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A REVIEW OF THE ANALYTICAL METHODS APPLIED TO THE SEPARATED TURBULENT BOUNDARY-LAYER PROBLEM

J. P. Wallace

ARO, Inc.

November 1968

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*Consultant

FOREWORD

The work reported herein was sponsored by Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 65401F.

The results of the research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, under Contract F40600-69-C-0001. The research was conducted from June 5 to September 1, 1967, by Dr. J. P. Wallace of the Tennessee Technological University, College of Engineering, Cookeville, Tennessee, who was a consultant to ARO, Inc. The report was submitted for publication on August 26, 1968.

This technical report has been reviewed and is approved.

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ABSTRACT

An investigation of the various parameters that affect separation was undertaken to determine if scale effects exist. For example, in incompressible flow a full, thick, turbulent boundary layer will separate more readily in an adverse pressure gradient than a thin boundary layer. The emphasis throughout is on the two-dimensional, forward facing step and compression corners and the shock wave boundary-layer interaction.

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NOMENCLATURE

C	Chapman-Rubesin constant
C_1	Velocity power-law constant of proportionality (Ref. 29)
C_I, C_F and C_{2a}	Crocco numbers of the initial, final, and local external flow field, respectively, $\sqrt{\left(\frac{\gamma - 1}{2} M^2\right) \Big/ \left(1 + \frac{\gamma - 1}{2} M^2\right)}$
C_p	Pressure coefficient, $(p_s - p_\infty)/q_\infty$ where $C_{p_{crit}}$ is the is the critical pressure coefficient for separated turbulent boundary layers

E	Boundary-layer velocity profile exponent function (Ref. 29)
$F(M_{2a})$	Mach number function (see Section 2.2)
H_i	Incompressible shape factor (δ^*/θ)
I_1, I_2	Integral parameters (see Section 2.2)
K	Correlation parameter for dividing streamline of a separated flow
ℓ_m	Mixing length, in.
M_{2a}	Local external Mach number
M_∞	Free-stream Mach number
m	Velocity profile exponent function (see Section 2.1)
N	Reattachment point pressure ratio correlation parameter
n	Velocity profile exponent
P	Mean pressure in the boundary layer, psia
p	Fluctuating component of the pressure in the boundary layer, psia
p_b	Base pressure, psia
p_o	Stagnation pressure, psia
p_r	Pressure at the reattachment point, psia
p_s	Pressure at the boundary-layer separation point, psia
p_∞	Free-stream pressure, psia
q_∞	Free-stream dynamic pressure, psia
Re_X	Reynolds number based on the length, X
Re_θ	Reynolds number based on the boundary-layer momentum thickness
U, U_∞ or u_1	Velocity external to the boundary layer and the free-stream velocity, respectively, ft/sec
u	Local velocity, within the boundary layer, ft/sec
u_τ, u_*	Equivalent frictional velocity, $\sqrt{\tau_w/\rho}$, ft/sec
$\overline{u^2}$	Square of the velocity fluctuations, $(ft/sec)^2$
v	Normal velocity component, ft/sec

X, Y	Local stream coordinate, in.
X′, Y′	Free-stream coordinate, in.
x_s	Location of profile relative to the forward facing step, in.
α	Angular change in the local flow-field direction, deg
γ	Ratio of specific heats
Δu	Velocity, defect, (u-U), ft/sec
δ	Boundary-layer thickness, in.
δ^*	Boundary-layer displacement thickness, in.
η, η_m, η_s	Nondimensionalized "Y" coordinate where η_m is the shifted coordinate and η_s is the dividing streamline coordinate (Ref. 21)
θ	Boundary-layer momentum thickness, in.
θ_I, θ_F	Initial and final momentum thickness, respectively, in.
ρ	Density, $lb_m/in.^3$
σ	Jet spreading parameter
τ_w	Friction value, $lb_f/in.^2$
ψ, ψ_D	Velocity ratio, ψ_D is the dividing stream velocity ratio

SECTION I
INTRODUCTION

In this report, an investigation of the various parameters that affect separation was undertaken to determine if scale effects exist. For example, in incompressible flow a full, thick, turbulent boundary layer will separate more readily in an adverse pressure gradient than a thin boundary layer (Refs. 1 and 2). The emphasis throughout is on the two-dimensional, forward facing step and compression corners and the shock wave boundary-layer interaction.

Early experiments of Bogdonoff, et al. (Ref. 3), were directed toward investigating the interaction of shock waves with tunnel wall boundary layers. In their experiments, the impinging shock strengths were varied while the free-stream Mach number was kept at three. Interactions were classified according to the impinging shock strength and the length of the interaction zone. The wall pressure distribution tended to show that as the shock strength increased, there was an increase in plateau length, and the point of coincidence of the reflected, induced shock moved farther from the wall.

Kuehn (Ref. 4) experimentally studied turbulent boundary-layer separation on curved surfaces, compression corners, and incident shock waves. His results show a Mach number effect on all models. At high Mach numbers, a strong influence of the lower Reynolds number on incipient separation was noted. For low Mach numbers and high Reynolds number, only a weak effect of Reynolds number was noted. At the same time, he reported that only the small separation zones were steady. The degree of unsteadiness increased as the separation zone increased.

Chapman, Kuehn, and Larson (Ref. 5) studied separation and noted that the effect of transition greatly influenced the geometry of the separation zone. They pointed out that the shock-laminar boundary-layer interaction is a free interaction, whereas the shock-turbulent boundary-layer interaction is not.

A number of authors, e.g., Refs. 6 and 7, have proposed that, based on experimental evidence, Reynolds number effects for the shock-turbulent boundary-layer interaction are not important to the separation problem except in the development of the boundary layer itself.

Attempts to analyze the separation zone in the shock-turbulent boundary-layer interaction have been less than satisfactory to this point. Thus far, none has yielded a complete solution. Of the analyses

reported, the flow field is generally divided into three regions: the region up to the separation point, the plateau region, and the reattachment region. These regions interact with one another and provide the boundary conditions for each other. Thus far, the complete solution is mathematically intractable; as a consequence, each regime is treated separately and the solutions superimposed. A summary of these regions is given in Section II.

SECTION II
ANALYTICAL DESCRIPTION OF A TURBULENT BOUNDARY-LAYER SEPARATION

2.1 PRESSURE RISE TO SEPARATION

Gadd (Ref. 8) derived an expression for the separation pressure by assuming a relationship between the geometry of the interaction lengths. Reasonable results were obtained for small interaction lengths and Mach numbers of 2 and 3. The experimental results deviate at large interaction lengths at the above Mach numbers, and the results at Mach 4 show considerable discrepancies.

Mager (Ref. 9) deduced an expression for the separation pressure coefficient from the results of a modified Dorodnietzyn-Stewartson Transformation by Van Le (Ref. 10) and a linearized Mach number relationship for the pressure rise across the shock. Experimental comparison indicated good results for the Mach number range from 1.7 to 3.0. In addition, this analysis predicts that shock separation will not occur for Mach numbers less than 1.2. Reynolds number effect is excluded from his analysis.

Honda (Ref. 11) used the incompressible auxiliary equation of Wiegardt, together with a transformation to incompressible form, to derive and numerically solve the pressure distribution in the initial pressure rise regime. His analysis is compared with the data of Gadd (Ref. 8) and Fage and Sargent (Ref. 12). For low ratios of local wall pressure to stagnation pressure, close agreement is obtained. At higher pressure ratios, that is, when the separation point is approached, the analysis exceeds the values of the experimental data. Since the analysis does not predict separation, the discrepancy may be due to the actual flow being separated and, thus, yielding the less severe pressure gradient indicated by the experimental data.

Erdos and Pallone (Ref. 13), using similarity arguments and an empirically determined correlation function, predict the wall pressure, separation and reattachment points, and the length of the interaction zone. Experimental comparison is with the data of Chapman, et al. (Ref. 5), Drougge (Ref. 14), and Sterrett and Emery (Ref. 15). The trend of the semiempirical theory agrees with the experimental data, but considerable scatter is observed. The separation pressure coefficient and the plateau pressure coefficient are plotted in the form $C_p Re_x^{1/10}$ versus Mach number, where C_p is the pressure coefficient and Re_x is the length Reynolds number to the location of separation. These parameters would necessitate knowing the length boundary-layer characteristics if the tunnel wall boundary layer is to be used in a shock-boundary-layer interaction study.

