NUMERICAL SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS

P. C. WATERMAN
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P. C. Waterman
and
C. V. McCarthy

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ABSTRACT

The purpose of this work is to describe a theoretical formulation, including a documented computer program, for the evaluation of electromagnetic scattering by perfectly conducting bodies having an axis of rotational symmetry. The main body of the work gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listings of the computer program. Finally, Appendix III gives the computer printout for a numerical example, scattering by a conducting sphere-cone-sphere obstacle, as obtained on the IBM 7030 digital computer.
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**TABLE OF CONTENTS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SECTION I</strong></td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>GENERAL DISCUSSION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>COMPUTATIONAL ASPECTS</td>
<td>2</td>
</tr>
<tr>
<td><strong>SECTION II</strong></td>
<td>THEORY</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>MATRIX FORMULATION</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>EVALUATION OF THE TRANSITION MATRIX</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>APPLICATION TO SPECIAL GEOMETRIES</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>INTERPRETATION OF NUMERICAL RESULTS</td>
<td>29</td>
</tr>
<tr>
<td><strong>APPENDIX I</strong></td>
<td>ORGANIZATION OF THE COMPUTER PROGRAM</td>
<td>37</td>
</tr>
<tr>
<td>1.0</td>
<td>INTRODUCTION</td>
<td>37</td>
</tr>
<tr>
<td>2.0</td>
<td>GLOSSARY OF THE SUBROUTINES</td>
<td>38</td>
</tr>
<tr>
<td>3.0</td>
<td>THE INPUT ROUTINE</td>
<td>40</td>
</tr>
<tr>
<td>4.0</td>
<td>CALCULATION OF END POINTS AND SPACING FOR INTEGRATION</td>
<td>42</td>
</tr>
<tr>
<td>5.0</td>
<td>THE FIRST CONTROL ROUTINE</td>
<td>43</td>
</tr>
<tr>
<td>6.0</td>
<td>ASSOCIATED LEGENDRE FUNCTIONS</td>
<td>46</td>
</tr>
<tr>
<td>7.0</td>
<td>BESSEL FUNCTIONS</td>
<td>48</td>
</tr>
<tr>
<td>8.0</td>
<td>RECURSION RELATIONSHIPS FOR BESSEL AND NEUMANN FUNCTIONS</td>
<td>48</td>
</tr>
<tr>
<td>9.0</td>
<td>GENERATING THE BODY SHAPE</td>
<td>50</td>
</tr>
<tr>
<td>10.0</td>
<td>FIRST MATRIX PRINTOUT</td>
<td>51</td>
</tr>
<tr>
<td>11.0</td>
<td>PRINTOUT OF AN ARRAY</td>
<td>52</td>
</tr>
<tr>
<td>12.0</td>
<td>GENERATING THE Q MATRIX AND THE T MATRIX</td>
<td>52</td>
</tr>
</tbody>
</table>
### TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.0 NORMALIZING MATRICES</td>
<td>55</td>
</tr>
<tr>
<td>14.0 CONDITIONING MATRICES</td>
<td>56</td>
</tr>
<tr>
<td>15.0 PRINTING THE T MATRIX</td>
<td>57</td>
</tr>
<tr>
<td>16.0 FINAL CONTROL ROUTINE</td>
<td>58</td>
</tr>
<tr>
<td>17.0 MULTIPLYING A MATRIX TIMES A VECTOR</td>
<td>61</td>
</tr>
<tr>
<td>18.0 CORE DUMP</td>
<td>61</td>
</tr>
<tr>
<td>19.0 STORAGE ARRANGEMENTS</td>
<td>61</td>
</tr>
<tr>
<td>APPENDIX II: THE FORTRAN IV PROGRAM LISTING</td>
<td>65</td>
</tr>
<tr>
<td>APPENDIX III: A NUMERICAL EXAMPLE: THE SPHERE-CONE-SPHERE</td>
<td>95</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>125</td>
</tr>
</tbody>
</table>
SECTION I
INTRODUCTION

GENERAL DISCUSSION

In recent years work has begun to appear in the literature on the numerical solution of electromagnetic scattering problems by digital computer. For the most part these methods have involved numerical solution of a vector surface integral equation. In any case, the basic procedure in all methods requires numerical generation of the elements of an \( N \times N \) matrix, followed by subsequent inversion. Because \( N \) increases roughly linearly with the size of the target (quadratically for bodies that are not axially symmetric) there are practical limitations on the sizes that can be treated successfully. Hence such computations, exact in the sense that in principle any desired accuracy may be attained, are extremely useful in the Rayleigh region and some portion of the resonance region, but must ultimately be supplemented by high frequency approximate techniques in order to obtain the complete frequency response of a given target. Observe that exact numerical computations may play a useful role in establishing the usefulness and accuracy of approximation techniques, and also in providing experimental targets for more comprehensive range calibration than is presently possible using spheres and dipoles.

An exact formulation of scattering of electromagnetic waves by perfectly conducting obstacles was given in 1949 by Maue, who obtained a pure integral equation and, alternatively, an integro-differential
equation, either of which suffices for determination of the unknown surface currents on the obstacle.\(^1\) Both equations have been discussed in the excellent review article on diffraction by Hönl, Maué, and Westpfahl (HMW),\(^2\) and a derivation of the pure integral equation has been presented by Van Bladel.\(^3\) The integro-differential equation has been programmed and solved numerically on the digital computer by Andreasen,\(^4\) who considered axially symmetric targets. Similarly, numerical analysis has been performed by Oshiro and co-workers, employing the pure integral equation for more general shapes.\(^5\)

An alternative theoretical approach, also leading to numerical results, has been given by Waterman.\(^6\) The purpose of this paper is to document a computer program for the implementation of this method. Section II gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listing of the computer program. Finally, Appendix III gives the computer printout for a numerical example, scattering by a conducting sphere-cone-sphere obstacle.

**COMPUTATIONAL ASPECTS**

In addition to their role in the present work, it should be noted that certain of the subroutines contained in this report may be of interest for other applications.
Principal among these are those routines for generating the spherical Bessel and Hankel functions by a combination of power series and recursion techniques, noting that both precision checks and alternative procedures are included for those cases where precision is difficult to maintain. The subroutine for generating associated Legendre functions, and their derivatives, by recursion is also essentially self-contained. Finally, certain of the matrix processing operations, e.g., orthogonalization, may prove of use elsewhere, perhaps with modifications.
SECTION II
THEORY

MAXWELL FORMULATION

Consider an incident electromagnetic wave $E_i(x), H_i(x)$ impinging on the closed, perfectly conducting surface $\sigma$ of Figure 1 in otherwise free space. It is assumed throughout that $\sigma$ is sufficiently regular that Green's theorem is applicable, and that $\sigma$ possesses a continuous single-valued normal $\hat{n}$ at each point. Only simple harmonic time dependence at angular frequency $\omega$ is considered; a factor $\exp(-i\omega t)$ is suppressed in all field quantities. Field behavior is described by Maxwell's equations in the form

$$\nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = 0,$$

with an identical equation governing $\mathbf{H}$. In these equations $k=\omega/c=2\pi/\lambda$ is the free-space propagation constant.

Because the surface conductivity is infinite on $\sigma$, no tangential components of electric field can be supported. Currents are induced in the surface, the electric field of which must precisely cancel the tangential components of $E_i$ at each point on $\sigma$. HMW have given a representation of the fields for this problem in terms of surface current. After minor modification their formulas may be written

$$\mathbf{E}(x) = E_i(x) + \int d\sigma' \nabla \times \mathbf{J}(x') g_0(k|\mathbf{r}-\mathbf{r}'|),$$

(2a)
Figure 1. Geometry for a Plane Wave, Propagation Vector \( \mathbf{k}_{in} \) Illuminating a Perfectly Conducting Obstacle Bounded by the Surface \( \sigma \)
\[
H(r) = H^\dagger(r) - ik \int d\sigma \nabla \times J(r') g_0(k|r-r'|),
\]  

(2b)

where \( E, H \) is the total field, \( g_0(kR) = (4\pi R)^{-1} \exp(ikR) \) is the (scalar) free space Green's function appropriate to outgoing waves, and the curl operators are with respect to the unprimed (i.e., not the integration) variables. The integrals represent the \( E \) and \( H \) fields, respectively, due to a surface distribution of electric dipoles, as one would anticipate on physical grounds. The quantity \( J(r) \), which we identify with induced surface current, stands for the jump discontinuity in magnetic field encountered in crossing the surface, i.e.

\[
J = -(1/ik) \hat{n} \times [H_+ - H_-] \text{ on } \sigma.
\]  

(2c)

In the course of obtaining Eqs. (2), the boundary conditions appropriate to conducting surfaces were employed, namely, that \( n \times E_+ = n \times E_- = 0 \) on \( \sigma \).

The nature of jump discontinuities in the field vectors across \( \sigma \) can be shown directly from Eqs. (2), giving

\[
E_+ - E_- = (\nabla \cdot J) \hat{n} = \hat{n} \mu_0 \rho
\]  

(3a)

\[
H_+ - H_- = ik \hat{n} \times \hat{d}
\]  

(3b)
In the first of these equations, the surface divergence \( \nabla_s \cdot \mathbf{j} \) of the current may be defined by the physical requirement that it equal the net flow of charge out of infinitesimal element of area (per unit area per unit time). The second equality, involving the surface charge density \( \rho \), follows from the continuity equation \( \nabla_s \cdot \mathbf{j} = - \partial \rho / \partial t \).

The extended boundary condition, requiring that the total electromagnetic field vanish identically in the interior (thus in particular \( \mathbf{E} = 0 \) on \( \partial \)), is from Eq. (3a) sufficient to guarantee the usual exterior boundary condition \( \hat{n} \times \mathbf{E} = 0 \). Applying the extended boundary condition in Eq. (2a) gives

\[
\int d\sigma' \nabla \times \mathbf{J}(\mathbf{r}') \cdot \mathbf{K}(\mathbf{r} - \mathbf{r}') = - \mathbf{E}(\mathbf{r}) ,
\]

an "extended" integral equation that is to hold for all points \( \mathbf{r} \) in the small dashed sphere in Figure 1. By taking the curl of both sides of this equation, it follows that the total magnetic field \( \mathbf{H} \) will also vanish in this region, once Eq. (4) is satisfied. Equation (4) is equivalent to three scalar equations for the two unknown tangential components of \( \mathbf{j} \); only two of the equations are independent, however, in consequence of the fact that each side of Eq. (4) must have zero divergence.

Equation (4) may be satisfied by expanding both sides in regular vector eigenfunctions of the vector Helmholtz Eq. (1).
To treat the integral one writes \( \varphi_0 = \varphi \cdot \varphi_0 \); the expansion of the "free space Green's dyad" \( \varphi_0 \) has been given by Morse and Feshbach.\(^{(9)}\) Because of orthogonality over any spherical surface about the origin shown in Figure 1, corresponding coefficients may be equated on both sides of Eq. (4) to give, for incident plane \( E^i(r) = \hat{e}_0 e^{ik \cdot r} \),

\[
\int d\sigma \varphi(r) \cdot \mathcal{M}^\sigma_{\sigma mn}(r)
= - (4\pi/ik^3) i^{\frac{\sigma}{2}} [\ell (n(n + 1))]^{\frac{\sigma}{2}} \hat{e}_0 \cdot C^\sigma_{mn}(\hat{k}) ,
\]

\[
\int d\sigma \varphi(r) \cdot \mathcal{N}^\sigma_{\sigma mn}(r)
= + (4\pi/ik^3) i^{\frac{\sigma}{2}} [\ell (n(n + 1))]^{\frac{\sigma}{2}} \hat{e}_0 \cdot iB^\sigma_{mn}(\hat{k}) .
\]

(5)

The \( \mathcal{M}^\sigma, \mathcal{N}^\sigma \) are the outgoing wave functions, and dependence on the direction of incidence \( \hat{k} \) is contained in the vector spherical harmonics \( C^\sigma_{mn}, B^\sigma_{mn} \).\(^{(10)}\) These equations are to hold for each triplet of values \( (\sigma, m, n) \), with \( \sigma = e, o \) (even, odd), \( m = 0, 1, \ldots, n, n = 1, 2, \ldots \).

These are the conditions under which the total \( E, H \) field will vanish identically in that volume consisting of the largest sphere inscribable within \( \sigma \) about the coordinate origin employed. As has been shown elsewhere,\(^{(6)}\) because of analytic continuability this is adequate to guarantee that \( E \) and \( H \) will vanish identically throughout the entire interior volume.
The surface current is next approximated by expansion in the assumed complete set of tangential vector functions $\hat{n} \times M$ and $\hat{n} \times N$; one writes

$$I(r) = \frac{(4/\omega k)}{s} \sum \sigma^\prime m^\prime n^\prime \left[ a_{\sigma^\prime m^\prime n^\prime} \hat{n}(r) \times N_{\sigma^\prime m^\prime n^\prime}(r) \right]$$

$$+ b_{\sigma^\prime m^\prime n^\prime} \hat{n}(r) \times N_{\sigma^\prime m^\prime n^\prime}(r)$$

(6)

where the expansion coefficients remain to be determined. At this point one can expedite the discussion by introducing a matrix notation. First, the triplet of indices appearing in Eqs. (5) and (6) are regrouped into a single index $\nu$ by the ordering $(\sigma mn) = e01, o01, e11, o11, e02, \ldots$. The vector spherical harmonics may then be written as column matrices $C, B$, having as their $\nu$th elements $i^{n(n+1)} \frac{c_{\sigma m n}}{\nu}$, and $i^{n(n+1)} \frac{b_{\sigma m n}}{\nu}$, respectively. The undetermined expansion coefficients of Eq. (6) are simply designated by the column matrices $a, b$.

In this notation, substitution of the expansion Eq. (6) into Eq. (5) yields a pair of coupled matrix equations

$$\begin{bmatrix}
I & J \\
K & L
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
= 
\begin{bmatrix}
\hat{e} \cdot C \\
-e \cdot iB
\end{bmatrix}$$

(7)
for the determination of \(a\) and \(b\). The matrix elements of \(I\) are given, after rewriting the triple scalar product that appears, by

\[
I_{\nu j} = \left(\frac{k^2}{\pi}\right) \int d\Omega_1(r) \cdot \left[ \mathbf{H}_{\nu mn}(r) \times \mathbf{H}_{\sigma m'n'}(r) \right],
\]

(8a)

and the four matrices \(I, J, K, L\) differ from each other only in the vector products appearing in the integrand of Eq. (8) which are, respectively, \(\mathbf{H}_{\nu 1} \times \mathbf{H}_{\nu 1}, \mathbf{H}_{\nu 1} \times \mathbf{H}_{\nu 1}, \mathbf{H}_{\nu 1} \times \mathbf{H}_{\nu 1},\) and \(\mathbf{H}_{\nu 1} \times \mathbf{H}_{\nu 1}.\) By inspection of the integrands, in view of the fact that \(\mathbf{H} = \text{Re} \mathbf{H}\) and \(\mathbf{N} = \text{Re} \mathbf{N}\), it is clear that \(\text{Re} I\) and \(\text{Re} L\) are skewsymmetric, whereas \(\text{Re} J\) and \(\text{Re} K\) are symmetric. The surface integrals of Eq. (8a) must, in general, be done numerically and are most conveniently performed in spherical coordinates \(\theta, \phi\), for which the appropriate radial coordinates to employ may be given by the parametric specification \(r = r(\theta, \phi)\) of the surface. In view of Green's second vector identity

\[
\int d\Omega \cdot [\mathbf{A} \times \nabla \times \mathbf{B} - \mathbf{B} \times \nabla \times \mathbf{A}] = \int d\Omega \cdot [\nabla \cdot \mathbf{B} \times \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}]
\]

the matrices may be seen to be interrelated by

\[
K = -J + i(D_1)^{-1},
\]

\[
L = -I
\]

(8b)
where the diagonal matrix $D_+$ (and $D_-$, employed below) has $v$th elements defined by

$$(D^v_+) = (\pm 1)^n \frac{C_m (2n + 1) (n - m)!}{4n(n + 1) (n + m)!}. \quad (8c)$$

The Neumann factor $C_m$ is given by $C_0 = 1$, $C_m = 2$ otherwise.

It is also desired to compute the scattered field $E^a$, $H^a$ given by the surface integrals in Eq. (2). Specifically for the electric field, one has

$$E^a (x) = 4 \sum_{\sigma mn} [f_{\sigma mn} H^a_{\sigma mn} (x)$$

$$+ g_{\sigma mn} H^a_{\sigma mn} (x)]; \quad r > r' \text{ max on } \sigma$$

$$\sim E^a_{\text{out}} e^{ikr}; \quad kr \gg 1. \quad (9)$$

The vector scattering amplitude $F$, depending both on direction of incidence $\hat{k}_i$ and observation $\hat{k}_o$, is obtained by introducing asymptotic forms of the outgoing partial waves $H^a$, $H^d$ in the preceding expression for $E^a$ to get

$$F(\hat{k}_o, \hat{k}_i) = (4/ik) [C'(\hat{k}_o) D_+ f + iB'(\hat{k}_o) D_+ g], \quad (10)$$

11
where \( C' \) is the transpose of \( C \) (and hence a row matrix). The outgoing partial wave expansion coefficients \( f, g \) are expressed in terms of surface currents \( a, b \) by

\[
\begin{bmatrix}
  f \\
  g
\end{bmatrix}
= -\text{Re}
\begin{bmatrix}
  I & J \\
  K & L
\end{bmatrix}
\begin{bmatrix}
  a \\
  b
\end{bmatrix}.
\]  

(11)

These formulas have been obtained by employing that expansion of the free space Green's dyad valid in the exterior region outside the large dashed sphere of Figure 1.

