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AUTOMATIC CONTROL SYSTEMS FOR LEGGED LOCOMOTION MACHINES

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ELECTRONIC SCIENCES LABORATORY

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Technical Report

AUTOMATIC CONTROL SYSTEMS FOR LEGGED LOCOMOTION MACHINES

by

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Further research along the lines established by this investigation may, therefore, ultimately lead to the development of improved prosthetic and orthotic appliances.

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ABSTRACT

The evident advantages enjoyed by animals over wheeled vehicles for locomotion over rough terrain has frequently been noted by the designers of military vehicles and others and has led to speculation relating to the possibility of constructing legged machines for off-the-road transportation. One of the principal difficulties which have retarded the development of such machines is the absence of a theory of legged locomotion adequate to permit the synthesis of control laws capable of producing the coordinated limb motion necessary for stable forward movement of a vehicle. This dissertation is aimed at a solution of this difficult control problem.

In order to treat the limb coordination control problem in a quantitative fashion, it is necessary to develop some new mathematical models for legged locomotion systems. The simplest such model represents an extension of previous discrete models to include the kinematic aspects of leg motion. It is shown that such a kinematic model can produce previously unsuspected theoretical results relating to the dynamic stability of certain types of gaits.

The results of the kinematic stability theory are used to design a very simple finite state controller to implement a quadruped crawl.

This controller, which is a variety of asynchronous sequential machines, solves the quadruped limb coordination problem by simply turning control motors on and off in the proper sequence in response to discrete feedback signals. The validity of this solution is demonstrated by the presentation of experimental results obtained with a small artificial quadruped. A trot gait was also successfully realized in experiments with this machine.

While a finite state controller is appropriate for the control of slow, stable gaits, more complicated control may be necessary for higher speed locomotion. To investigate this question, a six degree of freedom dynamic model for a locomotion machine is developed. This model has been programmed for a digital computer to permit the study of arbitrary gaits and control laws without the necessity of constructing additional machines. The results of this simulation show that it is possible to obtain stable locomotion by means of an automatic control system even for gaits in which the machine never finds itself in a statically stable condition.

While this dissertation is concerned primarily with quadruped locomotion, it appears that some of the concepts introduced and results obtained have application to the study of biped locomotion.

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Chapter 1

INTRODUCTION

1.1 Historical Background

Legged locomotion has been studied by man throughout recorded history. The earliest studies were probably performed by cave men as is apparent from the drawings found on the walls of their homes. Crude as many of these studies were, they nevertheless attempted to depict the essential elements of legged locomotion. Eventually man attempted to construct walking machines. One of the earliest walking machines, according to legend, was built during the Huang Ming Dynasty (220-265 A.D.). A team of wooden horses was reputedly constructed to carry supplies for the army [1][1]. In 1889 and 1893 Muybridge published his work on animal motion as studied by stop motion photography. This work, in combination with new energy conversion equipment, apparently generated much interest in constructing legged machines. A number of patents have been issued for mechanical walking devices. The earliest of these was issued in 1898 [2]. The limited popularity of such machines has been due to a lack of adaptability and relative inefficiency.

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[1] Bracketed numbers refer to the list of references collected at the end of the text.

The motivations for studying and constructing legged machines have remained essentially the same; they are 1) to provide transportation over unimproved terrain, 2) to provide paraplegics and partial paraplegics with a means of locomotion, and more recently 3) to provide transportation vehicles for use on the moon and other planets.

Theoretical studies have been perfc.:med in order to find an algcrithm, or set of rules, which will enable man to satisfactorily construct a legged locomotion device [3, 4, 5, 6, 7, 8]. The technical difficulties in the construction of a legged machine have existed for a long period of time [2]. The major problem lies in the control of the legs so that the machine can adapt to varying environmental conditions.

1.2 The General Legged Locomotion Problem

The study of leg control for walking machines and animals has been the subject of a number of papers in the past [4, 7, 9, 10, 11, 12 ̄. Tomovic and McGhee have postulated the study of leg control by finite state computer techniques. Using these techniques McGhee has been able to describe manv of the possible gaits (a gait is a time sequence for cycling the legs) existing for multi-legged machines. The objective of McGhee's and Tomovic's work is to make possible the synthesis of a finite state controller (digital computer) for leg control of a machine. One of the results of this thesis is the design of a more general finite state controller incorporating error correcting capacity. This is accomplished by a form of discrete feedback.

The next area of importance is that of stability of a legged machine. The question has been generally neglected in past investigations. Perhaps this is due to a lack of theory and motivation. The concept of an ideal machine will be introduced in this dissertation. The ideal machine is at best an approximation of a realistic machine. However, it serves as a model with which much insight can be obtained about the real system. The requirements for locomotion for this model or ideal machine are developed.

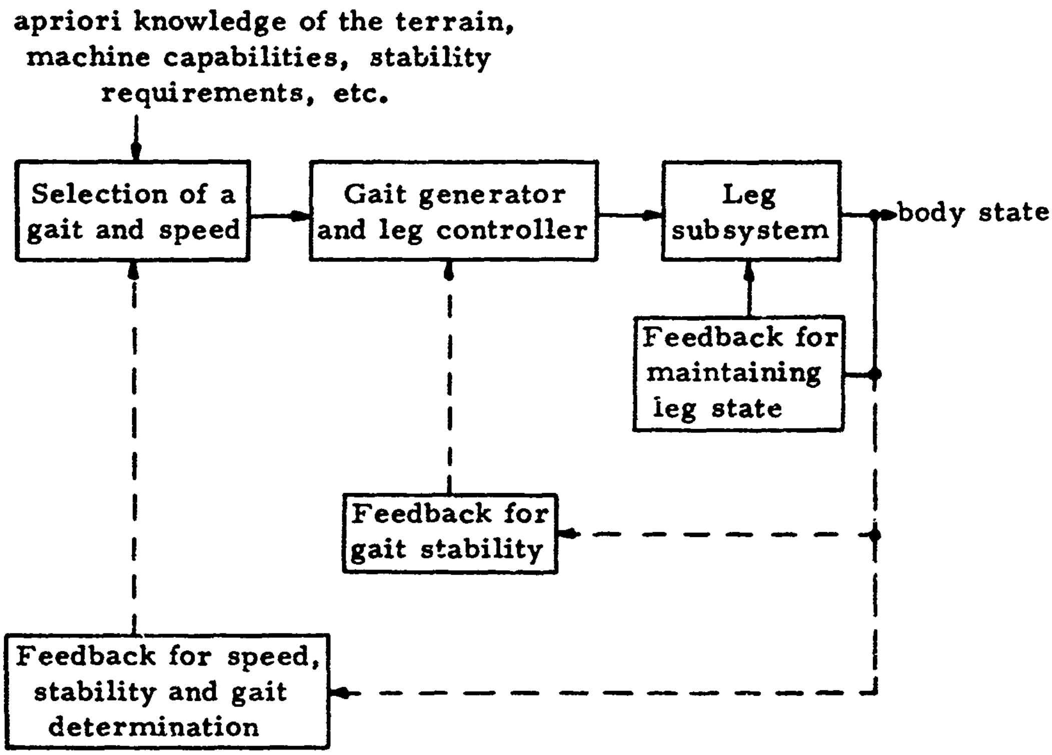
At this point it becomes clear that we should investigate the general organization of a legged machine in terms of its functions and its environment. A given machine and a given environment dictate the following control requirements:

1) the decision for selecting a gait and speed dependent upon terrain, energy requirements and physical limitations

2) the generation of a gait sequence for the legs

3) control of the leg subsystem

This hierarchy can be diagrammed as follows:



The Organization of a General Legged Locomotion Machine
Figure 1.1

This thesis deals primarily with the gait generator-leg controller, the leg subsystem and the stability requirements. The gait and speed selector, or decision processor, in most animals is the brain. In it, the decision on how locomotion is to be accomplished and at what speed is made according to the information the brain receives from its various sensors, i.e. eyes, ears, inner ear, propriocep- tive mechanisms, etc. A machine may be designed with or without this outer loop. With this outer loop the machine is able to adapt to its environment. Without it, the machine is designed for apriori determined environmental conditions. To control this machine for different environments, an operator or driver is required. The gait generator-leg controller mechanizes the direction from the brain or operator. It decides when to activate muscles or actuators from brain information and measured information from the state of the body. Here the state of the body is defined as the positions and velocities (both rectangular and angular) of the body structure and the positions and velocities of the various leg parts. The generation of gaits will be discussed and some specific examples will be analyzed.

Finally the leg subsystem controls the direction and magnitude of the forces applied to the ground whose reactions result in forces on the body. These subsystems must be capable of transmitting the desired forces according to the commands of the gait generator-leg controller.

The necessity for feedback in the various loops is dependent upon the particular machine design. For example, it is conceivable that a machine can be designed with no feedback. The machine applicability however, must clearly be limited since no feedback implies no regulation of any of the sub-blocks in the machine.

Childrens' toys are a good example. Many walking toys exist today. Their mechanical designs are clever, but in general they all suffer from a lack of adaptability, regulation, etc.

1.3 Basic Objectives of the Dissertation

The dissertation seeks to formulate the problems of legged locomotion and to obtain solutions to some of the most fundamental of these problems. Hopefully, these solutions will assist in the synthesis of practical legged locomotion machines. The results should also be useful in the design of prosthetic - orthotic appliances and the rehabilitation of those people suffering from a loss of lower limb functioning. The problems and solutions will be formulated for a general model with n-legs. They will be illustrated specifically for a quadruped machine.

1.4 Organization of the Dissertation

Chapter 1 provides a historical background and outline of the general problem of legged locomotion. The general problem treated in this dissertation is defined.

Chapter 2 provides the basic definitions and concepts required for the analysis of legged locomotion. The general problem of legged machine design is discussed. Notions such as energy consumption, and overall efficiency are clarified for legged machines. Concepts of stability and the control of such machines are introduced. The state of the machine is defined. The problems of leg control implementation by computers are introduced. Various computer systems are discussed. The possibility of using a finite state algorithm or a digital computer for the control of legs is presented.

Chapter 3 provides the core of the theoretical considerations in this dissertation. The dynamics of the problem are investigated in general, and demonstrated for an idealized quadruped performing a trot. Formal definitions are made for a mathematical model of a legged machine. Necessary and sufficient conditions for a specific mode of locomotion are derived for the general machine. The concept of state and dynamic stability is introduced. Computer algorithms to satisfy these general locomotion theorems are discussed.

Chapter 4 presents the construction of a practical machine. The mechanical design is discussed pointing out the design criterion used in the construction. Two distinct finite state computers are presented for two different gaits, the trot and the crawl. The simplicity of the trot gait allowed the design of a relay computer. The crawl gait, owing to the fact that at least three legs are on the ground at any one time, requires a more complicated control algorithm. A solid state computer was used to construct the required logic for control. The discrete computers required the feedback of discrete information from the legs. The amount of feedback, i.e., the number of feedback points per leg, determines the complexity of the computer and depends on the gait.

Chapter 5 provides the conclusions and extensions of the dissertation. In it are discussed the applications of the theory developed to the design of machines and prosthetic and orthotic devices.

Chapter 2

CONCEPTS AND DEFINITIONS

2.1 Definition of Legged Locomotion

2.1.1 General Definition of a Locomotion Machine

For purposes of this dissertation, a machine capable of moving itself from one position in a given fluid 3 space to another, constrained in its path by a given surface and subjected to a gravitational field, is a locomotion machine. The position is referenced to a particular point on the body of the machine. The surface is the terrain over which this machine is to move. The fluid in which the machine is to propagate may be air, water, oil, etc. From this definition terrestrial animals are classified as locomotion machines [13].

2.1.2 Definition of a Legged Locomotion Machine

A locomotion machine consisting of a body and a number of legs is to be defined. The legs are mechanisms with the following properties:

1) A leg is attached to the body of the machine at one end and contacts the ground at the other. The ground contact is made by a foot attached to the leg.

2) The feet are placed on the ground periodically at desired locations. When they are off the ground the feet must clear the ground until the desired location is reached.

3) The legs are capable of supplying a force between the body and the ground when the feet are on the ground.

In terms of this definition for legs, a locomotion machine supported and propelled by legs is a legged locomotion machine.

2.1.3 The Concept of a Gait

For a multi-legged machine, the sequence with which the legs are cycled may establish a periodic pattern. If a distinct repeatable pattern can be established for the foot falls, and if each leg completes one to and fro motion, in any complete cycle of the pattern, then the pattern is called a gait [3]. Note that locomotion can exist without the existence of a gait. For example, the legs may be allowed to cycle according to a random terrain, or one leg may complete two to and fro motions in each pattern, etc. The concept of a gait is useful in classifying the footfall patterns so that each can be examined for their particular properties such as stability, energy requirements, etc.

2.2 Mechanical Concepts Required for Legged Locomotion

2.2.1 Basic Mechanical Requirements

The legs of a machine must satisfy the following three basic mechanical requirements:

1) they must supply a consistent support of the body above the terrain (This does not imply a leg has to be in contact with the ground at all times.)
2) they must supply a driving force to the body in the direction of the desired motion
3) they must be able to return after the support and driving phase without interfering with the terrain.