Several of the preceding authors have used a compressible-to-incompressible transformation of the boundary-layer equations and used the results of incompressible flow separation. Two difficulties are encountered in this approach. The first deals with the applicability of the boundary-layer equations when a severe longitudinal pressure gradient exists since this gradient will introduce large transverse pressure gradients. As a consequence, more general equations than these boundary-layer equations are needed (see Refs. 16 and 17). The second difficulty concerns the question of what is the correct method of transforming the horizontal length scale, assuming that the boundary-layer equations hold. Mager (Ref. 18) proposes a modified Stewartson transformation; whereas, Coles (Ref. 19) proposes a more general approach. The experimental verification of these transformations has not been carried out to date.

Recent works by Childs, et al. (Ref. 20) and Paynter (Ref. 21) have approached the problem by utilizing a control volume which joins the upstream undisturbed boundary layer with the assumed free interaction region, as shown in Fig. 1.

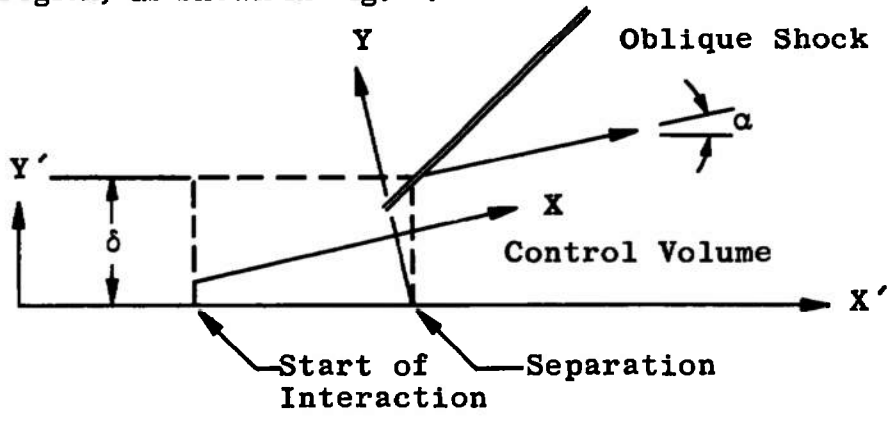

Fig. 1 Interaction Region Upstream of Separation

Using the criteria of Erdos and Pallone (Ref. 13) for the separation pressure, the free interaction flow conditions are determined through the application of mass and momentum conservation. It should be pointed out that the "Y" direction momentum equation is not satisfied in Refs. 20 and 21.

Lighthill's (Ref. 22) analytic study of disturbance effects in the boundary layer obtained from the Orr-Sommerfeld equation calculated an "inner viscous sub-layer"[1] which, in turn, allowed the upstream influence to be calculated. Based on the perturbation analysis, he concluded that for flows without separation:

1. the upstream influence decreases with Mach number, and
2. the upstream influence for the turbulent boundary layer decreases with increasing Reynolds number.

Although the above results of Lighthill are for small perturbations, Ray (Ref. 23) applied these results to a strong shock wave and derived an expression for the critical pressure rise to separation in the form

$$C_{p_{crit}} = 0.473\, Re_X^{-1/10} \left(1 + \frac{M_\infty^2}{5}\right)^{5/6} \left(M_\infty^2 - 1\right)^{-1/4} C^{2/3}$$

where

Re_X = length Reynolds number to location of interaction

M_∞ = free-stream Mach number

C = Chapman-Rubesin constant appearing in the temperature viscosity law

In Ray's analysis, as well as that of Erdos and Pallone (Ref. 13), the critical pressure rise is a one-tenth power of the Reynolds number, which indicates a very weak dependence of separation on this parameter.

---

[1]Lighthill considered the boundary layer to be composed of two regions: the "inner viscous layer" and the "outer viscous layer." In the "inner viscous layer," the perturbations to the viscous forces are of the same order as the perturbation to the inertial forces. In the "outer viscous layer," the perturbation to the viscous forces is much smaller than the perturbed inertial force.

Further evidence of the importance of the momentum deficient layer adjacent to the wall is given in the subsonic studies of Stratford (Ref. 24) and Townsend (Ref. 25). In these studies, the outer portion of the boundary layer is assumed to remain similar through the adverse pressure gradient, whereas in the inner layer the smaller inertia forces are such that the pressure gradient distorts the velocity profile. The agreement with low speed experiments is excellent.

A survey article by Cooke (Ref. 26), which deals with separated flows, tabulates a number of other formulas for calculating the separation pressure. All of these are empirical. For Mach number 3.0, the results of these empirical equations independent of Reynolds number are:

Chapman, Kuehn and Larson (Ref. 5)

$$\frac{p_s}{p_\infty} = 2.30$$

Gadd (Ref. 8)

$$\frac{p_s}{p_\infty} = 2.50$$

Mager (Ref. 9)

$$\frac{p_s}{p_\infty} = 2.01$$

The length Reynolds number must be known in applying the separation pressure formulations of Erdos and Pallone (Ref. 13) and Ray (Ref. 23). In attempting to determine the length Reynolds number, the boundary-layer data from Bell (Ref. 27) and Strike and Rippey (Ref. 28) were treated as though the boundary layer had developed on a flat plate, and an equivalent length Reynolds number was estimated from the theoretical analysis presented in Ref. 29, namely,

$$Re_\theta = \left(\frac{1+m}{2} E\, Re_X\right)^{\frac{1}{1-m}}$$

where

Re_θ = momentum thickness boundary layer

Re_X = length Reynolds number

$m = \frac{2n}{1+n}$

n = velocity profile exponent

$E = 2C_1^{-\left(\frac{2}{1+n}\right)}\left(\frac{(1+n)(1+2n)}{n}\right)^{-m}$

C_1 = tabulated constant in Ref. 29

Substituting this momentum thickness Reynolds number for the length Reynolds number in Ray's analysis (Ref. 23) and using the measured momentum thickness from Ref. 28, the following separation pressure ratios are obtained:

$$\frac{p_s}{p_\infty} = 1.905 \text{ for } p_o = 5.0 \text{ psia, } Re_\theta = 7.0 \times 10^3, \ \delta = 1.8 \text{ in.}$$

$$\frac{p_s}{p_\infty} = 1.710 \text{ for } p_o = 60 \text{ psia, } Re_\theta = 5.8 \times 10^3, \ \delta = 1.4 \text{ in.}$$

As indicated, these calculations lead to a reasonable estimate of the separation pressure, but the trend in the pressure ratios with boundary-layer thickness is contrary to the statements in Refs. 1 and 2 (see Section I). The assumption of an equivalent flat-plate-developed boundary-layer is questionable in this analysis. A check on this is through the calculated growth value assuming a linear profile[2] and the measured values from Refs. 27 and 28, which were separated by a distance of 21.25 in. For a free-stream stagnation pressure of 5.19 psia at Mach number 3.0,

$$\delta(\text{Ref. 28}) - \delta(\text{Ref. 27}) \sim \begin{array}{l}\text{experimental} = 0.61 \\ \text{calculated} = 0.48\end{array}$$

The historical development of the boundary layer is neglected in assuming a flat-plate-developed boundary layer. As pointed out by Clauser (Ref. 30) and Townsend (Ref. 1), the defect portion of the boundary layer is very slow to adjust to disturbances, whereas the wall region adjusts rapidly and is considered to be dependent on local conditions. It is the outer part that determines the local overall velocity profile (or parameter n). A clear demonstration of this phenomenon is shown in Fig. 2 (from Ref. 30) where a disturbance was introduced into the turbulent boundary layer and the return to similarity observed. The boundary layer profile has been successfully calculated for turbulent flow in the presence of a pressure gradient but only for an equilibrium boundary layer profile (that is, $\left(\delta \frac{dp}{dx} / \tau_w\right)$ = constant).