The scattering cross section \( \sigma^{\text{scat}} \) is given by

\[
\sigma^{\text{scat}} = (16\pi/k^2) \left( f^* D^f + g^* D^g \right). 
\]  

(12a)

As a numerical check on accuracy, one may also compute the total cross section

\[
\sigma^{\text{tot}} = (4\pi/k) \text{Im} \left[ \hat{e}_o \cdot F (\hat{k}_{\text{in}}, \hat{e}_{\text{in}}) \right], 
\]  

(12b)

which must equal \( \sigma^{\text{scat}} \) by the forward amplitude theorem \((11)\). The radar cross section, defined as \( 4\pi \) times the back-scattered power per steradian divided by incident power per unit area, is given by

\[
\sigma^{\text{radar}} = (64\pi/k^2) \left| \hat{e}_o \cdot C' (-\hat{k}_{\text{in}}) D_{-f} 
+ i \hat{e}_o \cdot B' (-\hat{k}_{\text{in}}) D_{-g} \right|^2. 
\]  

(12c)
If the return signal is regarded as resolved into two orthogonal linearly polarized modes, then this equation gives a measure of the power carried in that mode having polarization aligned with the original incident wave, whereas the cross-polarized return is given by replacing \( \hat{\mathbf{e}}_o \) by \( \hat{\mathbf{e}}'_o = \mathbf{K}_i \times \hat{\mathbf{e}}_o \) in Eq. (12c).

**EVALUATION OF THE TRANSITION MATRIX**

Instead of first solving Eq. (7) for the currents \( a, b \), then substituting in Eq. (11) to obtain the scattered wave \( f, g \), the currents may be formally eliminated to obtain the scattered wave directly from the incident wave as

\[
\begin{pmatrix}
  f \\
g
\end{pmatrix}
= - \begin{pmatrix}
  D_{+}^{-1} & 0 \\
  0 & D_{+}^{-1}
\end{pmatrix}
\begin{pmatrix}
  T_1 & T_2 \\
  T_3 & T_4
\end{pmatrix}
\begin{pmatrix}
  D_{+}^{1/2} & 0 \\
  0 & D_{+}^{1/2}
\end{pmatrix}
\begin{pmatrix}
  \mathbf{e}_o \cdot \mathbf{C} \\
  -\mathbf{e}_o \cdot \mathbf{B}
\end{pmatrix}.
\]

The block matrix,

\[
T = \begin{pmatrix}
  T_1 & T_2 \\
  T_3 & T_4
\end{pmatrix},
\]

is known as the **transition matrix**, and is both symmetric (i.e. \( T_1' = T_1, T_2' = T_3, T_4' = T_4 \)) and has the property \( T^* T = \text{Re} T \), i.e.,

\[
\begin{pmatrix}
  T_1^* & T_2^* \\
  T_3^* & T_4^*
\end{pmatrix}
\begin{pmatrix}
  T_1 & T_2 \\
  T_3 & T_4
\end{pmatrix}
= \begin{pmatrix}
  T_1^* T_1 + T_2^* T_3 & T_1^* T_2 + T_2^* T_4 \\
  T_3^* T_1 + T_4^* T_3 & T_3^* T_2 + T_4^* T_4
\end{pmatrix}
= \text{Re} \begin{pmatrix}
  T_1 & T_2 \\
  T_3 & T_4
\end{pmatrix}.
\]
The property Eq. (15) is a consequence of unitarity of the scattering matrix \( S = 1 - 2T \), as may be verified by substitution in the unitarity condition \( S'^*S = 1 \).

If one now defines the matrix \( Q \) as

\[
Q = \begin{pmatrix}
Q_1 & Q_2 \\
Q_3 & Q_4
\end{pmatrix} = \begin{pmatrix}
D_{+}^{\frac{1}{2}} & 0 \\
0 & D_{+}^{\frac{1}{2}}
\end{pmatrix} \begin{pmatrix}
J' & L' \\
I' & K'
\end{pmatrix} \begin{pmatrix}
D_{+}^{\frac{1}{2}} & 0 \\
0 & D_{+}^{\frac{1}{2}}
\end{pmatrix},
\]

then by comparison with Eqs. (7) and (11) the transition matrix is determined by the matrix equation

\[
QT = \text{Re } Q,
\]

which in general must be solved numerically.

Instead of working with the 2 by 2 block form of Eq. (17), involving in truncation four \( N \times N \) matrices, it is convenient, for the numerical processing, to change over to single \( 2N \times 2N \) matrices. Thus, define the \( 2N \times 2N \) matrix \( \hat{Q} \) by

\[
\begin{align*}
\hat{Q}_{(2m-1)(2n-1)} &= (Q_1)_{mn} \\
\hat{Q}_{(2m-1)(2n)} &= (Q_2)_{mn} \\
\hat{Q}_{(2m)(2n-1)} &= (Q_3)_{mn} \\
\hat{Q}_{(2m)(2n)} &= (Q_4)_{mn}
\end{align*}
\]

\( m, n = 1, 2, \ldots, N \).
The matrices $\hat{T}$, and $\hat{S} = 1 - 2\hat{T}$ are defined in exact analogy to this. At this point, Eq. (16b) may be written in terms of $\hat{S}$ as

$$\hat{Q}\hat{S} = -\hat{Q}^*.$$  

(19)

Because of the behavior of the radial (Hankel) functions that appear in the matrix elements of $\hat{Q}$, the imaginary parts of the elements of $Q$ will tend to grow very large numerically above the diagonal. In order to avoid loss of precision due to the finite precision arithmetic employed by the digital computer, it is convenient at this stage to reset all the mentioned elements to zero, by Gaussian elimination. This process is straightforward, the net effect being to premultiply $\hat{Q}$ by a real upper triangular matrix (all elements zero below the main diagonal). Suppose this conditioning to have been performed on Eq. (19), which we continue to employ without change of notation.

To Eq. (19) are adjoined the constraints of symmetry and unitarity mentioned above, which are unaffected by the $S \rightarrow \hat{S}$ transformation and thus given by

$$\hat{S} = \hat{S}'$$  

(20)

and

$$\hat{S}'\hat{S} = 1.$$  

(21)
Two extremes of view with regard to the system of Eqs. (19), (20) and (21) are as follows: first, one might truncate the matrix Eq. (19), solve numerically by digital computer, then compare the resulting solution with Eqs. (20) and (21'), the latter thus being employed as consistency checks. On the other hand, one might attempt to treat all three equations from a unified point of view from the onset, obtaining a solution in some sense of Eq. (19) subject to the constraints of Eqs. (2n) and (21). The first approach has been employed in earlier work on the computer for bodies of rotational symmetry, and works quite satisfactorily for a restricted range of body shapes and sizes. The second approach is employed in the present work in order to extend the range of bodies that can be handled, in view of the fact that the constraints essentially determine three quarters of the solution
[i.e., of the $8N^2$ real parameters appearing in the $2N \times 2N$ (truncated) complex matrix $\hat{S}$, it can be shown that only $N(2N + 1)$ are independent, if $\hat{S}$ satisfies Eqs. (20) and (21)].

To develop a unified analysis, observe first that if $\hat{S}$ could be constructed in the form

$$\hat{S} = U' U$$

(22)

where $U$ is unitary, then both constraints would be satisfied by inspection. This suggests that, rather than inverting $\hat{Q}$ directly in Eq. (19), it be
made unitary. Thus consider the upper triangular matrix $M$ (i.e., all elements are zero below the main diagonal) which by premultiplication makes $Q$ into a unitary matrix $Q_{\text{unit}}$, viz.

$$MQ = Q_{\text{unit}} \quad \quad (23)$$

Premultiplying Eq. (19) by $M$, one can write

$$\hat{Q}^* \hat{S} = -M\hat{Q}^* = -MM^* \hat{Q}_{\text{unit}}^* \quad \quad (24)$$

Upon solving for $\hat{S}$, there now results

$$\hat{S} = -\hat{Q}_{\text{unit}}^* (MM^*)^{-1} \hat{Q}_{\text{unit}}^* \quad \quad (25)$$

Substituting this result in Eq. (20), the symmetry constraint, it follows without difficulty that the matrix product $MM^*$ must be symmetric. But each of the matrices appearing in the product is upper triangular, and their product is again upper triangular. Consequently the product must be a diagonal matrix. Further, the diagonal elements can be written out explicitly, giving

$$MM^* = \begin{bmatrix}
M_{11}/M_{11}^* & 0 & \ldots \\
0 & M_{22}/M_{22}^* & \ldots \\
0 & 0 & \ddots \\
\end{bmatrix}$$
From Eq. (24) the $S$-matrix is now given by

$$S = - Q^* \text{unit} \hat{Q}^* \text{unit},$$

which is of the required form Eq. (22). Substituting Eq. (26), along with the identity $Q^* \text{unit} \hat{Q}^* \text{unit} = 1$, back in the relation $\hat{S} = 1 - 2\hat{T}$, the desired transition matrix is given by

$$\hat{T} = Q^* \text{unit} \text{Re} (\hat{Q}^* \text{unit}),$$

and the block form of $T$ is readily obtained by reversing the transformation of Eq. (18).

Returning to $M$ for a moment, Eq. (25) states simply that $M$ is real. Thus the process may be summed up in the (formal) theorem: given the matrix Eq. (19), with constraints, Eq. (20) and (21), on the solution, it follows that the given matrix $Q$ cannot be arbitrary, but must be such as to be factorizable into the product of a real upper triangular matrix and a unitary matrix, namely

$$\hat{Q} = N^{-1} \hat{Q} \text{unit}.$$
The transformation of $\mathbf{Q}$ into a unitary matrix, as required in Eq. (23), is done by Schmidt orthogonalization of the $2N$ vectors given by the rows of $\mathbf{Q}$, beginning with the bottom row and working up. The procedure is straightforward, and details are described in a subsequent section.

APPLICATION TO SPECIAL GEOMETRIES

In order to apply the equations to bodies having an axis of rotational symmetry, the axis of symmetry is chosen as polar axis for our spherical coordinates and, without loss of generality, the direction of incidence taken in the plane of azimuth $v = 0$, so that $\hat{k}_{in} = \hat{k}_{in}(u, 0)$. A reduced index notation may be employed for those matrix elements that do not vanish under the azimuthal integration, writing

$$I_{mnn'} = I_{ommenn'} = -I_{ennomn'}$$

$$J_{mnn'} = J_{emmenn'} = J_{omnomn'}$$

$$K_{mnn'} = -J_{mnn'} + \iota \delta_{nn'} D_{mnn}^{-1}$$

$$L_{mnn'} = I_{mnn'}$$

(29)
The independent matrix elements, written out, are

\[ I_{mnn'} = m \int_0^\pi d\theta \frac{\partial}{\partial \theta} \left( \frac{\partial^{m_m^m} P^m_n P^m_{n'} \left( kr \right)^2}{\sin^2 \theta} \right) \left( \frac{\partial^{m'_m'} P^{m'}_{n'} \partial^{m'_m'} P^{m'}_{n'} \left( kr \right)^2}{\sin^2 \theta} \right) \left( kr \right) h_n(kr) j_{m'}(kr) \]

(30)

\[ J_{mnn'} = \frac{-2}{\epsilon_m} \int_0^\pi d\theta \sin \theta \left( \frac{\partial^{m_m^m} P^m_n \partial^{m'_m'} P^{m'}_{n'} \left( kr \right)^2}{\sin^2 \theta} \right) \left( kr \right) \frac{d}{dkr} \left[ kr j_{m'}(kr) \right] \]

Observe that the real parts of all these matrices are symmetric. Also, because of the vanishing of all matrix elements with different azimuthal mode indices \((m \neq m')\), there is no coupling and each azimuthal mode \(m = 0, 1, 2, \ldots\) may be evaluated separately.

From the defining Eq. (16), the only non-vanishing elements of the Q matrix may now be written in reduced index notation as

\[ (Q_1)_{mnn'} = (Q_1)_{enmnen'} = (Q_1)_{omnmonn'} \]
\[ (Q_2)_{mnn'} = (Q_2)_{omnmonn'} = - (Q_2)_{emnomn'} \]
\[ (Q_3)_{mnn'} = (Q_3)_{enmnen'} = - (Q_3)_{omnmonn'} \]
\[ (Q_4)_{mnn'} = - (Q_4)_{enmnen'} = - (Q_4)_{omnmonn'} \]

(31)
In addition, the reduced index elements are related by

\[
(Q_3)_{mnn'} = (Q_2)_{mnn'}
\]

\[
(Q_4)_{mnn'} = -(Q_1)_{mnn'} + i \frac{1}{n'}
\]  \hspace{1cm} (32)

Finally, examination of Eq. (17) reveals that the non-vanishing elements in the four blocks of the T matrix are interrelated exactly as in Eq. (31), but not Eq. (32), so that the complete solution may be obtained by solving Eq. (17) once, using the reduced index quantities.

A further important reduction occurs in the preceding equations for obstacles (e.g., finite cylinder) having a plane of mirror symmetry normal to the axis of rotational symmetry. For this geometry the radius vector \( r(\theta) \) specifying the shape of the obstacle will be even about \( \theta = \pi/2 \), i.e.,

\[
r(\theta) = r(\pi - \theta)
\]  \hspace{1cm} (33)

Inspection of the parity of the integrands giving rise to matrix elements in Eq. (30) readily reveals that a checkerboard pattern of zeros has emerged, i.e.,

\[
I_{mnn'} = 0; \ (n + n') \text{ even}
\]

\[
J_{mnn'} = 0; \ (n + n') \text{ odd}
\]  \hspace{1cm} (34)

These elements can hence be set to zero without performing the numerical integrations.
Prolate (and oblate) spheroids have a mirror symmetry plane normal to their rotational symmetry axis, so that both mode and parity decompositions may be made, as discussed above. There is another reduction that occurs here, however, which from a theoretical standpoint lays the Rayleigh expansion out in full view, and for numerical purposes yields extremely well-conditioned matrices for inversion.

To see this, let us examine the matrix elements as given by Eqs. (30). The numerical magnitude of these elements is influenced mainly by the radial functions appearing in the integrand. For $I_{mn}'$, for example, one has

$$I_{mn}' \sim (kr)^2 \left[ h_n(kr) j_n'(kr) = (kr)^2 \left[ j_n h_n' + i n j_n \right] \right].$$

For a given argument $x$, the Bessel functions $j_n(x)$ decrease rapidly in magnitude, and the Neumann functions $n_n(x)$ increase, roughly as soon as the index $n$ exceeds $x$. Thus the real part of $I$, which is obviously symmetric, will eventually decrease rapidly in magnitude as one proceeds along any row or column. The numerical behavior of $I$ is dominated by its imaginary part, for which elements again decrease going out any row, but increase going down any column, at such a rate that diagonal elements remain relatively constant. These large numerical values presumably strongly influence the truncated matrix inversion procedure.
One can show, however, that for prolate or oblate spheroids this behavior, specifically the arbitrarily large values by which elements of \( I \) below the diagonal exceed corresponding elements above, vanishes identically. \( I \) and \( J \) become completely symmetric, and dominant terms lie only on the diagonal once either row or column index exceeds \( kr_{\text{max}} \) where \( r_{\text{max}} \) is the radius of the circumscribing sphere.

Based on results given by Watson\(^{(12)}\) one can show that the radial factor for an element below the diagonal in the imaginary part of \( I_{mn} \) is of the form

\[
x^{2n+2s+1} = x^{2n+1} + \frac{1}{x^{2s}} + \frac{1}{x^{2s-2}} + \cdots + 1
\]

where the equivalence symbol (\( \sim \)) indicates that the exact coefficients of inverse powers of \( x^2 \) have not been included, as they are not required in the present discussion. The first term on the right-hand side corresponds precisely to the symmetrically placed element above the diagonal; we must thus show that the inverse powers of \( x^2 \) contribute nothing to the integral

\[
\text{Im} I_{n(n+2s+1)\bar{m}} = \pi \int_0^\pi d\theta \frac{\partial}{\partial \theta} (P^m_{n+2s+1} P^n_m) (kr)^{2n+2s+1} j_n(kr) j_{\bar{m}}(kr) .
\]
For a prolate (oblate) spheroid, having semi-axes $a$, $b$, one has

$$kr = ka \left[ \cos^2 \theta + (a/b)^2 \sin^2 \theta \right]^{-\frac{1}{2}}, \quad (37)$$

which may be rewritten (identifying $x$ with $kr$)

$$1/x^2 = P_0 + P_2. \quad (38)$$

Now the product of two Legendre functions may itself be expanded in a series of Legendre polynomials, with indices ranging from the difference to the sum of the original indices, i.e.,

$$P_n^m P_{n'}^{m'} = \sum_{p=n-n'}^{n+n'} P_p \quad (39)$$

where again explicit numerical coefficients have been ignored.

Substituting Eq. (38) in the series of inverse powers of $x^2$ appearing in Eq. (35), then employing Eq. (39) repeatedly, one can write

$$x^2 n_{n+2s+1}^{n+1} = x^2 n_{n+2s+1}^{n+1} + \sum_{q=0}^{s} P_{2q}. \quad (40)$$
This result may be put in Eq. (36), recalling also that Re$I$ is symmetric, to get

\[ I_{m(n+2s+1)n} - I_{mn(n+2s+1)} = m \int_0^\pi d\theta \; \frac{\partial}{\partial \theta} \left( P_{n+2s+1}^m P_n^m \right) \sum_{q=0}^{s} P_{2q} \]

\[ = -m \int_0^\pi d\theta \; \sin \theta \; P_{n+2s+1}^m P_n^m \sum_{q=1}^{s} P_{2q-1} \]

\[ = -m \int_0^\pi d\theta \; \sin \theta \; \sum_{p=s}^{s+n} P_{2p+1} \sum_{q=1}^{s} P_{2q-1} \]

\[ = 0 \quad (41) \]

where in the second step we have integrated by parts, then employed Eq. (39), and finally observed that the highest Legendre polynomial appearing in the second sum is $P_{2s-1}$, while the first sum begins at $P_{2s+1}$; because of orthogonality, all the resulting integrals vanish.

