a transaction is completed given it was initially in a particular state is obtained from the U Matrix.

a. The probability is .56 that a first pass transaction will become an MRO and .08 that a subsequent pass transaction will become an MRO.

b. The probability is .04 that a first pass transaction will become a passing order and .12 that a subsequent pass transaction will become a passing order.

c. The probability is .08 that a first pass transaction will become a reject to customer and .29 that a subsequent pass transaction will become a reject to customer.

d. The probability is .31 that a first pass transaction will go on back-order and .52 that a subsequent pass transaction will go on back-order.

5. The number of cycles required to reduce the fraction of subsequent pass transactions to 1% or less is obtained from computation of the AM(K) vector.

a. Assuming initially that 20% of the transactions to be processed are subsequent pass, it can be expected to take three (3) cycles for the fraction of subsequent pass transactions to drop to 1% or less. In two (2) cycles the fraction of subsequent pass transactions can be expected to drop to about 4%.

b. The same percentages as above essentially hold even if initially 100% of the transactions to be processed are subsequent pass transactions.

E. A Variation of Parameter Study on Model B.

1. Initially, Model B was run with transition Matrix values as depicted in the earlier section showing the transition matrix diagram. The computer print-out is included in a later section.

2. A variation of parameter study was conducted on Model B. The purpose of the study is to develop charts and graphs depicting relationships between the input and output of Model B over a complete range in values of particular parameters. The parameters involved, their possible range in variation, and the parametric values studied are shown in the following table:

Parameter	Possible Range	Parametric Values Studied
First Pass to	0.0 to 1.0	.01, .05, .10, .15,
Subsequent Pass		.20, .25, .50, .75,
Transition Probability		.99, .999
Subsequent Pass to	0.0 to 1.0	.05, .10, .25, .50
Subsequent Pass		
Transition Probability		

INPUT: Model B was studied for three initial vector conditions.

- (1) (0, 1, 0): 100% first pass transactions and 0% subsequent pass transactions.
- (2) (0, .80, .20): 80% first pass transaction and 20% subsequent pass transactions.
- (3) (0, 0, 1): 0% first pass transactions, and 100% subsequent pass transactions.

OUTPUT: Model B was studied for the following Outputs:

(1) The model was studied for the number of cycles required to yield an output vector such that the percent of subsequent pass transactions (incompleted transactions) is 1% or less. This is equivalent to the number of cycles required to yield an output vector such that the percent of completed transactions is 99% or more.

(2) Also, the average quantity of cycles expected for a first pass transaction and a subsequent pass transaction to be completed were studied.

3. The results are presented on the following charts. Chart E-1 summarizes the results obtained for 100% first-pass input and also for the mixed input (80% first-pass and 20% subsequent pass). The solid curves represent the 100%

first-pass input and the broken curves represent the mixed input.

The other input condition, 100% subsequent pass, is shown on Chart E-2.

The average quantity of cycles expected for a first-pass transaction to be completed is presented on Chart E-3. Chart E-4 shows the average quantity of cycles expected for a subsequent-pass transaction to be completed.

The basic data for preparing Charts E-1 and E-2 is presented in a tabular format on Charts E-5 thru E-8.

Discussion and interpretation of the charts and other findings is being withheld at this time pending further study of the model.

CHART E-1

QUANTITY OF CYCLES REQUIRED SUCH THAT THE PERCENT OF OUTPUT TRANSACTIONS FOR SUBSEQUENT-PASS (INCOMPLETE) IS 1% OR LESS

INITIAL CONDITIONS:

100% FIRST PASS INPUT (0, 1, 0) AND MIXED INPUT (0, .80, .20)

SUBPAS TO SUBPAS VALUE:

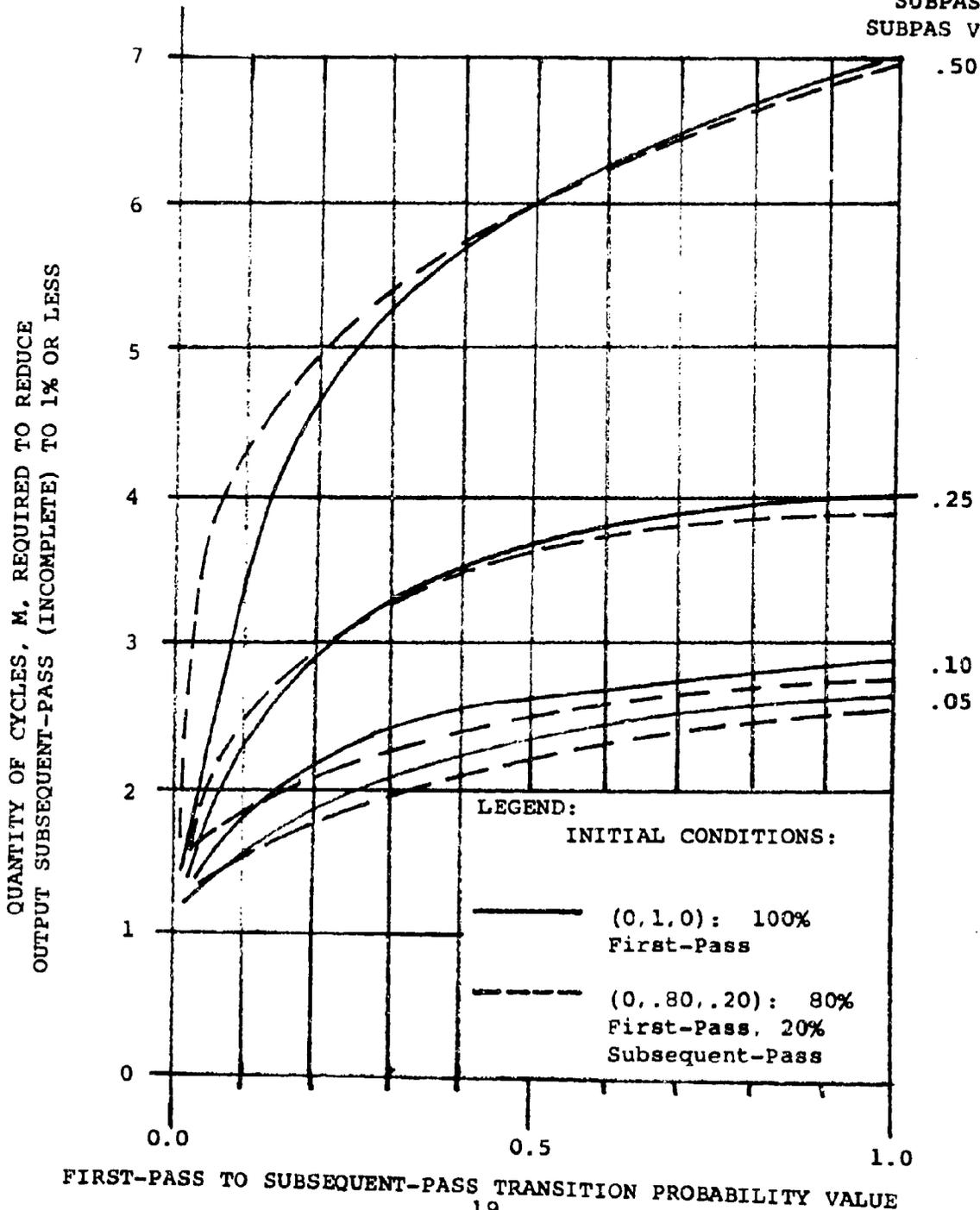
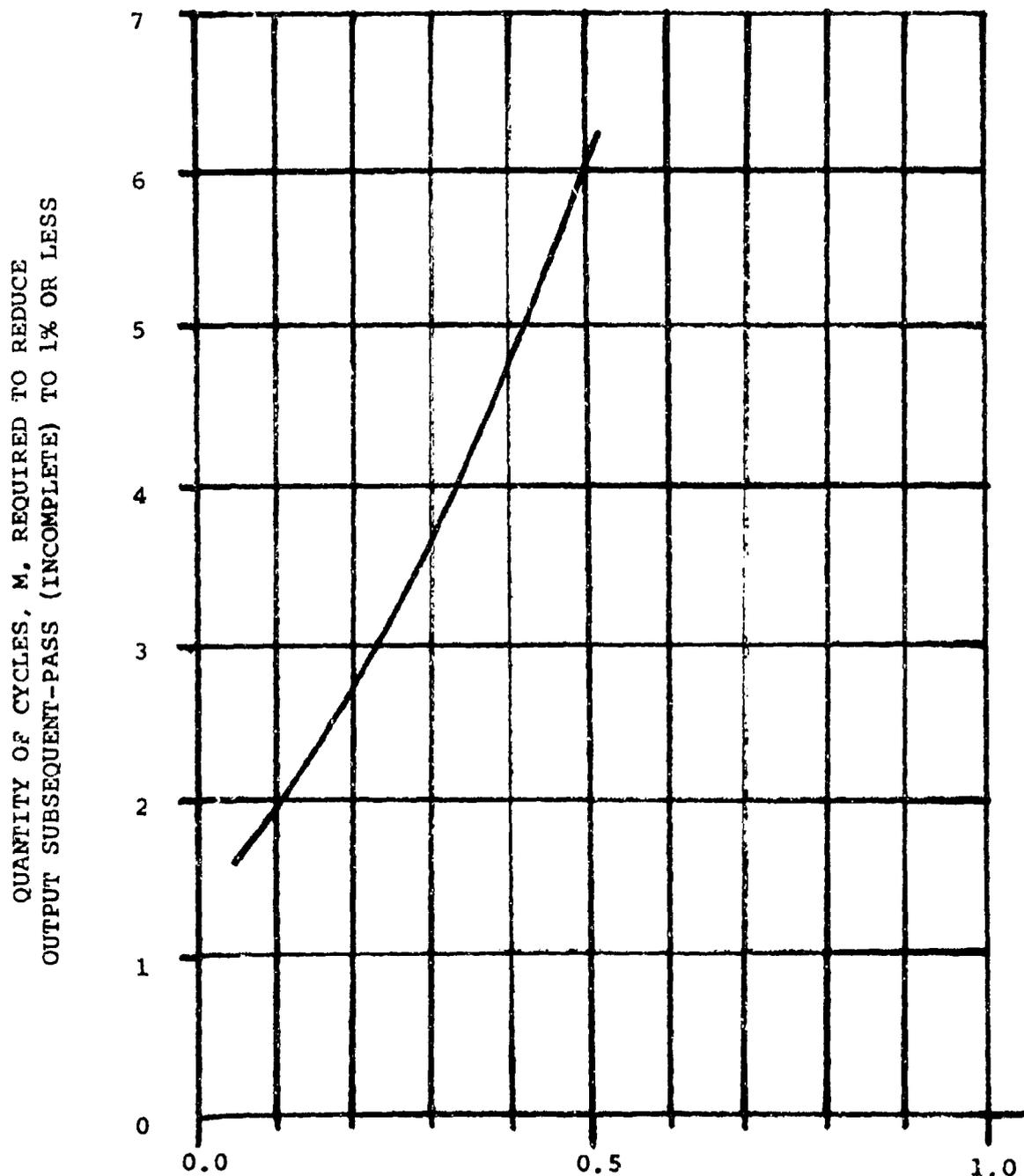


CHART E-2

QUANTITY OF CYCLES REQUIRED SUCH THAT THE PERCENT OF OUTPUT TRANSACTIONS FOR SUBSEQUENT-PASS (INCOMPLETE) IS 1% OR LESS

INITIAL CONDITIONS:

100% SUBSEQUENT-PASS INPUT (0, 0, 1)