2.2.2 Environmental Effects

The above requirements must be consistent with the environment, i.e. if the legs do not adapt to the environment, locomotion will cease. The environment in this case includes the roughness of the terrain, measured by the vertical excursion of the soil between its maximum height and the minimum height of the foot depression within the length of the vehicle [14], the 3 space fluid properties, i.e. viscosity, velocity, density, and the gravity field direction and magnitude. The environment dictates mechanical properties such as leg length, foot dimension, the method used for terrain clearance, and the positions of the legs relative to the body. The purpose of the machine is to transport a load over a given terrain. This determines other parameters such as the strength of the legs, the power required to provide locomotion over the varying terrain at the desired velocity, and the dimensions of the feet. To provide for these basic requirements, the leg systems of a machine can be constructed in a number of interesting ways. For example, the legs can be a piston-cylinder arrangement in which the piston is extended during the support and drive phase and retracted during the return phase [15]. The dimensions of the piston and the stroke of the cylinder are readily determined by the above parameters and a knowledge of the working fluid; e.g. hydraulic or pnuematic. In physiological systems, the same problem is solved by bending joints. The advantage of joint mechanization is that it allows a distribution of the power devices over the leg; further, each power device can also be of smaller capacity. To demonstrate this, consider a simple example. In Figure 2.1a, the torque $T_a = m\ell^2\ddot{\alpha}$. Suppose in Figure 2.1b that the masses are a distance $\ell/2$ from their respective pivots and that $m_1 = m_2 = m/2$. In computing the torque required to accelerate the upper leg by $+\ddot{\alpha}$ and the lower leg by $-\ddot{\beta}$, let $T_{b2} = 0$. Then, if the angles α, β are relatively small,

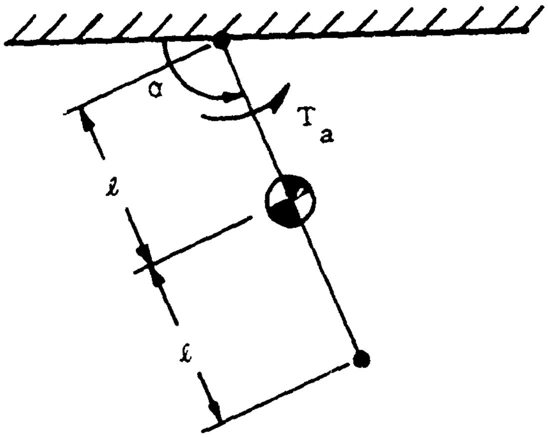


Figure 2.1a

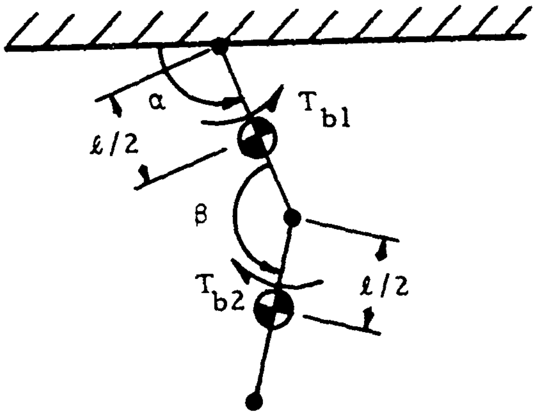


Figure 2.1b

The Purpose of Bending Joints

$$T_{b1} = m/2(\ell/2)^2\ddot{\alpha} + m/2(3\ell/2)^2(\ddot{\alpha} + \ddot{\beta})$$

$$= 5/4(m\ell^2\ddot{\alpha}) + 9/8(m\ell^2\ddot{\beta}) \qquad (2.1)$$

$$= m\ell^2\ddot{\alpha}(1.25 + 1.125\ddot{\beta}/\ddot{\alpha})$$

In the event that $\ddot{\beta}$ is negative and approximately $\ddot{\alpha}$ in magnitude, $T_{b1} = (1/8)T_a$. If, in addition, tl torque T_{b2} is added to the right hand side of the equation, T_{b1} is further reduced as T_{b2} would be negative. The reduction of torque implies that for the same angular acceleration less power is required. Therefore peak power requirements for the same response can be reduced substantially when this method is used.

Other ways to obtain ground clearance are achieved by lifting or retracting the entire leg into the body, by hopping or jumping, and by lateral rotation of the body. The advantages and disadvantages of each of these methods have to be weighed with respect to a particular machine.

2.2.3 Energy Transfer

The previous discussion has presented a number of approaches to the design of a leg to satisfy the basic requirements for locomotion. The overall design of a particular locomotion machine is dependent on many other factors, one of them being the total energy expended for locomotion. To conserve energy it is necessary, where possible, to store energy in order to release it at a more desirable time. A spring has this capability; i.e., it can store kinetic energy as potential energy, then, on release, the kinetic energy can be retrieved. Raising and lowering masses against gravity is another way of achieving this same effect.

Energy transfer can take place in many parts of a legged machine. The leg systems themselves should contain some form of energy transfer since they oscillate. In addition to oscillation, they also must strike the ground. Both these functions can be made more energy efficient if energy transfer devices are included in a leg system. Without such devices, the oscillatory motion would require energy to start and to stop the to and fro motion. The kinetic energy when the foot strikes the ground must be dissipated in heat in the leg or in soil displacement. If such devices are not incorporated on the body, each time the body falls, its loss of potential energy must be absorbed by the system. Thus, for a reasonably energy efficient machine, energy transfer devices are necessary. Notice that a wheeled vehicle on a smooth road suffers from none of these problems. If a legged machine were able to achieve all of its energy transfers in an optimal way it could conceivably rival a wheeled vehicle in efficiency, even on a smooth road.

2.2.4 Instantaneous Power Requirements

A high amount of instantaneous power is likely to be required for cycling the legs, accelerating the mass, and maintaining a velocity against a gravity field; e.g. climbing a hill. The average power required for locomotion of a highly efficient machine on a level (level meaning perpendicular to the gravity field) is the power required to overcome internal and the fluid dynamic friction of the 3 space. This average power can be relatively low. The instantaneous power, however, can be many times greater than the average power. For example, the power required to drive the legs fore and aft is the product of torque and angular velocity. If the motion of a leg is idealized to that of a simple harmonic oscillator, then assuming no friction, the angular velocity is $\omega = \omega_0 \cos \omega t$, the torque is $T = T_0 \sin \omega t$ and the power is $P(t) = (T_0 \omega_0 / 2) \sin 2\omega t$. It is obvious that the average power required is zero and yet the peak instantaneous power is $T_0 \omega_0 / 2$, which is positive twice and negative twice for each leg cycle. This instantaneous power is required and can be obtained from an energy storage device (a spring), an active source (an actuator, motor or muscle), or some suitable combination. The torque required in this simple example is determined by the mass of the leg. Therefore it is desirable to have as light a leg as possible for an efficient machine. A good example of this is a horse, which has a massive body but rather light legs. The average power required to accelerate and decelerate the body mass may also be close to zero for a system with an energy converter, but the instantaneous requirements may be quite high. Of course, for systems without long term energy storage capability, positive instantaneous power is required for both acceleration and deceleration.

2.2.5 Overall Machine Efficiency

Efficiency of a legged locomotion machine can be measured by the total energy required for a specific task. The task can be any specified set of motions in a given environment. The factors influencing the efficiency of one design relative to another are simply the management of internal power and energy. For example, a machine without any energy storage devices may require a hundred times more energy to perform the same task than a machine with energy storage capabilities. The harmonic oscillator leg cycling system with an energy storage device in the above example required zero total energy per cycle. If it had no energy storage device, it would require an average energy input of

$$E_{ave} = 4\int_{0}^{\pi/2\omega} P(S)dS$$
$$= 4T_0 \text{ ft lbs per cycle} \tag{2.2}$$

In this example no friction is assumed, no difference between forward drive and return stroke, etc., but it does illustrate the point.

2.3 Control System Concepts: Stability and Control

In reference to the block diagram of Chapter 1, the concepts of stability and control relate to the first (decision) block.

2.3.1 Systems Under Consideration

In order to make any positive statements about stability, a system must at least be capable of operation in an environment in which the gravity field is perpendicular to the terrain. The terrain will be assumed to be smooth and level (smooth implying a roughness factor very much less than the dimensions of the legs) and the 3 space fluid stationary until disturbed by the machine. Capable of operation implies that locomotion can take place. Note the legs need not cycle with any regularity for locomotion to take place; i.e. a gait need not exist.

2.3.2 Concept of Body State

In order to investigate the stability and control of any system, a definition of the state of the system is required. A legged locomotion machine consists of many dynamical elements. Thus the total state of the machine is rather complex. However, the most important element is the body of the machine. Consequently, the dynamics of the body will be considered as the state to which stability and control theory will be applied. Thus the body state will be defined as the translational position and velocity of the center of gravity and the angular position and velocity of the body to which the legs are attached. The body state thus has 12 elements.

2.3.3 Stability of the System State

There are two distinct concepts of stability required for this class of machines. The first is concerned with obtaining locomotion from a machine. The second is concerned with the relative stability of a moving system to perturbations. It is clear that the second concept is dependent on the first, but satisfying the first concept gives little insight into the second. To avoid confusion of terms the following definitions will be made; a legged machine will be called practical if locomotion can be achieved. It will be called practically stable [16], if it can sustain perturbations while in motion.

The concept of a practical locomotion machine will allow the determination of the required combination of legs on the ground and applied forces for locomotion. A criterion can be developed which, if satisfied, will allow a legged machine to be practical. The criterion should account for those cases in which all legs are off the ground as well as when various combinations of legs are on the ground.

If the practical machine has some degree of freedom in its body state, its leg placements and its leg forces, then another criterion is required to make a judgment of its relative merits. The concept of practical stability is one criterion which can be used. The perturbations may be construed as variations in terrain, a sudden gust of the fluid, or a sudden change in systems parameters such as mass. Another criterion equally valid to judge these merits may be to use total energy consumption. Such a criterion is dependent on the particular machine and gen al statements are difficult to make.

The dual definition of stability is due to the nonlinear and time varying nature of the basic system.

2. 3. 4 Control of the System State

After conditions of stability are satisfied, the control of such machines to achieve some desired state can be considered. It is obvious from the outset that not all elements of state can be controlled without bound. Control of such machines, because of the restrictions placed on the forcing elements, and the bounds on the body state may be difficult to achieve in the classical (Kalman) sense [17]. An alternative notion of controllability will be proposed for this class of machines. The body state of a legged locomotion system can be controlled if a specified objective function involving some components of the system state can be minimized. This corresponds roughly to the concept of regional controllability as proposed by Tou [18].

The objective function will be picked by the designer of the machine. The arguments of the objective function are the parameters to be controlled. Thus the objective function may include components such as, direction or path, velocity, altitude above the terrain, vertical acceleration or jerk (smoothness of ride), energy dissipation and body attitude. Since only a minimum is required on certain elements of the state, the control may not always be precise; or, the desirable state may not be completely achieved, but it may be consid ed satisfactory.

An example of a suitable objective function is a quadratic combination of the error between the parameters of concern and their desired values. Such a function is

$$
\begin{aligned}
Q = \int_0^T \Big[& a(x \quad x_d)^2 + b(y - y_d)^2 + c(x - x_d)^2 \\
& + d(y - y_d)^2 + e(h - h_d)^2 + f(h)^2 + gP(t) \\
& + h(\alpha - \alpha_d)^2 + i(\beta - \beta_d)^2 + j(\gamma - \gamma_d)^2 \ldots \\
& \ldots\ldots\ldots\ldots \Big] dt
\end{aligned}
$$

where

$a, \ldots, n$ = weighting constants

x, y, h = translational dimensions of the body state

α, β, γ = rotational dimensions of the body state

h = jerk measurement

$P(t)$ = power consumed

If Q can be minimized, then the system is controlled. The minimization of Q will lead to the determination of control laws. These laws determine the thrust vectors of the legs as a function of the state of the system. It is not always possible to minimize Q. In these cases the system is termed uncontrolled.

With this formulation, it is possible to consider two distinct problems of control. For a smooth terrain and no disturbances the problem is deterministic. For a random terrain and random disturbances the problem becomes stochastic.

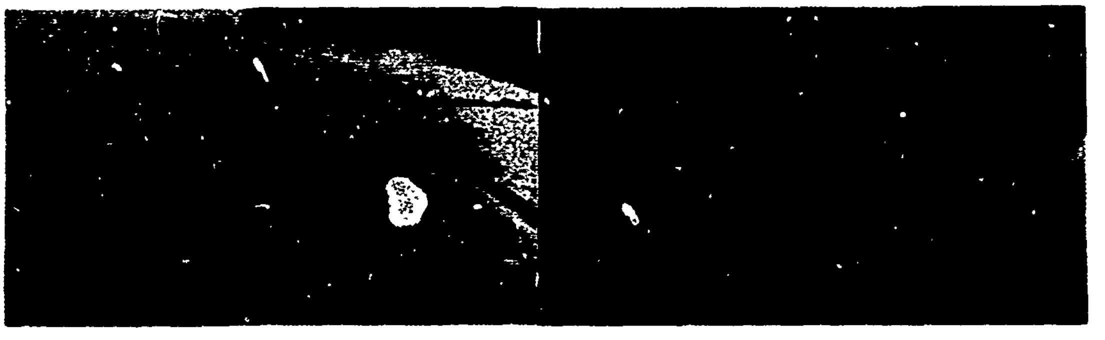
2.4 Computer system Concepts

Various ways of implementing a leg sequencer will be presented.

2.4.1 Mechanical Computers

One obvious way to enable cycling of the four legs is to mechanically gear the legs together [19, 20]. This technique is attractive from the standpoint of apparent simplicity. The actual construction of gears to do this job however, turns out to be rather complex. The designer is usually forced to use either cams or elliptical gears [21], neither of which lend themselves to easy change. Thus the disadvantage of such systems is their lack of flexibility. One of the prime requirements of such a system is that it be able to provide locomotion on unimproved terrain. Since all sorts of terrain exist, a useful machine would require an adaptive or changeable mechanism.

An example of this simple type of computer is shown in the following figure:



A Mechanical Computer Example
Figure 2.2

This machine is able to "crawl" but that is all it is able to do. It is constructed of an undulating pelvic section linked to a fore-aft oscillator which drives the four legs.

It is possible to conceive of more sophisticated mechanical computers such as Miratori's project the golden horse [22] with a hydraulic sequencer. But once the system becomes this complicated it would perhaps be more advantages to go to a electromechanical system with an electronic computer controlling a mechanical system. The same information must be processed regardless of the construction so no more mention of how to mechanize the desired leg control signals will be made.

2.4.2 Analog or Continuous Leg Control Systems

It is possible to generate a voltage, for example, as a function of time and require each leg system to follow such a voltage wave form. Leg sequencing will then be obtained by shifting the wave forms in time, or phase, between each other. The sequences can be stored and called when desired. Such a system is again a relatively simple concept for cycling the legs. The system can be synchronized to a clock. The clock frequency determines the speed of the machine and is an input to the computer. The frequency need not be constant and may vary with the speed of the machine.

Feedback may be requ' ed from the legs to determine when a leg has struck an obstacle, slipped, or fallen into a hole, so that the program can be altered accordingly. Without this feedback the system is open loop and if an obstacle is encountered which stalls one leg, the entire machine will go out of sequence and may become unstable.

The disadvantage of this method is that active control is required to maintain the leg synchronization. This implies energy consumption. It may be possible to construct a leg system in which little total energy is used and yet the system is analog, i.e. continuous. This may well be an excellent method of leg control for some machines. All such systems have a clock, either implicitly due to the existence of a ramp generator, or explicitly.

2.4.3 Digital or Discrete Leg Control Systems

The fact that the legs must cycle in some specified sequence and the fact that they are either on the ground and supplying a control force to the body, or off the ground and returning, lends itself to the suggestion that it is possible to construct a digital computer for the control of the legs. The concept of this binary action however, must be used in conjunction with specific actuators. That is to say, the actuator dynamics and mechanics determine to a large degree how to implement this type of control. The advantage of a discrete system over an analog system is that of having a noise free, highly reliable, simplified system, which is flexible yet adequate. Another advantage is that programs for leg sequences can be stored and switched to at will. These programs may be stored in a very small permanent memory and called for when needed. It is possible to construct a digital counterpart to the analog computer using essentially a digital-to-analog converter.

However, such complexity losses much of the advantage of having a digital system. Another feature of utilizing the actuator characteristics is that the system can be constructed asynchronously, i.e. the command for each actuator can be made dependent upon the position of the other legs, or actuators, of the system. This implies a very elaborate feedback network. An important advantage of such a system is apparent when failure modes are considered; the disadvantage is the lack of continuous control of the system. This disadvantage can be reduced by increasing the number of discrete feedback points associated with the legs.

