AIR FORCE MISSILE DEVELOPMENT CENTER
TECHNICAL REPORT

THE APPARENT INPUT AXIS MISALIGNMENT ERROR
CAUSED BY ANGULAR ROTATION ABOUT THE OUTPUT AXIS
OF A SINGLE-DEGREE-OF-FREEDOM, RATE-INTEGRATING-GYRO

BUD J. WIMBER

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"CENTRAL INERTIAL GUIDANCE TEST FACILITY"

HOLLOMAN AIR FORCE BASE
NEW MEXICO

MARCH 1969
FOREWORD

This report was presented at the Fourth Inertial Guidance Test Symposium, 6-8 November 1968. The report presents evidence of a very significant error source in single-degree-of-freedom rate integrating gyros that has not been previously reported. Because of its potential impact on strapdown gyro technology, the paper is published as a technical report for the exchange and stimulation of ideas.

I wish to acknowledge Dr. M. G. Jaenke, Technical Director of the CIGTF; Dr. Daniel P. Petersen, CIGTF Consultant, University of New Mexico; and Major John Kalish, former Gyroscope Test Branch Chief, for their encouragement and guidance during the time I was privileged to work with them. There are many others too numerous to mention specifically who have given of their time and talents to whom I am also indebted. But none of the work could have been done without the excellent technical assistance of SSgt Ronald Hanna who performed almost all of the tests for me. Finally, I am grateful to the Air Force for the opportunity given to me to accomplish this work.

This Technical Report has been reviewed and is approved for publication.

APPROVED:
ROBERT B. SAVAGE, Colonel, USAF
Director of Guidance Test
ABSTRACT

Results of laboratory testing of inertial grade strapdown single-degree-of-freedom gyroscopes has revealed that a deterministic error, proportional to rate squared, is generated when the gyro is subjected to angular rates about the output axis (OA). Tests further indicate that the rate error is related in some way to OA friction effects. This is evident from tests conducted on gyros where an attempt was made to reduce the OA friction coefficient by use of dithering jewel and taut wire suspension. Additional laboratory tests are being conducted at the Central Inertial Guidance Test Facility in order to identify the exact source of the error.
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I. INTRODUCTION

For many years, single-degree-of-freedom rate integrating (SDFRI) gyros have found wide use in military applications and more recently in civilian applications. Their use has been primarily to stabilize gimballed platforms containing other instruments whose function is to measure inertial quantities during the guidance and control aspect of navigation. Because of the benign environment these gyros are exposed to, the effect of angular motion about the output axis or spin axis causes little or no concern.

However, with the present trend toward strapdown navigation systems, where the inertial sensors are directly exposed to the missile or aircraft frame environment, additional error sources become important. In the platform application, the primary errors of interest were the non-acceleration, acceleration and acceleration-squared sensitive errors. When used in a strapdown application, the errors caused by angular motion take on a larger significance. Up to now the major design effort has been to minimize such errors as anisoinertia, rectification and angular acceleration.

The purpose of this paper is to show that there exists a very significant additional error which is caused by angular rotation about the gyro's output axis. Since from all outward appearances this error behaves as though the input axis (IA) is misaligned with respect to the output axis (OA) about the spin axis (SA), the error is referred to as an "Apparent IA-Misalignment Error". Although at this time the exact source of the error has not been discovered, some mechanisms that could cause the error are discussed.

II. THE NATURE OF THE ERROR

During the course of a test program that was conducted at the CIGTF, angular rates were applied about the output axis of Gyro A in both a centrifuge test and in a verification test on a high speed rate table. It was observed in both cases that even though the gyro's IA had been carefully aligned in a plane perpendicular to the motion prior to the test, the output pulse count of the gyro had a different
value for positive rates about the OA compared with negative rates. This caused a suspicion that the mount axes were shifting whenever the table (or centrifuge) motion was reversed. However, an investigation into the stability of the table and centrifuge revealed that the mount axes could not deviate more than an arc second. The gyro tests indicated that deflections of the mount axes of at least two to three arc minutes would be necessary to cause the errors observed. Therefore it was concluded that some mechanism in the gyro must be causing the discrepancies, and an intensive investigation was begun to determine the nature of the error and ultimately its cure.

