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THE WORTH OF INDIVIDUALIZING INSTRUCTION

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THE FIRST SEMIANNUAL TECHNICAL REPORT (WHICH COVERS THE PERIOD MAY 1965 THROUGH OCTOBER 1966) OF WORK PERFORMED UNDER CONTRACT NUMBER AF 49 (638) - 1744, ARPA ORDER NUMBER 833, BY THE SHUFORD - MASSENGILL CORPORATION, P.O. BOX 26, LEXINGTON, MASSACHUSETTS, 02173

DECISION-THEORETIC PSYCHOMETRICS: AN INTERIM REPORT, NOVEMBER 1966

Emir H. Shuford, Jr. and H. Edward Massengill

ABSTRACT

In Section A, A Logical Analysis of Guessing, appropriate test-taking strategies are derived for six major test-scoring procedures. Three commonly used definitions of guessing are interpreted as corresponding degree-of-confidence distributions. The ability of the testing procedures to separate these distributions from those representing higher degrees of knowledge is considered with the major result that only admissible probability measurement performs satisfactorily.

In Section B, The Effect of Guessing on the Quality of Personnel and Counseling Decisions, the fundamental probability distributions for total test scores are derived by assuming that each person knows the answers to some items and guesses on the remaining items. Analysis of a 10-item test shows that guessing levels encountered in practice (a) seriously degrade the value of selection, placement, and counseling decisions, (b) significantly impair test reliability and validity, and (c) magnify the influence of testwiseness.

In Section C, The Worth of Individualizing Instruction, equations are developed for expressing the cost and gain for applying an instructional sequence. The expected return from assigning instruction on the basis of (1) admissible probability measurement, (2) admissible choice testing, (3) conventional choice testing, (4) prior information only, and (5) matching the average student is computed for each of seven distributions of state of knowledge. The performance of (1) is outstanding; that of (2), (3), and (4) is disappointing, while (5) does surprisingly well.

The Worth of Individualizing Instruction

It would be surprising if the development of a decision-theoretic psychometrics were not able to provide some useful insights into the process of instruction. Instruction, whether guided externally or internally is basically a cybernetic process and an essential feature of this process is information concerning the current state of a person's knowledge (Shuford & Massengill, 1965). Now, a decision-theoretic psychometrics should be concerned, generally, with techniques for obtaining and utilizing information about the current state of a person's knowledge. A sufficiently developed decision-theoretic psychometrics could be viewed as a theory of instruction. However, in our judgement, though decision theory would provide an excellent framework within which to develop a theory of instruction, too much is buried in the utility functions of decision theory. It is quite feasible to develop a theory of utility functions for a certain area of application (Toda & Shuford, 1965) Such a theory serves to provide a rational basis for deriving utility functions from rather simple premises and greatly reduces the measurement problems involved. A theory of utility functions to be used in the area of instruction clearly must incorporate a theory of learning. Theories of learning logically consistent and coherent with decision-theoretic psychometrics have been developed to a considerable extent (Shuford, 1964; Watanabe, 1960; 1966). The real potential of a decision-theoretic theory of instruction will be realized when it has been developed to the extent that it can deal efficiently with complex structures with the hierarchical organization of knowledge (Toda & Shuford, 1965; Watanabe, 1966). Unfortunately, we are some distance away from achieving such a full-fledged decision-theoretic theory of instruction.

So, now let us return to a more mundane level of where we are now. In previous work (Massengill & Shuford, 1965), we considered a very simple instructional situation where the teacher had to decide whether a person was uninformed or well-informed. The formal explication just allowed two states of knowledge for the person and the two instructional sequences were specifically tailored for these two states of knowledge. Comparison of admissible with conventional choice procedures indicated a superiority for the former which was of considerable significance under a wide variety of conditions.

The results in Section B of this report concerned with placement decisions can also be viewed as relevant to instruction, since the two groups separated by the placement process can be given different instructional sequences. In contrast to our earlier paper, the placement problem allows a greater number of degrees of

knowledge and for more general utility functions. Still, the utility functions are rather arbitrary and empirical and we propose that in this section of the report to use more rationally derived utility functions, but with essentially a one-item test. Though these results are still some distance away from the full-fledged decision-theoretic theory of instruction, they should provide some insight into many instructional situations and they should provide the basis for a more thorcughly developed theory of instruction.

PRECISELY TAILORED INSTRUCTION

Suppose that instruction can be precisely adjusted to a person's state of knowledge as represented by p. For example, if a person is thoroughly misinformed, i.e. p=0, then the appropriate instructional sequence devotes considerable effort attempting to get the person to unlearn false notions and to building up the person's confidence in the correct concept. If, on the other hand, the person is uninformed, i.e., p is approximately equal to .50, then no unlearning is necessary and the instruction concentrates on building up the person's confidence in correct concepts. And finally, at the other extreme, if a person is completely confident or certain of the correct answer, i.e., p=1, no instruction is required. Now, the cost of these instructional procedures decreases as the person's degree of confidence increases. The cost of instruction decreases down to the point of no cost if the person is completely confident in the correct answer. Now, cost may be composed of many components, such as the student's and the teacher's time which, at least, implies that time is used that might be devoted to learning additional topics. There is also the cost of instructional materials. This latter cost may be considered as sunk cost and, therefore, would not enter into the calculations. Thus, student and/or teacher time is the primary component of this cost function and might be approximated by assuming that it is proportional to $1 - p$. If c is the maximum cost incurred when the student is thoroughly misinformed ($p = 0$), then the cost function may be expressed as $-c(1 - p)$ and graphically represented as shown in Graph A of Figure 1.

Consider now the gain resulting from instruction. Assume first that all precisely tailored instructional sequences are completely effective. As a consequence of experiencing a precisely tailored instructional sequence, a student with an initial state of knowledge represented by p will terminate the sequence with a state of knowledge represented by $p = 1$, i.e. he will be certain of the correct concept.