SUBSEQUENT-PASS TO SUBSEQUENT-PASS TRANSITION PROBABILITY VALUE

CHART E-3

AVERAGE QUANTITY OF CYCLES EXPECTED FOR A  
FIRST-PASS TRANSACTION TO BE COMPLETED

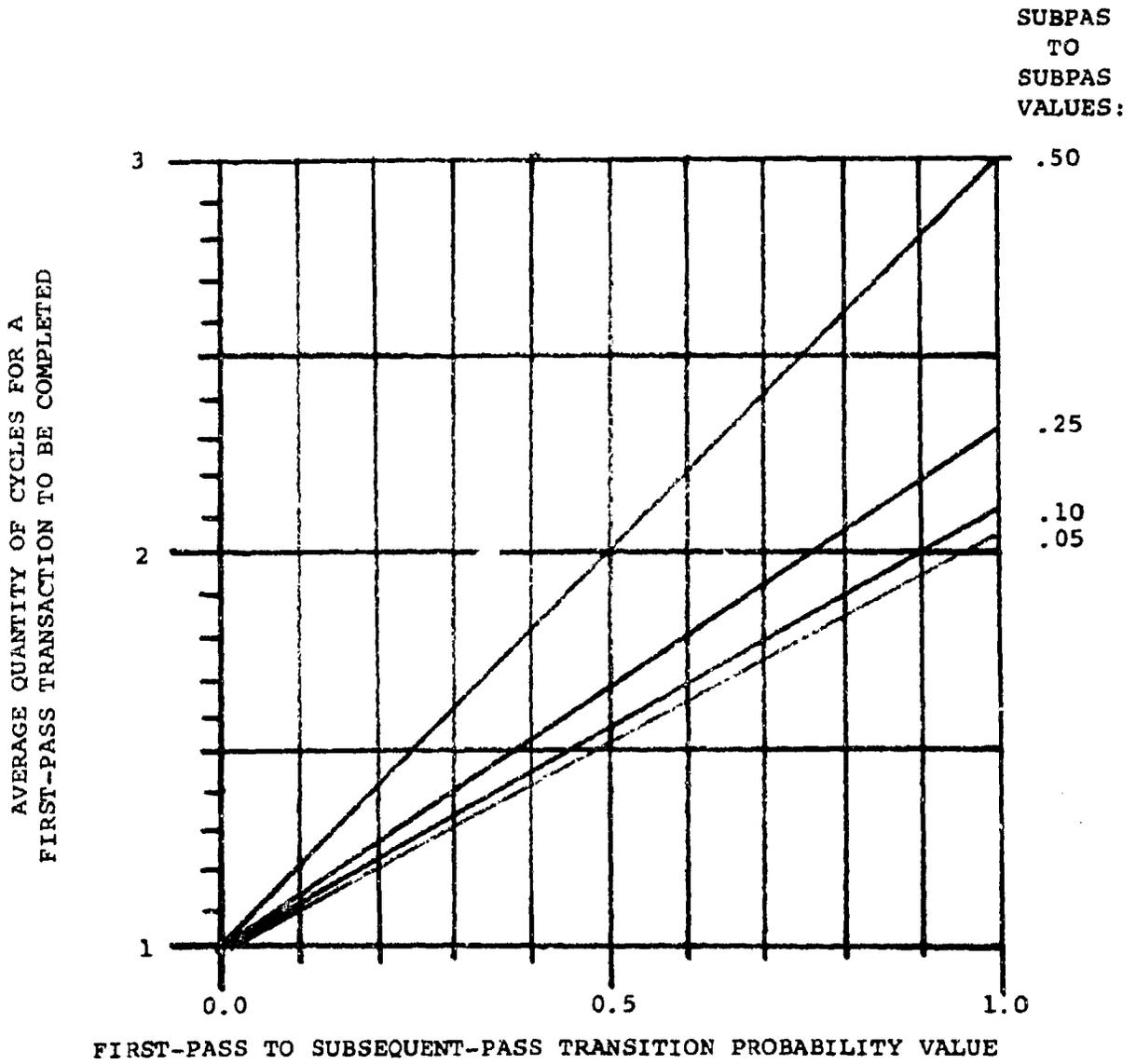


CHART E-4

AVERAGE QUANTITY OF CYCLES EXPECTED FOR A  
SUBSEQUENT-PASS TRANSACTION TO BE COMPLETED

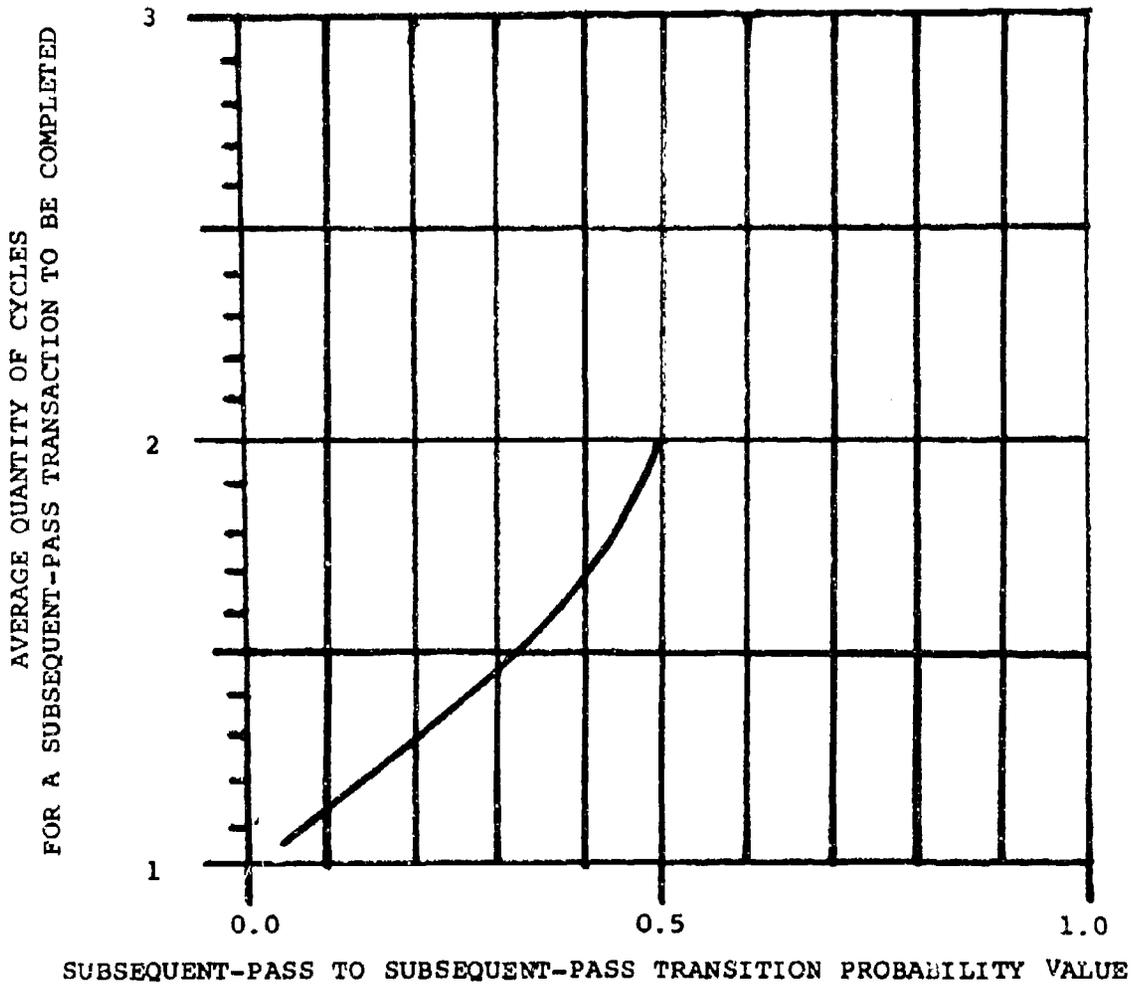


CHART E-5

TABULATION OF BASIC DATA  
 SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE=.05

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
INITIAL CONDITION *: (0, 1, 0)										
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.050	.049	.037	.025	.012	.010	.007	.005	.002	.000
3	.002	.002	.002	.001	.001					
4										
5										
6										
7										
8										
INITIAL CONDITION *: (0, .80, .20)										
1	.810	.810	.610	.410	.210	.170	.130	.090	.050	.002
2	.040	.040	.030	.020	.010	.008	.007	.004	.002	.000
3	.002	.002	.002	.001	.001					
4										
5										
6										
7										
8										
INITIAL CONDITION *: (0, 0, 1)										
1	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
2	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002
3										
4										
5										
6										
7										
8										

\*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-5

TABULATION OF BASIC DATA  
 SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE = .10

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
INITIAL CONDITION *: (0, 1, 0)										
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.100	.100	.075	.050	.025	.020	.015	.010	.005	.001
3	.010	.010	.007	.005	.002	.002				
4										
5										
6										
7										
8										
INITIAL CONDITION *: (0, .80, .20)										
1	.820	.820	.620	.420	.220	.180	.140	.100	.060	.030
2	.082	.082	.062	.042	.022	.018	.014	.010	.006	.003
3	.008	.008	.006	.004	.002					
4										
5										
6										
7										
8										
INITIAL CONDITION *: (0, 0, 1)										
1	.100	.100	.100	.100	.100	.100	.100	.100	.100	.100
2	.010	.010	.010	.010	.010	.010	.010	.010	.010	.010
3										
4										
5										
6										
7										
8										

\*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-7

TABULATION OF BASIC DATA  
 SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE=.25

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
INITIAL CONDITION *: (0, 1, 0)										
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.250	.250	.187	.125	.062	.050	.037	.025	.012	.002
3	.062	.062	.047	.031	.016	.012	.009	.006	.003	.001
4	.016	.015	.012	.008	.004					
5	.004	.003								
6										
7										
8										
INITIAL CONDITION *: (0, .80, .20)										
1	.850	.850	.650	.450	.250	.210	.170	.130	.090	.060
2	.212	.210	.162	.112	.062	.050	.042	.032	.022	.014
3	.053	.053	.041	.028	.016	.013	.010	.008	.006	.004
4	.014	.012	.010	.007	.004					
5	.003									
6										
7										
8										
INITIAL CONDITION *: (0, 0, 1)										
1	.250	.250	.250	.250	.250	.250	.250	.250	.250	.250
2	.062	.062	.062	.062	.062	.062	.062	.062	.062	.062
3	.016	.016	.016	.016	.016	.016	.016	.016	.016	.016
4	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004
5										
6										
7										
8										

\*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

CHART E-8

TABULATION OF BASIC DATA  
 SUB-PASS TO SUB-PASS TRANSITION PROBABILITY VALUE= .50

QTY CYCLES (STEPS)	FIRST PASS TO SUBSEQUENT PASS TRANSITION PROBABILITY VALUES									
	.999	.99	.75	.50	.25	.20	.15	.10	.05	.01
INITIAL CONDITION *: (0, 1, 0)										
1	.999	.990	.750	.500	.250	.200	.150	.100	.050	.010
2	.499	.499	.375	.250	.125	.100	.075	.050	.025	.005
3	.250	.250	.187	.125	.062	.050	.038	.025	.012	.002
4	.125	.125	.094	.062	.031	.025	.019	.012	.006	.001
5	.062	.062	.047	.031	.016	.012	.009	.006	.003	.001
6	.031	.031	.023	.016	.008	.006	.005	.003	.002	
7	.016	.015	.012	.008	.004	.003	.002	.001	.001	
8	.008	.008								
INITIAL CONDITION *: (0, .80, .20)										
1	.892	.892	.700	.500	.300	.260	.220	.180	.140	.110
2	.450	.450	.350	.250	.150	.130	.110	.090	.070	.054
3	.225	.225	.175	.125	.075	.065	.055	.045	.035	.027
4	.112	.112	.087	.062	.037	.032	.028	.022	.017	.013
5	.056	.056	.044	.031	.019	.016	.013	.011	.009	.007
6	.028	.028	.022	.016	.010	.008	.007	.006	.005	.003
7	.014	.014	.011	.008	.005	.004	.003	.003	.001	
8	.007	.007								
INITIAL CONDITION *: (0, 0, 1)										
1	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
2	.250	.250	.250	.250	.250	.250	.250	.250	.250	.250
3	.125	.125	.125	.125	.125	.125	.125	.125	.125	.125
4	.062	.062	.062	.062	.062	.062	.062	.062	.062	.062
5	.031	.031	.031	.031	.031	.031	.031	.031	.031	.031
6	.016	.016	.016	.016	.016	.016	.016	.016	.016	.016
7	.008	.008	.008	.008	.008	.008	.008	.008	.008	.008
8										

\*INITIAL CONDITION (% Computed., % FIRST PASS, % SUBSEQUENT PASS)

F. Mathematical Terminology and Formulae

1. Transition Probability Matrix: A square array of numbers all of whose entries for each row add up to one.

1.1 Example:

		NEXT STATE							
		HOME	2	3	4	5	6	7	BAR
PRESENT STATE	HOME	1	0	0	0	0	0	0	0
	2	3/4	0	1/4	0	0	0	0	0
	3	0	3/4	0	1/4	0	0	0	0
	4	0	0	3/4	0	1/4	0	0	0
	5	0	0	0	3/4	0	1/4	0	0
	6	0	0	0	0	3/4	0	1/4	0
	7	0	0	0	0	0	3/4	0	1/4
	BAR	0	0	0	0	0	0	0	1

1.2 Explanation: The entries for each particular row represent the probabilities of an item going to the corresponding column state given it was initially in the state corresponding to the particular row. i.e., The probability of going from state 4 to state 2 is 0, to state 3 is .750, and to state 5 is .250.

2. Absorbing States: Row probability vectors in the transition probability matrix having "1s" on the diagonal of the matrix and all the other entries are "0s" are referred to as absorbing states, i.e., row "home" and row "bar" are absorbing states.

3. Transient States (non-absorbing): Row probability vectors in the transition probability matrix not having "1s" on the diagonal of the matrix are referred to as transient states. i.e., rows "2", "3", "4", and "5" are transient states.

4. Canonical Form of Transition Probability Matrix: When in the transition probability matrix, all the absorbing states (rows) are grouped together at the top of the matrix with all the "1s" composing an identity matrix and all of the transient states (rows) are together at the bottom of the matrix, then, the transition probability matrix is said to be in canonical form.

4.1 Example:

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.750	0.000	0.000	.250	0.000	0.000	0.000	0.000
CØRNR3	0.000	0.000	.750	0.000	.250	0.000	0.000	0.000
CØRNR4	0.000	0.000	0.000	.750	0.000	.250	0.000	0.000
CØRNR5	0.000	0.000	0.000	0.000	.750	0.000	.250	0.000
CØRNR6	0.000	0.000	0.000	0.000	0.000	.750	0.000	.250
CØRNR7	0.000	.250	0.000	0.000	0.000	0.000	.750	0.000

5. Partitioning Canonical Matrix: The canonical matrix is subdivided as follows:

$$C = \begin{pmatrix} CI & & CØ \\ & & \\ CR & & CQ \end{pmatrix}$$

5.1 CI: Identity Matrix: The number of rows equals the number of columns equals the number of absorbing states.

5.11 Example:

	STATE	HØME	BAR
CI =	HØME	1.000	0.000
	BAR	0.000	1.000

5.2 CØ: Zero Matrix: A matrix containing all zeros and having the number of rows equal to the number of absorbing states and the number of columns equal to the number of transient states.

5.21 Example:

$$CØ = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5.3 CR: Matrix of Transient to Absorbing Probabilities: The entries represent the probability of going from transient state to absorbing state. The number of rows equals the number of transient states and the number of columns equals the number of absorbing states.

5.31 Example:

	STATE	HØME	BAR
CR=	CØRNR2	.750	0.000
	CØRNR3	0.000	0.000
	CØRNR4	0.000	0.000
	CØRNR5	0.000	0.000
	CØRNR6	0.000	0.000
	CØRNR7	0.000	0.250

5.4 CQ: Matrix of Transient to Transient Probabilities:

The entries represent the probability of going from transient state to transient state. The number of rows equals the number of columns equals the number of transient states.

5.4.1 Example:

	STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
CQ	CØRNR2	0.000	.250	0.000	0.000	0.000	0.000
	CØRNR3	.750	0.000	.250	0.000	0.000	0.000
	CØRNR4	0.000	.750	0.000	.250	0.000	0.000
	CØRNR5	0.000	0.000	.750	0.000	.250	0.000
	CØRNR6	0.000	0.000	0.000	.750	0.000	.250
	CØRNR7	0.000	0.000	0.000	0.000	.750	0.000

6. FN: Fundamental Matrix: Each entry, FN (I,J), is the expected number of times in state J (column) before being absorbed, given that the present state is I (row). The number of rows equals the number of columns equals the number of transient states.

$$FN = (NI - CQ)^{-1}$$

NOTE: NI is an identity matrix, established for the computation of FN. NI has the number of rows equal to the number of columns equal to the number of transient states.

6.1 Computation of FN: FN is computed, using the following power series approximation:

$$FN = NI + CQ + (CQ)^2 + (CQ)^3 + \dots$$

6.2 Example:

	STATE	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
FN =	CØRNR2	1.332	.443	.146	.048	.015	.004
	CØRNR3	1.328	1.771	.586	.190	.059	.015
	CØRNR4	1.317	1.757	1.903	.618	.190	.048
	CØRNR5	1.284	1.713	1.855	1.903	.586	.146
	CØRNR6	1.186	1.581	1.713	1.757	1.771	.443
	CØRNR7	.889	1.186	1.284	1.317	1.328	1.332

7. T: Matrix of Absorption Times: Each entry is the mean time to absorption (number of states passed through, including final state and not including initial state, in order to be absorbed). This matrix is a column vector with the number of rows equal to the number of transient states.