In order to determine if the error existed on other gyro units, and to determine the effects of not providing precision alignment prior to test, it was decided to perform the test on a different gyro unit (Gyro B) of the same gyro type. The results of this test are presented in Figure 1 where the resulting rate error in °/hr is plotted versus angular input rate about the gyro OA. As is shown in the figure, the rate error seems to be nearly proportional to angular output axis rates at least for positive rates.

For an explanation of the phenomenon, let us assume that the rate characteristic shown in Figure 1 can be attributed to an actual deflection or misalignment of the IA. Figure 2 shows the resultant misalignment angle as a function of input rate which would be required to cause the error. As shown in the figure, this angle approaches two minutes and has a nearly linear, deterministic, symmetrical characteristic with an initial misalignment angle of 50 arc seconds. The characteristic shown in Figure 2 has been measured repeatedly for different sensor orientations and initial conditions.

This same rate error characteristic has also been observed on another gyro (Gyro C) of similar design and construction. Figure 3 shows the misalignment angle as a function of output axis rate for the gyro for various mechanical alignments. It is interesting to note that realignment merely shifts but does not alter the rate error characteristic. It should be pointed out that this particular gyro was not designed for strapdown use and was tested in an analog configuration, and yet the same phenomena as observed for Gyros A and B (designed for strapdown and in the pulse-torque configuration) resulted from the test.
FIGURE 2

APPARENT IA MISALIGNMENT ANGLE VERSUS OUTPUT AXIS RATE (10 to 70°/sec)
FIGURE 3

APPARENT IA MISALIGNMENT ANGLE VERSUS OUTPUT AXIS RATE FOR GYRO C
The most significant fact to be obtained from this information is that there appears to be some definite deterministic aspect to the error (even the anomalies are repeatable) thus indicating that the error could be compensated for directly by calibration. Repeated tests of the same specimen over wide output axis rates have indicated that the phenomenon is repeatable to within a reasonable uncertainty.

Thus far we have limited our discussion to effects of angular motion about the OA for very high rates. In their actual mission application, Gyros A and B are not expected to perform with angular rates exceeding 12.5°/sec about the gyro's IA. In a system configuration, then, not more than 12.5°/sec should occur about the output axis of any gyro. The results of tests performed at rates from one to 10°/sec on Gyro B are presented in Figure 4. Even in this limited test interval (±10°/sec) there is a change of the apparent misalignment angle which approaches one arc minute, and there is a very deterministic character with what appears to be some initial offset between positive and negative OA rates of about 20 arc seconds. Observe that the misalignment angle for input rates of 10°/sec seems to indicate a change in the trending. To see how this trend fits into the overall picture, the test inputs were extended to ±70°/sec and the composite results are presented in Figure 5. In the 1 to 10°/sec region two tests were performed, one with monotonically increasing positive rates, then negative monotonically decreasing rates. The second test was performed by alternating polarities at each rate before going on to the next higher rate. It is significant to note that the slope of the data in the 1 to 10°/sec region for both tests conforms with the slope of up to 70°/sec, confirming the deterministic character of the phenomenon.

III. POSSIBLE CAUSES OF THE ERROR

It has been a very simple matter to present the nature of the error as observed in carefully controlled laboratory tests thus far. It is another story, however, to explain the cause of the error. As presented in the previous section, the error acts as if the gyro's IA were misaligned with respect to output-axis motion. It now becomes necessary to explore the physical phenomena that takes place whenever an SDFRI gyro is subjected to angular motion about the OA in an attempt to discover some mechanism that would allow an error torque of the observed magnitude to be developed about the OA.
NOT REPRODUCIBLE
For the purpose of discussion, our gyro is in a torque-rebalance configuration which means simply that the pickoff error signal is fed through a finite gain amplifier back to the torquer in a servo loop operation. An error signal from the pickoff is created by initial motion of the case of the gyro about the OA and returned to the gyro torquer. The torquer causes a torque about the OA which forces the float to precess about the gyro's IA. The float pivot moves through the small pivot-jewel clearance and eventually bears against the jewel. This causes a reaction torque of sufficient magnitude to precess the float about the OA thus nulling the initial error. It is instructive to calculate the steady-state reaction torque and determine the resultant force acting on the output axis supports. Figure 6 is a line schematic of the gyro showing the configuration involved: The reaction torque \( M_{IA} \) is obtained from the equation:

\[
M_{IA} = H (\omega_{OA} + \dot{\phi}_0)
\]

where

\[
H = \text{Spin Angular Momentum}
\]

\[
\omega_{OA} = \text{Angular rate of the case with respect to inertial space about the OA}
\]

\[
\phi_0 = \text{Hangoff angle of float to case (difference between SA and SRA)}
\]

