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A PARAMETRIC STUDY OF MULTI-STAGE GUN LAUNCHED ROCKETS

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SUMMARY

In order to assess the potential of multi stage gun-launched rockets, a study of the vehicle and trajectory parameters was undertaken. A digital computer program for trajectories was written and was used in an experimental manner to approach optimum performance within various sets of restricting assumptions. The approach was found to be effective and a useful orbital potential was demonstrated with reasonable design parameters.

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INTRODUCTION

During recent years interest has been growing in the use of guns to deliver payloads to high altitudes. A substantial amount of development in this area has been carried out by the Ballistic Research Laboratories of the U.S. Army and by McGill University with support from the U.S. Army through the Army Research Office and B.R.L. (Refs. 1 & 2). The latter effort has been concerned with a 16 inch gun located on Barbados (Refs. 3 & 4). The gun is smooth bored and is mounted to allow firing elevations from about 60° to near vertical. Initial developments were concerned with sub-caliber fin stabilized saboted ballistic projectiles. This class of vehicle has been fairly well proven at this stage, and operations are phasing into scientific data gathering flights. At the same time, a sub-caliber rocket assisted vehicle is being developed. (Ref. 5) This vehicle will eventually be useful for research purposes with higher performance requirements than can be achieved with pure ballistic vehicles. It is also a stage in the development of a multi-stage full bore rocket powered vehicle with orbital capabilities.

In order to provide a basis for orientation and design guidance of a program to develop such a vehicle, it was necessary to indicate achievable performance, based on suitable assumptions and corresponding design requirements.

In particular it was necessary to determine the type of trajectory which should be flown and the corresponding sizing of the rocket stages. It was decided that experimental use of a computer program for trajectory calculation was a promising approach. The intention was to sub-optimize what appeared to be the more important parameters, and then to determine the effects of other variables, with some checking to ensure that no important cross effects were neglected.

Although many computer programs for trajectory computation are available, all have certain restrictions and many include detail which would not be required in a study of this nature. It was decided that a program designed specifically for this task would provide benefits far out weighing the time and expense of writing and debugging the program, particularly since a large high speed computer (IBM 7040) with an advanced and efficient language system (Fortran IV) was available at McGill at reasonable cost. Accordingly, the program was written and a full description of it is included in this report. It has proven to be extremely useful for this study, and it can be modified with little effort for other uses, such as the production of tracking data.

The technique of experimental optimization has proven to be quite effective. Usually the combined effects of two parameters could be determined with sufficient accuracy for these studies from two computer runs comprising nine to twelve cases

(trajectories and vehicles) each. This would cost of the order of $20. in computer time and about the same for preparation and analysis. All told about 500 cases have been run to date and all of the variables considered to be of major importance have been investigated. Although it is not claimed that performance increases could not be achieved by further manipulations of the parameters, it is felt that the results to date represent a close approximation to the ultimate, such that any improvement in performance could be considered a detail refinement and not a fundamental change in design philosophy.

DESCRIPTION OF COMPUTER PROGRAM

The program calculates trajectories for multi-stage gun launched rockets. It is restricted to the three translational degrees of freedom, and to axially symmetric vehicles with constant thrust motors. The earth is assumed to be spherical, and the following functional relationships are pre-programmed: zero lift drag coefficient and normal force coefficient-angle of attack derivative vs Mach number (Fig. 1), ambient pressure and sonic speed vs altitude (Figs. 2 & 3). Internal ballistics are not calculated. Ignition of each stage occurs at a requested value of altitude, absolute flight path angle, or time after burnout of the previous stage (or launch, in the case of the first stage). The attitude of each stage during engine firing may be in the direction of the relative velocity (as for an aerodynamically stabilized vehicle), in the direction of the absolute velocity (tangent to the absolute trajectory), a fixed attitude tangent to the absolute trajectory at ignition, or a fixed attitude specified by elevation and azimuth at ignition.

The equations of motion are written with respect to axes defined as follows:

U - along the radius vector
V - along the horizontal velocity vector
W - to form a right handed orthogonal triad

Position is related through direction cosines to a set of non-rotating axes defined as follows:

X - in the equatorial plane toward the launch longitude at the time of launch
Y - to form a right handed orthogonal triad
Z - normal to the equatorial plane toward the north

The equations are integrated by the Kutta-Gill method (Ref. 6). The symbols used are defined in Appendix I. The mathematical relationships are listed in Appendix II, corresponding in order to the complete program listing in Appendix III.

The inputs to the program are listed in Appendix IV, along with the card formats, and the instructions for deck make-up are in Appendix V. There are twelve general inputs on the first data card for each case, followed by a card for each rocket stage containing eleven inputs. All data fields are five columns and decimal points need not be punched.

The outputs of the program are listed in Appendix VI, which also includes a sample print-out. Inputs are listed automatically at the start of each case. In addition, messages are provided at ignition, burnout, apogee and impact. Latitude and longitude are also provided at ignition, burnout and impact. Perigee and apogee heights are calculated if burnout occurs at an altitude greater than 250,000 feet and the orbit is not hyperbolic. The program continues post final stage burnout to impact only if the calculated perigee height at final stage burnout is below 50 nm. Each case is terminated by a listing of the errors in the U and W unit vectors, and the value of the UW unit vector scalar product, followed by messages concerning calculation error conditions (underflow may be neglected) and computing time.

EXPERIMENTS AND RESULTS

The work completed so far has been restricted to orbital characteristics of a three stage rocket, with variations in gun elevation, first and second stage ignition points, first and second stage thrust direction, and third stage weight (with fixed all up weight). It is obvious that specific impulse, mass fraction and muzzle velocity are major factors in determining performance, but their primary contribution is to the ideal velocity increment. In fact, specific impulse and mass fraction may be traded off against one another or against payload with comparative impunity by maintaining the ideal velocity increment of each stage at the original value. Errors would arise due to relatively slight changes in ballistic coefficient after first stage ignition, but most of the drag loss has been incurred prior to this and these errors should not be significant. Similar calculations with muzzle velocity as an additional parameter are not likely to be as accurate due to drag losses at low altitude and trajectory changes, but could be attempted as a first approximation (i.e. an increase in muzzle velocity could be interpreted as a decrease in all up weight, an increase in payload weight, or decreases in specific impulse or mass fraction, by maintaining the same total ideal velocity increment).

The general approach consisted of selecting arbitrarily an apparently reasonable set of parameters, and then varying two parameters at a time with three or four values each.

One of the variables was always gun elevation, and the apogee and perigee heights after third stage burnout were plotted against gun elevation to determine the achievable circular orbit heights and the corresponding gun elevations. Typical results are sketched in Fig. 4. Apogee and perigee heights are represented by two curves against gun elevation. Where circular orbits are possible, the curves cross, and exchange identification because of the definitions of apogee and perigee. Both curves however are continuous provided that the flight path angle at third stage burnout is zero (i.e. the velocity vector at this point is horizontal). One curve represents the height at burnout, the other represents the height half-way around the earth. Where the ignition time and thrust direction of the third stage are not adjusted correctly to produce zero flight path angle at burnout, the curves cannot quite meet and are modified in the region near the intersection such that apogee and perigee heights respectively are continuous functions of gun elevation.

The following parameters were held constant at the indicated values (latitude, longitude, azimuth, [redacted] launch weight and cross section area refer to the present 16.4 inch gun installation at Barbados).

Muzzle Height	150 ft.
Latitude	13.07 deg.
Longitude	-59.48 deg.
Azimuth	118.23 deg.
Muzzle Velocity	4500 ft. sec.

Launch Weight	2000 lbs.
Number of Stages	3
Cross Section Area	1.478 ft.^{2}
Nozzle Exit Area	0.785 ft.^{2}
Burning Time	20 sec.
Mass Fraction	0.8
Specific Impulse	300 sec.

The third stage was constrained to fire horizontally, and the absolute flight path angle at ignition was adjusted to the nearest 0.01 deg. to keep the absolute flight path angle at burnout within 0.01 deg.

The initial set of parameters included a fairly light payload of 16 lbs., geometric staging (equal ideal velocity increments), first stage ignition at 100,000 ft., and first and second stage thrust direction type 0 (see list of firing indicator values, Appendix IV). The first parameter varied in addition to gun elevation was the second stage ignition point, and the results are shown in Fig. 5. It is obvious that the second stage should be ignited as soon as possible after first stage burnout. A similar effect for first stage ignition is indicated in Fig. 6 (16 lbs. payload) and Fig. 7 (40 lbs. payload), although in the latter case the increase in drag losses due to ignition at low altitudes evidently is a limiting factor, emphasized by the low launch angle. Fig. 6 also shows checkpoints indicating the effect of second stage ignition delay and first stage firing direction. It is apparent that performance is not extremely sensitive to these factors.

The effect of second stage ignition delay was rechecked for a 40 lb. payload, and at the same time the effect of second stage firing angle was investigated (Fig. 8). Again, the effects are not of major importance within reasonable ranges of values; the desirability of early second stage firing is still evident, and it is also clear that a g-turn trajectory is close to the optimum.

Attention was then focussed on stage sizing. Since the first and second stages should be fired in quick succession, it was assumed that geometric sizing (equal velocity increments) would be near optimum for these stages, and the more significant effect would be the weight of the third stage (and thus the contribution of the third stage to the total ideal velocity increment). This effect is shown in Figs. 9, 10 & 11. Again, performance is not extremely sensitive to parameter variations. The optimized configuration yields a slight increase in circular orbit height, and exhibits rather large variations from geometric staging (Fig. 12). Comparison of Fig.6 & Fig.7 indicates that the first stage ignition altitude should increase as the payload increases. Thus the performance indicated in Fig. 11 (constant first stage ignition altitude at 25,000 ft) is probably less than that achievable with the heavier payloads. In fact, the point at 50 lb. payload required lowering the second stage firing angle, and stage weights were not optimized since it probably

would have been necessary to consider the weights of all three stages as well as the second stage firing angle and the gun elevation. It may be possible to retain a g-turn trajectory with later ignition.

It is interesting to note that as the parameter values approach the optimum, the gun elevation approaches the neighbourhood of 45°, and the slopes of the curves of apogee and perigee height vs. gun elevation approach equal magnitude (with opposite signs). Conversely, for any condition which is far from optimum, these characteristics are not apparent.

It is of interest to examine the losses involved in the trajectories considered. Considering two payloads (16 and 40 lb.), optimized third stage weight and first stage ignition at 25,000 ft. (not optimum for 40 lb. payload) we have the following energy balances:-

Payload Weight - Lb.		16	40
Input			
Muzzle Velocity	ft/sec	4500	4500
Rocket Velocity Increment	ft/sec	29019	25338
Total Velocity Increment	ft/sec	33519	29838
Energy	10^6 ft^2/sec^2	561.8	445.2
Initial State			
Radius	10^6ft	20.93	20.93
Velocity	ft/sec	1492	1492
Potential Energy	10^6 ft^2/sec^2	-672.7	-672.7
Kinetic Energy	10^6 ft^2/sec^2	1.1	1.1
Total	10^6 ft^2/sec^2	-671.6	-671.6

Final State

Radius	10^6 ft	30.18	23.11
Velocity	ft/sec	21598	24682
Potential Energy	10^6 ft^2/sec^2	-466.4	-609.2
Kinetic Energy	10^6 ft^2/sec^2	233.2	304.6
Total	10^6 ft^2/sec^2	-233.2	-304.6

Energy Loss

Absolute	10^6 ft^2/sec^2	123.4	78.2
Percent of Input		22.0	17.6

In order to show the effects of specific impulse and mass fraction, it seems reasonable to select a total ideal rocket velocity increment which is representative of that required for circular orbit entry using the 16 inch gun at Barbados. This has been done for a velocity of 24,400 ft/sec and the results are shown in Fig. 14. Geometric staging was assumed.

Typical trajectory characteristics are shown in Fig. 15 (height vs. range), Fig. 16 (height vs. airspeed) and Fig. 17 (height and airspeed vs. time). Typical vehicle parameters are listed below.

STAGE	Weight -- Lb. Propellant	Remainder	Total
1	1152	288	1440
2	320	80	400
3	96	24	120
Payload	0	40	40
3 & Payload	96	64	160
2, 3 & Payload	416	144	560
1, 2, 3 & Payload	1568	432	2000

CONCLUSIONS AND RECOMMENDATIONS

The orbital capabilities of a multi-stage gun launched rocket have been demonstrated. In particular, it has been shown that with the existing 16 inch gun installation at Barbados, launching a 2000 lb. three stage vehicle at 4500 ft/sec., a payload of 40 lb. can be placed in a 400 nm circular orbit (based on a specific impulse in vacuum of 300 sec. and a propellant mass fraction of 0.8)

Optimum trajectories for entry into circular orbits are close to the g-turn type, with gun elevation in the neighborhood of 45°, first stage ignition in the 25-100,000 ft. range and second stage ignition as soon as possible after first stage burnout. Optimum vehicles involve third stage velocity increments somewhat less than one-third of the total (for three stage vehicles) but the penalty of equal velocity increment staging is small.

Further work in this area could be directed toward heavier payloads (or lower specific impulse, or lower mass fraction), lower muzzle velocity, long burning times, higher gun elevations, and variations in drag (or atmospheric properties), launch location, and azimuth.

It is hoped that the optimization procedure will be programmed in the near future, so that the optimum vehicle and/or trajectory will be produced automatically, at least for a limited number of parameters.

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   McGill Engineering Technical Note 62-1

F I G U R E 1

AERODYNAMIC CHARACTERISTICS

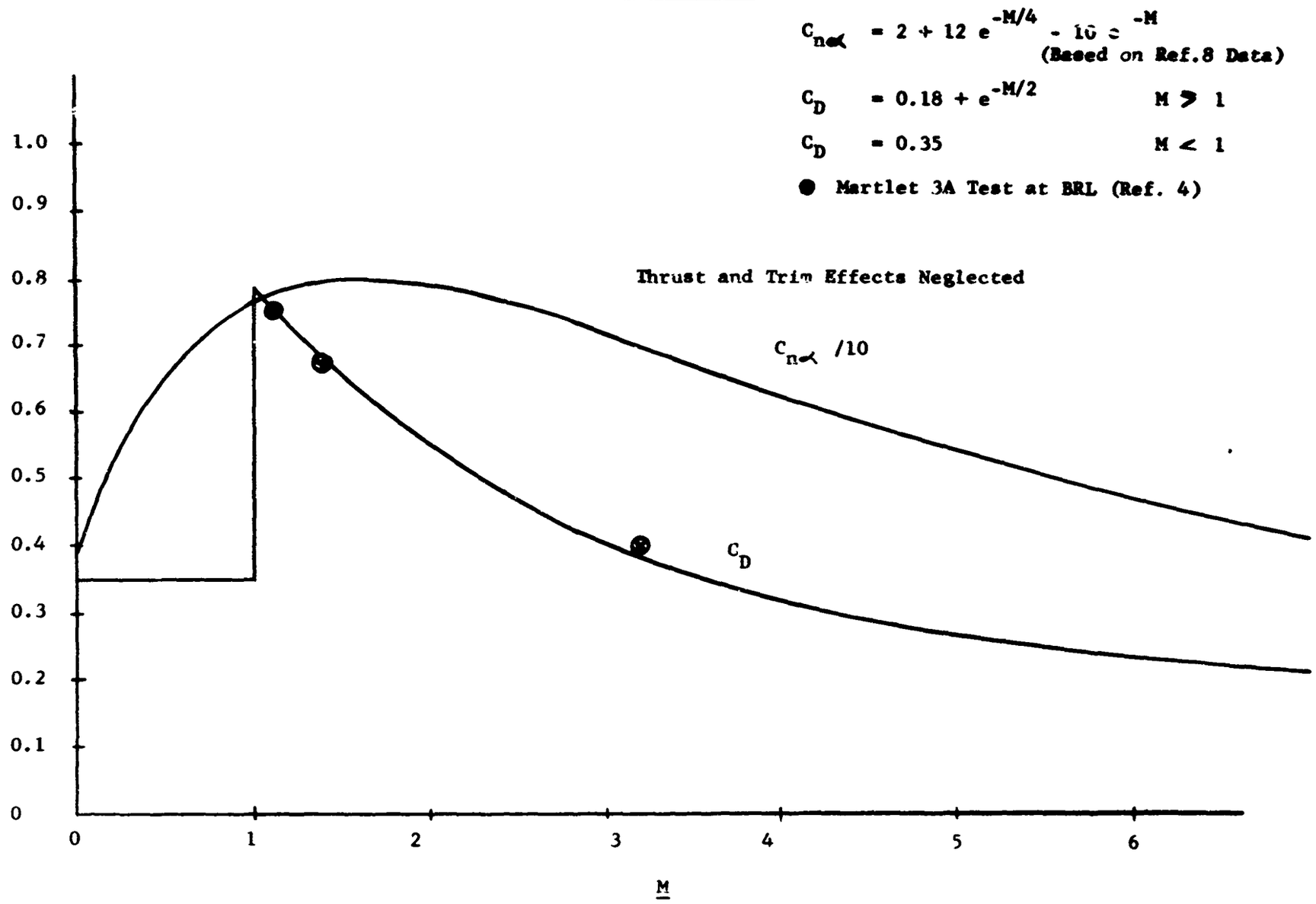


FIGURE 2

ATMOSPHERIC PRESSURE APPROXIMATION ERROR

Based on U.S. 1962 Standard Atmosphere (Ref. 5)