To perform the analogous calculation for $J_{mnn}'$, one proceeds by first employing Green's identity to rewrite $J_{mnn}'$ in the more symmetric form

\[ J_{mnn}' = -\frac{1}{\varepsilon_m} \int_0^\pi d\theta \; \sin \theta \; B_{mnn}'(\theta) \frac{d}{dx} \left[ x^2 h_n(x) j_n(x) \right]_{x=kr(\theta)} \]

\[ + \frac{1}{2\varepsilon_m} \int_0^\pi d\theta \; \sin \theta \; C_{mnn}'(\theta) \left[ x^2 h_n(x) j_n(x) \partial(1/x^2)/\partial \theta \right]_{x=kr(\theta)} \quad (42) \]
valid for \( n \neq n' \), with

\[
B_{mnn'}(\theta) = \frac{m^2 P_m^n P_{m'}^{n'}}{\sin^2 \theta} + \frac{\partial P_m^n}{\partial \theta} \frac{\partial P_{m'}^{n'}}{\partial \theta},
\]

\[ (43a) \]

\[
C_{mnn'}(\theta) = n'(n'+1) \frac{\partial P_m^n}{\partial \theta} P_{m'}^{n'} + n(n+1) P_m^n \frac{\partial P_{m'}^{n'}}{\partial \theta}.
\]

\[ (43b) \]

It is convenient this time to write

\[
1/x^2 = \text{const.} + \sin^2 \theta.
\]

Using this in conjunction with the inverse polynomial expression

\[
x^n_{n+2s} = x^n_{n+1} + \frac{1}{x^{2s-1}} + \frac{1}{x^{2s-3}} + \cdots + \frac{1}{x^n}
\]

Eq. (42) may be reduced to

\[
J_{m(n+2s)n} - J_{mn(n+2s)}
\]

\[
= \frac{(2s-1)/\epsilon_m}{\pi} \int_0^\pi d\theta \sin \theta B_{m(n+2s)n} \sum_{q=1}^{s} (\sin \theta)^{2q},
\]

\[
+ \frac{1/\epsilon_m}{\pi} \int_0^\pi d\theta \sin^2 \theta \cos \theta C_{m(n+2s)n} \sum_{q=0}^{s-1} (\sin \theta)^{2q}
\]

\[ (44) \]
where in the first term the constant term in the summation has been dropped because of the additional orthogonality relations

\[ \int_0^\pi d\theta \sin \theta \delta_{mnop} = 0, \ n \neq n' \]

At this point, using the standard recursion formulas for the Legendre functions one can write

\[ \sin^2 \theta B_{m(n+2s)n} = P_m^{n+2s-1}P_m^{n+1} + P_m^{n+2s-1}P_m^{n-1} + P_m^{n+2s}P_m^{n} \]

\[ + P_m^{n+2s+1}P_m^{n-1} - P_m^{n+2s+1}P_m^{n+1} \]  \hspace{1cm} (45a)

\[ \sin \theta \cos \theta C_{m(n+2s)n} = P_m^{n+2s-1}P_m^{n+1} + P_m^{n+2s-1}P_m^{n-1} \]

\[ + P_m^{n+2s+1}P_m^{n-1} + P_m^{n+2s+1}P_m^{n+1} \]  \hspace{1cm} (45b)

The polynomials in \( \sin^2 \theta \) appearing in Eq. (44) may be expanded in Legendre polynomials of highest index \( 2(s-1) \) [Note that a factor \( \sin^2 \theta \) has been taken out in the first case to employ in Eq. (45a)]. By examination of Eq. (39) it may be seen because of orthogonality that only the first term on the right-hand side of Eqs. (45a) and (45b)
will make a non-zero contribution to their respective integrals. Writing out these non-vanishing terms in Eq. (44) explicitly, one finally obtains

\[
J_{m(n+2s)n} - J_{mn(n+2s)}
\]

\[
\pi \left( \frac{2s-1)(n+2s+1)(n+2s+m)(m-m+1)}{e_m(2n+4s+1)(2n+1)} \right) \int_0^{\pi} \sin \phi F^m_{n+2s-1} F^m_{n+1} (\sin \phi)^{2s-2}
\]

+ same expression

= 0  \hspace{1cm} (46)

Thus I and J are symmetric, and one need only compute elements on and above the diagonal in Eqs. (30). The matrices are expected to be well-conditioned in the sense that numerical results will converge rapidly to final values versus truncation. From the point of view of the Rayleigh expansion in powers of ka, valid at low frequencies, observe that all matrices may be expanded in powers of ka, e.g., writing

\[
(J^{(m)})_{nn'} = J_{mn'}
\]

one has

\[
J^{(m)} = A + B(ka) + C(ka)^2 + \ldots
\]

\[
= A \left[ I + A^{-1} (J^{(m)} - A) \right]
\]

(47)
where \( A \) is diagonal, \( B \) is tridiagonal (all elements zero except on, one above, and one below the diagonal) and so forth. The inverse, expanded in powers of \( ka \), is readily obtainable by the binomial theorem as

\[
(J^{(m)})^{-1} = [1 - A^{-1} (J^{(m)} - A) + \ldots ] A^{-1} .
\] (48)

INTERPRETATION OF NUMERICAL RESULTS

In order to provide some insight into the behavior in practice of the various matrices discussed above, the numerical printout for an example has been included (Appendix III). In addition to providing a test case for use with the computer program, many features of matrix behavior are most conveniently described by reference to this printout.

The obstacle to which the results refer consists of a sphere-cone-sphere, as shown in Figure 2. The analytical description of this shape as inputted to the computer is detailed in Paragraph 9.0 of Appendix I. It will be seen that the printout consists almost entirely of matrix quantities, as an aid in gauging the numerical effectiveness of the truncation being employed.

The first page lists input parameters. Thus, four cases \((m = 0, 1, 2, 3)\) were evaluated consecutively, with truncation to \(6 \times 6\) matrices. The body is described analytically in three sections. The body shape "9" indicates that the body does not possess mirror symmetry normal to the axis of rotational symmetry. \(U\) vector indicates that 46 aspect
angles will be evaluated. The body size is $kA = 1.0$ (Figure 2), and the ratio of sphere radii is $b/a = (1 + \sin \alpha)^{-1} \approx 0.794$ (correct to nine significant figures in the computer), with cone half-angle $\alpha = 15$ degrees. Numerical integration is performed by Bode's rule using 64 equally spaced divisions in each section, and the angular end points in degrees are indicated for each section. (14)

The basic quantities shown, for each $m$ value successively, are the $Q$ matrix, the orthogonalized $\hat{Q}$ matrix, the transition matrix $T$, and the cumulative cross section quantities.

Consider first the case $m = 0$. The blocks $Q_2$ and $Q_3$ are zero for this case, and hence are not shown. The remaining blocks, $Q_1$ and $Q_4$, are obtained by numerical integration from the defining equations (16, 29, 30, 31, 32). Because of the vanishing of the blocks $Q_2, Q_3$ it turns out, as one can verify with some study, that the remaining two blocks actually are processed with no interactions, so that behavior can be discussed by examining say, $Q_1$, alone.

Considering the imaginary part of $Q_1$, which is the numerically dominant portion, one observes that elements of each of the six row vectors increase in numerical magnitude as one moves to the right. It is immediately clear that row vectors must be orthogonalized from the bottom up, in order that elements of the resulting vectors may settle down to constant values independent of truncation (that is, if one began orthogonalizing with the top row, then it can be seen that after
normalizing, the first element is smaller by a factor $14.1/11.1 \approx 13$ than it would have been in 5 x 5 truncation). Observe also that the bottom row is the best candidate for a "unit vector in the six direction," in that the sixth element is larger than the first by a factor of about $10^7$, whereas the corresponding factor for the top row is only $2 \times 10^2$.

The orthogonalized matrix $\hat{Q}_{\text{unit}}$, Eq. (23), is shown next, having dimension 12 x 12. Odd numbered rows have come from the original 6 x 6 $Q_1$, whereas the even numbered rows are associated with $Q_4$. It is striking to observe that each of the original matrices (and hence the entire imaginary part of $\hat{Q}_{\text{unit}}$) has become nearly diagonal. This occurred because the main effect of the sixth row vector was to reduce the last entry of each preceding row to nearly zero. The new fifth row vector, in similar fashion, then served primarily to reduce the fifth entry in each of the preceding four rows, and so on. Thus the first row of $Q_1$, which originally increased by a factor of about 170 from first to last entry, now decreases by a factor (see row one of the imaginary part of $\hat{Q}_{\text{unit}}$) of about $7 \times 10^{-8}$. The total relative reduction is of order $4 \times 10^{-10}$.

It is not difficult to study the behavior of the row vectors, or the individual matrix elements, versus truncation. For example, if a 5 x 5 truncation had been employed, then the first entry in the fifth row vector would have been, after normalization, $(-1.278 + 10.5861) \times 10^{-5}/(0.5646) = (-2.263 + 11.038) \times 10^{-5}$, whereas the 6 x 6 truncation (see row nine of $\hat{Q}_{\text{unit}}$) actually gives $(-2.251 + 11.036) \times 10^{-5}$. One
can verify that the sixth row vector has even less effect on rows earlier than the fifth. In particular, orthogonalizing the first row to the sixth row will change the first element of the first row from 0.848 to approximately 0.848 - (144/0.551) (6.99 x 10^-8), which constitutes a change in the sixth significant figure.

The blocks of the transition matrix, computed from Eq. (27) and the reverse transformation of Eq. (18), are shown next. Again, the blocks $T_2 = T_3'$ vanish identically and are not shown. Both $T_1$ and $T_4$ are seen to be exactly symmetric to the number of digits given, and the unitary-related condition of Eq. (15) is readily verified on the desk computer to within round-off error in the last place shown. Observe that the elements of both $T_1$ and $T_4$ fall off rapidly in magnitude moving away from the upper left hand corner, so that the scattering behavior would be efficiently and accurately describable in this instance using only the first two rows and columns of $T_1$ and $T_4$.

Finally, the accumulated (over $m = 0$ only) far field quantities are shown for $E\parallel$ (class 1) and $E\perp$ (class 2) polarizations. The first column gives the incident aspect angle measured from the axis of rotational symmetry. For each aspect, subsequent columns give the scattering cross section [Eq. (12a)], forward amplitude [the complex quantity appearing in Eq. (12b)], backscattered amplitude [the complex quantity appearing in Eq. (12c) before squaring], and finally the radar cross section and phase of the backscattered amplitude. Observe that
both energy conservation (equality of the second and fourth columns) and reciprocity (symmetry of the third and fourth columns about the aspect angle of ninety degrees) are satisfied to seven significant figures.

Turning to the case \( m = 1 \), the blocks \( Q_1 \) and \( Q_2 \) are shown, \( Q_3 \) and \( Q_4 \) then being given by [see Eqs. (8b, 16, 29)] \( Q_3 = Q_2, Q_4 = -Q_1 + iI \). A partial check on the precision of numerical integration is available for this and all subsequent \( m \) values. From the Wronskian relation

\[
x^2 [ \gamma_n(x) h_{n-1}(x) - \gamma_{n-1}(x) h_n(x) ] = i \quad \text{and the first of Eqs. (30)}
\]

it is immediately clear that the imaginary parts of the first off-diagonal elements of \( Q_2 \) should be symmetric, a result not used in the program (whereas symmetry of \( \text{Re}Q_1 \) and \( \text{Re}Q_2 \) is always enforced). The expected symmetry is seen to obtain to seven significant figures for the \((1, 2)\) and \((2, 1)\) elements. Precision subsequently deteriorates slightly so that discrepancies have appeared in the fifth significant figure between the \((5, 6)\) and \((6, 5)\) elements.

Numerical behavior of both the \( Q \) and the orthogonalized \( \hat{Q} \) matrices appears to proceed substantially as in the case discussed above for \( m = 0 \), although the details are of course considerably more complex because of the presence of all four non-zero blocks. The near-diagonal nature of the orthogonalized \( \hat{Q} \) matrix is still evident by inspection, however. In the resulting transition matrix the blocks \( T_1 \) and \( T_4 \) are seen to be symmetric, and the block \( T_3 \) to be equal to the transpose of \( T_2 \). Comparison of the far field results with those for \( m = 0 \) reveals significant changes at all aspect angles.
For the two subsequent cases \( m = 2, 3 \) a new effect is seen over and above the previously discussed features, due to the vanishing of the associated Legendre functions \( P^m_n \) for \( n < m \). In consequence, the first row and column of each block of the \( Q \) matrix (for \( m = 3 \) the first two rows and columns) are identically zero. The behavior versus truncation at \( n = 6 \), as judged by the near-diagonal results after orthogonalization, appears unaffected, however. The net result is that the computation becomes gradually simpler as \( m \) increases; \( m = 2 \) requiring treatment of \( 5 \times 5 \) blocks, and \( m = 3 \) requiring only \( 4 \times 4 \) blocks.

A measure of error incurred by stopping at \( m = 3 \) may be obtained by comparing the far field results with those obtained at \( m = 2 \) (except for incidence along the axis of rotational symmetry, 0 degrees or 180 degrees, for which scattering behavior is completely determined from the \( m = 1 \) results only). At incidence 80 degrees from the axis, for example, and for either polarization, the scattering cross section is seen to be unchanged to about six significant figures. For the same cases the radar cross section, however, has changed in about the third significant figure. Such precision is nevertheless quite adequate in most practical applications.
APPENDIX I

ORGANIZATION OF THE COMPUTER PROGRAM

1.0 INTRODUCTION*

The "EMSCAT" Program has been written in FORTRAN IV language for the IBM 7030 Computer to produce solutions to the electromagnetic scattering problems which are outlined above in Section II. Several factors were given consideration in the design of the program:

Efficient coding to reduce computer run time as much as possible. The routine VECMUL for matrix by vector multiplication was coded in machine language to take advantage of specialized coding available at that level. This routine is also available in FORTRAN (though less efficient and accurate) so that the program can be run on machines other than the 7030 Computer.

Full single word accuracy of a 7030 register and where necessary double precision accuracy was utilized in the calculation of special functions. Single precision accuracy on the 7030 maintains 15 digits of accuracy.

Maximum use of core storage. The size of the solution matrices (60 x 60 complex) was determined so that secondary storage devices such as tapes do not have to be utilized in running the program.

The matrices are stored and manipulated from 1 of 3 large blocks of common storage. Each block is dimensioned 120 x 120. However, through various equivalence statements in the proper routines, these major blocks are resegmented and renamed for ease of programming.

*The paragraphs in Appendix I have been numbered to facilitate cross-referencing.
2.0 GLOSSARY OF THE SUBROUTINES

The program operates via a MAIN routine and 15 auxiliary routines which are briefly described and listed below. Standard I/O and mathematical routines, e.g., SIN, LOG, etc. are assumed to be available through the FORTRAN operating system.

2.1 The MAIN routine controls overall run processing and computes the I, J, K and L matrices.

Routines called are:

- RDDATA
- GENLP
- GENKR
- GENBSL
- PRTMTX
- PRCSSM

2.2 Subroutine RDDATA reads the user's control parameters and sets up preliminary output heading information.

Routine called is: CALENP

2.3 Subroutine CALENP computes the sections of \( \theta \), the polar angle, and the step size for numerical integration.

2.4 Subroutine GENLP computes the associated Legendre functions over the necessary range.

2.5 Subroutine GENBSL controls backward recursion of Bessel functions and forward recursion of Neumann functions.

2.6 Subroutine BESSEL computes the Bessel function for a specific argument and order.

2.7 Subroutine GENKR computes the parameter "kr" and its derivative with respect to the polar angle \( \theta \).
2.8 Subroutine PRTMTX prints the headings and controls the printout of the I, J, K and L complex matrices.

Routine called is: PRINTM

2.9 Subroutine PRINTM prints the elements of a specified matrix of specified rank.

2.10 Subroutine PRCSSM generates the Q matrices from the I, J, K and L matrices, and transforms the Q matrix into the T matrix.

Routines called are: NRMQNX
                    CNDTNQ
                    PRTRIT
                    ADDPRC

2.11 Subroutine NRMQNX normalizes the I, J, K and L matrices to obtain the Q matrices.

2.12 Subroutine CNDTNQ conditions the Q matrix before transforming it into the T matrix.

2.13 Subroutine PRTRIT prints headings and controls printout of the T matrix.

Routine used is: PRINTM

2.14 Subroutine ADDPRC does final processing of the T matrix to provide the scattering results.

Routines called are: GENLGP
                    VECMUL

39
2.15 **Subroutine VECMUL** multiplies a matrix times a vector.

2.16 **Subroutine DUMP** gives a listing of core storage when an error condition occurs.

Subsequent paragraphs detail the above routines where necessary.

It should be noted at this point that standard mathematical notation is not necessarily followed, e.g., program notation labels Bessel functions as $B$ instead of $j$. This was done for ease of relating program mnemonics to mathematical notation. When necessary, parameters have been labeled which have notation different from the earlier text.

3.0 THE INPUT ROUTINE

Subroutine **RDDATA** reads the user's control information, prints out headings and obtains information for numerical integration. The input cards and their formats are listed below.

3.1 Card 1

<table>
<thead>
<tr>
<th>NM, NRANK, NSECT, IBODY, NUANG</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FORMAT</strong> (5112)</td>
</tr>
</tbody>
</table>

**NM**

No. of values of "m". See Card 3.

**NRANK**

Rank of matrices $I$, $J$, $K$ and $L$

**NSECT**

No. of sections defining body shape and integration intervals. See Subroutine **CALENP** for fuller description of body shapes.

**IBODY**

Case No. or body shape identifier

- 7 : Spheroid
- 8 : Mirror Symmetry
- 9 : General Axisymmetric Case

**NUANG**

No. of aspect angles "u". See Card 5. 40
3.2 Card 2.