7.1 Computation of T: Each entry of T is equal to the row sum of each row of FN. This is accomplished by establishing a column vector,  $\mathbf{1}$  of 1's having the same number of rows equal to the number of transient states.

$$T = (FN) \cdot (\mathbf{1})$$

7.2 Example:

STATE	
T =	CØRNR2 1.987
	CØRNR3 3.949
	CØRNR4 5.833
	CØRNR5 7.487
	CØRNR6 8.450
	CØRNR7 7.337

8. U: Matrix of Absorption Probabilities: Each entry is the probability of being absorbed, given it was initially in a transient state. The number of rows equals the number of transient states, and the number of columns equals the number of absorbing states.

8.1 Computation of U:

$$U = (FN) \cdot (CR)$$

8.2 Example:

	STATE	HØME	BAR
U =	CØRNR2	.999	.001
	CØRNR3	.996	.004
	CØRNR4	.988	.012
	CØRNR5	.963	.037
	CØRNR6	.889	.111
	CØRNR7	.667	.333

9. CQMP: Matrix of Transition Probabilities for M

Steps: Each entry is the probability of going from state to state. The number of rows equals the number of columns equals the number of absorbing and transient (non-absorbing) states.

9.1 Computation of CQMP:

$$CQMP = (C)^M$$

9.2 Example:

For M = 3

CQMP =

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.891	0.000	0.000	.094	0.000	.016	0.000	0.000
CØRNR3	.562	0.000	.281	0.000	.141	0.000	.016	0.000
CØRNR4	.422	0.000	0.000	.422	0.000	.141	0.000	.016
CØRNR5	0.000	.016	.422	0.000	.422	0.000	.141	0.000
CØRNR6	0.000	.062	0.000	.422	0.000	.422	0.000	.094
CØRNR7	0.000	.297	0.000	0.000	.422	0.000	.281	0.000

10. A: Initial State Space Probability Row Vector:

Initial values for each of the possible states (columns).  
The number of rows is one, and the number of columns equals  
the number of absorbing and transient states

10.1 Example:

$$A = (0, 0, 1/3, 1/6, 1/6, 1/6, 1/6, 1/6)$$

11. AM: State Space M Steps Later: Each entry is the  
value for each of the states, M steps later, given initial  
values A.

11.1 Computation of AM:

$$AM = (A) \bullet (QMP)$$

11.2 Example:

For M=3.

STATE	HOME	BAR	CORNR2	CORNR3	CORNR4	CORNR5	CORNR6	CORNR7
	.313	.063	.117	.157	.164	.097	.073	.018

12. Summary: For absorbing Markov chains, the following  
three questions are usually of interest:

- a. What is the probability that the process will end  
up in a given absorbing state?

Answer: Entries in Matrix U.

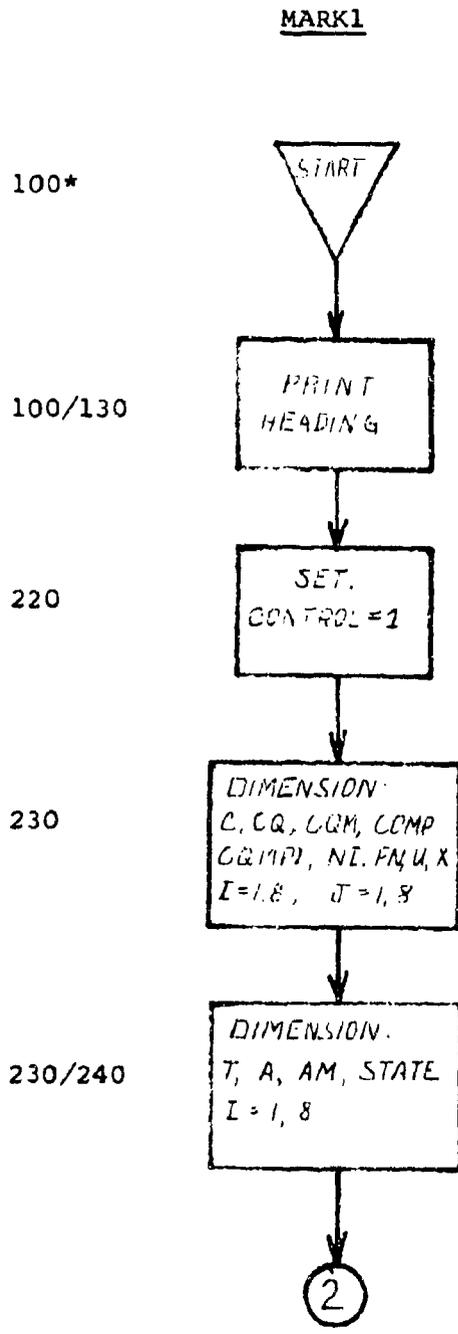
b. On the average, how long will it take for the process to be absorbed?

Answer: Entries in Matrix T.

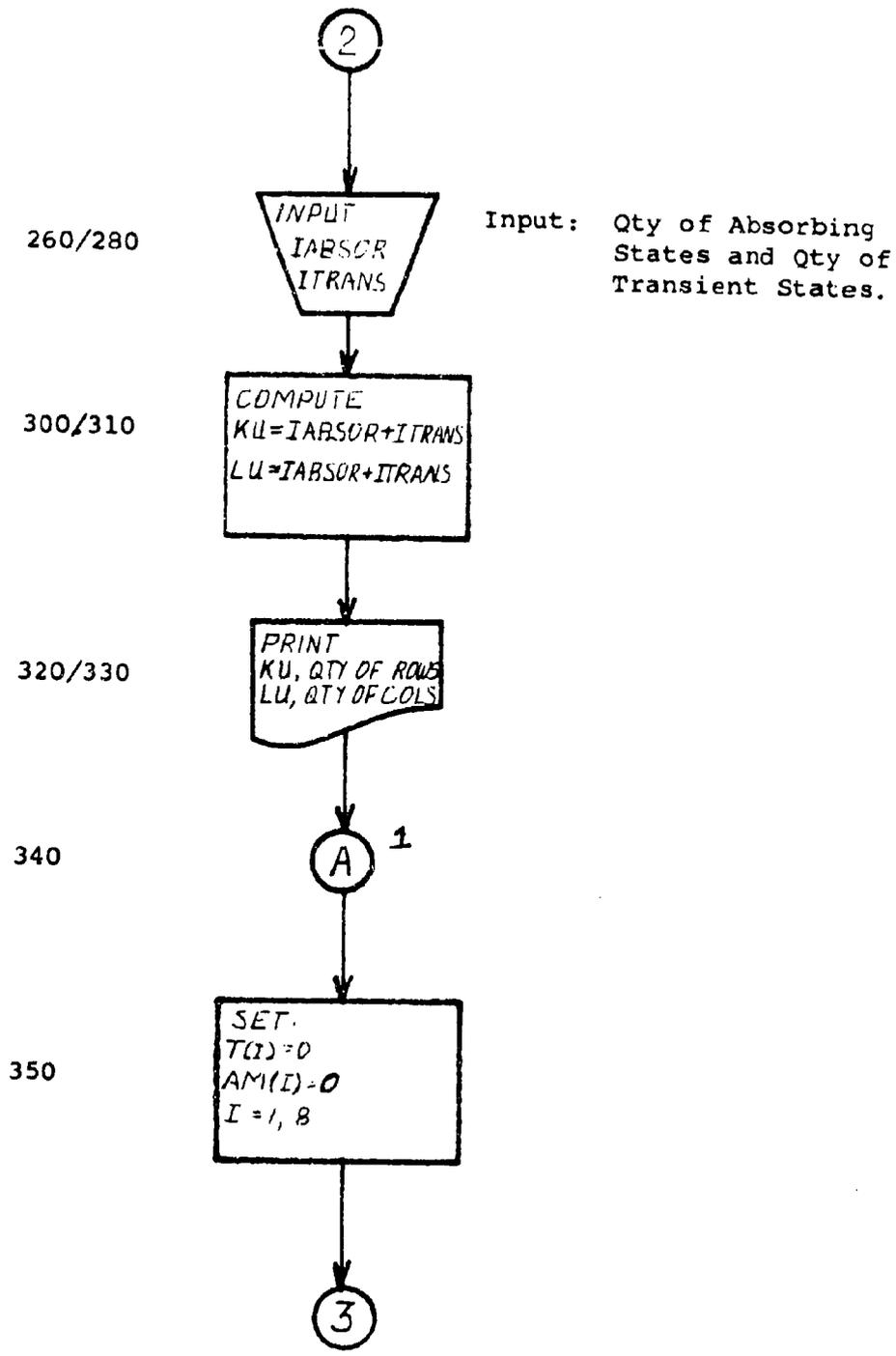
c. On the average, how many times will the process be in each transient (non-absorbing) state?

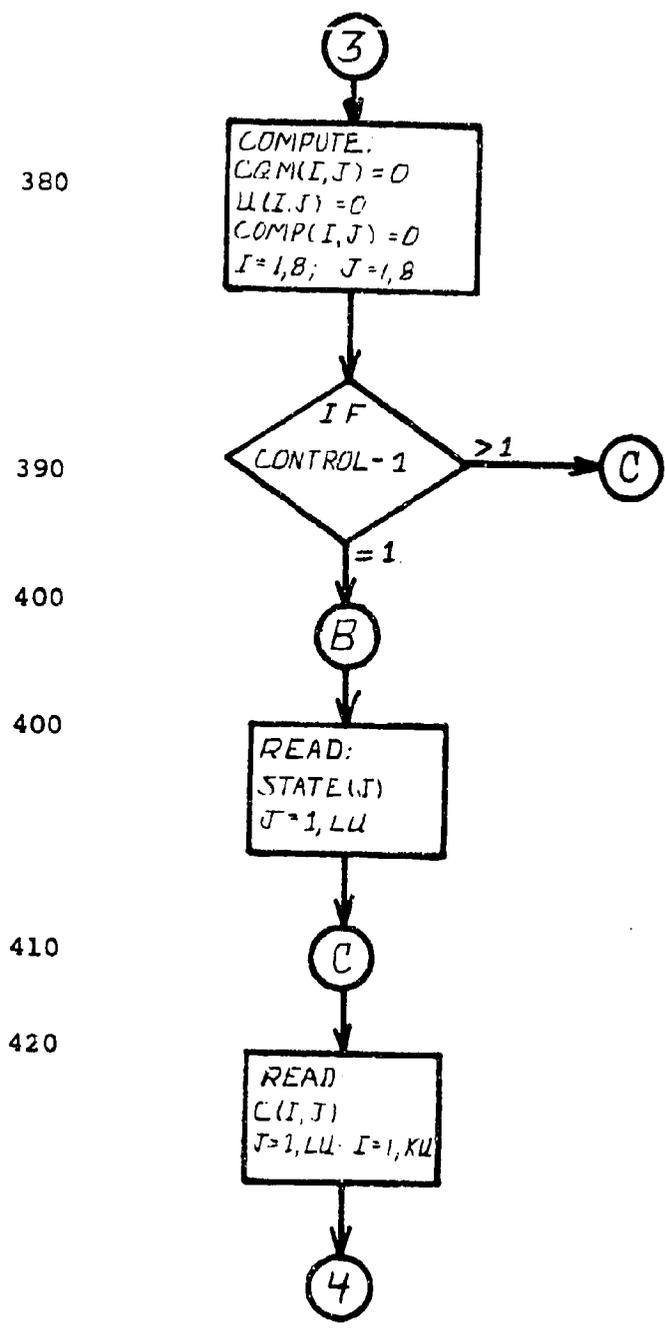
Answer: Entries in Matrix FN.

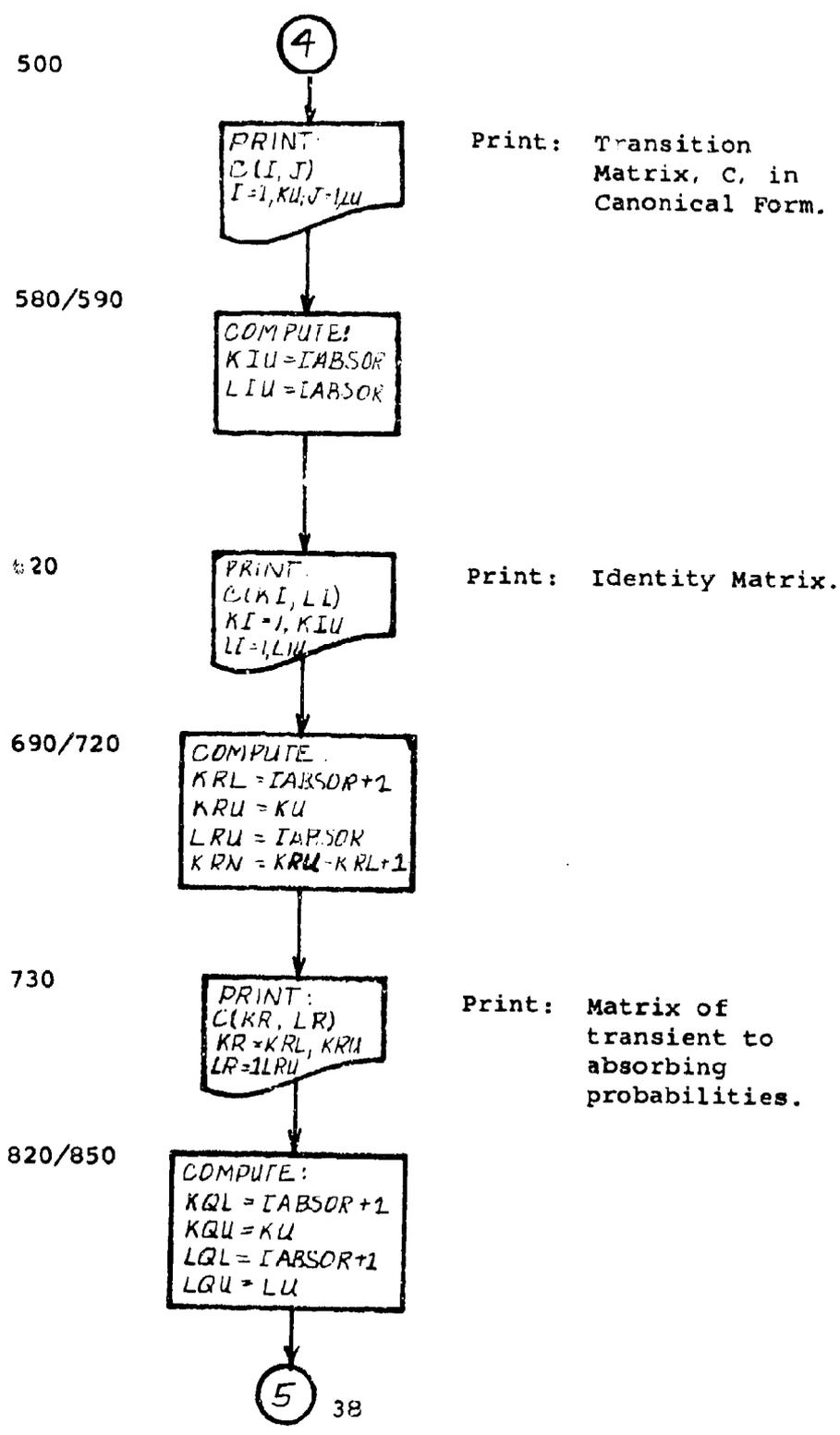
G. Flow chart (Fortran Program - MARK1/2/3)  
Simulation with Markov  
Transition Matrix Model of  
A Requisition Processing System