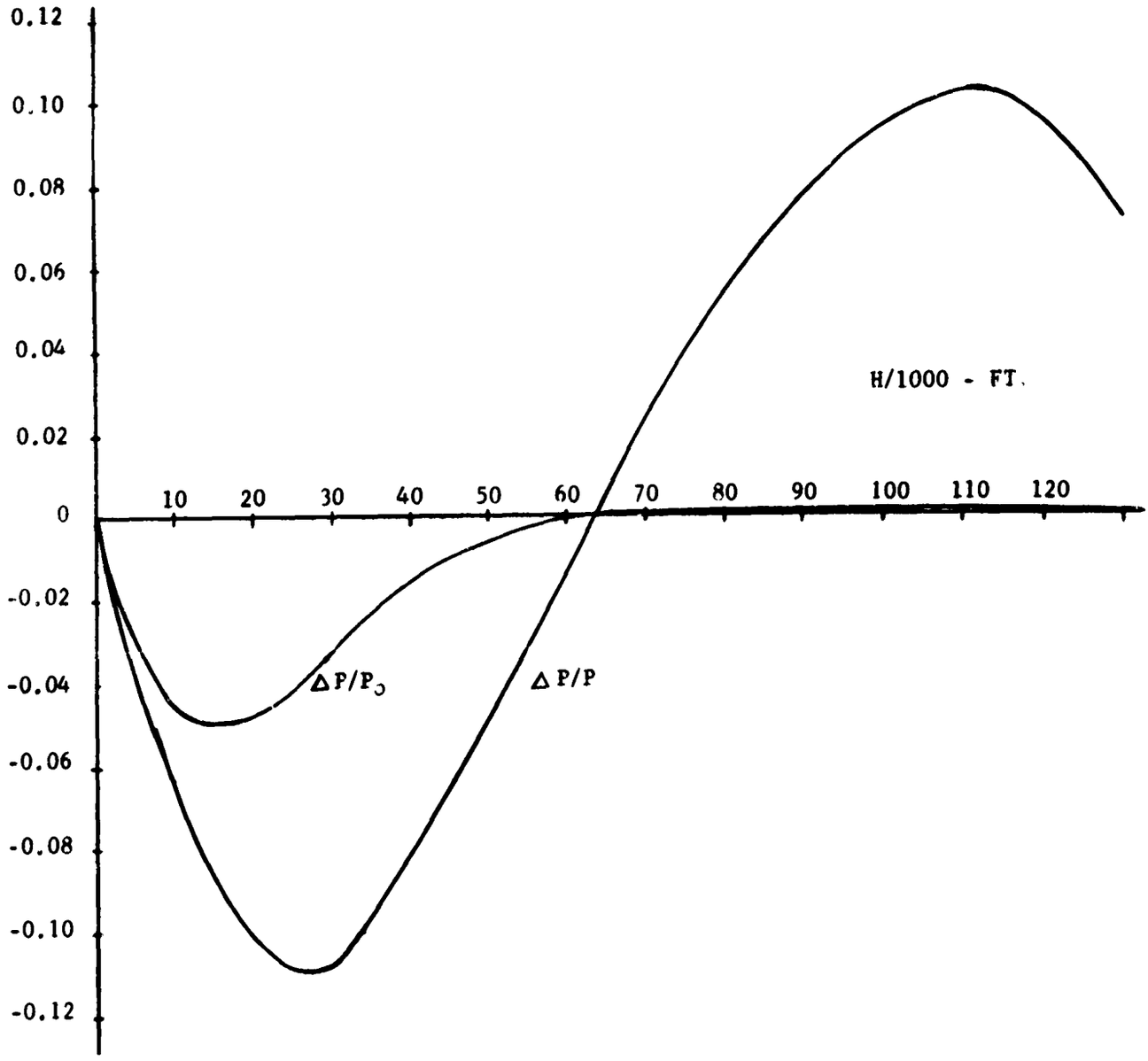


FIGURE 3

SONIC SPEED APPROXIMATION ERROR

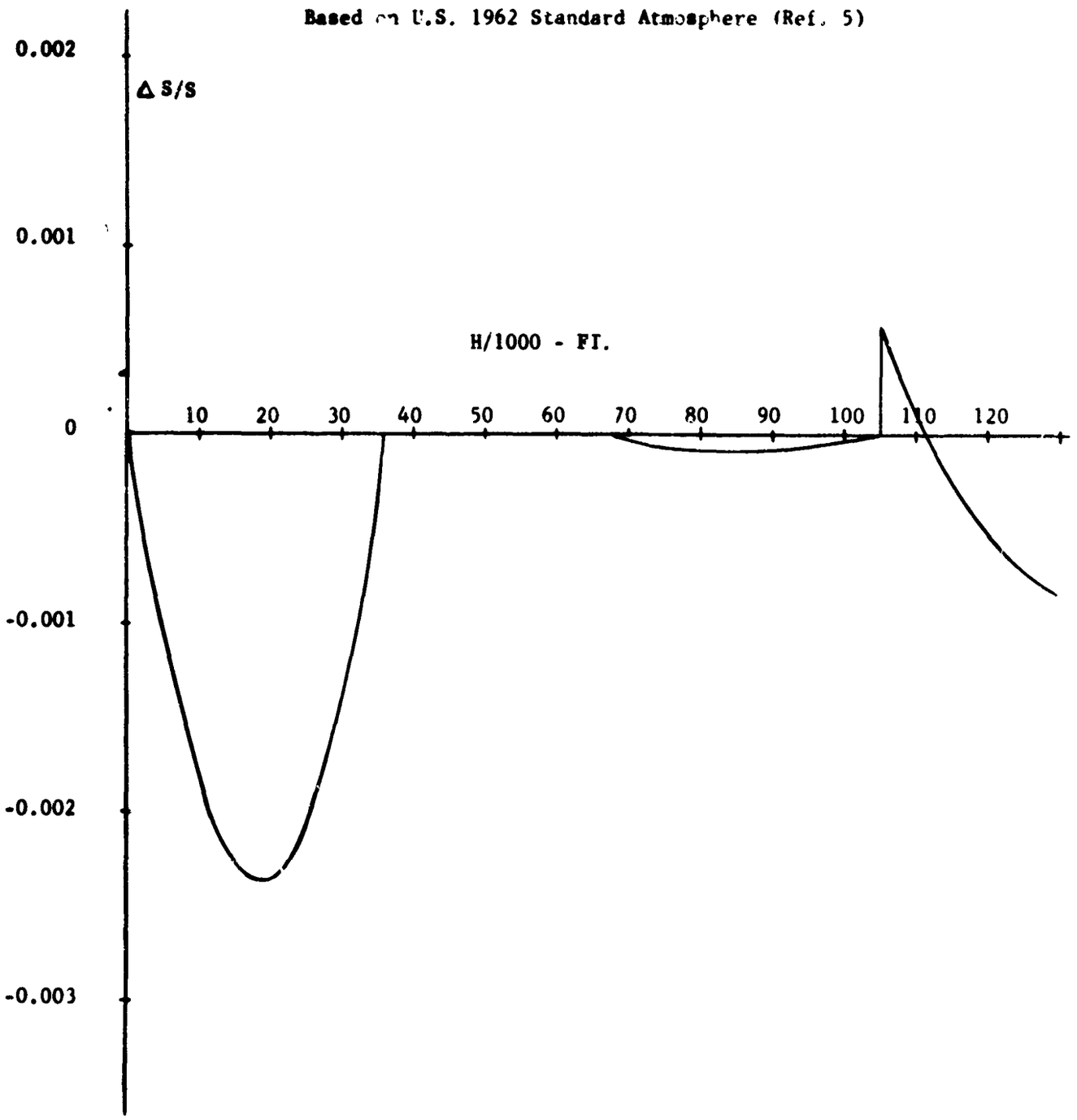
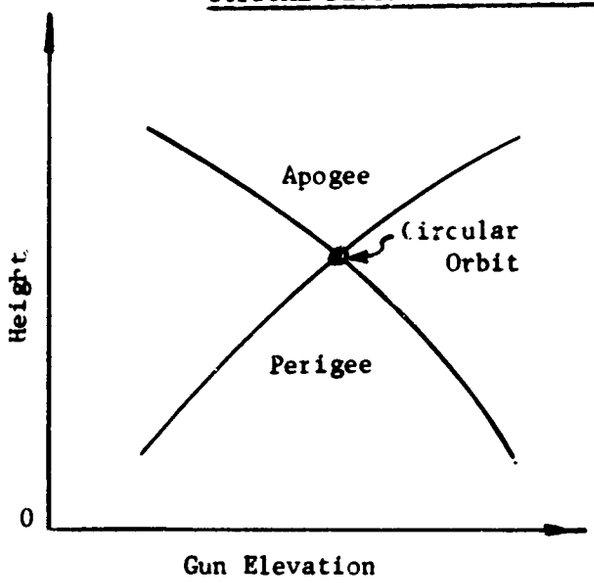
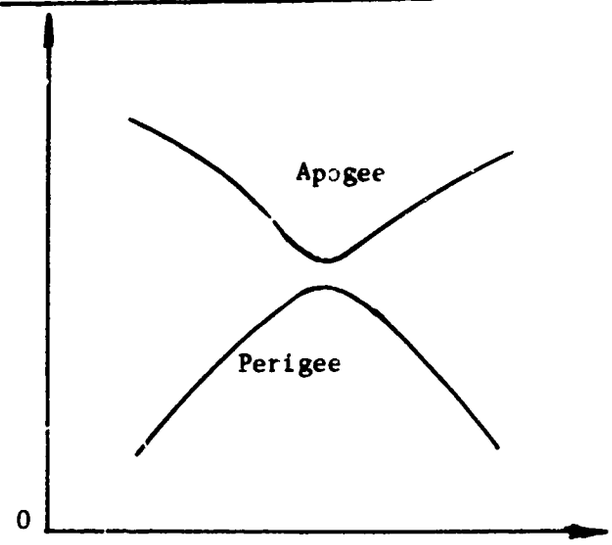


FIGURE 4

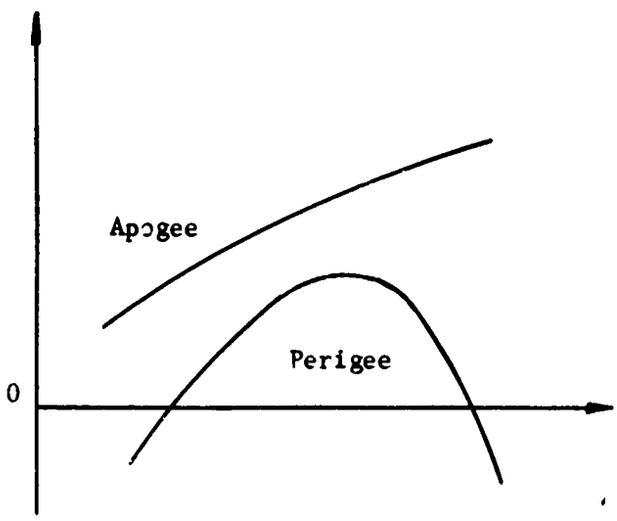
TYPICAL PLOTS OF APOGEE AND PERIGEE HEIGHT VS. GUN ELEVATION



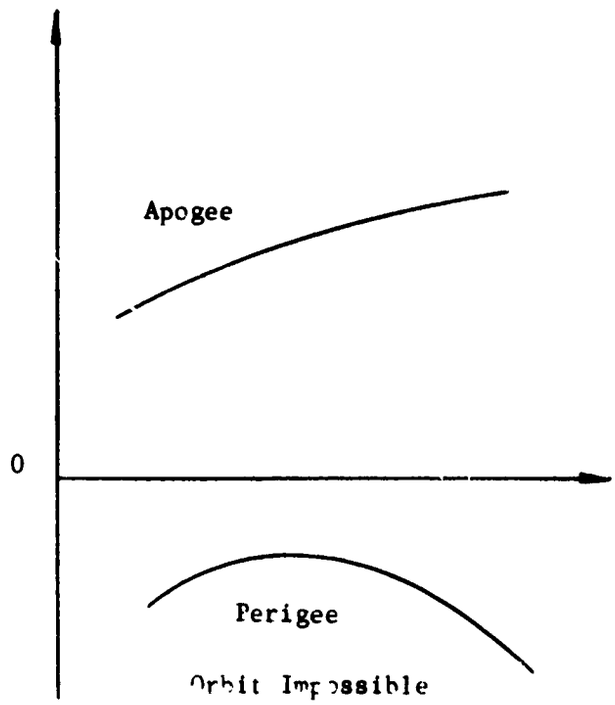
Correctly Adjusted for Circular Orbit



Incorrect Adjustment of Last Stage; Non-Zero Flight Path Angle at Burnout



Circular Orbit Impossible with these Values of the Parameters; Orbit is Possible



Orbit Impossible

FIGURE 5

EFFECT OF SECOND STAGE IGNITION POINT

Payload 16 Lb.
Third Stage Weight 64 Lb. (Geometric Staging)
First Stage Ignition at 100,000 Ft.
First & Second Stage Thrust Direction Type 0

