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STATISTICAL-PROBABLE MODELING OF RANDOM PROCESSES

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ABSTRACT
Operational and tactical calculations, as well as scientific and technical investigations, often involve consideration of random phenomena and processes which can be characterized by statistical parameters calculated as averages of observed (selected) values. The statistical probability modelling method for random processes suggested uses the Monte Carlo method as its mathematical base, and the article proceeds to describe its application to two practical examples, one of which is the calculation of the process involved in a missile closing a target; the other reproducing an actual noise process in accordance with the normal distribution law. The mathematical method described involves the use of an electronic computer to obtain the statistical evaluation of characteristics with required accuracy and to reproduce the probability models of the subjects under investigation using experimental data based on distribution laws and correlated ties in the elements of those subjects, without waiting for the development of an analytical theory and the disclosure of the physical content of those subjects. Orig. art. has: 8 formulas and 1 figure.

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STATISTICAL-PROBABLE MODELING OF RANDOM PROCESSES

A. S. Zhernenko

During operational-tactical calculations, as well as during the fulfillment of scientific-technical investigations it is often necessary to discuss random phenomena and occurrences.

Parameters of the movement of rockets and missiles, characteristics of a naval medium, elements of motion of a target – all these are values, which during observations (measurements) may acquire various discrete or continuous values.

In a random occurrence instead of the rigid cause of communication with a fixed complex of external conditions of its realization is only observed a statistical bond, which is revealed at a greater series of observations and is expressed in a probable form. Correspondingly characteristic feature of a random process appears in the fact, that it cannot be previously predicted, which of the multitude describing its functions will be observed at an alternate experiment.

Random values and processes are characterized by statistical parameters, which are calculated as an average of observed (selective) values.

A random value ξ is considered known, when all the values are known, values which it can accept (x_1, x_2, ..., x_n...), as well as probabilities of realizing each of these values ($P(x_1)$, $P(x_2)$,..., $P(x_n)$...). The exhaustive characteristic of a random value ξ serves either the function of probability distribution $F(x)$, or the density of probabilities $W(x)$.

An arbitrary function $\{x(t)\}$ can be represented as a sequence of an infinite multitude of random values $x(t_k)$ – instantaneous values of its realization in a discrete series of points of argument (t). Hence it follows, that statistical properties of a random process can be fully determined only by an infinite sequence of distribution functions. To obtain the mentioned sequence is in a majority of cases practically impossible. Therefore searches are conducted for methods of investigating random processes, allowing to confine oneself only to investigations of distribution functions of the first order. The method of statistical-probable modeling of random processes,[1] pertaining to their number, is based in its mathematical basis on the known Monte-Carlo method.

A statistical-probability model of a stationary random process is represented in form of an ensemble of discrete realizations, obtained on the basis of two statistical characteristics of the process: correlation function $\psi_x(\tau)$ and monodimensional distribution function $F_1(x)$.

The correlation function as a time averaged statistical parameter of stationary processes is equal to mathematical expectation of a product of two instantaneous values – $x(t)$ and $x(t + \tau)$, – shifted in time to interval τ. Being a statistical average of second order, the correlation function represents a simplified reflection of time

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[1]This method was developed at the Cybernetics Inst. of the Acad. of Sc. Georgian SSR under the leadership of Dr. of Phys. Math. Sc. V. V. Chavchanidze and explained in detail in his reports "Method of random testing (the Monte-Carlo method)" transaction of the Physics Inst. of the Acad. of Sc. Georgian SSR, Vol. 3, pp. 107-122, "Statistical-probable modeling of physical processes and structures," transactions of the Inst. of Cybernetics of the Acad. of Sc. Georgian SSR, Vol. 1, pp. 13-18.

function x(t), containing no phase information and bound with the spectral density of the prower of the process S(f) by the Fourier transformation $\psi(\tau) = \int_{-\infty}^{\infty} S(f) e^{j\omega\tau} df$.

At a statistical-probability modeling of any random process the first its section, equal to the correlation interval,[1] is represented in form of a system of n random values, distributed in accordance to the given monodimensional function $F_1(x)$ and statistically bound between themselves in conformity with its correlation function $\psi(\tau)$ at $n = \frac{\tau_0}{\Delta t}$, where Δt — step between discrete values of the argument, corresponding to modeled sections of the process. All subsequent instantaneous realization values of the process from moment $t = \tau_0 + \Delta t$ and further are found with the aid of a drawing of random values by the rule, which appears to be an n-dimensional conditional distribution function and is determined by given $F_1(x)$ and $\psi(\tau)$ at known n of previous instantaneous values of that realization.