Any test should incorporate the determination that the profile is indeed fully turbulent. Thompson (Ref. 17) recommends that a two-parameter profile and the Clauser inner profile be used for this determination.

---

[2]The linear profile was used so that an explicit calculation could be made. In using Tucker's (Ref. 31) development equation, a closed development could not be obtained.

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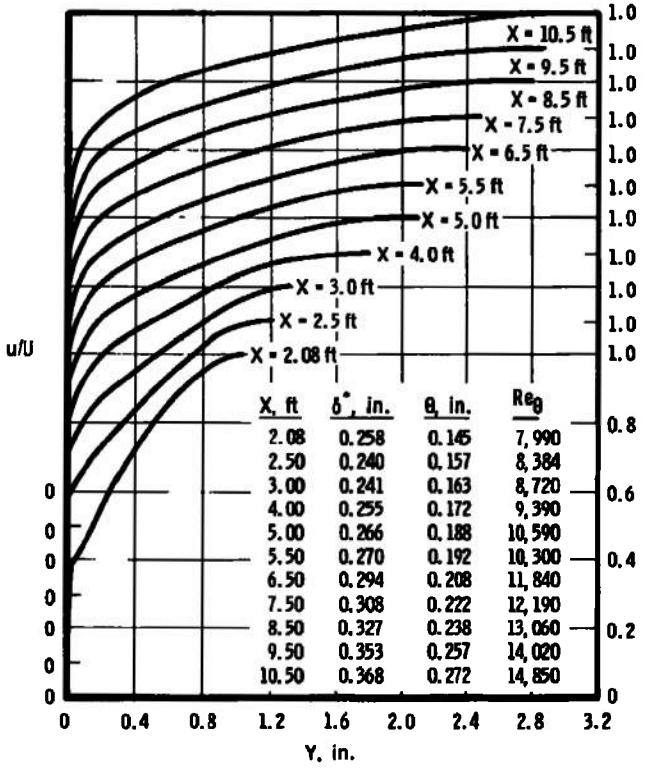
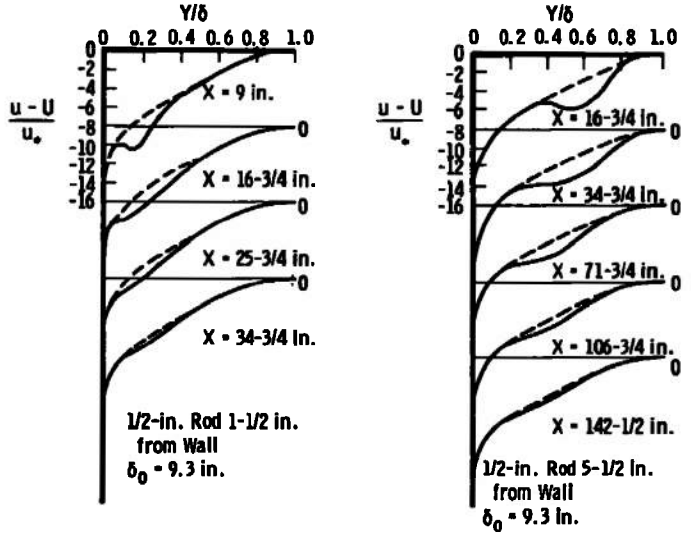
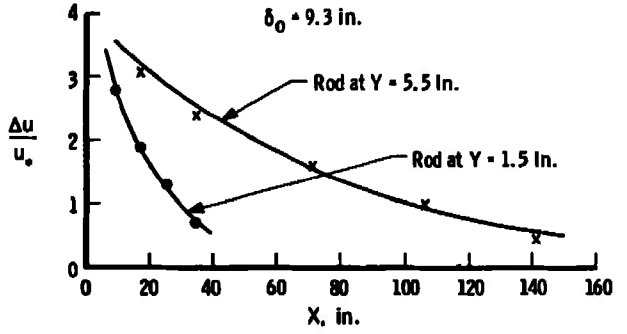


X, ft	δ^*, in.	θ, in.	Re_θ
2.08	0.258	0.145	7,990
2.50	0.240	0.157	8,384
3.00	0.241	0.163	8,720
4.00	0.255	0.172	9,390
5.00	0.266	0.188	10,590
5.50	0.270	0.192	10,300
6.50	0.294	0.208	11,840
7.50	0.308	0.222	12,190
8.50	0.327	0.238	13,060
9.50	0.353	0.257	14,020
10.50	0.368	0.272	14,850

a. Response of a Turbulent Boundary Layer to Wall Disturbances (Ref. 30, Fig. 14)



b. Response of a Turbulent Boundary Layer to Disturbances Introduced in the Layer (Ref. 30, Fig. 12)



c. Decay of Velocity Distortion in a Turbulent Boundary Layer (Ref. 30, Fig. 13)

Fig. 2 Turbulent Boundary-Layer Profile Disturbances

After it is established that a fully turbulent profile exists and an adverse pressure gradient is applied, the velocity profile becomes distorted, as shown in Fig. 3 from Coles (Ref. 32). In this region, the terms in the turbulent boundary-layer integral equations which are normally neglected are as follows:

$$-\frac{1}{\rho v^2}\frac{d}{dX}\int_0^\delta (P - p)\,dY \text{ and } \frac{1}{v^2}\frac{d}{dX}\int_0^\delta \overline{u^2}\,dY$$

These terms should be included as a first-order correction to the boundary-layer equations. An indication of the rate of change of $\overline{u^2}$, the turbulent intensity, with distance in subsonic step separation is shown in Fig. 4. These results were obtained by the author in a subsonic tunnel operating at a nominal velocity of 65 ft/sec with a turbulent boundary-layer thickness of 0.45 in. on the tunnel wall. The height of the step was 1.75 in., and the profiles were obtained at 2.1 and 4.1 in. upstream of the step. The functional form for these terms is not known and would have to be experimentally determined.

For the same reasons as specified above, the use of a shape factor value to predict the separation point is questionable.

