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NARROWBAND INTERFEROMETER IMAGING

N. M. Tomljanovich
H. S. Ostrowsky
J. F. A. Ormsby

NOVEMBER 1968

Prepared for

SPACE DEFENSE AND COMMAND SYSTEMS PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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Project 4966
Prepared by
THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF19(628)-5165

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FOREWORD

This technical report has been prepared by the MITRE Corporation, Department D-85, under Contract AF 19(628)-5165, Project 4966. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, L. G. Hanscom Field, Mass.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

HENRY J. MAZUR, Colonel, USAF
Director, Space Defense & Command Systems Pgm Ofc
Deputy for Surveillance & Control Systems

ABSTRACT

The use of narrowband interferometry at small bistatic angles to obtain two and three dimensional images as scattering center plots is demonstrated. Considerations on resolution and ambiguity for range and cross range are covered for both ideal and actual conditions. The methods utilize Fourier transform phase from Doppler resolved scattering center returns at each site.

TABLE OF CONTENTS

		Page
SECTION I	INTRODUCTION	1
SECTION II	THE IDEAL CASE	3
SECTION III	SOME CONSIDERATIONS CONCERNING ASSUMPTIONS	8
SECTION IV	AMBIGUITY AND ERROR CONSIDERATIONS	14
SECTION V	CONCLUSIONS	21

SECTION I

INTRODUCTION

This preliminary investigation will try to examine how a narrowband interferometry system, operating at small bistatic angles, and employing its accurate phase measurement capability, can be used to obtain two or three dimensional plots of an array of scattering centers located in space, without "a priori" knowledge of the motion of the array.

The approach presented can be summarized briefly as follows: once the system has resolved the scattering centers in Doppler, the phase difference information between any two resolved centers at two or more stations is used to obtain the magnitude and direction of the relative position vector between the pair.

In this report, we shall first consider the "ideal case" and see whether for this perfect situation the measured quantities can be used to obtain the relative position of each center of the array.

For the "ideal case," the following set of assumptions are made:

1) each scattering center is resolved in Doppler, and the "modulus" Doppler maps are homothetic (this term used to indicate similarity of shape) from site to site,

2) the atmosphere is homogeneous and non dispersive,

3) the phase difference between resolved Doppler peaks is the phase difference between scattering centers, due only to their relative position.

In the next section, we examine in detail these assumptions and their limitations, and consider the difficulties that might arise if the assumptions were not fulfilled.

The "phase ambiguity", neglected so far, is then investigated and it is shown that it could be resolved if the interferometer can be operated at two different frequencies.

The report will also discuss how range and cross range resolution of the obtained plots is affected by errors in the measurement of phase.

Finally, the conclusion summarizes the feasibility of the interferometry system in obtaining three-dimensional images of arrays of scattering centers, which for high frequency scattering is the way that typical space objects look.

SECTION II

THE IDEAL CASE

In this section the "ideal case" is investigated by postulating the set of assumptions stated in the introduction. The justification of such assumptions is postponed to another section.

Under such an ideal situation, let us consider a typical pair of scattering centers A and B of the array and let us observe them with a two-site small-angle narrowband interferometry system (Figure 1).

If site S is active (transmitting and receiving) and S' a passive (only receiving) system, the phases associated with each Doppler peak, at each site are

$$\phi_A = \frac{2\pi}{\lambda}\left(2\,\vec{r}_A \cdot \hat{r}\right) + \delta_{SAS} \tag{1}$$

$$\phi_B = \frac{2\pi}{\lambda}\left(2\,\vec{r}_B \cdot \hat{r}\right) + \delta_{SBS} \tag{2}$$

$$\phi_A' = \frac{2\pi}{\lambda}\left(\vec{r}_A \cdot \hat{r} + \vec{r}_A' \cdot \hat{r}'\right) + \delta_{SAS'} \tag{3}$$

$$\phi_B' = \frac{2\pi}{\lambda}\left(\vec{r}_B \cdot \hat{r} + \vec{r}_B' \cdot \hat{r}'\right) + \delta_{SBS'} \tag{4}$$

where the δ's are the atmospheric phase contributions along each propagation path. At each site, the measured phase difference between the two resolved Doppler peaks A and B, is

