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August 1968

• applied • mathematics • •

Numerical Evaluation of the Series Expansion Method for the Prediction of the Far Field Intensity from Near Field Cross-Power Measurements

by

John L. Butler
Dorothy A. Moran

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Abstract

An evaluation of a series expansion method for predicting the far-field from near-field cross-power measurements of partially coherent sound fields is given. This series method is based on an expansion in spherical wave functions truncated to include only the number of terms equal to the number of near field measuring points (which need not be restricted to the usual separable surfaces). The evaluation is through a set of numerical examples where the predicted far field is compared with the exact far field from a theoretical source of partially coherent sound. It is shown that good results can be obtained for incoherent, partially coherent, and coherent sound fields.

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I. Introduction

In this report we shall demonstrate, through a numerical evaluation, that the far field can be predicted from the near field of coherent, partially coherent, and incoherent sources by a finite series method. The series method has been described in some detail in an earlier memorandum[1] where an evaluation of the method for only coherent sources was given. In order to test the method for various degrees of coherency it was necessary to devise a theoretical model for a partially coherent source distribution. The details of this model are the subject of another technical memorandum[2].

The numerical evaluation presented in this memorandum is based on a comparison of the exact and predicted far field intensity distribution. The exact far field is calculated directly from the theoretical source distribution while the predicted far field is obtained by the series method using near field calculations of the cross power from the theoretical source distribution. These near field samplings simulate the near field measurements that would be made by the hydrophones about an actual source.

We have confined our attention in this report to distributions which have axial symmetry. A brief review of the series method is given along with a brief description of the theoretical model of the partially coherent source distribution used to test the method. The results of the far field predictions are then compared with the exact values for various hydrophone and source distributions.

II. Review of the Series Method

If we consider axially symmetric sources, the solution for the cross-power function may be written as

$$G_{12}(\bar{r}_i, \bar{r}_j, \omega) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} h_n^{(2)}(kr_i) P_n(\cos\theta_i) C_{nm} h_m^{(1)}(kr_j) P_m(\cos\theta_j) \tag{1}$$

where the conventional notation for spherical wave functions has been adopted, and the C_{nm} are the coefficients to be determined. It is assumed that the fields of interest may be adequately represented by a finite number of terms, N. Consider now N measuring hydrophones (which need <u>not</u> be on the usual separable surfaces). The hydrophones would measure the auto spectral density (intensity) $G_{11}(\bar{r}_j, \bar{r}_j, \omega)$ at the N positions, and the cross-power $G_{12}(\bar{r}_i, \bar{r}_j, \omega)$ between all the pairs of the N hydrophones can be determined yielding a total of N^2 measurements at each frequency. With the two summations in the above equation each truncated to include only the first N terms there will then be N^2 coefficients which can be determined.

Under the conditions of only N terms in the sums, the above equation may be written more compactly in the matrix notation. Adopting the summation convention the above may be written (in terms of the elements) as

$$g_{ij} = a^*_{ni} C_{nm} a_{mj} \tag{2}$$

where

$$a_{mj} = h_m^{(1)}(kr_j) P_m(\cos\theta_j) \tag{3}$$

and the asterisk means complex conjugate. With the upper case letters denoting the corresponding matrices we have, in matrix notation

$$G = A^{\dagger} C A \tag{4}$$

where $A^{\dagger}$ is the transpose of the complex conjugate of A. The solution for the N^2 coefficients C_{nm} can then be obtained through matrix inversion so that

$$C = A^{-1\dagger} G A^{-1} . \tag{5}$$

A solution for the entire field is obtained with the evaluated C'_{nm} substituted into the expansion. The degree to which the result is an accurate prediction depends on the number and location of the hydrophones.

III. Description of the Source Model

The development of the source model is given in detail in an earlier memorandum[2]. The model specialized for axial symmetry consists of a linear distribution of small spherical acoustical radiators; that is, point sources. The time dependence for the nth source is assumed to be a general function $f_m(t)$. The solution for the pressure p at $\bar{r}_j$ can then be written as

$$p(\bar{r}_j, t) = \sum_{m=1}^{M} \frac{f_m(t - r_{jm}/c)}{r_{jm}} \tag{6}$$

where M is the number of sources, r_{jm} is the distance from the nth source to the field point at $\bar{r}_j$, and c is the velocity of sound.

The cross-power $G_{12}(\bar{r}_i, \bar{r}_j, \omega)$ in the field was obtained from the

product of the Fourier transform of p at $\bar{r}_j$, and the complex conjugate of the transform at $\bar{r}_i$. On separating the factors which depend on the sources only and those which relate the sources to the field the result may be written in the form

$$G_{12}(\bar{r}_i, \bar{r}_j, \omega) = \sum_{m=1}^{M} \sum_{n=1}^{M} q_{nm}(\omega) \frac{e^{i\omega/c(r_{jm} - r_{in})}}{r_{jm}\, r_{in}} \tag{7}$$

where $q_{nm}(\omega)$ is the cross power amongst the sources. By writing the expansion in this way we need not specify the exact phase of the sources and, in fact, they may be random. What is specified is the intensity of each source and the cross-power or correlations between the sources. Thus, for a chosen $q_{nm}(\omega)$ the near field as well as the far field cross G_{12} may be calculated through Eq. (7) for n sources at frequency ω.

We would like to emphasize that this calculation does <u>not</u> imply the sources are necessarily single frequency (and thus, coherent). What is meant by a calculation at a single frequency is that this is one frequency, of a possible multitude, at which we focus our attention. In actual practice this can be done by frequency filters which ideally do not change the degree of coherency.

One of the difficult problems is to choose a representative q_{nm}. As an initial test we chose

$$q_{nm} = e^{-\frac{1}{2}(m-n)^2 (s/\ell)^2} \tag{8}$$

where we assumed the sources to be spaced an equal distance s. The

quantity $s|m-n|$ represents the separation between the sources and we call ℓ the coherence length. For $\ell >> s|m-n|$ it can be seen that q_{nm} is unity and all the sources are coherent, while for $\ell << s|m-n|$ q_{nm} vanishes for $n \neq m$ and the sources are incoherent. For intermediate values of ℓ the sources are partially coherent.

IV. Numerical Evaluation

Here we present some of the results of a numerical evaluation of the series method. The calculations were programmed for computation on our IBM 1130. Because of limited core storage it was convenient to evaluate arrays corresponding to only eleven hydrophones. The program for the numerical evaluation can be separated into three parts:

1) The "source program" - from which the exact near and far field values are calculated using various source distributions described by equations (7) and (8).

2) The "coefficient program" - from which the C coefficients are evaluated using matrix equation (5) along with the near field cross power values from the "source program".

3) The "field prediction program" - from which the far field is calculated using equation (1) along with the coefficient values obtained from the "coefficient program".