2.5 Summary

In this chapter a fundamental definition for a legged locomotion machine was presented. With this definition, the stage is set to formally investigate legged locomotion in the next chapter.

Discussed were some important mechanical design considerat... for machine construction. The question of efficiency is outlined. Power and energy requirements are discussed. These points illustrate the ultimate capability of such machines, mechanically speaking.

To provide the basis for application of control theory, a definition of the state of the machine is made. With this definition notions about the stability and control of such machines were introduced. Formal results are presented in the next chapter.

Once the problems of stability are solved, the problem of leg control to satisfy these solutions requires investigation. This problem is shown to be solved by a computer to provide leg coordination and force commands. The construction of such computers is discussed. Some basic schemes are presented.

Chapter 3

THEORY

3.1 Dynamics and Kinematics

3.1.1 Statement of the Problem

As previously defined, a legged locomotion system consists of a number of legs connected to a body and oscillating fore and aft. Also mentioned were some ways in which the legs could satisfy the requirements of fore-aft motion while providing the necessary support and driving forces to the body. The particular kinematic solution for these requirements determines the number of leg segments involved, their masses, degrees of freedom, and the magnitude and direction of the applied torques and forces of each segment. Once the kinematics are selected, the power devices can be designed into the system. The system design will dictate the masses and inertias of each leg segment. Having this information, the dynamic behavior of the entire system can be analyzed. The problem to be solved is the determination of body movement as a function of time. The body will be considered as all of the dynamic elements of the machine. Two methods for obtaining the equations of motion are presented. The first is Lagrange's formulation which allows use of the minimum number of equations; the second is the application of Newton's laws of motion. The second method generally involves more equations but is often simpler in concept. Both methods are illustrated for a quadruped. The gait chosen for the comparison is the trot. The set of equations derived from Newton's laws of motion have been programmed on a computer and the results are presented in this chapter.

3.1.2 Lagrange Formulation of the General Problem

Given a legged locomotion machine, it is desirable to write a set of differential equations to describe the motion. The motion of the body will be continuous; however, the equations describing this motion are discontinuous. The discontinuity arises from the fact that forces can only be applied into the ground. When a leg comes off the ground the force is zero. This nonlinearity is analogous to an ideal diode in an electrical circuit. Neglecting other subtleties for the present, the order of magnitude of the problem of using Lagrange's formulation can be investigated. Consider a general legged locomotion machine. Suppose it consists of a body with a mass and rotational inertias and some number of legs, each leg having n segments and each segment having its own mass and rotational inertias. Further, assume the machine is operating in a gravity field normal to the terrain, which is very smooth, and assume there is no slippage of the feet. Geometrically such a machine might appear as shown in Figure 3.1. The leg system on the ground can be described as shown in Figure 3.2. Now the Lagrangian energy function can be written for the entire leg as

$$L = \sum_{i=1}^{m} \tfrac{1}{2} [m_i(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) + I_{pi}\dot{p}^2 + I_{qi}\dot{q}^2 + I_{ri}\dot{r}^2] - m_i g z_i \tag{3.1}$$

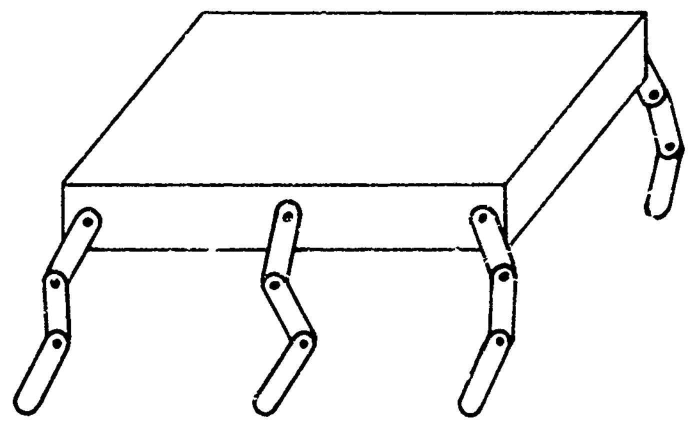
Since the coordinates x, y, z, p, q, r of each segment can be expressed in terms of the angles θ_i and φ_i (if each joint allows only two rotational degrees of freedom), then each segment will have two differential equations of the forms

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \left(\frac{\partial L}{\partial \theta_i}\right) = T_{\theta_i} \tag{3.2}$$

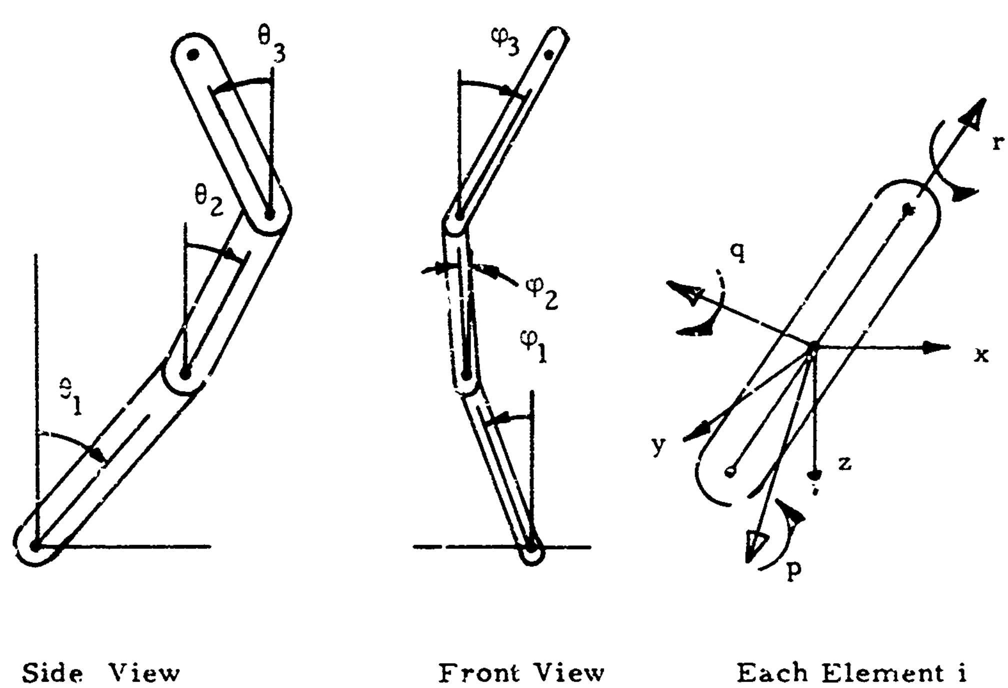
and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}_i}\right) - \left(\frac{\partial L}{\partial \varphi_i}\right) = T_{\varphi_i} \tag{3.3}$$

where T_{θ_i}, T_{φ_i} = Torques applied in the θ and φ directions. Rotation of the elements adds one more equation. The equations must be modified when the leg is off the ground. The angles θ_i and φ_i are still measured with respect to an inertial axis, however, the frame on which the elements are attached is in motion. Thus, in forming



A General Legged Locomotion Machine
Figure 3.1



The Parameters of Leg Analysis
Figure 3.2

the Lagrangian function, the velocities include the frame motion as well. The differential equations will be formed in the same manner as above and will reflect this frame motion. Thus, if no rotation of the segments about their longitudinal axes is allowed, that is simple hinge joints in two dimensions are assumed, each leg segment requires four second order differential equations to describe its motion.

The body has a Lagrangian function like the leg segments; however, the free coordinates depend on the constraints imposed by the legs. For example, if a machine is solidly supported by four stick legs then the body has only two degrees of freedom if the legs are constructed with simple two dimensional hinge joints. The differential equations would be expressed in terms of θ_i and φ_i of the legs.

The number of legs on the ground may change the number of equations required. Thus, for each configuration of legs on the ground a different set of equations will have to be developed. For a multi-leg machine this can be a large number. For a specific locomotion machine such as a quadruped which has two one dimensional joints per leg, the number of equations for the body may be as high as fifty-six. The number fifty-six is derived by considering the combinations of the legs on the ground and the possible degrees of freedom associated with each combination. It is now clear that even for the simplest of machines a large number of equations may be required.

Consider now a highly simplified model of a quadruped. The Lagrange formulation will be applied. The simplification made possible by its use will be illustrated. Let the assumptions be:

1) the legs are constrained to move in a plane parallel to the longitudinal axis of the body
2) the legs have only one joint located at the body
3) the legs are massless and have no spring properties
4) each leg, with a point foot, is of length ℓ
5) torque is applied to the joint of those legs on the ground
6) the body has a mass concentrated at the geometrical center of the four legs and in the same plane as the four joints
7) the body has a moment of inertia I about cg in the "pertinent axis"
8) the machine will be assumed to be symmetrical and the legs will be assumed placed so that only the pertinent axis is required.
9) the legs do not impact upon the surface but gradually make contact

10) there is no possibility of slippage of any leg on the ground

11) the machine is performing a "singular" trot [23], i.e. a trot where at any time there exists exactly two legs on the ground

Pictorially see Figure 3.3. The only dynamic element in this model is the body. The Lagrangian energy function is

$$L = [m/2(\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2) + I_e/2(\ddot{\alpha})^2 + mgz] \tag{3.4}$$

where α is rotation about the axis 1 - 4. If the angle the legs make relative to the body is θ, then L can be expressed in terms of θ, θ, α, and α. The two differential equations resulting from L are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = T_\theta \tag{3.5}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \left(\frac{\partial L}{\partial \alpha}\right) = 0 \tag{3.6}$$

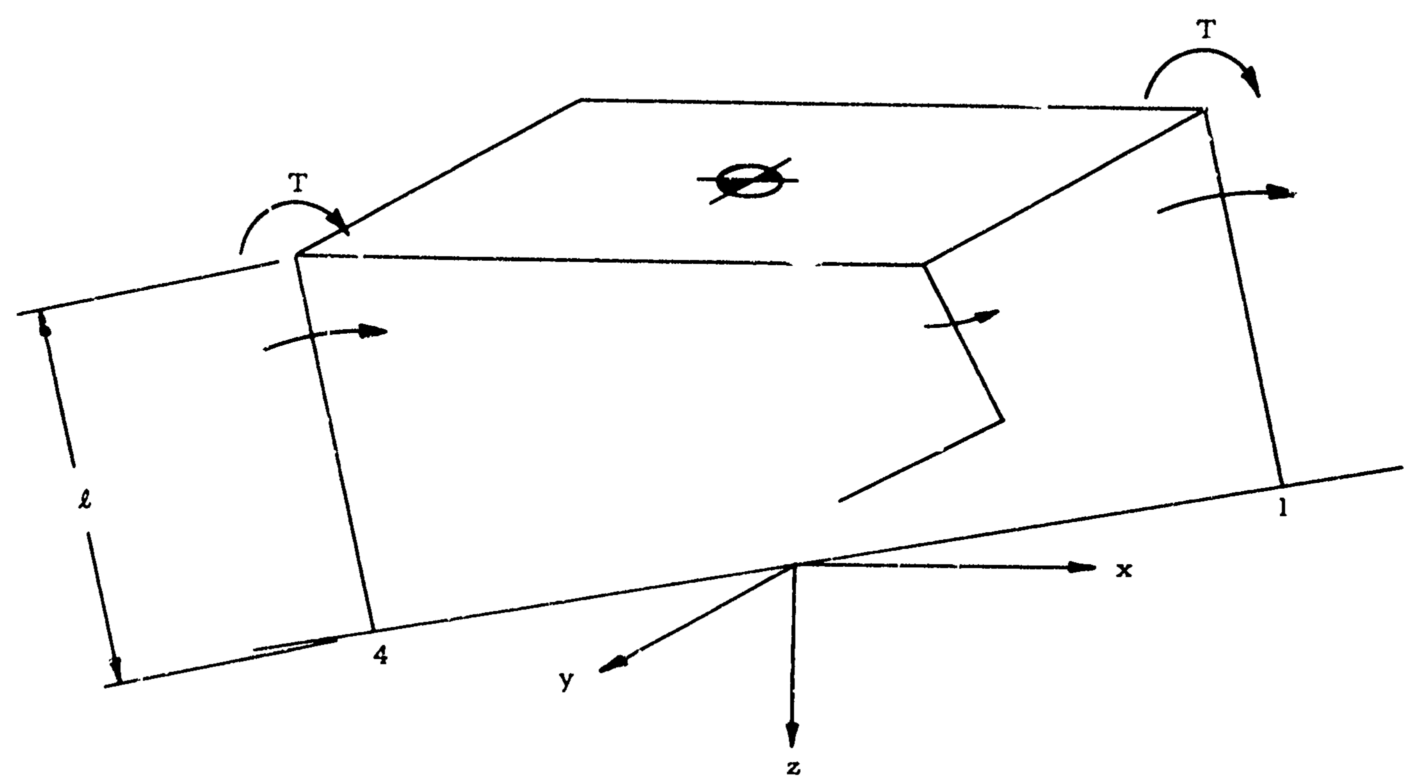
To obtain the Lagrangian function consider Figure 3.4. Let the following parameters be:

b = body length

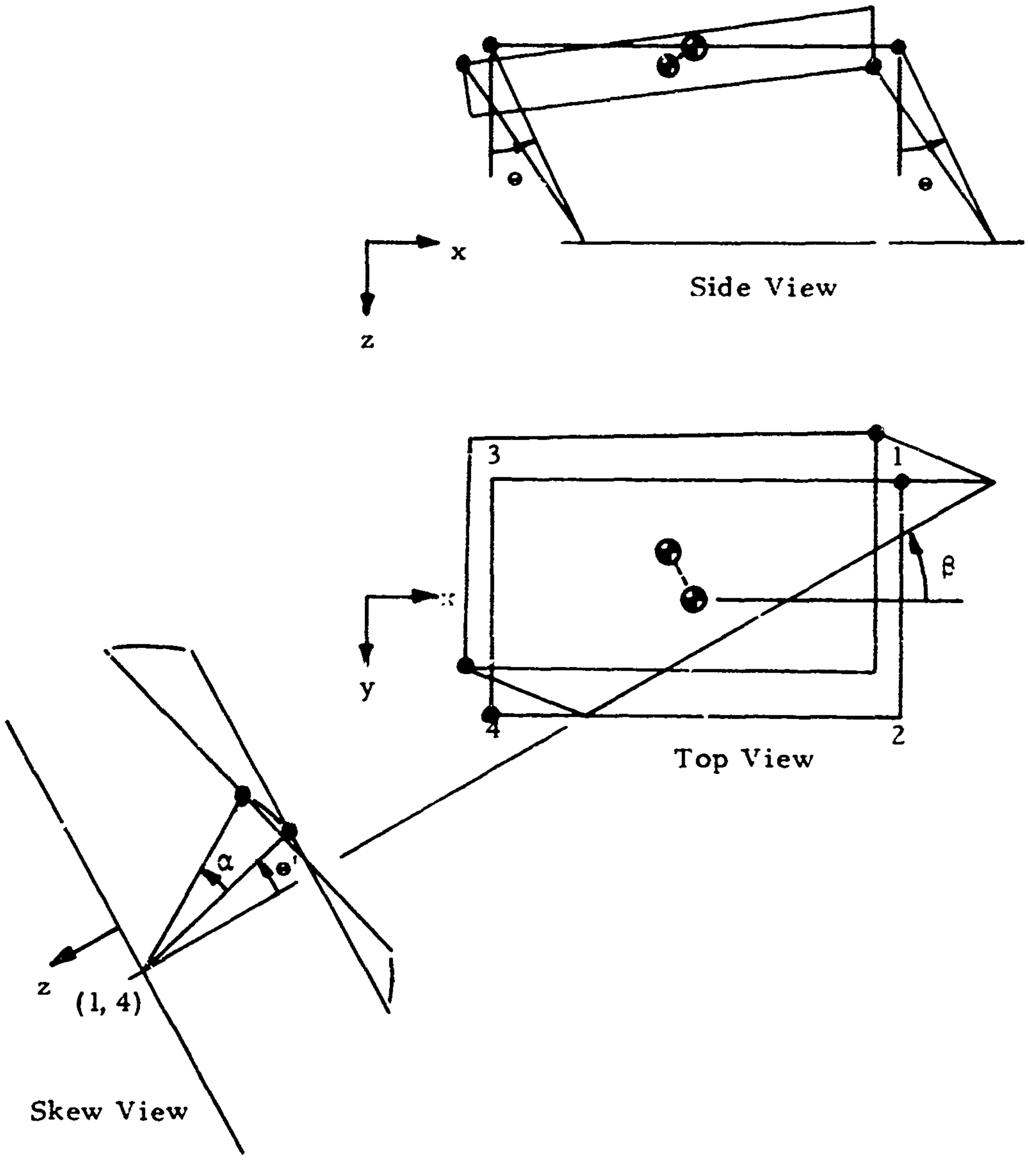
w = body width

θ = angle of the legs with respect to the body

α = angle of the body with respect to the horizontal



A Mathematical Quadruped Performing a Singular Trot
Figure 3.3



Motions of a Fixed Leg Length Trotting Quadruped
Figure 3.4

$$I_e = I_{cg} + m\ell^2$$

Then the position of the body in inertial space is:

$$x = x_0 - \ell\sin\theta - \ell\sin\beta[\sin(\theta' + \alpha) - \sin\theta'] + b/2 \qquad (3.7)$$

$$y = \ell'\cos\beta[\sin(\theta' + \alpha) - \sin\theta'] \qquad (3.8)$$

$$z = \ell'[\cos\theta' - \cos(\alpha + \theta')] - \ell\cos\theta \qquad (3.9)$$

where

$$\ell' = \ell\cos\theta\sec\theta'$$

$$\theta' = \tan^{-1}(\tan\theta\sin\beta)$$

Simplifying the above expressions

$$\begin{aligned} x &= x_f + b/2 - \ell\cos^2\beta\sin\theta - \ell\sin^2\beta\sin\theta\cos\alpha \\ &\quad - \ell\sin\beta\cos\theta\sin\alpha \qquad (3.10) \\ &= A - B\sin\theta - C\sin\theta\cos\alpha - D\cos\theta\sin\alpha \end{aligned}$$

$$\begin{aligned} -z &= \ell\cos\theta\cos\alpha - \ell\sin\beta\sin\theta\sin\alpha \\ &= E\cos\theta\cos\alpha - F\sin\theta\sin\alpha \end{aligned} \qquad (3.11)$$

$$\begin{aligned} y &= \ell(\sin 2\beta/2)\sin\theta(\cos\alpha - 1) + \ell\cos\beta\cos\theta\sin\alpha \\ &= G\sin\theta(\cos\alpha - 1) + H\cos\theta\sin\alpha \end{aligned} \qquad (3.12)$$

where

$$A = x_f + b/2$$

$$B = \ell\cos^2\beta$$

$$C = \ell\sin^2\beta$$

$$D = \ell \sin\beta$$

$$E = \ell$$

$$F = \ell \sin\beta$$

$$G = (\ell \sin 2\beta)/2$$

$$H = \ell \cos\beta$$

The rates are

$$\begin{aligned} \dot{x} &= -B\cos\theta\dot{\theta} + C\sin\theta\sin\alpha\dot{\alpha} - C\cos\theta\dot{\theta}\cos\alpha \\ &\quad -D\cos\theta\cos\alpha\dot{\alpha} + D\sin\theta\dot{\theta}\sin\alpha \\ &= [-B\cos\theta - C\cos\theta\cos\alpha + D\sin\theta\sin\alpha]\dot{\theta} \\ &\quad + [C\sin\theta\sin\alpha - D\cos\theta\cos\alpha]\dot{\alpha} \end{aligned} \tag{3.13}$$

$$\begin{aligned} -\dot{z} &= -E\cos\theta\sin\alpha\dot{\alpha} - E\sin\theta\dot{\theta}\cos\alpha - F\sin\theta\cos\alpha\dot{\alpha} \\ &\quad -F\cos\theta\dot{\theta}\sin\alpha \\ &= [-E\sin\theta\cos\alpha - F\cos\theta\sin\alpha]\dot{\theta} + [-E\cos\theta\sin\alpha \\ &\quad F\sin\theta\cos\alpha]\dot{\alpha} \end{aligned} \tag{3.14}$$

$$\begin{aligned} \dot{y} &= -G\sin\theta\sin\alpha\dot{\alpha} + G\cos\theta\dot{\theta}(\cos\alpha - 1) + H\cos\theta\cos\alpha\dot{\alpha} \\ &\quad -H\sin\theta\dot{\theta}\sin\alpha \\ &= [G\cos\theta(\cos\alpha - 1) - H\sin\theta\sin\alpha]\dot{\theta} + [-G\sin\theta\sin\alpha \\ &\quad + H\cos\theta\cos\alpha]\dot{\alpha} \end{aligned} \tag{3.15}$$

and

$$L = m/2(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + (\dot{\alpha})^2 I_e/2 - mgy \tag{3.16}$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{x}\,\frac{\partial \dot{x}}{\partial \dot{\theta}} + m\dot{y}\,\frac{\partial \dot{y}}{\partial \dot{\theta}} + m\dot{z}\,\frac{\partial \dot{z}}{\partial \theta} \tag{3.17}$$

$$\frac{\partial L}{\partial \theta} = m\dot{x}\,\frac{\partial \dot{x}}{\partial \theta} + m\dot{y}\,\frac{\partial \dot{y}}{\partial \theta} + m\dot{z}\,\frac{\partial \dot{z}}{\partial \theta} + \frac{\partial I_e}{\partial \theta}\,\dot{\alpha}^2 - mg\,\frac{\partial y}{\partial \theta} \tag{3.18}$$

Now equations (3.5) and (3.6) can be solved. To solve these two simultaneous differential equations a computer will have to be used. These equations are valid only when legs 1 and 4 are on the ground. When legs 2 and 3 are on the ground an identical set of equations is obtained, however, the initial conditions for this set of equations are the final set of conditions from the first set of equations. The continuous solution for the problem is obtained by alternating between these two sets of equations. The time to switch must be determined by a criteria dependent upon the driving torque $T_0(t)$, time and positions of the legs θ, or the state of the body. The criteria is computed by a controller.

3.1.3 Newtonian Formulation of the General Problem

To illustrate the use of Newtonian physics for the same problem consider the body as having six degrees of freedom and use rectangular coordinates. The advantage of this method is simplicity and order of thought; the disadvantage being more equations to mechanize. When using a computer however, the problem of a few more equations is insignificiant. The equations, using Newtonian physics and Eular angles will be developed and then programmed on the computer. To do this the following additional assumptions must be made:

1) the moments of inertia about the three principle body axes are I_{xx}, I_{yy}, and I_{zz}

2) the legs are damped springs producing a force along each leg of

$$F_{\ell} = k_{\ell}(\ell - \ell_0) + k_{\ell}\ell \tag{3.19}$$

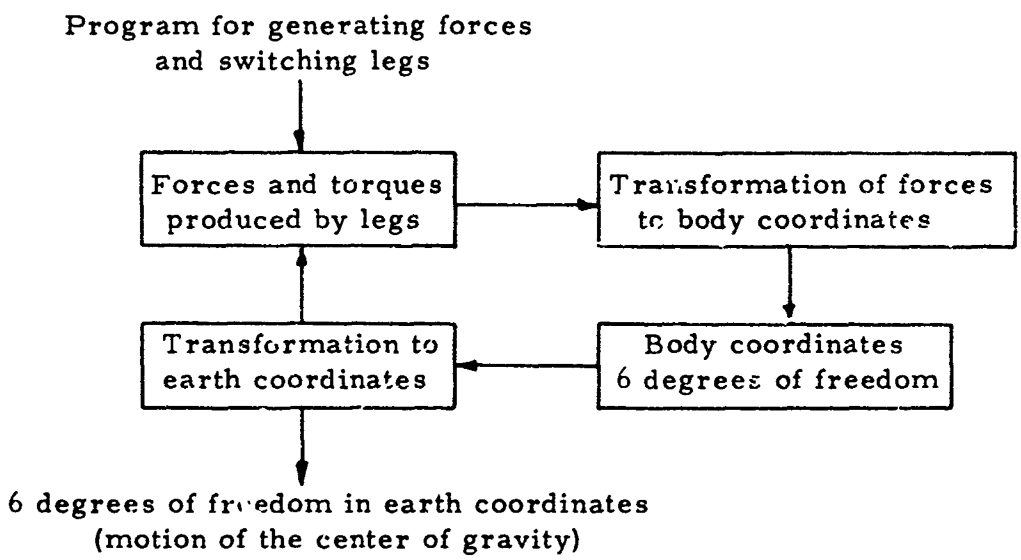
3) the legs have two angular degrees of freedom, longitudinal and lateral

4) a driving torque in the longitudinal direction is applied to the leg body joints

5) lateral forces are restrained by a lateral spring-damper centering system on each leg

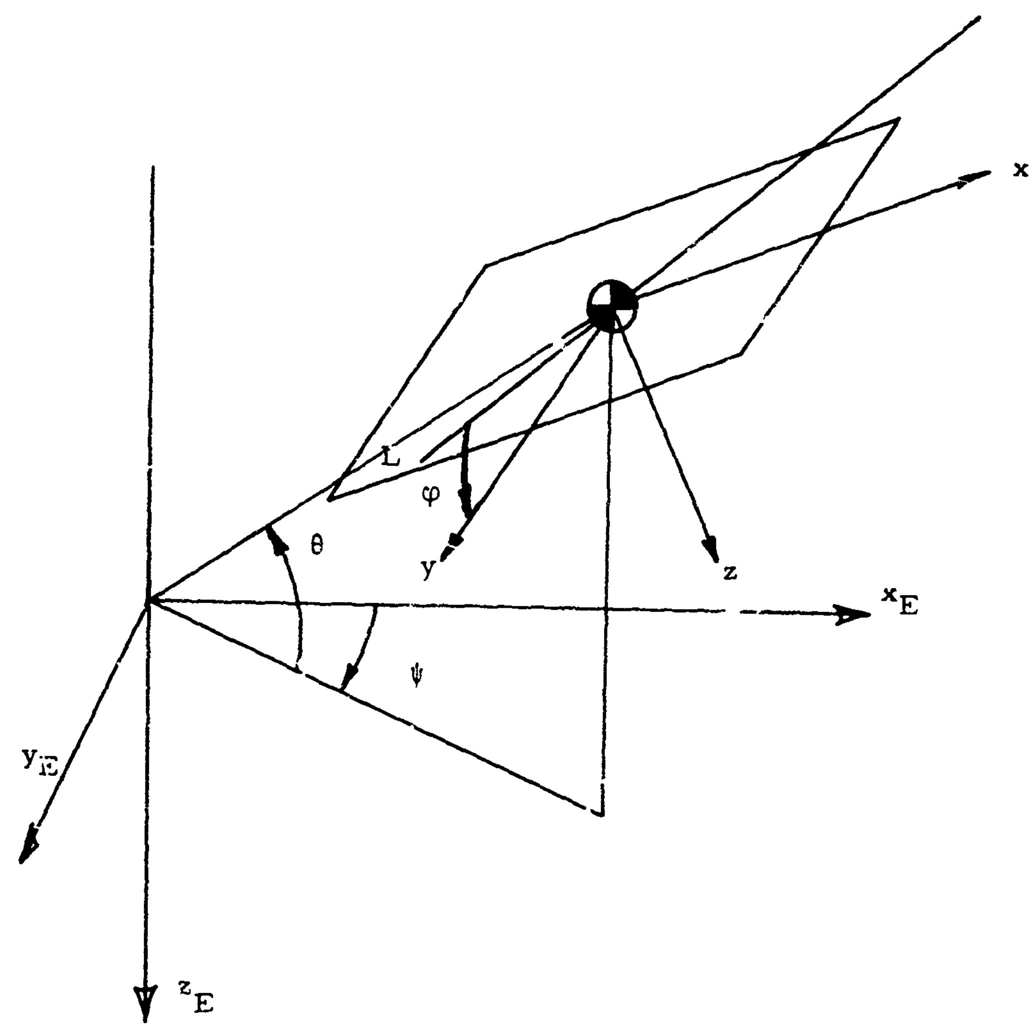
The equations for this system are presented in Figure 9 of the Appendix in computer printout form. A block diagram of these equations is shown in Figure 3.5.

There are 88 equations used for each pair of legs. Each equation however, is considerably simpler than the two equations of the Lagrange formulation. The choice of the method for representing the dynamics will depend on the problem, the constraints and the desires of the programmer. Geometrically, the block diagram represents a picture as in Figure 3.6. The body forces induced by the legs, the body velocity and the angular rates are represented in Figure 3.7.



Block Diagram for Computation of Legged Machine Motions
Figure 3.5

The objective is to compute the forces and torques produced by the legs to the body in body coordinates, then using Newton's laws compute the six velocities of Figure 3.7. Then these velocities are transformed to earth coordinates. Next, using Figure 3.6, these velocities are integrated to obtain angles and positions with respect to earth coordinates. A detail of the flow chart to do this is presented in Figure 3.8.