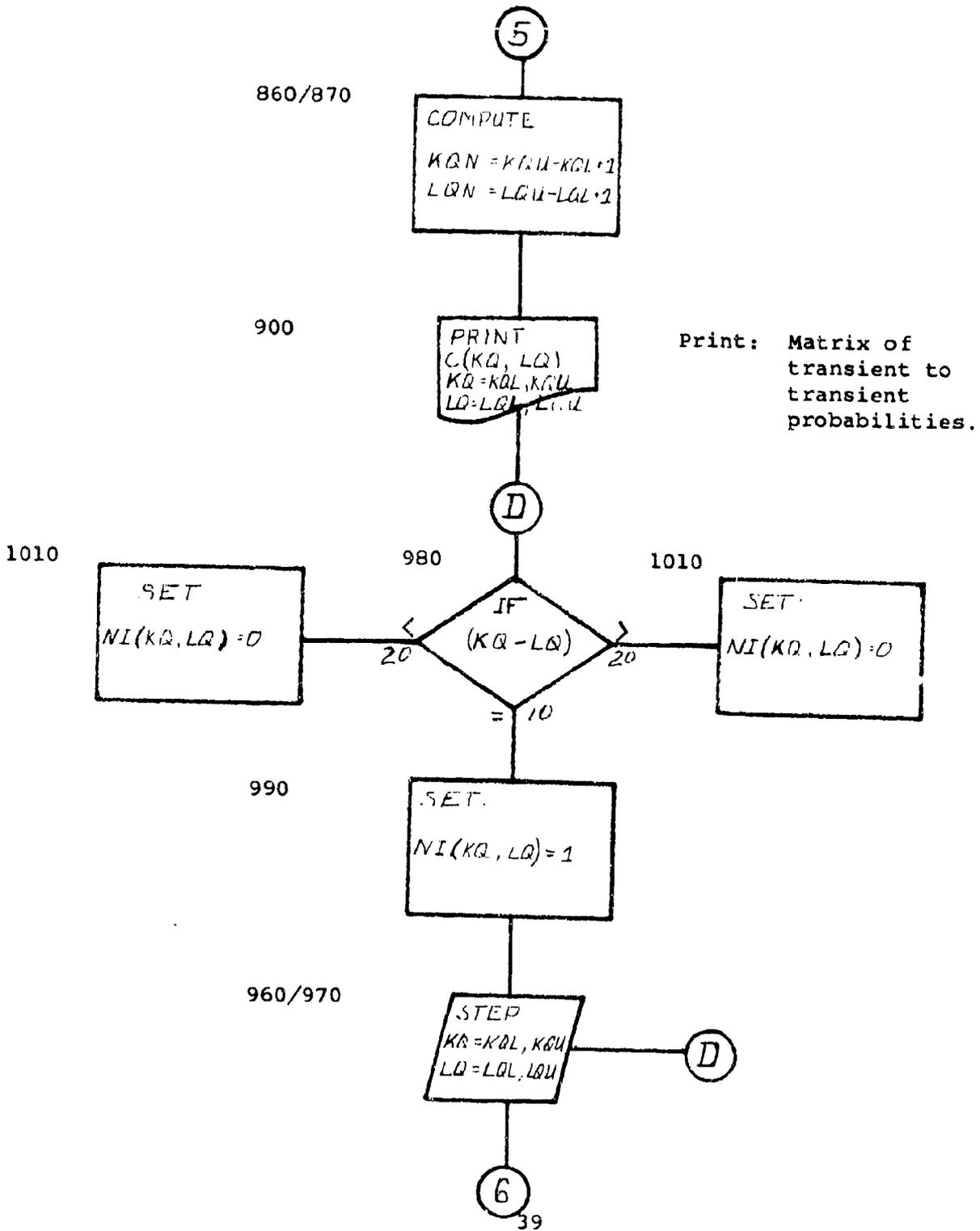


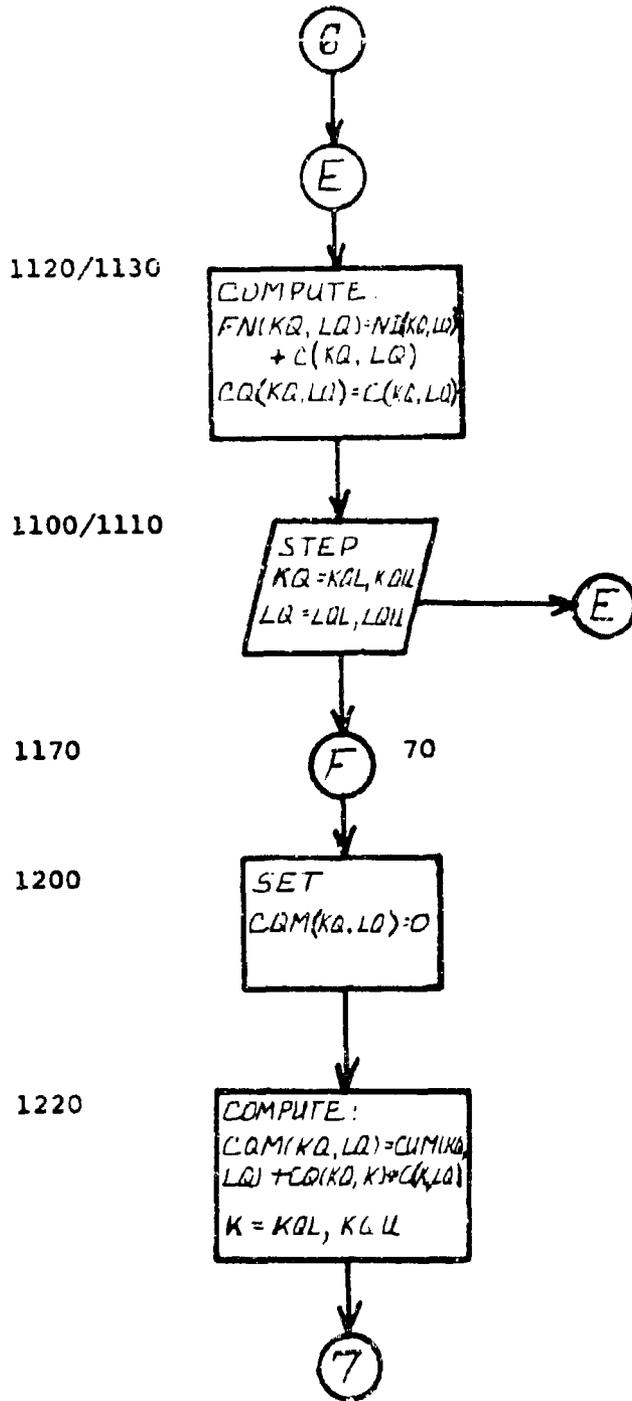
\*Program Line Numbers

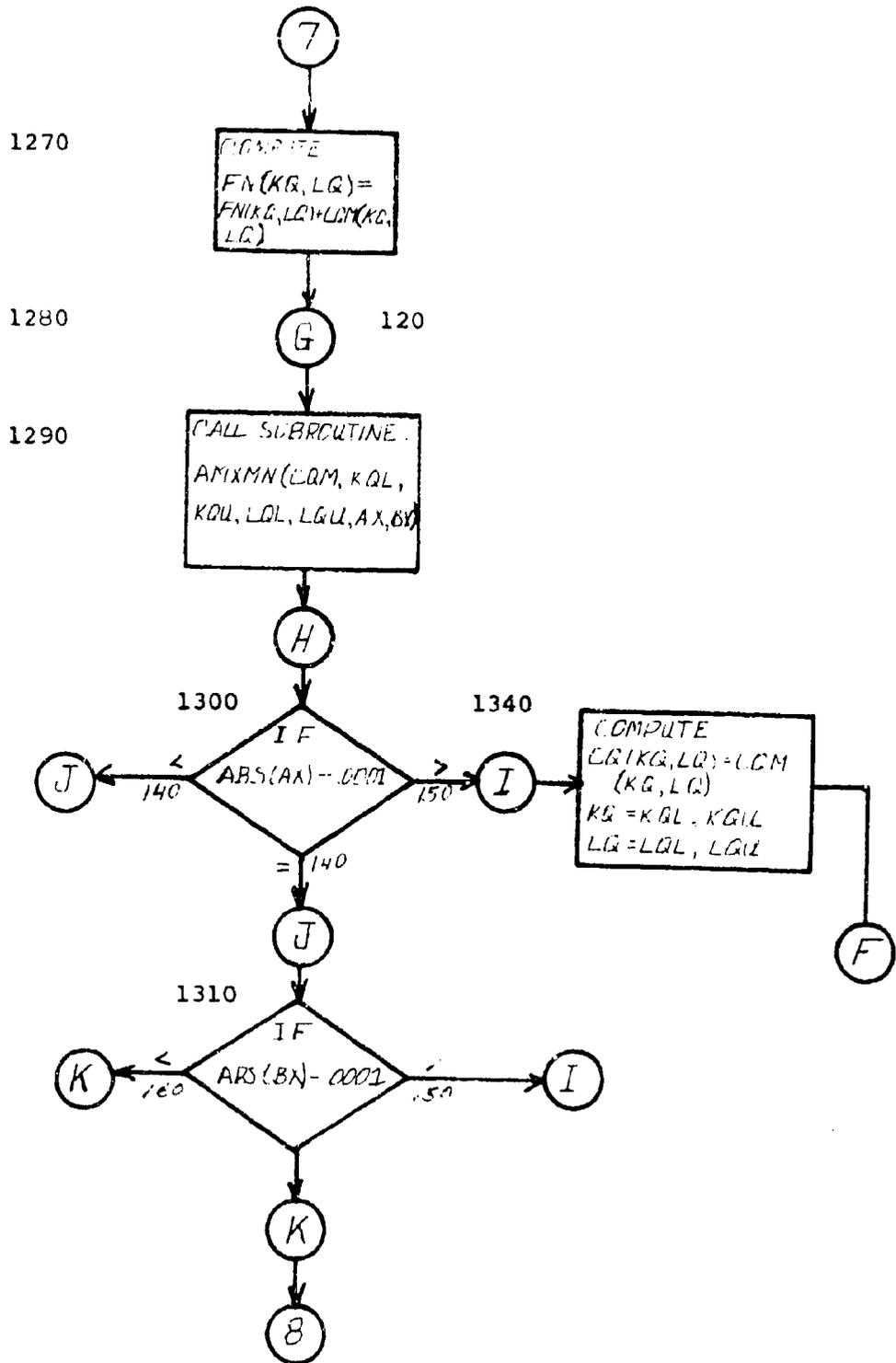




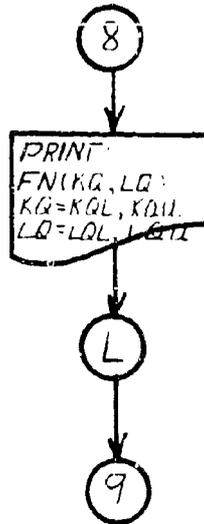








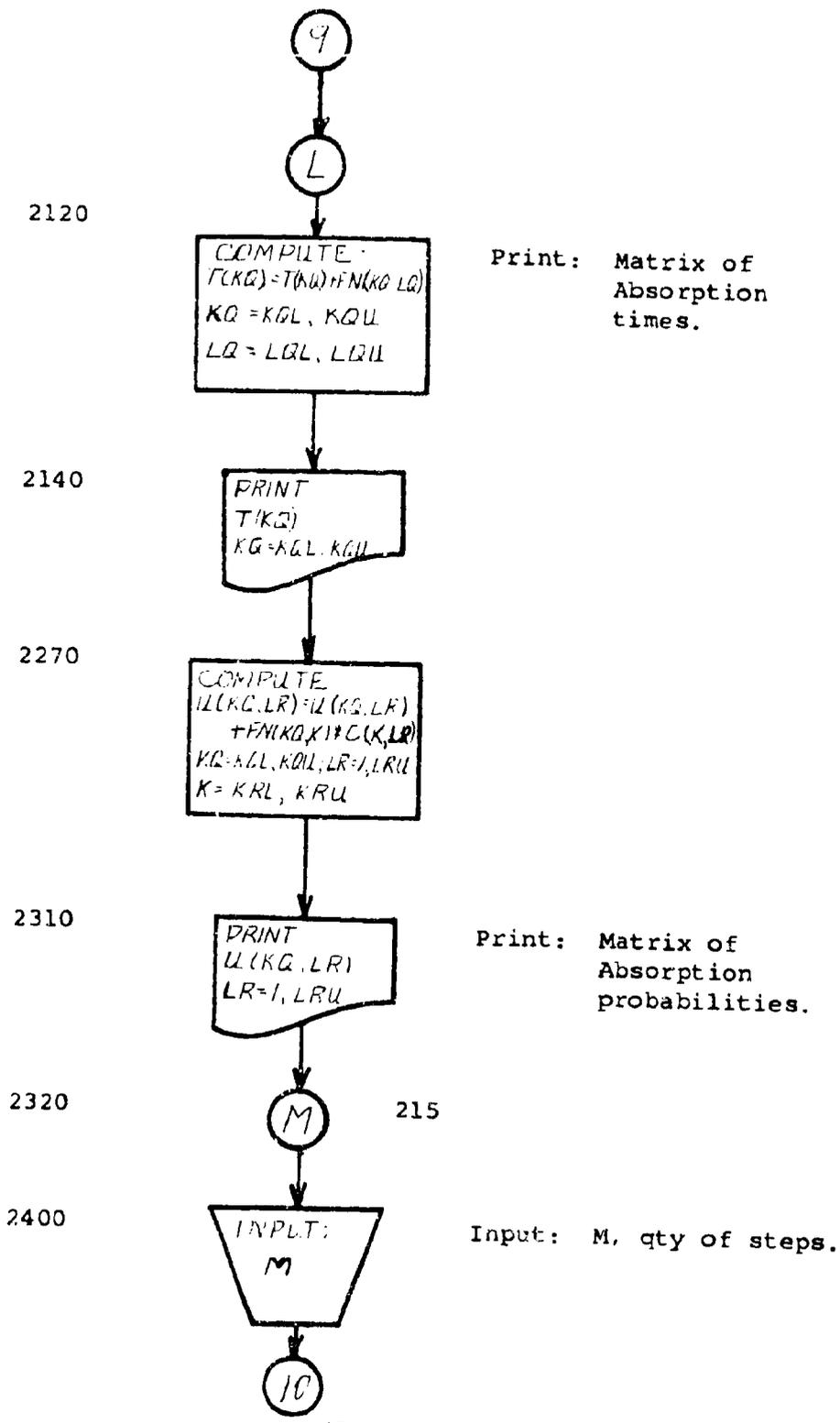
1370

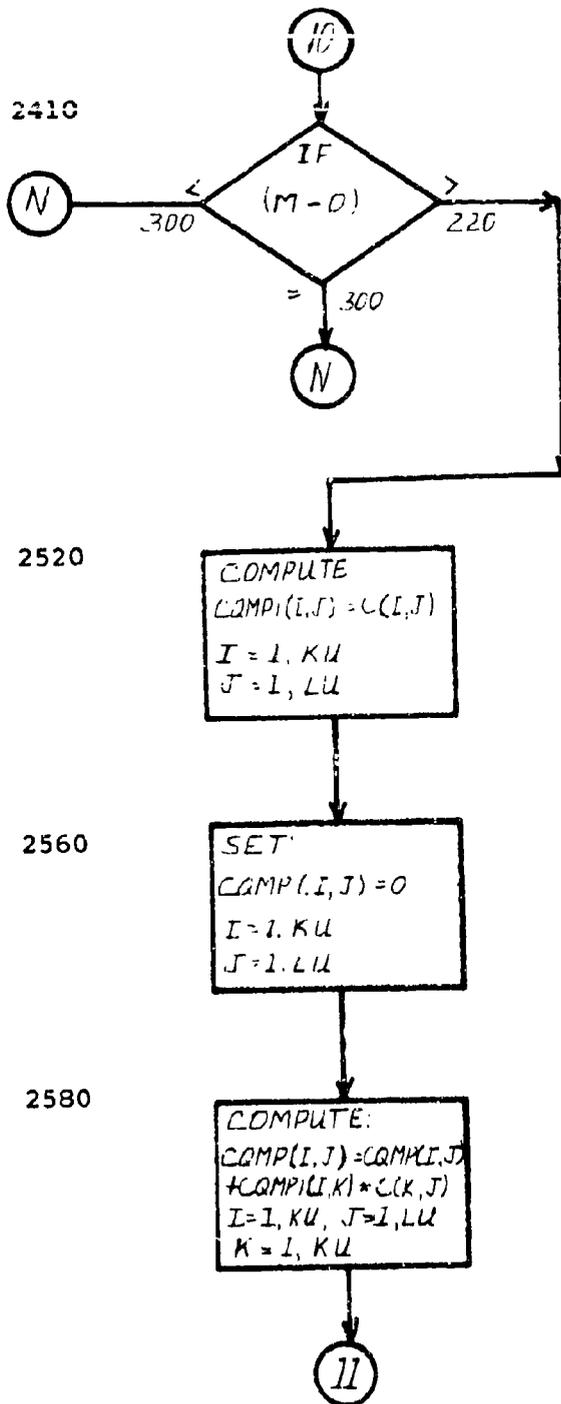


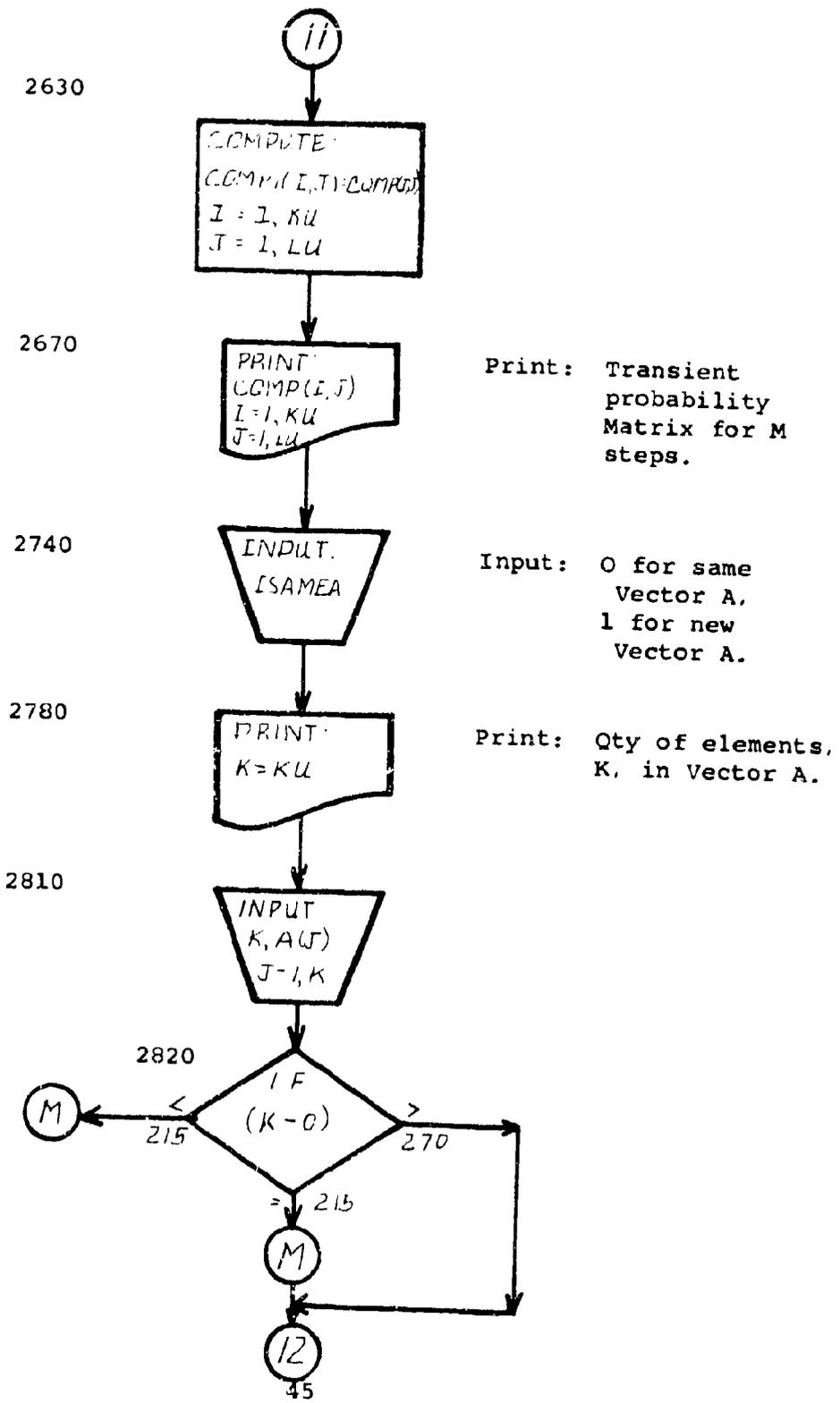
Print: Fundamental Matrix