CONK, BRXT, ALPHA

FORMAT (3E12.7)

CONK  

$\lambda_a$, scale factor for $r$, the polar radius, in determining body shape.

BRXT  

variable parameter to be used in computing body shapes.

ALPHA  

$\alpha$, or a variable parameter, to be used in calculating body shapes. For a fuller description of its usage see Subroutines CALENP and GENKR described below.

3.3 Card(s) 3.

CMI(I), I = 1, NM

FORMAT (6E12.7)

CMI(I)  

$I^{th}$ value of "$m\prime\prime$ to be used in current solution of scattering problem. As many as 30 values of "$m\prime\prime$, the azimuthal index, may be read in; "$m\prime\prime$ is any integer $\geq 0$.

3.4 Card 4.

NDPS(I), I = 1, NSECT

FORMAT (6I12)

NDPS(I)  

No. of divisions for integration in $I^{th}$ section of the body shape. The body may be divided into as many as 6 sections. These parameters are used to calculate spacing for numerical integration, and they must be a multiple of 4.

3.5 Card(s) 5.

UANG(I), I = 1, NUANG

FORMAT (6I12)

UANG(I)  

$I^{th}$ value of "$u\prime\prime$, a member of a table of aspect angles (in degrees). As many as 60 values of "$u\prime\prime$ may be read in.
4.0 CALCULATION OF END POINTS AND SPACING FOR INTEGRATION

Subroutine CALENP is one of two special routines that have to be written into the program for specific body shapes. This routine calculates "NSECT" values of the polar angle $\theta$, which provide boundaries for dividing the body into sections for numerical integration. With each boundary point a value of $\theta$ is associated. The spacing for integration is then determined by dividing the range of $\theta$ by the correct value of "NDPS". Note that the number of divisions does not have to remain constant from one section to the next, but it must be a multiple of 4.

Since the computations for each special version of CALENP may vary, the following parameters, "ALPHA", "BRXT", "QB", "SNALPH", and "CSALPH" may be used for communicating between routines special values associated with a particular body shape. Note the use of QB below for a variable peculiar to the sphere-cone-sphere shape.

In Appendix II, a listing of the routines CALENP and GENKR are given for a sphere-cone-sphere body (Figure 2.).

For the sphere-cone-sphere body, three end points for $\theta$; $\theta_1$, $\theta_2$, $\theta_3$, are computed as follows:

$$\theta_1 = \tan^{-1} \left[ \frac{\sin \alpha}{q - \sin \alpha} \right] ; \quad 0 \leq \theta_1 \leq 105^\circ$$

NOTE: $\alpha$ (Figure 2) is an input parameter stored at "ALPHA"

$b/a$ (Figure 2) is an input parameter stored at "BRXT"
\[ q = \frac{(1 - b/a)(1 - \sin \alpha)}{2} \]

and is computed and stored in QB.

\[ \theta_2 = \tan^{-1} \left[ \frac{(b/a) \sin \alpha \cos \theta}{1 - q - (b/a) \cos \alpha} \right]; \quad \theta_1 < \theta_2 < \pi \]

\[ \theta_3 = \pi \]

5.0 THE FIRST CONTROL ROUTINE

The MAIN Routine controls the general flow of the program and computes the real and imaginary parts of the partitioned scattering matrices, I, J, K and L. After the user's control data has been read in, a numerical integration system utilizing Bode's 3rd order rule is utilized. The program computes the I and J complex matrix elements for the three cases, axisymmetric, mirror-symmetric and spheroidal as follows:

5.1

\[ I_{ij} = \sum_{m} \int_{0}^{\pi} (\sin \theta)^2 B_j(kr) H_1(kr) \left\{ (i + m) \cos \theta P_i^m P_j^m - (i + m) P_i^m P_{j-1}^m \right\} \, d\theta \]

\[ B_j(kr) : \text{Bessel function of the first kind of order } "j" \text{ and argument } "kr" \]
$H_i(kr)$: Hankel functions which are defined as

$$B_i(kr) + i^{-1} N_i(kr)$$

$N_i(kr)$: Neumann functions of order "i" and argument "kr".

$$p^m_i = \frac{p^m_i (\cos \theta)}{\sin \theta}$$

where

$$p^m_i (\cos \theta)$$

is the associated Legendre function, of rank $m$, and order $i$.

$r$ : Polar radius used in calculating body shape.

$i$ : Subscript notation for $i$th row of the matrix,

$j$ : Subscript notation for $j$th column of the matrix.

$$J_{ij} = \sum_{m=-i}^i \int_0^\pi (\sin \theta) H_i(kr) \left( \frac{p^m_i}{p^m_j} \right) \left( \frac{d(kr)}{d\theta} \right) B_j(kr) \sin \theta \cos \theta$$

$$+ i j (j+1) \frac{d(kr)}{d\theta} B_j(kr) \sin \theta \cos \theta$$

$$-(i + m) \frac{d^2(kr)}{d\theta^2} B_j(kr) \sin \theta$$
\[ + (j + m) P^m_{j-1} \int \left[ \left( kr B^1_j (kr) - j B_j (kr) \right) \left( (i + m) P^m_{i-1} \cos \phi P^m_i \right) \right] \, d\phi \]

where:

\[ e_m = \begin{cases} 1 & \text{for } m = 0 \\ 2 & \text{for } m \geq 1 \end{cases} \]

Within the program each element of the \( I \) and \( J \) arrays is used as an accumulator for numerical integration under Bode's rule. Thus, for a specified value of \( \theta \), all necessary functions are computed and added to the correct matrix element.

To save computer time, computations which would produce a null contribution to the integration are eliminated, and the following symmetries are taken advantage of in the direct computation of the \( I \) and \( J \) matrices.

5.3 General axisymmetric bodies:

\[
Re(I_{ij}) = Re(I_{ij}) \\
Re(J_{ij}) = Re(J_{ij})
\]

5.4 Mirror-symmetric bodies: use paragraph 5.3 plus

\[
I_{ij} = 0; \text{ if } i \text{ and } j \text{ are } \begin{cases} \text{both odd} \\ \text{both even} \end{cases} \\
J_{ij} = 0; \text{ if } i \text{ and } j \text{ are } \begin{cases} \text{odd, even} \\ \text{even, odd} \end{cases}
\]
5.5 Spheroids: use paragraphs 5.3, 5.4, plus

\[ \text{Im}(I_{ij}) = \text{Im}(I_{ij}) \]
\[ \text{Im}(J_{ij}) = \text{Im}(J_{ij}) \]

The \text{K} and \text{L} matrices are then calculated from the following relationships with the \text{I} and \text{J} matrices:

5.6 \[ \text{Re}(K_{ij}) = -\text{Re}(J_{ij}) \]
\[ \text{Im}(K_{ij}) = -\text{Im}(K_{ij}) + b \cdot D_{ij} \]

where \[ b = \begin{cases} 
1.0 & \text{General Axisymmetric bodies} \\
0.5 & \text{Mirror-Symmetric or Spheroidal bodies} 
\end{cases} \]

\[ D_{ij} = \begin{cases} 
0.0; & i \neq j \\
\frac{\varepsilon_{m} (2i + 1) (i - m)!}{4i(i + 1)(i + m)!}; & i = j 
\end{cases} \]

5.7 \[ \text{Re}(L_{ij}) = -\text{Re}(I_{ij}) \]
\[ \text{Im}(L_{ij}) = -\text{Im}(I_{ij}) \]

The \text{I}, \text{J}, \text{K} and \text{L} complex matrices are then printed by Subroutine \text{PRTMTX} and control passes to Subroutine \text{PRCSSM} for further processing.

6.0 ASSOCIATED LEGENDRE FUNCTIONS

Subroutine \text{GENLGP} generates the associated Legendre functions, \( P_{\text{m}}^{\text{m}}(x) \) for a given argument \( x \), a given value of the azimuthal index \( m \),
and for all values of degree $i$ from 0 to "NRANK", the input-specified rank of the matrix. The first two values of $P$ are generated by formula, then the remaining values of $P$ are generated by a recursion relationship.

The following formulae are used to generate $P_i^{(x)}$. Note that for this particular program, the functions always appear in the context

$$\frac{P_i^{(x)}}{\sin^0}$$

$$\frac{P_i^{(x)}}{\sin^0} = 0.0 \quad ; \quad i < m$$

$$\frac{P_m^{(x)}}{\sin^0} = \frac{(2m)! \sin^{m-1}(\theta)}{2^m \cdot m!} \quad ; \quad i = m$$

$$\frac{P_0^{(x)}}{\sin^0} = \frac{1.0}{\sin^0} \quad ; \quad i = m = 0$$

$$\frac{P_1^{(x)}}{\sin^0} = \frac{\cos^0}{\sin^0}$$

Recursion relationship:

$$\frac{P_n^{(x)}}{\sin^0} = \frac{(2n-1)\cos^0 \left[ \frac{P_{n-1}^{(x)}}{\sin^0} \right]^{(n + m - 1)} \left[ \frac{P_{n-2}^{(x)}}{\sin^0} \right]}{n - m}$$

47
7.0 BESSEL FUNCTIONS

Subroutine BESSEL generates a Bessel function of the first kind \( B_n(x) \), for a specified argument \( x \), and order \( n \), by means of an infinite series. To preserve accuracy, the computations are performed in double precision arithmetic and truncation error due to neglected terms in the series is \( < 10^{-20} \). If the series has not converged to the aforementioned accuracy before the computation of the 100\(^{th} \) term, an error indication is given.

The following infinite series is used to compute a Bessel function:

\[
B_n(x) = \frac{x^n}{1 \cdot 3 \cdot 5 \cdots (2n + 1)} \sum_{i=0}^{\infty} a_i
\]

where:

\[
a_0 = 1.0
\]

\[
a_{i+1} = \frac{-x^2}{2i(2n + 2i + 1)} a_{i-1}
\]

8.0 RECURSION RELATIONSHIPS FOR BESSEL AND NEUMANN FUNCTIONS

Subroutine GENBSL calls Subroutine BESSEL to obtain two successive BESSEL functions for a specified argument, and then uses these first two values to recurse backward over the range of \( i \) from NRANK to 0. If the two computed functions of order NRANK and NRANK-1 do not satisfy the accuracy requirements mentioned in paragraph 7.0,
the routine will increase the order of the computed BESSEL function to 4(NRANK). If this fails to produce a satisfactory pair of functions, the run will abort and a dump of core memory is taken.

The recursion relation used for computing BESSEL functions is:

\[ B_{n-1}(x) = (2n + 1)x^{-1} B_n(x) - B_{n+1}(x) \, . \]

This routine also computes Neumann functions by a forward recursion formula after the first two values are computed by the following formulae:

\[ N_0(x) = \frac{-\cos x}{x} \, , \]
\[ N_1(x) = \frac{-\cos x}{x^2} - \frac{\sin x}{x} \, . \]

The recursion relation used for computing the remaining Neumann functions is:

\[ N_{n+1}(x) = (2n+1)x^{-1} N_n(x) - N_{n-1}(x) \, . \]

To test the accuracy of the functions over the range of computed Bessel and Neumann functions for a given argument, two tests are performed in the MAIN Routine after the vector of functions from 0 to NRANK is computed. If the following relations are not satisfied to an accuracy of $10^{-10}$, an error message indicating such a condition is printed, and the program continues. Though the tests are performed
In the **MAIN** Routine after the call to Subroutine **GENBSL**, for convenience they are listed here:

**Bessel Test:**
\[ | x^2 \left[ B_1(x)N_0(x) - B_0(x)N_1(x) \right] - 1 | < 10^{-10} \]

**Neumann Test:**
\[ | x^{2r} \left[ B_{NRANK}(x)N_{NRANK}(x) - B_{NRANK-1}(x)N_{NRANK-1}(x) \right] - 1 | < 10^{-10} \]

9.0 GENERATING THE BODY SHAPE

Subroutine **GENKR** is one of two custom written routines which are adapted to the particular body shape in question. As noted above in Subroutine **CALENP** certain parameters are available to the programmer to use as he sees fit to communicate information from one routine to another. The basic function of all versions of **GENKR** is to compute the polar radius \( r \) as a function of the polar angle \( \theta \), to compute \( \frac{dr}{d\theta} \) and to scale these values by the input constant \( ka = CONK \).

To illustrate the use of this routine, a sphere-cone-sphere body shape is used (Figure 2). As a result of subroutine **CALENP** the major divisions of the body as a function of \( \theta \) have been recorded. This routine, given a value of \( \theta \) now computes \( (ka)r \) and \( ka(\frac{dr}{d\theta}) \); the scale factor \( ka \) is an input to the program.

9.1 Section 1 \( 0 \leq \theta \leq \theta_1 \)

\[ r = \frac{a \cos \theta}{\sin \alpha} + \left[ 1 - \left( \frac{a \sin \theta}{\sin \alpha} \right)^2 \right]^{1/4} \]
\[ \frac{dr}{d\theta} = -\frac{a \sin\theta}{\sin\alpha} - \left(\frac{a}{\sin\alpha}\right)^2 \sin\theta \cos\theta \left[ 1 - \left(\frac{a \sin\theta}{\sin\alpha}\right)^2 \right] \]

NOTE: q was computed in Subroutine CALCPB and stored in location QB.

9.2 Section 2 \( \theta_1 < \theta \leq \theta_2 \)

\[ r = \frac{1 - q}{\sin(\theta - \alpha)} \]

\[ \frac{dr}{d\theta} = -\frac{(1-q) \cos(\theta - \alpha)}{\sin^2(\theta - \alpha)} \]

9.3 Section 3 \( \theta_2 < \theta \leq \pi \)

\[ r = \left[ \frac{1 - (b/a) - q}{\sin\alpha} \right] \cos\theta + \left[ (b/a)^2 - \left( \frac{1 - (b/a) - q}{\sin\alpha} \right)^2 \sin^2\theta \right]^{1/2} \]

\[ \frac{dr}{d\theta} = \left[ \frac{1 - (b/a) - q}{\sin\alpha} \right] \sin\theta - \left( \frac{1 - (b/a) - q}{\sin\alpha} \right)^2 \sin\theta \cos\theta \left[ (b/a)^2 - \left( \frac{1 - (b/a) - q}{\sin\alpha} \right)^2 \sin^2\theta \right]^{1/2} \]

10.0 FIRST MATRIX PRINTOUT

Subroutine PRTMTX controls the printout of the I, J, K and L matrices. Both the real and imaginary arrays comprising each of these matrices are labeled and printed out on the community output tape.

This output, which was originally intended as an intermediate printout for checking the program, may be eliminated by removing the "CALL PRTMTX"
statement which follows Fortran statement 860 in the MAIN Routine.

11.0 PRINTOUT OF AN ARRAY

Subroutine PRINTM will print out a specified square array of given rank.

12.0 GENERATING THE Q MATRIX AND THE T MATRIX

Subroutine PRCSSM is the second major control routine and it controls the transformation of the I, J, K and L matrices to the "Q" matrices, and the subsequent solution of a matrix equation which provides the "T" matrix.

Subroutine NRM3MX (see below) normalizes the I, J, K and L matrices to produce the Q matrix.

For notational convenience we define:

\[ Q = \text{Re}(Q) + i \text{Im}(Q) = \begin{pmatrix} Q_1 & Q_2 \\ Q_2 & Q_4 \end{pmatrix} \]

where

\[ i = \sqrt{-1}. \]

The method currently used by the program to transform the Q matrix into the T matrix involves orthogonalizing the Q matrices. After the complex Q matrix has been generated by normalizing the I, J, K and L matrices it is in the form noted in paragraph 12. From these Q matrices, a new complex Q matrix of rank 2N is generated from the following relations:
\[ Q_{(2m-1) (2n-1)} = (Q_1)_{m \, n} \]
\[ Q_{(2m-1) (2n)} = (Q_2)_{m \, n} \]
\[ Q_{(2m) (2n-1)} = (Q_3)_{m \, n} \]
\[ Q_{(2m) (2n)} = (Q_4)_{m \, n} \]

The new \( Q \) matrix is next conditioned as outlined in Subroutine \texttt{CNDTNQ} of paragraph 14.0 below.

Orthogonalization then proceeds as follows.

1) Consider each row of \( Q \) as a vector with \( 2N \) components; e.g. the components of the first vector \( Q_1 \) would be:

\[ Q_1 \, 1, Q_1 \, 2, Q_1 \, 3, \ldots, Q_1(2N) \]

Orthogonalization will proceed from the bottom or \( 2N^{th} \) vector upward.

2) Normalize the \( 2N^{th} \) vector as follows:

\[ Q_{2N} = \frac{Q_{2N}}{(Q_{2N}^* \cdot Q_{2N})^{\frac{1}{2}}} \]

where the scalar product of the complex conjugate \( Q_{p}^* \) by another vector \( Q_{q} \) is defined as follows:
\[
Q^* \cdot Q = \sum_{r=1}^{2N} Q^*_{p1} Q_{qr} = Q^*_{p1} Q_{q1} + Q^*_{p2} Q_{q2} + \cdots + Q^*_{p(2N)} Q_{q(2N)}
\]

3) Orthogonalize \( \hat{Q}_{(2N-1)} \) to \( \hat{Q}_{(2N)} \):

\[
\hat{Q}_{2N-1} = \hat{Q}_{2N-1} - [Q^*_{2N} \cdot \hat{Q}_{2N-1}] \hat{Q}_{2N}
\]

4) Normalize \( \hat{Q}_{2N-1} \):

\[
\hat{Q}_{2N-1} = \frac{\hat{Q}_{2N-1}}{\sqrt{Q^*_{2N-1} \cdot \hat{Q}_{2N-1}}}
\]

5) Orthogonalize \( \hat{Q}_{2N-2} \) to both \( \hat{Q}_{2N} \) and \( \hat{Q}_{2N-1} \):

\[
\hat{Q}_{2N-2} = \hat{Q}_{2N-2} - [Q^*_{2N-1} \cdot \hat{Q}_{2N-2}] \hat{Q}_{2N-1} - [Q^*_{2N} \cdot \hat{Q}_{2N-2}] \hat{Q}_{2N}
\]

6) Normalize \( \hat{Q}_{2N-2} \):

\[
\hat{Q}_{2N-2} = \frac{\hat{Q}_{2N-2}}{\sqrt{Q^*_{2N-2} \cdot \hat{Q}_{2N-2}}}
\]

7) Continue the orthogonalization and normalization process until \( \hat{Q}_1 \) has been orthogonalized to all subsequent rows.
8) A complex matrix \( \hat{T} \) is now generated from the complex matrix \( \hat{Q} \) by the following relation:

\[
\hat{T} = \hat{Q}^* \text{Re}(\hat{Q})
\]

9) The \( \hat{T} \) matrix is then decomposed into the matrices \( T_1, T_2, T_3 \) and \( T_4 \) by the reverse of the procedure in paragraph 12.1.