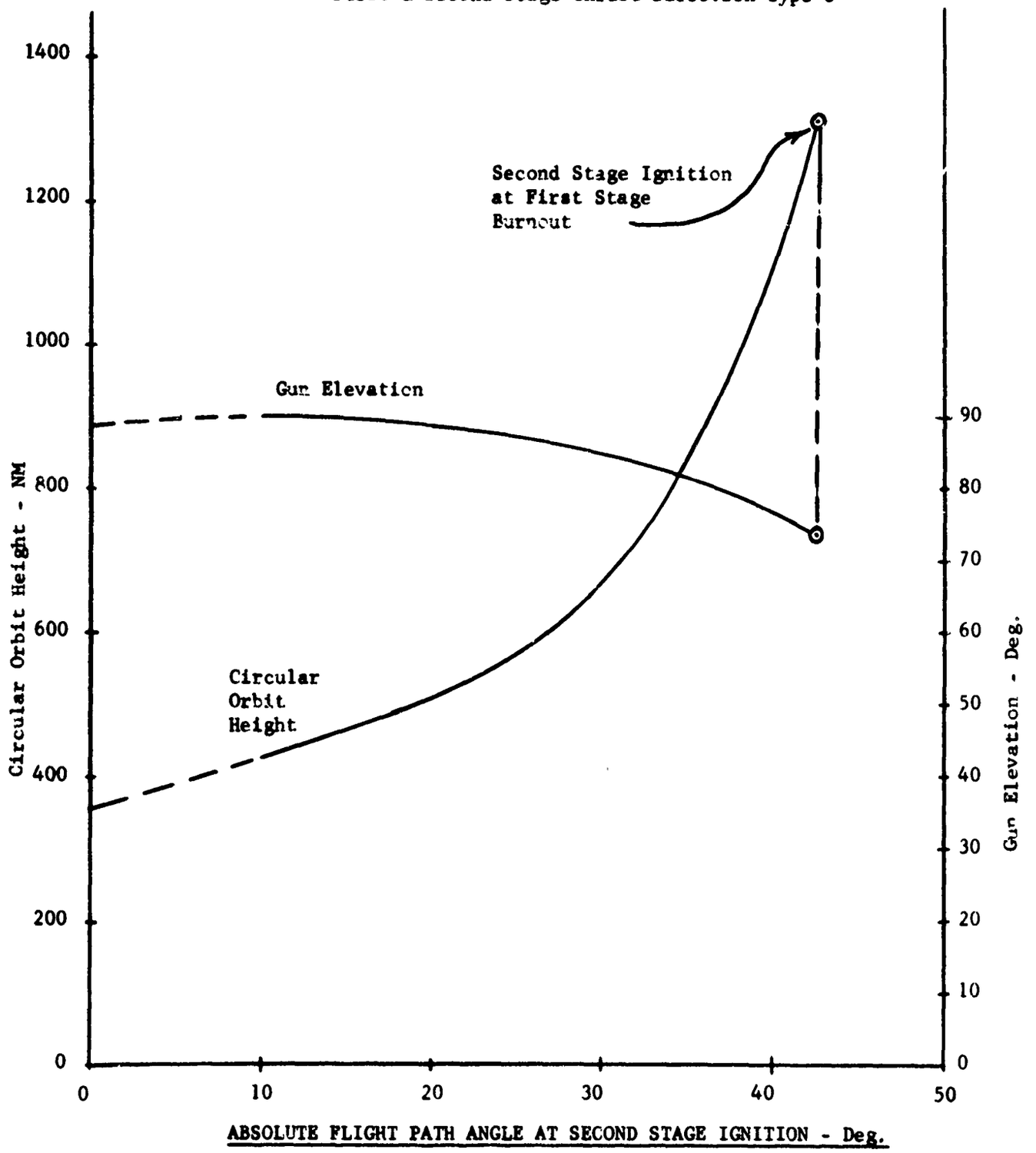


FIGURE 6

EFFECT OF FIRST STAGE IGNITION POINT

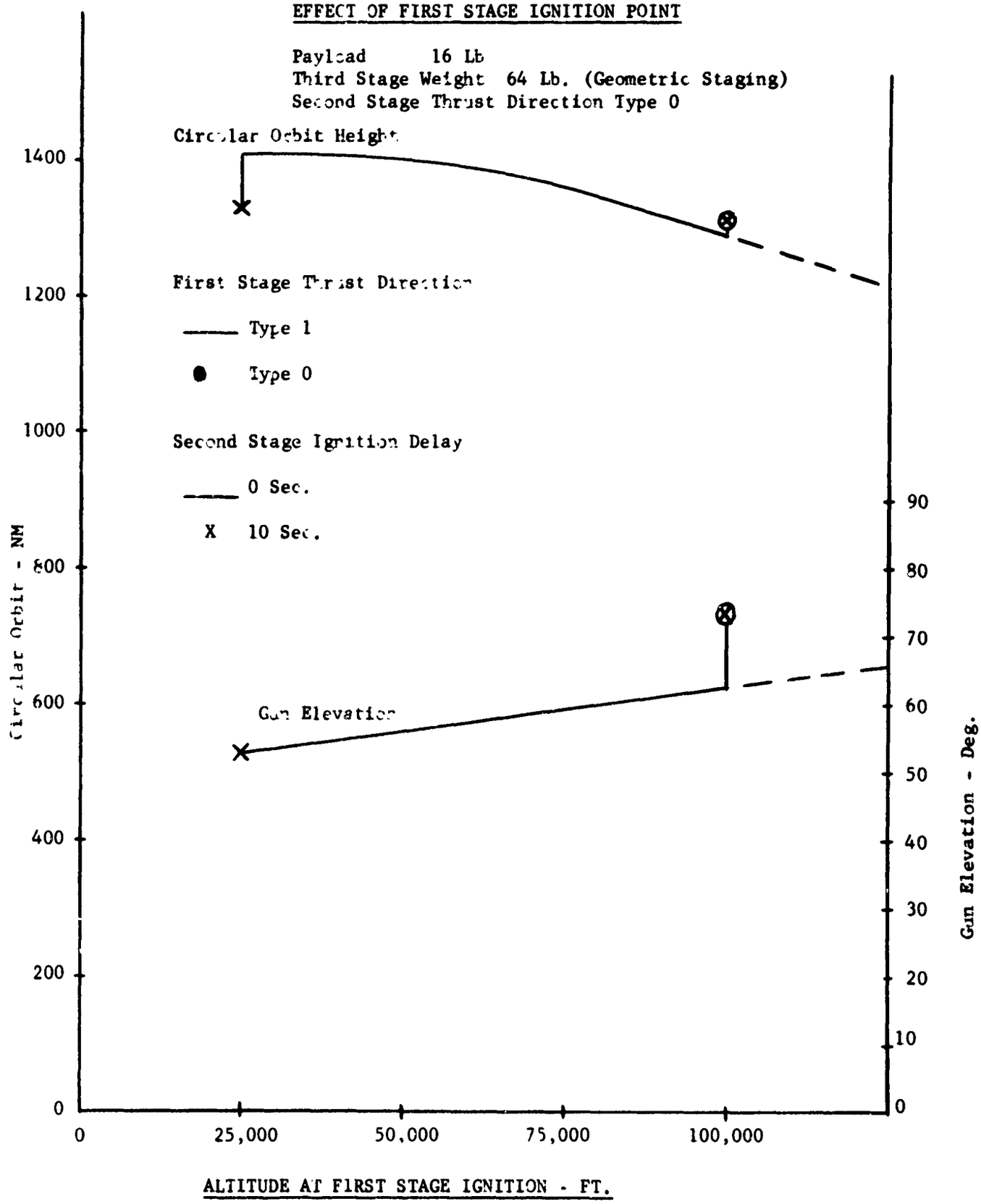


FIGURE 7

EFFECT OF FIRST STAGE IGNITION POINT

Payload 40 Lb.
Third Stage Weight 140 Lb.

Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0

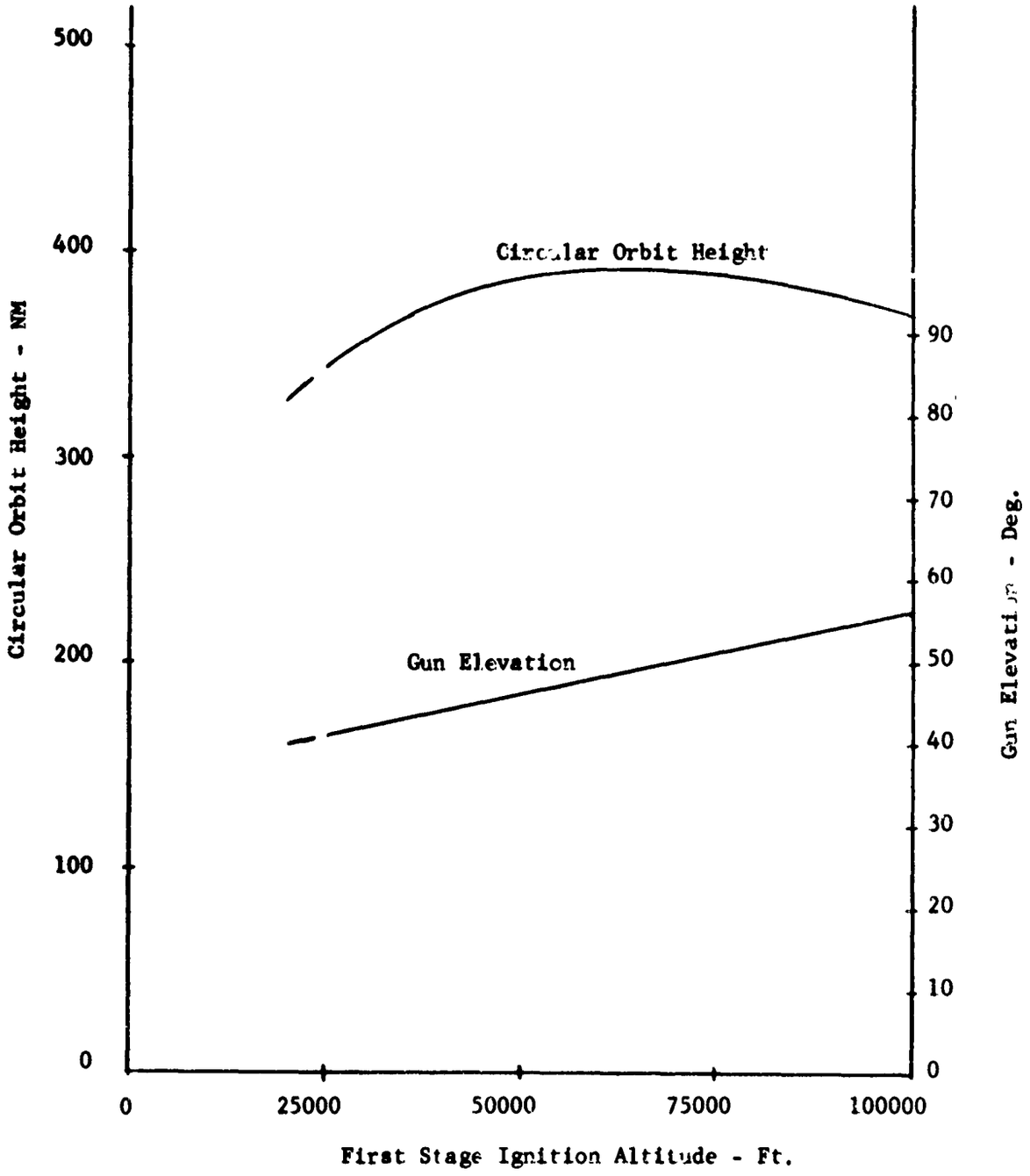


FIGURE 8

EFFECT OF SECOND STAGE IGNITION DELAY AND THRUST DIRECTION

Payload 40 lb.
Third Stage Weight 140 Lb.

First Stage Ignition at 25000 Ft.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 2

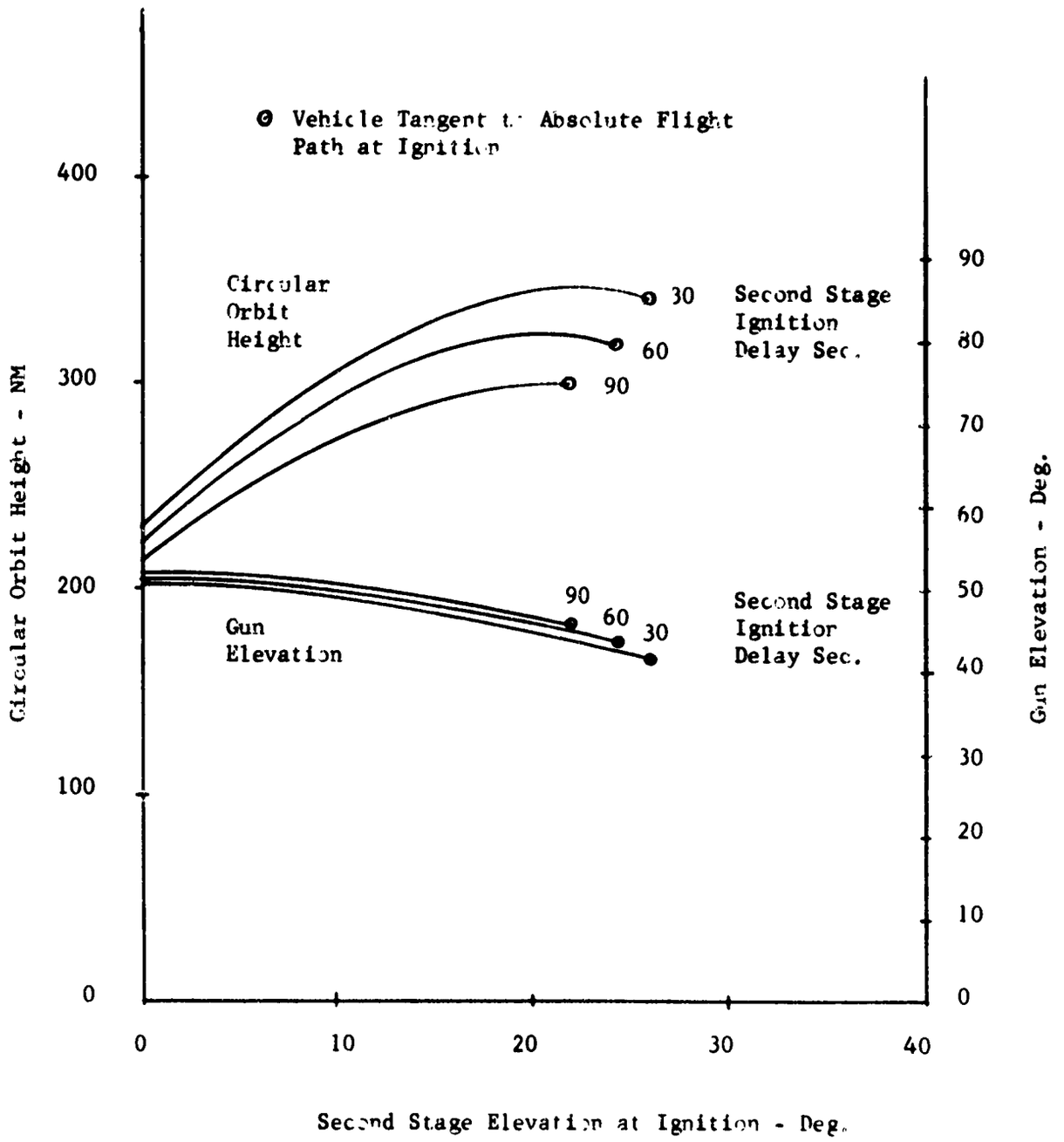
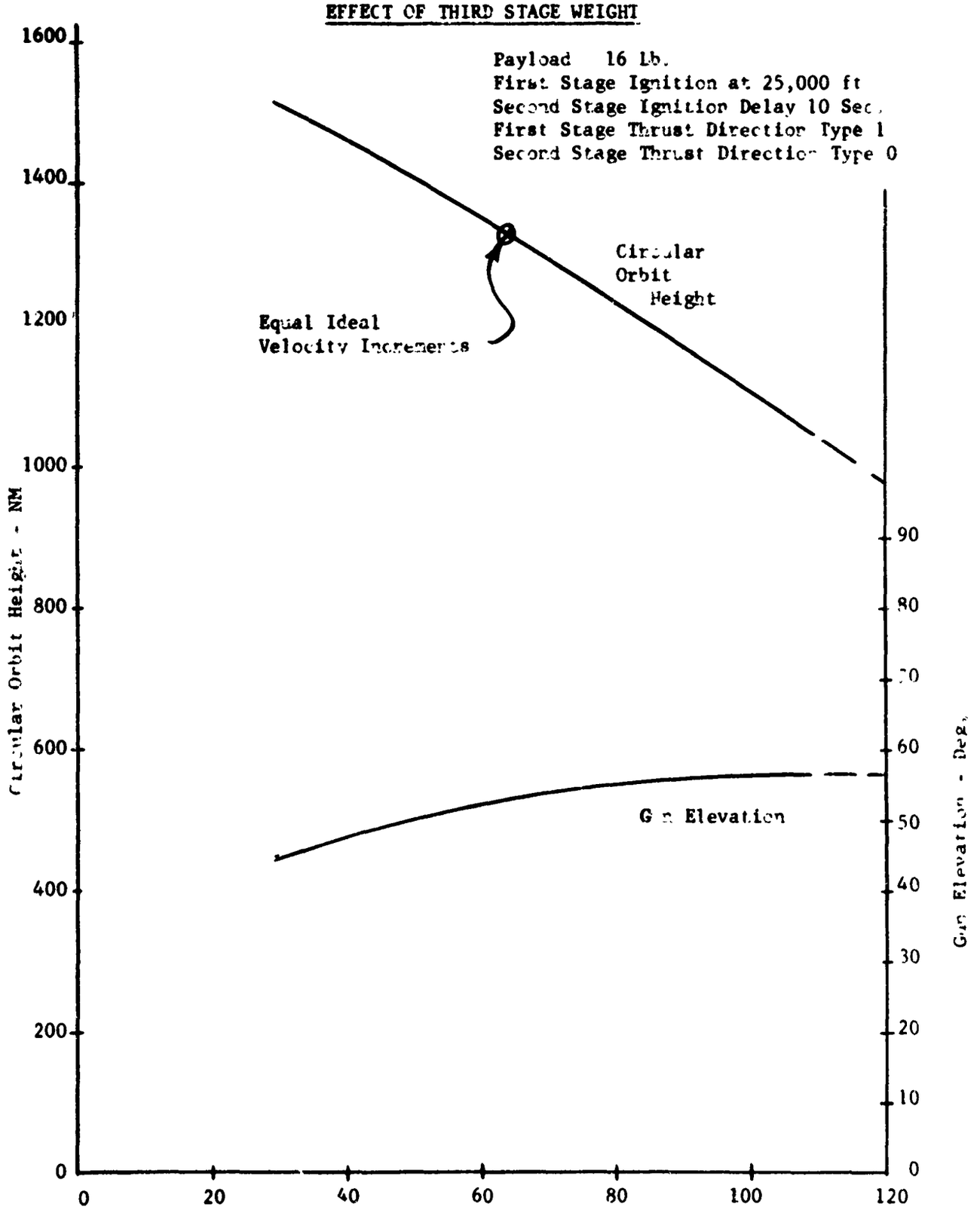


FIGURE 9

EFFECT OF THIRD STAGE WEIGHT



THIRD STAGE WEIGHT - LB

FIGURE 10

EFFECT OF THIRD STAGE WEIGHT

Payload 40 Lb.
First Stage Ignition at 25,000 ft.
Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0