Considering the discrete nature of statistical-probability modeling of random processes and the greater volume of calculations, bound with the formation of a sequence of random values — instantaneous realization values, for mathematical modeling is usually used an universal or special electron digital computer.[2]

The below discussed two instances illustrated practical methods of applying the statistical-probability modeling method.

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[1]The correlation interval of the process is called the section of time τ_0, beyond the limits of which statistical bonds between sections of the process are practically inexistent.

[2]When studying noncomplex situations, when the drawing is made with the aid of random number tables, the numerical calculation can be carried out also with the aid of a key computer.

Calculation of Parameters of the Process of Approaching of a Rocket and Target

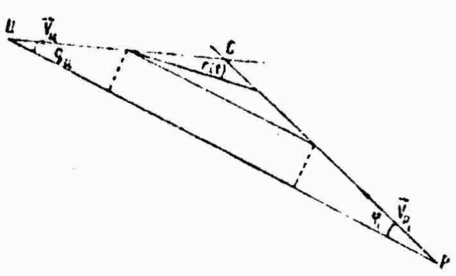
It is known,[1] that if data, obtained from the system of target indication to assure rocket – target encounter, are insufficiently accurate, and the rocket has a highly exact, but possessing a small radius of action control apparatus, then from the viewpoint of calculated evaluation of tactical effectiveness of the armament a decisive value acquire such kinematic parameters, as the distance between the rocket and target, bearing to the target, etc.

We will calculate the minimum distance between rocket and target for the following conditions:

the target moves rectilinearly at constant velocity V_t;

velocity of the rocket V_{pi} for every i-realization is unchanged;

movement of rocket and target occurs in a stationary plane, formed by vectors $\vec{V}_t$ and $\vec{V}_{pi}$.



The relative position of rocket and target at each moment of time (t) will be determined by the distance between same r(t), angles between vectors $\vec{V}_t$ and $\vec{V}_p$ and by the direction of rocket – target.

The value of the leading angle at the moment of firing (at t = 0) is found from the correlation

$$\varphi_0 = \arcsin\left(\frac{V_{u_i}}{V_p}\sin q_{u_i}\right), \tag{1}$$

where V_{t_i} and q_{u_i} – measured velocity values and course angle of the target; V_p – nominal value of rocket velocity.

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[1]See for example, A. S. Lokk. Missile control, GITTL, 1957, pp. 327 and 498.

Results of measuring target movement elements, as already mentioned, appear to be random values, which are distributed by the normal law

$$V_{ui} \to N(V_u, \sigma_u) \text{ and } q_{ui} \to N(q_u, \sigma_q).$$

where V_t and q_t – actual values of important magnitudes; σ_t and σ_q – mean quadratic velocity deviations and of course angle from average V_t, q_t.

At a normal distribution of random value X with a probability, close to unity, is valid ratio

$$\sigma_x = \frac{1}{3} \Delta X_{max}, \tag{2}$$

where ΔX_{max} – maximum possible deviation of random value from mean value $\bar{x}$; σ_x – corresponding standard or mean square deviation.

The actual value of rocket velocity V_{p_i} and angle between vector $\vec{V}_{p_i}$ and target direction at an i-discharge differ from calculated V_p, ϕ_0, because they appear to be random values, which are distributed by the normal law $V_{p_i} \to N\ (V_p,\ \sigma_p)$ and $\phi_i \to N\ (\phi_0,\ \sigma_\phi)$, where σ_p and σ_ϕ – mean square velocity deviations of course angle of the rocket from mean V_p and ϕ_0.

The problem about, what namely random value magnitude X is being realized at an alternate "theoretical experiment," is decided either by drawing with the use of random numbers table, or with the aid of a sensing element of pseudo random numbers.

The drawing with the use of random numbers[1] is based on a

[1]Detailed information about this is given in the above mentioned report of V. V. Chavchanidze "Method of random tests."

probability ratio, according to which any monodimensional random value X, transformed by a natural integral distribution law F(x), becomes uniformly distributed in the range of from 0 to 1.