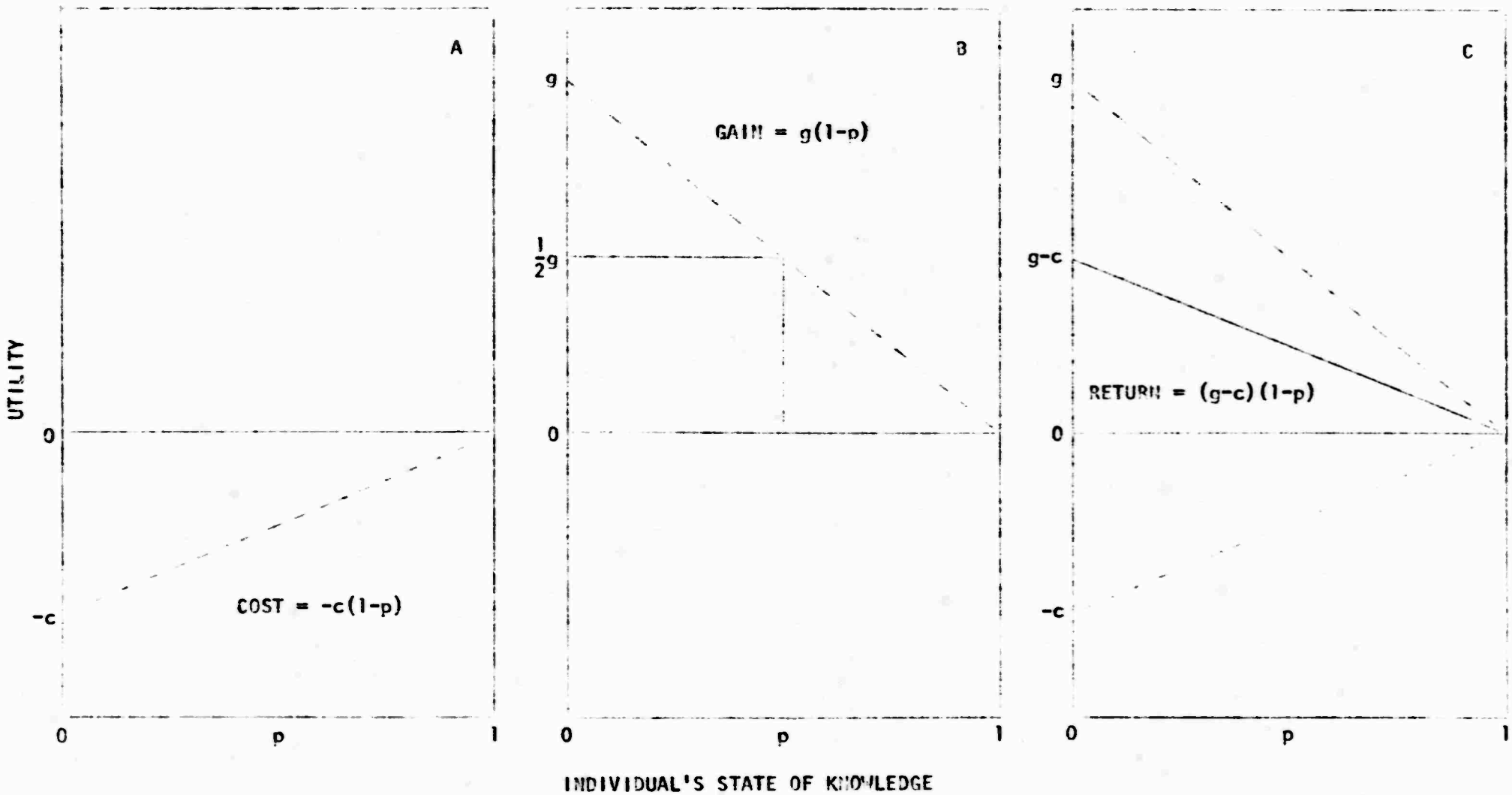

Figure 1. Illustrative cost, gain, and return functions for precisely tailored instruction.

A misinformed person is less effective than an uninformed person, since a misinformed person will go ahead and confidently take action on the basis of his misinformation, while an uninformed person will be motivated by his uncertainty to seek more information before taking action or, if this is not possible, he prefers to take less extreme action. And more clearly, perhaps, a well-informed person is more effective than an uninformed person (more effective in taking actions which better serve his own ends). Taking all this into account, it is evident that the gain from using a precisely tailored instructional sequence is greatest for misinformed students and declines down to zero for completely informed students. For the sake of simplicity, assume that the gain from the use of a precisely tailored instructional sequence is proportional to $1 - p$. In other words, this gain function can be represented by a straight line, with the gain associated with changing a thoroughly misinformed student, $(p = 0)$, into a thoroughly informed student, $(p = 1)$, twice that of changing an uninformed student, $(p = \frac{1}{2})$, into a thoroughly informed student. [Note: This assumption is not always appropriate, since there are probably instances in which being misinformed is many times worse than being uninformed.] This gain function can be represented graphically as in Graph B of Figure 1 with g equal to the maximum possible gain.

Now, by adding the cost and gain functions in order to subtract the cost from the gain, a linear total return or net gain function is obtained, which is proportional to $1 - p$ with a maximum of $g - c$ at $p = 0$ and with a minimum of 0 at $p = 1$. This is represented graphically in Graph C of Figure 1.

If a student's state of knowledge, p, is determined with some precision, as for example, by using an admissible probability measurement procedure, then it is possible to think of precisely tailoring instruction to his state of knowledge. Instruction can be matched precisely to the needs of the student. Such a procedure has often been considered a desirable goal. In the remainder of this section, the hypothetical desirability of this goal will be computed and shown to be a function of the student's states of knowledge. The actual quantities obtained are not important in themselves and do not represent the real value of precisely tailored instruction, since utility values will be assigned arbitrarily without empirical determination or without derivation from a theory of learning. What is important, however, is that in later sections the performance of other procedures will be computed and compared to this best possible, ideal performance of an instructional system. The relative values of these performance indices are

meaningful and remain valid once the actual utilities are empirically determined.

The return from the use of precisely tailored instructional sequences is a function of a student's state of knowledge as represented by p. Therefore, the total return realizable by processing a number of students depends upon the distribution of p for this group of students.

If there are N students in this group and if the state of knowledge of the i-th student is represented by p_i, then the total return, R, from precisely tailored instruction is

$$(1) \quad R = \sum_{i=1}^{N} (g-c)(1-p_i) = (g-c)(K-\sum_{i=1}^{N} p_i).$$

For dealing with large numbers of students and for theoretical purposes, it is convenient to represent the distribution of the student's states of knowledge by a continuous probability distribution defined over the interval [0,1]. A reasonably flexible distribution is the Beta function which depends upon two parameters, a and b, which must be greater than zero.

$$(2) \quad P(p) = f_\beta(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1}$$

$$= \frac{(a+b-1)!}{(a-1)!(b-1)!} p^{a-1}(1-p)^{b-1}.$$

The mean of this distribution is

$$(3) \quad \bar{p} = E(p) = \frac{a}{a+b}$$

and the variance is

$$(4) \quad V(p) = \frac{ab}{(a+b)^2(a+b+1)}.$$

Some representative Beta functions are graphed in Figure 2.

The expected return per individual, r, from precisely tailored instruction with a group of students whose states of knowledge are represented by the Beta function is