LL is the intersection of y, z plane and a plane parallel to $x_E \cdot y_E$ plare

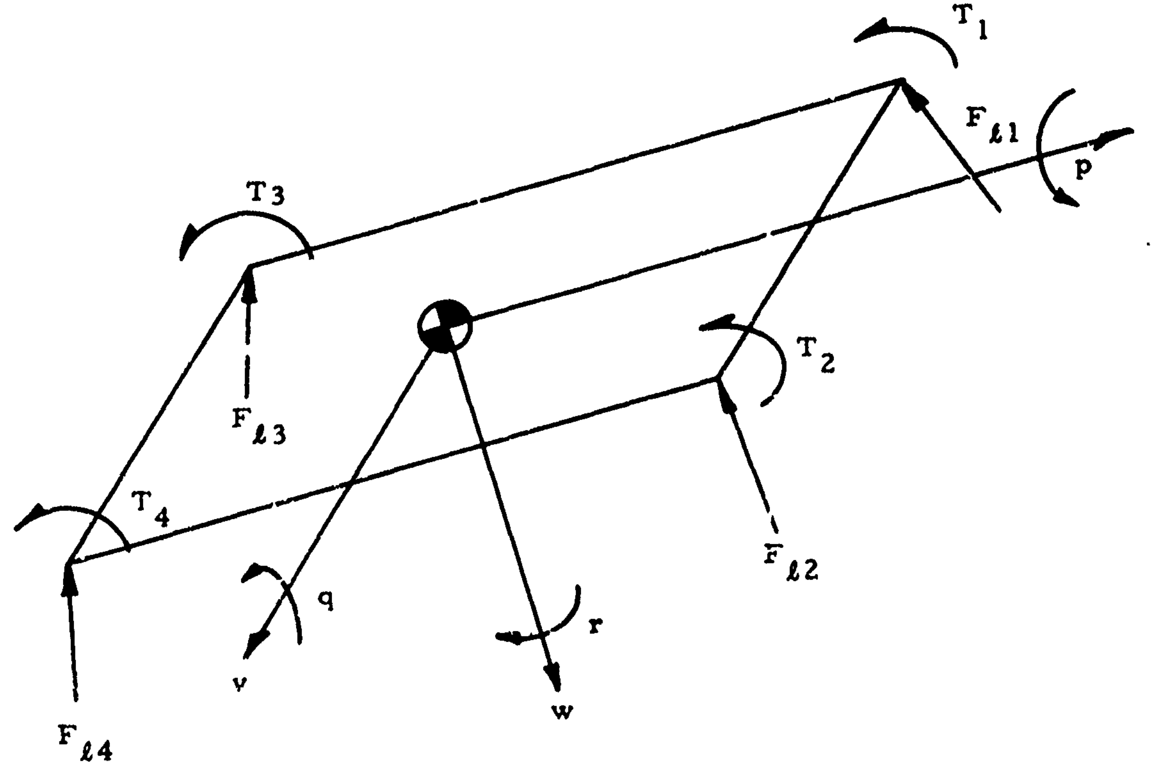
$x_E y_E z_E$ are earth coordinates

θφψ are pitch, roll and yaw of body with respect to earth.

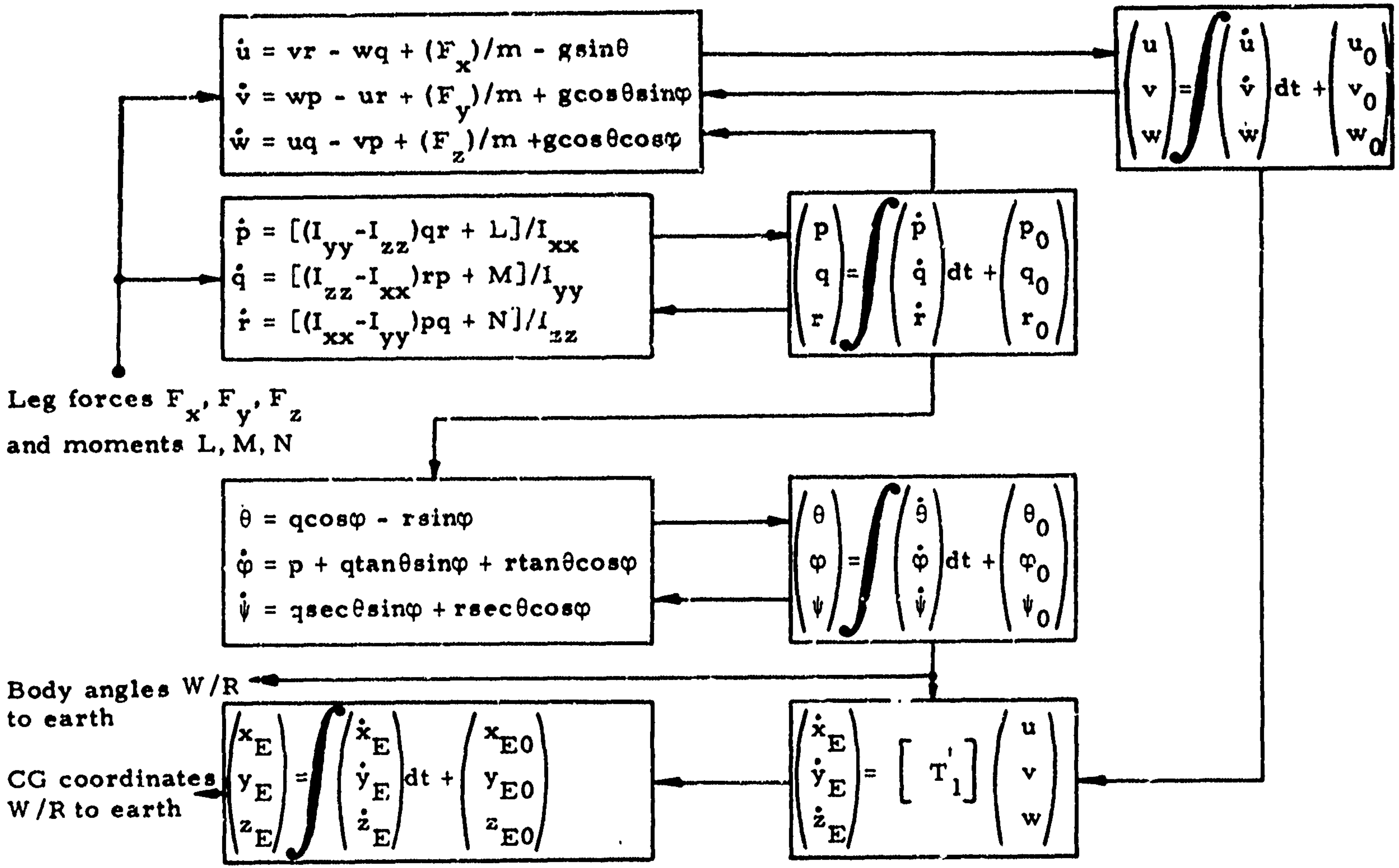
xyz are body coordinates

Body and Earth Coordinate Systems

Figure 3.6



Applied Forces, Torques, Linear Velocity and Angular Velocity
Of the Rigid Body of a General Quadruped
Figure 3.7



Computation of body angles and positions from applied leg forces and torques

Figure 3.8

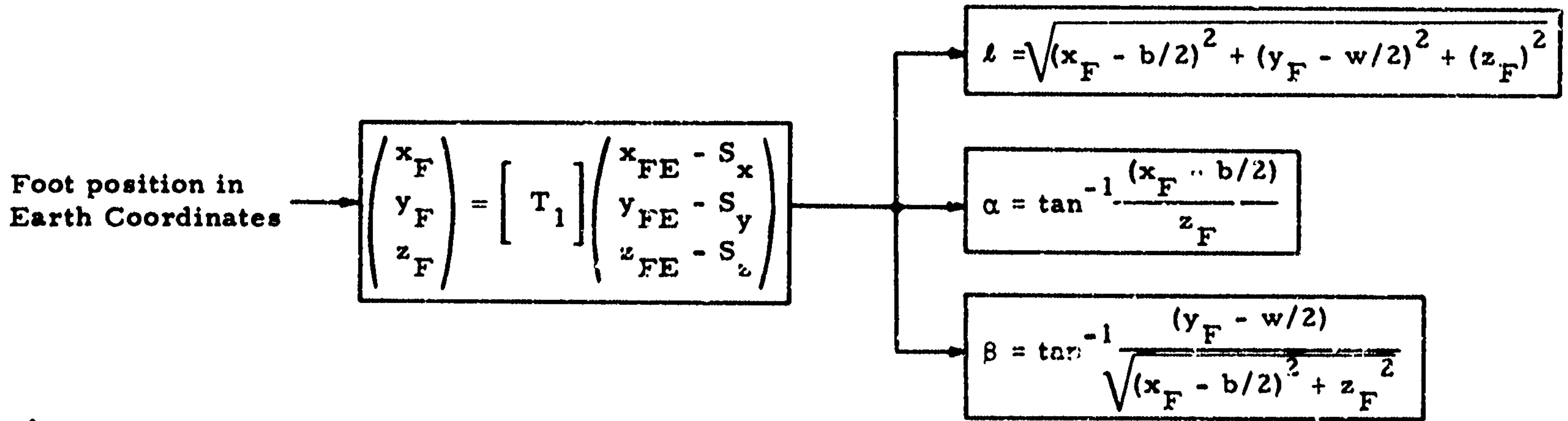
The lateral torque produced by the legs will be assumed proportional to the deflection and deflection rate. The following equation expresses this torque,

$$T_\beta = C_\beta \beta + C_{\dot{\beta}} \dot{\beta} \qquad (3.20)$$

In order to compute the torques and forces of equations (3.19) and (3.20), the quantities ℓ, ℓ, β, β must be computed. This can be done by knowing the body position and angles and the foot positions on the ground. Thus there are two ways these variables can be computed. The first is to compute the foot positions relative to the body, the second is to compute the hip position relative to the feet. The first method is chosen, as this will readily provide forces and torques relative to the body. The position of the feet will be assumed to be precise, i.e. the feet will be placed in ideal precalculated positions. Knowing these positions then the position of the feet relative to the body can be computed. To compute α, $\dot{\alpha}$, β, $\dot{\beta}$, ℓ and $\dot{\ell}$ the flow chart in Figures 3.9 and 3.10 will be used. Here, since the feet are fixed on the ground, the angular velocities are determined by the radial and tangential velocities of the hip sockets.

Now the forces and moments at the center of gravity can be computed. To do this the flow diagram of Figure 3.11 is used.

The computation of leg force and torques is presented for one

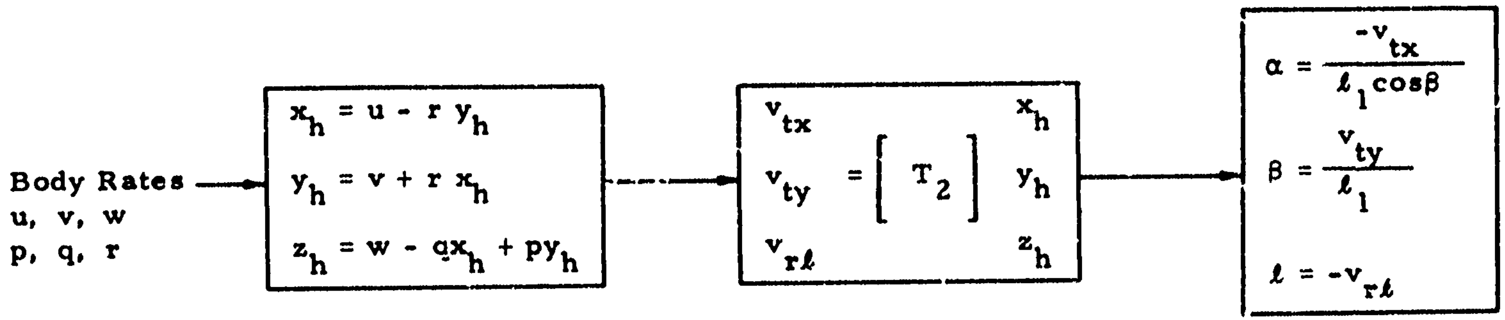


where

$$T_1 = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi\cos\theta & -\sin\theta \\ (\cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi) & (\cos\psi\cos\varphi + \sin\psi\sin\theta\sin\varphi) & \cos\theta\sin\varphi \\ (\sin\psi\sin\varphi + \cos\psi\sin\theta\cos\varphi) & (\sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi) & \cos\varphi\cos\theta \end{bmatrix}$$

b = length of the body
w = width of the body
x_F, y_F, z_F = foot position relative to the body
S_x, S_y, S_z = body position relative to earth

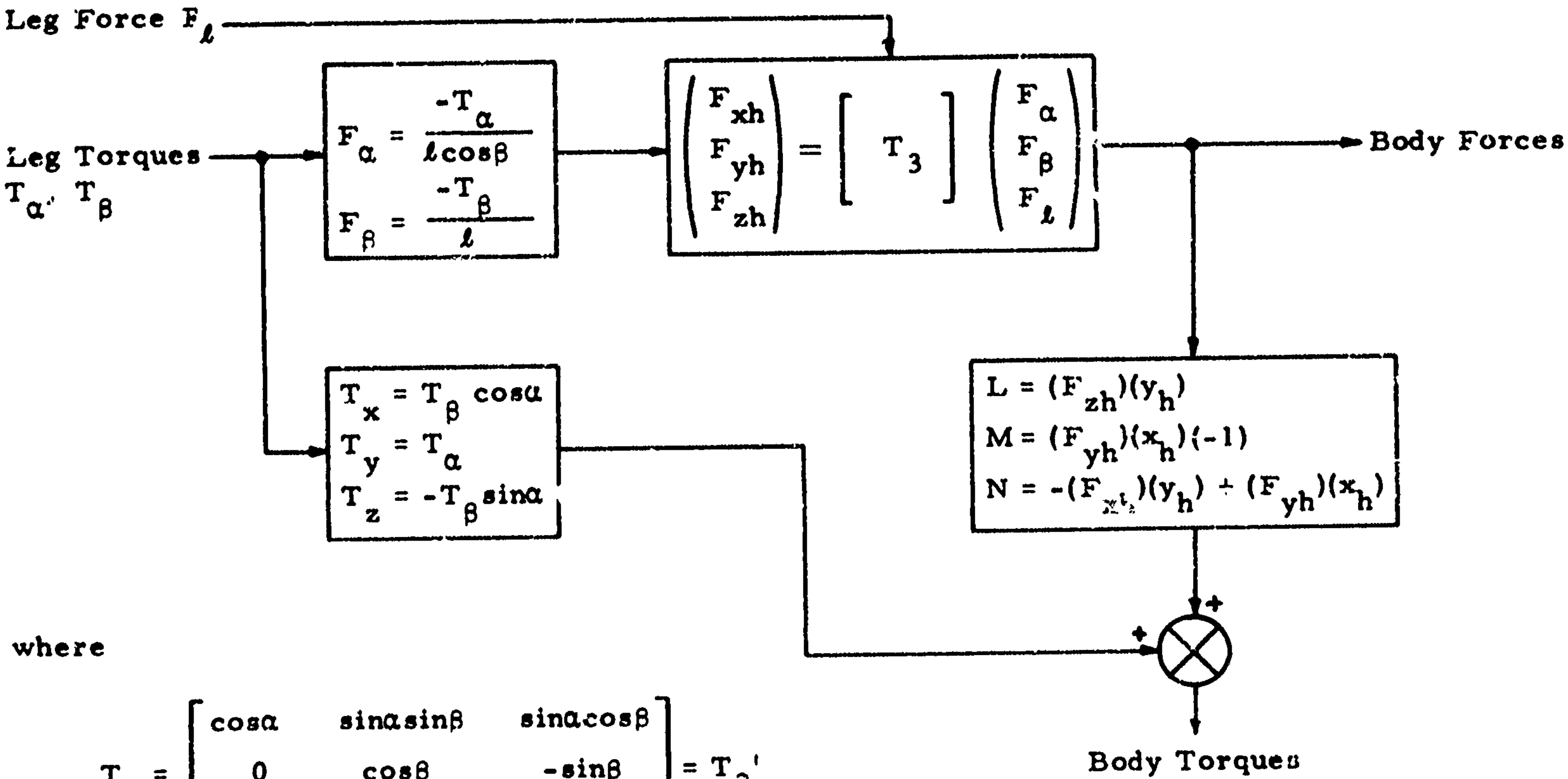
Computation of Leg Angles and Lengths
Figure 3.9



where

$$T_2 = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ \sin\alpha\sin\beta & \cos\beta & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta & -\sin\beta & \cos\alpha\cos\beta \end{bmatrix}$$

Computation of Leg Rates
Figure 3.10



where

$$T_3 = \begin{bmatrix} \cos\alpha & \sin\alpha\sin\beta & \sin\alpha\cos\beta \\ 0 & \cos\beta & -\sin\beta \\ -\sin\alpha & \cos\alpha\sin\beta & \cos\alpha\cos\beta \end{bmatrix} = T_2{}'$$

Computation of Body Forces and Torques
Figure 3.11

leg. The total forces and torques to the body will be the sum of the forces and torques produced by those legs on the ground.