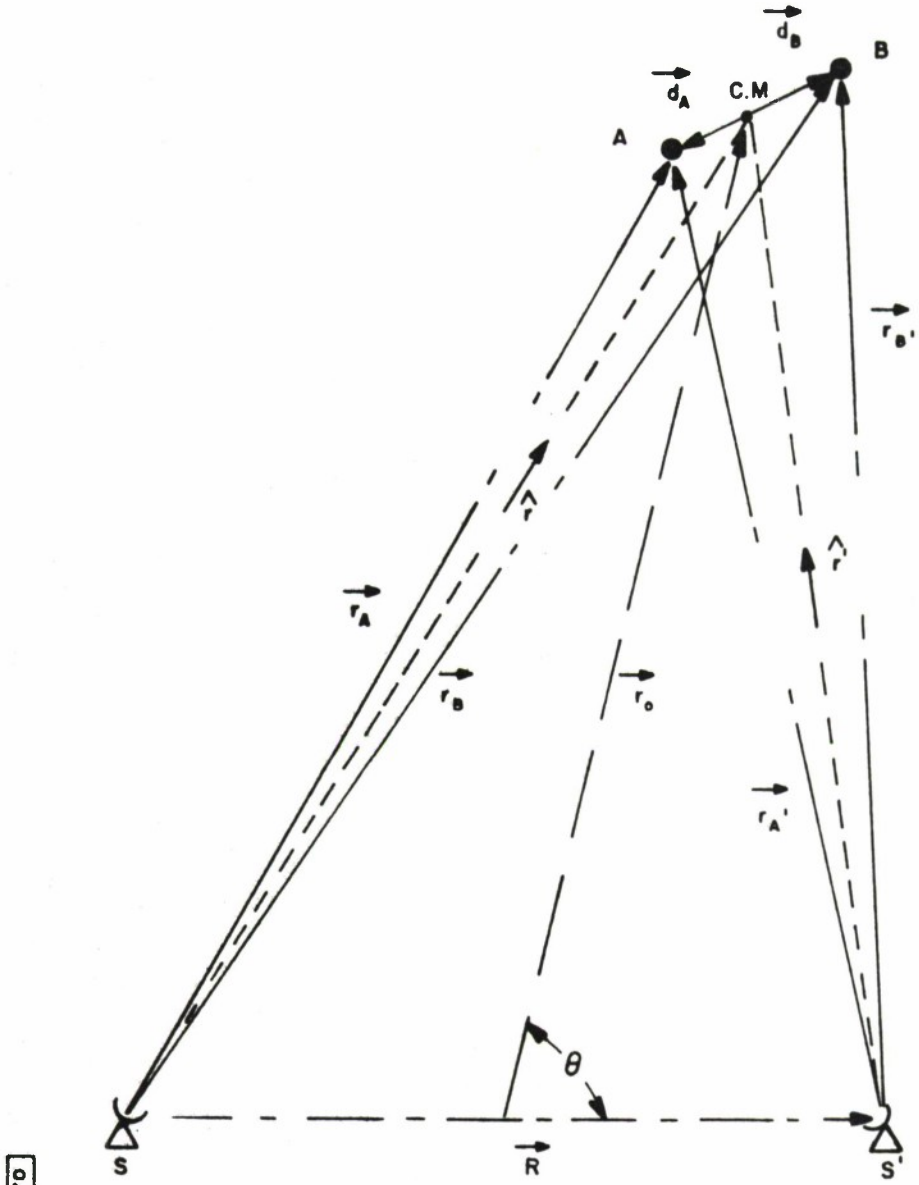

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Figure 1. Geometry for a Two-Site Interferometer

$$\alpha = \Delta\phi_{BA} = \phi_B - \phi_A = \frac{4\pi}{\lambda}\left(\vec{r}_B - \vec{r}_A\right) \cdot \hat{r} + \underbrace{\delta_{SBS} - \delta_{SAS}}_{0} \tag{5}$$

$$\alpha' = \Delta\phi_{BA}' = \phi_B' - \phi_A' = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \left(\hat{r} + \hat{r}'\right)\right] + \underbrace{\delta_{SBS}' - \delta_{SAS}'}_{0} \tag{6}$$