$$(5) \quad r = \int_0^1 (g-c)(1-p)\ f_\beta(p|a,b)dp$$

$$= (g-c)\frac{(a+b-1)!}{(a-1)!(b-1)!} \int_0^1 p^{a-1}(1-p)^b dp$$

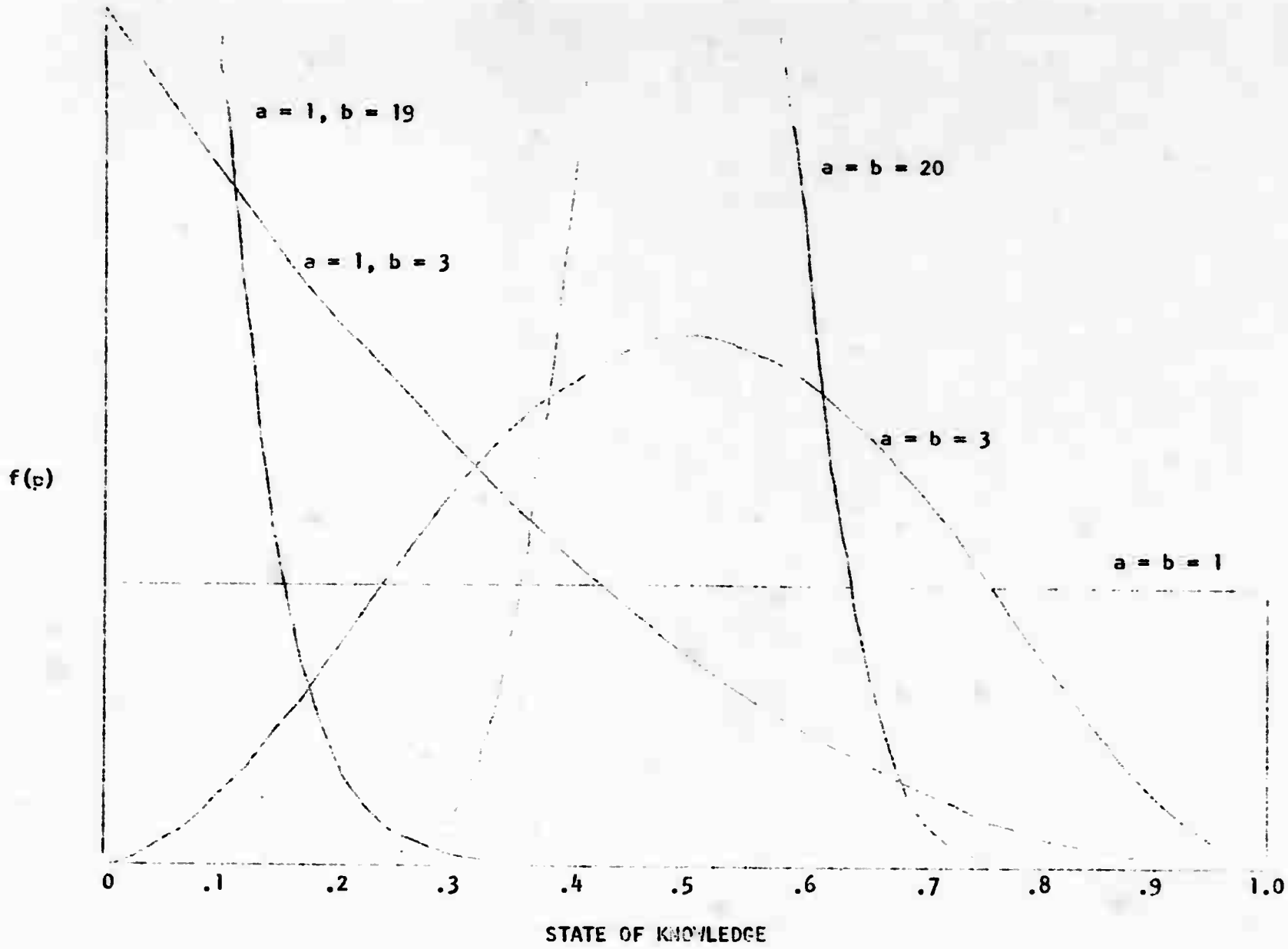

Figure 2. Illustrative distributions of state of knowledge concerning topic to be taught.

$$= (g-c)\frac{(a+b-1)!}{(a-1)!(b-1)!} \cdot \frac{(a-1)!(b)!}{(a+b)!}$$

$$= (g-c)(1-\bar{p}).$$

The expected return per individual, r, depends only on the mean of the distribution of the students' states of knowledge and is a decreasing function of this mean. It does not depend upon the variance of the Beta function. However, if the return function were non-linear then the expected return per individual would depend upon the variance and, possibly, upon other moments of the distribution of the students' states of knowledge. This is true regardless of the exact mathematical form of the distribution.

A PRIORI CHOICE BETWEEN INSTRUCTIONAL SEQUENCES

Now, consider the use of choice methods to indicate a student's state of knowledge. In abandoning precise measurementof a student's state of knowledge, we must give up all hope of using precisely tailored instructional sequences, since there is no basis for matching them to the student's state of knowledge. We must choose from among several instructional sequences and there is no point in choosing more than the number of discriminations we can make in a student's state of knowledge. Remember from Section A above that choice methods are generally capable of discriminating at most three states of knowledge; informed, uninformed, and misinformed. Thus, conventional choice testing can only make the discrimination <u>not misinformed</u> vs. <u>not well-informed</u>. However, we will be considering admissible choice procedures in the next section and since admissible choice can distinguish three states of knowledge, we will use three instructional sequences for both the conventional choice and the admissible choice procedures.

As for the instructional sequences, we will select

I_1 ≡ the precisely tailored instructional sequence for p=1 (No instruction).

$I_{1/2}$ ≡ the precisely tailored instructional sequence for p=1/2 (A moderate amount of instruction, no unlearning).

I_0 ≡ The precisely tailored instructional sequence for p=0 (Extensive unlearning of material, moderate instruction in the correct concepts).

Now, we must make some assumptions about what happens when an instructional

sequence is misapplied (There was no reason to do this before when we could be certain of precisely matching instructional sequences with the student's state of knowledge).

The case of no instruction, I_1, is relatively straightforward. It should result in no cost and no gain and can be graphed as shown in Figure 3. It is, of course, a horizontal line, $r = 0$.

For the case of moderate instruction, $I_{1/2}$, the cost will be constant at a value of $\frac{1}{2}c$, while the gain will decline from a maximum of $\frac{1}{2}g$ at $p = \frac{1}{2}$ as p increases toward 1. As to what happens for $p < \frac{1}{2}$, it seems reasonable to assume that instruction would become less effective as a student becomes more convinced of an incorrect concept. If a student were thoroughly convinced in the correctness of a wrong notion, it appears likely that such training would have no effect at all on the student's state of knowledge, since any demonstration of the wrongness of his idea would be lacking. Thus, the gain from applying $I_{1/2}$ decreases as p goes from 1/2 down to 0 and the gain at $p = 0$ is nil. For simplicity, assume that this decrease is linear. Then, the resulting function can be graphed as shown in Figure 3. It is a triangular-shaped function with one maximum at $p = 1/2$ and with two minima at $p = 0$ and $p = 1$. The minimum value is $-\frac{1}{2}c$.

Finally, consider the effect of applying the instructional sequence, I_0, tailored for the thoroughly misinformed student. The cost is constant at c (This is obviously an approximation since, in some cases, a better-informed student can complete a sequence more rapidly, leading to a lower cost, but there probably exists some additional cost in lost motivation or boredom as a result of suffering through inappropriate instruction). Now, as p increases, the result of applying I_0 is over-instruction, in the sense that the instruction is more than sufficient to carry the student's state of knowledge from p up to $p = 1$. Since complete certainty, represented by $p = 1$, is the limit on a student's knowledge with respect to a particular concept or topic, the gain from applying I_0 must decrease as the student's initial state of knowledge, p, moves toward $p = 1$. Again, we assume for the sake of simplicity that this decrease is linear in $1 - p$. Adding these cost and gain functions we obtain the return function for I_0 shown in Figure 3. Notice that it is a linear function with a negative slope and with a maximum of $g - c$ at $p = 0$ and a minimum of $-c$ at $p = 1$.