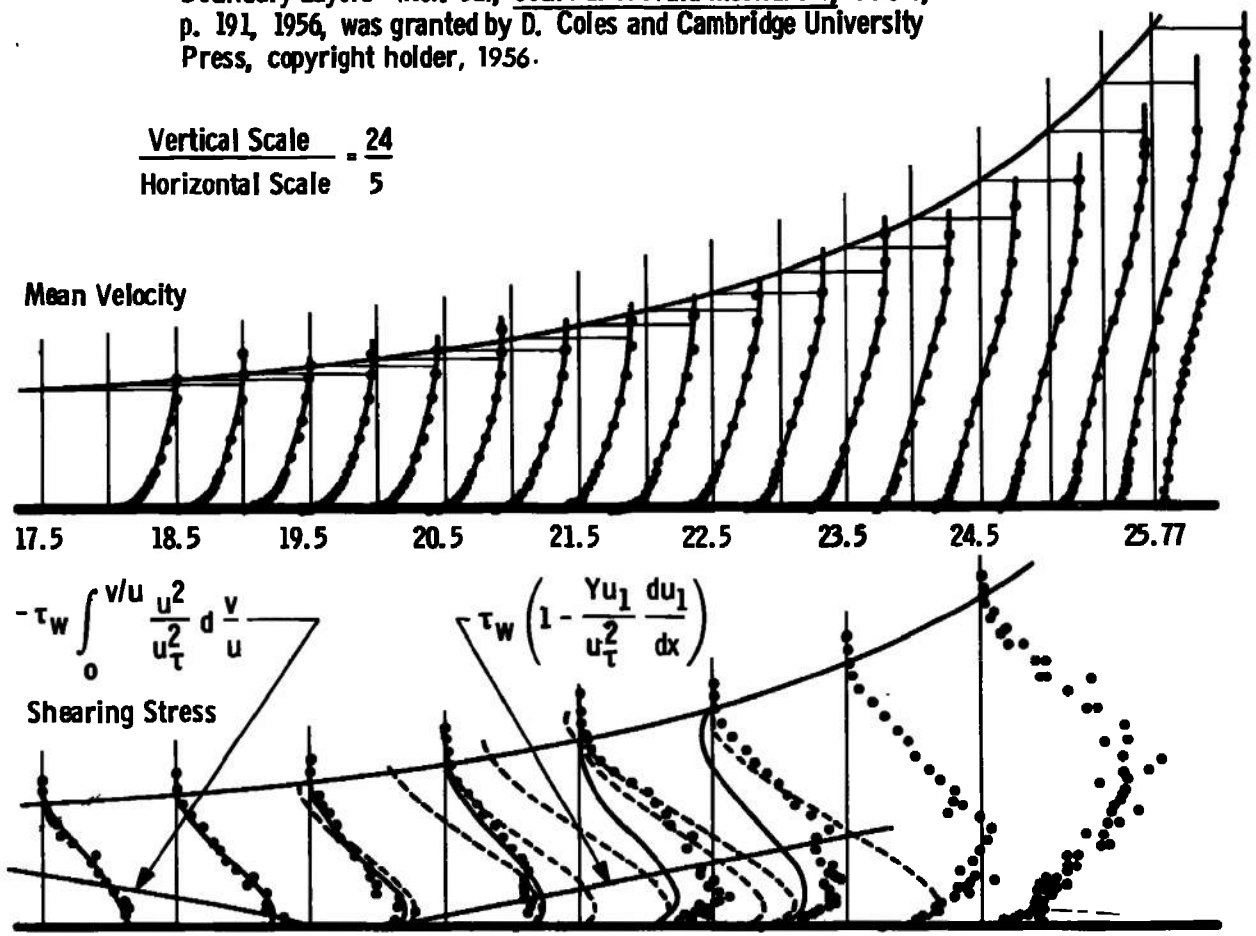
2.2 PLATEAU PRESSURE

In this region, the flow fields consist of the separated boundary layer and the reversed flow region. Experimental data (Ref. 5) show that a gradual rise in pressure takes place from the separation point to the plateau pressure over a distance on the order of one boundary-layer thickness. The separated boundary layer is assumed to behave like a constant pressure free jet (Refs. 7, 20, 21, and 33). This assumption neglects the pressure increase to the plateau level. Experimental evidence for this assumption is quoted in Ref. 21 in which measured velocity profiles did not differ appreciably from the constant pressure velocity profile. By using a modified form of the turbulent jet mixing theory of Korst, et al. (Ref. 34), Nash (Ref. 35), who improved on the method of Kirk (Ref. 36), developed a technique to account for the finite thickness of the boundary layer at separation.[3] This modified theory considers the separated layer as having originated at a distance upstream such that the mass and momentum flux at the separation point for the separated boundary layer and the equivalent jet are the same.

---

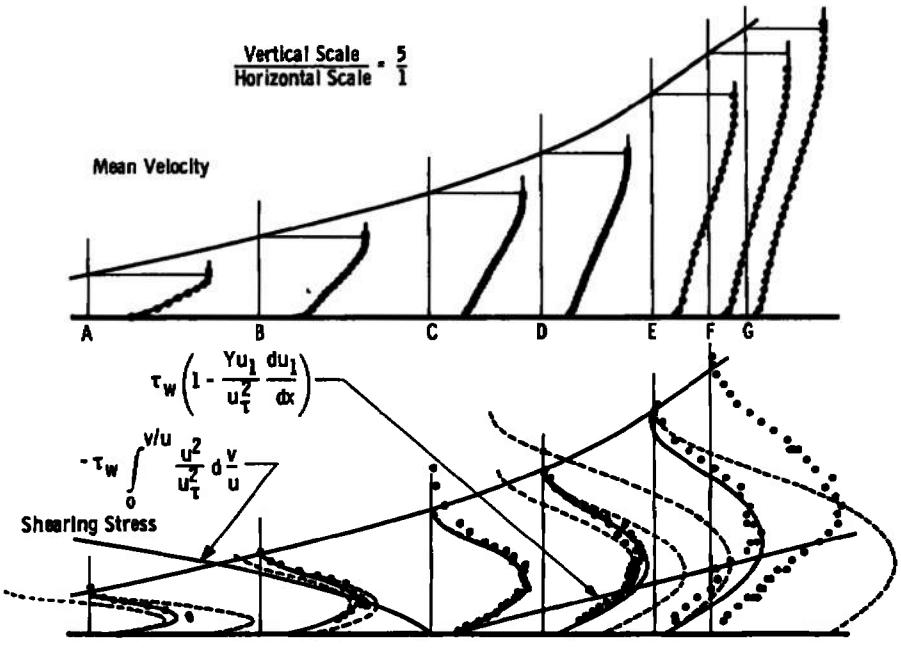
[3]Recent work by Hill (Ref. 37) has incorporated a "Y" shift as well as the "X" shift of Nash. The "Y" term is, in general, small and can be neglected in analyzing the shift.

Permission to reproduce the data in Figs. 3a, b, and c, below, taken from D. Coles' article, "The Laws of the Wake in Turbulent Boundary Layers" (Ref. 32), Journal of Fluid Mechanics, Vol. I, p. 191, 1956, was granted by D. Coles and Cambridge University Press, copyright holder, 1956.

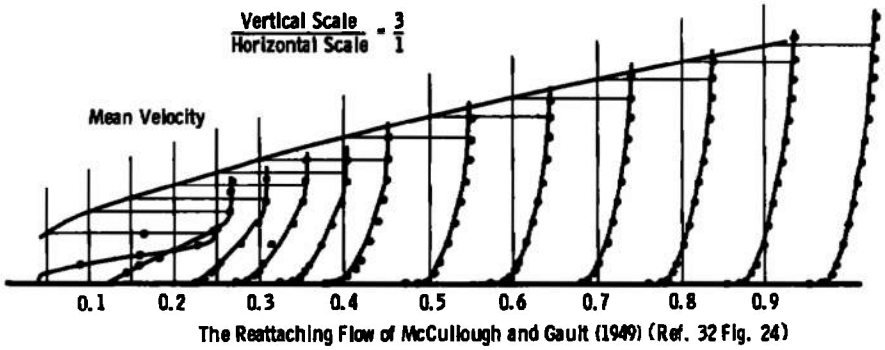


a. The Separating Flow of Schubauer and Klebanoff (1950) (Measured Values of Shearing Stress Have Been Reduced by 31 percent) (Ref. 32, Fig. 22)

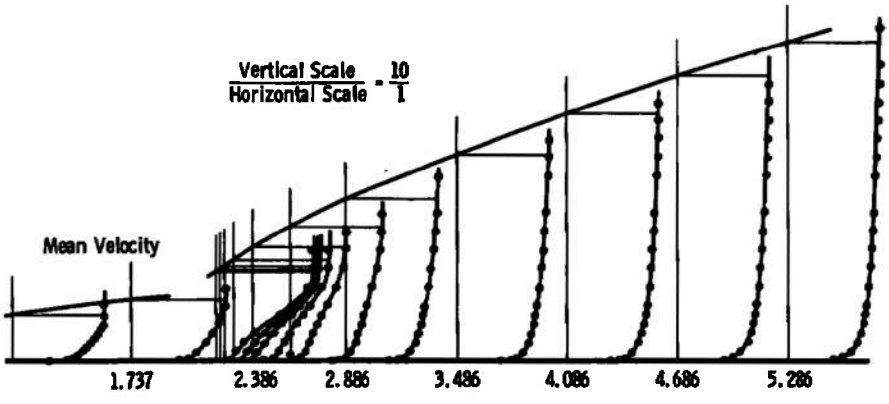
Fig. 3 Examples of Distorted Velocity Profiles Resulting from Adverse Pressure Gardients



b. The Separating Flow of Newman (1951) (Ref. 32, Fig. 23)