Let us consider the two quantities

$$\alpha' \text{ and } \beta \equiv \alpha - \alpha'$$

$$\alpha' = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \left(\hat{r} + \hat{r}'\right)\right] = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \vec{u}_r\right] \tag{7}$$

$$\beta = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \left(\hat{r} - \hat{r}'\right)\right] = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \vec{u}_\ell\right] \tag{8}$$

since $\vec{u}_r$ and $\vec{u}_\ell$ are orthogonal vectors, the above equations then indicate that α' and β are linearly proportional to the projections of the unknown quantity $\vec{r}_B - \vec{r}_A$, the quantity which we seek to determine, along two orthogonal vectors and thus one could, in principle, find the vector $(\vec{r}_B - \vec{r}_A)$, if α' and β are known.

To complete our investigation, the direction and magnitude of the two orthogonal vectors $\vec{u}_r$ and $\vec{u}_\ell$ must be established.

Using Figure 1, where r_o is the distance of the center of mass or center of rotation of the target, relative to the center of the interferometer, and θ is the elevation angle of the vector $\hat{r}_o$ and the two equations:

$$\vec{u}_r = \hat{r} + \hat{r}' \tag{9}$$

$$\vec{u}_\ell = \hat{r} - \hat{r}' \tag{10}$$

one can see that for $R \ll r_o$, or neglecting terms of order $\frac{R^2}{r_o^2}$ and higher,

$$\vec{u}_r = 2\,\hat{r}_o \tag{11}$$

$$\vec{u}_\ell = \frac{R}{r_o} \sin\theta\, \hat{u}_\ell \tag{12}$$

where the direction of $\vec{u}_\ell$ is perpendicular to $\hat{r}_o$. Hence the measured quantities α' and β are

$$\alpha' = \frac{4\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \hat{r}_o\right] \tag{13}$$

$$\beta = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \hat{u}_\ell\right] \frac{R \sin\theta}{r_o} \quad . \tag{14}$$

The range and cross-range distances of the two scattering centers A and B are then obtained by inverting the above equations:

$$\mathcal{R}_{BA} = \left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \hat{r}_o\right] = \frac{\alpha'}{4\pi}\lambda \tag{15}$$

$$\mathcal{L}_{BA} = \left[\left(\vec{r}_B - \vec{r}_A\right) \cdot \hat{u}_\ell\right] = \frac{\beta}{2\pi}\,\frac{r_o}{R \sin\theta}\,\lambda \tag{16}$$

In the case that S and S' are both active systems (transmitting and receiving in sequence), the following equations, analogous to Equations 5 and 6, can be derived:

$$\alpha = \Delta\phi_{BA} = \frac{4\pi}{\lambda}\left(\vec{r}_B - \vec{r}_A\right)\cdot \hat{r} \tag{17}$$

$$\alpha'' = \Delta\phi_{BA}' = \frac{4\pi}{\lambda}\left(\vec{r}_B - \vec{r}_A\right)\cdot \hat{r}' \tag{18}$$

and the two calculated quantities, related to the projection of unknown vector $\vec{r}_B - \vec{r}_A$ along the two orthogonal basis vectors, are

$$f = \frac{\alpha + \alpha''}{2} = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right)\cdot \vec{u}_r\right] = \alpha' \tag{19}$$

$$f' = \frac{\alpha - \alpha'}{2} = \frac{2\pi}{\lambda}\left[\left(\vec{r}_B - \vec{r}_A\right)\cdot \vec{u}_\ell\right] = \beta \tag{20}$$

For the "ideal case", we have here shown the relations between the phase differences at each site and the range and cross range location of a typical pair of Doppler resolved scattering centers.