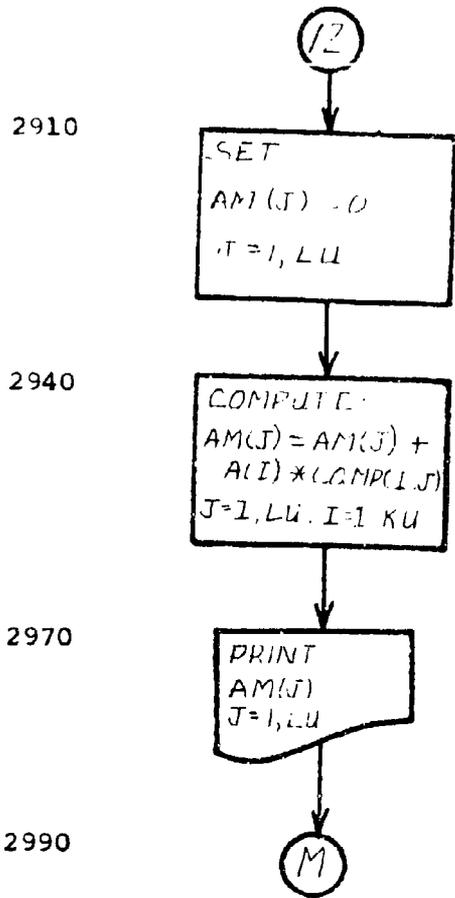
1400

MARK2

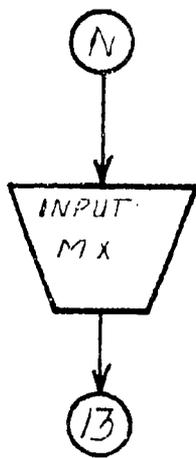




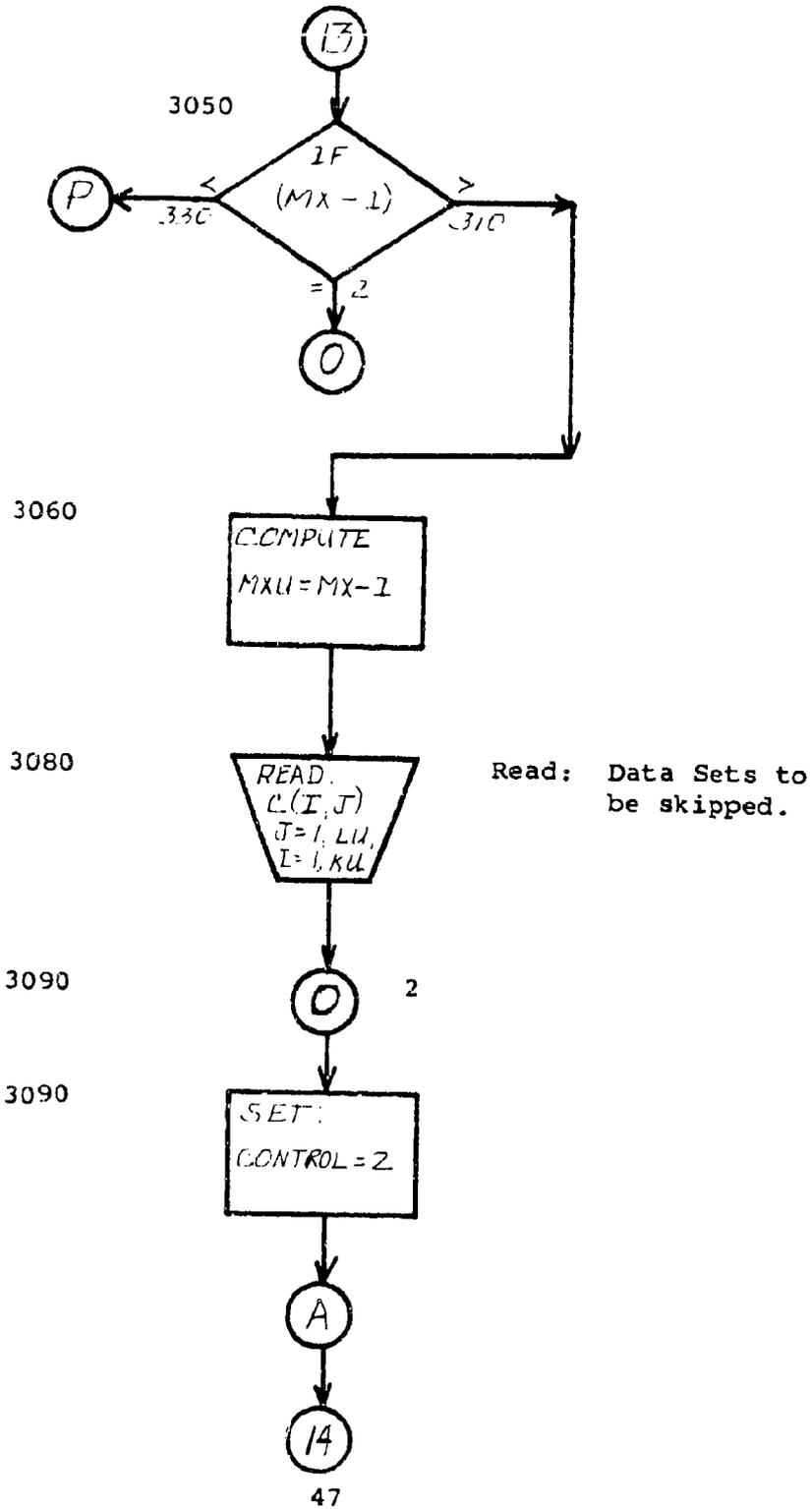




Print: Vector, AM,  
for State Space  
M steps later.



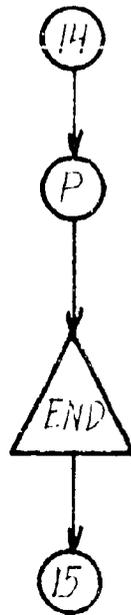
Input: MX  
0 to end  
Computation,  
1 to Do next  
set data,  
S+1 to skip S  
sets of data  
and do next  
set of data.