The complex \( T \) matrix is printed by Subroutine \text{PRTRIT} and then the final processing is performed by Subroutine \text{ADDPRC}.

13.0 NORMALIZING MATRICES

The Subroutine \text{NRM3MX} normalizes the I, J, K and L matrices to obtain the Q matrix. The Q matrix is blocked as noted above in paragraph 12.0 and the following procedure is used:

\[
Q_1 = (Z_2)^{-\frac{1}{2}} J' (Z_2^{-\frac{1}{2}}) = Q_{1j} = \frac{J_{1j}}{\sqrt{Z_{ij}} \cdot \sqrt{Z_{kj}}}
\]

\[
Q_2 = -(Z_2)^{-\frac{1}{2}} L' (Z_2^{-\frac{1}{2}})
\]

\[
Q_3 = (Z_2)^{-\frac{1}{2}} I' (Z_2^{-\frac{1}{2}})
\]

\[
Q_4 = (Z_2)^{-\frac{1}{2}} K' (Z_2^{-\frac{1}{2}})
\]

55
The expression for $Z_n^2$ is the same as that used in computing the $K$ matrix of paragraph 5.6. The prime on $J$, etc. denotes matrix transpose as seen from the second half of the equality statement.

14.0 CONDITIONING MATRICES

After the matrix $\hat{Q}$ of rank $2N$ has been formed, the matrix is conditioned starting with the last row $\hat{Q}_{2N}$ and working towards row $\hat{Q}_1$.

14.1

$$\left(\hat{Q}_{2N}\right)_1 = \left[\frac{1}{\text{Im}(\hat{Q}_{2N}^2)}\right] \left(\hat{Q}_{2N}\right)_1 ; \ i = 1, 2, \ldots 2N$$

The notation $(\hat{Q}_{2N})_1$ refers to the $i$th element of the $(2N)^{th}$ (last) row vector. Now set

$$\hat{Q}_m = \hat{Q}_m - \left[\text{Im}(\hat{Q}_m)_{2N}\right] \hat{Q}_N$$

where the equivalence is performed for each of the $2N$ elements of $\hat{Q}_m$, and repeated for all rows $m = 1, 2, \ldots, 2N-1$.

14.2 Redefine

$$\hat{Q}_{2N-1} = \left[\frac{1}{\text{Im}(\hat{Q}_{2N-1})_{2N-1}}\right] \hat{Q}_{2N-1},$$
then compute
\[ \hat{\delta}_m = \hat{\delta}_m - \left[ \text{Im}(\hat{\delta}_m)_{2N-1} \right] \hat{\delta}_{2N-1} \]

for all rows \( m = 1, 2, \ldots 2N-2 \).

14.3 Continue the process of paragraphs 14.1 and 14.2 for all the remaining rows. The final step in the process is to generate

\[ \hat{\alpha}_2 = \left[ \frac{1}{\text{Im}(\hat{\alpha}_2)} \right] \hat{\alpha}_2, \]

\[ \hat{\alpha}_1 = \hat{\alpha}_1 - \left[ \text{Im}(\hat{\alpha}_1) \right] \hat{\alpha}_2. \]

14.4 Set

\[ \text{Im}(\hat{\delta}_m)_i = 0.0; \ i = m + 1, m + 2, \ldots, 2N; \ m = 1, 2, 3, \ldots, 2N-1. \]

15.0 PRINTING THE T MATRIX

Subroutine PRTRIT controls the printout of the T matrix, both real and imaginary elements, in the same manner as Subroutine PRMTM (paragraph 11.0 above) controls the printout of the I, J, K and L matrices. The community output tape is used. Since this printout is used mainly for checkout, it can be eliminated by removing the "CALL PRTRIT" statement following FORTRAN statement 140 in Subroutine PRCSSM.

57
16.0 FINAL CONTROL ROUTINE

Subroutine *ADDPRC* is the third and last control routine which converts the *T* matrix to the final set of results. Two sets of results are generated, a set of answers for the current value of *m* and an accumulated set of answers for all values of *m* up to and including the present value of *m*.

To generate the final results, the following procedure is followed: The *T* matrix is normalized

\[ T(1.) = (Z2_1)^{1/2} T_{1j}(k) (Z2_j)^{-1/2} \]

where: \( k \) indicates 1 of 4 blocks;

NOTE:

\[ T = \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix} \]

\( Z2_n \) is as defined in Subroutine **NRMNMX** under paragraph 13.0.

NOTE: For the reader who is relating the mathematics to the program listing in Appendix II, the mapping of COMMON storage in paragraph 19.0 should be consulted.

The associated Legendre functions of form

\[ \frac{P_n^m(\cos u)}{\sin u} \]
are generated for each value of the aspect angle \( u \), and for \( n = 1 \) to \( \text{NRANK} \). The derivatives of the Legendre functions are computed from:

\[
\frac{d}{du} P_n^m(\cos u) = n \cos u \left[ \frac{P_n^m(\cos u)}{\sin u} \right] - (n + m) \left[ \frac{P_{n-1}^m(\cos u)}{\sin u} \right].
\]

Values of the vectors \( \mathbf{F}^1, G^1 \) and \( \mathbf{F}^2, G^2 \) are generated by Subroutine \textsc{vecmul}. These vectors are defined as:

16.1

\[
\begin{pmatrix}
F^1 \\
G^1
\end{pmatrix} = -i \begin{pmatrix}
T_1 & T_2 \\
T_3 & T_4
\end{pmatrix} \begin{pmatrix}
(i)^n m P_n^m(\cos u)/\sin u \\
-(i)^{n+1} \frac{d}{du} P_n^m(\cos u)
\end{pmatrix}
\]

and

16.2

\[
\begin{pmatrix}
F^2 \\
G^2
\end{pmatrix} = i \begin{pmatrix}
T_1 & T_2 \\
T_3 & T_4
\end{pmatrix} \begin{pmatrix}
(i)^{n+1} \frac{d}{du} P_n^m(\cos u) \\
(i)^n m P_n^m(\cos u)/\sin u
\end{pmatrix}
\]

The final sets of answers are generated from the following equations:

\[
\text{SCATT 1, 2} = \frac{16}{(ka)^2} \sum_{n=1}^{\text{NRANK}} (22)^{-1} \left[ |F_n^{1,2}|^2 + |G_n^{1,2}|^2 \right]
\]

59
\[ \text{TOTAL } 1, 2 = \frac{16}{(ka)^2} \sum_{n=1}^{\text{NRANK}} (Z2)^{-1}(-i)^n \left[ \int P_{n}^{1, 2} \frac{m P_n^m(\cos u)}{\sin u} + ig_{n}^{1, 2} \frac{dP_n^m(\cos u)}{du} \right] \]

\[ \text{RTRAD } 1, 2 = \frac{8}{(ka)} \sum_{n=1}^{\text{NRANK}} (Z2)^{-1}(i)^n \left[ \int P_{n}^{1, 2} \frac{m P_n^m(\cos u)}{\sin u} - ig_{n}^{1, 2} \frac{dP_n^m(\cos u)}{du} \right] \]

The final results are divided into two classes as noted above by the quantities SCATT 1, 2 etc. The classes are two different incident polarizations. Class 1 is the E-parallel incidence; class 2 is the E-perpendicular incidence.

Appendix III contains a listing of a sample output of the sphere-cone-sphere-body shape. The printout titles and their meanings are:

- **ANGLE**
  - \( u \), the aspect angle (degrees)

- **SCATT 1, 2**
  - Scattering cross-section for each class, normalized by \([na^2]^{-1}\)

- **TOTAL 1, 2**
  - Complex forward amplitude, normalized by \([na^2]^{-1}\)

- **RTRAD 1, 2**
  - Complex back scattered amplitude normalized by \([na^2]^{-1}\)

- **RCS 1, 2**
  - Radar cross section normalized by \([na^2]^{-1}\)

**NOTE:** RCS \(= |\text{RTRAD}|^2\) only is computed for the accumulative case.
PHASE ANGLE 1, 2

For the accumulative case, a phase angle is computed:

\[
\text{PHANG} = \tan^{-1} \left[ \frac{\text{Im}(R\text{RAD})}{\text{Re}(R\text{RAD})} \right]
\]

This is the phase angle of the back scattered amplitude (Degrees).

17.0 MULTIPLYING A MATRIX TIMES A VECTOR

Subroutine \texttt{VECMUL} is one of two routines coded in both machine language and FORTRAN. It multiplies a matrix times a vector to compute the vectors \( F^1, G^1 \) and \( F^2, G^2 \) of paragraphs 16.1 and 16.2. The machine coded version has the advantages of higher speed and accuracy.

18.0 CORE DUMP

If an abnormal or uncorrectable error condition occurs, Subroutine \texttt{DUMP} gives a dump of core memory as an aid in debugging the error condition. The Subroutine \texttt{LBPDMP} is a system routine for dumping core between specified limits.

19.0 STORAGE ARRANGEMENTS

To conserve and fully utilize core storage, three large matrix arrays of dimension 120 x 120 have been set up in an area of COMMON storage named "MTXCOM". To aid in programming, various routines use EQUIVALENCE statements to resegment these large arrays into manageable blocks.
The FORTRAN array names of the three major blocks are:

CMTXRL (120, 120)
CMTXIM (120, 120)
SPRMTX (120, 120)

Within the MAIN Routine the following overlays are made:

\[
\begin{align*}
\text{CMTXRL} & : \begin{cases} 
AMXIR (60, 60) : \text{RE(I)} \\
AMXJR (60, 60) : \text{RE(J)} \\
AMXKR (60, 60) : \text{RE(K)} \\
AMXLR (60, 60) : \text{RE(L)} 
\end{cases} \\
\text{CMTXIM} & : \begin{cases} 
AMXIL (60, 60) : \text{IM(I)} \\
AMXJI (60, 60) : \text{IM(J)} \\
AMXKI (60, 60) : \text{IM(K)} \\
AMXLI (60, 60) : \text{IM(L)} 
\end{cases}
\end{align*}
\]

The SPRMTX block is unused.

Subroutine NRMQMX and Subroutine FRCSSM, the second control routine, allocated storage as follows:

\[
\begin{align*}
\text{CMTXRL} & : \begin{cases} 
QMTXII (60, 60) : \text{QI1 (60, 60)} : \text{IM(Q)}_1 \\
QMTXJI (60, 60) : \text{QI2 (60, 60)} : \text{IM(Q)}_2 \\
QMTXKI (60, 60) : \text{QI3 (60, 60)} : \text{IM(Q)}_3 \\
QMTXLI (60, 60) : \text{QI4 (60, 60)} : \text{IM(Q)}_4 
\end{cases} \\
\text{SPRMTX} & : \begin{cases} 
QMTXIR (60, 60) : \text{QE1 (60, 60)} : \text{RE(Q)}_1 \\
QMTXJR (60, 60) : \text{QR2 (60, 60)} : \text{RE(Q)}_2 \\
QMTXKR (60, 60) : \text{QR3 (60, 60)} : \text{RE(Q)}_3 \\
QMTXLR (60, 60) : \text{QR4 (60, 60)} : \text{RE(Q)}_4 
\end{cases}
\end{align*}
\]
NOTE: After Subroutine NRM3MX normalizes and moves the Q matrix (complex) into the SPRMTX and CMTXRL areas, the processing which transforms the Q to the T matrix follows the procedure outlined in paragraph 12.0. The storage allocation is noted above in the eight itemized steps.

Subroutine INVMBL always assumes the block matrix which is to be processed is stored in the CMTXRL area. The intermediate steps as outlined in paragraph 14.0 are performed in the CMTXIM area.

The third and last control routine, Subroutine ADDPRC makes the following storage allocations:

\[
\begin{align*}
\text{CMTXRL} & : \\
QMTXII (60, 60) & : \text{FGVECT} (2, 120, 2) \\
QMTXJI (60, 60) & : \text{FGMUL} (120, 2) \\
QMTXKI (60, 60) & : \text{FGANS} (60, 10) \\
QMTXLI (60, 60) & : \text{(unused)} \\
\text{CMTXIM} & : \\
\text{SPRMTX} & : \\
\text{TCMPLX} (2, 120, 120) & : \text{Real and imaginary components of the T matrix.}
\end{align*}
\]

NOTE: That TCMPLX overlays both the CMTXIM and SPRMTX areas. As noted in paragraph 16.0, FGVECT contains the $F^1$, $G^1$ and $G^2$, $F^2$ vectors. The first subscript refers to the real and imaginary components of the vectors, the second subscript refers to the dimension of the vectors which is 2-NRANK and the last subscript differentiates the 2 vectors.

FGMUL contains the vectors which post-multiply the T matrix to generate FGVECT. The first and second subscripts correspond to
the second and third subscripts of $\text{FGVECT}$.  

$\text{FGANS}$ contains the final answers. The first subscript corresponds with the value of aspect angle which generated it and the second subscript refers to the answers in the following manner:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCATT$^1$</td>
<td>Re(TOTAL$^1$)</td>
<td>Im(TOTAL$^1$)</td>
<td>Re(RRAD$^1$)</td>
<td>Im(RRAD$^1$)</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>SCATT$^2$</td>
<td>Re(TOTAL$^2$)</td>
<td>Im(TOTAL$^2$)</td>
<td>Re(RRAD$^2$)</td>
<td>Im(RRAD$^2$)</td>
</tr>
</tbody>
</table>

64
APPENDIX II

THE FORTRAN IV PROGRAM LISTING
SUBTYPE FORTRAN, LMAP, LSTRAP

SCATTERING FROM AXISYMMETRIC CONDUCTORS FOR CASES 7, 9 AND 9.

COMMON DTR, RTR, CPI
COMMON /CMWCOM/ NM, CM1(31), CMV, CM2, CMF, TMW, PRDM
COMMON /PNCOM/ PMMLE1(61), RSSLSP(61), CMFNM(61)
COMMON /HTHCOM/ THETA, NTHETA, DLTHTA, SINTH, CUS1TH, ISMR1L, ISWTCH1, 3R
1MUL, SMULSS1(7), COH(6), DNM, NSFCT, NDFPS(6), EPPS(6), KSECT
COMMON /MTXCOM/ NRANK, NPAR1, AMXIR(60), AMXJ(60, 32), AMXKR(60, 32)
1L, AMXLK(60, 32), AMXII(60, 40), AMXJII(60, 40), AMXLII(60, 40), AMXII(60, 40)
2. SPRMTXI(120, 120), CMSINNM(60)
COMMON /BCYCOM/ CKR, DCKR, CKR2, CSXR, CSNKR, CNK, ARKR, ALPH, ARDY, OR
1. SNALPH, CSALPH

DIMENSION CLRMTX(16320)
EQUIVALENCE (AMXIN, CLRMTX)

SET PROGRAM CONSTANTS.
DTR = 1.74532923194686-02
RTR = 57.2957951311
CPI = 1.141592653589R
ISWTCH11 = 2
ISWTCH12 = 3
ISWTCH13 = 4
ISWTCH41 = 1
SMULSS11 = 32.0
SMULSS12 = 12.0
SMULSS13 = 32.0
SMULSS41 = 14.0

CALL ROUTINE TO READ DATA AND PRINT HEADINGS FOR OUTPUT
20 CALL RDATA
IF (IR0DV=9)24, 22, 24
22 ADFYCT = 1.0
GO TO 26
24 ADFYCT = 0.5
GO TO 26

C SET UP A LOOP FOR M AND SFT VARIABIES WHICH ARE A FUNCTION OF M.
26 DO 900 IM = 1, NM
CMV = CM1(IM)
CM2 = CMV*CMV
PRDM = 1.0
IF (CMV>100.0)43, 44
40 EM = 1.0
GO TO 60
44 EM = 2.0
QUANM = CMV
DO 52 IFFT = 1, KNY
QUANM = QUANM1.0
PRDM = QUANM*PRDM/2.0
52 CONTINUE
60 QEM = -2.0/EM
TMW = CHWCVVM
C INITIALIZE ALL MATRIX AREAS TO 0RN
DO 80 I = 1, 128BD0
CLRMXII = 0.0
80 CONTINUE

C SET UP A LOOP FOR ALL VALUES OF THETA.
THETA = 0.0
C SET UP GENERAL LOOP FOR CORRECT NUMBER OF INTEGRATION SECTIONS.
DO 800 ISECT = 1, NSFCT
KSECT = ISECT
NTHETA = NDFPS(ISECT+1)
DLTHTA = CMH(ISECT)

66
RI\[A/7;,
C
00   700
MT
■
L.NME\[A/7;,
S?
SWIir.HtS
ANf)   M|)l
I^BS
fu
SMC^ilNS
r
Nlf»
ÄTION
MFTHnn.
IF( ITt'M • lll?ti 1? '.1 >?
12J
S\[A/7;,
12J
IF( I\[A/7;,
144
144
GO   TO   HO
200    IS\[A/7;,
360    IF ( OUAN\[A/7;,
12J
S\[A/7;,
12J
IF( I\[A/7;,
360    IF ( OUAN\[A/7;,
12J
S\[A/7;,
12J
IF( I\[A/7;,
&37