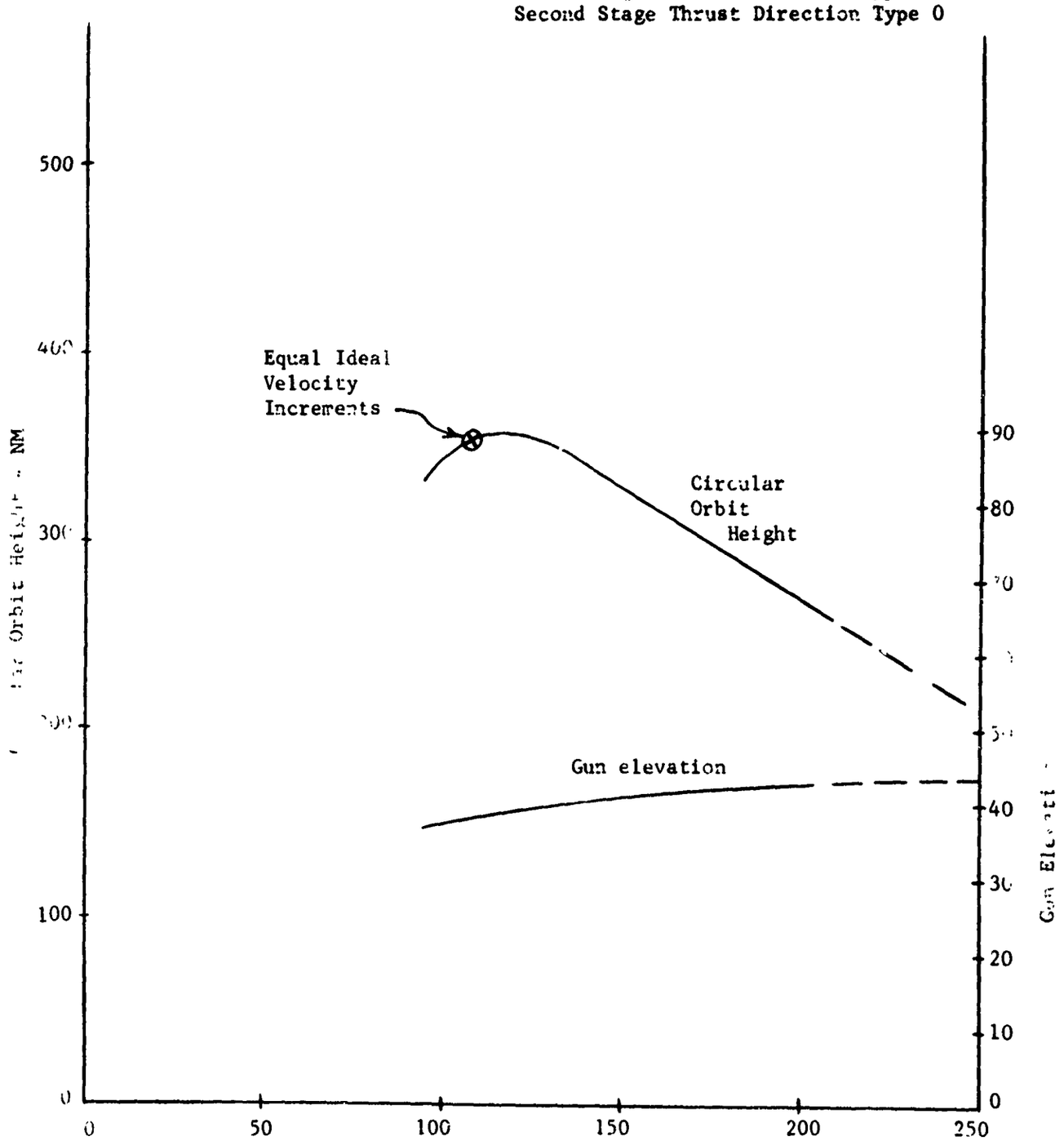


FIGURE 11

EFFECT OF OPTIMIZING THIRD STAGE WEIGHT

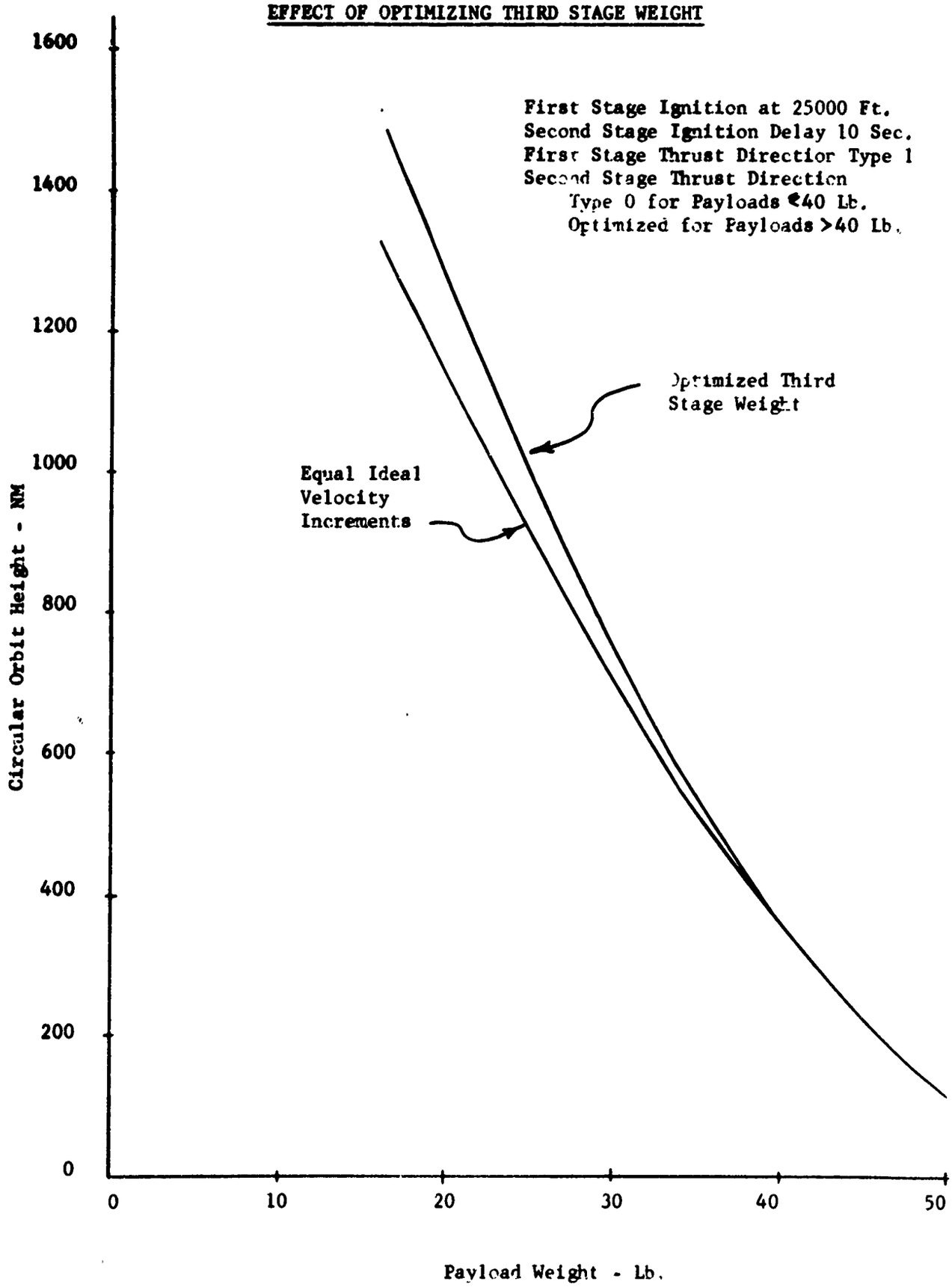


FIGURE 12

COMPARISON OF NOMINAL AND OPTIMIZED THIRD STAGE WEIGHT

First Stage Ignition at 25000 Ft.
Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0

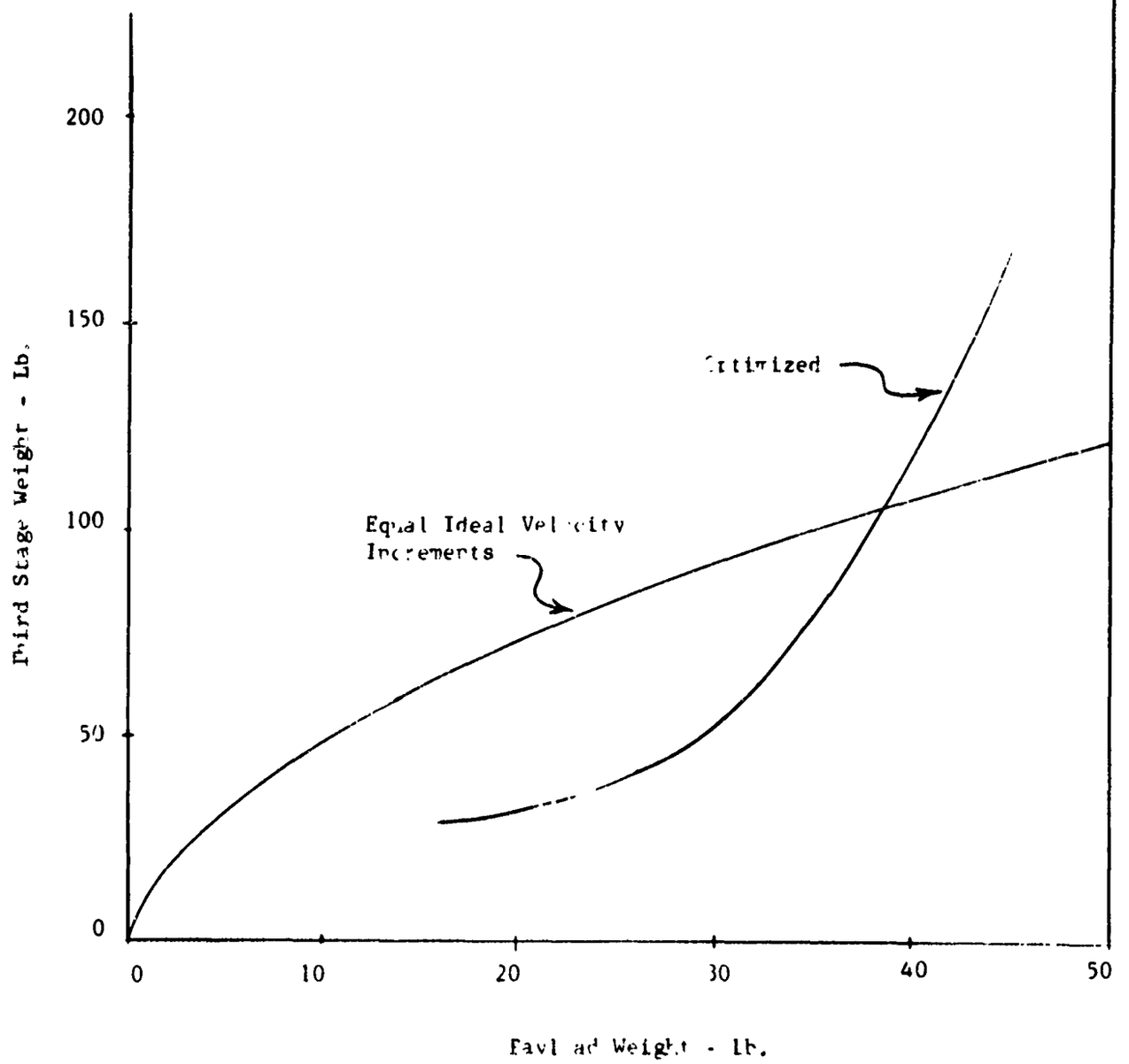


FIGURE 13

EFFECT OF SECOND STAGE THRUST DIRECTION

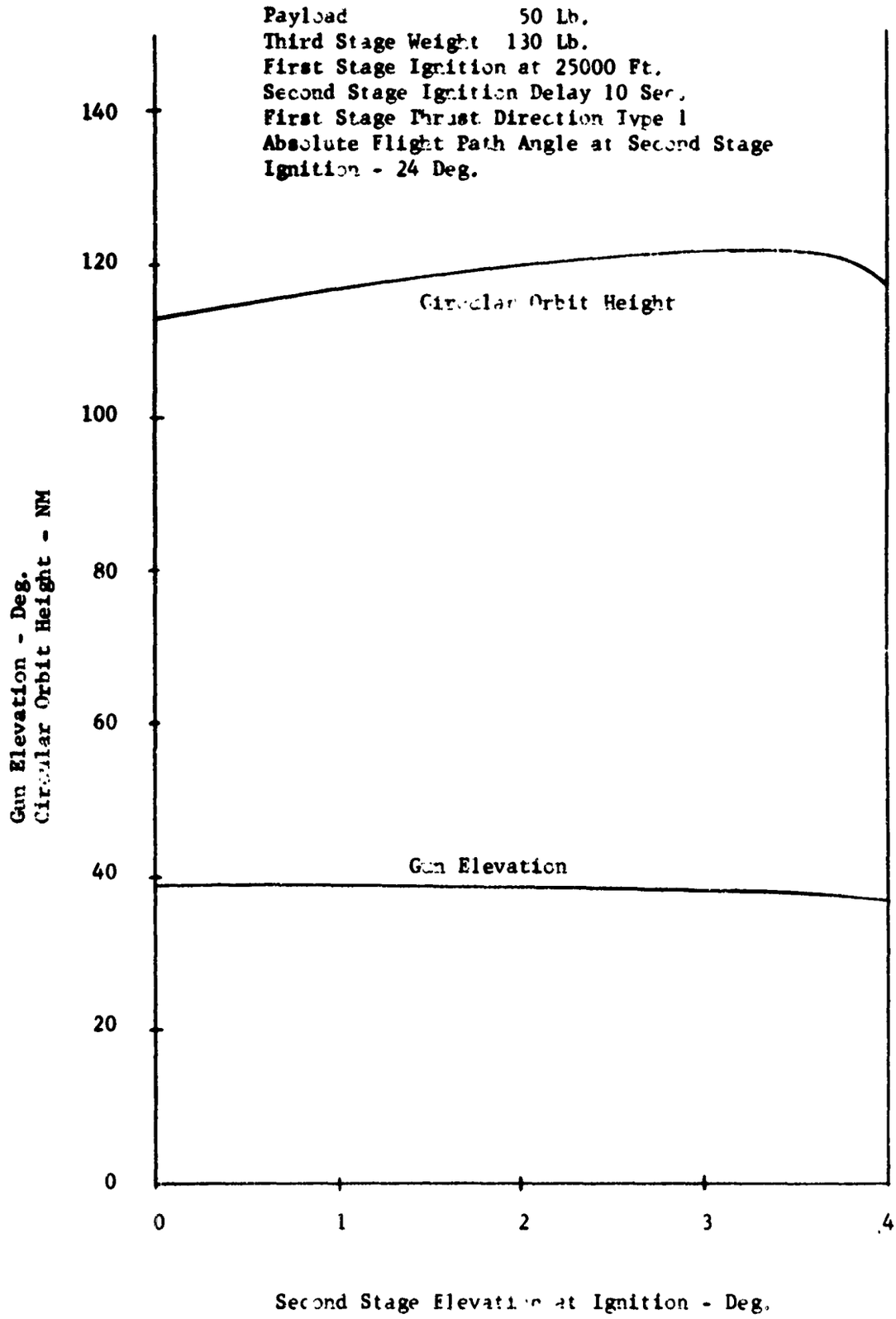


FIGURE 14

EFFECT OF SPECIFIC IMPULSE AND MASS FRACTION

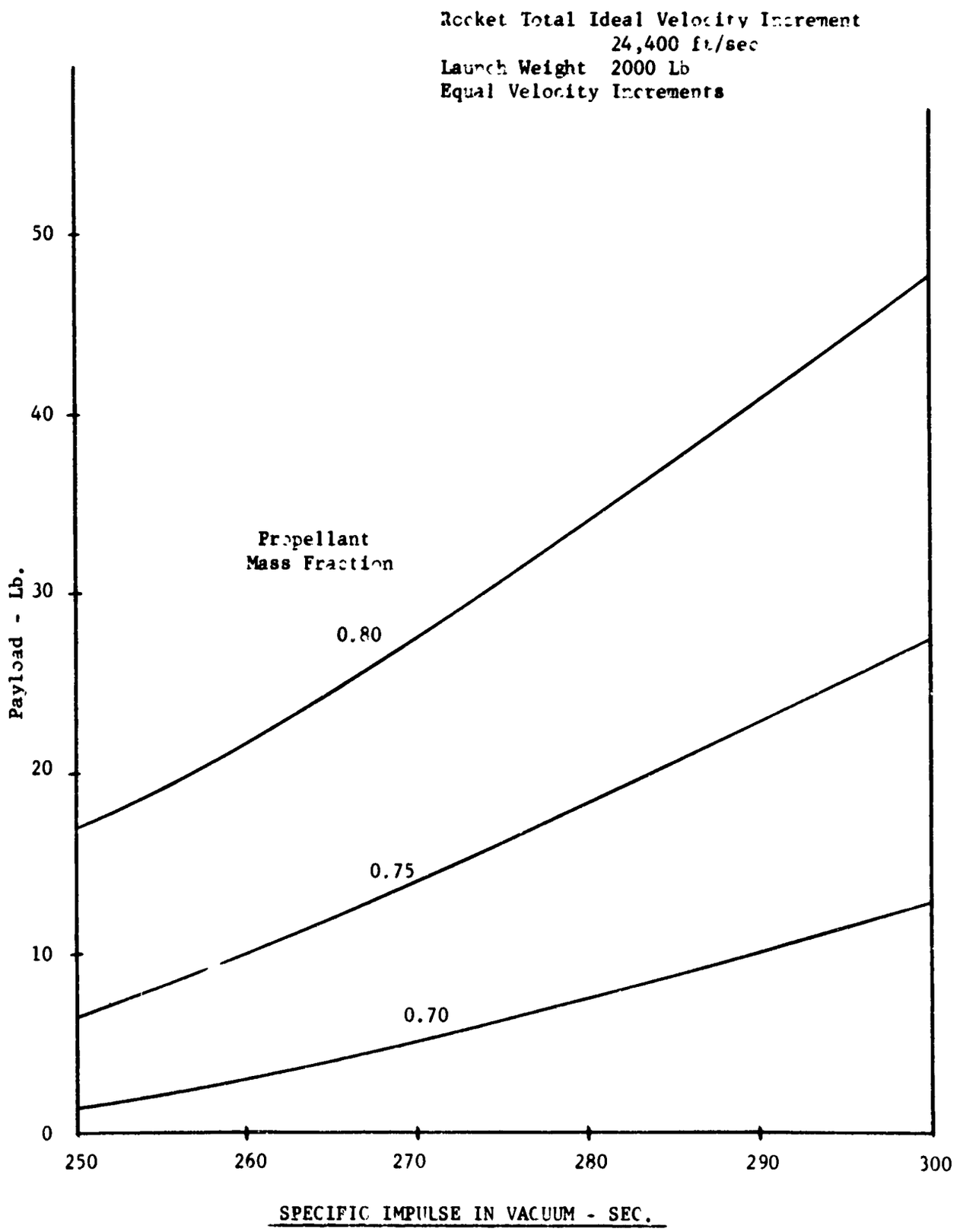


FIGURE 15

TYPICAL TRAJECTORY - HEIGHT VS. RANGE

Payload 40 Lb.
Specific Impulse 300 Sec.
Mass Fraction 0.8
Burning Times 20 Sec.
Gun Elevation 39 Deg.
First Stage Ignition at 25,000 Ft.
First Stage Direction Type 1
Second Stage Ignition Delay 10 Sec.
Second Stage Direction Type 0