From the above equations, it is clear that the interferometer does not have to maintain a coherent baseline, but needs only to be coherent at each site (except as it applies to the removal of orbital motion).

The results can be naturally extended to a three-site interferometer where each pair of sites obtains a cross range and a range plot. For a non-colinear interferometer, the two cross-range plots can be combined to obtain a planar plot of the scattering centers and furthermore together with the range plot, a three-dimensional plot of the illuminated scattering centers can be constructed.

SECTION III

SOME CONSIDERATIONS CONCERNING ASSUMPTIONS

The preceding formalism was worked out for an "ideal case" under a set of assumptions set forth at the beginning of Section I. It is time now to take a careful look at these assumptions, to consider their validity and their necessity to the results obtained, and in particular to see if there are any unstated or hidden assumptions which are necessary for the success of this "ideal" solution. Let us examine the numbered assumptions as given on Page 1.

Both parts of assumption (1) are essential in regard to mapping the locations of all scattering centers. Any scattering centers which are not resolved in Doppler will interfere with one another, so that the argument corresponding to this unresolved peak in the modulus of the "Doppler map" function will not be simply related to the position of its supposed "scattering center". This is no serious problem with respect to body-motion determination, since we need utilize only those scatterers which are well resolved. But for scatterer mapping, if the number of scattering centers is of the order of the number of Doppler resolution cells across the target, then it is quite likely that at any given time the orientation may be such as to place two or more scatterers in the same resolution cell. The only sure defense against this problem is to decrease the size and increase the number of the Doppler cells; unfortunately, this is not a practical solution since it would require an increase in the integration time, which is unacceptable due to a condition (to be developed shortly) restricting the length of the integration time. Perhaps a possible remedy is to examine sequences of Doppler maps covering a period of time, and to choose to process further only those maps which seem not to contain merged peaks.

Equally necessary is the requirement that the moduli of the Doppler maps be homothetic from site to site. This assures that we are able to associate particular peaks in one map with the corresponding peaks in the other maps, in order to compare their phases. If this association cannot be made of course the entire process in unavailing; since there is no getting around it, we must assume that somehow or other the association can be made.

Assumption (2) can in essence be removed completely. Because of the quite small bistatic angle (at the target) between sites atmospheric inhomogeneities usually make no significant contribution to phase differences, and may be discounted. And since we are talking about narrowband interferometry (bandwidth approximately 1 kHz) we need not pay any attention to atmospheric dispersion. But if we are using dual frequencies, both narrowband and separated by ~50 MHz, dispersion may be a problem. Some consideration should be given to the effects of dispersion on multi-frequency work.

Assumption (3) is the one that requires the most careful consideration because under it are subsumed several distinct points. First of all, it presupposes that we are not dealing with unresolved scatterers, but this has already been postulated under assumption (1). Secondly, it requires that all of the scattering centers must cause the same intrinsic phase change upon scattering. Generally, this requirement is physically unreasonable since the scattering centers may be of different sorts and will be located in different orientations. It is more reasonable to require only that the intrinisic phase shift of a given scatterer appear the same at all receiver sites; this will probably be true most of the time. With this assumption it may be possible, using dual frequencies, to extract the intrinisic phase shifts from the arguments of the Doppler map, at least for the term involving differences of phase differences.

Finally, these other requirements being postulated, there remains the question whether the theory tells us that the difference in argument of the Doppler map corresponding to two different peaks in the map's modulus is in fact determined only by the relative positions of the respective scattering centers. The answer (as we shall see) is yes, provided certain further requirements are met. Consider the configuration shown in Figure 1. By tracing the path of a radar pulse from S to A and back to S, and similarly from S to B and back to S, it is not hard to see that if all the previously discussed conditions are met, then the total signal received back at S has the form

$$S = e^{j\phi_o}\left[H_A e^{-\frac{4\pi}{\lambda} j\hat{r}\cdot\vec{d}_A} + H_B e^{-\frac{4\pi}{\lambda} j\hat{r}\cdot\vec{d}_B}\right] \tag{21}$$

where ϕ_o is some initial phase constant and H_A and H_B represent the "strengths" of the respective scattering centers.