376
378
3fH
388
3<»0
392
I
1
I
396
398
«00
600
700
800
808
812
816
C
c
820
BJBJ1 = CKR*(CKR*ASSLSPLICOL1-CCOL*ASSLSPLICOL1))
IFI(BODY-9)374,376,374
374 GO TO (376,3921,JMR
C TEST FOR N = 0.
376 IFICMV1388,394,378
C CALCULATE THE TERM FOR THE CURRENT ELEMENT IN THE I MATRIX.
378 TERM1 = SINTHCRK2*ASSLSPLICOL1*ICOSTH*PNMLLG(1ROW1)*PNMLLG(1)
1LG1*CR1J-CRNW*PNMLLG(1COL1)*PNMLLG(1ROW1-CCOL)*PNMLLG(1ROW1)*P
2NMLLG(1COL1)
AMXJ1(1ROW,ICOL1 = AMXJ1(1ROW,ICOL1)*CNF1NV(1ROW1)*TERM1
IFI(COL-1ROW)388,394,394
394 AMXJ1(1ROW,ICOL1 = AMXJ1(1ROW,ICOL1)*ASSLSPL(1ROW1)*TERM1
398 IFI(BODY-9)390,392,390
390 JMR = 2
GO TO 460
C TEST FOR CURRENT ELEMENT IN THE J MATRIX.
392 PTJ1 = PNMLLG(1ROW1)*PNMLLG(1COL1)*B1J1J1*(CMZCRSSJ1*ICOSTH**2)
1CRSSJ1*ICOSTH*1XP)
PTJ2 = CRNM*PNMLLG(1ROW1)*PNMLLG(1COL1)*ICOSTH*1BJ1J1J1*XP)
PTJ3 = CCOL*PNMLLG(1COL1)*1BJ1J1*(CMZCRSSJ1)1ROW1-CROW1*ICOSTH*PN
1LG1(1ROW1)
AMXJII(1ROW,ICOL1 = AMXJII(1ROW,ICOL1)*SINTH*CNF1NV(1ROW1)*PT
1J1-PTJ2PTJ3)
IFI(COL-1ROW)398,396,396
396 AMXJII(1ROW,ICOL1 = AMXJII(1ROW,ICOL1)*SINTHASSLSPL(1ROW1)*PT
1J1-PTJ2PTJ3)
398 JMR = 1
CONTINUE
400 CONTINUE
400 CONTINUE
700 CONTINUE
800 CONTINUE
C SYMMETRIZE REAL MATRICES AND IMAGINARY SPHERICAL MATRICES.
DO 816 IRON = 2,NRANK
IE: DSY = IRON-1
DO 812 ICOL = 1,ENDSY
AMXRI(1ROW,ICOL1 = AMXRI(1ROW,ICOL1)
AMXRI(1ROW,ICOL1 = AMXRI(1ROW,ICOL1)
C TEST FOR SPHERICAL BODIES.
IFI(BODY-7)12,808,812
808 AMXJ1(1ROW,ICOL1 = AMXJ1(1ROW,ICOL1)
AMXJ1(1ROW,ICOL1 = AMXJ1(1ROW,ICOL1)
812 CONTINUE
816 CONTINUE
C SUMMATION FOR ALL MATRIX ELEMENTS COMPLETE, FINISH PROCESSING THEM
DO 880 JROW = 1,NRANK
DO 820 JCOL = 1,NRANK
AMXRI(JROW,JCOL1 = CMXV*AMXRI(JROW1,JCOL1)
AMXJ1(JROW,JCOL1 = CMXV*AMXJ1(JROW1,JCOL1)
AMXRI(JROW,JCOL1 = QEM*AMXRI(JROW1,JCOL1)
AMXJ1(JROW,JCOL1 = QEM*AMXJ1(JROW1,JCOL1)
C COMPUTE K MATRIX AS A FUNCTION OF THE J MATRIX.
AMXKRI(JROW,JCOL1 = -AMXKRI(JROW,JCOL1)
AMXKJ1(JROW,JCOL1 = -AMXKJ1(JROW,JCOL1)
C COMPUTE THE L MATRIX AS A FUNCTION OF THE K MATRIX.
AMXLR1(JROW,JCOL1 = -AMXLR1(JROW,JCOL1)
AMXLL1(JROW,JCOL1 = -AMXLL1(JROW,JCOL1)
820 CONTINUE
C COMPUTE ADDITIONAL TERM FOR THE IMAGINARY PART OF THE K MATRIX.
CR20M = JROW
IFI(MV124,824,826
68
A24 FCTKI = 1.0
GO TO 840
826 IF(JROW-KMV)<>2H, 828, 830, 930
828 CMXNRM(JROW) = 1.0
GO TO 840
830 IFACT = JROW-KMV
EFFCT = JROW-KMV
FPF=1 = IFCT
FCTKI = IFACT
CD 832 LFCT = IFACT, IEFCT
FCTKI = FCTKI*FPF
IFCT = IFACT
832 CONTINUE
B40 CMXNRM(JROW) = 4.0*CKROW*(CKROW, 0)*FCTKI/(FM*(CKROW,CKROW, 0))
AMK(JROW, JROW) = AMK(JROW, JROW)*FKFCT*CMXNRM(JROW)
CMXNRM(JROW) = SORT(CMXNRM(JROW))
860 CONTINUE
C PROCESS COMPUTED MATRICES
CALL PRESSM
400 CONTINUE
GO TO 20
END
SUBTYPE,FORTRAN,LMAP,LSTRAP

A program to read input data for the scattering program.
SUBROUTINE RDATA
COMMON DTR,RDT,CPI
COMMON /CMVCOM/ NM,CM(130),CMV,KMV,CM2,EM,OM,TMV,PRDNN
COMMON /FNCCOM/ PINL,LI(61),RSSLSPI(61),CNP(1M(61)
COMMON /MTXCOM/ KPNK,NRANKI,CMTXRI(120,120,1),CMTXI(120,120,1),SPRTM
LI(120,120,1),CMXNRM(160)
COMMON /THTCOM/ THETA,NTHTA,DLTHTA,SINTH,COSTH,ISMR,ISWCH(7),SR
LMSL,SMSL(57),CND(6),DMM,NSECT,NDFP(6),EPPS(6),KSECT
COMMON /RTYCOM/ CKR,OCKR,SKR,SNKR,X,CONK,R,ALPHA,IR00Y,OM,OR
1,SNALP,CSALP
COMMON /VCTCIM/ ACANS(15G,101),STSFC,RTSFCT
COMMON /UVCOM/ UANG(160),NUANG
DIMENSION CLRTOT(600)
EQUIVALENCE (ACANS(1,1),IR00Y)
COMMON /EPDCOM/ EPDEG(10)

READ NECESSARY INPUT DATA
PRINT 40
40 FORMAT(1H111X,3HCVM///////////////////////IHO^X
,40H»«*«««*«*«**«•••«•••*•••*•••*•••*•••*•••*•••*•••*/IHO2RX
63MELECTROMAGNFTIC SCATTFRINC
2 FROM GENERAL AXSYMMETRIC CONDUCTORS/IHO39X,40H************
3***************
)
READ SO,NM,NRANK,NSECT,IR00Y,NUANG
80 FORMAT(612L)
NRANK = NRANK1
PRINT 80
88 FORMAT(1H129X,5SHCASES MATEX RANK SECTIONS
1 BODY SHAPE U VECTOR)
PRINT 92,NM,NRANK,NSECT,IR00Y,NUANG
92 FORMAT(1H029X,5S11)
READ 96,CONK,BRXT,ALPHA
96 FORMAT(6F12.1)
RTSFCT = 0.0/CONK
STSFCT = 2.0/RTSFCT/CONK
PRINT 100
100 FORMAT(1H029X,60HRFl2.4n
1 ALPHA)
PRINT 104,CONK,BRXT,ALPHA
104 FORMAT(1H044X,3F15.3)
READ 96,(CM(111),I = 1,NM)
READ 80,INDP5,I = 1,NSECT)
PRINT 120,INDP5(I),I = 1,NSECT)
120 FORMAT(15H INTEGRATIONS/SECTIONR112/(1H023X,R112)
READ 96,(UANG(I),I = 1,NUANG)
C CLEAR AREA WHICH CONTAINS RUNNING TOTALS.
ON 136: I = 1,600
CLRTOT(I) = 0.0
136 CONTINUE
C COMPUTE END POINTS FOR THFTA.
ALPHA = 0TR#ALPHA
CALL CALEN
DO 14C I = 1,NSECT
EPDEG(I) = RTDEPS(I)
140 CONTINUE
PRINT 16A(EPDEG(I),I = 1,NSECT)
168 FORMAT(24H END POINTS8FI2.4,/(1H023X,RF12.4)
RETURN
END

70
SUBROUTINE BESSEL(NORDER, ARGMT, ANSMR, IFRROR)
DOUBLE PRECISION ARGMT, ANSMR, X, CN, SUM, APR, TOPR, CI, CNI, ACR, PROD,
               I, N, X

N = NORDER
X = ARGMT
CN = N
SUM = 1.0
APR = 1.0
TOPR = -0.500*X*X
CI = 1.0
CNI = 2*N+3
DO 60 I = 1, 100
ACR = TOPR*APR/(CI*CNI)
SUM = SUM+ACR
IF (DABS(ACR/SUM)<1.0D-20) GO TO 100
APR = ACR
CI = CI+1.0
CNI = CNI+2.0
60 CONTINUE
IFRROR = 1
GO TO 200

C THE SERIES HAS CONVERGED.
100 PROD = 2*N+1
FACT = 1.0
IFN160, 160, 120
120 DO 140 IFACT = 1, N
FACT = FACT*X/PROD
PROD = PROD*2.0
140 CONTINUE
160 ANSMR = FACT*SUM
200 RETURN
END
SUBROUTINE FORTRAN, LMAP, LSTRAP

A ROUTINE TO GENERATE LEGENDRE POLYNOMIALS.

COMMON /CFCOM/ XM, CM1(30), CMV, KMV, CM2, EM, DEM, TMM, PRTNM
COMMON /FCNCOM/ PNMLLG(I1), BSLSP(61), CNFIMN(61)
COMMON /TFCOM/ NRANK, NRANMK, CMXKL(120,120), CMXTK(120,120), SR
COMMON /NCFCOM/ THETA, NTHETA, SINTH, CSTH, ISNTH, ISOTH, SR
COMMON /MCFCOM/ NULM, SS(7), CMH(6), DMN, NSECT, NPS(6), KSECT
COMMON /BFCOM/ CKR, DCKR, CKR2, CMXKX, SMKR, CONK, BRT, ALPHA, IMODY, OR

1, SNALPH, CSMALPH

DO W = TW(1), 0
IF (THETA(I) < 0, 16, 4)
IF (KMV < 20, 20, 40)
THE SPECIAL CASE WHEN M = 0.
20 PLA = 1.0 * SINTH
PLB = COSH * PLA
PNMLLG(I) = PLA
PNMLLG(2) = PLB
IBEG = 3
GO TO 60
16 IF (KMV > 20, 20, 40)
C GENERAL CASE FOR M NOT EQUAL TO 0.
40 DO 44 ILG = 1, KMV
PNMLLG(I) = 0.0
44 CONTINUE
PLA = PROD CMP SINH(KMV-1)
PNMLLG(KMV) = PLA
48 PLB = DTW * COSH * PLA
PNMLLG(KMV2) = PLB
IBEG = KMV3
C DO RECURSION FORMULA FOR ALL REMAINING LEGENDRE POLYNOMIALS.
60 CNMUL = 1REG(I) + 3
CNM = 2.0
CNMM = DTW
DO 80 ILG = IBEG, NRANK
PLC = (CNMUL * COSTH * PLC-CNMM * PLA) / CNM
PNMLLG(I) = PLC
PLA = PLC
PLB = PLC
CNMUL = CNMUL2 + 3
CNM = CNM + 1.0
CNMM = CNMM + 1.0
80 CONTINUE
88 RETURN
END
SUBROUTINE ADOPRC
COMMON DTR, RTO, CPI
COMMON /CMVCOM/ NM, CMH, 30, CHV, KMV, CM2, EM, OPM, TWH, PRDN
COMMON /FNCCDM/ PNMLLGI, 61, BSSL, SP, Sll, TCFUMNI, 61
COMMON /MTXCOM/ NRANK, NRANKU, OMTXJ, I60,60, OMTXJ, I60,60, PMX1(60,60), PMX2(60,60), PMX3(60,60), PMX4(60,60)
2, OMTXK(60,60), OMTXR(60,60), OMTXLR(60,60), CMXRM(60)
COMMON /VCMCOM/ ISVBG, JSVBG, KSVBG, NSYMT
DIMENSION FGVECT(2,120,2), TCMPLX(2,120,120), FGMUL(120,120), FGANS(60,10)
EQUIVALENCE (OMTXK, CMXRM, TCMPLX, TCMPLX, TCMPLX, TCMPLX, TCMPLX, TCMPLX, TCMPLX)
COMMON /THTCOM/ THE, THETA, THETA, SIN, COST, ISW, ISWR, ISW (17), SR
MUL, SM Ul, SISI, CDTH, DTH, NSECT, NDP (6), FPPS, KSFT
COMMON /TOTCOM/ ACANS(60,10), STSFCT, RSFC
COMMON /UVCCOM/ UANG, NUANG
COMMON /RCVCOM/ CKR, OKCR, CKR2, COKX, SXR, X, SNX, X, CKN, BRXT, BPHA, IBOV, OB
1, SNLPH, CSALPH
C NORMALIZE AND STORE SECTIONS T1 AND T3 OF THE COMPLEX T MATRIX.
DO 40 IC = 1, NRANK
DO 20 IR = 1, NRANK
JR = IRCNRANK
QUANN = CMXRM(1, IR)/CMXRM(1, IC)
TCMPLX1(1, IR, IC) = QUANN*OMTXK(1, IR, IC)
TCMPLX1(1, JR, IC) = QUANN*OMTXLR(1, IR, IC)
TCMPLX1(1, JR, IR) = QUANN*OMTXJR(1, IR, IC)
TCMPLX1(1, JR, IC) = QUANN*OMTXJ(1, IR, IC)
20 CONTINUE
40 CONTINUE
C NORMALIZE AND STORE SECTIONS T2 AND T4 OF THE COMPLEX T MATRIX.
DO 80 IC = 1, NRANK
JC = IRCNRANK
JR = IRCNRANK
QUANN = CMXRM(1, IR)/CMXRM(1, IC)
TCMPLX1(1, IR, IC) = QUANN*OMTXK(1, IR, IC)
TCMPLX1(1, JR, IC) = QUANN*OMTXLR(1, IR, IC)
TCMPLX1(1, JR, IC) = QUANN*OMTXJR(1, IR, IC)
TCMPLX1(1, JR, IC) = QUANN*OMTXJ(1, IR, IC)
80 CONTINUE
60 CONTINUE
C SET UP A LOOP FOR ALL VALUES OF THE ANGLF U.
DO 400 IU = 1, NUANG
C GENERATE LEFENDRE POLYNOMIALS AND DERIVATIVES, RESET THE LIST.
IF (IUANG(IU)) 196, 89
99 COST = 1.0
92 SINTH = 0.0
THETA = 0.0
GO TO 112
96 IF (IUANG(IU)) 180, 01104, 100, 104
100 COST = -1.0
GO TO 92
104 THETA = DTR*IUANG(IU)
SINTH = SINTH(THETA)
COST = COST(THETA)
CALL GENGP
DO 120 IPS = 1, NRANK

73
FGMUL(IPS, 1) = CMV*PNMLLG(IPS1)
CPS = IPS
FGMUL(IPS, 2) = CPS*COST#*PNMLLG(IPS1)-(CPS*CMV)*PNMLLG(IPS)
JPS = IPS*NRANK
FGMUL(IPS, 1) = FGMUL(IPS, 2)
FGMUL(IPS, 2) = FGMUL(IPS, 1)

120 CONTINUE
C
MULTIPLY THE T COMPLEX MATRIX TIMES THE LFGFNDRF VECTORS,

KMVM1 = (KMVM-1)/4
KMVM = 4*KMVM1
IF(KMVM1)132 132, 124

124 DO 128 12 = 1,KMVM1
FGVECT(1, 12, 1) = 0.0
FGVECT(2, 12, 1) = 0.0
FGVECT(1, 12, 2) = 0.0
FGVECT(2, 12, 2) = 0.0
IF(ISYBG)126 0
J = [IZRANK
FGVECT(1, J, 1) = 0.0
FGVECT(2, J, 1) = 0.0
FGVECT(1, J, 2) = 0.0
FGVECT(2, J, 2) = 0.0

128 CONTINUE
ISYBG = 2*KMVM1
JSYBG = KMVM1
KSYBG = 2*KMVM1
NSYMT = NRANK-KMVM1
GO TO 136

132 ISYBG = 0
JSYBG = 0
KSYBG = 0
NSYMT = NRANK

136 CALL VECMUL
C A LOOP TO PEEL CURRENT SUMS OF SCATT1,2, TOTAL1,2 AND RTRAD1,2.
DO 140 ICMS = 1,NRANK
FGANS(1U, 17) = 0.0