The parameters for the computer simulation are chosen so that the following conditions are met:

1) mass = 20 slugs, w = [illegible], B = 5 ft, ℓ_0 = 3 ft, leg cycle time = 2 sec, stride = 2 ft

2) vibration axis (2 legs on the ground), frequency and damping:

vibration axis	frequency	damping
vertical	12 cps	1.0
fore-aft	3 cps	1.0
lateral	7 cps	1.0
pitch	5 cps	1.0
roll	15 cps	1.0
yaw	4 cps	1.0

These parameters, of course, were selected using a linearized approximate model. From these assumptions, the spring and damping constants of the leg systems are computed. The inertias about the various axes are also selected to meet these assumptions.

It is important to note here that it is possible to write the total force and moment equations for a general n-legged machine with n-massless legs of the form assumed in the above model in matrix form. Here

$$(a) = (A) (t) \tag{3.21}$$

where the components of (a), (A) and (t) are presented in Figure 3.12. Figure 3.13 again illustrates the meaning of equation (3.21).

3.2 Legged Locomotion Theory

As described in the previous section, the general problem of legged locomotion can be rather complex. To obtain theoretical results for a general locomotion machine some simplifications are required to reduce the problem to one which is tractable. The results, then, are for a specific mathematical model, which at best is an approximation of the real situation. This is the modern philosophy adopted in engineering.

3.2.1 Formal Definitions of Legged Locomotion

Legged locomotion will be studied using the assumption of a smooth level surface in which the coefficient of friction is infinite. The fluid is assumed to be a vacuum.

The vector quantities (a) and (t) are:

$$(a) = \begin{pmatrix} mu \\ mv \\ m(w+g) \\ I_{xx}p \\ I_{yy}q \\ I_{zz}r \end{pmatrix} = \begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m(\ddot{z}+g) \\ I_{xx}p \\ I_{yy}q \\ I_{zz}r \end{pmatrix} = \text{Reaction force vector}$$

$$(t) = \begin{pmatrix} T_{\alpha 1} \\ T_{\beta 1} \\ F_{\ell 1} \\ \vdots \\ T_{\alpha n} \\ T_{\beta n} \\ F_{\ell n} \end{pmatrix} = \text{Applied torques and forces vector from each leg i, i=1------n}$$

The matrix (A) is:

$$(A) = \begin{pmatrix} (F_1) & \text{------} & (F_n) \\ & & \\ (M_1) & \text{-------} & (M_n) \end{pmatrix} = \text{Force and moment matrix}$$

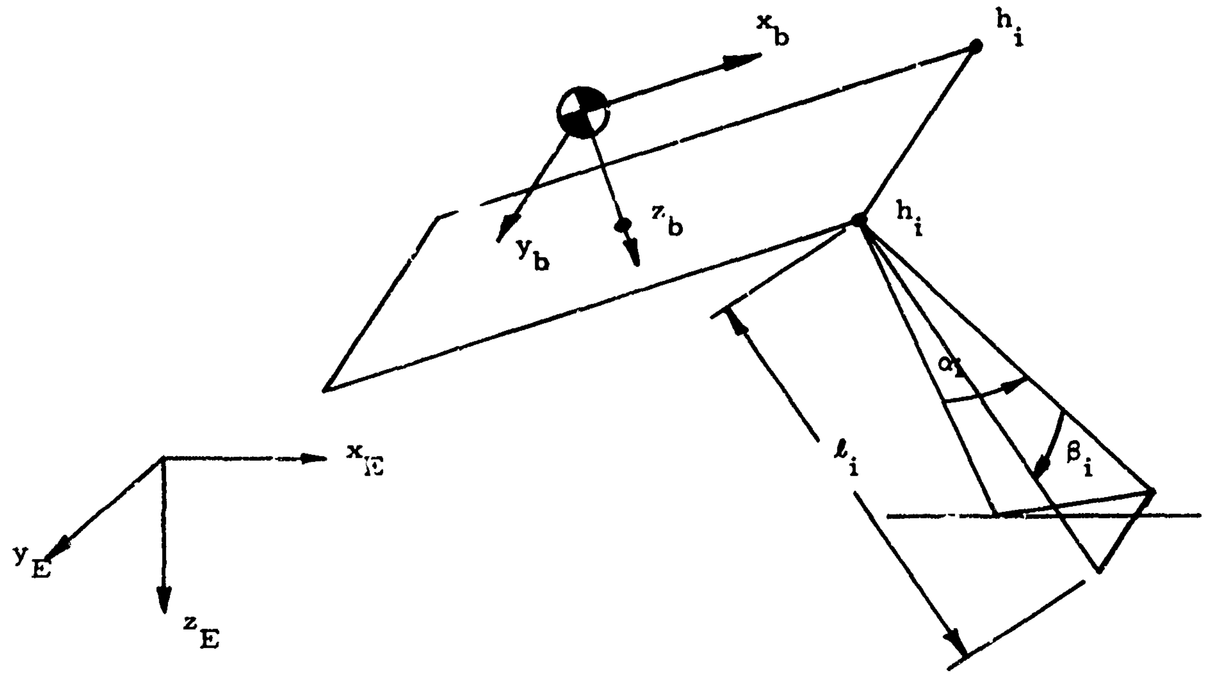
The Generalized Matrix Equation for the Dynamics
of an Ideal Locomotion Machine

Figure 3.12

$$F_i = \begin{pmatrix} -\dfrac{\cos\alpha_i}{\ell_i\cos\beta_i} & -\dfrac{\sin\alpha_i\sin\beta_i}{\ell_i} & -\sin\alpha_i\cos\beta_i \\ 0 & -\dfrac{\cos\beta_i}{\ell_i} & -\sin\beta_i \\ \dfrac{\sin\alpha_i}{\ell_i\cos\beta_i} & -\dfrac{\cos\alpha_i\cos\beta_i}{\ell_i} & -\cos\alpha_i\cos\beta_i \end{pmatrix} = \text{force matrix}$$

$$M_i = \begin{pmatrix} \dfrac{\sin\alpha_i}{\ell_i\cos\beta_i}y_{hi} & \dfrac{\cos\alpha_i\cos\beta_i}{\ell_i}y_{hi} + \dfrac{\cos\beta_i}{\ell_i}z_{hi} + \cos\alpha_i & \cos\alpha_i\cos\beta_i y_{hi} + \sin\beta_i y_{hi} \\ 1 - \dfrac{\sin\alpha_i}{\ell_i\cos\beta_i}x_{hi} - \dfrac{\cos\alpha_i}{\ell_i\cos\beta_i}z_{hi} & \dfrac{\cos\alpha_i\cos\beta_i}{\ell_i}x_{hi} - \dfrac{\sin\alpha_i\sin\beta_i}{\ell_i}z_{hi} & -\cos\alpha_i\cos\beta_i y_{hi} + \sin\alpha_i\cos\beta_i z_{hi} \\ \dfrac{\cos\alpha_i}{\ell_i\cos\beta_i}y_{hi} & \dfrac{\sin\alpha_i\sin\beta_i}{\ell_i}y_{hi} - \dfrac{\cos\beta_i}{\ell_i}x_{hi} - \sin\alpha_i & -\sin\alpha_i\cos\beta_i y_{hi} - \sin\beta_i x_{hi} \end{pmatrix}$$

The Generalized Matrix Equation for the Dynamics of an Ideal Locomotion Machine
Figure 3.12 (continued)



Leg Coordinate System Relative to the Body

Figure 3.13

Consider the following definitions for a legged locomotion machine:

Definition I. An ideal leg has zero mass, arbitrary controllable length, arbitrary compression force, (tension force is impossible), and an arbitrary moment vector applied to the body subject only to the constraint that the net force applied to the supporting surface by any leg can never point in an upward direction. It will be assumed for the following analysis that moment vectors are applied only normal to the length of the leg and no rotational torque along its length can exist.

Definition II. An ideal legged locomotion machine is a rigid body, with mass and rotational inertia, to which are attached a set of ideal legs.

Definition III. Ideal locomotion is any mode of locomotion which results in zero rotational velocity and acceleration about all body axes and maintains zero vertical acceleration and velocity of the center of gravity.

Definition IV. Constant velocity ideal locomotion is ideal locomotion with the added constraint that all horizontal components of acceleration are also zero. This implies constant horizontal velocity.

Definition V. A leg is in a position of support if it is on the ground and supplying a vertically directed force greater than zero to the body.

The foot of a leg in a position of support supplies a load into the ground. If the foot has a finite area then it can be represented as in Figure 3.14 where f is the force distribution into the ground and F is the resultant of f determined by the surface integral

$$F = \int_{(s)} f\,ds \tag{3.22}$$

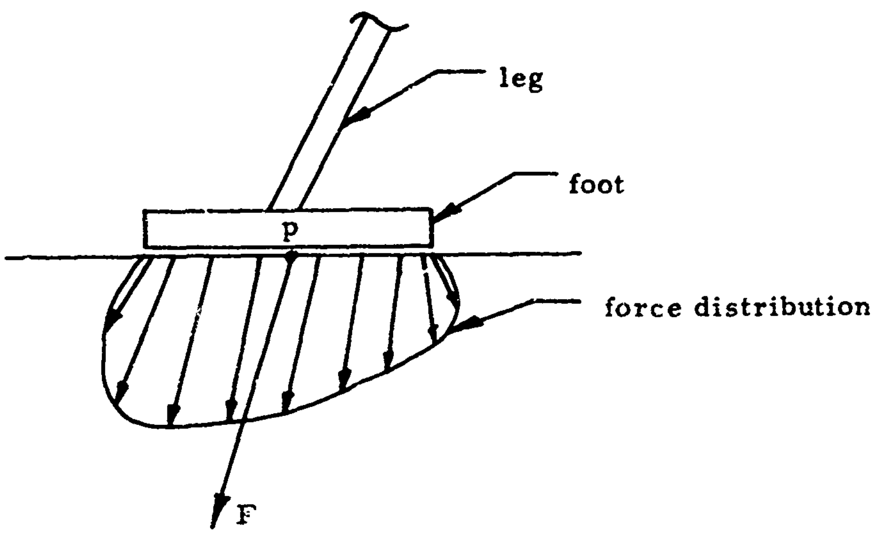
The force F acts at a point p. Thus effectively the legs supply a force into the ground at a single point. Since all feet lie in a plane, the following definition can be made:

Definition VI. The figure formed by constructing the minimum area convex polygon containing all feet is called a support polygon.

Note that this definition implies the support polygon may change with time since the force distribution can change as the machine propagates.

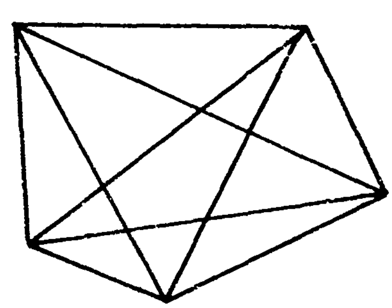
Definition VII. An n-sided support polygon can be partitioned by drawing lines between pairs of feet on opposite sides of the polygon. A polygon so partitioned is a set of triangles (Figure 3.15).

Definition VIII. A machine supported at any instant of time by more



Foot Force Distribution

Figure 3.14



A Partitioned Support Polygon

Figure 3.15

than 3 point feet is said to be redundantly supported. The determination of the load on each foot is arbitrary and has to be selected according to some criterion.

Theorem 1.

Constant velocity ideal locomotion is possible with an ideal locomotion machine if and only if the vertical projection of the center of gravity of the machine lies within the support zone.

Proof.

If the projection of the center of gravity lies within the support polygon, then it is to be shown that constant velocity ideal locomotion is possible. Constant velocity ideal locomotion implies that equation (3.21) must be satisfied with the vector

$$(a) = \begin{pmatrix} 0 \\ 0 \\ -mg \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (3.23)$$

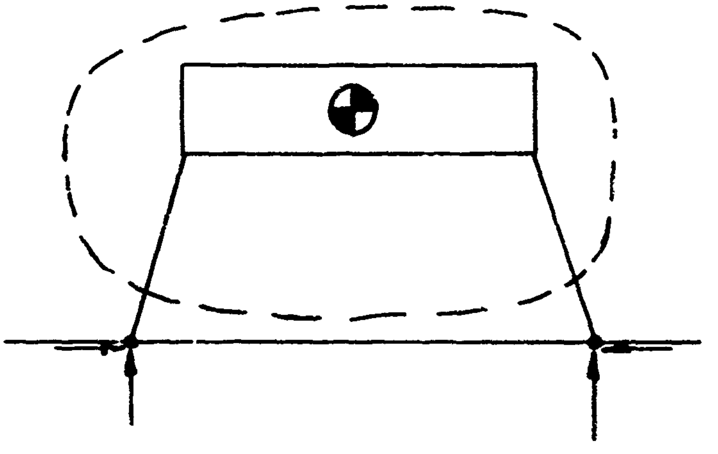
and

$$0 < F_{zi} \leq mg \qquad \text{for every } i = 1, \ldots, n$$

which says the sum of all moments applied to the body is zero; the sum of all forces in the x and y directions must be zero, and the sum of all vertical forces must exactly equal mg.