This result is for one pulse only. To see how the received signal varies from pulse to pulse, consider the time interval $t_o - \frac{T}{2} < t < t_o + \frac{T}{2}$. If T is sufficiently small (what this means will be examined shortly) we can expand $\hat{r}(t)$ and $\vec{d}(t)$ in Taylor series about t_o and keep only the first two terms. For $\hat{r}$ we write

$$\hat{r}(t) \simeq \hat{r}(t_o) + (t - t_o)\,\vec{\omega}_T(t_o) \times \hat{r}(t_o)\,, \tag{22}$$

where $\vec{\omega}_T$ is the angular velocity of the radar line-of-sight to the target. In most cases $\vec{\omega}_T$ is sufficiently small and slowly varying that we can with little error ignore higher order terms in the Taylor expansion.

For $\vec{d}_A$ we write

$$\vec{d}_A(t) \simeq \vec{d}_A(t_o) + (t - t_o)\,\vec{\omega}(t_o) \times \vec{d}_A(t_o)\,, \tag{23}$$

and similarly for $\vec{d}_B$. Here we are ignoring terms of order $(\omega \frac{T}{2})^2$, where ω is the angular velocity of the target's rotation. This is approximately equivalent to the assumption that $\omega \frac{T}{2} < 0.1$, or

$$T < 0.03\tau\,, \tag{24}$$

where τ is the period of the target's rotation. This is a condition on the length of the integration interval which may be used in the Doppler mapping procedure; if T is too large the effect is one of "smearing out" the Doppler map, due (physically) to rotation through too large an angle.

We must further assume that H_A and H_B are such slowly varying functions of scatterer aspect angle that they can be considered constant during the integration time. This assumption is generally reasonable, although there are usually some regions of aspect angle in which the "strength" of a scattering center will change abruptly (such as, to give an extreme example, when entering a shadow). Since there is no good way of dealing with such pathological cases, we might as well develop the remainder of the theory under an assumption that we may disregard them.

Now, using the above equations we can write the received signal as a function of time in the form:

$$S(t) \simeq H_A e^{j\phi_o} \exp\left\{ -\frac{4\pi j}{\lambda}\left[\hat{r}(t_o) \cdot \vec{d}_A(t_o) - (t - t_o)\,\hat{r}(t_o) \times \vec{d}_A(t_o) \cdot \vec{\Omega}(t_o)\right]\right\} \tag{25}$$

$$+ \text{ similar terms in } H_B \text{ and } \vec{d}_B$$

where

$$\vec{\Omega} \equiv \vec{\omega} - \vec{\omega}_T \ .$$

Taking the Fourier transform relative to t on $\left[t_o - T/2 \ , \ t_o + T/2\right]$ gives

$$\tilde{S}(f) \equiv \int_{t_o - T/2}^{t_o + T/2} S(t) e^{-2\pi jf(t - t_o)} dt \qquad (26)$$

or

$$\tilde{S}(f) = TH_A e^{j\phi_o} e^{-\frac{4\pi}{\lambda} j\hat{r}(t_o) \cdot \vec{d}_A(t_o)} \operatorname{sinc}\left[T(f - 2/\lambda \ \hat{r}(t_o) \times \vec{d}_A(t_o) \cdot \vec{\Omega}(t_o)\right] \qquad (27)$$

$$+ \text{ similar term in } H_B \text{ and } \vec{d}_B$$

A similar calculation can obviously be done for the bistatic receiver, and also for the case of more than two scattering centers; the extensions are obvious.