140 CONTINUE
C
SET UP LOOP FOR CURRENT VALUES OF THE SUMS.

C
IPTH = 1
DO 200 ICMS = 1,NRANK
JCMS = ICMS*NRANK

C
COMPUTE SCATT1 AND SCATT2 SUMS
FGANS(1U, 1) = FGANS(1U, 1)*(FGVECT(1, ICMS, 1)**2+FGVECT(2, ICMS, 1)**2)
1FGVECT(1, ICMS, 1)**2+FGVECT(2, ICMS, 1)**2)/(CMXNRM(ICMS)**2
FGANS(1U, 6) = FGANS(1U, 6)*(FGVECT(1, ICMS, 2)**2+FGVECT(2, ICMS, 2)**2)
1FGVECT(1, ICMS, 2)**2+FGVECT(2, ICMS, 2)**2)/(CMXNRM(ICMS)**2

C
FORM THE REAL AND IMAGINARY PARTS OF TOTAL1,2 AND RTRAD1,2
PFR1 = FGVECT(1, ICMS, 1)*FGMUL(ICMS, 1)
PFL1 = FGVECT(1, ICMS, 1)*FGMUL(ICMS, 1)
PF2 = -FGVECT(2, ICMS, 1)*FGMUL(ICMS, 1)
PGL1 = -FGVECT(2, ICMS, 1)*FGMUL(ICMS, 1)
PG1 = FGVECT(1, ICMS, 1)*FGMUL(ICMS, 1)
PG1 = FGVECT(1, ICMS, 1)*FGMUL(ICMS, 1)
PGR2 = -FGVECT(2, ICMS, 1)*FGMUL(ICMS, 1)
PGR2 = -FGVECT(2, ICMS, 1)*FGMUL(ICMS, 1)

GO TO (150, 154, 15, 162), IPTH

150 SGN = 1.0
IPTH = 2
GO TO 170

154 SGN = -1.0
IPTH = 3

74
2(IMAG) RTRAD(REAL) RTRAD(IMAG) RCS PHASE ANGLE
3/
JREG = 165*(JPR-1)
JEND = JREG+6
00 560 JUP = 1,NUANG
C COMPUTE PHASE ANGLE AND RCS 1 OP 2.
PHANG = RTRAD*TAN2(ACANS(JUP,JEND),ACANS(JUP,JEND-1))
RCS12 = ACANS(JUP,JEND-1)*2*ACANS(JUP,JEND)**2
PRINT 440,(JANG(JUP),(ACANS(JUP,LPI),LPI = JREG,JEND),RCS12,PHANG
540 FORMAT(1H,F14.3,1P7E15.6)
560 CONTINUE
600 CONTINUE
RETURN
END
SUBROUTINE PRINT

COMMON /MTXCOM/ NRANK, NRANK1, QMTXI(60,60), QMTXJI(60,60), QMTXII(60,60), QMTXIII(60,60), PMX(60,60), PMX1(60,60), PMX2(60,60), PMX3(60,60), PMX4(60,60), QMTXIR(60,60,60), QMTXJR(60,60,60), QMTXKL(60,60,60), CMXNRM(60,60)

FOUVALENCE (QMTXI, QMTXRI, QMTXII, QMTXIII, QMTXIV, QMTXV)

DIMENSION CMTXI(120,120), CMTXII(120,120), CMTXIII(120,120), CMTXIV(120,120), CMTXV(120,120)

PRINT 20
20 FORMAT(1H1///1H052X,16HMATRIX T(1), RFAL)
CALL PRINT(QMTXIR, NRANK)
PRINT 120
120 FORMAT(1H1///1H052X,16HMATRIX T(2), RFAL)
CALL PRINT(QMTXJR, NRANK)
PRINT 220
220 FORMAT(1H1///1H052X,16HMATRIX T(3), RFAL)
CALL PRINT(QMTXKL, NRANK)
PRINT 320
320 FORMAT(1H1///1H052X,16HMATRIX T(4), RFAL)
CALL PRINT(QMTXJR, NRANK)
PRINT 420
420 FORMAT(1H1///1H052X,16HMATRIX T(5), IMAGINARY)
CALL PRINT(QMTXII, NRANK)
PRINT 520
520 FORMAT(1H1///1H052X,16HMATRIX T(6), IMAGINARY)
CALL PRINT(QMTXIIJ, NRANK)
PRINT 620
620 FORMAT(1H1///1H052X,16HMATRIX T(7), IMAGINARY)
CALL PRINT(QMTXIII, NRANK)
PRINT 720
720 FORMAT(1H1///1H052X,16HMATRIX T(8), IMAGINARY)
CALL PRINT(QMTXIV, NRANK)
RETURN
END
SUPTVPEtWKAP

A ROUTINE TO GET THE F1,G1 AND F2,G2 VECTORS FROM T = COS U.

PUNREL
PUNCPC,LAST,COMM

VECMUL FENTER,SXVRS

PUNCPC

MTXCOM COMBLOCK,FINAL

VECMUL COMBLOCk,FINALV

SXVRS SX*12,XR12
SX*13,XR13
SX*14,XR14
SX*15,XR15
SX*16,XR16
SX*17,XR17
SX*18,XR18
SX*19,XR19
SX*20,XR20
SX*21,XR21
SX*22,XR22
SX*23,XR23
SX*24,XR24

SAVF INDEX REGISTERS 12.

* 13.

* 14.

COMPLETE ADDRESS FACTOR T OF G1 OR G2.

SHFL 3R

FI(I1),3R

STIU(I),GIF?A

LX*12,TMKWR

VG*12,ISYHRG**32

LC*12,2.0

SX*12,TMKWR

VG*13,ISYHRG**32

LC*13,ASYHRG**32

SX*13,TMKWR

LI(I),NSYMT

GI(I),NSYMT

LX*14,FGXWR

VG*14,ISYHRG**32

LC*14,6G0.32

SX*14,FGXWR

FZIRU:1.81,RFGRGD0.20

* SET SIGN FOR ANSWER STORAGE, F1 OR G1

FZIRU:1.81,RFGRGD0.20

OD THE REAL PART OF F1 OR G1, G2 OR F2.

MGANLP LX*12,§12

VG*12,GIF2AC.32

LV*13,§13

VG*13,NPANKX.32

SRFL1R KC*14,NSYMTG.32

RXF,NCCMPR

VG*12,KSYHRG.32

SV*12,TMKWR

VG*14,KSYHRG.32

LV*12,§12

NLCCMPR LV*10,NIF1R

OLIU),ZFPD

SMPL1R R0,001§10

NIFIR LMPR(I),G0(12)

* IN MMON 4 = 1 OR 1.

LMPR(I),1.012)

* IN MMON 4 = -1 OR 1.

N2FIR LMR(I),0.0112)

* IN MMON 4 = 1 OR 1.

LMR(I),0.0121)

LNR(I),0.0121)
**N3F1R**

* N MOD 4 = 1 OR 1.0

**N4F1R**

* N MOD 4 = 1 OR 1.0

**EML1R**

* STORE REAL PART OF F1 OR G1, G2 OR F2.

**REFGF**

* DO THE IMAGINARY PART OF F1 OR G1, G2 OR F2.

**SMFII**

* N MOD 4 = 1 OR 1.0

**NIFII**

* N MOD 4 = 1 OR 1.0

**N2F11**

* NMOD 4 = -1.0 OR 1

**N3F11**

* NMOD 4 = -1.0 OR 1.0

**N4F11**

* N MOD 4 = 1.0 OR -1

**FML11**

* STORE IMAGINARY PART OF F1, G1, G2, F2.

**IFGFG**

* SET UP FOR NEXT ITEM.
VAR, »3, NRANKED, 32
LV, »12, TMXW
VC, »12, X2.0
SV, »12, TMXW
LV, »2, »12
VC, »2, G12GO, 32
CARL, »14, SMLFIR
VC, »14, MTXSF
LV, »12, TMXW
VC, »12, ISYRGEO, 32
SV, »12, TMXW
VC, »13, VCTSZE
SV, »13, PMXW
F11NU, »1, »11, FGFGEQ, 20
F11NU, »1, »11, FGFGEQ, 20
CA, »12, BCANLP
LX, »2, XR2
LX, »3, XR3
LX, »10, XR10
LX, »12, XR12
LX, »13, XR13
LX, »14, XR14
8, 0, 0, (10)

*STORAGE REQUIREMENTS FOR VECTOR MULTIPLICATION ROUTINE*

**XR12** WM, 0, 0, 0
**XR13** WM, 0, 0, 0
**XR14** WM, 0, 0, 0
**XR2** WM, 0, 0, 0
**XR3** WM, 0, 0, 0
**XR10** WM, 0, 0, 0
**TMXW** WM, TCMPLX, 0, TMXW
**PMXW** WM, FMGULT, 0, PMXW
**FGXW** WM, FVCECT, 0, FGXW
**TMXW** WM, TCMPLX, 0, TMXW
**PMXW** WM, FMGULT, 0, PMXW
**FGXW** WM, FVCECT, 0, FGXW
**G12F2** OR(U), 1
**TMFRTY** (F10), DD(U), 240.0, DX3R
**ZERO** DD(N), 0
**MTXSF** SYN, 240.0
**VCTSZE** SYN, 120.0
**LAST** SYN, 5
**SLCRCOM**
**COMMON** SYN, 5
**SLCRCOM, MTXCOM**
**NRANK** DR(U), 1
**NRANK1** DR(U), 1
**FVCT** DR(N), 480
**QMIJR** DR(N), 3120
**FVGUL** DR(N), 240
**QM1JR** DR(N), 3360
**FGUNS** DR(N), 600
**QMJTR** DR(N), 3600
**QMJTL** DR(N), 3600
**TCMPLX** DR(N), 28800
**CNXNSR** DR(N), 60
**FINAL** SYN, 5
**SLCRCOM, VCMCOM**
**ISYBG** DR(U), 1
**JSYBG** DR(U), 1

80
SUBROUTINE PRINTM(P,N)
DIMENSION P(60,60)
NR = N
100 I = 1,NR
10 IR = I
20 IF = IAE7
24 IF = NR
28 IF(I-1)36,36,60
36 PRINT 44,1,P(I,J),J = IR,IE
44 FORMAT(5HO,RM13,2X,1PF15.6)
GO TO 60
60 PRINT 66,(P(I,J),J = IR,IE)
GO TO 60
68 FORMAT(1H9X,1PF15.6)
80 IR = IEE1
84 IF(I-NR)20,20,120
100 CONTINUE
RETURN
END
END
SUBTYPE, FORTRAN, LMAP, LSTRAP
C A ROUTINE FOR BESSEL FUNCTIONS, DERIVATIVES AND NEUMANN FUNCTIONS.
SURROUTINE GENFSL
COMMON DTR, RTO, CP1
COMMON /CMVCOM/ NM, CNM(130), CNV, KMY, CM2, EM, QFM, TMI, PRADD
COMMON /FNCOM/ PMHLLG(61), ASSLSP(61), CNFUMN(61)
COMMON /TRXCOM/ NANK, NANK1, CMTXRL(120,120), CMXIM(120,120), SRRMT
1X(120,120), CNMPM(60)
COMMON /THCOM/ THETA, ATHETA, DLTHTA, SINHT, COSHT, ISMLA, ISWTCM1, SR
1ML, SMULS(71), CDFM(61), DMN, NSECT, NOPS(61), EPSL(61), KSFCT
COMMON /ROVCOM/ CKR, OCKR, CKR2, CSALPH, SNKR, CONK, BRXT, ALPHA, IPND, OR
1, SNALPH, CSALPH
DOUBLE PRECISION PCKR, ANSWR, ANSB, ANSC, CONN
C SET UP A LOOP TO GET 2 SUCCESSIVE BESSEL FUNCTIONS.
NVAL = NANK-1
PCKR = CKR
DO 40 I = 1, 4
CALL RFSSEL(NVAL, PCKR, ANSWR, IERORR) IF(IERRORR) 20, 20, 32
40 ANSA = ANSWR
NVAL = NVAL+1
CALL RFSSEL(NVAL, PCKR, ANSWR, IERORR) IF(IERRORR) 24, 24, 72
72 ANSB = ANSWR
GO TO 60
20 NVAL = NVAL-1
32 NVAL = NVAL+NRANK
40 CONTINUE
C PROGRAM (UNABLE TO) GENERATE BESSEL FUNCTION.
CALL DUMP
C SET UP FOR PROPER RECURSION OF THE BESSEL FUNCTIONS.
60 IF(NVAL-NANK) 110, 110, 64
64 NEND = NVAL-NANK
CONN = 2*(NVAL-1)E1
DO 72 IF = 1, IFND
ANSC = CUNAA/ANSA/PCKR-ANSR
CONN = CONN-2.0DO
ANSR = ANSA
ANSA = ANSC
72 CONTINUE
C PROGRAM IS READY TO RECURSE DOWNWARD INTO BESSEL FUNCTION VECTOR.
100 ASSLSP(NANK1) = ANSA
ASSLSP(NANK1-1) = ANSA
CONN = NANK1-NANK-1
IF = NANK1-2
JF = IF
DO 120 JA = 1, JF
ANSC = CONNAA/PCKR-ANSR
ASSLSP(IF) = ANSC
ANSB = ANSA
ANSA = ANSC
IF = IF-1
CONN = CONN-2.0DO
120 CONTINUE
C GENERATE THE NEUMANN FUNCTIONS.
CMULN = 3.0
SNSA = -CSKRX
SNSA = -CSKRX/PCKR-SKRX
CNFUMN(1) = SNSA
CNFUMN(2) = SNSA
83
DO 290 I = 3, NRANK
SNSC = CMULN*SNSH/CR0-SNSA
CMFUMN(I) = SNSC
SNSA = SNSH
SNSH = SNSA
CMULN = CMULN2.0
290 CONTINUE
RETURN
END
SUBTYPE, STRAP

A ROUTIN TO DUMP CORF.

PUNREL

PUNRF, LAST, COMMON

DUMP

ENTER, START

XW, $, 0, 0

START

R, $MCP

$AREDJ

H, 0, 0 (115)

LAST

SYN, $5

SLCCTOM

COMMON

SYN, $5

END
SUBTYPE FORTRAN LMAP LSTRAP
C A ROUTINE TO ORTHOGONALIZE THE Q MATRICES TO PRODUCE T MATRICES.

SUBROUTINE PROCSS
COMMON /MTXMCM/ NR, NR, OJ, O1, P1(60, 60), P12(60, 60), P13(60, 60), O14(60, 60)
O1, O2(60, 60), O3(60, 60), O4(60, 60), OR(60, 60), OR2(60, 60), OR3(60, 60), CMXRMX(60, 60)
COMMON /RHX/ RHX, OR1, OR2(60, 60), OR3(60, 60)
COMMON /ORH/ OR1, OR2, OR3
COMMON /MN/ PM
EQUIVALENCE (OR1, OR2), (PM, ORH)
DIMENSION RR1(120, 120), OR1(120, 120), OR2(120, 120), CMXRMX(120, 120)

C NORMTAE AND TRANPOSE THE I, J, K, L MATRICES TO OBTAIN Q MATRICES.
C CALL NORMXM
C SET UP REAL AND IMAGINARY MATRICES FOR GENERAL CASE.
DO 6 I = 1, NR
MM = I
DO 4 J = 1, NR
NN = J
TP1 = RR1(MM, NN) = G11(I, J)
TP2 = RR1(MM, NN) = O12(I, J)
TP3 = RR1(MM, NN) = OR1(I, J)
TP4 = RR1(MM, NN) = OR2(I, J)
4 CONTINUE
6 CONTINUE
NBGR = NRGR
DO 10 I = 1, NGR
MM = I
DO 8 J = 1, NBGR
RR1(I, J) = TMMX(I, J)
8 CONTINUE
10 CONTINUE
NBGR = NBGR - 1
DO 12 J = 1, NR
NN = J
TP1 = RR1(MM, NN) = OR1(I, J)
TP2 = RR1(MM, NN) = OR2(I, J)
TP3 = RR1(MM, NN) = OR3(I, J)
TP4 = RR1(MM, NN) = OR4(I, J)
12 CONTINUE
14 CONTINUE
DO 16 I = 1, NGR
MM = I
DO 14 J = 1, NBGR
RR1(I, J) = TMMX(I, J)
14 CONTINUE
16 CONTINUE
C CONDITION Q MATRICES BEFORE ORTHOGONALIZING THEM.
C CALL CNDTNO
C NORMALIZE THE NTH ROW OF AN N BY N MATRIX
SUM1 = 0.0
DO 20 K = 1, NGR
SUM = RR1(NGR, K)**2 + RR1(NGR, K)**2 + SUM
20 CONTINUE
SUM1 = SQUARE(SUM1)
DO 22 K = 1, NGR
RR1(NGR, K) = RR1(NGR, K) / SUM1
RR1(NGR, K) = RR1(NGR, K) / SUM1
22 CONTINUE
C SET UP A LOOP FOR THE N-1 REMAINING ROWS.
NM = NGR - 1
NROW = NGR
DO 100 I = 1, NM
NROW = NROW - 1
86
DO 36 K=1, NGR
TMMX(1,K) = R11(NROW,K)
TMMX(2,K) = R11(NROW,K)
36 CONTINUE
DO 80 J = NROW,NMI
SR1 = 0.0
SI1 = 0.0
MROW = MROWC1
DO 40 K = 1, NGR
SR1 = SR1*RR1(NROW,K)*SL1*RR1(NROW,K)
SI1 = SI1*RR1(MROW,K)*SL1*RR1(MROW,K)
TMMX(1,J) = TMMX(1,J)-SR1*RR1(MROW,K)+SI1*RR1(MROW,K)
TMMX(2,J) = TMMX(2,J)-SR1*RR1(NROW,K)-SI1*RR1(NROW,K)
40 CONTINUE
DO 88 K = 1, NGR
TMMX(2,K) = TMMX(2,K)-SR1*RR1(NROW,K)-SI1*RR1(NROW,K)
88 CONTINUE
SUM1 = 0.0
DO 84 K = 1, NGR
SUM1 = SUM1+TMMX(1,K)**2+TMMX(2,K)**2
84 CONTINUE
SUM1 = SUM1/SORT(SUM1)
DO 98 K = 1, NGR
R11(NROW,K) = TMMX(1,K)/SUM1
SI1(NROW,K) = TMMX(2,K)/SUM1
98 CONTINUE
103 CONTINUE
C PRINT OUT ORTHOGANLIZED Q MATRCES
PRINT 120
120 FORMAT (1H14X,40HREAL SECTION OF ORTHOGANLIZED Q MATRX.),
CALL PRNOUT(RR1, NGR),
PRINT 128
128 FORMAT (1H14X,45HIMAGINARY SECTION OF ORTHOGANLIZED Q MATRX.),
CALL PRNOUT(R11, NGR),
C PERFORM 0 TRANSPOSE R REAL(Q) TO GET T MATRX.
DO 160 I = 1, NGR
DO 152 J = 1, NGR
TMMX(I,J) = J*0
152 CONTINUE
160 CONTINUE
DO 180 I = 1, NGR
DO 176 J = 1, NGR
DO 172 K = 1, NGR
TMMX(I,J) = TMMX(I,J)-R11(K,1)*RR1(K,J)
172 CONTINUE
176 CONTINUE
180 CONTINUE
DO 196 I = 1, NR
NN = I*I
DO 192 J = 1, NR
NN = JJ
Q11(I,J) = TMMX(NN-1,NN-1)
Q12(I,J) = TMMX(NN-1,NN)
Q13(I,J) = TMMX(NN,NN-1)
Q14(I,J) = TMMX(NN,NN)
192 CONTINUE
196 CONTINUE
DO 204 J = 1, NGR
TMMX(I,J) = 0.0
204 CONTINUE
204 CONTINUE
208 CONTINUE
DO 220 I = 1, NAGR
212 CONTINUE
DO 216 J = 1, NAGR
DO 212 K = 1, NAGR
TMMX(I, J) = TMMX(I, J) + (K, I) + (K, J)
212 CONTINUE
DO 216 J = 1, NP
220 CONTINUE
MM = IC(I)
DO 232 J = 1, NM
NN = J6(J)
CRI(I, J) = TMMX(MM - 1, NN - 1)
CHR(I, J) = TMMX(MM - 1, NN)
CRS(I, J) = TMMX(MM, NN - 1)
CR4(I, J) = TMMX(MM, NN)
232 CONTINUE
23A CONTINUE
C PRINT THE T MATRIX
CALL PRINT
C DO FINAL PROCESSING
CALL ADDPRC
RETURN
END
SUBTYPE: FORTRAN, LMAP, L STRAP

A ROUTINE TO CONDITION Q MATRICES BEFORE ORTHOGONALIZING THEM.