The scheme for the display of the results may be understood by referring to Fig. 1. The separation and relative location of the

sources and hydrophones (points at which the near-field values are used) is shown pictorially in the upper left-hand quadrant. As shown here, the evaluation is for three sources each separated by one-half wavelength (λ) with near-field values sampled at the eleven hydrophone locations. The hydrophone array is symmetrical and at a normal distance of 1.59 wavelengths with inter-element spacing given by 0.318 wavelengths.

The results for three degrees of coherency are shown in the remaining quadrants of the page. The exact values for the far field intensity $G_{11}(\bar{r}_i, \bar{r}_j, \omega)$ are given as a solid line and the predicted values are given by the dots. The three degrees of coherency were obtained using Eq. (8) with the values of ℓ/s equal to 0.1, 1.0, and 10.0; and calling these cases incoherent, partially coherent, and coherent, respectively. Except for Fig. 2 coherency is essentially reached for ℓ/s equal to ten.

The results for the sources have been normalized so that each source separately yields an amplitude value unity in the far field. Thus, for three incoherent sources we have an omnidirectional pattern with an intensity value of three since here the intensity is simply the sum of the intensities of each source. For complete coherency the far field intensity result at 90° is the square of the sum of amplitude values yielding a value 9 for three sources; moreover, we see that here we get the usual pattern structure with side lobes and nulls. The case of partial coherency is characterized by a main lobe without deep nulls and separate side lobes. The exact values for the far-field

cross-power for this source distribution are given in an earlier memorandum[2] (in which the coordinate system was rotated 90° relative to the one in this memorandum).

Inspection of the predicted and exact far-field intensity results shown in Fig. 1 indicate quite clearly that for this case good agreement is obtained for all three degrees of coherency. In fact, for the case of incoherency the predicted values of the intensity at 90°, 45° and 0° are 3.00001, 3.00028 and 3.00249 respectively. If we compare the worst predicted value at 0° (i.e., along the axis of the source) with the exact value 3, the worst percentage error is less than one-tenth of one-percent. This very good agreement is partly due to the large number of hydrophones (i.e., near field sampling points) compared to the ratio of the overall source length to wavelength. For best results we would expect (for reasonable distributions and array locations) that the number of hydrophones exceed $5(L/\lambda)$ where L is the length of the hydrophone line. In this particular case L is equal to λ and the eleven hydrophones safely exceed the number five.

In Fig. 2, the hydrophone arrangement, overall source length, and ℓ/s ratios are the same; however, here eight additional sources have been inserted between the two outer sources. In this case the separation between sources is one-tenth of one-wavelength, and the source acts as a nearly continuous line one-wavelength in length. Again the agreement between the exact and the predicted values of the intensity is good, and the added complication of more sources does not appear to

produce any serious problems providing that the overall length of the source line is not increased. We should note that "coherency" has not been actually reached in this case since the coherence length is now on the order of the overall source length rather than much greater than the source length as it was in Fig. 1.

In Fig. 3 the same hydrophone array is used at the same location; however, here three sources are used again, and this time the spacing is one wavelength for an overall length of two wavelengths. We see that in all three cases the prediction is quite good in the arc from 50° to 90°. This might be expected since this region is covered quite adequately by the hydrophone array. The results are for the most part in poor agreement in the remaining region for the coherent case; however, we see that there is a trend toward better agreement in the axial direction (0°) as the coherency decreases.

For the case under discussion the number of hydrophones (11) is approximately equal to $5L/\lambda$ rather than considerably greater as in the previous cases. The results can, however, be improved by increasing the separation between the hydrophones. This improvement is shown in Fig. 4 where now the separation between hydrophones is one wavelength whereas in the previous cases it was approximately one-third of one-wavelength. The prediction deteriorates as the hydrophone separation increases much beyond one and one-quarter wavelengths.

The results for a shorter hydrophone array in closer to the three sources, with source separation $\lambda/2$, are shown in Figs. 5 and 6. The

separation between the hydrophone elements is now .064 λ so that the total length of the hydrophone array is less than the overall length of the source. In Fig. 5 the normal distance to the hydrophone array is slightly beyond the smallest circle drawn through the two outer sources; whereas, in Fig. 6 the same array is close enough so that all the hydrophones are entirely within this circle.

We see that the hydrophone line is too close and too short for an accurate prediction throughout the field. Although the more remote line location gives better results in the direction of the main lobe, the line location within the circle yields a better average prediction. It appears, therefore, that a reasonable prediction of the far field can be obtained even if all the hydrophones are within the smallest circle through the outer sources. Satisfactory agreement has also been obtained for cases where part of the hydrophone array was within, and the remaining part outside of this circle. The fact that a reasonable prediction can be obtained even though the sampling points (hydrophone locations) are within this circle, is of theoretical as well as practical interest and requires further study.

V. Summary and Conclusions

The series expansion method for the prediction of the far field from near field measurements of coherent, partially coherent, and incoherent acoustic fields has been evaluated through a set of numerical examples. It was found that good agreement can be obtained

for all degrees of coherency. The shortcomings of this series method for cross-power measurements appear to be no worse than those found for the series method for pressure amplitude and phase measurements reported earlier[1].

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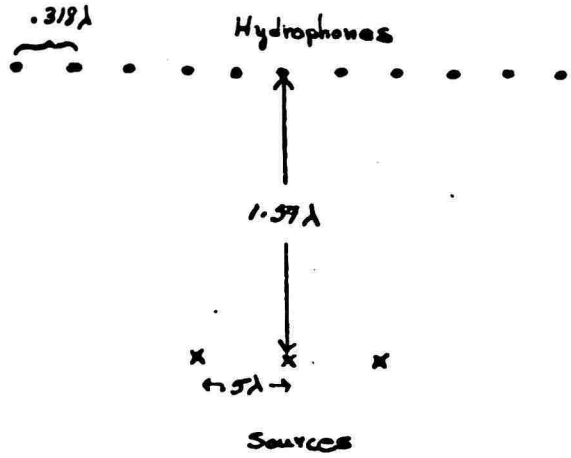

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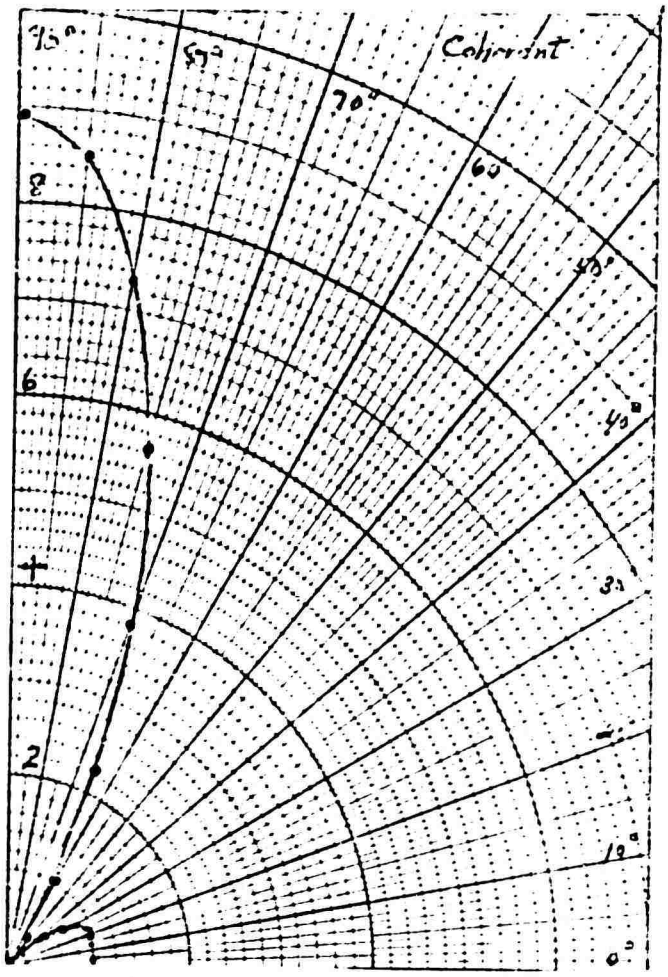