Notice that the modulus of $\tilde{S}$ has two sharp peaks at appropriate values of f, and that the difference in the arguments corresponding to these peaks is

$$\Delta\phi = \frac{4\pi}{\lambda} \hat{r}(t_o) \cdot \left[\vec{d}_A(t_o) - \vec{d}_B(t_o)\right], \qquad (28)$$

as required. However, it is important to note that a serious difficulty might occur here, due to the sidelobes of the sinc function causing mixing of the arguments. The magnitude of the first sidelobe of the sinc function is about one-fifth that of the central peak; this means that unless the "strengths" of all the scattering centers are of the same order of magnitude,

serious mixing of arguments could occur. However, further investigation seems to indicate that such mixed-up arguments can in principal be untangled to reveal the correct phase terms for individual scatterers, although the calculation may involve solution of coupled transcendental equations. Perhaps after all it would be better to add to the other assumptions a requirement that all the scattering centers have approximately equal "strengths". Note that if appropriate weighting is used during integration to reduce the sidelobe levels, this would lighten the necessity of the above requirement; it would also have the effect of relaxing the condition that $T < 0.03\tau$, perhaps to something more like $T < 0.05\tau$ or thereabouts. The exact improvements would of course depend on the type of weighting used.

SECTION IV

AMBIGUITY AND ERROR CONSIDERATIONS

Since the determination of the location of scattering centers by this approach depends on the phase difference measurements at each site, it is extremely important to be able to remove any ambiguity in such phase measurements.

The ambiguity in phase is the consequence of the fact that any phase difference is measured only up to a modulus of 2π , and hence the actual phase difference could be the measured value plus any integer number of 2π radians.

If the actual phase difference is ϕ , whereas the measured phase difference is ϕ_*, one can write

$$\phi = \phi_* + n\, 2\pi \tag{29}$$

where n is an unknown integer, which introduces the ambiguity.

If such ambiguity in phase is not resolved, it will introduce ambiguities in range and cross range, according to Equations 15, 16, which will make the whole approach inapplicable.

Using Equations 15, 16, relations between the actual and the apparent range or cross range distances for a typical pair of scattering centers are

$$\mathcal{R}_{BA} = \mathcal{R}_{BA*} + n\,\frac{\lambda}{2} \tag{30}$$

$$\mathcal{L}_{BA} = \mathcal{L}_{BA*} + m\,\frac{r_o}{R \sin\theta}\,\lambda \tag{31}$$

where the subscript "*" refers to the apparent quantities.

According to the above equations, the unambiguous range region is only $\lambda/2$, whereas the unambiguous cross range region is $(r_o/R \sin \theta)\lambda$.

Examining the ambiguity in cross range, it is quite clear that, since the unambiguous region is $(r_o/R \sin\theta)\lambda$, phase can be measured unambiguously for targets or scattering centers arrays whose size is of the order of $(r_o/R \sin \theta)\lambda$.

For a typical value of $r_o/R \sin \theta$ of the order of 30, the unambiguous cross range region is then $\sim 30\lambda$.

If it is assumed that the interferometry system itself can resolve regions of space in sectors with a cross range resolution of about 30λ , then the only ambiguous phase which enters is that which is used to obtain range.

Furthermore, it is not necessary to remove completely the ambiguity, but only to extend the unambiguous range region to that region which cannot be resolved by the interferometer without using phase information.

The extension of the unambiguous range region can be accomplished by operating the interferometry system at two different frequencies.

For a pair of Doppler resolved scattering centers, the apparent phase differences for two different frequencies f_1 and f_2 $(f_1 > f_2)$ are related to the apparent range distances for the pair by Equation 13.

$$\alpha_{1*}' = \frac{4\pi}{\lambda_1} \mathcal{R}^{(1)}_{BA*} = \frac{4\pi}{\lambda_1} \mathcal{R}_{BA} \tag{32}$$

$$\alpha_{2*}' = \frac{4\pi}{\lambda_2} \mathcal{R}^{(2)}_{BA*} = \frac{4\pi}{\lambda_2} \mathcal{R}_{BA} \tag{33}$$

where the right-hand equality is valid because of Equation 29 and Equation 30.