SUBROUTINE CNNDND

COMMON /XTRCOL/ NNR, NRM, D11(60,60), D12(60,60), D14(60,60), D16(60,60), D18(60,60), D20(60,60), OR3(60,60), OR5(60,60), OR7(60,60), OR9(60,60), OR11(60,60), OR13(60,60), OR15(60,60), OR17(60,60), OR19(60,60), OR21(60,60), OR23(60,60), OR25(60,60), OR27(60,60), OR29(60,60), OR31(60,60), OR33(60,60), OR35(60,60), OR37(60,60), OR39(60,60), OR41(60,60), OR43(60,60), OR45(60,60), OR47(60,60), OR49(60,60), OR51(60,60), OR53(60,60), OR55(60,60), OR57(60,60), OR59(60,60)

EQUIVALENCE (Q11, Q11), (OR1, OR1), (TMMX, PI)

DIMENSION RR1(126,126), RR2(126,126), RR3(126,126)

C SET UP LOOPS FOR ALL BUT THE FIRST ROW.

NRGR = NNR + 1
NROW = NRGR
DO 60 KR = 2, NRGR

C RESCALE THE CURRENT ROW.

SCLE = 1.0/RR1(NROW,NROW)
DO 8 IC = 1, NRGR
RR1(NROW, IC) = SCLE * RR1(NROW, IC)
RR2(NROW, IC) = SCLE * RR2(NROW, IC)

8 CONTINUE

C RESCALE ALL THE ROWS UP TO THE CURRENT ROW.

NROW = NROW - 1
DO 22 IC = 1, NROW

SCLE = 1.0/RR1(NROW, NROW)
DO 16 IC = 1, NROW
RR1(MR, MC) = RR1(MR, MC) - SCLE * RR1(NROW, MC)
RR2(MR, MC) = RR2(MR, MC) - SCLE * RR2(NROW, MC)

16 CONTINUE

DO 22 CONTINUE

NROW = NROW - 1
20 CONTINUE

C SET IMAGINARY ELEMENTS ABOVE THE MAIN DIAGONAL = 0.

NROW = NRGR - 1
DO 72 J = 1, NROW
RA = J + 1
DO 72 I = 1, NROW
RI(I, J) = 0.0
72 CONTINUE

DO 80 CONTINUE
RETURN
END
SUBROUTINE NRMMX
COMMON AMXIR(60,60), AMXJR(60,60), AMXKR(60,60)
CIMNTX(I60,60), QMTEXJ(60,60), QMTEXL(60,60), CHMNRMMX(60,60)
EQUIVALENCE (AMXIR(60,60), AMXJR(60,60), AMXKR(60,60))
DIMENSION QMTEX(I60,60), QMTEXJ(60,60), QMTEXL(60,60)

C SET UP LOOPS TO PROCESS ALL ROWS AND COLUMNS FOR THE REAL MATRICES.
ON 200 IR = 1, NRMXK
ON 100 IC = 1, NRMNK
QUANNM = CMNRMMX(1,1) + CMNRMMX(2)
QMTEX(R, IC) = AMXIR(R, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
QMTEX(J, IC) = AMXIR(1, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
QMTEXL(IR, IC) = AMXIR(R, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
100 CONTINUE
200 CONTINUE

C SET UP LOOPS FOR THE IMAGINARY MATRICES.
ON 400 IR = 1, NRMXK
ON 300 IC = 1, NRMNK
QUANNM = CMNRMMX(1,1) + CMNRMMX(2)
QMTEX(R, IC) = AMXIR(R, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
QMTEX(J, IC) = AMXIR(1, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
QMTEXL(IR, IC) = AMXIR(R, IC) + QMTEX(J, IC) + QMTEXL(IR, IC)
300 CONTINUE
400 CONTINUE

C PRINT OUT NORMALIZED AND TRANSPOSED Q MATRICES
PRINT 420
420 FORMAT(1H40X,3HREAL PART OF Q1(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEX, NRMK)
PRINT 420
428 FORMAT(1H40X,3HREAL PART OF Q2(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXJ, NRMK)
PRINT 428
436 FORMAT(1H40X,3HREAL PART OF Q3(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXL, NRMK)
PRINT 436
444 FORMAT(1H40X,3HREAL PART OF Q4(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEX, NRMK)
PRINT 444
452 FORMAT(1H40X,3HREAL PART OF Q1(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXJ, NRMK)
PRINT 452
460 FORMAT(1H40X,3HREAL PART OF Q2(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXL, NRMK)
PRINT 460
468 FORMAT(1H40X,3HREAL PART OF Q3(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXJ, NRMK)
PRINT 468
476 FORMAT(1H40X,3HREAL PART OF Q4(NORMALIZED,TRANSPOSED))
CALL PRINTM(QMTEXL, NRMK)
RETURN
END
SUBROUTINE FORTRAN, LMAP, LSTRAP
C
A ROUTINE TO PRINT OUT A MATRIX ARRAY

SUBROUTINE PRTQUT(P,N)
DIMENSION P(120,120)
NR = N
DO 100 I = 1, NR
IB = I

10 IF (IB>NR) GO TO 100
PRINT 20, (P(I,J), J = 1, NR)
20 IF (IB-NR) 29, 24
24 IF (NR-NR) 29, 24

PRINT 36, (P(I,J), J = 1, NR)
36 PRINT 44, (P(I,J), J = 1, NR)
44 FORMAT (5HO, R11W13, 2X, 1P4F15.6)
GO TO 80
80 FORMAT (6R, (P(I,J), J = 1, NR)
60 FORMAT (1H, 9X, 1P4F15.6)
80 IF (NR-NR) 100, 100
100 CONTINUE
RETURN
END
SUBROUTINE FINDTRAN, MAP, L, LSTRAP

C THIS ROUTINE CALCULATES END POINTS AND SPACING FOR FINDTRAN.

SUBROUTINE CALEND

COMMON DTP, DTDP, CPT

COMMON /CMVCM/ TN, CM(1170), CVR, KMN, CMZ, FM, DFM, TNM, PHN

COMMON /VENCM/ PMULG(11), RSSI, SPI(11), NFIMM(11)

COMMON /MIXCM/ NPMAX, NPMAX, CMXRI (123, 123), CMXI (123, 123), SDMRT

COMMON /FINDCM/ THEI, NTHEI, MTHI, SINTH, COSTH, SCONTH, SWITCHTH, SP

COMMON /MULCM/ SMULS(T), CMHAI, UMM, REFCT, NDDPS(1), EPPS(1), PSCT

COMMON /P leadership/ CNP, NCNR, CRP, CRPS, CNVR, CRPA, INPA, NRT, ALPHA, T, MNY, ON

C

1. SNALPH = CSALPH

SNAI PH = SIN(ALPHA)

CSALPH = COS(ALPHA)

QB = (1.0-COSX(1.0))/SNALPH

C

CALCULATE THE FIRST END POINT AND STEP SIZE:

TANGAM = SNALPH*CSALPH/(UR-SNALPH*SNALPH)

GAMMA = ATAN(TANGAM)

IF(GAMMA)20,32,32

20 GAMMA = GAMMA(CPI)

3: EPPS(1) = GAMMA

C

COVD = NDDPS(1)

C

CALCULATE THE SECOND END POINT AND STEP SIZE:

TANPSI = -AXT*SNALPH*CSALPH/(1.0-AXT*CSALPH*SNALPH)

PSI = ATAN(TANPSI)

IF(EPPS(1)-PSI)72,72

72 PSI = PSI+CPI

72 EPPS(2) = PSI

C

COVD = NDDPS(2)

C

CALCULATE THIRD END POINT AND STEP SIZE:

EPPS(3) = PSI

COVD = NDDPS(3)

C

RETURN

END
SUBTYPE, FORTRAN, LMAP, LSTRAP

COMMON /CMVCMD/ NM, CM1101, CMV, KM2, EM, DFNM, TWM, PRONM
COMMON /FNCMD/ RNML1101, ASSHI11, CNFM1111
COMMON /MTXCM/ NRANK, RNANK1, CMTXRL12, SPrMT
C
COMMON /THMCMD/ THETA, THETA0, THET, CMTH17, SUP1, SLUMS, SLULS, \nCMTD, SNALPH, CSMUL, CMOD, CMODN, NSEC, NPS516, EPPS16, KSECT
COMMON /AYMCMD/ CKR, DCKR, CKP2, CSKPX, SNKRX, CONK, ARCT, ALPHA, IRDY, OR
1, SNALPH, CSALPH
KSECT = KSect
C
SECTION FOR INTEGRATION
IF (IKSECT = 214) GO TO 1140, 240
C
SECTION 1
4J QUAN1 = SQRT (1.0 - (OR * SINTH / SNALPH) ** 2)
CKV = CONK * (OR * COSTH / SNALPH * QUAN1)
DCKR = - CONK * (OR * SINTH / SNALPH * (1.0 - OR * COSTH / (SNALPH * QUAN1)))
GO TO 360
C
SECTION 2
140 QUAN2 = THETA - ALPHA
SNON2 = SINTH(SQUAN2)
CKP = CONK * (1.0 - OR) / SNON2
DCKR = - CONK * (1.0 - OR) * COSTH (QUAN2 / 1.0 - SNON2 / SNON2)
GO TO 330
C
SECTION 3
243 QUAN3 = (1.0 - OR * XR - OR) / SNALPH
QNSO = SQRT (XR * XR - QUAN3 * SINTH * OR ** 2)
CKR = CONK * (QNSO - QUAN3 * COSTH)
DCKR = CONK * (QNSO - QUAN3 * SINTH - QUAN3 * QUAN3 * SINTH * COSTH / QNSO)
330 RETURN
END
APPENDIX III

A NUMERICAL EXAMPLE: THE SPHERE-CONE-SPHERE
ELECTROMAGNETIC SCATTERING FROM GENERAL AXISYMMETRIC CONDUCTORS

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97
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| ROW 3 | -4.63018E-04 | 3.66236E-04 | -8.20267E-06 | 7.07945E-06 | -1.82918E-07 | 2.82127E-08 |
| ROW 4 | 6.26730E-05 | -1.20622E-05 | 1.77292E-06 | -1.82918E-07 | 2.82127E-08 |
| ROW 5 | -2.90607E-06 | 1.77292E-06 | -1.82918E-07 |
| ROW 6 | 1.99669E-07 | -6.17791E-08 | 1.20622E-05 | 6.26730E-05 |

**IMAGINARY PART OF Q1(NORMALIZED, TRANSPOSED):**

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| ROW 3 | -4.63018E-04 | 3.66236E-04 | -8.20267E-06 | 7.07945E-06 | -1.82918E-07 | 2.82127E-08 |
| ROW 4 | 6.26730E-05 | -1.20622E-05 | 1.77292E-06 | -1.82918E-07 | 2.82127E-08 |
| ROW 5 | -2.90607E-06 | 1.77292E-06 | -1.82918E-07 |
| ROW 6 | 1.99669E-07 | -6.17791E-08 | 1.20622E-05 | 6.26730E-05 |

**REAL PART OF O2(NORMALIZED, TRANSPOSED):**

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| ROW 3 | -4.63018E-04 | 3.66236E-04 | -8.20267E-06 | 7.07945E-06 | -1.82918E-07 | 2.82127E-08 |
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| ROW 6 | 1.99669E-07 | -6.17791E-08 | 1.20622E-05 | 6.26730E-05 |

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115
| \( N = 3 \) | REAL PART OF \( Q_1 \) (NORMALIZED, TRANSPOSED) |
|---|---|---|---|---|---|---|---|
| ROW 1 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 |
| ROW 2 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 | -0.000000E 00 |
| ROW 3 | -0.000000E 00 | -0.000000E 00 | -5.635017E-04 | -1.916494E-05 | -2.359892E-06 | -5.793659E-09 | -5.793659E-09 |
| ROW 4 | -0.000000E 00 | -0.000000E 00 | -1.916494E-05 | -9.799395E-06 | -4.366426E-07 | -6.590991E-08 | -6.590991E-08 |
| ROW 5 | -0.000000E 00 | -0.000000E 00 | -2.595092E-06 | -4.366426E-07 | -1.700041E-07 | -4.173203E-09 | -4.173203E-09 |
| ROW 6 | -0.000000E 00 | -0.000000E 00 | -5.793659E-09 | -6.590991E-08 | -6.173203E-09 | -1.991914E-09 | -1.991914E-09 |

| \( N = 3 \) | IMAGINARY PART OF \( Q_1 \) (NORMALIZED, TRANSPOSED) |
|---|---|---|---|---|---|---|
| ROW 1 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 |
| ROW 2 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 |
| ROW 3 | 0.000000E 00 | 0.000000E 00 | 5.515189E-01 | 1.784495E-02 | 5.820831E-02 | -9.612511E+00 | -9.612511E+00 |
| ROW 4 | 0.000000E 00 | 0.000000E 00 | 3.514700E-03 | 5.524111E-01 | 1.011439E-03 | 4.181259E-01 | 4.181259E-01 |
| ROW 5 | 0.000000E 00 | 0.000000E 00 | 1.210924E-04 | 4.141077E-04 | 9.504908E-01 | -3.343593E-02 | -3.343593E-02 |
| ROW 6 | 0.000000E 00 | 0.000000E 00 | -3.800205E-06 | 9.529041E-05 | 1.432907E-04 | 5.446234E-01 | 5.446234E-01 |

| \( N = 3 \) | REAL PART OF \( Q_2 \) (NORMALIZED, TRANSPOSED) |
|---|---|---|---|---|---|---|---|
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| ROW 3 | -0.000000E 00 | -0.000000E 00 | 3.026242E-03 | 6.085272E-06 | 1.712412E-08 | 2.118473E-08 | 2.118473E-08 |
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| ROW 6 | -0.000000E 00 | -0.000000E 00 | 2.118473E-08 | -2.537229E-10 | 1.228086E-09 | 1.319774E-11 | 1.319774E-11 |

| \( N = 3 \) | IMAGINARY PART OF \( Q_2 \) (NORMALIZED, TRANSPOSED) |
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| ROW 2 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 | 0.000000E 00 |
| ROW 3 | 0.000000E 00 | 0.000000E 00 | 5.739959E-03 | -1.640457E-03 | -1.775943E-03 | -1.010184E-01 | -1.010184E-01 |
| ROW 4 | 0.000000E 00 | 0.000000E 00 | -1.640457E-03 | -1.345388E-04 | -6.470731E-04 | -1.359665E-01 | -1.359665E-01 |
| ROW 5 | -0.000000E 00 | -0.000000E 00 | 6.919463E-05 | -8.470731E-04 | 3.719961E-04 | -6.494914E-04 | -6.494914E-04 |
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121
REFERENCES


5. F. K. Oshiro, Private Communication.


9. Ref. 8, p. 1875.

10. Ref. 8, pp. 1898-1901.


**NUMERICAL SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS**

The purpose of this work is to describe a theoretical formulation, including a documented computer program, for the evaluation of electromagnetic scattering by perfectly conducting bodies having an axis of rotational symmetry. The main body of the work gives the theory, which has been modified considerably from that given earlier. Appendix I gives the analysis and logic which forms the basis for the various subroutines of the computer program. Appendix II gives the complete FORTRAN listings of the computer program. Finally, Appendix III gives the computer printout for a numerical example, scattering by a conducting sphere-cone-sphere obstacle, as obtained on the IBM 7030 digital computer.
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