The dashed line in Figure 3 is the return function, I_p, for precisely tailored instruction as shown previously in Figure 1. In applying one of these

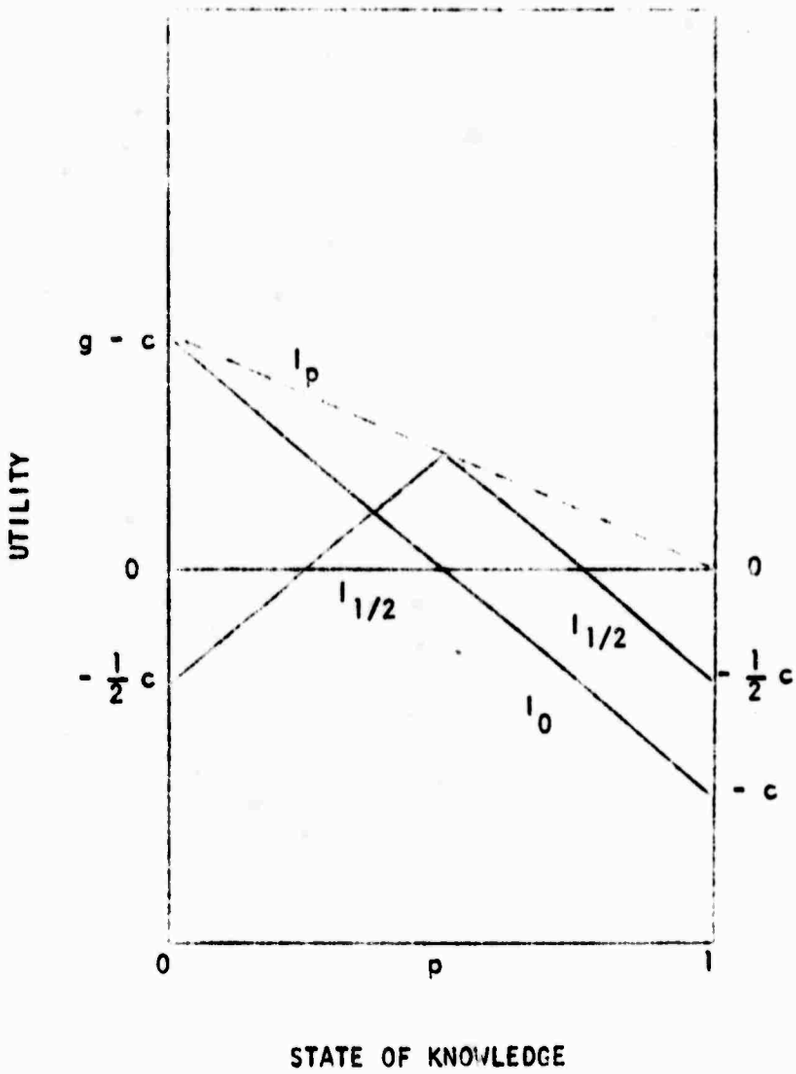

Figure 3. Return functions from three rigid instructional sequences, I_0, $I_{1/2}$, I_1, and for precisely-tailored instruction, I_p.

three rigid sequences, there is no loss relative to precisely tailored instruction at the three values of $p = 0$, $p = 1/2$, or $p = 1$. At other values of p there is some loss and the use of these three rigid instructional sequences treat both moderately misinformed and moderately well-informed students rather badly, although we can do no better without having more precise information (such as that obtained through admissible probability measurement) about each student's state of knowledge.

Sometimes it may happen that we have precise information about a student's state of knowledge, but because of economic or other reasons we may be able to apply just one of the instructional sequences, for example, we may have available three programmed texts at different difficulty levels or three textbooks, elementary, intermediate and advanced. In this instance, examination of Figure 3 clearly indicates that, for students who are more than moderately misinformed, the best assignment is the instructional sequence I_0, whereas for students who are more than moderately well-informed, the best instructional sequence is I_1, which usually represents no instruction, while, for all other students, the return would be maximized by assigning them instructional sequence $I_{1/2}$. Consider now that we have no test information on each student, but we have the overall distribution of ability levels as expressed by the Beta function. Which is the best of the three instructional sequences to apply? This same sequence will, of course, be applied to all of the students. This analysis will serve to illustrate the logic of the institutional decision involved and can also provide a baseline against which we can compare the performance of the choice testing procedures which will be considered later. We need to compute the expected return from applying each of the three instructional sequences. Consider now applying I_1, that is, no instruction, the action most appropriate if a student is thoroughly informed. Since the return for all values of p of applying I_1 is 0, the expected return will also be zero, regardless of the shape of the distribution of states of knowledge,

$$(6a) \quad E\, I_1 = 0.$$

The expected return from applying I_0 is obtained by weighting the return at each level of p by the corresponding probability density and integrating this function over the interval [0,1]. Thus,

$$(6b \quad E\, I_0 = \int_0^1 [g(1-p)-c] f_\beta(p|a,b)\,dp = g(1-\bar{p}) - c.$$

Here again the expected return depends only upon the mean and not upon any other moment of the distribution of the students' states of knowledge.

Consider now applying instructional sequence $i_{1/2}$. As before, we wish to integrate the product of the return function of the probability density over the whole range. Since this is a triangular function with one discontinuity at $p = 1/2$, we can break the integral into two parts and perform the corresponding integration by parts

$$\text{(6c)} \quad E\, i_{1/2} = \int_0^{\frac{1}{2}} (gp - \tfrac{c}{2}) f_\beta(p|a,b)dp + \int_{\frac{1}{2}}^{1} [g(1-p) - \tfrac{c}{2}] f_\beta(p|a,b)dp$$

$$= g\{\bar{p}G_b(a+1|\tfrac{1}{2},a+b) + (1-\bar{p})[G_b(b+1|\tfrac{1}{2},a+b)]\} - \tfrac{c}{2}.$$

The second line of this equation is given for convenience in obtaining actual numerical values for these integrals. G_b stands for the right-hand cumulative of the binomial distribution. The incomplete integral of the Beta distribution can now be solved explicitly in terms of a and b. Thus, tables have been computed and published for the cumulative of the Beta distribution, however, it is more convenient and more accurate in most cases to make use of the fact that the cumulative of a Beta distribution corresponds to a cumulative of a binomial distribution. Thus, for example, the integral from 0 to 1/2 of the Beta distribution, with parameters a and b, corresponds to the sum of the individual terms of the binomial distribution, with parameters 1/2 and a + b, and the sum is over all terms greater than or equal to a + 1. The binomial distribution has been extensively tabled in a number of places. The one we find most convenient is Tables of the Cumulative Binomial Probability Distribution, The Annals of the Computation Laboratory, Harvard University, Vol. XXXV, Cambridge, Mass.: Harvard University Press, 1955.

Thus, the expected return from applying the instructional sequence optimally suited to uninformed students is determined by first finding the two parameters of the Beta distribution, which represent the states of knowledge of the population of students, and determining the gain, g, and the cost, c, and then performing some computations to find the mean, $\bar{p} = a/(a+b)$, of the distribution of the states of knowledge and looking up two values of the cumulative of the binomial distribution.

CONVENTIONAL CHOICE TESTING

Consider now the possibility of asking each student a question before we assign him one of the three instructional sequences. Suppose further that this question were administered by one of the conventional choice methods which allow the student only two responses, where his choice of a response is determined by whether his state of knowledge, p, is greater than or less than 1/2. Thus, his response to the question is a two-valued random variable, X, where X = 0 indicates t that the student's p is less than 1/2 and X = 1 indicates that the student's p is greater than 1/2.