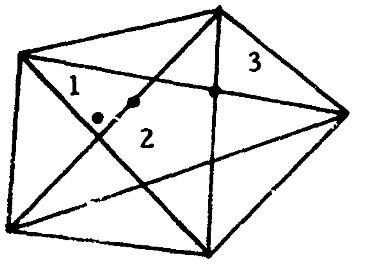
Before proceeding, it should be emphasized that since the legs are massless and the feet can not transmit a moment from the ground, then the free body diagram of the machine can be considered as shown in Figure 3.16. With this diagram relationship equation (3.23) is to be satisfied by the body within the dotted lines. The foot forces are reactions to the moments and the longitudinal force applied to the legs by the body. This says the net force in the horizontal plane at the feet must be zero; thus it is only necessary to consider the vertical forces. To compute these vertical forces consider the support polygon, its partitions and the center of gravity contained within.

As shown in Figure 3.17, three conditions can exist: (1) the center of gravity is supported by one or more triangles of support, (2) the center of gravity lies on a partition line, (3) the center of gravity lies on two or more partition lines. Suppose the center of gravity is in condition (1) and it has k support triangles supporting the center of gravity. Since only one triangle is sufficient to support the center of gravity it is redundantly supported. Thus a criterion is needed to determine the load on each triangle. Suppose the criterion chosen is that each triangle of support is to share the load equally. Then the load carried by each foot is the sum of the loads for that foot over the number of supporting triangles, that is,



Free Body Diagram of an Ideal Locomotion Machine

Figure 3.16



Three Possible Positions of the Center of Gravity Contained in a Support Polygon

Figure 3.17

given k triangles supporting the center of gravity, each triangle carries $1/k$ portion of the load. Each triangle has the configuration shown in Figure 3.18, where the normalized load applied at the center of gravity is $1/k$. The following force equation must be satisfied if the vertical force at the corners is a_1, a_2, and a_3,

$$a_1 + a_2 + a_3 = 1/k \tag{3.24}$$

The load at each foot can be computed by taking moments about the respective sides of the triangle (1, 2, 3) and equating these to zero. This is possible since an ideal locomotion machine can apply no moments at any foot. Thus,

$$\begin{aligned} a_1 \ell_2 \sin\theta_3 &= (1/k)x_1 \\ a_1 &= x_1/(k\ell_2 \sin\theta_3) \end{aligned} \tag{3.25}$$

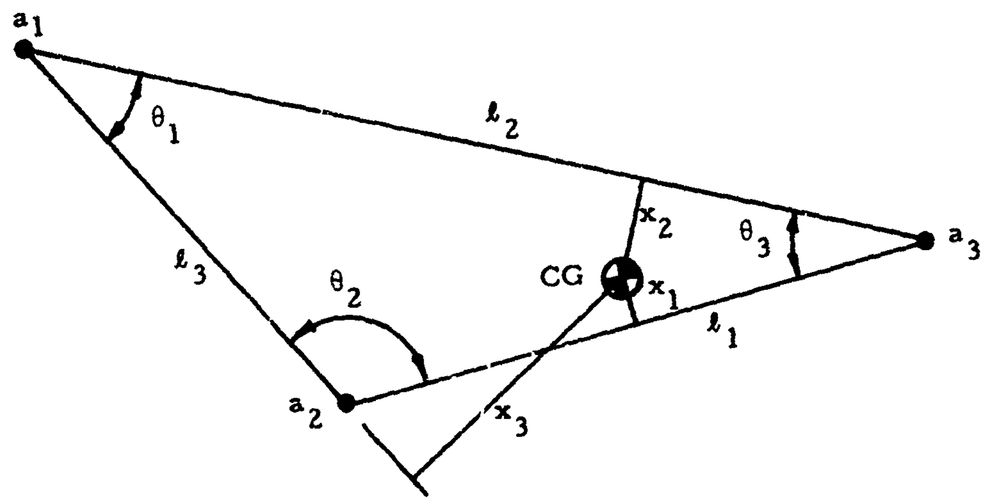
$$\begin{aligned} a_2 \ell_1 \sin\theta_3 &= (1/k)x_2 \\ a_2 &= x_2/(k\ell_1 \sin\theta_3) \end{aligned} \tag{3.26}$$

$$\begin{aligned} a_3 \ell_2 \sin\theta_1 &= (1/k)x_3 \\ a_3 &= x_3/(k\ell_2 \sin\theta_1) \end{aligned} \tag{3.27}$$

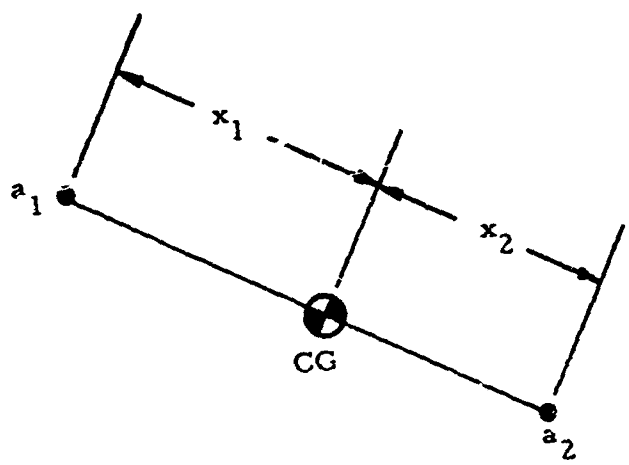
then,

$$\begin{aligned} a_1 + a_2 + a_3 &= (1/k)[x_1/(\ell_2 \sin\theta_3) + x_2/(\ell_1 \sin\theta_3) + x_3/(\ell_2 \sin\theta_1)] \\ &= 1/k \end{aligned}$$

The quantity $[x_1/(\ell_2 \sin\theta_3) + x_2/(\ell_1 \sin\theta_3) + x_3/(\ell_2 \sin\theta_1)]$ is



A Support Triangle
Figure 3.18



A Support Partition
Figure 3.19

always 1 when the center of gravity is contained in the triangles since $x_1\ell_1 + x_2\ell_2 + x_3\ell_3 = \ell_1\ell_2\sin\theta_3 = \ell_3\ell_2\sin\theta_1$.

The total load on each foot is the sum of the loads on the feet of the triangles in which the foot is common, that is

$$\frac{F_{zi}}{mg} = \sum_{j=1}^{k} a_{j(T)} \tag{3.28}$$

where the subscript T is used to indicate the triangle. As all $a_{j(T)}$ are positive, and the sum of a set of positive numbers is still positive, all foot forces are positive. Thus equation (3.23) is satisfied.

Consider now condition (2) where the center of gravity is on a partition, or two legs support the center of gravity. One triangle of support is now reduced to a line. Again redundancy of support exists since other triangles of support are also present. To determine the forces on the feet a criteria must be selected. The same criteria for condition (1) can be selected, i.e. each triangle including this special line carries an equal load. Thus again, if there are k - 1 triangles and one partition boundary then each triangle can, by the criteria, carry 1/k of the load and the partition boundary can carry 1/k of the load. To insure the load carried by the partition boundary introduces no moments to the body, consider the situation shown in Figure 3.19. The normalized forces again can be obtained by summing moments since all moments in the body must be zero. The resulting normalized forces are:

$$\begin{aligned} a_1 &= [x_1/(x_1 + x_2)](1/k) \\ a_2 &= [x_2/(x_1 + x_2)](1/k) \end{aligned} \qquad (3.29)$$

Thus the total force on each foot is again expressed by equation (3.28) where the subscript T now extends to partitions as well. The total force again is positive.

When two or more partitions support the center of gravity as in condition (3), the same analysis as equation (3.29) can be used and it is clear again that all feet forces can be made positive and produce no resulting moments.

Thus it is concluded if the center of gravity is contained in the support polygon at any time the equation (3.23) can be satisfied and constant velocity ideal locomotion is possible.

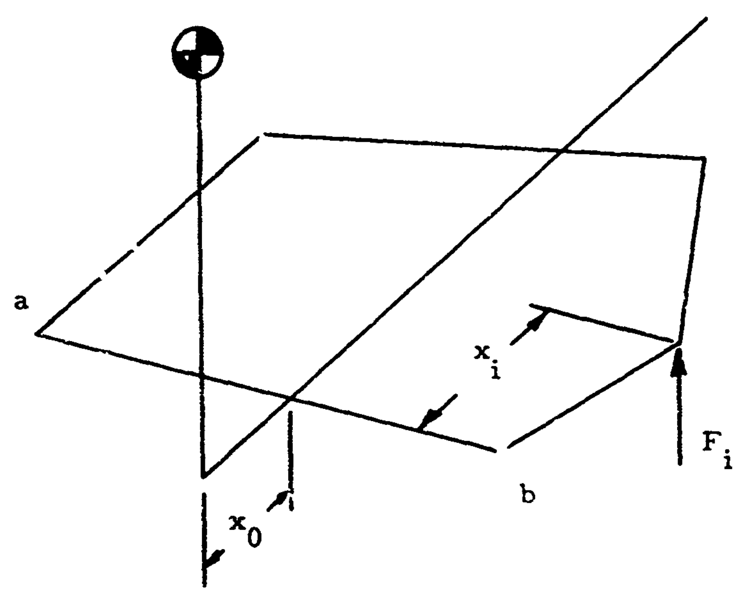
To prove that locomotion can be satisfied only if the center of gravity is contained by the support polygon will require showing that if the center of gravity is outside the support polygon the conditions of equation (3.23) can not be satisfied by an ideal locomotion machine. This means that to have the (a) vector specified it will be required to violate the condition $0 < F_{zi} \leq mg$ for some i.

Consider the configuration shown in Figure 3.20 where the center of gravity is outside the support polygon adjacent to a side (a, b). If the center of gravity is outside the support polygon there will always exist such a side due to convexity. The side may be just a corner. The conditions of equation (3.23) specify all moments to be zero. Thus moments about the line (a, b) must also be zero. Suppose there are k feet on the right side of (a, b) and the distance of these feet to the line (a, b) is x_i. Suppose they each produce force F_{zi} into the ground. Then summing moments

$$-mgx_0 = \sum_{i=1}^{k} F_{zi}x_i \tag{3.30}$$

But to balance the center of gravity, there must be at least one F_{zi} which is negative. But this violates Definition I. Thus constant velocity ideal locomotion is not possible with the center of gravity outside the support polygon. This completes the proof.

It has been shown that constant velocity ideal locomotion requires that the center of gravity always be contained by the support polygon. The question to be asked is, what is the minimum number of legs required to provide such a support polygon always under the center of gravity as the machine propagates. Before proceeding suppose the following definitions are made:



Center of Gravity Not Supported By a Support Polygon

Figure 3.20

Definition IX. Ideal locomotion for an ideal locomotion machine is stable if, when perturbed, ideal locomotion continues. The perturbations are in a class P, where P is a class of horizontal forces, velocities, or displacements.

Definition X. The stability margin of a mode of ideal locomotion is determined by the relative size of P. A positive number greater than zero implies the magnitude of this relative stability.

Definition XI. Ideal locomotion for an ideal locomotion machine is statically stable if the stability margin remains constant for an arbitrarily slow velocity.

Definition XII. A non-singular gait is a gait having the property that no two feet are ever required to be placed or lifted simultaneously [23].

These definitions lead to the following conclusion:

Theorem 2.

The minimum number of point feet required for statically stable constant velocity locomotion with a stability margin greater than zero in every direction and with a non-singular gait, is four. Furthermore, there are exactly three quadruped gaits with this property.

Proof.

From Theorem 1 it was found that for a constant velocity ideal locomotion to exist, it is necessary to have the center of gravity supported by the support polygon. With two feet this can only occur if the center of gravity is over the line between the two feet on the ground. Since the center of gravity must move forward then these two feet must be placed in the direction of motion. To take a step, the center of gravity must be over one foot or a point. But this must occur for a finite time since the velocity is constant and the gait is non-singular. This is possible with a properly shaped foot. However, there does not exist a positive stability margin greater than zero for such a machine, since there can not be a perturbation applied when the machine is on one leg if the machine has no sensors. With sensors the direction of motion can change depending upon the direction of perturbation, provided the feet have the proper characteristics. Thus two legged machines can not satisfy this theorem if they are to be ideal.

A three legged machine again must, at some time, be on two legs implying the two legs must be in the direction of motion. Thus the three feet all must somehow be in a line. There exists no lateral stability to such a machine and it perhaps is clumsier than the two legged machine. that is, there can exist a positive stability margin, however this stability margin only applies in the direction of motion.

Consider now a four legged machine in which there always exists at least three legs on the ground. The question to be answered is, can three of the four legs always be placed in such a way that the center of gravity is continually supported. In this way Theorem 1 can be satisfied and stable constant velocity ideal locomotion is possible. There are six possible gaits which can be implemented by the four legs. The objective is to discover the gaits and their parameters which can satisfy Theorem 1. These parameters are:

β = duty factor; the time a leg spends on the ground relative to the time in the air

φ = the relative phase; the time a leg is placed on the ground relative to the time a reference leg was placed on the ground

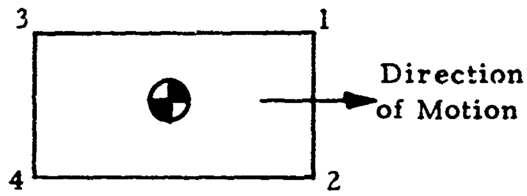
s = mechanical stroke of the legs

a = the initial position of the ideal legs relative to the center of gravity

The assumption of constant velocity implies velocity is equal to $S/(\tau\beta)$ where τ is the time required to complete one four leg cycle. The duty factor β is a measure of the time a leg spends on the ground with respect to the cycle time τ [23]. The relative phase φ_i is the time leg i is placed down on the ground relative to the time the first leg (defined as th. left front) is placed down. Also assume a normalization of time and space so that $\tau = 1$ and $S = \beta$, then the velocity is β, $\varphi_1 = 0$ and $\varphi_i < 1$.[2] Figure 3.21 shows the machine configuration used for the analysis and lists the six possible gaits.

Suppose the stability margin S_M during a cycle period τ, is defined for this simple system as the minimum distance between the center of gravity and a support line. The configuration when a leg is in the air and returning is as shown in Figure 3.22. It should be noticed that minimum x or y occurs either when a leg (in this case leg 2) is about to be placed down or when it has just left the ground. From Figure 3.22 it is clear then that x is a minimum just before leg 2 touches down and y is a minimum just before leg 2 lifts off the ground. Thus it is only necessary to consider these two cases for the analysis. Further, as the duty factor β increases the stability margin S_M also increases since the motion of the center of gravity, when the leg is in the air, is reduced. The

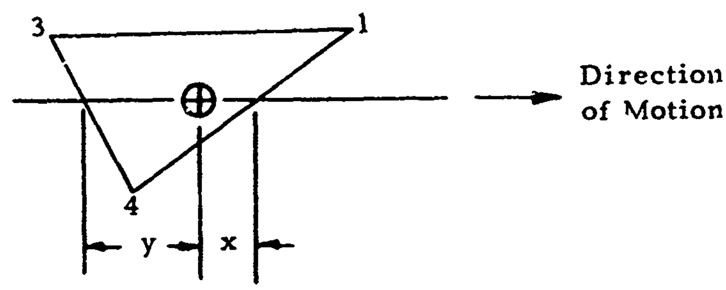
[2] It should be noted at this point, that the following analysis provides gaits and variables such that locomotion can be accomplished at an arbitrary, slow velocity. The gaits and conditions found unsuitable here however, in no way imply locomotion can not take place.