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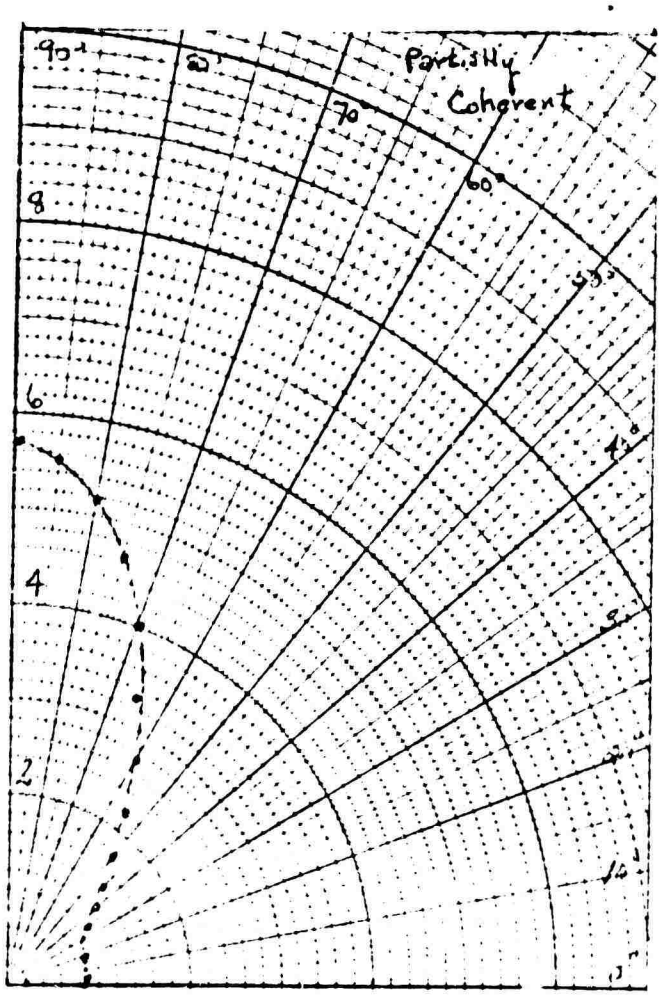

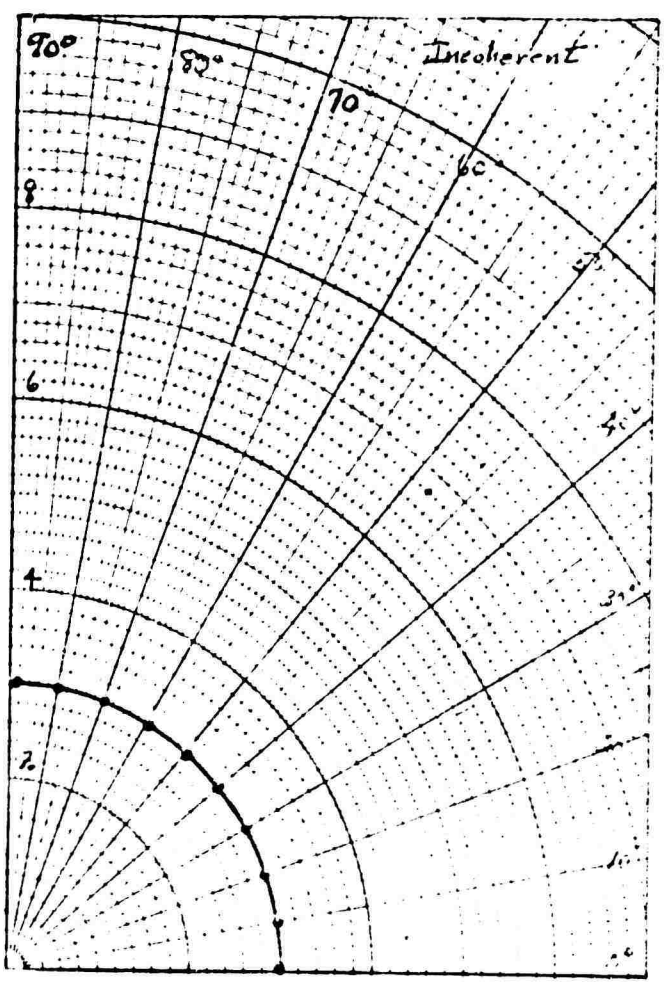

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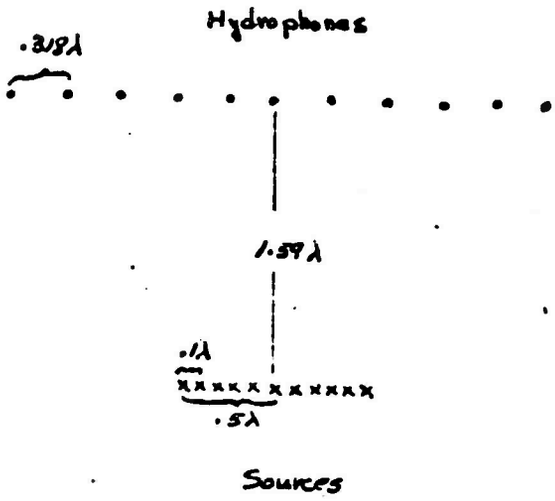