Now, we can compute the expected return given that the student responds and that one of the instructional sequences is applied. Knowing the student's response tells us whether to apply the left half or the right half of the Beta distribution of ability levels in computing the expected return. If we consider applying i_0, the return is always 0. Therefore, the expected return from i_1 is always 0 whether or not the student responds X = 0 or X = 1. Thus, we have

$$\text{(7a)} \quad E_X i_1 = 0.$$

For applying i_0, if a student responds X = 0, the expected return is the integral from 0 to 1/2 of the product of the return function times the distribution of ability levels, while if the student responds X = 1, it is the integral from 1/2 to 1 of the product of the return function times the distribution of ability levels. Thus,

$$\text{(7b)} \quad E_X i_0 = \begin{cases} \int_0^{\frac{1}{2}} [g(1-p) - c] f_\beta(p|a,b)dp & \text{if } X = 0 \\ \int_{\frac{1}{2}}^{1} [g(1-p) - c] f_\beta(p|a,b)dp & \text{if } X = 1 \end{cases}$$

$$= \begin{cases} g(1-\bar{p})G_b(a|\frac{1}{2}, a+b) - cG_b(a|\frac{1}{2}, a+b-1) & \text{if } X = 0 \\ g(1-\bar{p})G_b(b+1|\frac{1}{2}, a+b) - cG_b(b|\frac{1}{2}, a+b-1) & \text{if } X = 1 \end{cases}$$

By a similar process, we obtain conditional, expected return for applying $i_{1/2}$

$$\text{(7c)} \quad E_X i_{1/2} = \begin{cases} \int_0^{\frac{1}{2}} (gp - \frac{c}{2}) f_\beta(p|a,b)dp & \text{if } X = 0 \\ \int_{\frac{1}{2}}^{1} [g(1-p) - \frac{c}{2}] f_\beta(p|a,b)dp & \text{if } X = 1 \end{cases}$$

$$= \begin{cases} g\bar{p}G_b(a+1|\frac{1}{2},a+b) - \frac{c}{2}G_b(a|\frac{1}{2},a+b-1) & \text{if } X = 0 \\ g(1-\bar{p})G_b(b+1|\frac{1}{2},a+b) - \frac{c}{2}G(b|\frac{1}{2},a+b-1) & \text{if } X = 1 \end{cases}$$

And finally, the probability of a student responding, X, is

$$\text{(7d)} \quad P(X) = \begin{cases} \int_0^{\frac{1}{2}} f_\beta(p|a,b)dp & \text{for } X = 0 \quad = G_b(a|\frac{1}{2},a+b-1) \\ \int_{\frac{1}{2}}^{1} f_\beta(p|a,b)dp & \text{for } X = 1 \quad = G_b(b|\frac{1}{2},a+b-1) \end{cases}$$

ADMISSIBLE CHOICE TESTING

Remember from Section A above that the use of an admissible choice procedure yields, at least, three different responses, Y = 0,1,2. If a student answers Y = 0, he is misinformed about the topic under consideration. If the student answers Y = 1, he is neither very misinformed nor very well-informed about the topic under consideration. If the student answers Y = 2, he is fairly well-informed about the topic under consideration. By varying the size of the penalty for a wrong response, we can vary the critical values of p, where the student will make one or another response. To be explicit, we will consider the case in which the student will respond Y = 0, if p is less than 1/4; he will respond Y = 1, if p is between 1/4 and 3/4; and he will respond Y = 2, if p is greater than 3/4. Remember that the middle category can be broadened or narrowed and would have some effect on the performance of admissible choice as a guide to instruction, depending upon the exact values of g and c. However, 1/4 and 3/4 are not too unreasonable to use in general with admissible choice testing and, in reality, we are probably not free to change the penalty (and, thus, the middle range) from time to time and expect the students to adjust immediately to the change.

Consider now the expected return from first asking a question about the topic under consideration by using an admissible choice procedure and then applying one of the three instructional sequences depending upon the answer that the student gives. As before, the expected return from applying I_1 is 0

$$\text{(8a)} \quad E_v I_1 = 0$$

We obtain the expected return by applying I_0 in a similar fashion to that given for conventional choice testing, but for three different responses

$$(8b)\quad E_Y I_0 = \begin{cases} \int_0^{\frac{1}{4}} [g(1-p) - c] f_\beta(p|a,b)dp & \text{if } Y = 0 \\ \int_{\frac{1}{4}}^{\frac{3}{4}} [g(1-p) - c] f_\beta(p|a,b)dp & \text{if } Y = 1 \\ \int_{\frac{3}{4}}^{1} [g(1-p) - c] f_\beta(p|a,b)dp & \text{if } Y = 2 \end{cases}$$

$$= \begin{cases} g(1-\bar{p})G_b(a|\frac{1}{4},a+b) - cG_b(a|\frac{1}{4},a+b-1) & \text{if } Y = 0 \\ g(1-\bar{p}) - c - E_{Y=0}I_0 - E_{Y=2}I_0 & \text{if } Y = 1 \\ g(1-\bar{p})G_b(b+1|\frac{1}{4},a+b) - cG_b(b|\frac{1}{4},a+b-1) & \text{if } Y = 2 \end{cases}$$

and the expected return from applying $I_{1/2}$

$$(8c)\quad E_Y I_{1/2} = \begin{cases} \int_0^{\frac{1}{4}} [gp - \frac{c}{2}] f_\beta(p|a,b)dp & \text{if } Y = 0 \\ \int_{\frac{1}{4}}^{\frac{1}{2}} [gp - \frac{c}{2}] f_\beta(p|a,b)dp + \int_{\frac{1}{2}}^{\frac{3}{4}} [g(1-p) - \frac{c}{2}] f_\beta(p|a,b)dp & \text{if } Y = 1 \\ \int_{\frac{3}{4}}^{1} [g(1-p) - \frac{c}{2}] f_\beta(p|a,b)dp & \text{if } Y = 2 \end{cases}$$

$$= \begin{cases} g\bar{p}G_b(a+1|\frac{1}{4},a+b) - \frac{c}{2}G_b(a|\frac{1}{4},a+b-1) & \text{if } Y = 0 \\ g\bar{p}G_b(a+1|\frac{1}{2},a+b) - \frac{c}{2}G_b(a|\frac{1}{2},a+b-1) - E_{Y=0}I_{1/2} \\ \quad + g(1-\bar{p})G_b(b+1|\frac{1}{2},a+b) - \frac{c}{2}G_b(b|\frac{1}{2},a+b-1) - E_{Y=2}I_{1/2} & \text{if } Y = 1 \\ g(1-\bar{p})G_b(b+1|\frac{1}{4},a+b) - \frac{c}{2}G_b(b|\frac{1}{4}\ a+b-1) & \text{if } Y = 2 \end{cases}$$

Likewise, the probability of students responding $Y = 0,1,2$ is

$$(8d)\quad P(Y) = \begin{cases} \int_0^{\frac{1}{4}} f_\beta(p|a,b)dp & \text{for } Y = 0 \quad G_b(a|\tfrac{1}{4},a+b-1) \\ \int_{\frac{1}{4}}^{\frac{3}{4}} f_\beta(p|a,b)dp & \text{for } Y = 1 \quad G_b(a|\tfrac{1}{2},a+b-1) + G_b(b|\tfrac{1}{2},a+b-1) \\ \int_{\frac{3}{4}}^{1} f_\beta(p|a,b)dp & \text{for } Y = 2 \quad G_b(b|\tfrac{1}{4},a+b-1) \end{cases}$$

MATCHING INSTRUCTION TO THE AVERAGE STUDENT

Consider now the last instructional strategy, that of matching instruction to the average student. To be more explicit, the idea is to determine the average state of knowledge of the students and then to select and apply that instructional sequence precisely tailored to this mean value. Though there is some logic to this procedure, it has been widely criticized, probably, most often because people object to the idea of using one instructional sequence for pupils of varying states of knowledge. It has been criticized somewhat less often because people object to matching instruction to the mean rather than to some other value. In this report, we will just consider matching instruction to the mean and in a later report we may consider matching instruction to other aspects of the distribution of states of knowledge.