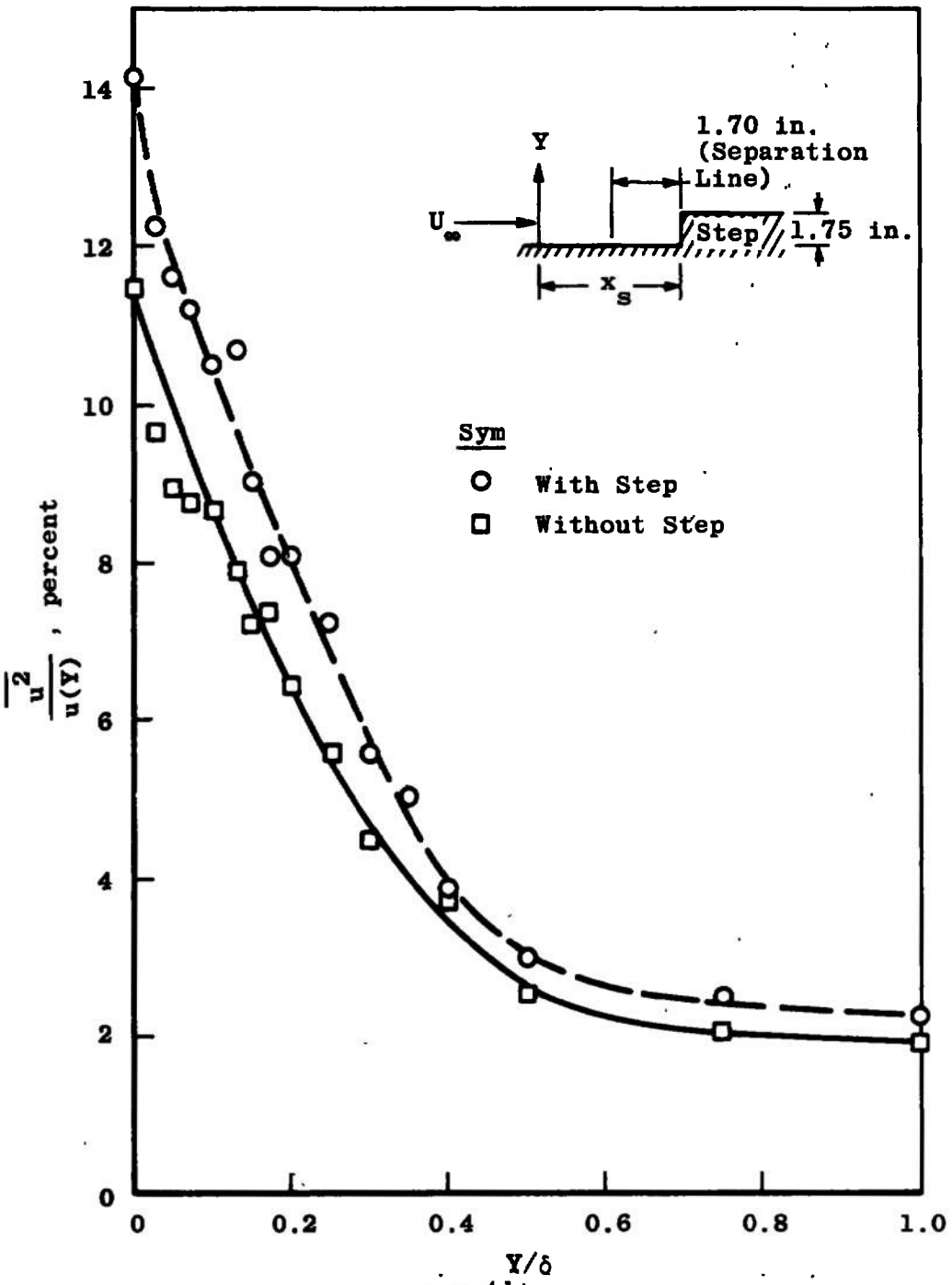


The Reattaching Flow of McCullough and Gault (1949) (Ref. 32 Fig. 24)



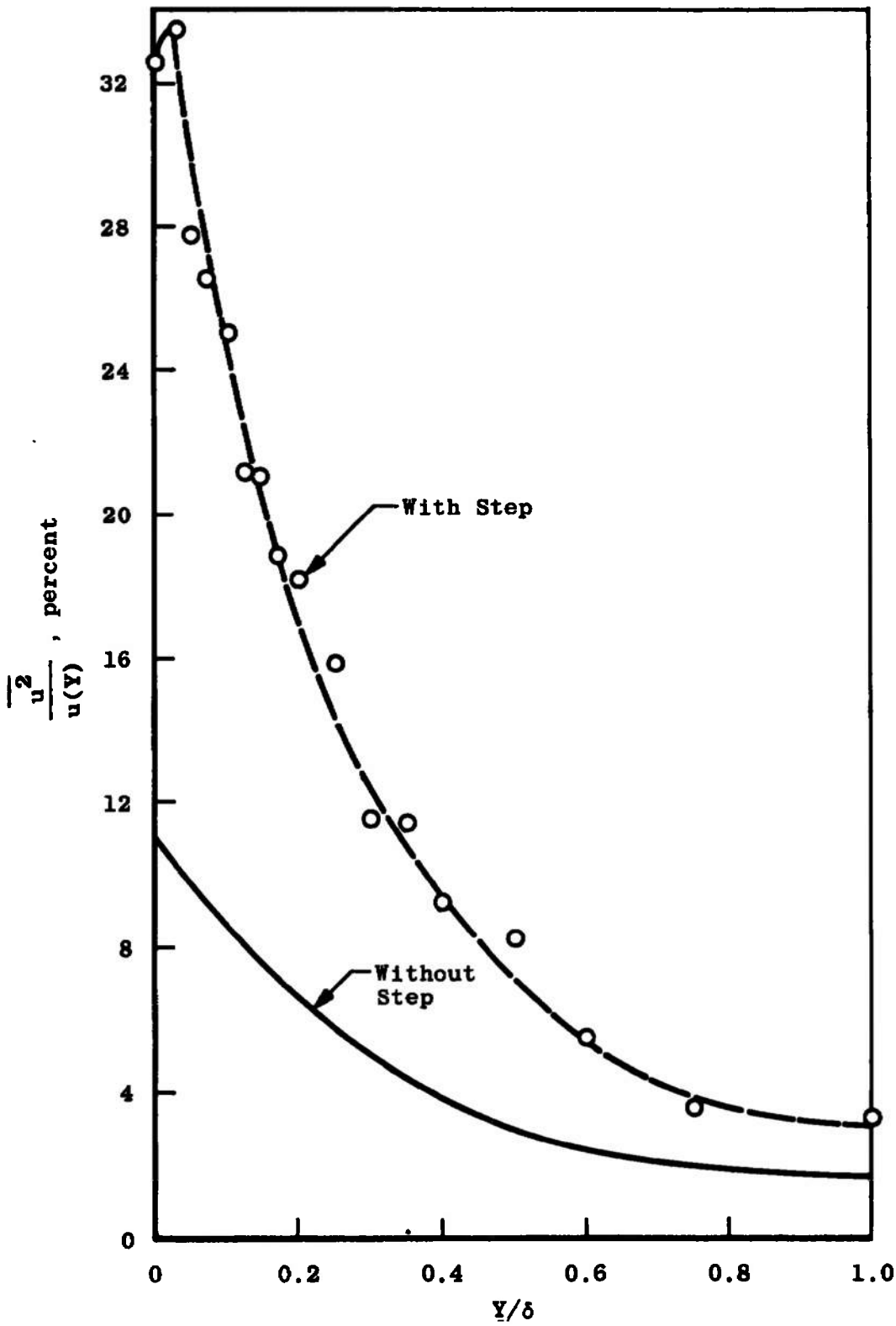
c. The Reattaching Flow of Tillmann (1945) (Ref. 32, Fig. 25)

Fig. 3 Concluded



a. x_s = 4.1 in.

Fig. 4 Turbulent Intensity Measurement in a Subsonic Stream (U_∞ = 65 ft/sec) Upstream of a Step



b. x_s = 2.1 in.