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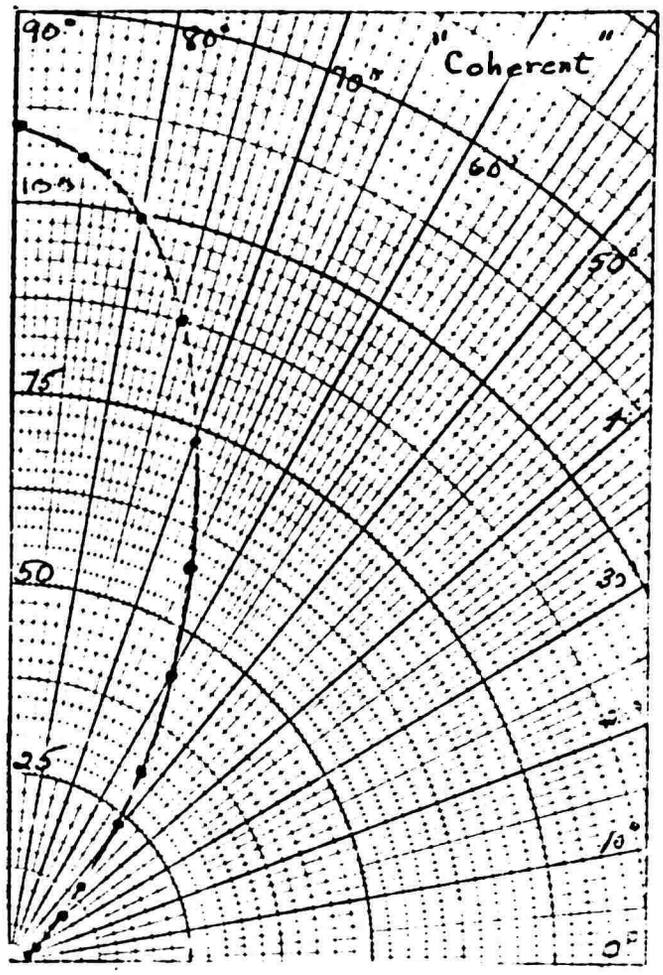

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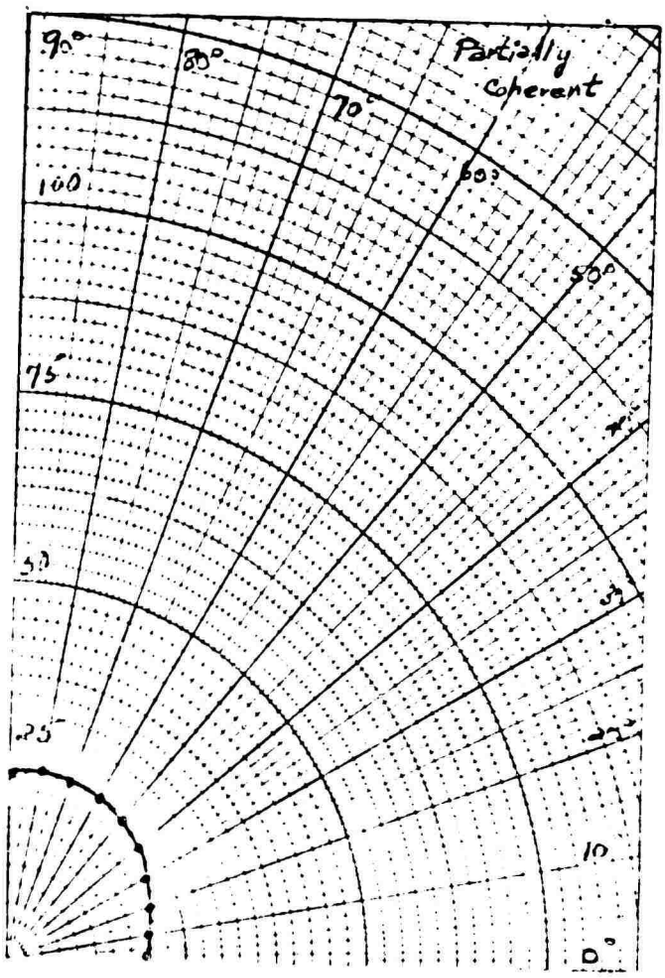