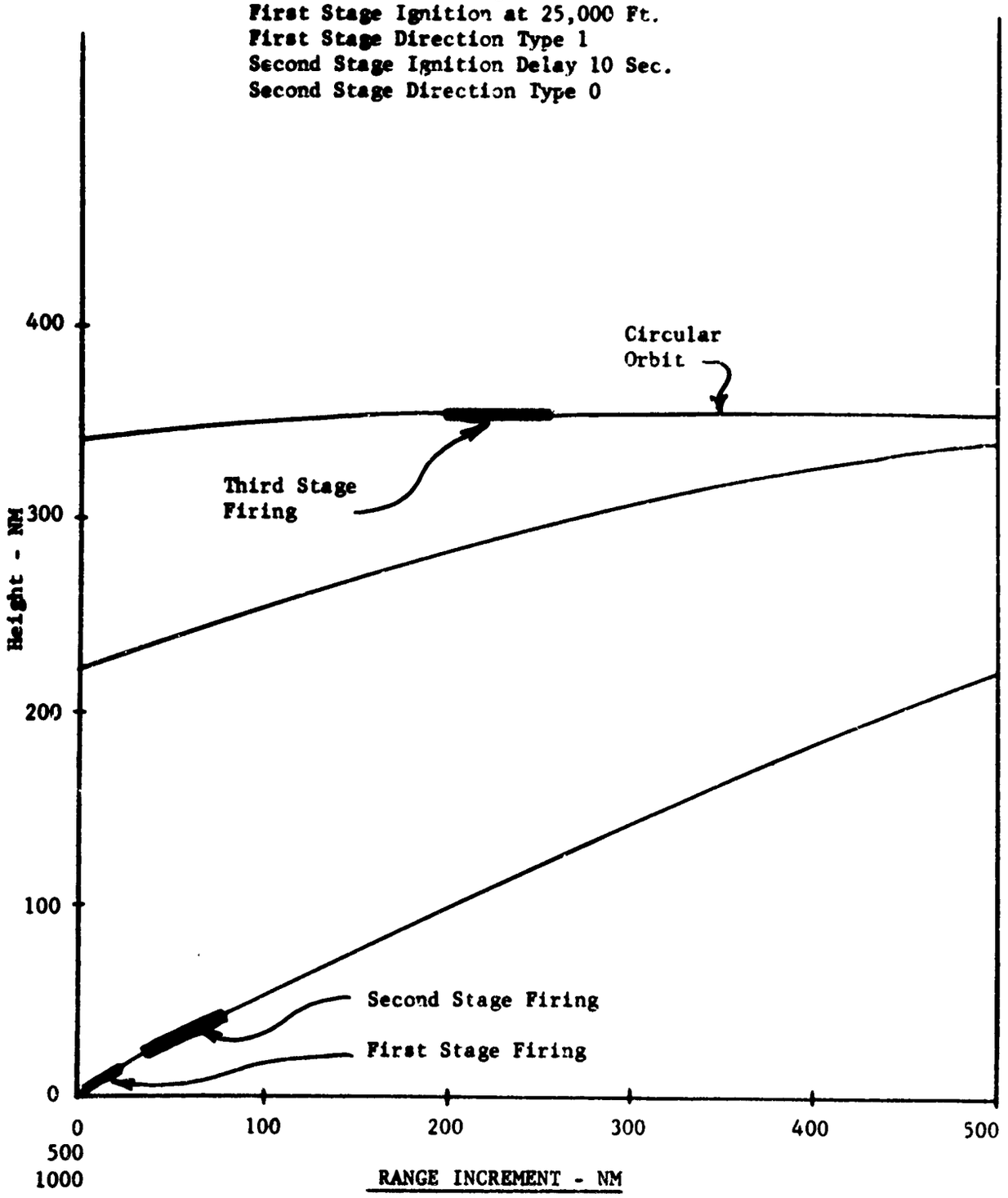
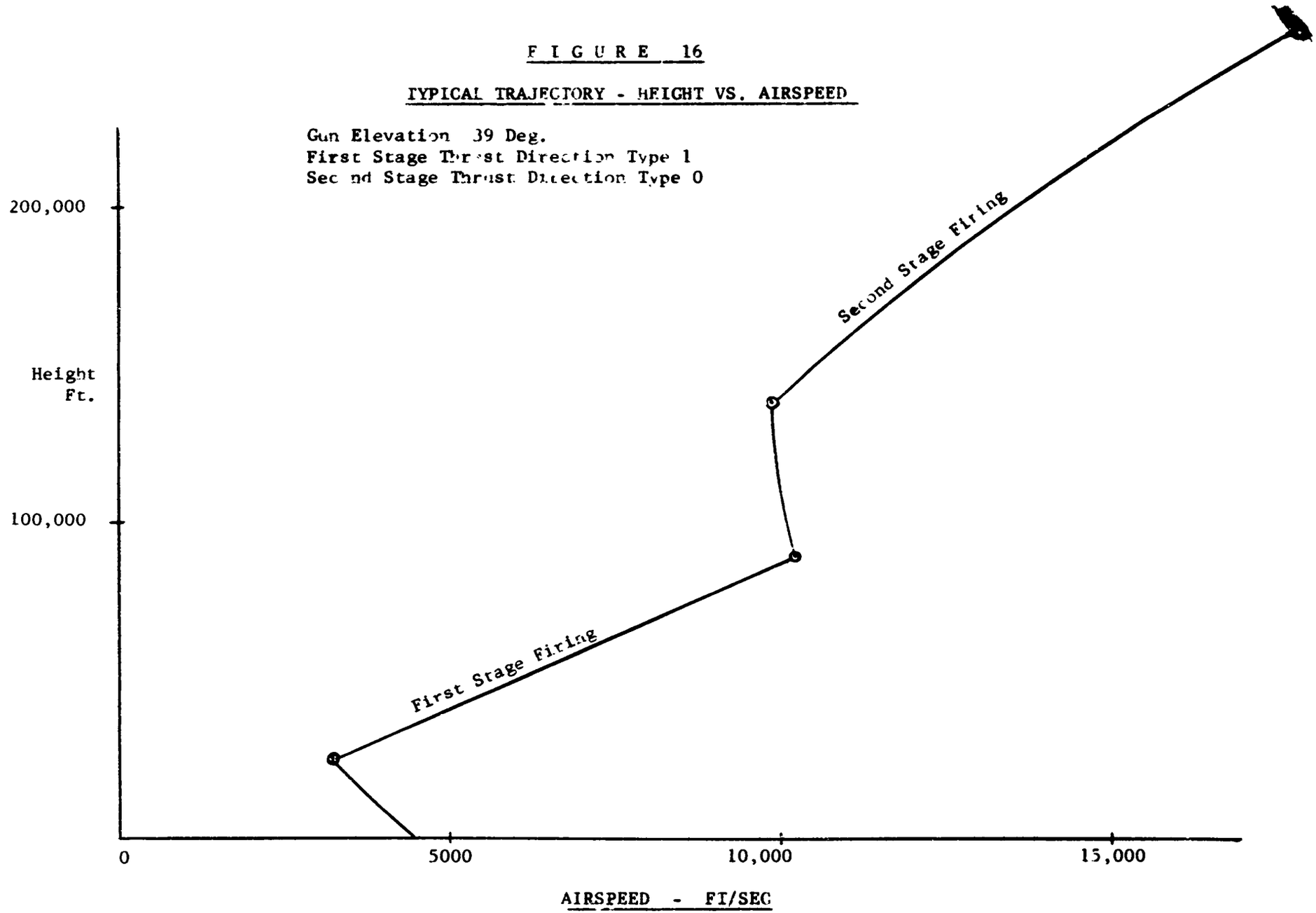


FIGURE 16

TYPICAL TRAJECTORY - HEIGHT VS. AIRSPEED



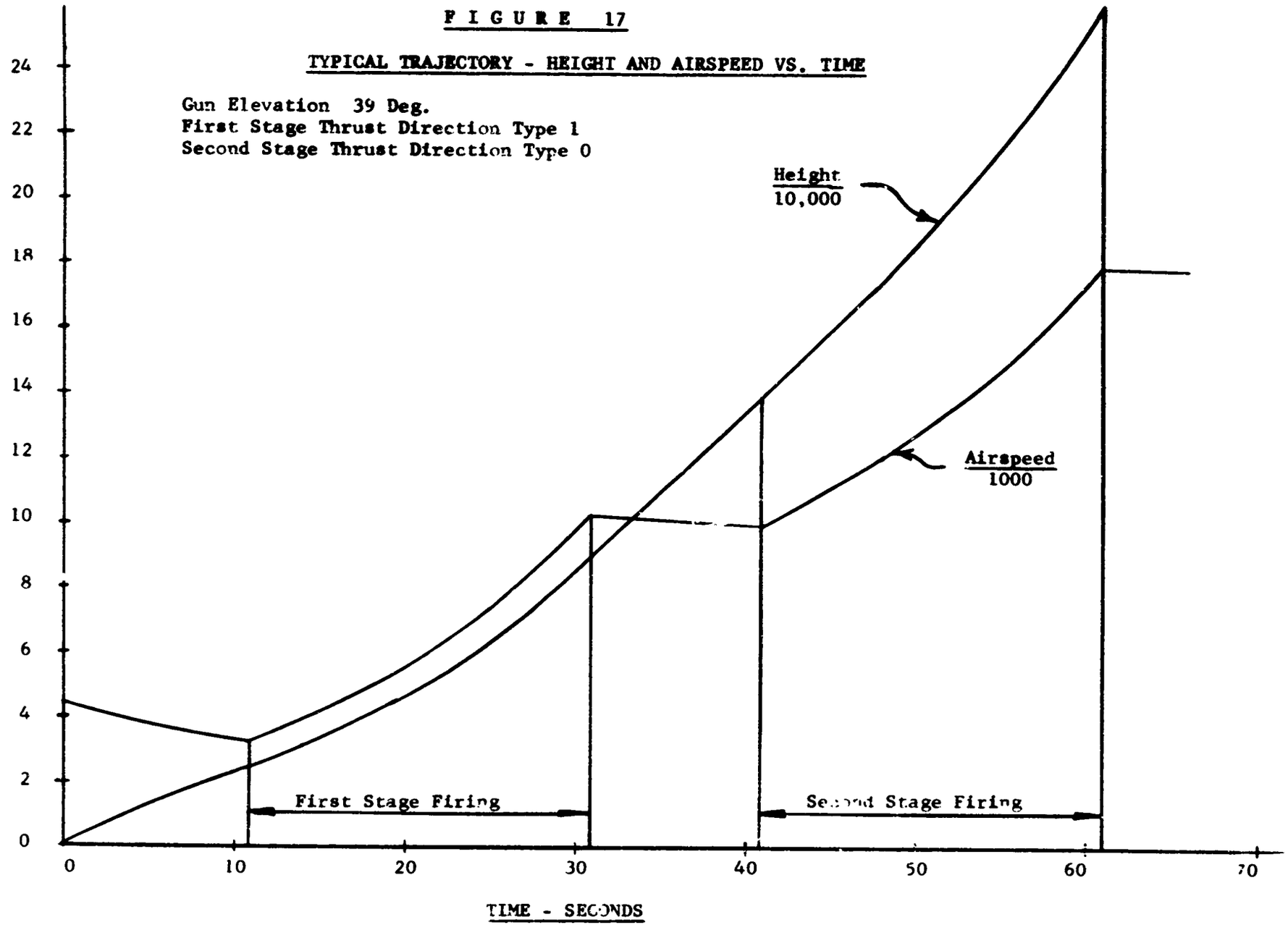


FIGURE 17
TYPICAL TRAJECTORY - HEIGHT AND AIRSPEED VS. TIME
Gun Elevation 39 Deg.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0
Height / 10,000
Airspeed / 1000
First Stage Firing
Second Stage Firing
24
22
20
18
16
14
12
10
8
6
4
2
0
0
10
20
30
40
50
60
70
TIME - SECONDS

APPENDIX I

LIST OF SYMBOLS

FORTRAN	MATH	DEFINITION
AP	P	Atmospheric Pressure
AT	ϕ	Latitude (See Note (2))
ATD	ϕ°	Latitude (Degrees)
ATG	ϕ_g°	Latitude of Gun (Degrees)
ATO	ϕ_g	Latitude of Gun
AVU		Length of Unit Vector on U Axis Minus 1.
AVW		Length of Unit Vector on W Axis Minus 1.
AX	a	Semi-Major Axis of Orbit
AZ	β	Azimuth (See Note (1))
AZD	β°	Azimuth (Degrees)
AZG	β_g°	Azimuth of Gun (Degrees)
AZS (N)	β_s	Azimuth of Thrust Vector at Ignition (Degrees)
C (I)	K_i	Kutta-Gill K for Variable I
CAZ	$\cos \beta$	
CD	C_d	Drag Coefficient
CEL	$\cos \delta$	
CLA	$\cos \phi$	
CNA	C_{n_α}	Normal Force Coefficient-Angle of Attack Derivative
CT	C	Iteration Count
DL		Computing Time in 60ths of a Second
DLS		Computing Time in Seconds
DR	D	Drag

FORTRAN	MATH	DEFINITION
DT	Δt	Step Size
DTB (N)	Δt_b	Burning Time
DTD	Δt_p	Possible Step Size; Previous Step Size
DTI	Δt_I	Nominal Step Size
DTP	Δt_p	Nominal Print-Out Interval
DU	δ_u	U-Direction Cosine of Thrust Vector (Body Axis)
DV	δ_v	V-Direction Cosine of Thrust Vector (Body Axis)
DW	δ_w	W-Direction Cosine of Thrust Vector (Body Axis)
DX	δ_x	X-Direction Cosine of Thrust Vector (Body Axis)
DY	δ_y	Y-Direction Cosine of Thrust Vector (Body Axis)
DZ	δ_z	Z-Direction Cosine of Thrust Vector (Body Axis)
EA (N)	A	Exhaust Area
EL	ϵ	Elevation
ELD	ϵ^o	Elevation (Degrees)
ELG	ϵ_g^o	Elevation of Gun (Degrees)
ELS (N)	ϵ_s^o	Elevation of Thrust Vector at Ignition (Degrees)
EM	M	Mach Number
F	F	$108/\pi$
FN	F_n	Normal Force
FNU	F_{nu}	U-Component of Normal Force
FNV	F_{nv}	V-Component of Normal Force
FNW	F_{nw}	W-Component of Normal Force

FORTRAN	MATH	DEFINITION
FR (N)	f	Mass Fraction
GC	k_e^2	Gravitational Constant
GO	g_o	Acceleration due to Gravity at S.L.
HA	H_A	Apogee Height
HP	H_p	Perigee Height
HT	H	Height (Feet)
HTN		Height (Nautical Miles)
I	i	Variable Number
I1 (N)	I_1	Ignition Indicator
I2 (N)	I_2	Firing Indicator (Direction)
J	N	Integration Step
K		Error Indicator
L		Computer Clock Reading at End of Run
LO		Computer Clock Reading at Beginning of Run
M		Case Number
N	n	Stage Number
NL	n_L	Number of Lines Printed this Page
NN	n^1	Previous Stage Number
NP	n_p	Page Number
NS	n_s	Number of Stages
ØN	θ	Longitude (See Note (4))
ØND	θ^o	Longitude (Degrees)
ØNG	θ_g^o	Longitude of Gun (Degrees)
ØNO	θ_g	Longitude of Gun
PA	γ	Absolute Flight Path Angle (See Note (2))

FORTRAN	MATH	DEFINITION
PAD	γ°	Absolute Flight Path Angle (Degrees)
PAP	γ'	Absolute Flight Path Angle (Previous Step)
Q (I)	Q_i	Kutta-Gill Q for Variable I
QN		
QS	QS	
QT (N)	Q_n	Ignition Value (See Note (5))
RA	d	Range (Feet)
RAN		Range (Nautical Miles)
RE	r_e	Earth Radius
RP	r_p	Perigee Radius
SA (N)	S	Cross Section Area
SAZ	$\sin \beta$	
SEL	$\sin \delta$	
SI (N)	I_{∞}	Specific Impulse in Vacuum
SL	p	Semi-Latus Rectum of Orbit
SLA	$\sin \phi$	
SP	Ω	Angular Rate of Earth
SPWU		Scalar Product of Unit Vectors on U and W Axes
SS	c	Speed of Sound
S3	S_3	Event Indicator
S4	S_4	Firing Indicator (Condition)
T	t	Time
TB	t_B	Time of Burnout
TH	T	Thrust
THO	T_{∞}	Thrust in Vacuum

FORTRAN	MATH	DEFINITION
TI	t_I	Time of Ignition
TO	t_o	Time of Last Print-Out
V	V	Absolute Velocity
VR	V_r	Relative Velocity (Referred to Earth)
VRU	V_{ru}	U-Component of Relative Velocity
VRV	V_{rv}	V-Component of Relative Velocity
VRW	V_{rw}	W-Component of Relative Velocity
VX	V_x	X-Component of Unit Vector on V Axis
VY	V_y	Y-Component of Unit Vector on V Axis
VZ	V_z	Z-Component of Unit Vector on V Axis
WAZD	β_w	Azimuth of Relative Velocity
WELD	δ_w	Elevation of Relative Velocity
WI (N)	W	Total Weight Before Ignition
WU	ω_u	U-Component of Angular Velocity Vector
Y (I)	Yi	Variable I
YD (I)	$\dot{Y}i$	Time Derivative of Variable I

NOTES:

1. Azimuth as measured from the gun (the input gun direction) is measured in a horizontal plane, clockwise from North as viewed from above (farther from the earth center).