To consider this case, we have to specify our cost, gain, and return functions more generally, since any precisely tailored instructional sequence, I_p, may be chosen and misapplied to students with various states of knowledge.

It is only logical to use the same cost function that we used before for a precisely tailored instructional sequence, since it depends upon the instructional sequence and not upon the state of knowledge of the student. Thus,

$$(9)\quad \text{Cost} = c(1-\bar{p})$$

We must, however, more completely specify the return function. To see this, suppose we apply $I_{3/4}$. For students whose states of knowledge, p, are greater than 3/4, the return function will be similar to those specified before and, due to the limit on the maximum state of knowledge, the return will decline as p approaches 1. For students whose states of knowledge, p, are less than 3/4, the return will decline as p increases. Assuming that instruction is progressively less effective in increasing the students' states of knowledge as the instruction is tailored for

students who are at progressively higher levels of knowledge, then we can approximate this behavior by a linear function which declines at a certain rate. The decline progresses down to where the gain is zero and then remains at zero for all smaller values of p. Thus, we can derive a set of return functions which includes those used earlier

$$(10)\quad I_{\bar{p}}(p) = \begin{cases} -c(1-\bar{p}) & \text{for } p \leq 2\bar{p} - 1 \\ gp - g(2\bar{p}-1) - c(1-\bar{p}) & \text{for } 2\bar{p} - 1 \leq p \leq \bar{p} \\ g(1-p) - c(1-\bar{p}) & \text{for } p \geq \bar{p} \end{cases}$$

Three different return functions are illustrated in Figure 4 along with the return function for precisely tailored instruction.

Using this general Equation 10 for the return function, we can compute the expected return from using an instructional sequence precisely tailored to the mean state of knowledge of the students. Thus,

$$\begin{aligned}(11)\quad E\, I_{\bar{p}}(p) &= \int_0^1 I_{\bar{p}}(p) f_\beta(p|a,b)dp \\ &= \int^{2\bar{p}-1} [-c(1-\bar{p})] f_\beta(p|a,b)dp \\ &+ \int_{2\bar{p}-1}^{\bar{p}} [gp-g(2\bar{p}-1) - c(1-\bar{p})] f_\beta(p|a,b)dp \\ &+ \int_{\bar{p}}^1 [g(1-p) - c(1-\bar{p})] f_\beta(p|a,b)dp \\ &= g\bar{p}\{F_\beta(\bar{p}|a+1,b) - F_\beta(2\bar{p}-1|a+1,b)\} \\ &- g(2\bar{p}-1)\{F_\beta(\bar{p}|a,b) - F_\beta(2\bar{p}-1|a,b)\} \\ &+ g(1-\bar{p})G_\beta(\bar{p}|a,b+1) - c(1-p)\end{aligned}$$

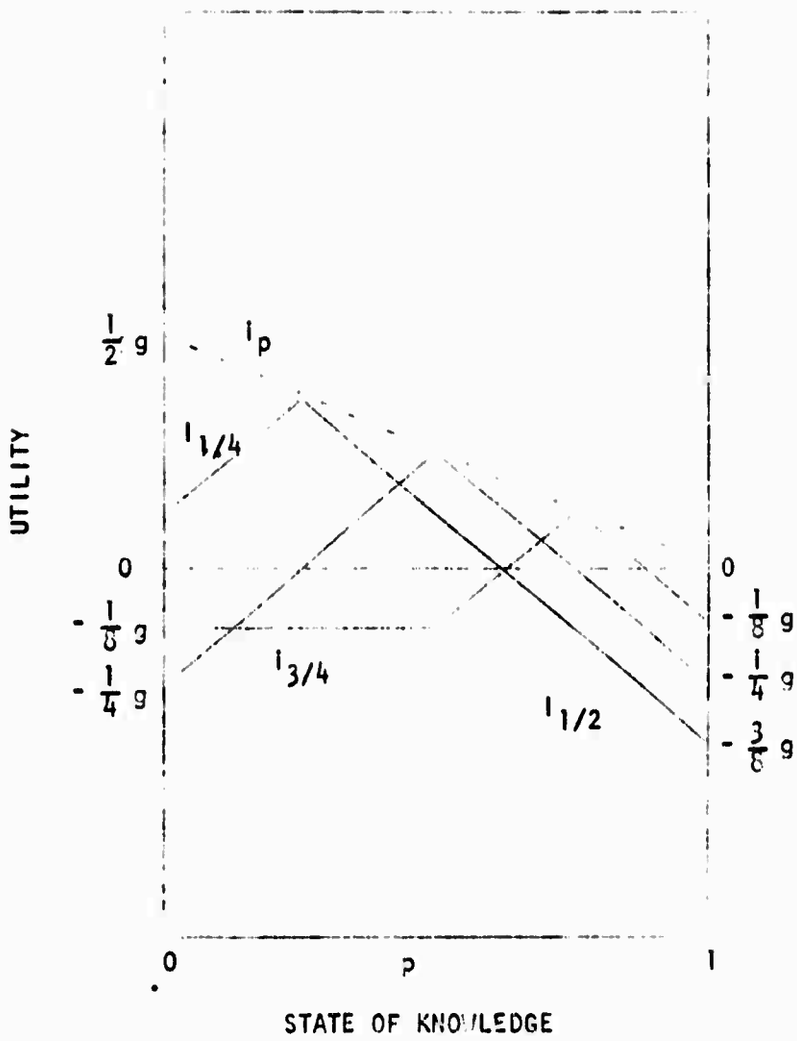

Figure 4. Illustrative return functions for matching instruction to average state of knowledge.

SOME COMPUTED VALUES BASED ON SELECTED PARAMETERS

In order to get some numbers from these equations, we have to give numerical values to four parameters. Two of them, a and b, specify the distribution of states of knowledge of the students under instruction. The other two, g and c, represent gain and cost, respectively, of the instruction. We will fix $g = 4$ and $c = 2$ for all of the following computations. The results are a little more general than they seem, since they also apply proportionally to all cases where $g = 2c$.

We use seven different distributions of states of knowledge. Three of them are symmetric about $p = 1/2$, but with different degrees of spread about this value. Two are skewed with many of the students being misinformed, while the other two distributions are skewed in the other direction, with many of the students being well-informed. The following pages give the parameters and the computed values along with comments on the results.

Parameters: $a = b = 1$, $g = 4$, $c = 2$, $\bar{p} = 1/2$.

Case I: $E\, I_p = 1$

Case II: $E\, I_0 = 0$

$E\, I_{1/2} = 0$

$E\, I_1 = 0$

Case III:

	(X = 0)	(X = 1)	$E_X I^* = .5$
$E_X I_0$	= .5	-.5	
$E_X I_{1/2}$	= 0	0	
$E_X I_1$	= 0	0	
P(X)	= .5	.5	

Case IV:

	(Y = 0)	(Y = 1)	(Y = 2)	$E_Y I^* = .625$
$E_Y I_0$	= .375	0	-.375	
$E_Y I_{1/2}$	=-.125	.25	-.125	
$E_Y I_1$	= 0	0	0	
P(Y)	= .25	.50	.25	

Case V: $E\, I_{\bar{p}} = 0$

Comments: Here the distribution of states of knowledge corresponds to the rectangular distribution shown in Figure 2. Effectively, the teacher has no information whatsoever concerning the student's state of knowledge. Notice that precisely tailored instruction yields a return of some size, while tailoring instruction with no testing either by choosing one of the three sequences or by matching instruction to the average student yields a return of 0. The use of choice testing to guide instruction yields a return of half of that possible by the use of admissible probability measurement procedures while the use of admissible choice procedures yields a return somewhat higher than that of conventional choice testing. In summary, in this case the teacher has no information whatsoever about the students, so not surprisingly, testing pays off.