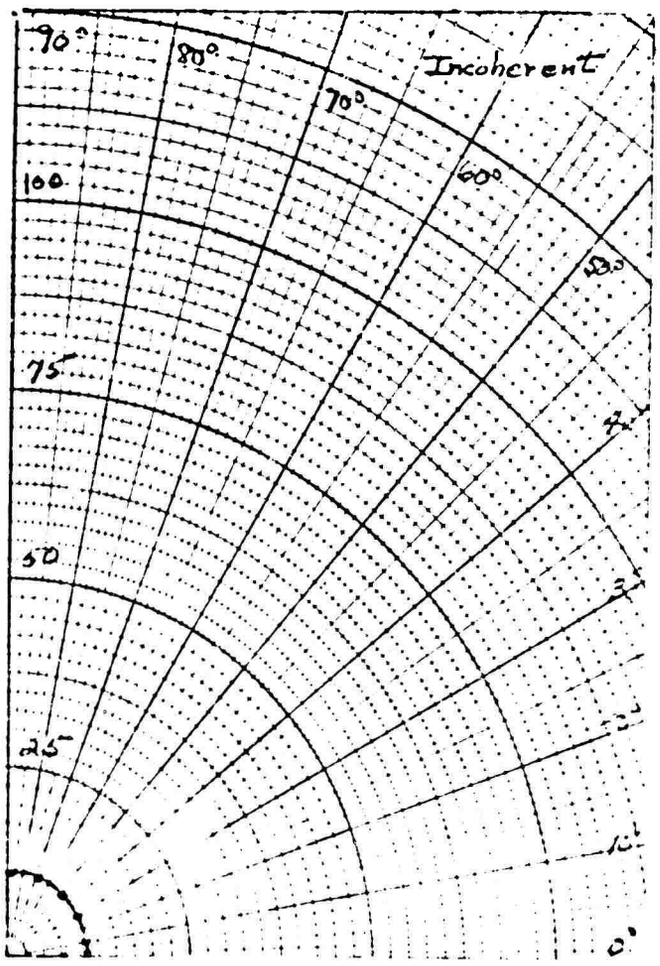

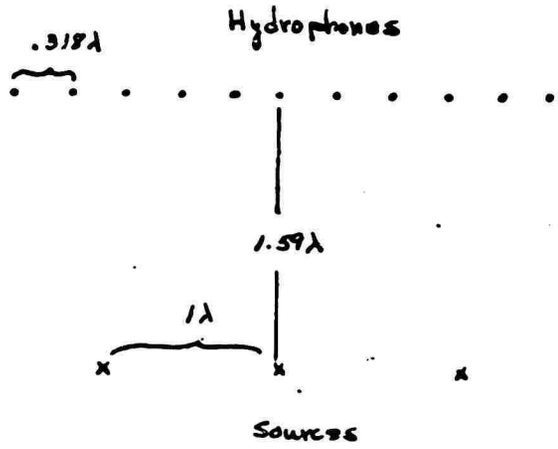

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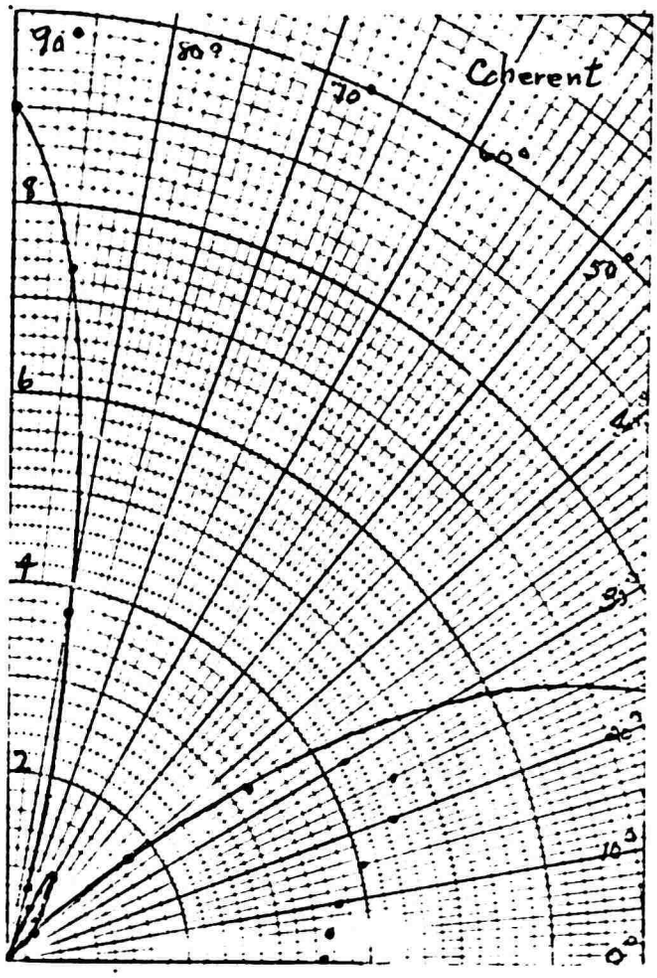

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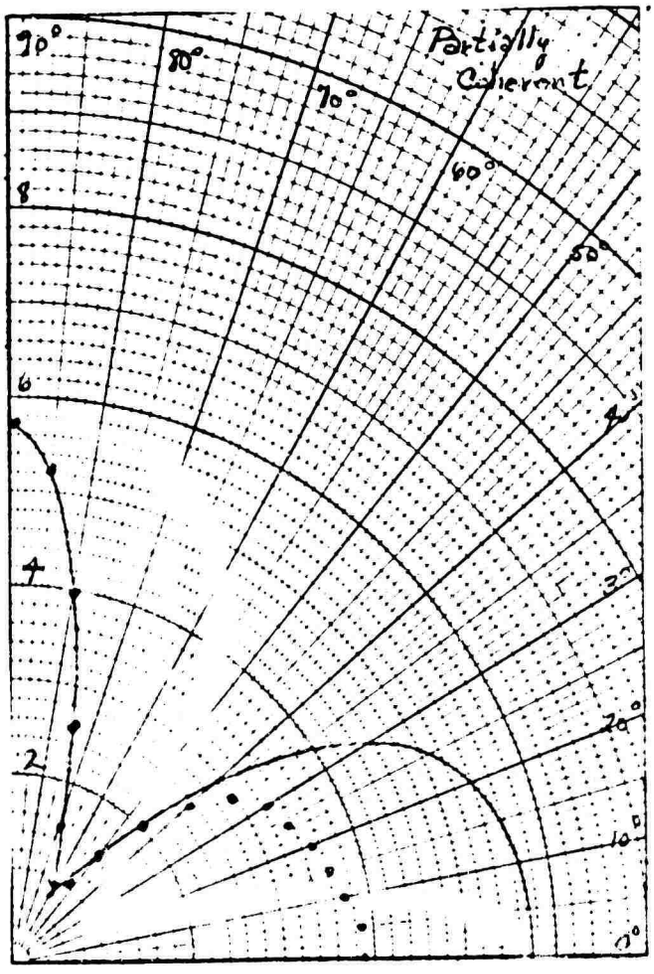

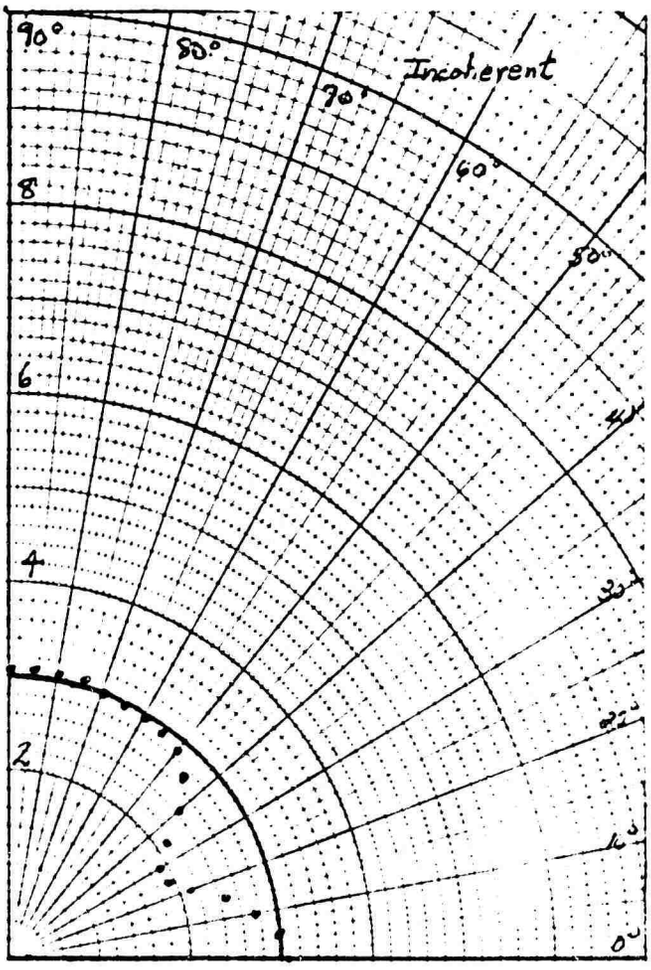

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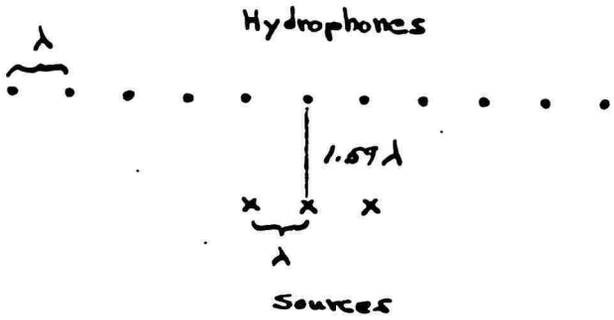