From Figure 6,

\[
M_{IA} = Fl
\]
FIGURE 6

LINE SCHEMATIC OF SDFRI GYRO
For constant angular rate, $\theta_0 = 0$ and the resulting equation is:

$$F = \frac{H_{\omega_{OA}}}{k}$$

For a typical length of seven (7) cm and angular momentum of $5 \times 10^5$ dyne cm sec, an angular rate of one (1) radian per second will cause a reaction torque of $5 \times 10^5$ dyne cm or a force on each support of $7.15 \times 10^4$ dynes. For the same gyro, a drift rate of $10^6$/hr can be caused by a torque about the OA of about 24 dyne cm. Thus we see that a comparatively very large reaction torque (and force) is developed due to OA angular rotation.

An examination of the structure involved in the "OA rate-reaction torque" phenomena reveals that there can be no motion of the float about the SA (which is the only motion that would allow the IA to move into the angular environment) unless there is anisoeelasticity of the output axis support. A very interesting theory proposed by Dr. Daniel D. Petersen, University of New Mexico, Consultant to the CIGTF suggests that if the output axis support is anisoeelastic, the deflections at the pivots may not be colinear with the imposed forces. Thus, the angular rotation of the float may have a component about the spin axis, rotating the input axis out of the plane perpendicular to the output axis and thus into the angular environment. Calculations indicate that in order to produce the deflections observed the anisoelastic spring constants would have to be about 1000 times smaller than those claimed by the manufacturer. Also the amount of deflection required would be almost prohibitive considering the very small clearance in the fluid gap. Thus, anisoeelasticity of the OA supports does not appear to be the answer, at least for the case of Gyros A and B. There does not appear to be any other structural mechanism that would allow deflection of the IA about the spin axis and therefore actual misalignment of the IA as a cause of error is improbable.

The other aspect of the OA rotation-reaction torque phenomena that is yet subject to further examination, is the OA friction torque effect that occurs as the pivots bear against the jewels. It is well known in practice that static friction plays an important role in the determination of threshold and scale factor irregularities, and designs such as dithering jewels, taut wire or magnetic suspensions are introduced to reduce the amount of friction torque uncertainty. However,
a search of the literature indicates that friction torque as a possible deterministic error source in an SDFRI gyro has not been considered. At first glance, it would seem that static friction in a gyro feedback loop should not cause deterministic errors but should manifest itself as a random variation of rate error. Viscous friction could cause a deterministic error but this requires a relative velocity of the float with respect to the case which does not occur.

Thus it would appear that output axis friction has little to do with the phenomenon observed. To investigate this assertion, tests were performed on both a dithering jewel gyro (Gyro D) and a taut-wire suspended gyro (Gyro E). The test was performed on the dithering jewel gyro with the jewel on and off and the results are presented in Figure 7. The results shown in Figure 7 were taken from test data provided by the gyro manufacturer. Figure 8 shows the results of tests performed on the taut-wire suspended gyro. Both Figures 7 and 8 illustrate quite dramatically the effects of reducing the coefficient of friction. As can be seen in Figure 7, with the jewel on, the IA misalignment angle remains relatively constant for OA rates up to 20°/sec. However, there still appears to be a significant difference in the angle (approximately 25 arc seconds) between positive and negative OA rates. After 20°/sec the misalignment angle deviates rapidly to values which are below those reached without dithering. Figure 8 shows a similar characteristic but for lower input rates (the taut-wire suspended gyro was not designed for strapdown configuration).

Therefore, it appears that the rate error is related to friction in the float support in a way which is presently not explained.