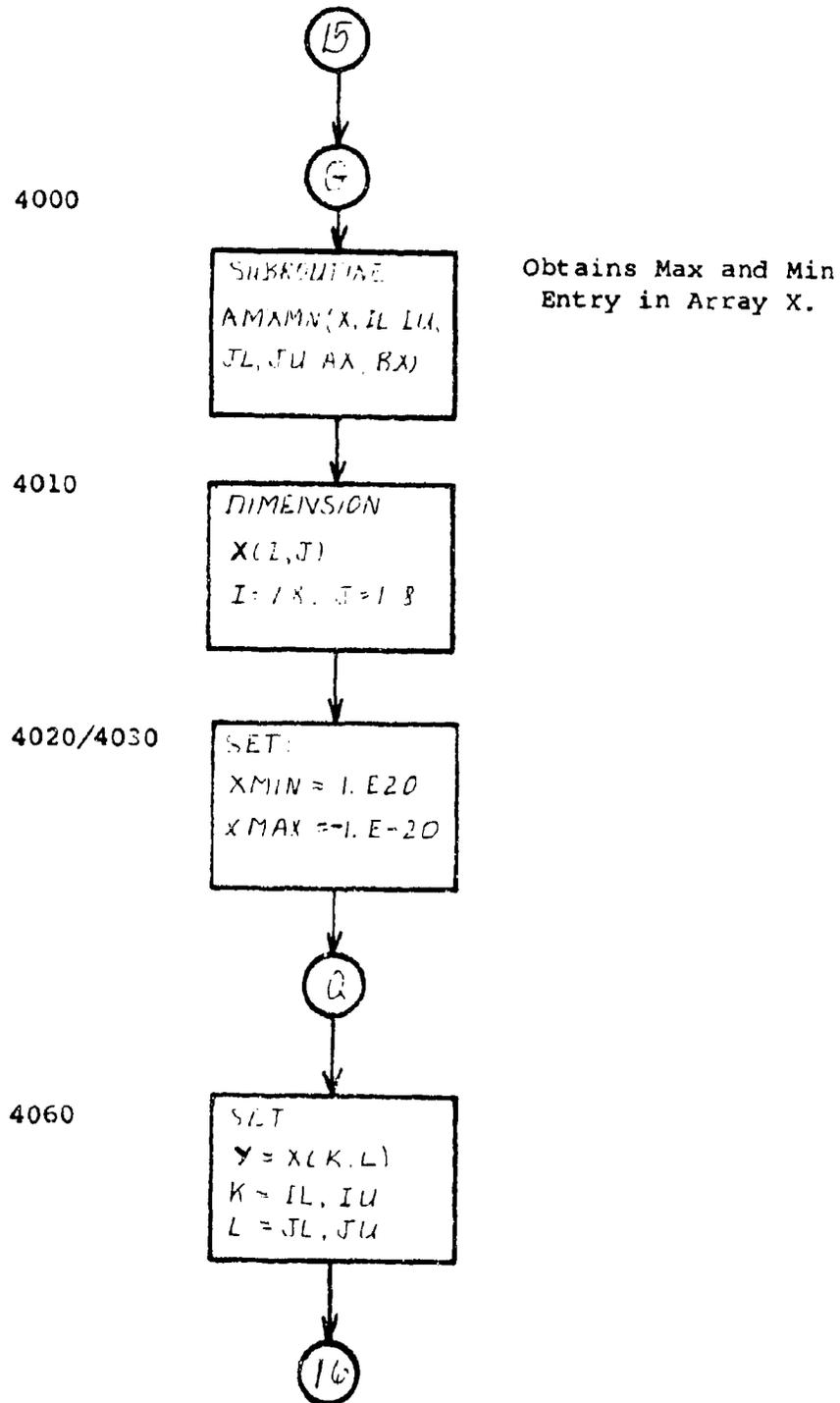
3110

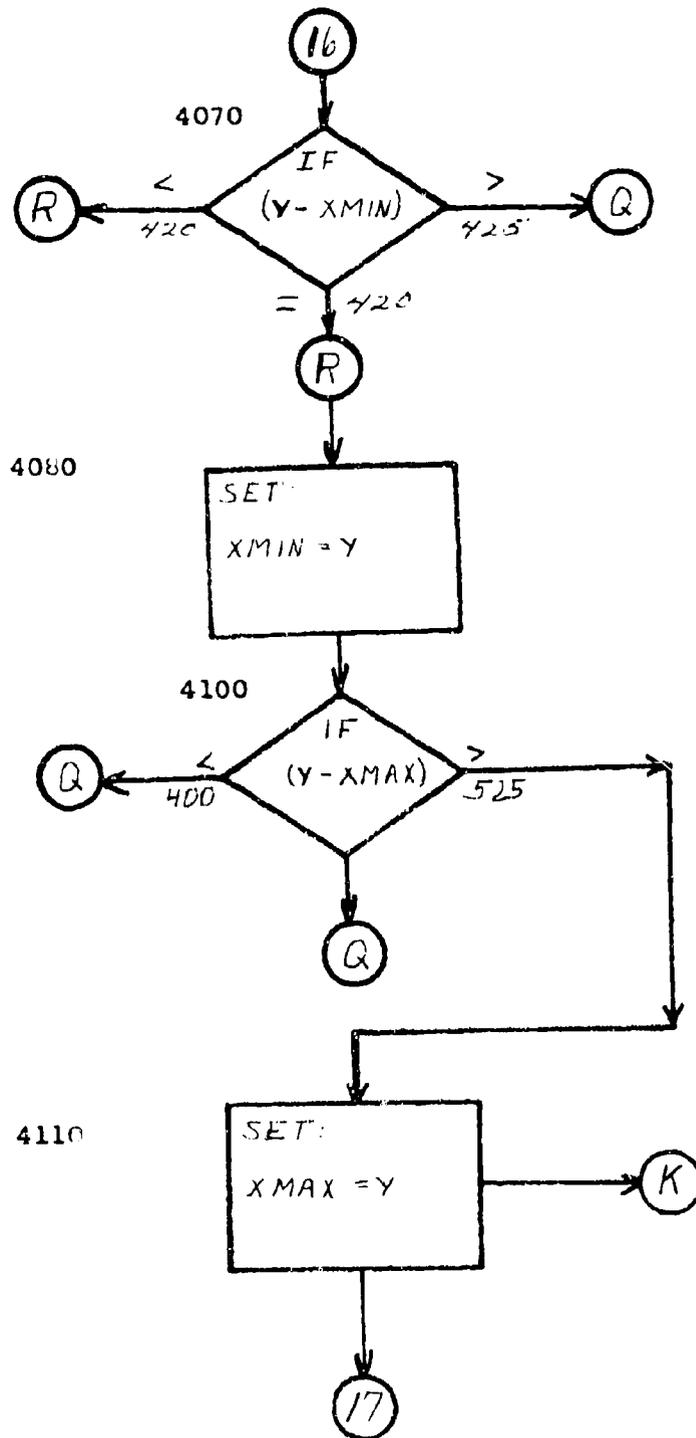
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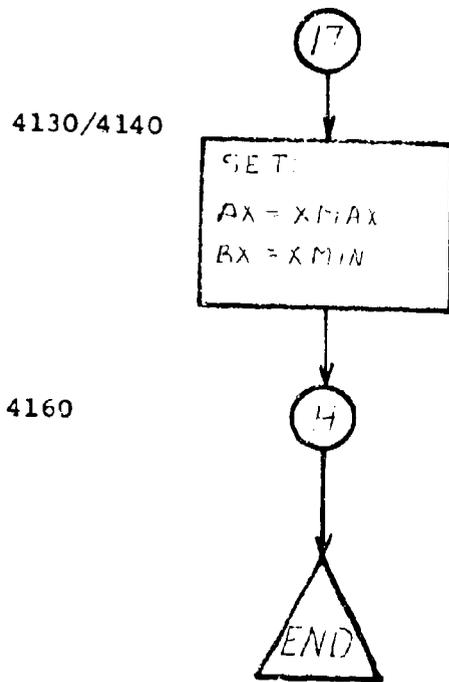
3110



MARKS







II. COMPUTER FORTRAN PROGRAM MARK 1/2/3

MARK I

```
100 PRINT "SIMULATION WITH"  
110 PRINT "MARKOV TRANSITION MATRIX MODEL OF"  
120 PRINT "A REQUISITION PROCESSING SYSTEM"  
130 PRINT "BY IRWIN GOODMAN"  
140 PRINT "INPUT, C, TRANSITION MATRIX IN CANONICAL FORM"  
150 PRINT "IN LINES 1420 THRU 1800 BY ROWS"  
160 PRINT "PRECEDE C MATRIX DATA WITH NAMES OF STATES"  
170 PRINT "RIGHT JUSTIFY BETWEEN COMMAS WITH SPACES ADDED FOR"  
180 PRINT "7 CHARACTER FORMAT."  
190 PRINT "EXAMPLE: HOME, BAR, CORNER, ETC. FOLLOWED BY DATA"  
200 CONTROL = 1  
230 DIMENSION C(8,8),CQ",CQM",CQMP",CQMP1",NI",FN",U",X",T(8),A",AM"  
240 DIMENSION STATE(8)  
250 PRINT "INPUT QTY OF ABSORBING STATES=IABSOR"  
260 INPUT, IABSOR  
270 PRINT "INPUT QTY OF TRANSIENT STATES=ITRANS"  
280 INPUT, ITRANS  
290 C COMPUTE QTY OF ROWS AND COLUMNS IN MATRIX C,  
300 KU=IABSOR + ITRANS  
310 LU=IABSOR + ITRANS  
320 PRINT "QTY OF ROWS IN MATRIX C =",KU  
330 PRINT "QTY OF COLUMNS IN MATRIX C =",LU  
340 I D0 65 I=1,8  
350 T(I)=0; 65 AM(I)=0  
360 D0 66 I=1,8  
370 D0 66 J=1,8  
380 CQM(I,J)=0; U(I,J)=0; 66 CQMP(I,J)=0  
390 G0 T0 (3,4), CONTROL  
400 3 READ, (STATE(J),J=1,LU)  
410 4  
420 READ, ((C(I,J),J=1,LU),I=1,KU)  
430 PRINT  
440 PRINT  
450 PRINT "TRANSITION MATRIX, C, IN CANONICAL FORM"  
460 PRINT  
470 PRINT BB, "STATE ",(STATE(J),J=1,LU)  
480 BB:FORMAT(10A7)  
490 D0 B, I=1,KU  
500 B: PRINT AA, STATE(I),(C(I,J),J=1,LU)  
510 AA: FORMAT(A7,10F7.3)  
520 PRINT  
530 PRINT  
540 PRINT "PARTITION TRANSITION MATRIX (CANONICAL FORM)"  
550 PRINT "INTO FOLLOWING MATRICIES:"  
560 PRINT "CI: IDENTITY MATRIX"  
570 PRINT  
580 KIU=IABSOR  
590 LIU=IABSOR  
600 PRINT BB, "STATE ",(STATE(LI),LI=1,LIU)  
610 D0 D, K
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK1 CONTINUED

```
620 D: PRINT AA, STATE(KI), (C(KI, LI), LI=1, LIU)
630 PRINT
640 PRINT
650 PRINT "CR: MATRIX OF TRANSIENT TO ABSORBING PROBABILITIES-"
660 PRINT "PROBABILITY OF GOING FROM TRANSIENT STATE"
670 PRINT "TO ABSORBING STATE"
680 PRINT
690 KRL=IABSOR + 1
700 KRU=KU
710 LRU=IABSOR
720 KRN=KRU-KRL + 1
730 PRINT BB, "STATE ", (STATE(LR), LR=1, LRU)
740 DØ E, KR=KRL, KRU
750 E: PRINT AA, STATE(KR), (C(KR, LR), LR=1, LRU)
760 PRINT
770 PRINT
780 PRINT "CQ: MATRIX OF TRANSIENT TO TRANSIENT PROBABILITIES-"
790 PRINT "PROBABILITY OF GOING FROM TRANSIENT STATE"
800 PRINT "TO TRANSIENT STATE"
810 PRINT
820 KQL=IABSOR + 1
830 KQU=KU
840 LQL=IAESOR + 1
850 LQU=LU
860 KQN=KQU - KQL + 1
870 LQN=LQU - LQL + 1
880 PRINT BD, "STATE ", (STATE(LQ), LQ=LQL, LQU)
890 DØ F, KQ=KQL, KQU
900 F: PRINT AA, STATE(KQ), (C(KQ, LQ), LQ=LQL, LQU)
910 PRINT
920 PRINT
930 PRINT "COMPUTATION OF THE FUNDAMENTAL MATRIX"
940C ESTABLISH IDENTITY MATRIX, NI, FOR COMPUTATION OF
950C FUNDAMENTAL MATRIX, FN.
960 DØ 40 KQ=KQL, KQU
970 DØ 30 LQ=LQL, LQU
980 IF (KQ-LQ) 20, 10, 20
990 10: NI(KQ, LQ)=1
1000 GØ TØ 30
1010 20: NI(KQ, LQ)=0
1020 30: CONTINUE
1030 40: CONTINUE
1040 PRINT "FN: FUNDAMENTAL MATRIX-"
1050 PRINT "EACH ELEMENT IS THE EXPECTED NUMBER OF TIMES"
1060 PRINT "IN STATE J(COLUMN) BEFORE BEING ABSORBED"
1070 PRINT "GIVEN THAT THE PRESENT STATE IS I(ROW)"
1080 PRINT
1090 PRINT BB, "STATE ", (STATE(LQ), LQ=LQL, LQU)
1100 DØ 50 KQ=KQL, KQU
1110 DØ 50 LQ=LQL, LQU
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONT'D.)  
MARK1 CONTINUED

```
1120 FN(KQ,LQ)=NI(KQ,LQ) + C(KQ,LQ)
1130 CQ(KQ,LQ)=C(KQ,LQ)
1140 50 CONTINUE
1150C COMPUTE THE FUNDAMENTAL MATRIX, FN, EQUAL TO THE INVERSE
1160C OF FNI=NI-CQ, BY SERIES APPROXIMATION
1170 70
1180 D0 90 KQ=KQL,KQU
1190 D0 90 LQ=LQL,LQU
1200 CQM(KQ,LQ)=0
1210 D0 80 K=KQL,KQU
1220 CQM(KQ,LQ)=CQM(KQ,LQ)+CQ(KQ,K)*C(K,LQ)
1230 80 CONTINUE
1240 90 CONTINUE
1250 D0 120 KQ=KQL,KQU
1260 D0 120 LQ=LQL,LQU
1270 FN(KQ,LQ)=FN(KQ,LQ)+CQM(KQ,LQ)
1280 120 CONTINUE
1290 CALL AMXMN(CQM,KQL,KQU,LQL,LQU,AX,BX)
1300 IF (ABS(AX)-.0001) 140,140,150
1310 140 IF(ABS(BX)-.0001) 160,160,150
1320 150 D0 500 KQ=KQL,KQU
1330 D0 500 LQ=LQL,LQU
1340 500 CQ(KQ,LQ)=CQM(KQ,LQ)
1350 G0 T0 70
1360 150 D0 G,KQ=KQL,KQU
1370 G: PRINT AA, STATE(KQ), (FN(KQ,LQ),LQ=LQL,LQU)
1380 PRINT
1390 PRINT
1400 $USE MARK2
1410 $DATA
1411 HOME, BAR, CORNR2, CORNR3, CORNR4, CORNR5, CORNR6, CORNR7
1412 1.,7*0.
1413 0.,1.,6*0.
1414 .75,2*0.,.25,4*0.
1415 2*0.,.75,0.,.25,3*0.
1416 3*0.,.75,0.,.25,2*0.
1417 4*0.,.75,0.,.25,0.
1418 5*0.,.75,0.,.25,0.,.25,4*0.,.75,0.
1419 COMPLT, FRSPAS, SUBPAS
1440 1.,0.,0.
1441 .99,0.,.01
1442 .75,0.,.25
1450 1.,0.,0.
1451 .95,0.,.05
1452 .75,0.,.25
1460 1.,0.,0.
1461 .90,0.,.10
1462 .75,0.,.25
1470 1.,0.,0.
1471 .85,0.,.15
```

II. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK1 CONTINUED

```
1472 .75,0.,.25
1480 1.,0.,0.
1481 .80,0.,.20
1482 .75,0.,.25
1490 1.,0.,0.
1491 .75,0.,.25
1492 .75,0.,.25
1500 1.,0.,0.
1501 .50,0.,.50
1502 .75,0.,.25
1510 1.,0.,0.
1511 .25,0.,.75
1512 .75,0.,.25
1520 1.,0.,0.
1521 .01,0.,.99
1522 .75,0.,.25
1530 1.,0.,0.
1531 .001,0.,.999
1532 .75,0.,.25
1801 MRØ, PASØRD, REJCUS, BACKØR, FRSPAS, SUBPAS
1802 1.,5*0.,0.,1.,4*0.,2*0.,1.,3*0.,3*0.,1.,2*0.
1803 .55,.02,.03,.22,0.,.18,.06,.09,.22,.40,0.,.23
```

MARK2

```
2000 PRINT "COMPUTATION OF THE MATRIX OF ABSORPTION TIMES"
2010 PRINT "I: MATRIX OF ABSORPTION TIMES-"
2020 PRINT "EACH ELEMENT IS THE MEAN TIME TO ABSORPTION"
2030 PRINT "(NUMBER OF STATES PASSED THROUGH, INCLUDING FINAL STATE"
2040 PRINT "AND NOT INCLUDING INITIAL STATE, INORDER TO BE ABSORBED)"
2050 PRINT
2060C COMPUTE T=FN TIMES C WHERE C IS A COLUMN VECTOR OF 1'S.
2070C THE ELEMENTS IN MATRIX T ARE EQUAL TO THE ROW SUM FOR EACH
2080C ROW OF FN.
2090 PRINT BB, STATE
2100 DØ 180 KQ=KQL,KQU
2110 DØ 170 LQ=LQL,LQU
2120 T(KQ)=T(KQ) + FN(KQ,LQ)
2130 170 CONTINUE
2140 PRINT AA, STATE(KQ),T(KQ)
2150 180 CONTINUE
2160 PRINT
2170 PRINT
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK2 CONTINUED

```
2180 PRINT "COMPUTATION OF THE MATRIX OF ABSORPTION PROBABILITIES"
2190 PRINT "U. MATRIX OF ABSORPTION PROBABILITIES"
2200 PRINT "PROBABILITY OF BEING ABSORBED,"
2210 PRINT "GIVEN IT WAS INITIALLY IN A TRANSIENT STATE"
2220 PRINT
2230C MULTIPLY MATRIX FN TIMES CR.
2240 DØ 190 KQ=KQL,KQU
2250 DØ 190 LR=1,LRU
2260 DØ 190 K=KRL,KRU
2270 U(KQ,LR)=U(KQ,LR) + FN(KQ,K)*C(K,LR)
2280 190 CONTINUE
2290 PRINT BB, "STATE ", (STATE(LR),LR=1,LRU)
2300 DØ GG, KQ=KQL,KQU
2310 GG: PRINT AA, STATE(KQ), (U(KQ,LR),LR=1,LRU)
2320 215
2330 PRINT
2340 PRINT
2350 PRINT "COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS"
2360 PRINT "INPUT NUMBER OF STEPS, M,"
2370 PRINT "THAT MATRIX OF TRANSITION PROBABILITIES"
2380 PRINT "SHOULD BE COMPUTED FOR"
2390 PRINT "INPUT M=0 IF COMPUTATION NOT DESIRED"
2400 INPUT, M
2410 IF(M=0) 300,300,220
2420C GØ TØ END FØR M=0
2430 220 PRINT
2440 PRINT
2450 PRINT "CMP: MATRIX OF TRANSITION"
2460 PRINT "PROBABILITIES FOR M STEPS,M=",M
2470 PRINT "PROBABILITY OF GOING FROM STATE TØ STATE"
2480 PRINT
2490C COMPUTE C TØ THE M PØWER.
2500 DØ 225 I=1,KU
2510 DØ 225 J=1,LU
2520 225 CQMP(I,J)=C(I,J)
2530 DØ 260 L=2,M
2540 DØ 250 I=1,KU
2550 DØ 250 J=1,LU
2560 CQMP(I,J)=0
2570 DØ 230 K=1,KU
2580 CQMP(I,J)=CQMP(I,J)+CQMP(I,K)*C(K,J)
2590 230 CONTINUE
2600 250 CONTINUE
2610 DØ 255 I=1,KU
2620 DØ 255 J=1,LU
2630 255 CQMP(I,J)=CQMP(I,J)
2640 260 CONTINUE
2650 PRINT BB, "STATE ", (STATE(J),J=1,LU)
2660 DØ H,I=1,KU
2670 H: PRINT AA, STATE(I), (CQMP(I,J),J=1,LU)
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONTD)

MARK2 CONTINUED

```
2680 PRINT
2690 PRINT
2700 PRINT "COMPUTATION OF STATE SPACE, M STEPS LATER,"
2710 PRINT "GIVEN INITIAL STATE"
2720 PRINT "INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED"
2730 PRINT "INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT."
2740 INPUT,ISAMEA
2750 IF(ISAMEA-1) 270,265,270
2760 265
2770 PRINT "INPUT INITIAL STATE AS A ROW VECTOR IN FOLLOWING FORM"
2780 PRINT "K= ,KU
2790 PRINT "INPUT K=0 IF COMPUTATION NOT DESIRED"
2800 PRINT "INPUT K,A(1),A(2),A(3),...,A(K)"
2810 INPUT,K,(A(J),J=1,K)
2820 IF(K-0) 215,215,270
2830C REPEAT M STEP COMPUTATION.
2840 270 PRINT
2850 PRINT
2860 PRINT "AM; ROW VECTOR FOR STATE SPACE M STEPS LATER,M=",M
2870 PRINT "GIVEN INITIAL STATE A"
2880C COMPUTE AM=A TIMES CQMP.
2890 PRINT
2900 D0 275 J=1,LU
2910 275 AM(J)=0
2920 D0 290 J=1,LU
2930 D0 290 I=1,KU
2940 AM(J)=AM(J)+A(I)*CQMP(I,J)
2950 290 CONTINUE
2960 PRINT BB, "STATE ",(STATE(J),J=1,LU)
2970 PRINT CC,(AM(J),J=1,LU)
2980 CC: FORMAT(7X,10F7.3)
2990 G0 T0 215
3000 300 PRINT "INPUT MX=0 TO END COMPUTATION"
3010 PRINT "MX=1 TO D0 NEXT SET OF DATA"
3020 PRINT "MX=S+1 TO SKIP S SETS OF DATA"
3030 PRINT "AND D0 NEXT SET OF DATA"
3040 INPUT,MX
3050 IF(MX-1) 330,2,310
3060 310 MXU=MX-1
3070 D0 320 K=1,MXU
3080 320 READ,((C(I,J),J=1,LU),I=1,KU)
3090 2 CONTR0L = 2
3100 G0 T0 1
3110 330 END
3120 $USE MARKS
```

H. COMPUTER FORTRAN PROGRAM MARK 1/2/3 (CONT'D)

MARK3

```
4000 SUBROUTINE AMXMN(X,IL,IU,JL,JU,AX,BX)
4010 DIMENSION X(8,8)
4020 XMIN=1.E20
4030 XMAX=-1.E-20
4040 DO 400 K=IL,IU
4050 DO 400 L=JL,JU
4060 Y=X(K,L)
4070 IF(Y-XMIN) 420,420,425
4080 420 XMIN=Y
4090 425 CONTINUE
4100 IF(Y-XMAX) 400,400,525
4110 525 XMAX=Y
4120 400 CONTINUE
4130 AX=XMAX
4140 BX=XMIN
4150 RETURN
4150 END
```

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS

I.1 Classical Random Walk

MARK1 13:11 MON.---07/08/68

IN MARK2  
IN .FIRST  
IN MARK3  
IN .FIRST

SIMULATION WITH  
MARKOV TRANSITION MATRIX MODEL OF  
A REQUISITION PROCESSING SYSTEM  
BY IRWIN GOODMAN  
INPUT, C, TRANSITION MATRIX IN CANONICAL FORM  
IN LINES 1420 THRU 1800 BY ROWS  
PRECEDE C MATRIX DATA WITH NAMES OF STATES  
RIGHT JUSTIFY BETWEEN COMMAS WITH SPACES ADDED FOR  
7 CHARACTER FORMAT.  
EXAMPLE: HOME, BAR, CORNR2, ETC. FOLLOWED BY DATA  
INPUT QTY OF ABSORBING STATES=IABSOR  
? ?2  
INPUT QTY OF TRANSIENT STATES=ITRANS  
? ?6  
QTY OF ROWS IN MATRIX C = 8  
QTY OF COLUMNS IN MATRIX C = 8

TRANSITION MATRIX, C, IN CANONICAL FORM

STATE	HOME	BAR	CORNR2	CORNR3	CORNR4	CORNR5	CORNR6	CORNR7
HOME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CORNR2	.750	0.000	0.000	.250	0.000	0.000	0.000	0.000
CORNR3	0.000	0.000	.750	0.000	.250	0.000	0.000	0.000
CORNR4	0.000	0.000	0.000	.750	0.000	.250	0.000	0.000
CORNR5	0.000	0.000	0.000	0.000	.750	0.000	.250	0.000
CORNR6	0.000	0.000	0.000	0.000	0.000	.750	0.000	.250
CORNR7	0.000	.250	0.000	0.000	0.000	0.000	.750	0.000

PARTITION TRANSITION MATRIX (CANONICAL FORM)  
INTO FOLLOWING MATRICIES:  
CI: IDENTITY MATRIX

STATE	HOME	BAR
HOME	1.000	0.000
BAR	0.000	1.000

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS (CONTD)

I.1 Classical Random Walk

CR: MATRIX OF TRANSIENT TO ABSORBING PROBABILITIES-  
 PROBABILITY OF GOING FROM TRANSIENT STATE  
 TO ABSORBING STATE

STATE	HOME	BAR
CORNR2	.750	0.000
CORNR3	0.000	0.000
CORNR4	0.000	0.000
CORNR5	0.000	0.000
CORNR6	0.000	0.000
CORNR7	0.000	.250

CQ: MATRIX OF TRANSIENT TO TRANSIENT PROBABILITIES-  
 PROBABILITY OF GOING FROM TRANSIENT STATE  
 TO TRANSIENT STATE

STATE	CORNR2	CORNR3	CORNR4	CORNR5	CORNR6	CORNR7
CORNR2	0.000	.250	0.000	0.000	0.000	0.000
CORNR3	.750	0.000	.250	0.000	0.000	0.000
CORNR4	0.000	.750	0.000	.250	0.000	0.000
CORNR5	0.000	0.000	.750	0.000	.250	0.000
CORNR6	0.000	0.000	0.000	.750	0.000	.250
CORNR7	0.000	0.000	0.000	0.000	.750	0.000

COMPUTATION OF THE FUNDAMENTAL MATRIX

FN: FUNDAMENTAL MATRIX-  
 EACH ELEMENT IS THE EXPECTED NUMBER OF TIMES  
 IN STATE J(COLUMN) BEFORE BEING ABSORBED  
 GIVEN THAT THE PRESENT STATE IS I(ROW)

STATE	CORNR2	CORNR3	CORNR4	CORNR5	CORNR6	CORNR7
CORNR2	1.332	.443	.146	.048	.015	.004
CORNR3	1.328	1.771	.586	.190	.059	.015
CORNR4	1.317	1.757	1.903	.618	.190	.048
CORNR5	1.284	1.713	1.855	1.903	.586	.146
CORNR6	1.186	1.581	1.713	1.757	1.771	.443
CORNR7	.689	1.186	1.284	1.317	1.328	1.332

COMPUTATION OF THE MATRIX OF ABSORPTION TIMES

T: MATRIX OF ABSORPTION TIMES-  
 EACH ELEMENT IS THE MEAN TIME TO ABSORPTION  
 (NUMBER OF STATES PASSED THROUGH, INCLUDING FINAL STATE  
 AND NOT INCLUDING INITIAL STATE, IN ORDER TO BE ABSORBED)

STATE	
CORNR2	1.987
CORNR3	3.949
CORNR4	5.833
CORNR5	7.487
CORNR6	8.450
CORNR7	7.337

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS (CONTD)

I.1 Classical Random Walk

COMPUTATION OF THE MATRIX OF ABSORPTION PROBABILITIES  
 U. MATRIX OF ABSORPTION PROBABILITIES  
 PROBABILITY OF BEING ABSORBED,  
 GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE	HØME	BAR
CØRNR2	.999	.001
CØRNR3	.996	.004
CØRNR4	.988	.012
CØRNR5	.963	.037
CØRNR6	.889	.111
CØRNR7	.667	.333

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? ?2

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 2  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.750	0.000	.187	0.000	.062	0.000	0.000	0.000
CØRNR3	.562	0.000	0.000	.375	0.000	.062	0.000	0.000
CØRNR4	0.000	0.000	.562	0.000	.375	0.000	.062	0.000
CØRNR5	0.000	0.000	0.000	.562	0.000	.375	0.000	.062
CØRNR6	0.000	.062	0.000	0.000	.562	0.000	.375	0.000
CØRNR7	0.000	.250	0.000	0.000	0.000	.562	0.000	.187

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.