When using tables of random numbers to assure independence of selective x_1 values the selection of random numbers for all situations is necessary to realize by one very same previously established law, beginning with any arbitrary selected number.

To calculate the distance between rocket and target at any moment of time (t) it is possible to utilize expression

$$r_i(t) = \sqrt{(r_0 - a_i t)^2 + (b_i t)^2}, \qquad (3)$$

where r_0 — distance between objects at moment t = 0,

$$a_i = V_{pi}\cos\varphi_i + V_u\cos q_i, \quad b_i = V_{pi}\sin\varphi_i - V_u\sin q_i.$$

The minimum distance between rocket and target at i-discharge is determined from conditions $\frac{dr_i(t)}{dt} = 0$ and is expressed by formula

$$r_{mi} = \frac{r_0 |b_i|}{\sqrt{a_i^2 + b_i^2}}. \qquad (4)$$

Carrying out a sufficiently great number of "theoretical discharges" M and having determined for each of them the value r_{m_1} it is possible to calculate the statistical probability of trapping the target at any given radius of reaction R_0 of an autonomous control system[1]

$$P^*(r_m \leq R_0) = \frac{m}{M}. \qquad (5)$$

[1]Under the reaction radius R_0 is understood the distance of noncontact detection of the target by the guidance system.

where m — number of "experiments," at which is fulfilled the inequality $r_m < R_n$.

Example. Initial data are given: $V_t = 8$ m/s, $q_t = 40°$, $V_p = 25$ m/s, $r_0 = 12{,}000$ m, $\Delta V_t = 1.8$ m/s, $\Delta q_t = 18°$, $\Delta V_p = 0.6$ m/s, $\Delta \varphi = 9.0°$.

<u>To find</u> values r_{m_1} for the given situation.

<u>Solution</u> 1. We determine the "measured" values of elements of target movement (V_{t_1} and q_{t_1}): using a table of random numbers,[1] we find, that $\beta_1 = 0806$ and $\beta_2 = 5603$; with the aid of integral functions table for the normal law N (0.1) we determine, that $Z_1 = F^{-1}[0.0806] = -1.40$ and $Z_2 = F^{-1}$ [0.5603] = 0.15; we calculate $V_{t_1} \to N$ (8; 0.6) and q_{t_1} N (40; 6) and we find, that V_{t_1} = 0.6) (-1.40) + 8.0 = 7.2 m/s, $q_{t_1} = 6.0 \cdot 0.15 + 40 = 40.9°$.

2. We calculate by formula (1) value of the lead angle

$$\varphi_0 = \arcsin\left[\frac{7{,}2}{25{,}0}\sin(40{,}9°)\right] = 10°50'.$$

3. We determine real values of velocity and course angle of the rocket V_{p_1} and ϕ_1 for which:

from the table of random numbers we select two more numbers, we will assume, $\beta_3 = 9138$ and $\beta_4 = 0285$; by table of integral function we determine the corresponding values $Z_3 = F^{-1}$ (0.9138) = 1.36 and $Z_4 = F^{-1}$ (0.0285) = -1.93; we calculate values of sought for parameters of rocket motion

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[1]See for example, A. K. Mitropol'skiy. — Technique of statistical calculations. Moscow, Fizmatgiz, 1961.

$$V_{p_i} = 0{,}2 \cdot 1.36 + 25{,}0 = 25{,}272 \text{ m/s},$$
$$\varphi_i = 3 \cdot (-1{,}93) + 10^\circ 30' - 5{,}0^\circ.$$

4. We calculate by formula (4) the value of minimum distance from the rocket to the target at an i-discharge

$$r_{m_i} = \frac{12\,000 \cdot 2{,}810}{\sqrt{7{,}842 + 986{,}75}} = 1060 \text{ m}.$$

Reproducing the Realization of Noise Process with Normal Law of Distribution

The physical fields spreading in real media, as a rule, are formed under the effect of many factors, having a random nature. As result of this also the fields represent random processes, the distribution of which is subject to the Gauss law.