IV. SOME PRACTICAL ASPECTS

The apparent IA misalignment error of a SDFRI gyro affects gyro test philosophy in the following way:

First, because of the deterministic character of the error, an additional term should be added to the performance model for an SDFRI gyro. Such a term might be

\[ \omega_E = D \omega_O^2 \]
where

\[(\omega_0)\]

\[\omega_E\] is the rate error caused by rotation about the OA \((\degree/hr)\)

\[D(\omega_0)\] is the drift coefficient \((\degree/hr)/(\degree/sec)^2\)

\[\omega_0\] is the angular velocity of the gyro case with respect to inertial space about the OA

Secondly, the addition of the term to the performance model will require that changes be made in test philosophy for certain dynamic tests such as centrifuge tests and tests to measure the effects of angular acceleration on a high speed rate table. There is certainly no question that centrifuge tests with the gyro's OA parallel to the rotation rate vector are invalid because of the high correlation of IA physical misalignment error to the apparent IA misalignment error and uncertainty. Any further investigation of the linearity of the \(g^2\) - sensitive coefficients to high \(g\) levels should definitely be carried out on a centrifuge with a counter-rotating platform. Unless the angular-acceleration sensitive term is very large for the accelerations of interest, (such is not generally the case) tests to measure the term on a high speed rate turn table should be avoided.

Another aspect that should be mentioned here is the possibility of an error torque caused by angular oscillation about the OA. The error results because of the rectification characteristics of the apparent IA misalignment angle. Qualitative tests at the CIGTF indicate that at very low frequencies (.01 to .07 Hz) there is significant rectification of the output waveform (See Figure 9-a). As the frequency is increased above .07 Hz, the tendency to rectify also decreases and the waveform looks like a badly distorted sinusoid with a high degree of phase shift (Approximately 120\(^\circ\)) (See Figures 9-b, c, d). At the frequency of approximately 1.5 Hz, the distortion and phase shift disappear and the gyro output signal represents a true picture of the input motion (See Figure 9-e). The output waveform then conforms with the gyro frequency response characteristic (output response to output motion).

Test conducted on a strapdown system also revealed a significant gyro error when the system was oscillated at .05 Hz about one or more
Figure 9a

Output Waveform versus Output Axis Oscillation
at Frequency of .05 Hz
Figure 9b
Output Waveform versus Output Axis Oscillation at Frequency of .08 Hz
Figure 9c
Output Waveform versus Output Axis Oscillation at Frequency of 0.15 Hz
Output Waveform versus Output Axis Oscillation at Frequency of .4 Hz
Figure 9e
Output Waveform versus Output Axis Oscillation at Frequency of 1.5 Hz
of the gyro's OA. The error was evident even though the system's gyros had dithering jewels. More detailed investigation revealed that the apparent misalignment error was very non-linear.

Since the forcing inputs to the gyro have been held to a more or less "static" nature, the potential dynamic characteristics have yet to be explored. Also, until the source mechanism is discovered and can be modeled, the use of simulation to investigate dynamic characteristics will be severely limited.

V. FUTURE EFFORT REQUIRED

As indicated in this paper the exact source mechanism of the apparent IA misalignment error has not been determined though, some possibilities have been considered.

It has been shown that the error is related in some way to OA friction. Laboratory tests are being conceived to investigate the OA friction effects on different types of gyros. Computer simulation is being used wherever possible to further understand gyro phenomena when the gyro is exposed to forcing functions not easily obtained in the laboratory. Further tests to determine OA suspension and support dynamic characteristics are also being considered.

VI. CONCLUSIONS

Results of laboratory testing of inertial grade strapdown SDFRI gyros has revealed that a deterministic rate error is generated when the gyro is subjected to angular rates about the output axis. Tests further indicate that the rate error is related in some way to OA friction effects. This is evident from tests conducted on gyros where an attempt was made to reduce the OA friction coefficient by the use of dithering jewel, and taut wire suspension. Additional laboratory tests are being conducted at the CIGTF in order to identify the exact source of the error.
Results of laboratory testing of inertial grade strapdown single-degrees-of-freedom gyroscopes has revealed that a deterministic error, proportional to rate squared, is generated when the gyro is subjected to angular rates about the output axis (OA). Tests further indicate that the rate error is related in some way to OA friction effects. This is evident from tests conducted on gyro's where an attempt was made to reduce the OA friction coefficient by use of dithering jewel and taut wire suspension. Additional laboratory tests are being conducted at the Central Inertial Guidance Test Facility in order to identify the exact source of the error.
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