Fig. 4 Concluded

The solution of Korst yields an error function profile in the form

$$\psi = \frac{u}{u_\infty} = \frac{1 + \mathrm{erf}\,(\eta)}{2}$$

where

$$\eta = \frac{\sigma y}{x}$$

u_∞ = free-stream velocity

σ = jet spread parameter

x,y = longitudinal and transverse intrinsic coordinates

The jet spread parameter, σ,[4] reported in the literature shows considerable scatter, as is evident in the compiled data presented in Fig. 12 of Ref. 38. Kessler (Ref. 7) and Childs, et al. (Ref. 20), use Korst's empirical relationship:

$$\sigma = 12 + 2.758\, M_\infty \quad (M_\infty = \text{free-stream Mach number})$$

McDonald (Ref. 39) biases his correlation to the measurements of Maydew and Reed (Ref. 38) through Mach 2 and obtains

$$\frac{\sigma}{\sigma_{(\mathrm{incomp})}} = 1 + \frac{\gamma - 1}{2} M_\infty^2 (1 + 0.0035\, M_\infty^2)$$

---

[4]In interpreting the jet-spread parameter, it should be borne in mind that this term physically represents the lateral eddy diffusion of momentum and, as a consequence, it is sensitive to the turbulence structure parameters of the boundary layer. In particular, the transverse turbulence intensity determines to a large extent the lateral eddy diffusion. The history of formation of the turbulent boundary layer determines this structure in using the jet mixing model for the separating layer. The experimental tests to date have ignored the significance of these parameters, and the author feels strongly that a correlation with the turbulence intensity of Reynolds stresses would reduce the apparent scatter in the values of σ. A subsonic anemometry experiment to investigate the effects of structure and finite thickness of the turbulent boundary layer in jet mixing is being conducted by the author.

The upstream shift of the apparent origin of the free jet is given in Ref. 21:

$$\frac{x'}{\sigma\theta} = (1 - C_{2a}^2 (I_1\{C_{2a,\infty}\} - I_2\{C_{2a,\infty}\}))^{-1}$$

where

x' = the origin shift

θ = momentum thickness at separation

C_{2a} = Crocco number of the external stream

$$I_1(C_{2a,\infty}) = \int_{-\infty}^{\infty} \frac{\psi}{1 - C_{2a}^2 \psi^2} d\eta$$

$$I_2(C_{2a,\infty}) = \int_{-\infty}^{\infty} \frac{\psi^2}{1 - C_{2a}^2 \psi^2} d\eta$$

$$\psi = \frac{u}{u_{2a}}$$

A plot of $I_1(C_{2a,\infty}) - I_2(C_{2a,\infty})$ from the data of Bauer (Ref. 40) is shown in Fig. 5. From this curve the parameter $\frac{X'}{\sigma\theta}$ is readily determined.

In the actual development of the jet profile, a preasymptotic range of nonsimilar velocity profile exists (see Refs. 35 and 41). Because of the increased complexity, the preasymptotic range is neglected, and similar profiles throughout the separated regime are assumed. Using the analysis of Nash (Ref. 35) and the error function velocity profiles of Korst, et al. (Ref. 34), Paynter (Ref. 21) derives an expression for the dividing streamline in the form

$$I_1(C_{2a,\eta_s}) = I_1(C_{2a,\eta_m}) - \frac{\sigma\left(1 + \frac{\gamma - 1}{2} M^2_{2a}\right)}{\frac{X}{\theta} + \sigma F(M_{2a})}$$