   Azimuth as measured from the vehicle (the input direction of the thrust vector at ignition and the output body axis and wind directions) is measured in a horizontal plane, clockwise from the horizontal absolute velocity vector as viewed from above.

2. Elevation and flight path angle are measured from the horizontal plane, positive upwards.

NOTES (Cont'd)

3. Latitude is measured positive North from the equator.

4. Longitude is measured positive East from Greenwich.

5. The ignition value is interpreted as

Height in thousands of feet if	$I_1 = 0$
Flight path angle in degrees if	$I_1 = 1$
Time after previous burnout if	$I_1 = 2$

BASIC VARIABLES

VARIABLE NO. I	MATH SYMBOL	DEFINITION
1	W	Weight
2	q	Radius Rate
3	r	Radius
4	ω_w	W-Component of Angular Velocity
5	U_x	X-Component of U-Direction
6	U_y	Y-Component of U-Direction
7	U_z	Z-Component of U-Direction
8	W_x	X-Component of W-Direction
9	W_y	Y-Component of W-Direction
10	W_z	Z-Component of W-Direction

Variable Y (I)

Rate of Change of Variable YD (I)

LIST OF SYMBOLS (Cont'd)

INDICATORS

INDICATOR	VALUE	MEANING
I1	0	Ignition at Preset Height
	1	Ignition at Preset Flight Path Angle
	2	Ignition at Preset Time After Burnout*
	NOT 0,1,2	Ignition at Burnout*
I2	NOT 1,2,3	Firing at Constant Attitude Initially Tangent to Trajectory
	1	Firing in Wind Direction
	2	Firing at Preset Attitude
	3	Firing Along Trajectory
S3	-1.	Impact
	0	Burnout
	1.	Ignition or Launch or Apogee
	2.	No Event
S4	-1.	Not Firing
	0	No More Stages to Fire
	1.	Firing

* Burnout refers to the previous stage, or in the case of first stage ignition, the muzzle.

APPENDIX II

MATHEMATICS

Initialize Run

$$q = V_R \sin \delta$$

$$r = r_e + H$$

$$\omega_w = \left[(V_R \cos \delta \sin \beta + \Omega r \cos \phi)^2 + (V_R \cos \delta \cos \beta)^2 \right]^{\frac{1}{2}} / r$$

$$U_x = \cos \phi$$

$$U_y = 0$$

$$U_z = \sin \phi$$

$$W_x = -(V_R \cos \delta \sin \beta + \Omega r \cos \phi) \sin \phi / r \omega_w$$

$$W_y = -V_R \cos \delta \cos \beta / r \omega_w$$

$$W_z = (V_R \cos \delta \sin \beta + \Omega r \cos \phi) \cos \phi / r \omega_w$$

Initialize Step

$$V_x = U_z W_y - U_y W_z$$

$$V_y = U_x W_z - U_z W_x$$

$$V_z = U_y W_x - U_x W_y$$

$$V_{RU} = q$$

$$V_{RV} = r(\omega_w - \Omega W_z)$$

$$V_{RW} = r \Omega V_z$$

$$V_R = (V_{RU}^2 + V_{RV}^2 + V_{RW}^2)^{\frac{1}{2}}$$

$$H = r - r_e$$

$$V = \left[q^2 + (r \omega_w)^2 \right]^{\frac{1}{2}}$$

$$\gamma = \tan^{-1} (q / r \omega_w)$$

$$\begin{aligned}
\Delta t^1 &= (Q/F - \gamma) \Delta t / (\gamma - \gamma^1) && t \neq t_B \quad n < n_s \quad I_1 = 1 \\
&= (1000Q - H)/q && n < n_s \quad I_1 = 0 \\
&= Q - t + t_B && n < n_s \quad I_1 = 2 \\
&= -H/q && n = n_s \quad q < 0 \\
&= -q/\dot{q} && t \neq t_B \quad n = n_s \quad q > 0
\end{aligned}$$

$$\left.\begin{aligned} S_u &= q/V \\ S_v &= r\omega_w/V \\ S_w &= 0 \end{aligned}\right\} \quad I_2 = 1,\ 2 \text{ or } 3$$

$$\left.\begin{aligned} S_u &= \sin S \\ S_v &= \cos S \cos\beta \\ S_w &= -\cos S \sin\beta \end{aligned}\right\} \quad I_2 = 2$$

$$\left.\begin{aligned} S_x &= S_u U_x + S_v V_x + S_w W_x \\ S_y &= S_u U_y + S_v V_y + S_w W_y \\ S_z &= S_u U_z + S_v V_z + S_w W_z \end{aligned}\right\} \quad I_2 = 1 \text{ or } 3$$

$$\dot{W} = f\,(W_{n+1} - W_n)/\Delta t_B$$

$$T_o = -I\dot{W}$$

$$\theta = \tan^{-1}(U_y/U_x) - \Omega t + \theta_o$$

$$\phi = \tan^{-1}\left[U_z/(U_x^2 + U_y^2)^{\frac{1}{2}}\right]$$

<u>Calculate Variables</u>

$$V_x = U_z W_y - U_y W_z$$

$$V_y = U_x W_z - U_z W_x$$

$$V_z = U_y W_x - U_x W_y$$

$$V_{RU} = q$$

$$V_{RV} = r\,(\omega_w - \Omega W_z)$$

$$V_{RW} = r\,\Omega V_z$$

$$V_R = (V_{RU}^2 + V_{RV}^2 + V_{RW}^2)^{\frac{1}{2}}$$

$$H = r - r_e$$

$$P = 2116.22e^{-4.42 \times 10^{-5} H} \qquad H < 10^6$$

$$P = 0 \qquad H > 10^6$$

Calculate Thrust

$$T = T_o - AP$$

$$\left.\begin{aligned} S_u &= S_x U_x + S_y U_y + S_z U_z \\ S_v &= S_x V_x + S_y V_y + S_z V_z \\ S_w &= S_x W_x + S_y W_y + S_z W_z \end{aligned}\right\} \quad I_2 \neq 1 \text{ or } 3$$

$$\left.\begin{aligned} S_u &= V_{RU}/V_R \\ S_v &= V_{RV}/V_R \\ S_w &= V_{RW}/V_R \end{aligned}\right\} \quad I_2 = 1$$

$$\left.\begin{aligned} S_u &= q/V \\ S_v &= r\omega_w/V \\ S_w &= 0 \end{aligned}\right\} \quad I_2 = 3$$

Calculate Drag and Normal Force

$$M = V_R/c$$

$$C_d = 0.35 \qquad M < 1$$

$$C_d = 0.18 + e^{-M/2} \qquad M > 1$$

$$QS = 0.7\ PM^2\ S$$

$$D = C_d\ QS$$

$$C_{n_\alpha} = 2 + 12\ e^{-M/4} - 10\ e^{-M}$$

$$Q_n = C_{n_\alpha}\ QS/V_R$$

$$F_{nu} = Q_n \left[S_u\ (S_v V_{RV} + S_w V_{RW}) - V_{RU}\ (S_v^2 + S_w^2)\right]$$

$$F_{nv} = Q_n \left[S_v\ (S_u\ V_{RU} + S_w V_{RW}) - V_{RV}\ (S_u^2 + S_w^2)\right]$$

$$F_{nw} = Q_n \left[S_w\ (S_u\ V_{RU} + S_v V_{RV}) - V_{RW}\ (S_u^2 + S_v^2)\right]$$

$$F_n = (F_{nu}^2 + F_{nv}^2 + F_{nw}^2)^{\frac{1}{2}}$$

Calculate Rates

$$\dot{q} = r\omega_w^2 + (g_o/W)\ (T\delta_u - D\ V_{RU}/V_R + F_{nu}) - k_e^2/r^2$$

$$\dot{r} = q$$

$$\dot{\omega}_w = \left[-2\ q\omega_w + (g_o/W)\ (T\delta_v - D\ V_{RV}/V_R + F_{nv})\right]/r$$

$$\omega_u = (T\delta_w - D\ V_{RW}/V_R + F_{nw})\ g_o/Wr\omega_w$$

$$\dot{U}_x = V_x\omega_w$$

$$\dot{U}_y = V_y\omega_w$$

$$\dot{U}_z = V_z\omega_w$$

$$\dot{W}_x = -V_x\omega_u$$

$$\dot{W}_y = -V_y\omega_u$$

$$\dot{W}_z = -V_z\omega_u$$

Output

$$\theta = \tan^{-1}(U_y/U_x) - \Omega t + \theta_o$$

$$\phi = \tan^{-1}(U_z/\sqrt{U_x^2 + U_y^2})$$

$$\beta = \tan^{-1}\left[\sin(\theta - \theta_o)\cos(\phi - \phi_o)/\sin(\phi - \phi_o)\right]$$

$$d = r_e \tan^{-1}\left[|\sin(\theta - \theta_o)|\ /\ |\sin\beta|\cos(\theta - \theta_o)\right]$$

$$\delta_w = \tan^{-1}(V_{RU}/\sqrt{V_{RV}^2 + V_{RW}^2})$$

$$\beta_w = \tan^{-1}(-V_{RW}/V_{RV})$$

$$\left.\begin{aligned} \delta_B &= \delta_w \\ \beta_B &= \beta_w \end{aligned}\right\} \text{Not firing}$$

$$\left.\begin{aligned} \delta_B &= \tan^{-1}(\delta_u/\sqrt{\delta_v^2 + \delta_w^2}) \\ \beta_B &= \tan^{-1}(-\delta_w/\delta_v) \end{aligned}\right\} \text{Firing}$$

$$a = 1/(2/r - V^2/k_e^2)$$

$$p = (Vr\cos\gamma)^2/k_e^2$$

$$r_p = p/(1 + \sqrt{1-p/a})$$

$$H_p = r_p - r_e$$

$$H_a = ap/r_p - r_e$$

Integrate

Let X represent $W \quad q \quad r \quad \omega_w \quad U_x \quad U_y \quad U_z \quad W_x \quad W_y$ or W_z

For all Steps $\Delta x = \dot{x}\,\Delta t_I$

Step 1

$$x^1 = x + \Delta x/2$$

$$\Delta = \Delta x$$

$$t = t + \Delta t_I/2$$

Step 2

$$x^1 = x + (1 - 1/\sqrt{2})\,(\Delta x - \Delta)$$

$$\Delta = (2 - \sqrt{2})\,\Delta x + (-2 + 3/\sqrt{2})\,\Delta$$

Step 3

$$x^1 = x + (1 + 1/\sqrt{2})\,(\Delta x - \Delta)$$

$$\Delta = (2 + \sqrt{2})\,\Delta x + (-2 - 3/\sqrt{2})\,\Delta$$

$$t = t + \Delta t_I/2$$

Step 4

$$x^1 = x + \Delta x/6 - \Delta/3$$

After each step, $\dot{x}$ is recalculated for the next step.

APPENDIX III

COMPUTER PROGRAM

(SOURCE DECK LISTING)