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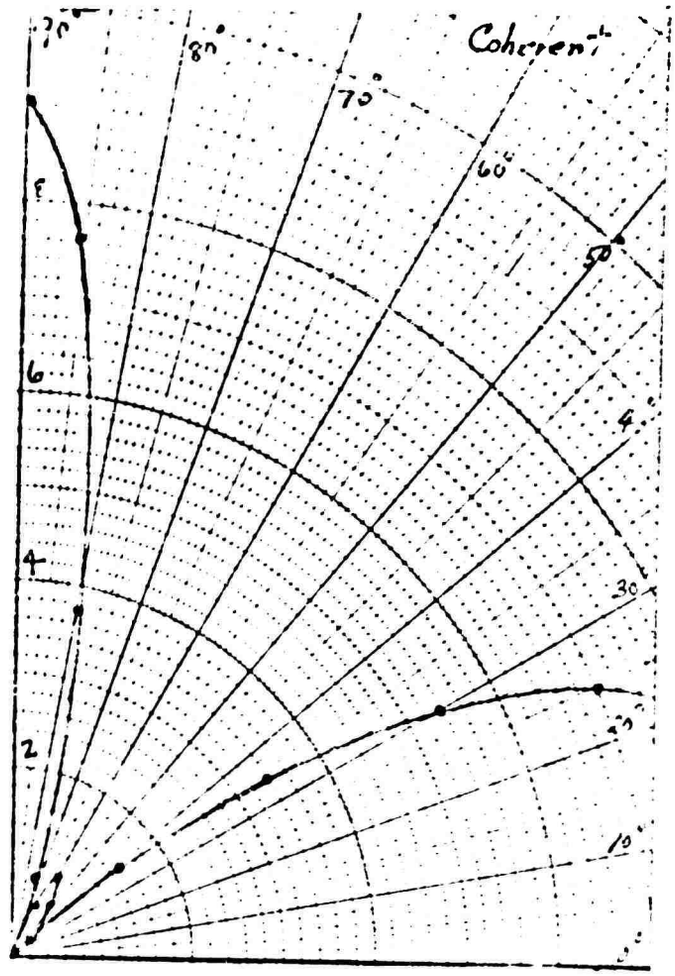

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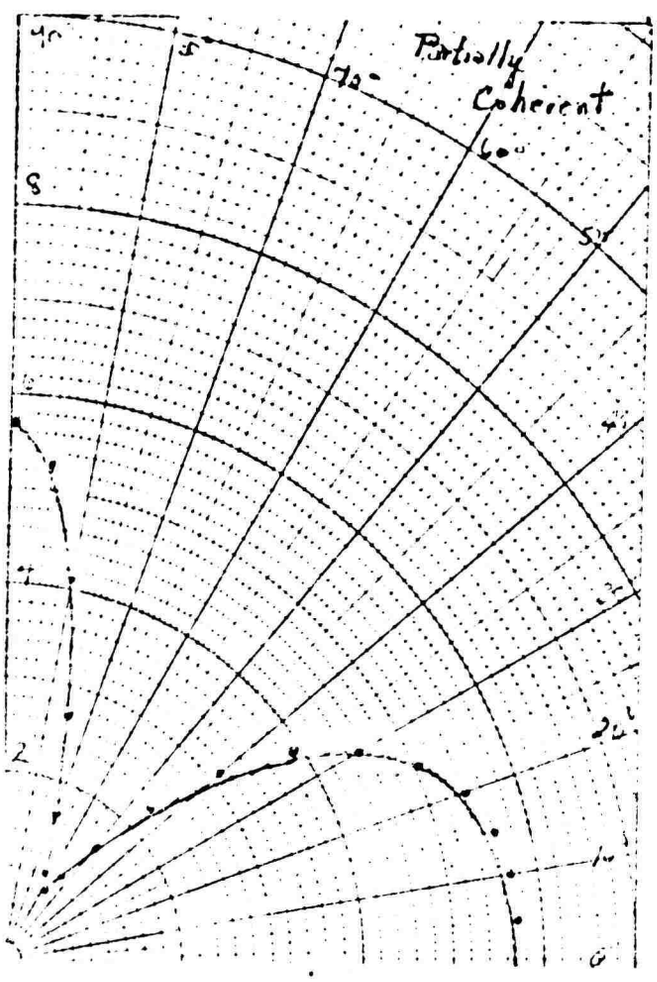

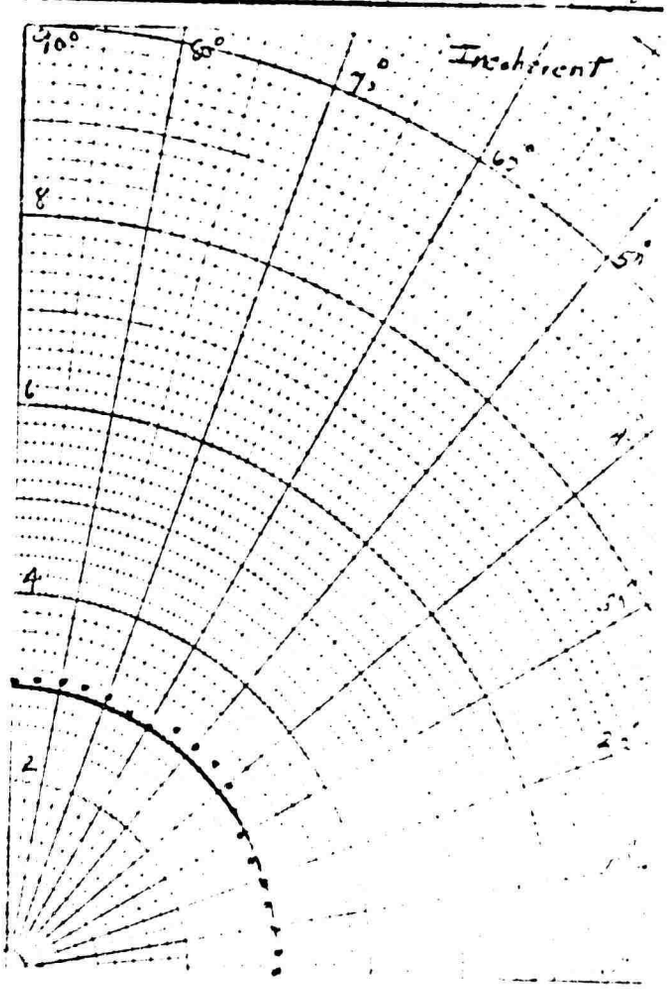

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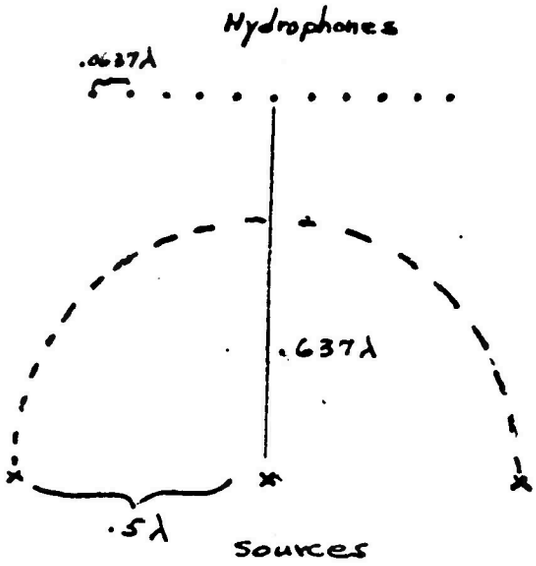