Therefore

$$\left[\alpha_{1*}' - \alpha_{2*}'\right] = \frac{4\pi}{\lambda_1} \mathcal{R}_{BA} - \frac{4\pi}{\lambda_2} \mathcal{R}_{BA} = \frac{4\pi}{c} \mathcal{R}_{BA} \left[f_1 - f_2\right] \quad (34)$$

$$= 4\pi \frac{\Delta f}{c} \mathcal{R}_{BA} = \frac{4\pi}{\lambda_\Delta} \mathcal{R}_{BA} \quad (35)$$

so that

$$\mathcal{R}_{BA} = \left[\frac{\alpha_1' - \alpha_2'}{4\pi}\right]_* \lambda_\Delta \quad (36)$$

Since $\left[\alpha_1' - \alpha_2'\right]_*$ is measured only modulus 2π , one can see that, operating at two frequencies, the unambiguous range region is extended to $\lambda_\Delta/2$ and it depends on the frequency difference Δf . If the desired unambiguous range region is $N\lambda$, a relation between the frequency difference Δ_f and one of the frequencies can be obtained by equating $\lambda_\Delta/2$ to the desired quantity; namely

$$\frac{\lambda_\Delta}{2} = N\lambda \quad (37)$$

or

$$\Delta f = \frac{f}{2N} . \quad (38)$$

For a general interferometer having an uncompressed pulse duration of τ seconds, a pulse compression ratio R_c , the range resolution obtained without phase information is:

$$\frac{c\ \tau}{R_c} , \quad (39)$$

where c is the speed of propagation of the pulse. Using two frequencies, the ambiguity in phase in such region can be resolved by using a maximum frequency shift Δf which, according to Equation (37) and the above desired unambiguous region, is

$$\Delta f = \frac{R_c}{2\tau} \quad . \tag{40}$$

Thus, in principle, using the interferometer at two different frequencies, it is possible to remove the ambiguity in range.

The report will next investigate the resolution capability of the interferometer in each of the two basic dimensions.

The cross range resolution depends almost entirely on how accurately one can measure phase.

From Equation 16, neglecting errors in the measurements of r_o, R and θ, one can establish a relation between cross range resolution $\delta \mathcal{L}_{BA}$ and the error in the difference of phase difference measurements at each site $\delta\beta$, which is:

$$\delta \mathcal{L}_{BA} = \frac{\delta\beta}{2\pi} \frac{r_o}{R \sin\theta} \lambda \tag{41}$$

As an example, for a typical set of values: $r_o = 500$ km, $R = 20$ km, at the zenith $\theta = 90^o$, if we assume the error in phase difference measurements at each site to be $\delta\alpha \sim 10^o$, then $\delta\beta = \delta\alpha - \delta\alpha' \simeq 20^o$ (at worst) and we get:

$$\delta \mathcal{L}_{BA} = \frac{20}{2\pi} 25\lambda \simeq 1.1\lambda \quad ; \tag{42}$$

a result which is quite acceptable.

Similarly, the range resolution $\delta \mathcal{R}_{BA}$, using the two frequencies to resolve the ambiguity, is related to errors in measuring the difference of phase differences at the passive site for the two frequencies and from Equation (36) is

$$\delta \mathcal{R}_{BA} = \frac{\delta(\alpha_1' - \alpha_2')}{2\pi} \lambda_\Delta , \tag{43}$$

where the "*" notation has been dropped since it is equal to the un "*" notation for the unambiguous region. Choosing the unambiguous range region $\lambda_\Delta/2$ equal to the unambiguous cross range $(r_o/R \sin \theta)\lambda$, one can see that their respective resolutions are about equal:

$$\frac{\delta \mathcal{R}_{BA}}{\delta \quad BA} = \frac{\dfrac{\delta(\alpha_1' - \alpha_2')}{2\pi} \dfrac{\lambda_\Delta}{2}}{\dfrac{\delta\beta}{2\pi} \dfrac{r_o}{R \sin \theta} \lambda} = \frac{\delta(\alpha_1' - \alpha_2')}{\delta\beta} = \frac{\delta(\alpha_1' - \alpha_2')}{\delta(\alpha - \alpha')} \simeq 1 . \tag{44}$$

However, from Equation 15 one can see that if phase was measured unambiguously the range resolution would be related to error in phase difference measurements by

$$\delta \mathcal{R}_{BA} = \frac{\delta\alpha'}{4\pi} \lambda \tag{45}$$

and range distances could be obtained to a much higher order of precision.