At statistical-probability modeling of a section of a stationary random process with normal distribution law is possible to use the method of canonical disposition of functions.[1] Considering, that really existing noise processes have a zero mean value, canonical decomposition of corresponding random function can be written in form

$$x(t) = \sum_{k} B_k \cdot \nu_k(t), \tag{6}$$

where B_k – random noncorrelated values (decomposition coefficients), distributed by normal law N (0, σ_k); $\nu_k(t)$ – coordinate time functions, bound with B_k by ratio

$$\nu_l(t) = \frac{1}{\sigma_l^2} [\overline{B_l \cdot x(t)}]. \tag{7}$$

[1]Detailed data about this, for example, in the V. S. Pugachev book "Theory of Random Functions," GIFML, 1962.

If B_ℓ is presented in form of a linear combination of instantaneous values of functions x(t) in the corresponding points of the modeled section τ_0 and formula (7) is considered, then is possible to obtain recurrent ratios for the calculation of dispersions of coefficients B_ℓ and coordinate functions $\nu_\ell(t)$

$$\text{at } l = 1 \qquad \sigma_1^2 = \psi_x(t_1, t_2) \text{ и } \nu_1(t) = \frac{1}{\sigma_1^2}\psi_x(t, t_1),$$

$$\text{at } l = 2, 3, \ldots \quad \sigma_l^2 = \psi_x(t_l, t_l) - \sum_{k=1}^{l-1}\sigma_k^2[\nu_k(t_l)]^2$$

$$\nu_l(t) = \frac{1}{\sigma_l^2}\left[\psi_x(t, t_l) - \sum_{k=1}^{l-1}\sigma_k^2\cdot\nu_k(t)\nu_k(t_l)\right], \qquad (8)$$

where $\psi_x(t_1, t_2)$ – correlation function of the process.

Practical application of the described method of modeling instantaneous values of realizing a random function will be examined for $\frac{\tau_0}{\Delta\tau} = 3$, where Δt – value of the step on the axis of time between discrete values of the argument in section points of the process. At first are made auxiliary calculations:

1. Calculated are values of elements of a triangular correlation matrix $|\psi_x(\tau)| = \begin{vmatrix} \psi_x(0), & \psi_x(\Delta t), & \psi_x(2\Delta t) \\ & \psi_x(0), & \psi_x(\Delta t) \\ & & \psi_x(0) \end{vmatrix}$

2. By recurrent ratios (8) are calculated three values σ_ℓ^2 and the corresponding to the correlation matrix number of elements of triangular matrix $||\nu_\ell(t)||(\ell = 1, 2, 3)$.

The obtained values are used during the calculation of three instantaneous values of function x(t).

1. By the selected method (with the aid of table of random numbers or a special generator) are obtained three groups of random numbers (12 numbers in every group), uniformly distributed in the interval (0, 1).

2. Calculated are three random numbers γ, distributed by normal law $N(0,1)-\gamma_j=\sum_{i=1}^{12}\beta_i-6$ (at $j=1, 2, 3$).

3. Determined are values of canonical decomposition coefficients $B_1 = \sigma_1 \cdot \gamma_1$; $B_2 = \sigma_2 \cdot \gamma_2$; $B_3 = \sigma_3 \cdot \gamma_3$, where σ_1, σ_2, σ_3 – previously calculated standard deviations of random values of coefficients B_ℓ at $\sigma_\ell = \sqrt{\sigma_l^2}$.

4. During the multiplication of elements of a triangular matrix $\|\nu_l(t_k, t_l)\|$ into corresponding values B_ℓ is obtained a triangular matrix $\|\nu_l(t_k, t_l) \cdot B_l\|$.

5. Instantaneous values of realizing function x(t) on section τ_0 is obtained by adding by columns of elements of the triangular matrix

$$\begin{aligned} x(t_1) &= B_1 \cdot \psi_x(0) \\ x(t_2) &= B_1 \cdot \psi_x(\Delta t) + B_2 \cdot \psi_x(0) \\ x(t_3) &= B_1 \cdot \psi_x(2\Delta t) + B_2\psi_x(\Delta t) + B_3 \cdot \psi_x(0). \end{aligned}$$

The described mathematical method allows with the aid of a TSVM[1] to obtain statistical evaluations of characteristics of needed accuracy and to reproduce probability models of investigated objects on the basis of experimental data about laws of distribution and correlation bonds in the elements of these objects, not expecting the formulation of an analytical theory and revealing their physical nature.

[1]Since the number of investigated realizations is limited only by rapid action and "memory" volume of the machine.