where

η_s = dividing streamline

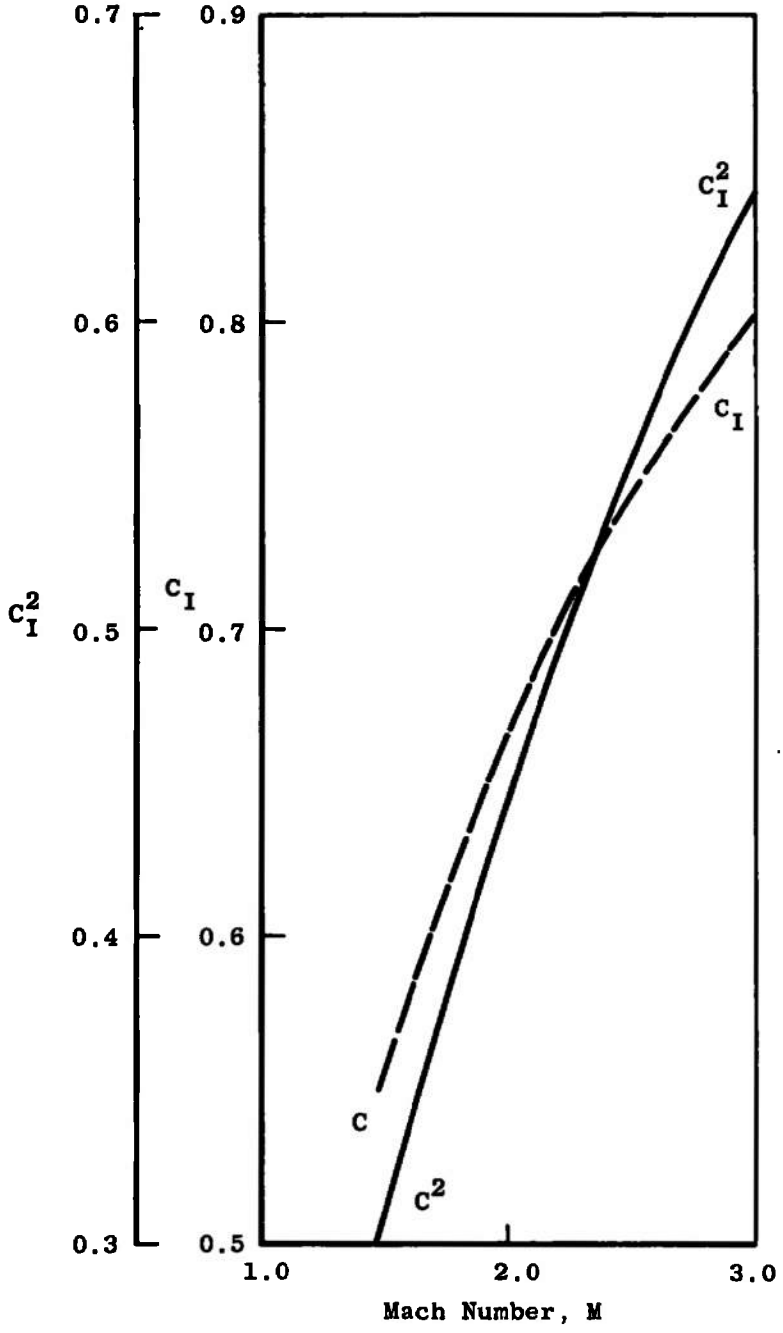
η_m = nondimensional "Y" shift of Korst

$I_1(C_{2a,\eta_s})$ = as defined previously but η_s is the upper limit

$I_1(C_{2a,\eta_m})$ = intergral upper limit, η_m

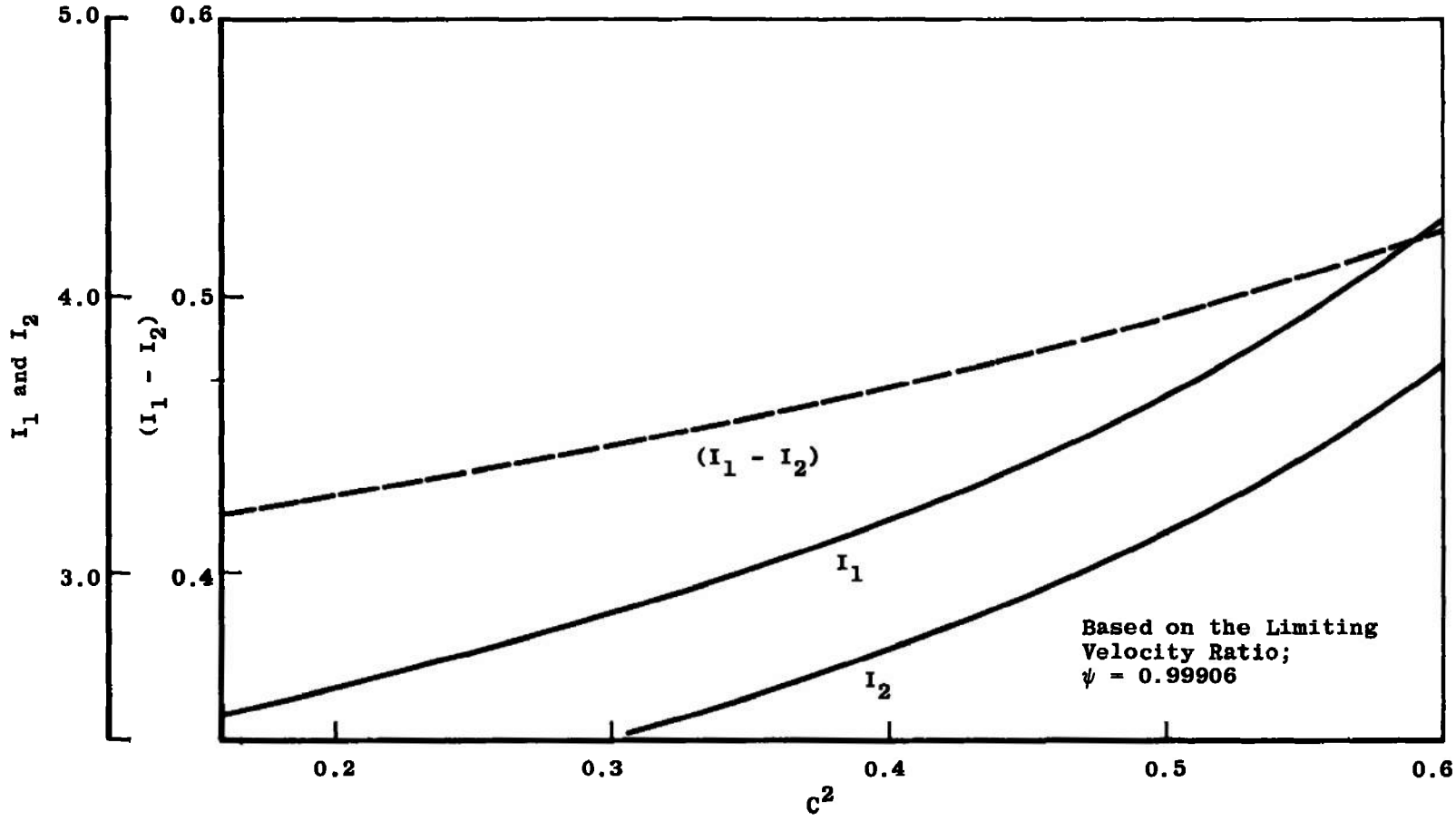
X = longitudinal dimension from point of separation

$F(M_{2a}) = \frac{x'}{\sigma\theta}$



a. Crocco Number

Fig. 5 Jet Spreading Parameters



b. Integral Parameters

Fig. 5 Concluded

For a given value of σ, $F(M_{2a})$, and $\frac{X}{\theta}$, the dividing streamline η_s may be obtained from the tabulated functions in Refs. 34 or 40.

Critical to the above shifting is the momentum thickness of the approaching boundary layer. The technique of Ref. 21 was the one selected for use. A comparison of this technique with that of White (Ref. 42) and Kessler (Ref. 7) is given by Kessler in Ref. 43 and shown in Fig. 6. These techniques show a wide divergence of calculated results for very high Mach numbers at a given shock strength. For Mach numbers below four, the results show fair agreement but appear to become progressively worse as the shock strength is increased.

The length of the separated zone is determined from the boundary condition imposed by the reattachment pressure and is determined by an iteration process, as in Refs. 7 and 39.

2.3 REATTACHMENT PRESSURE

When the separated layer reattaches, it enters into a region of high adverse pressure gradients. The fluid that cannot penetrate the gradient is reversed into the recirculation "bubble." For flows without external mass bleed, the streamline that separates must be the reattachment streamline. Korst, et al. (Ref. 34) used the criteria that the stagnation pressure on the dividing streamline be equal to the downstream pressure. As discussed by Nash (Ref. 35), only the limiting case of a zero thickness boundary layer approached the Korst value. This was also brought out by the experiments of Rosko and Thomke discussed in Ref. 29, where they obtained base pressures below that of Korst when a finite thickness boundary layer existed. In an attempt to explain the base pressure with Reynolds number, Nash (Ref. 35) attempted to locate the reattachment point with the parameter

$$N = \frac{p_r - p_b}{p_\infty - p_b}$$

where

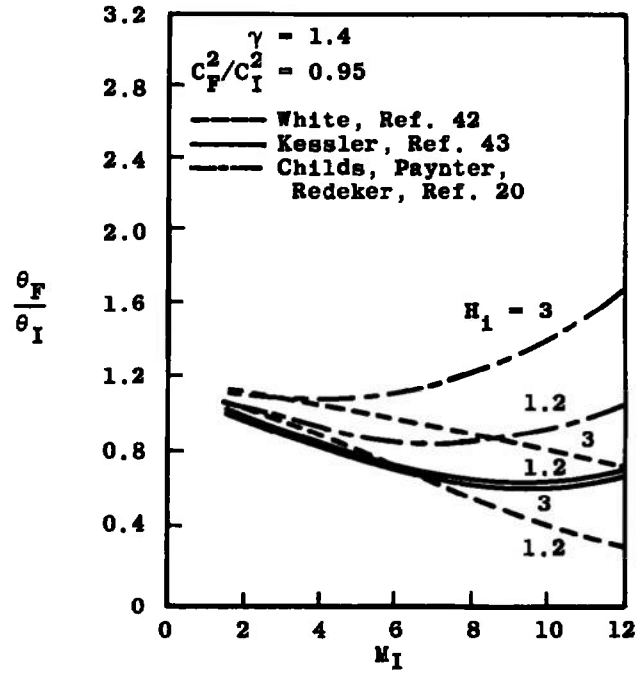
p_∞ = pressure upstream of reattachment

p_r = reattachment pressure

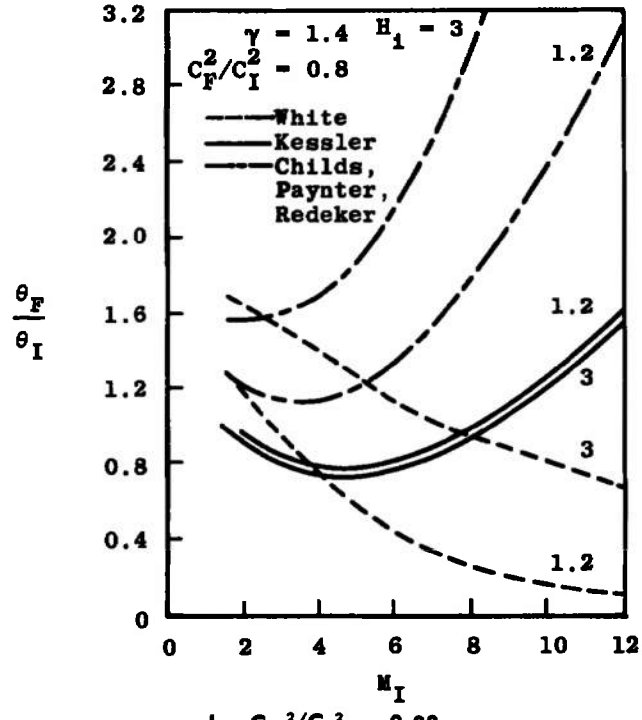
p_b = base pressure

In a later publication, Nash (Ref. 44) stated that N varies in an unknown manner with boundary-layer thickness and Mach number. McDonald (Refs. 39 and 45) used the Mager transformation to incompressible form and specified that the shape factor downstream of reattachment should be that of the flat plate value determined from an empirical equation formulated by Squire and Young and given in Refs. 45 and 46 by the method of Reshotko and Tucker (Ref. 47). In both instances, the analyses show a strong dependence on the upstream Reynolds number.