FORTRAN IV - IBM 7040

```
 ISN        SOURCE STATEMENT

      C
      C         HARP  TRAJECTORY  PROGRAM  -  R. M. MCKEE
      C
      C          INPUT
      C
    1         DIMENSION AZS(10),C(10),DTB(10),ELS(10),EA(10),FR(10),
             1 I1(10),I2(10),Q(10),QT(10),SA(10),SI(10),WT(10),Y(10),YD(10)
    2    1000 CALL CLOCK (L)
    3         READ(5,1001)M,NS,(WT(NS+1)),(SA(NS+1)),DTI,DTP,
             1            VR,ELG,AZG,ONG,ATG,HT
   16    1001 FORMAT (2I5,F5.2,F5.3,3F5.0,4F5.2,F5.0)
   17         IF (NS.EQ.0) GO TO 2
   22         READ (5,1002) (I1(N),I2(N),WT(N),SA(N),EA(N),DTB(N),QT(N),
             1              ELS(N),AZS(N),FR(N),SI(N),N=1,NS)
   27    1002 FORMAT (2I5,F5.1,2F5.3,5F5.2,F5.0)
   30       2 NP = 1
   31         WRITE (6,1003) M,NP
   32    1003 FORMAT (1H1,15X,41HHARP  TRAJECTORY  PROGRAM  -  R. M. MCKEE,
             1          35X,4HCASE,I5,5X,4HPAGE,I3)
   33         WRITE (6,1004)M,VR
   34    1004 FORMAT (1H-,10X, 11HCASE NUMBER,6X,I7,9X,15HMUZZLE VELOCITY,
             1         2X,F7.0)
   35         WRITE (6,1005)NS,ELG
   36    1005 FORMAT (1H ,10X,13HNO. OF STAGES,4X,I7,9X, 9HELEVATION,8X,F7.2)
   37         WRITE (6,1006)WT(NS+1),AZG
   40    1006 FORMAT (1H ,10X,15HPAYLOAD  WEIGHT,2X,F7.2,9X, 7HAZIMUTH,10X,F7.2)
   41         WRITE (6,1007)SA(NS+1),ONG
   42    1007 FORMAT (1H ,19X,4HAREA,4X,F7.3,9X,9HLONGITUDE,8X,F7.2)
   43         WRITE (6,1008)DTI,ATG
   44    1008 FORMAT (1H ,10X,  9HSTEP SIZE,8X,F7.0,9X,8HLATITUDE,9X,F7.2)
   45         WRITE (6,1009)DTP,HT
   46    1009 FORMAT (1H ,10X,15HOUTPUT INTERVAL,2X,F7.0,9X,6HHEIGHT,11X,F7.0)
   47         IF (NS.EQ.0)  GO TO 1
   52         WRITE (6,1010)(N,N=1,NS)
   57    1010 FORMAT (1H-,10X,12HSTAGE NUMBER,6X,6I15)
   60         WRITE (6,1011)(I1(N),N=1,NS)
   65    1011 FORMAT (1H0,10X,18HIGNITION INDICATOR,6I15)
   66         WRITE (6,1012)(I2(N),N=1,NS)
   73    1012 FORMAT (1H ,10X,16HFIRING INDICATOR,2X,6I15)
   74         WRITE (6,1013)(WT(N),N=1,NS)
  101    1013 FORMAT (1H ,10X,14HWEIGHT (TOTAL),4X,6F15.1)
  102         WRITE (6,1014)(SA(N),N=1,NS)
  107    1014 FORMAT (1H ,10X,4HAREA,14X,6F15.3)
  110         WRITE (6,1015)(EA(N),N=1,NS)
  115    1015 FORMAT (1H ,10X,12HEXHAUST AREA,6X,6F15.3)
  116         WRITE (6,1019)(DTB(N),N=1,NS)
  123    1019 FORMAT (1H ,10X,12HBURNING TIME,6X,6F15.2)
  124         WRITE (6,1016)(QT(N),N=1,NS)
  131    1016 FORMAT (1H ,10X,14HIGNITION VALUE,4X,6F15.2)
  132         WRITE (6,1017)(ELS(N),N=1,NS)
  137    1017 FORMAT (1H ,10X,16HFIRING ELEVATION,2X,6F15.2)
  140         WRITE (6,1018)(AZS(N),N=1,NS)
  145    1018 FORMAT (1H ,10X,14HFIRING AZIMUTH,4X,6F15.2)
  146         WRITE (6,1026) (FR(N),N=1,NS)
  153    1026 FORMAT (1H ,10X,13HMASS FRACTION,5X,6F15.2)
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN        SOURCE STATEMENT

      154        WRITE (6,1020)(SI (N),N=1,NS)
      161   1020 FORMAT (1H ,10X,16HSPECIFIC IMPULSE,2X,6F15.0)
      162      1 WRITE (6,1025)
      163   1025 FORMAT (1H-, 7X,4HTIME,6X,6HHEIGHT,7X,5HRANGE,4X,8HVELOCITY,
               1  4X,8HAIRSPEED,2X,10HPATH ANGLE,3X,9HELEVATION,5X,7HAZIMUTH,
               2  5X,7HWIND EL,5X,7HWIND AZ/1H )
      164        NL = 28
      165        IF (NS.EQ.0) NL = 13
          C
          C          INITIALIZE  RUN
          C
      170     10 GC = 1.407639E16
      171        GO = 32.1465
      172        RE = 2.092564E7
      173        SP = 7.31958  E -5
      174        F = 57.295780
      175        ONO = ONG/F
      176        ATO = ATG/F
      177        AZ  = AZG/F
      200        EL  = ELG/F
      201        N = 1
      202        CT = 0
      203        S4 = -1.
      204        IF (NS.EQ.0) S4 = 0
      207        DT = DTI
      210        DTD = 0
      211        TH = 0
      212        YD(1) = 0
      213        T  = 0
      214        TO = 0
      215        TI = 0
      216        TB = 0
      217        Y(1) = WT(1)
      220        SLA = SIN(ATO)
      221        CLA = COS(ATO)
      222        SAZ = SIN(AZ)
      223        CAZ = COS(AZ)
      224        SEL = SIN(EL)
      225        CEL = COS(EL)
      226        Y(2) = VR*SEL
      227        Y(3) = RE+HT
      230        Y(4) = SQRT((VR*CEL*SAZ+SP*Y(3)*CLA)**2+(VR*CEL*CAZ)**2)/ Y(3)
      231        Y(5) =  CLA
      232        Y(6) =  0
      233        Y(7)=  SLA
      234        Y(8) = -(VR*CEL*SAZ+SP*Y(3)*CLA)*SLA/(Y(3)*Y(4))
      235        Y(9) = -VR*CEL*CAZ/(Y(3)*Y(4))
      236        Y(10) = (VR*CEL*SAZ+SP*Y(3)*CLA)*CLA/(Y(3)*Y(4))
          C
          C          INITIALIZE  STEP
          C
      237     20 VX = Y(7)*Y(9)-Y(6)*Y(10)
      240        VY = Y(5)*Y(10)-Y(7)*Y(8)
      241        VZ = Y(6)*Y(8)-Y(5)*Y(9)
      242        VRU = Y(2)
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN        SOURCE STATEMENT

      243        VRV = Y(3)*(Y(4)-SP*Y(10))
      244        VRW = Y(3)*SP*VZ
      245        VR =  SQRT(VRU**2+VRV**2+VRW**2)
      246        HT = Y(3)-RE
      247        V  = SQRT(Y(2)**2+(Y(3)*Y(4))**2)
      250        IF (T.NE.TB) PAP = PA
      253        PA = ATAN2(Y(2),Y(3)*Y(4))
      254        S3  = 2.
      255        IF (T.EQ.0.) S3 = 1.
      260        IF(HT.LT.0.)S4=0.
      263        DTD = DT
      264        DT = DTI
      265        IF (S4.EQ.0.)GO TO 27
      270        IF (S4.EQ.1.) GO TO 24
      273        IF (I1(N).EQ.1.AND.T.EQ.TB) GO TO 30
      276        IF (I1(N).EQ.1)     DTD = (QT(N)/F-PA)*DTD/(PA-PAP)
      301        IF (I1(N).EQ.0)     DTD = (1000.*QT(N)-HT)/Y(2)
      304        IF (I1(N).EQ.2  )  DTD = QT(N)-T+TB
      307        IF (I1(N).EQ.3) GO TO 21
      312        IF (ABS(DTD).GE.DTI) GO TO 30
      315        IF (ABS(DTD).LT..01) GO TO 22
      320        CT = CT+1.
      321        IF (CT.GT.20.)   GO TO 29
      324        DT = DTD
      325        GO TO 30
      326     22 CT = 0
      327        IF (T.GE.TB) GO TO 21
      332        NN = N-1
      333        WRITE (6,1107) N,NN
      334   1107 FORMAT (1H0,24HATTEMPTED IGNITION STAGE,I2,1X,
                1     20HBEFORE BURNOUT STAGE,I2)
      335        GO TO 70
      336     21 IF (I2(N).EQ.1.OR.I2(N).EQ.3) GO TO 23
      341        IF (I2(N).EQ.2  )  GO TO 25
      344        DU = Y(2)/V
      345        DV = Y(3)*Y(4)/V
      346        DW = 0
      347        GO TO 26
      350     25 EL  = ELS(N)/F
      351        AZ  = AZS(N)/F
      352        CEL = COS(EL)
      353        SEL = SIN(EL)
      354        CAZ = COS(AZ)
      355        SAZ = SIN(AZ)
      356        DU  = SEL
      357        DV  = CEL*CAZ
      360        DW  = -CEL*SAZ
      361     26 DX = DU*Y(5)+DV*VX+DW*Y(8)
      362        DY = DU*Y(6)+DV*VY+DW*Y(9)
      363        DZ = DU*Y(7)+DV*VZ+DW*Y(10)
      364     23 S4 = 1.
      365        S3 = 1.
      366        YD(1) =-FR(N)*(WT(N)-WT(N+1))/DTB(N)
      367        THO =-SI(N)*YD(1)
      370        TI = T
```

```
 ISN        SOURCE STATEMENT

 371        DT = DTB(N)/4.
 372        IF (DT.GT.DTI)  DT = DTI
 375        OND= F*(ATAN2(Y(6),Y(5))-SP*T+ONO)
 376        ATD= F*(ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2)))
 377        WRITE (6,1101)N,OND,ATD
 400   1101 FORMAT (1H ,5HSTAGE,I2,1X,20HIGNITION - LONGITUDE,F9.3,
           1  9H LATITUDE,F9.3)
 401        NL = NL +1
 402        GO TO 30
 403     24 IF ((T+DT).GE.(TI+DTB(N)))  DT = TI+DTB(N)-T
 406        IF (DT.LT..01) S3 = 0
 411        GO TO 30
 412     27 IF (Y(2).GE.0.) GO TO 102
 415        DTD = -HT/Y(2)
 416        IF (ABS(DTD).GT.DTI)  GO TO 30
 421        IF (ABS(DTD).LT..01) GO TO 28
 424        CT = CT+ 1.
 425        IF (CT.GT.20.) GO TO 29
 430        DT = DTD
 431        GO TO 30
 432     28 OND= F*(ATAN2(Y(6),Y(5))-SP*T+ONO)
 433        ATD= F*(ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2)))
 434        WRITE (6,1103) OND,ATD
 435   1103 FORMAT (1H ,18HIMPACT - LONGITUDE,F9.3,9H LATITUDE,F9.3)
 436        NL = NL+1
 437        S3 = -1.
 440        GO TO 30
 441    102 IF (T.EQ.TB) GO TO 30
 444        DTD = -Y(2)/YD(2)
 445        IF (ABS(DTD).GT.DTI) GO TO 30
 450        IF (ABS(DTD).LT..01) GO TO 103
 453        CT = CT+1.
 454        IF (CT.GT.20.) GO TO 29
 457        DT = DTD
 460        GO TO 30
 461    103 CT = 0
 462        WRITE (6,1109)
 463   1109 FORMAT (1H ,6HAPOGEE)
 464        NL = NL+1
 465        S3 = 1.
 466        GO TO 30
 467     29 WRITE (6,1104)
 470   1104 FORMAT (1H0,17HITERATION FAILURE)
 471        GO TO 70
      C
      C          CALCULATE VARIABLES
      C
 472     30 DO 65 J = 1,4
 473        IF (NL.LT.55) GO TO 31
 476        NP = NP+1
 477        WRITE (6,1003) M,NP
 500        WRITE (6,1025)
 501        NL = 5
 502     31 VX = Y(7)*Y(9)-Y(6)*Y(10)
 503        VY = Y(5)*Y(10)-Y(7)*Y(8)
```

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HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN        SOURCE STATEMENT

      504        VZ = Y(6)*Y(8)-Y(5)*Y(9)
      505        VRU = Y(2)
      506        VRV = Y(3)*(Y(4)-SP*Y(10))
      507        VRW = Y(3)*SP*VZ
      510        VR =  SQRT(VRU**2+VRV**2+VRW**2)
      511        HT = Y(3)-RE
      512        IF (HT.LT.1E6) GO TO 32
      515        AP = 0.
      516        DR = 0.
      517        GO TO 40
      520     32 AP = 2116.22*EXP(-4.42E-5*HT)
          C
          C          CALCULATE  THRUST
          C
      521     40 IF (S4.NE. 1.)  GO TO 50
      524        TH = THO-EA(N)*AP
      525        IF (I2(N).EQ.1 )   GO TO 41
      530        IF (I2(N).EQ.3) GO TO 43
      533        DU = DX*Y(5)+DY*Y(6)+DZ*Y(7)
      534        DV = DX*VX  +DY*VY  +DZ*VZ
      535        DW = DX*Y(8)+DY*Y(9)+DZ*Y(10)
      536        GO TO 50
      537     41 DU = VRU/VR
      540        DV = VRV/VR
      541        DW = VRW/VR
      542        GO TO 50
      543     43 DU = Y(2)/V
      544        DV = Y(3)*Y(4)/V
      545        DW = 0
          C
          C          CALCULATE  DRAG  AND  NORMAL  FORCE
          C
      546     50 IF (HT.LT.36200.)                    SS = 1116.45 - .0040986*HT
      551        IF (HT.GE.36200..AND.HT.LT.65800.)  SS =  968.08
      554        IF (HT.GE.65800..AND.HT.LT.105000.) SS =   924.28 + .0006658*HT
      557        IF (HT.GE.105000..AND.HT.LT.155500.)SS =  811.54 + .0017394*HT
      562        IF (HT.GE.155500..AND.HT.LT.172000.)SS =  1082.02
      565        IF (HT.GE.172000..AND.HT.LT.202000.)SS =  1291.40 - .0012173*HT
      570        IF (HT.GE.202000..AND.HT.LT.262500.)SS =  1584.72- .0026694*HT
      573        IF (HT.GE.262500.)                  SS =  884.0
      576        EM = VR/SS
      577        IF (EM.LT.1.) CD = .35
      602        IF (EM.GE.1.) CD = .18+EXP(-EM/2.)
      605        QS=.7*AP*EM**2*SA(N)
      606        DR = CD*QS
      607        IF (S4.NE.1..OR.I2(N).EQ.1) GO TO 51
      612        CNA = 2.+12.*EXP(-EM/4.)-10.*EXP(-EM)
      613        QN = CNA*QS/VR
      614        FNU = QN*(DU*(DV*VRV+DW*VRW)-VRU*(DV**2+DW**2))
      615        FNV = QN*(DV*(DU*VRU+DW*VRW)-VRV*(DU**2+DW**2))
      616        FNW = QN*(DW*(DU*VRU+DV*VRV)-VRW*(DU**2+DV**2))
      617        FN = SQRT(FNU**2+FNV**2+FNW**2)
      620        IF  FN/Y(1).LT.20.) GO TO 55
      623        WRITE (6,1115)
      624   1115 FORMAT (33H NORMAL ACCELERATION EXCEEDS 20 G)
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN        SOURCE STATEMENT

      625           GO TO 70
      626        51 FNU = 0.
      627           FNV = 0.
      630           FNW = 0.
          C
          C             CALCULATE  RATES
          C
      631        55 YD(2) = Y(3)*Y(4)**2+(GO/Y(1))*(TH*DU-DR*VRU/VR+FNU)-GC/Y(3)**2
      632           YD(3) = Y(2)
      633           YD(4) = (-2.*Y(2)*Y(4)+(GO/Y(1))*(TH*DV-DR*VRV/VR+FNV))/Y(3)
      634           WU    = (TH*DW-DR*VRW/VR+FNW)*GO/(Y(1)*Y(3)*Y(4))
      635           YD(5) = VX*Y(4)
      636           YD(6) = VY*Y(4)
      637           YD(7) = VZ*Y(4)
      640           YD(8) = -VX*WU
      641           YD(9) = -VY*WU
      642           YD(10) = -VZ*WU
          C
          C             OUTPUT
          C
      643        80 IF (S3.EQ.2..AND.T.LT.(TO+DTP).OR.J.NE.1) GO TO 60
      646           TO = T
      647           ON = ATAN2(Y(6),Y(5))-SP*T+ONO
      650           AT = ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2))
      651           IF (T.EQ.0.) GO TO 83
      654           AZ = ATAN2(SIN(ON-ONO)*COS(AT-ATO),SIN(AT-ATO))
      655           RA = RE*ATAN2(ABS(SIN(ON-ONO)),ABS(SIN(AZ))*COS(ON-ONO))
      656           GO TO 84
      657        83 RA = 0
      660        84 RAN = RA/6076.104
      661           HTN = HT/6076.104
      662           WELD = F*ATAN2(VRU,SQRT(VRV**2+VRW**2))
      663           WAZD = F*ATAN2(-VRW,VRV)
      664           IF (S4.EQ.1.) GO TO 81
      667           ELD = WELD
      670           AZD = WAZD
      671           GO TO 82
      672        81 ELD = F*ATAN2(DU,SQRT(DV**2+DW**2))
      673           AZD = F*ATAN2(-DW,DV)
      674        82 PAD = F*PA
      675           WRITE (6,1105)T,HTN,RAN,V,VR,PAD,ELD,AZD,WELD,WAZD
      676      1105 FORMAT (3F12.2,2F12.0,5F12.2)
      677           NL = NL +1
      700           IF (S3.NE.0.) GO TO 60
      703           OND = F*ON
      704           ATD = F*AT
      705           WRITE (6,1102)N,OND,ATD
      706      1102 FORMAT (1H ,5HSTAGE,I2,20H BURNOUT - LONGITUDE,F9.3,
                  1  9H LATITUDE,F9.3)
      707           NL = NL+1
      710           N=N+1
      711           S4 = -1.
      712           IF (N.GT.NS)    S4 = 0
      715            YD(1) = 0
      716           TH = 0
```

```
 ISN        SOURCE STATEMENT

 717        TB = T
 720        DT = DTI
 721        Y(1) = WT(N)
 722        IF (HT.LT.2.5E5) GO TO 20
 725        AX = 1./(2./Y(3)-V**2/GC)
 726        SL = (V*Y(3)*COS(PA))**2/GC
 727        IF (SL/AX.LT.1.) GO TO 85
 732        WRITE (6,1110)
 733   1110 FORMAT (17H HYPERBOLIC ORBIT)
 734        NL = NL+1
 735        GO TO 86
 736     85 RP = SL/(1.+SQRT(1.-SL/AX))
 737        HP = (RP-RE)/6076.104
 740        HA = (AX*SL/RP-RE)/6076.104
 741        WRITE (6,1106)HP,HA
 742   1106 FORMAT (21X,7HPERIGEE,F9.2,3X,6HAPOGEE,F9.2)
 743        NL = NL+1
 744        IF (HP.LT.50.)   GO TO 20
 747     86 IF (S4.EQ.0.) GO TO 70
 752        GO TO 20
      C
      C          INTEGRATE
      C
 753     60 IF (S3.EQ.-1.) GO TO 70
 756        DO 65 I=1,10
 757        C(I) = YD(I)*DT
 760        GO TO (61,62,63,64),J
 761     61 Y(I) = Y(I)+C(I)/2.
 762        Q(I) = C(I)
 763        IF (I.EQ.1) T = T+DT/2.
 766        GO TO 65
 767     62 Y(I) = Y(I)+.29289322*(C(I)-Q(I))
 770        Q(I) = .58578644*C(I)+.12132034*Q(I)
 771        GO TO 65
 772     63 Y(I) = Y(I)+1.7071068*(C(I)-Q(I))
 773        Q(I) = 3.4142136*C(I)-4.1213203*Q(I)
 774        IF (I.EQ.1) T = T+DT/2.
 777        GO TO 65
1000     64 Y(I) = Y(I)+C(I)/6.-Q(I)/3.
1001     65 CONTINUE
1004        GO TO 20
      C
      C          DIAGNOSTICS
      C
1005     70 AVU = SQRT(Y(5)**2+Y(6)**2+Y(7)**2)-1.
1006        AVW = SQRT(Y(8)**2+Y(9)**2+Y(10)**2)-1.
1007        SPWU = Y(8)*Y(5)+Y(9)*Y(6)+Y(10)*Y(7)
1010        WRITE (6,1108)AVU,AVW,SPWU
1011   1108 FORMAT (1H0,18HACCURACY CHECK - U,F12.8,3H  W,F12.8,5H  U.W,F12.8)
1012        CALL OVERFL (K)
1013        GO TO (71,73,72),K
1014     71 WRITE (6,1111)
1015   1111 FORMAT (9H OVERFLOW)
1016        GO TO 73
1017     72 WRITE (6,1112)
```

HARP TRAJ. PROG. R. M. MCKEE FORTRAN SOURCE LIST RMCKEE