Parameters: $a = b = 3$, $g = 4$, $c = 2$, $\bar{p} = 1/2$.

Case I: $E\, I_p = 1$

Case II: $E\, I_0 = 0$

$E\, I_{1/2} = \underline{.375}$

$E\, I_1 = 0$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = .5$
$E_X I_0 =$	$\underline{.3125}$	$-.3125$	
$E_X I_{1/2} =$	$.1875$	$\underline{.1875}$	
$E_X I_1 =$	0	0	
$P(X) =$	$.5$	$.5$	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = .56346$
$E_Y I_0 =$	$\underline{.13182}$	0	$-.13182$	
$E_Y I_{1/2} =$	$-.02832$	$\underline{.43164}$	$-.02832$	
$E_Y I_1 =$	0	0	$\underline{0}$	
$P(Y) =$	$.10352$	$.79296$	$.10352$	

Case V: $E\, I_{\bar{p}} = .375$

Comments: The distribution of the students' states of knowledge, in this case, may be seen in Figure 2. It is symmetric about $p = 1/2$ with a maximum at this value. The students' states of knowledge fall over a broad range, but relatively few are very well-informed or very misinformed. Notice that, in this case, precisely tailored instruction again yields a return of some size. In tailoring instruction without testing, $I_{1/2}$, is the best of the three and yields an expected return equal to the expected return of matching instruction to the average student. As before, the use of choice testing to guide instruction yields half the expected return from using admissible probability measurement. The use of an admissible choice procedure is somewhat better, but the gain is less than was found before.

Parameters: $a = b = 20$, $g = 4$, $c = 2$, $\bar{p} = 1/2$.

Case I: $E\,I_p = 1$

Case II: $E\,I_0 = 0$

$E\,I_{1/2} = \underline{.74924}$

$E\,I_1 = 0$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = .74924$
$E_X I_0 =$	.12538	-.12538	
$E_X I_{1/2} =$	$\underline{.37462}$	$\underline{.37462}$	
$E_X I_1 =$	0	0	
$P(X) =$	.5	.5	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = .74970$
$E_Y I_0 =$	$\underline{.00040}$	0	-.00040	
$E_Y I_{1/2} =$	-.00003	$\underline{.74930}$	-.00003	
$E_Y I_1 =$	0	0	$\underline{0}$	
$P(Y) =$	.00037	.99926	.00037	

Case V: $E\,I_{\bar{p}} = .74924$

Comments: This distribution of the students' states of knowledge can also be found in Figure 2. It is the very narrow distribution symmetric about $p = 1/2$, the great majority of students being in the region of being uninformed, but essentially no one is well-informed or misinformed. It represents a very homogeneous class which may have arisen either through a previously applied placement process or because of the nature of the subject matter under study, such as Greek, where relatively few students would possess prior knowledge of the subject. Here, the expected return from precisely tailored instruction is of some size which, again, is best of any of the procedures. Notice, however, that all the remaining procedures do about equally well. One can do as well without testing as by using a conventional choice test and the gain from admissible choice testing is really trivial. This is a reflection of the vast amount of information already known about the students' states of knowledge. In summary, as the class becomes more homogeneous, with relatively fewer well-informed and misinformed students, the value of precisely tailored instruction remains best and constant with the return from the other procedures increasing until, finally, the gain from choice testing is wiped out, but with considerable advantage still remaining to the use of admissible probability measurement.

Parameters: $a = 1$, $b = 3$, $g = 4$, $c = 2$, $\bar{p} = 1/4$.

Case I: $E\, I_p = 1.5$

Case II: $E\, I_0 = \underline{1}$

$E\, I_{1/2} = -.25$

$E\, I_1 = 0$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = 1.125$
$E_X I_0 =$	$\underline{1.0625}$	$-.0625$	
$E_X I_{1/2} =$	$-.1875$	$\underline{.0625}$	
$E_X I_1 =$	0	0	
$P(X) =$	.875	.125	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = 1.08982$
$E_Y I_0 =$	$\underline{.89451}$	.12502	$-.03126$	
$E_Y I_{1/2} =$	$-.31641$	$\underline{.19531}$	$-.00390$	
$E_Y I_1 =$	0	0	$\underline{0}$	
$P(Y) =$	.57813	.40624	.01563	

Case V: $E\, I_{\bar{p}} = .86721$

Comments: This distribution of students' states of knowledge is also shown in Figure 2. It is the moderately positively skewed distribution with relatively few of the students being well-informed. Here, the return from precisely tailored instruction is even greater than before. This is primarily a reflection of the characteristic of the return functions that the yield is greater from training a misinformed student. In the case of selecting one of the three instructional sequences without testing, it is best to treat all of the students as being misinformed and the return is 2/3 of that obtainable from precisely tailored instruction. Notice that matching instruction to the average student, i.e., applying $I_{1/4}$, yields almost as great a return as does using I_0. The use of choice testing is slightly better than applying I_0. The use of admissible choice testing, while very slightly better than applying I_0, is not quite as good as using conventional choice testing. This is undoubtedly due to the choice of the penalty score for the admissible choice testing procedure used. Admissible choice testing applies I_0 whenever a student picks the wrong answer, which is about 58% of the time and applies $I_{1/2}$ whenever the student skips the question, which is about 41% of the time, whereas conventional choice testing applies I_0 whenever the student gets the wrong answer, which is about 88% of the time and applying $I_{1/2}$ whenever the student gets the right answer, which is about 12% of the time. In summary, when many of the students are not well-informed, choice testing is of little value. The use of admissible probability measurement with precisely tailored instructional sequences still retains a considerable superiority.

Parameters: $a = 1$, $b = 19$, $g = 4$, $c = 2$, $\bar{p} = 1/20$.

Case I: $E\, I_p = 1.8$

Case II: $E\, I_0 = \underline{1.8}$

$E\, I_{1/2} = -.8$

$E\, I_1 = 0$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = 1.8$
$E_X I_0 =$	$\underline{1.8}$	0	
$E_X I_{1/2} =$	$-.8$	0	
$E_X I_1 =$	0	0	
$P(X) =$	1	0	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = 1.79998$
$E_Y I_0 =$	$\underline{1.79641}$	$\underline{.00357}$	0	
$E_Y I_{1/2} =$	$-.80063$	$-.00360$	0	
$E_Y I_1 =$	0	0	0	
$P(Y) =$	.99577	.00423	0	

Case V: $E\, I_{\bar{p}} = 1.75663$

Comments: This distribution of students' states of knowledge is also shown in Figure 2. It is the highly positively skewed distribution with almost all the students being more or less misinformed. Such a situation might be faced by a teacher given the task of political indoctrination or of teaching a group of students selected by a previously applied placement process. Here the return from precisely tailored instruction is somewhat greater than previously, primarily reflecting the greater number of students who are misinformed. Notice, however, that all the other instructional strategies yield almost the same return. This is so because they recommend essentially the same instructional sequence. They all recommend the application of I_0 except in the case of matching instruction to the average student. In this case, the recommendation, $I_{.05}$, is not too different from I_0. So, in this case, testing doesn't help, whether it is a choice procedure or admissible probability measurement. This is primarily because the amount of information already available to the teacher is just too great and additional information will not help.