Permission to reproduce the data in Figs. 6a and b, below, taken from T. J. Kessler's article, "Comment on the Effect of Sudden Compressions on the Turbulent Boundary Layer" (Ref. 43), AIAA Journal, p. 2109, November 1967, was granted by AIAA, copyright holder, 1967.



a. C_F^2/C_I^2 = 0.95



b. C_F^2/C_I^2 = 0.80

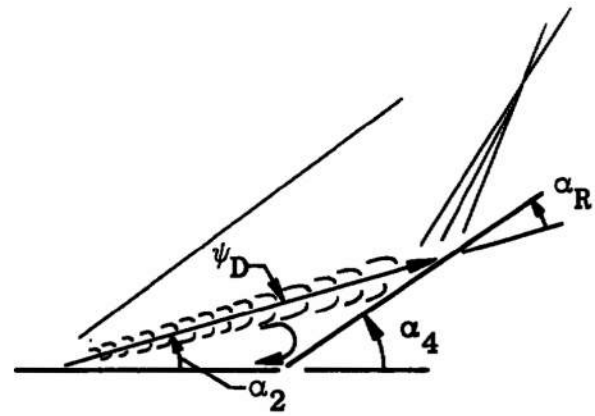
Fig. 6 Comparison of the Theoretical Estimates of the Momentum Thickness after Separation

Carrier and Sirieux (summarized in Ref. 7) in their reattachment studies specified that the reattachment angle is a more important parameter than the pressure rise. Kessler (Ref. 7), on the basis of Carrier's experiments, selected the reattachment angle as the parameter for reattachment and proposed a correlation based on the effective mixing length, ℓ_m, and the dividing streamline, ψ_D, that is, as shown in Ref. 48 (see the sketch below for the nomenclature for the angles in the separated flow region),

$$\alpha_R = K(\alpha_4 - \alpha_2)$$

and

$$K = K(\psi_D, \ell_m)$$



The functional relationships for K were given as

$$K = 0.5(1 - \cos(180\,\psi_D + 0.8))$$

In applying this correlation, only the limiting case of a zero thickness boundary layer was applied to the experimental data. The analysis predicts a decrease in length of the separated zone for a decrease in the boundary-layer momentum thickness (an implicit Reynolds number effect) and an increase in Mach number. The plot of the reattachment angle and dimensionless effective mixing length of Page, et al. is shown in Fig. 7. Recent work had indicated that this correlation held for mass bleed and feed experiments as well.

Paynter's (Ref. 21) reattachment model is based on a control volume analysis of the separation bubble. In this portion of his analysis, he obtains a relationship between the pressure rise to reattachment and the length of the shear layer.[5]

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[5]The assumptions used in this analysis are given on page 36 in Ref. 21.

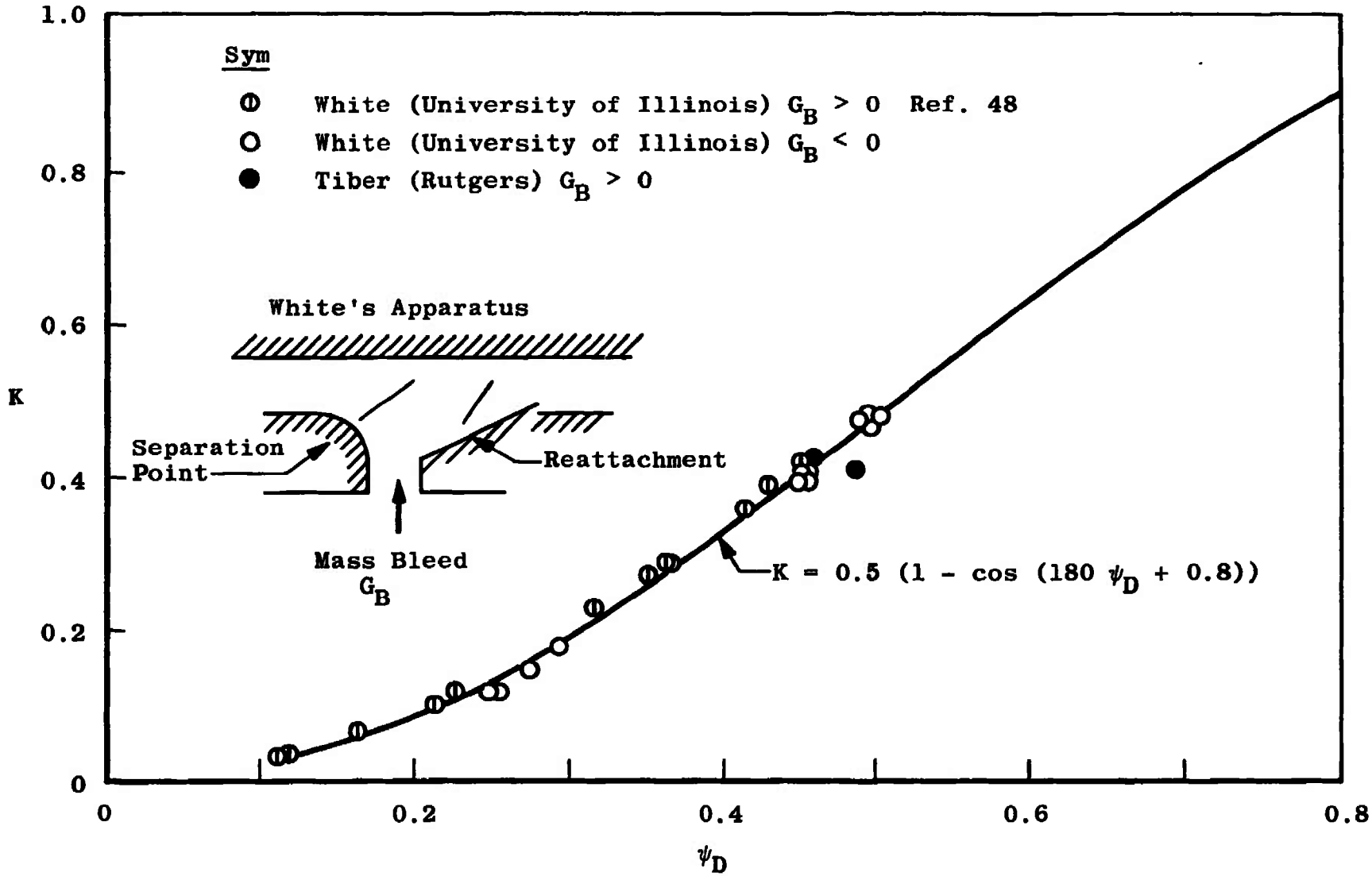

Fig. 7 Kessler's Correlation of the Reattachment Angle and the Effective Mixing Length for $\delta \to 0$

SECTION III
CONCLUDING REMARKS

The overall solution of the Navier-Stokes equations requires specification of boundary condition such as the pressure distribution about the domain of interest, but, in general, this is what is required. For this reason, the modeling technique treats each subdomain separately, and they are only coupled to each other through the interface boundary conditions. Unfortunately, even an adequate description of first-order effects is lacking. There is scatter in the existing data which is either due to experimental inaccuracies or lack of specifically determined initial conditions. In order to study the scale effects, it is imperative that the initial boundary-layer profile, before subjecting it to an adverse pressure gradient, be examined to ensure that it is a self-preserving form (that is, equilibrium form).

The analysis of the boundary layer to separation assumes the form of specification of the shape factor value with the precise value not known. It is felt that a numerical scheme to integrate from the undisturbed region to the separation point must be devised. This requires an auxiliary relationship to predict the separation pressure. In this respect, the author is attempting to devise a control volume approach using the momentum deficient layer as the prerequisite.

Since evidence does exist that the separated zone behaves as a half-infinite jet, the Korst (Ref. 34) mixing zone analysis with a Nash (Ref. 35) type shift is recommended. Illustrations of this technique are Refs. 7 and 34. Since this region involves an experimentally determined constant σ, the velocity profiles throughout this region should be measured to ensure a reasonable value for this parameter to be used in the calculations. In this region, experimental difficulties are experienced since an impact tube can seriously alter the flow field.

Two alternatives exist for the calculation of the reattachment zone. The first is that of Paynter (Ref. 21), in which a control volume is used, and the second is that of Kessler (Ref. 7), who uses the Carrier and Sirieux reattachment criteria.

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13. ABSTRACT

An investigation of the various parameters that affect separation was undertaken to determine if scale effects exist. For example, in incompressible flow a full, thick, turbulent boundary layer will separate more readily in an adverse pressure gradient than a thin boundary layer. The emphasis throughout is on the two-dimensional, forward facing step and compression corners and the shock wave boundary-layer interaction.

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