! ? ?1  
 INPUT INITIAL STATE AS A ROW VECTOR IN FOLLOWING FORM  
 K= 8  
 INPUT K=0 IF COMPUTATION NOT DESIRED  
 INPUT K, A(1), A(2), A(3), ..., A(K)  
 ? ?8, 0, 0, .167, .167, .167, .167, .167, .167

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 2  
 GIVEN INITIAL STATE A

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.219	.052	.125	.157	.167	.167	.073	.042

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS (CONT'D)  
 I.1 CLASSICAL RANDOM WALK

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? 225

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 25  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.999	.001	0.000	.000	0.000	.000	0.000	.000
CØRNR3	.995	.004	.001	0.000	.001	0.000	.000	0.000
CØRNR4	.986	.012	0.000	.002	0.000	.001	0.000	.000
CØRNR5	.959	.037	.003	0.000	.002	0.000	.001	0.000
CØRNR6	.884	.111	0.000	.004	0.000	.002	0.000	.000
CØRNR7	.660	.333	.003	0.000	.003	0.000	.001	0.000

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.  
 ? 20

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 25  
 GIVEN INITIAL STATE A

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.916	.083	.001	.001	.001	.000	.000	.000

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? 23

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 3  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.891	0.000	0.000	.094	0.000	.016	0.000	0.000
CØRNR3	.562	0.000	.281	0.000	.141	0.000	.016	0.000
CØRNR4	.422	0.000	0.000	.422	0.000	.141	0.000	.016
CØRNR5	0.000	.016	.422	0.000	.422	0.000	.141	0.000
CØRNR6	0.000	.062	0.000	.422	0.000	.422	0.000	.094
CØRNR7	0.000	.297	0.000	0.000	.422	0.000	.281	0.000

I. COMPUTER TIME SHARING TERMINAL PRINT-OUTS (CONTD)  
 I.1 CLASSICAL RANDOM WALK

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=! FOR NEW A(K) VECTOR INPUT.  
 ? ?0

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 3  
 GIVEN INITIAL STATE A

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.313	.063	.117	.157	.164	.097	.073	.018

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? ?5

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 5  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
HØME	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BAR	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
CØRNR2	.943	0.000	0.000	.044	0.000	.012	0.000	.001
CØRNR3	.773	.001	.132	0.000	.079	0.000	.015	0.000
CØRNR4	.659	.004	0.000	.237	0.000	.088	0.000	.012
CØRNR5	.316	.024	.316	0.000	.264	0.000	.079	0.000
CØRNR6	.237	.086	0.000	.396	0.000	.237	0.000	.044
CØRNR7	0.000	.314	.237	0.000	.316	0.000	.132	0.000

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.  
 ? ?0

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 5  
 GIVEN INITIAL STATE A

STATE	HØME	BAR	CØRNR2	CØRNR3	CØRNR4	CØRNR5	CØRNR6	CØRNR7
	.489	.072	.114	.113	.110	.056	.038	.009

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? ?0

1.2 Model A of Requisition Processing System

MARKI 13:45 MON.---07/08/68

IN 001  
 TO 01500  
 BY MARKI  
 TO 01500

SIMULATION WITH  
 MARKOV TRANSITION MATRIX MODEL OF  
 A REQUISITION PROCESSING SYSTEM  
 BY IRWIN GOODMAN  
 INPUT, C, TRANSITION MATRIX IN CANONICAL FORM  
 IN LINES 1420 THRU 1800 BY ROWS  
 PRECEDE C MATRIX DATA WITH NAMES OF STATES  
 RIGHT JUSTIFY BETWEEN COMMAS WITH SPACES ADDED FOR  
 7 CHARACTER FORMAT.  
 EXAMPLE: HOME, BAR, CORNR2, ETC. FOLLOWED BY DATA  
 INPUT QTY OF ABSORBING STATES=IABSOR  
 ? ?4  
 INPUT QTY OF TRANSIENT STATES=ITRANS  
 ? ?2  
 QTY OF ROWS IN MATRIX C = 6  
 QTY OF COLUMNS IN MATRIX C = 6

TRANSITION MATRIX, C, IN CANONICAL FORM

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
MRØ	1.000	0.000	0.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000	0.000	0.000
BACKØR	0.000	0.000	0.000	1.000	0.000	0.000
FRSPAS	.550	.020	.030	.220	0.000	.180
SUBPAS	.060	.090	.220	.400	0.000	.230

PARTITION TRANSITION MATRIX (CANONICAL FORM)  
 INTO FOLLOWING MATRICIES:  
 CI: IDENTITY MATRIX

STATE	MRØ	PASØRD	REJCUS	BACKØR
MRØ	1.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000
BACKØR	0.000	0.000	0.000	1.000

1.2 Model A of Requisition Processing System

OK: MATRIX OF TRANSIENT TO ABSORBING PROBABILITIES-  
PROBABILITY OF GOING FROM TRANSIENT STATE  
TO ABSORBING STATE

STATE	MRØ	PASØRD	REJCUS	BACKØR
FRSPAS	.550	.020	.030	.220
SUBPAS	.060	.090	.220	.400

CQ: MATRIX OF TRANSIENT TO TRANSIENT PROBABILITIES-  
PROBABILITY OF GOING FROM TRANSIENT STATE  
TO TRANSIENT STATE

STATE	FRSPAS	SUBPAS
FRSPAS	0.000	.180
SUBPAS	0.000	.230

COMPUTATION OF THE FUNDAMENTAL MATRIX  
FN: FUNDAMENTAL MATRIX-  
EACH ELEMENT IS THE EXPECTED NUMBER OF TIMES  
IN STATE J(COLUMN) BEFORE BEING ABSORBED  
GIVEN THAT THE PRESENT STATE IS I(ROW)

STATE	FRSPAS	SUBPAS
FRSPAS	1.000	.234
SUBPAS	0.000	1.299

COMPUTATION OF THE MATRIX OF ABSORPTION TIMES  
T: MATRIX OF ABSORPTION TIMES-  
EACH ELEMENT IS THE MEAN TIME TO ABSORPTION  
(NUMBER OF STATES PASSED THROUGH, INCLUDING FINAL STATE  
AND NOT INCLUDING INITIAL STATE, IN ORDER TO BE ABSORBED)

STATE	FRSPAS	SUBPAS
FRSPAS	1.234	
SUBPAS	1.299	

COMPUTATION OF THE MATRIX OF ABSORPTION PROBABILITIES  
U: MATRIX OF ABSORPTION PROBABILITIES  
PROBABILITY OF BEING ABSORBED,  
GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE	MRØ	PASØRD	REJCUS	BACKØR
FRSPAS	.564	.041	.081	.314
SUBPAS	.078	.117	.286	.519

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
INPUT NUMBER OF STEPS, M.  
THAT MATRIX OF TRANSITION PROBABILITIES  
SHOULD BE COMPUTED FOR  
INPUT M=0 IF COMPUTATION NOT DESIRED  
? 72

I.2 Model A of Requisition Processing System

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 2  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
MRØ	1.000	0.000	0.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000	0.000	0.000
BACKØR	0.000	0.000	0.000	1.000	0.000	0.000
FRSPAS	.561	.036	.070	.292	0.000	.041
SUBPAS	.074	.111	.271	.492	0.000	.053

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.  
 ? 71  
 INPUT INITIAL STATE AS A ROW VECTOR IN FOLLOWING FORM  
 K= 6  
 INPUT K=0 IF COMPUTATION NOT DESIRED  
 INPUT K, A(1), A(2), A(3), ..., A(K)  
 ? 76, 0, 0, 0, 0, .80, .20

AM, ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 2  
 GIVEN INITIAL STATE A

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
	.463	.051	.110	.332	0.000	.044

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? 73

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 3  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
MRØ	1.000	0.000	0.000	0.000	0.000	0.000
PASØRD	0.000	1.000	0.000	0.000	0.000	0.000
REJCUS	0.000	0.000	1.000	0.000	0.000	0.000
BACKØR	0.000	0.000	0.000	1.000	0.000	0.000
FRSPAS	.563	.040	.079	.309	0.000	.010
SUBPAS	.077	.115	.282	.513	0.000	.012

AM, ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 3  
 GIVEN INITIAL STATE A

STATE	MRØ	PASØRD	REJCUS	BACKØR	FRSPAS	SUBPAS
	.466	.055	.119	.349	0.000	.010

1.3 Model B of Requisition Processing System

MARK1 14:20 MON.---07/08/68

IN MARK2  
IN .FIRST  
IN MARK3  
IN .FIRST

SIMULATION WITH  
MARKOV TRANSITION MATRIX MODEL OF  
A REQUISITION PROCESSING SYSTEM  
BY IRWIN GOODMAN  
INPUT, C, TRANSITION MATRIX IN CANONICAL FORM  
IN LINES 1420 THRU 1800 BY ROWS  
PRECEDE C MATRIX DATA WITH NAMES OF STATES  
RIGHT JUSTIFY BETWEEN COMMAS WITH SPACES ADDED FOR  
7 CHARACTER FORMAT.  
EXAMPLE; HOME, BAR, CORNR2, ETC. FOLLOWED BY DATA  
INPUT QTY OF ABSORBING STATES=IABSOR  
? 71  
INPUT QTY OF TRANSIENT STATES=ITRANS  
? 72  
QTY OF ROWS IN MATRIX C = 3  
QTY OF COLUMNS IN MATRIX C = 3

TRANSITION MATRIX, C, IN CANONICAL FORM

STATE	COMPLT	FRSPAS	SUBPAS
COMPLT	1.000	0.000	0.000
FRSPAS	.820	0.000	.180
SUBPAS	.770	0.000	.230

PARTITION TRANSITION MATRIX (CANONICAL FORM)  
INTO FOLLOWING MATRICES:  
CI: IDENTITY MATRIX

STATE	COMPLT
COMPLT	1.000

CR: MATRIX OF TRANSIENT TO ABSORBING PROBABILITIES-  
PROBABILITY OF GOING FROM TRANSIENT STATE  
TO ABSORBING STATE

STATE	COMPLT
FRSPAS	.820
SUBPAS	.770

I.3 Model B of Requisition Processing System (contd)

CQ: MATRIX OF TRANSIENT TO TRANSIENT PROBABILITIES-  
PROBABILITY OF GOING FROM TRANSIENT STATE  
TO TRANSIENT STATE

STATE	FRSPAS	SUBPAS
FRSPAS	0.000	.180
SUBPAS	0.000	.230

COMPUTATION OF THE FUNDAMENTAL MATRIX  
FN: FUNDAMENTAL MATRIX-  
EACH ELEMENT IS THE EXPECTED NUMBER OF TIMES  
IN STATE J(COLUMN) BEFORE BEING ABSORBED  
GIVEN THAT THE PRESENT STATE IS I(ROW)

STATE	FRSPAS	SUBPAS
FRSPAS	1.000	.234
SUBPAS	0.000	1.299

COMPUTATION OF THE MATRIX OF ABSORPTION TIMES  
T: MATRIX OF ABSORPTION TIMES-  
EACH ELEMENT IS THE MEAN TIME TO ABSORPTION  
(NUMBER OF STATES PASSED THROUGH, INCLUDING FINAL STATE  
AND NOT INCLUDING INITIAL STATE, IN ORDER TO BE ABSORBED)

STATE	
FRSPAS	1.234
SUBPAS	1.299

COMPUTATION OF THE MATRIX OF ABSORPTION PROBABILITIES  
U: MATRIX OF ABSORPTION PROBABILITIES  
PROBABILITY OF BEING ABSORBED,  
GIVEN IT WAS INITIALLY IN A TRANSIENT STATE

STATE	COMPLT
FRSPAS	1.000
SUBPAS	1.000

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
INPUT NUMBER OF STEPS, M,  
THAT MATRIX OF TRANSITION PROBABILITIES  
SHOULD BE COMPUTED FOR  
INPUT M=0 IF COMPUTATION NOT DESIRED  
? ?2

CMP: MATRIX OF TRANSITION  
PROBABILITIES FOR M STEPS, M= 2  
PROBABILITY OF GOING FROM STATE TO STATE

STATE	COMPLT	FRSPAS	SUBPAS
COMPLT	1.000	0.000	0.000
FRSPAS	.959	0.000	.041
SUBPAS	.947	0.000	.053

I.3 Model B of Requisition Processing System (contd)

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.  
 ? ?1  
 INPUT INITIAL STATE AS A ROW VECTOR IN FOLLOWING FORM  
 K= 3  
 INPUT K=0 IF COMPUTATION NOT DESIRED  
 INPUT K,A(1),A(2),A(3),...,A(K)  
 ? ?3,0,.80,.20

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 2  
 GIVEN INITIAL STATE A

STATE	COMPLT	FRSPAS	SUBPAS
	.956	0.000	.044

COMPUTATION OF TRANSIENT PROBABILITY MATRIX FOR M STEPS  
 INPUT NUMBER OF STEPS, M,  
 THAT MATRIX OF TRANSITION PROBABILITIES  
 SHOULD BE COMPUTED FOR  
 INPUT M=0 IF COMPUTATION NOT DESIRED  
 ? ?3

CMP: MATRIX OF TRANSITION  
 PROBABILITIES FOR M STEPS, M= 3  
 PROBABILITY OF GOING FROM STATE TO STATE

STATE	COMPLT	FRSPAS	SUBPAS
COMPLT	1.000	0.000	0.000
FRSPAS	.990	0.000	.010
SUBPAS	.988	0.000	.012

COMPUTATION OF STATE SPACE, M STEPS LATER,  
 GIVEN INITIAL STATE  
 INPUT ISAMEA=0 IF SAME ROW VECTOR TO BE USED  
 INPUT ISAMEA=1 FOR NEW A(K) VECTOR INPUT.  
 ? ?0

AM. ROW VECTOR FOR STATE SPACE M STEPS LATER, M= 3  
 GIVEN INITIAL STATE A

STATE	COMPLT	FRSPAS	SUBPAS
	.990	0.000	.010

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UNCLASSIFIED

AD 688044

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) US Army Tank Automotive Command Warren, Michigan	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A
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3. REPORT TITLE  
Simulation with Markov Transition Matrix Model of a Requisition Processing System

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)  
Final

5. AUTHOR(S) (First name, middle initial, last name)  
Irwin F. Goodman

6. REPORT DATE May 1969	7a. TOTAL NO. OF PAGES 73	7b. NO. OF REFS 11
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8a. CONTRACT OR GRANT NO  b. PROJECT NO N/A c. N/A d.	9a. ORIGINATOR'S REPORT NUMBER(S) S-1001 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
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10. DISTRIBUTION STATEMENT  
Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY In-house
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13. ABSTRACT

A cursory review of the literature relating to the application of the Markov Transition Probability Matrix for the evaluation and analysis of problems was accomplished. A FORTRAN IV computer timing program, based upon the mathematics of Markov Transition Matrices, has been developed and documented. The program was initially developed with data based upon a classical random walk problem involving a drunk meandering from corner to corner between his home and a bar. The resulting Markov Model has been applied to a requisitioning system, an essentially equivalent problem. Some analysis results are presented following the application of the computer program to a requisitioning system. The computer program has been written generally enough for application to such other diverse problem areas as charge accounts, tank battles, and reliability and maintainability.

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Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Markov Process, Simulation, Inventory Theory						

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