```
 ISN        SOURCE STATEMENT

1020  1112 FORMAT (10H UNDERFLOW)
1021    73 CALL DVCHK (K)
1022       IF (K.EQ.2) GO TO 74
1025       WRITE (6,1113)
1026  1113 FORMAT (13H DIVIDE CHECK)
1027    74 LO = L
1030       CALL CLOCK (L)
1031        DL = L-LO
1032       DLS = DL/60.
1033       WRITE (6,1114)DLS
1034  1114 FORMAT (15H COMPUTING TIME,F8.2,8H SECONDS)
1035       GO TO 1000
     C
1036       END
```

PLUS ROUTINE FPT (UNLIMITED FLOATING POINT TRAPS)

APPENDIX IV

INPUTS

A complete list of inputs is given on the next page, with symbols, definitions and units. They are divided into two groups corresponding to the two types of data cards, and the order is the same as that on the data cards. The indicators and ignition value are defined as follows:-

Ignition Indicator	Meaning	Ignition Value
	Ignition at given	
0	- Height	Height (ft)/1000
1	- Flight Path Angle	Angle (deg)
2	- Time after Previous Burnout	Interval (sec)
Other	- Burnout of Previous Stage	

Firing Indicator	Meaning
	Direction of roll axis during firing
0	- fixed, tangent to absolute flight path angle at ignition
1	- parallel to relative velocity (aero-stab.)
2	- fixed, given elevation and azimuth at ignition
3	- parallel to absolute velocity
Other	- as 0

The layout of the data cards is shown on the page following the list of inputs. They are of two types, one for gun and general parameters, and one for each of the rocket stages.

All data fields are five columns wide. Decimal points need not be punched (and must not be in the first two fields). The assumed number of places to the right of the decimal is indicated. This format will be superseded by a decimal point if punched, but the formal of the input print-out will not then agree. These cards must be ordered according to the sequence of operation in the real system, for each case; i.e., a gun card followed by a card for the first stage, a card for the second stage, etc. The number of stage cards must be the same as the number of stages quoted on the gun card. The sets of cards for successive cases follow one another.

The integration interval is ideally one which produces the least errors. As the interval is increased, the mathematical errors predominate; as it is decreased, round off errors take over. For most purposes, 5 seconds should be about right.

SYMBOL			
FORTRAN	MATH	DESCRIPTION	
M	Case	Case Number	
NS	n_s	Number of Stages	
WT(NS+1)	W_{ns+1}	Payload Weight	(lb)
SA(NS+1)	S_{ns+1}	Payload Area	(ft^2)
DTI	Δt_I	Step Size for Integration	(sec)
DTP	Δt_p	Nominal Output Interval	(sec)
VR	V_R	Muzzle Velocity	(ft/sec)
ELG	δ_G	Gun Elevation from Horizontal	(deg)

SYMBOL FORTRAN	SYMBOL MATH	DESCRIPTION	
AZG	β_g	Gun Azimuth from North	(deg)
ON	θ_g	Gun Longitude (Positive East)	(deg)
ATG	ϕ_g	Gun Latitude (Positive North)	(deg)
HT	H_g	Muzzle Height Above S.L.	(ft)
I1(N)	I_{1n}	Ignition Indicator	
I2(N)	I_{2n}	Thrust Direction Indicator	
WT(N)	W_n	Total Weight Before Ignition	(lb)
SA(N)	S_n	Cross Section Area	(ft^2)
EA(N)	A_n	Exhaust Area	(ft^2)
DTB(N)	Δt_{Bn}	Burning Time	(sec)
QT(N)	Q_n	Ignition Value	
ELS(N)	δ_{sn}	Thrust Elevation above Horizontal	(deg)
AZS(N)	β_{sn}	Thrust Azimuth from Track	(deg)
FR(N)	f_n	Mass Fraction	
SI(N)	I_n	Specific Impulse in Vacuum	(sec

INPUT DATA FORMAT AND DECK MAKE-UP

Column	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60
General Data Card	Case	n_s	W_{ns+1}	S_{ns+1}	Δt_I	Δt_p	V_R	δ_g^o	β_g^o	θ_g^o	ϕ_g^o	H_g
Decimal Places	0	0	2	3	0	0	0	2	2	2	2	0
Stage Data Card	I_1	I_2	W	S	A	Δt_B	Q	δ_s	β_s	f	I	
Decimal Places	0	0	1	3	3	2	2	2	2	2	0	

Column	1 - 6	8 - 10	11 - 13	14 - 15	16 - 21	18 - 45
Deck Make-Up A						
Control Cards - 1	$JOB	(JOB CODE)	t	n		(Identificatic
2	$IBJOB				Nodeck	
3	$IBFTC	(DECK NAME)				
Source Deck						
FPT Deck						
Entry Card	$ENTRY				(Deck Name)	
Data Deck						
Final Card	$IBSYS					
Deck Make-Up B						
Control Cards - 1	$JOB	(JOB CODE)	t	n		(Identificatic
2	$IBJOB				No source	
Object Deck						
FPT Deck						
Entry Card	$ENTRY				(Deck Name)	
Data Deck						
Final Card	$IBSYS					

APPENDIX V

DECK MAKE-UP

The order of the complete deck and the details of the control cards for use on the IBM 7040 at McGill are shown on the preceding page. The first is for use with a source deck (as listed in Appendix III). Normally, this would be used only when making program changes; when these have proven satisfactory, the second card should not be punched NODECK as shown; an object deck will then be produced. The second shows the deck set-up for normal operation, with the object deck. The FPT deck is an object deck of a special routine which allows unlimited floating point traps; the normal procedure is for the monitor to terminate execution when the recorded number of floating point traps reaches a given amount.

On the first card "t" is the estimated computing time in minutes including compilation, and "n" is the estimated number of pages of output. If either of these is exceeded, the job will be terminated. The computer time may be estimated as $3.8 + t_f/18 \Delta t_I$ (or $t_f/10 \Delta t_p$, whichever is larger) seconds computing time per case where t_f is the expected flight time in seconds, plus compiling time of 4 minutes if a source deck is used. The number of pages of output may be estimated as $1 + t_f/55 \Delta t_p$ per case rounded to the next higher integer, plus 8 for the listing if a source deck is used, plus 3 for bookkeeping.

APPENDIX VI

OUTPUTS

The print-out for each case begins with a complete listing of all inputs, followed by the column headings for the periodic records which are printed at intervals of Δt_p and at events (launch, ignition, burnout, apogee and impact).

TIME	.....	time in seconds from launch
HEIGHT	.....	above sea level in nautical miles
RANGE	.....	at sea level (great circle) in nautical miles
VELOCITY	.....	absolute velocity in ft/sec
AIRSPEED	.....	velocity relative to rotating earth in ft/sec
PATH ANGLE	.....	angle of elevation of the tangent to the absolute flight path above the local horizontal in degrees
ELEVATION	.....	angle of elevation of the vehicle roll axis above the local horizontal, in degrees
AZIMUTH	.....	angle from the horizontal velocity vector to the horizontal projection of the vehicle roll axis, measured clockwise as viewed from above, in degrees
WIND EL	.....	as elevation, for the relative velocity in degrees
WIND AZ	.....	as azimuth, for the relative velocity, in degrees

In addition, messages are produced at the appropriate times to indicate ignition, burnout, apogee and impact. Except for apogee, these are accompanied by latitude (in degrees North) and longitude (in degrees East).

When burnout occurs at a height greater than 250,000 ft., perigee and apogee heights are calculated and presented in nautical miles, unless the orbit is hyperbolic, which fact is indicated.

Various error conditions are indicated by appropriate messages. "Attempted ignition (of a) stage before burnout (of the previous) stage" is caused by incompatible inputs. "Iteration failure", "overflow" or "divide check" require detailed examination of the program. "Underflow" is not significant. "Accuracy check" shows the errors in the unit vectors on the U and W axes and their scalar product after completion of the case. These should be of the order of the round off error, i.e. of the order of $t_f/10^8 \Delta t_I$. An arbitrary limit of 20G was placed on the normal acceleration, and exceeding this figure results in termination of the calculation with the appropriate message.

A sample output is shown on the following pages.

CASE NUMBER	-0	MUZZLE VELOCITY	4500.
NO. OF STAGES	3	ELEVATION	38.00
PAYLOAD WEIGHT	50.00	AZIMUTH	118.23
AP A	1.478	LONGITUDE	-59.48
STEP SIZE	5.	LATITUDE	13.07
OUTPUT INTERVAL	5.	HEIGHT	150.

STAGE NUMBER	1	2	3
IGNITION INDICATOR	0	2	1
FIRING INDICATOR	1	2	2
WEIGHT (TOTAL)	2000.0	600.0	180.0
AREA	1.478	1.478	1.478
EXHAUST AREA	0.785	0.785	0.785
BURNING TIME	20.00	20.00	20.00
IGNITION VALUE	25.00	10.00	0.38
FIRING ELEVATION	-0.	3.00	0.
FIRING AZIMUTH	-0.	0.	0.
MASS FRACTION	0.80	0.80	0.80
SPECIFIC IMPULSE	300.	300.	300.

TIME	HEIGHT	RANGE	VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	AZIMUTH	WIND EL	WIND AZ
0.	0.02	0.	5639.	4500.	29.43	38.00	8.26	38.00	8.26
5.00	2.06	2.74	4945.	3772.	26.84	36.29	9.22	36.29	9.22
10.00	3.75	5.16	4545.	3340.	24.44	34.25	9.86	34.25	9.86
STAGE 1 IGNITION - LONGITUDE -59.396 LATITUDE 13.026									
11.21	4.11	5.72	4472.	3260.	23.87	33.71	9.98	33.71	9.98
16.21	5.79	8.39	5558.	4350.	24.32	31.74	8.05	31.74	8.05
21.21	7.91	11.99	6916.	5710.	24.59	30.26	6.49	30.26	6.49
26.21	10.58	16.76	8678.	7474.	24.81	29.16	5.19	29.16	5.19
31.21	13.98	23.07	11058.	9857.	25.05	28.36	4.09	28.36	4.09
STAGE 1 BURNOUT - LONGITUDE -59.140 LATITUDE 12.891									
36.21	17.74	30.24	10834.	9625.	24.38	27.69	4.16	27.69	4.16
STAGE 2 IGNITION - LONGITUDE -58.931 LATITUDE 12.780									
41.21	21.35	37.32	10716.	9499.	23.73	3.00	-0.	27.00	4.20
46.21	24.81	44.95	11967.	10707.	20.09	3.14	0.12	22.58	3.58
51.21	28.15	53.86	13539.	12250.	17.27	3.31	0.15	19.15	3.07
56.21	31.44	64.31	15515.	14205.	14.94	3.50	0.15	16.35	2.61
61.21	34.75	76.68	18089.	16763.	12.95	3.72	0.14	14.00	2.19
STAGE 2 BURNOUT - LONGITUDE -58.346 LATITUDE 12.486									
66.21	38.05	90.17	18049.	16719.	12.70	13.73	2.20	13.73	2.20
71.21	41.28	103.62	18012.	16680.	12.45	13.47	2.21	13.47	2.21
76.21	44.44	117.04	17977.	16643.	12.20	13.20	2.22	13.20	2.22
81.21	47.53	130.43	17944.	16607.	11.95	12.93	2.23	12.93	2.23
86.21	50.56	143.79	17911.	16572.	11.70	12.66	2.24	12.66	2.24
91.21	53.51	157.12	17880.	16538.	11.45	12.39	2.25	12.39	2.25
96.21	56.40	170.42	17849.	16505.	11.19	12.12	2.26	12.12	2.26
101.21	59.21	183.69	17820.	16473.	10.94	11.85	2.27	11.85	2.27
106.21	61.96	196.94	17790.	16442.	10.68	11.57	2.28	11.57	2.28

TIME	HEIGHT	RANGE	VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	AZIMUTH	WIND EL	WIND AZ
111.21	64.64	210.16	17762.	16411.	10.43	11.30	2.28	11.30	2.28
116.21	67.25	223.35	17734.	16382.	10.17	11.02	2.29	11.02	2.29
121.21	69.80	236.52	17707.	16353.	9.91	10.74	2.30	10.74	2.30
126.21	72.27	249.66	17681.	16324.	9.65	10.47	2.31	10.47	2.31
131.21	74.68	262.78	17656.	16297.	9.40	10.19	2.32	10.19	2.32
136.21	77.01	275.88	17631.	16270.	9.14	9.91	2.32	9.91	2.32
141.21	79.28	288.95	17607.	16244.	8.87	9.63	2.33	9.63	2.33
146.21	81.48	302.01	17584.	16219.	8.61	9.34	2.34	9.34	2.34
151.21	83.62	315.04	17561.	16195.	8.35	9.06	2.35	9.06	2.35
156.21	85.68	328.04	17539.	16171.	8.09	8.78	2.35	8.78	2.35
161.21	87.68	341.03	17518.	16148.	7.83	8.49	2.36	8.49	2.36
166.21	89.61	354.00	17497.	16126.	7.56	8.21	2.37	8.21	2.37
171.21	91.47	366.95	17478.	16105.	7.30	7.92	2.37	7.92	2.37
176.21	93.26	379.88	17459.	16084.	7.03	7.64	2.38	7.64	2.38
181.21	94.98	392.79	17440.	16065.	6.77	7.35	2.39	7.35	2.39
186.21	96.64	405.68	17423.	16046.	6.50	7.06	2.39	7.06	2.39
191.21	98.23	418.56	17406.	16027.	6.23	6.77	2.40	6.77	2.40
196.21	99.75	431.42	17390.	16010.	5.97	6.48	2.41	6.48	2.41
201.21	101.20	444.27	17374.	15993.	5.70	6.19	2.41	6.19	2.41
206.21	102.59	457.10	17360.	15978.	5.43	5.90	2.42	5.90	2.42
211.21	103.91	469.91	17346.	15962.	5.16	5.61	2.42	5.61	2.42
216.21	105.16	482.71	17332.	15948.	4.89	5.32	2.43	5.32	2.43
221.21	106.34	495.50	17320.	15935.	4.62	5.03	2.44	5.03	2.44
226.21	107.46	508.27	17308.	15922.	4.35	4.73	2.44	4.73	2.44
231.21	108.50	521.03	17297.	15910.	4.08	4.44	2.45	4.44	2.45
236.21	109.48	533.78	17286.	15899.	3.81	4.15	2.45	4.15	2.45
241.21	110.40	546.52	17277.	15888.	3.54	3.85	2.46	3.85	2.46
246.21	111.24	559.24	17268.	15878.	3.27	3.56	2.46	3.56	2.46
251.21	112.02	571.96	17260.	15870.	3.00	3.26	2.47	3.26	2.47
256.21	112.73	584.66	17252.	15861.	2.73	2.97	2.47	2.97	2.47
261.21	113.37	597.36	17245.	15854.	2.46	2.67	2.47	2.67	2.47
266.21	113.95	610.05	17239.	15847.	2.19	2.38	2.48	2.38	2.48
271.21	114.45	622.73	17234.	15842.	1.91	2.08	2.48	2.08	2.48
276.21	114.89	635.40	17229.	15837.	1.64	1.79	2.49	1.79	2.49
281.21	115.27	648.06	17225.	15832.	1.37	1.49	2.49	1.49	2.49
286.21	115.57	660.72	17222.	15829.	1.10	1.19	2.49	1.19	2.49
291.21	115.81	673.37	17219.	15826.	0.83	0.90	2.50	0.90	2.50
296.21	115.98	686.02	17218.	15824.	0.55	0.60	2.50	0.60	2.50
STAGE 3 IGNITION - LONGITUDE -49.208 LATITUDE 7.768									
299.39	116.05	694.05	17217.	15823.	0.38	0.00	0.00	0.41	2.50
304.39	116.12	707.27	18721.	17326.	0.13	0.24	0.00	0.14	2.29
309.39	116.13	721.80	20504.	19108.	-0.02	0.50	0.00	-0.03	2.08
314.39	116.11	737.89	22694.	21296.	-0.07	0.78	0.00	-0.07	1.87
319.39	116.10	755.96	25531.	24133.	0.01	1.10	-0.00	0.01	1.65

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13. ABSTRACT
In order to assess the potential of multi stage gun-launched rockets, a study of the vehicle and trajectory parameters was undertaken. A digital computer program for trajectories was written and was used in an experimental manner to approach optimum performance within various sets of restricting assumptions. The approach was found to be effective and a useful orbital potential was demonstrated with reasonable design parameters.

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