Parameters: $a = 3$, $b = 1$, $g = 4$, $c = 2$, $\bar{p} = 3/4$.

Case I: $E\, I_p = .5$

Case II: $E\, I_0 = -1$

$E\, I_{1/2} = -.125$

$E\, I_1 = \underline{0}$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = .0625$
$E_X I_0$	$\underline{.0625}$	-1.0625	
$E_X I_{1/2}$	$\underline{.0625}$	$-.1875$	
$E_X I_1$	0	$\underline{0}$	
$P(X)$	$.125$	$.875$	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = .21483$
$E_Y I_0$	$\underline{.01952}$	$-.12498$	$-.89454$	
$E_Y I_{1/2}$	$-.00390$	$\underline{.19531}$	$-.31641$	
$E_Y I_1$	0	0	$\underline{0}$	
$P(Y)$	$.01563$	$.40624$	$.57813$	

Case V: $E\, I_{\bar{p}} = .07033$

Comments: This distribution of students' states of knowledge is not shown in Figure 2, but it is essentially the reverse of the distribution for $a = 1$, $b = 3$. In other words, it is negatively skewed with relatively few of the students being misinformed. Here, the return from precisely tailored instruction is less than before, essentially reflecting the fact that the students need less instruction than before. In the case of choice between the three instructional sequences without testing, the best choice is no instruction, but the yield is zero. Notice also that considerable loss results from applying either $I_{1/2}$ and, particularly, I_0. Matching instruction to the average student does almost as badly with a very small yield. Choice testing is even worse than matching to the average student, while admissible choice testing does pretty well by recommending I_0, whenever the student chooses the wrong answer; $I_{1/2}$, whenever the student skips the question; and I_1, whenever the student answers correctly.

Parameters: $a = 19$, $b = 1$, $g = 4$, $c = 2$, $\bar{p} = 19/20$.

Case I: $E\, I_p = .1$

Case II: $E\, I_0 = -1.8$

$E\, I_{1/2} = -.8$

$E\, I_1 = \underline{0}$

Case III:

	$X = 0$	$X = 1$	$E_X I^* = 0$
$E_X I_0 =$	$\underline{0}$	-1.8	
$E_X I_{1/2} =$	0	$-.8$	
$E_X I_1 =$	0	$\underline{0}$	
$P(X) =$	0	1	

Case IV:

	$Y = 0$	$Y = 1$	$Y = 2$	$E_Y I^* = .00063$
$E_Y I_0 =$	$\underline{0}$	$-.00357$	$-.79640$	
$E_Y I_{1/2} =$	0	$\underline{.00063}$	$-.80063$	
$E_Y I_1 =$	0	0	$\underline{0}$	
$p(Y)$	0	$.00423$	$.99577$	

Case V: $E\, I_{\bar{p}} = -.01905$

Comments: This distribution is not shown in Figure 2, but it is the reverse of the distribution for $a = 1$, $b = 19$. In other words, it is highly negatively skewed with almost all of the students being fairly well-informed. Here, the return from precisely tailored instruction is not very large compared to the values previously obtained. Application of one of the three instructional sequences without testing yields a return of zero from applying I_1, that is, no instruction. The misapplication of I_0 or $I_{1/2}$ yields very large losses, however. Choice testing is of no help with a truly trivial advantage for admissible choice testing over conventional choice testing. Matching instruction to the average student actually yields a negative return, but of very small value. In summary, none of the returns here are very large compared to those obtained previously. However, there are undoubtedly situations in which it is very important to have all the students very well-trained (e. g. maintaining proficiency for critical jobs) so that the return would become quite important. In such a case, it is evident that this type of training cannot be achieved efficiently without the use of the precisely tailored instruction based on admissible probability measurement, at least not by the use of any of the procedures considered here.

CONCLUSIONS

Precisely tailored instruction has evidenced a marked superiority to the other methods in all instances considered so far. Now, precisely tailored instruction is not characteristic of most formal, institutional procedures for instruction. It has certainly never been used in conjunction with admissible probability measurement. Precisely tailored instruction, in the spirit in which it is used here, is probably best approximated by very good tutorial instruction in the case of external instruction and by a graduate student or by a self-taught expert in the case of internally guided instruction. Most talk about individualized instruction is in the context of institutionalized instructional programs and, in this context, individualized instruction has been in the past, at most, something approximating the use of choice testing to guide the application of special instructional sequences. This kind of instruction has been approximated in some schools and by programmed textbooks and computer-based materials which branch on the basis of the students' choices. Most programmed instruction and computer-assisted instruction approximates the matching of instruction to the average student, since presumably, the instructional material is evolved through successive tryouts to do a fairly good job with most students. This also applies to some extent to the development of textbooks and other instructional material. However, the choice between textbooks and other instructional material, as faced in most schools, is approximated by a priori choice, where no testing is done and previously available information about the students is used to select one of three instructional sequences.

In general, we find that individualized instruction is at least as good as these other procedures already in use. In many cases, however, it is no better and, in other cases, it is probably not enough better to offset the additional cost of testing. Matching instruction to the average student is, frankly, better than we thought it would be. It is as good as individualized instruction in every case, but two. The most dramatic instance is where the teacher has essentially no information about the students' states of knowledge. The other, less dramatic, instance is where relatively few of the students are well-informed, but there is still a moderate spread in the students' states of knowledge with the majority of students tending to be somewhat misinformed. This latter instance seems somewhat less likely to occur in actual practice than the first instance, which might be

approximated by those situations in which the teacher is teaching a very large class of students or in which the teacher is beginning a course of instruction with new students about which he has no prior information. Thus, it would appear that individualized instruction, as conceived by most educators, would be of greatest benefit in these situations. In these and many other situations, however, the benefit from precisely tailored instruction would be much greater. One of the hurdles to achieving precisely tailored instruction in a formal, instructional context has been surmounted by the development of admissible probability measurement procedures. Another major hurdle probably lies in making the application of the precisely tailored instructional sequences economically and organizationally feasible. Computer based-instruction, tutorial testing, and independent self-study probably hold some promise here. We plan to investigate their potential in a future report.

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UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

The Shuford-Massengill Corporation
Box 26
Lexington, Massachusetts 02173

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

DECISION-THEORETIC PSYCHOMETRICS: THE WORTH OF INDIVIDUALIZING INSTRUCTION

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

Emir H. Shuford, Jr.
H. Edward Massengill

6. REPORT DATE

November 1966

7a. TOTAL NO. OF PAGES

30

7b. NO. OF REFS

20

8a. CONTRACT OR GRANT NO. AF 49(638)1744

b. PROJECT NO. 920F-9719

c. 6154501R

d. 681313

9a. ORIGINATOR'S REPORT NUMBER(S)

SMC R-6

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

AFOSR-68-2156

10. DISTRIBUTION STATEMENT

1. This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES

TECH, OTHER

12. SPONSORING MILITARY ACTIVITY

Air Force Office of Scientific Research
1400 Wilson Boulevard (SRLB)
Arlington, Virginia 22209

13. ABSTRACT

In Section C, The Worth of Individualizing Instruction, equations are developed for expressing the cost and gain for applying an instructional sequence. The expected return from assigning instruction on the basis of (1) admissible probability measurement, (2) admissible choice testing, (3) conventional choice testing, (4) prior information only, and (5) matching the average student is computed for each of seven distributions of state of knowledge. The performance of (1) is outstanding; that of (2), (3), and (4) is disappointing, while (5) does surprisingly well.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Guessing						
Counseling Decisions						
Individualizing Instruction						