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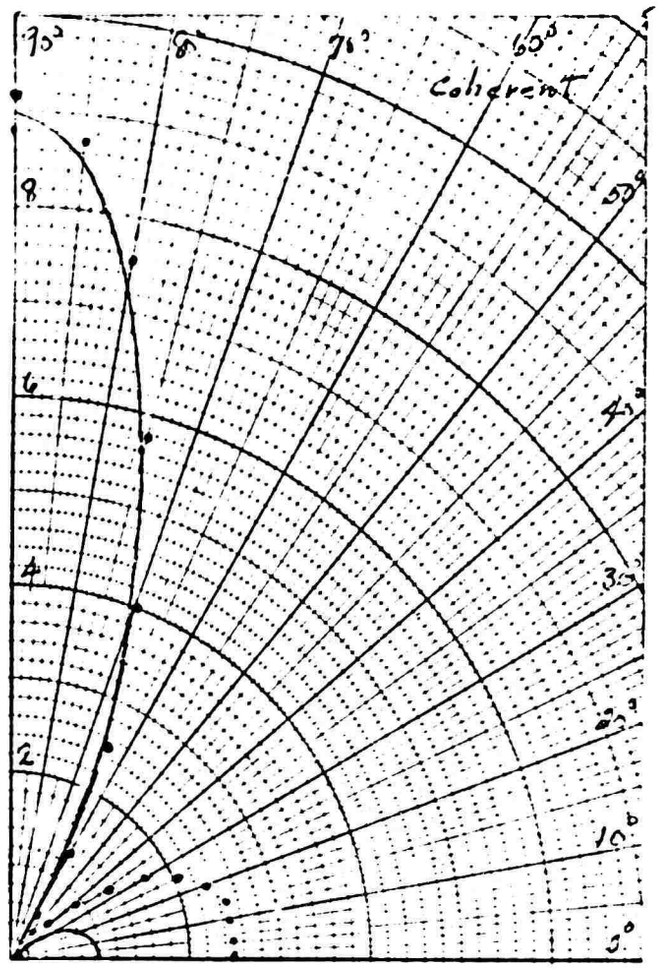