Indeed, in principle, a two frequencies system could resolve completely the ambiguity in the unambiguous range region if its range resolution is equal or better than the unambiguous range region of the one frequency system, or if

$$\delta \mathcal{R}_{BA} \leq \frac{\lambda}{2} \tag{46}$$

or

$$\frac{\delta(\alpha_1' - \alpha_2')}{4\pi} \lambda_\Delta \leq \frac{\lambda}{2} . \tag{47}$$

To achieve a higher order of precision in the calculation of range, the following condition is then imposed on $\delta(\alpha_1' - \alpha_2')$, the difference of phase differences for the two frequencies at the passive site S',

$$\delta(\alpha_1' - \alpha_2') \leq \frac{2\pi\lambda}{\lambda_\Delta} \tag{48}$$

As an example, setting the unambiguous range region equal to the unambiguous cross range region or

$$\lambda_\Delta = \frac{2r_o}{R \sin\theta} \lambda \quad . \tag{49}$$

the requirement on $\delta(\alpha_1' - \alpha_2')$ is

$$\delta(\alpha_1' - \alpha_2') \leq \frac{\pi R \sin\theta}{r_o} \tag{50}$$

For a typical value of $r_o/R \sin\theta$ of about 25 the requirement on $\delta(\alpha_1' - \alpha_2')$ is

$$\delta(\alpha_1' - \alpha_2') \leq 7.2^o \quad . \tag{51}$$

If the two frequencies system is unable to achieve such accuracy, the excellent range resolution of Equation 16 could still be obtained using a three frequencies system.

These are just some of the main considerations that must be explored when applying the above approach to a particular interferometry system.

By no means this is the only technique, using phase data from a small-angle narrowband interferometer that can be employed to obtain two or three-dimensional plots of scattering centers for an array.

From the computational point of view to improve on the accuracy of the approach, it might be better to use phase difference data at each site and integrate them for several aspects before combining them to get cross-range information.

SECTION V

CONCLUSIONS

A study was made on the use of small-angle narrowband interferometry phase data for obtaining two (or three) dimensional plots of Doppler resolved target scattering centers. In all cases, it is accepted that individual scattering centers are resolved and can be identified at each interferometer site using the Doppler maps.

The study is divided into first showing feasibility under a set of assumptions constituting an ideal situation, and next considering the difficulties that in a practical case may arise due to relaxations of the assumptions. Details such as problems associated with the Doppler processing, removal of phase ambiguities via a dual frequency capability, the effect of phase errors and intrinsic phase differences were examined. By measuring phase differences between resolved Doppler peaks at two different sites and at two different frequencies a two-site narrowband interferometer can obtain a two-dimensional plot of the location of all scattering centers.

In addition, of course, with two pairs of sites (e.g. giving a three-site short-baseline radar interferometer) the non-planar two-dimensional plots from all scattering centers allows a three-dimensional description. No additional information comes from the third pair of sites in the three site interferometer. The motion of scattering centers along edges is not critical because of the short baseline of the system.

Since general space objects under high frequency electromagnetic illumination resemble arrays of scattering centers, this report serves the purpose of indicating the feasibility of a general interferometer system in determining a three-dimensional image of such arrays.

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13. ABSTRACT

The use of narrowband interferometry at small bistatic angles to obtain two and three dimensional images as scattering center plots is demonstrated. Considerations on resolution and ambiguity for range and cross range are covered for both ideal and actual conditions. The methods utilize Fourier transform phase from Doppler resolved scattering center returns at each site.

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