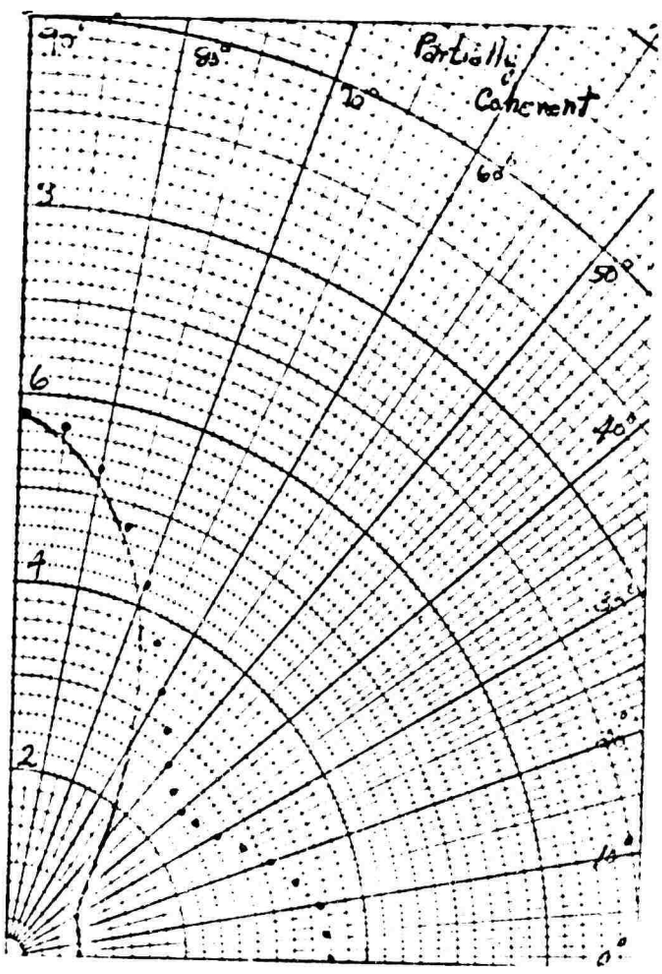

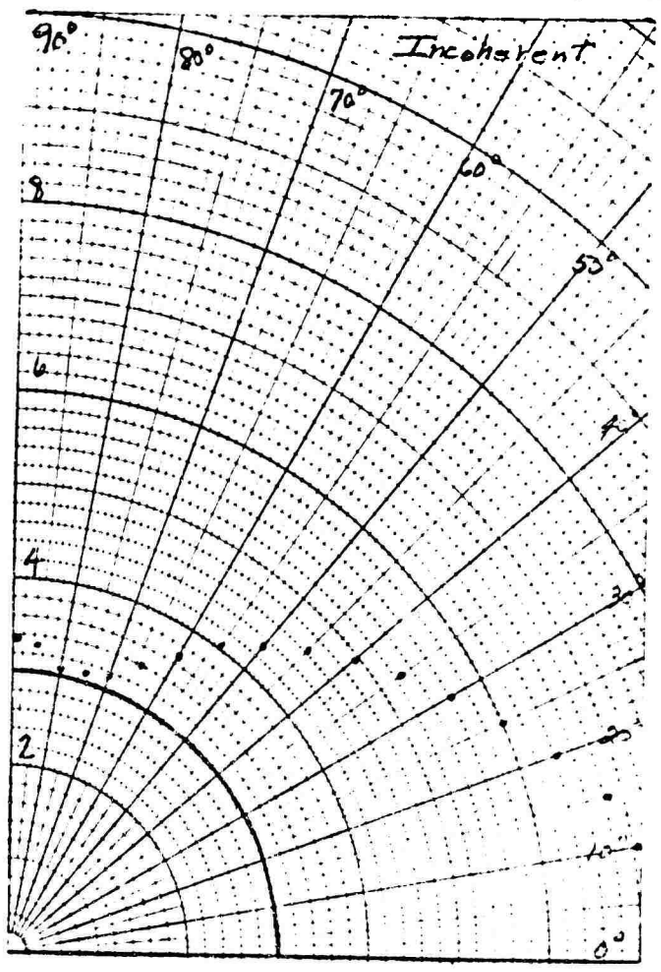

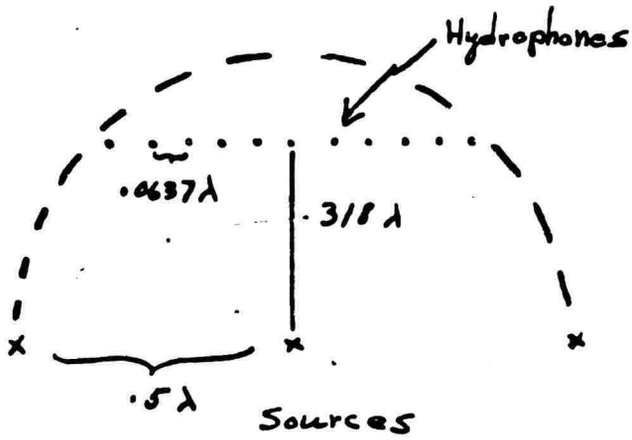

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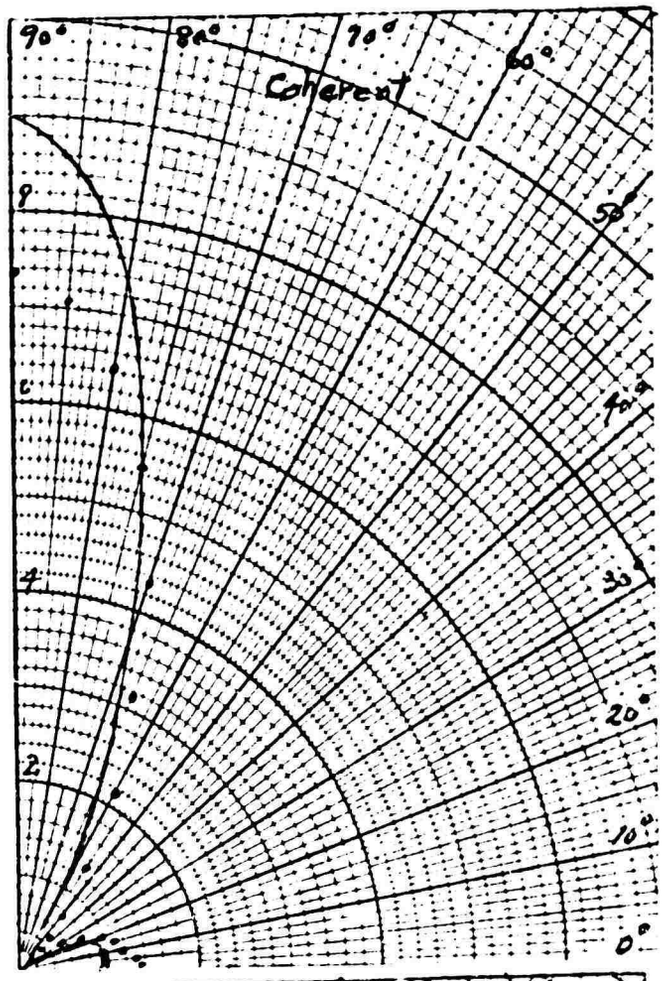

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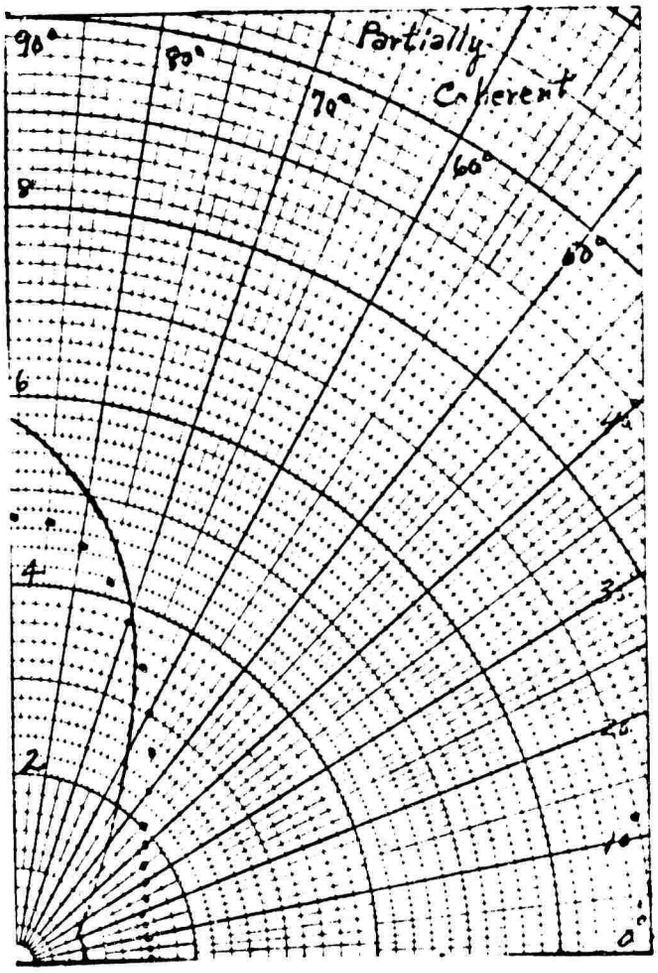

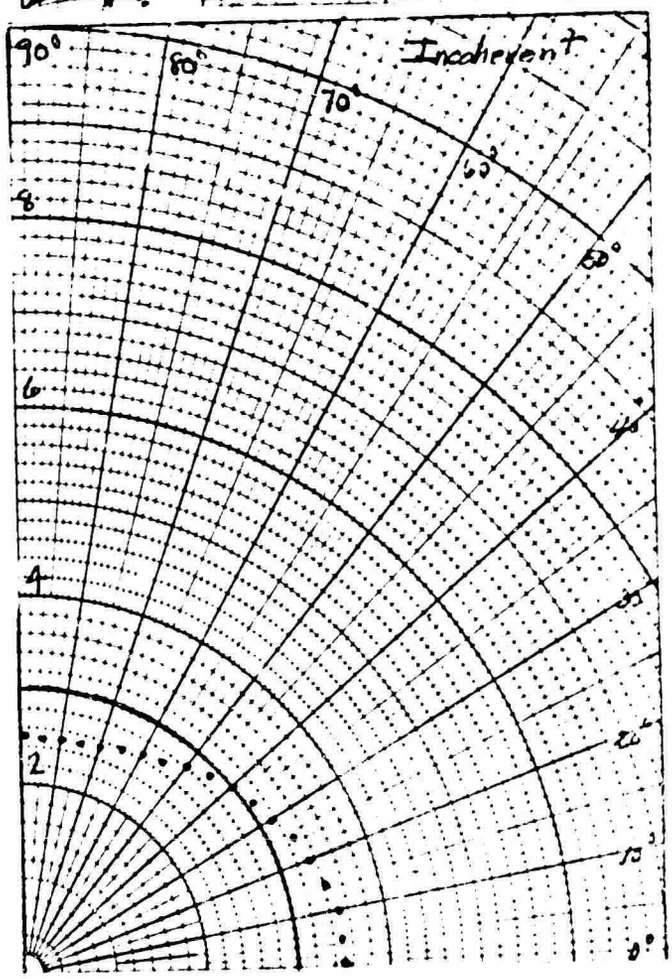

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13. ABSTRACT

An evaluation of a series expansion method for predicting the far-field from near-field cross-power measurements of partially coherent sound fields is given. This series method is based on an expansion in spherical wave functions truncated to include only the number of terms equal to the number of near field measuring points (which need not be restricted to the usual separable surfaces). The evaluation is through a set of numerical examples where the predicted far field is compared with the exact far field from a theoretical source of partially coherent sound. It is shown that good results can be obtained for incoherent, partially coherent, and coherent sound fields.

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