Possible Gaits For Crawling Quadrupeds	
No.	Gait
1	1423
2	1432
3	1324
4	1342
5	1234
6	1243

Crawling Quadrupeds

Figure 3.21



S_m = Min. (x, y) evaluated over one cycle

x = distance of the center of gravity to the front support line

y = distance of the center of gravity to the rear support line

Definition of Stability Margin

Figure 3.22

limit is $\beta = 1$ which means the legs must return in zero time. For quadrupeds this implies infinite available power, or zero mass in the leg.

Let the equations for the first gait (1423) be written with the center of gravity defined at the origin and the initial leg position relative to the center of gravity as a_i for $i = 1, 2, 3, 4$. The distance time normalization implies that the legs are at a position determined by the initial position a_i and the time elapsed. The time is measured with respect to the time leg 1 touches the ground. The distance travelled is determined by the φ's, a's, and β's. Further assume β is the same for all 4 legs. Then, at the time leg 1 lifts off the ground the positions of the supporting legs (2, 3, 4), relative to the center of gravity are:

$$x_2 = a_2 + \varphi_2 - 1 + (1 - \beta) \tag{3.31}$$

$$x_3 = a_3 + \varphi_3 - 1 + (1 - \beta) \tag{3.32}$$

$$x_4 = a_4 + \varphi_4 - 1 + (1 - \beta) \tag{3.33}$$

which yield the following conditions to be satisfied in order that stability exists:

$$a_2 + a_3 + \varphi_2 + \varphi_3 - 2 \geq 2(1 - \beta) \quad \text{(front support line)} \tag{3.34}$$

$$a_3 + a_4 + \varphi_3 + \varphi_4 - 2 \leq 2(1 - \beta) \quad \text{(rear support line)} \tag{3.35}$$

At touch down of leg 1 the equations are:

$$\left.\begin{array}{l} x_2 = a_2 + \varphi_2 - 1 \\ x_3 = a_3 + \varphi_3 - 1 \\ x_4 = a_4 + \varphi_4 - 1 \end{array}\right\} \Rightarrow \begin{array}{ll} a_2 + a_3 + \varphi_2 + \varphi_3 - 2 \geqslant 0 \text{ (front line)} & (3.36) \\ a_3 + a_4 + \varphi_3 + \varphi_4 - 2 \leqslant 0 \text{ (rear line)} & (3.37) \end{array}$$

But as noted above, the critical time occurs at the rear line, or lift off, and at the front line on touch down. Therefore, the two critical equations are (3.35) and (3.36).

Using the same reasoning for the other 3 legs the following set of equations and inequalities are obtained at:

leg 4 touch down and lift off

$$x_1 = a_1 - \varphi_4 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

$$a_1 + a_2 + \varphi_2 - 2\varphi_4 - 1 \geqslant 0 \qquad (3.38)$$

$$x_2 = a_2 + \varphi_2 - 1 - \varphi_4 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

$$a_2 + a_3 + \varphi_2 + \varphi_3 - 2\varphi_4 - 2 \leqslant 2(1-\beta) \qquad (3.39)$$

$$x_3 = a_3 + \varphi_3^{-1} - \varphi_4 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

leg 2 touch down and lift off

$$x_1 = a_1 - \varphi_2 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

$$a_1 + a_4 + \varphi_4 - 2\varphi_2 \geqslant 0 \qquad (3.40)$$

$$x_3 = a_3 + \varphi_3 - \varphi_2 - 1 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

$$a_3 + a_4 + \varphi_3 + \varphi_4 - 2\varphi_2 - 1 \leqslant 2(1-\beta) \qquad (3.41)$$

$$x_4 = a_4 + \varphi_4 - \varphi_2 - \begin{vmatrix} (1-\beta) \\ 0 \end{vmatrix}$$

leg 3 touch down and lift off

$$x_1 = a_1 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix}$$

$$a_1 + a_2 + \varphi_2 - 2\varphi_3 \geqslant 0 \qquad (3.42)$$

$$x_2 = a_2 + \varphi_2 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix}$$

$$a_1 + a_4 + \varphi_4 - 2\varphi_3 \leqslant 2(1-\beta) \qquad (3.43)$$

$$x_3 = a_4 + \varphi_4 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix}$$

The inequalities yield the following conditions:

$$\varphi_4 \geqslant -(1 - \beta) \qquad (3.44)$$

from equations (3. 37) and (3. 40), and

$$\varphi_2 \leqslant \varphi_3 + (1 - \beta) \qquad (3.45)$$

from equations (3. 41) and (3. 44). In addition to satisfying equations (3. 31) through (3. 44) the following inequality must be satisfied:

$$0 \leqslant \varphi_4 + (1 - \beta) \leqslant \varphi_2 + (1 - \beta) \leqslant \varphi_3 + (1 - \beta) \leqslant 1 \qquad (3.46)$$

Therefore, equations (3. 37), (3. 38), (3. 41), (3. 42), (3. 44), (3. 45) and (3. 46) need to be satisfied in order to assure a positive stability margin for this gait. Equation (3. 46) is imposed by the requirement for at least 3 legs on the ground and the 1423 gait.

An example of a selection of variables to satisfy the 7 inequalities is: $a_1 = a_2 = 1$, $a_3 = a_4 = -1$, $\varphi_4 = 1/4$, $\varphi_2 = 1/2$, $\varphi_3 = 3/4$ and $\beta = .9127$. This selection of variables allows a quadruped to continuously satisfy theorem 1 and move at an arbitrary speed. It should be noted here that Muybridge observed this as the dominant low speed gait used by animals in nature [3]. He has termed it the crawl.

Thus it is shown that with a minimum of four legs statically stable constant velocity ideal locomotion is possible with a machine constructed as modelled. Thus the first part of Theorem 1 is proved. The other five gaits will next be analyzed to discover which yield a possibility for statically stable locomotion.

Analysis of gait 2 (1432) follows:

leg 1 touch down and lift off

$$\begin{aligned}
x_2 &= a_2 + \varphi_2 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_3 &= a_3 + \varphi_3 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_4 &= a_4 + \varphi_4 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
\begin{aligned}
&a_2 + a_3 + \varphi_2 + \varphi_3 - 2 \geqslant 0 \\
&a_3 + a_4 + \varphi_3 + \varphi_4 - 2 \leqslant 2(1-\beta)
\end{aligned}
\tag{3.47}$$

leg 4 touch down and lift off

$$\begin{aligned}
x_1 &= a_1 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_2 &= a_2 + \varphi_2 - \varphi_4 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_3 &= a_3 + \varphi_3 - \varphi_4 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
\begin{aligned}
&a_1 + a_2 + \varphi_2 - 2\varphi_4 - 1 \geqslant 0 \\
&a_2 + a_3 + \varphi_2 + \varphi_3 - 2\varphi_4 - 2 \leqslant 2(1-\beta)
\end{aligned}
\tag{3.48}$$

leg 3 touch down and lift off

$$\begin{aligned}
x_1 &= a_1 - \varphi_3 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_2 &= a_2 + \varphi_2 - \varphi_3 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
a_1 + a_2 + \varphi_2 - 2\varphi_3 - 1 \geqslant 0
\tag{3.49}$$

$$x_4 = a_4 + \varphi_4 - \varphi_3 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| \qquad a_1 + a_4 + \varphi_4 - 2\varphi_3 \leqslant 2(1-\beta)$$

leg 2 touch down and lift off

$$\begin{aligned} x_1 &= a_1 - \varphi_2 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & a_1 + a_4 + \varphi_4 - 2\varphi_2 &\geqslant 0 \\ x_3 &= a_3 + \varphi_3 - \varphi_2 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & & \\ x_4 &= a_4 + \varphi_4 - \varphi_2 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & a_3 + a_4 + \varphi_3 + \varphi_4 - 2\varphi_2 &\leqslant 2(1-\beta) \end{aligned} \tag{3.50}$$

From equations (3.49) and (3.50)

$$\begin{aligned} \varphi_2 - \varphi_3 &\leqslant (1-\beta) \\ \varphi_2 &\leqslant \varphi_3 + (1-\beta) \end{aligned} \tag{3.51}$$

But this gait requires

$$0 \leqslant \varphi_4 + (1-\beta) \leqslant \varphi_3 + (1-\beta) \leqslant \varphi_2 + (1-\beta) \leqslant 1 \tag{3.52}$$

Thus equations (3.51) and (3.52) are in contradiction. Hence it is impossible to have a positive stability margin with this gait.

Analysis of gait 3 (1324) follows:

leg 1 touch down and lift off

$$\begin{aligned} x_2 &= a_2 + \varphi_2 - 1 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & a_2 + a_3 + \varphi_2 + \varphi_3 - 2 &\geqslant 0 \\ x_3 &= a_3 + \varphi_3 - 1 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & & \\ x_4 &= a_4 + \varphi_4 - 1 - \left|\begin{matrix}(1-\beta)\\0\end{matrix}\right| & a_3 + a_4 + \varphi_3 + \varphi_4 - 2 &\leqslant 2(1-\beta) \end{aligned} \tag{3.53}$$

leg 3 touch down and lift off

$$
\begin{aligned}
x_1 &= a_1 - \varphi_3 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_2 &= a_2 - \varphi_2 - \varphi_3 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_4 &= a_4 + \varphi_4 - \varphi_3 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
\begin{aligned}
&a_1 + a_2 + \varphi_2 - 2\varphi_3 - 1 \geqslant 0 \\
&a_1 + a_4 + \varphi_4 - 2\varphi_3 - 1 \leqslant 2(1-\beta)
\end{aligned}
\tag{3.54}
$$

leg 2 touch down and lift off

$$
\begin{aligned}
x_1 &= a_1 - \varphi_2 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_3 &= a_3 + \varphi_3 - \varphi_2 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_4 &= a_4 + \varphi_4 - \varphi_2 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
\begin{aligned}
&a_1 + a_4 + \varphi_4 - 2\varphi_2 - 1 \geqslant 0 \\
&a_3 + a_4 + \varphi_3 + \varphi_4 - 2\varphi_2 - 1 \leqslant 2(1-\beta)
\end{aligned}
\tag{3.55}
$$

leg 4 touch down and lift off

$$
\begin{aligned}
x_1 &= a_1 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_2 &= a_2 + \varphi_2 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \\
x_3 &= a_3 + \varphi_3 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right|
\end{aligned}
\qquad
\begin{aligned}
&a_1 + a_2 + \varphi_2 - 2\varphi_4 \geqslant 0 \\
&a_2 + a_3 + \varphi_2 + \varphi_3 - 2\varphi_4 \leqslant 2(1-\beta)
\end{aligned}
\tag{3.56}
$$

From equations (3.53) and (3.56)

$$\varphi_4 \geqslant \beta \tag{3.57}$$

From equations (3.54) and (3.55)

$$\varphi_2 \leqslant \varphi_3 + (1-\beta) \tag{3.58}$$

But the requirements for this gait are

$$0 \leqslant \varphi_3 + (1-\beta) \leqslant \varphi_2 + (1-\beta) \leqslant \varphi_4 + (1-\beta) \leqslant 1 \tag{3.59}$$

Equations (3.57), (3.58) and (3.59) are contradictory. Thus this gait can not provide constant velocity ideal locomotion.

Analysis of gait 4 (1342) follows:

The first two sets of equations are identical to gait 3. The other equations are:

leg 4 touch down and lift off

$$\begin{aligned} x_1 &= a_1 - \varphi_4 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\ x_2 &= a_2 + \varphi_2 - \varphi_4 - 1 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\ x_3 &= a_3 + \varphi_3 - \varphi_4 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \end{aligned} \qquad \begin{aligned} &a_1 + a_2 + \varphi_2 - 2\varphi_4 - 1 \geqslant 0 \\ &a_2 + a_3 + \varphi_2 + \varphi_3 - 2\varphi_4 - 1 \leqslant 2(1-\beta) \end{aligned} \tag{3.60}$$

leg 2 touch down and lift off

$$\begin{aligned} x_1 &= a_1 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\ x_3 &= a_3 + \varphi_3 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\ x_4 &= a_4 + \varphi_4 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \end{aligned} \qquad \begin{aligned} &a_1 + a_4 + \varphi_4 - 2\varphi_2 \geqslant 0 \\ &a_3 + a_4 + \varphi_3 + \varphi_4 - 2\varphi_2 \leqslant 2(1-\beta) \end{aligned} \tag{3.61}$$

Combining equations (3.53) and (3.60)

$$\varphi_4 \geqslant 1/2 - (1 - \beta) \tag{3.62}$$

Combining equations (3.54) and (3.61)

$$\varphi_2 \leqslant \varphi_3 + 1/2 + (1 - \beta) \tag{3.63}$$

For this gait if equations (3.53), (3.54), (3.60), (3.61), (3.62), (3.63) and

$$0 \leq \varphi_3 + (1 - \beta) \leq \varphi_4 + (1 - \beta) \leq \varphi_2 + (1 - \beta) \leq 1 \tag{3.64}$$

are satisfied then the system can provide stable constant velocity ideal locomotion. These conditions can be satisfied with variables $\varphi_2 = .7500$, $\varphi_3 = .4167$, $a_4 = .6667$, $\beta = .9167$, $S = 1.00$, $a_1 = a_2 = 2.0$ and $a_3 = a_4 = -1.0$.

Analysis of gait 5 (1234) follows:

leg 1 touch down and lift off

$$\begin{aligned}
x_2 &= a_2 + \varphi_2 - 1 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_3 &= a_3 + \varphi_3 - 1 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_4 &= a_4 + \varphi_4 - 1 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix}
\end{aligned}$$

$$\begin{aligned}
&a_2 + a_3 + \varphi_2 + \varphi_3 - 2 \geq 0 \\
&a_3 + a_4 + \varphi_3 + \varphi_4 - 2 \leq 2(1 - \beta)
\end{aligned} \tag{3.65}$$

leg 2 touch down and lift off

$$\begin{aligned}
x_1 &= a_1 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_3 &= a_3 + \varphi_3 - 1 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_4 &= a_4 + \varphi_4 - 1 - \varphi_2 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix}
\end{aligned}$$

$$\begin{aligned}
&a_1 + a_4 - \varphi_4 - 2\varphi_2 - 1 \geq 0 \\
&a_3 + a_4 + \varphi_3 + \varphi_4 - 2\varphi_2 - 2 \leq 2(1 - \beta)
\end{aligned} \tag{3.66}$$

leg 3 touch down and lift off

$$\begin{aligned}
x_1 &= a_1 - \varphi_3 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_2 &= a_2 + \varphi_2 - \varphi_3 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix} \\
x_4 &= a_4 + \varphi_4 - \varphi_3 - 1 - \begin{vmatrix}(1-\beta)\\ 0\end{vmatrix}
\end{aligned}$$

$$\begin{aligned}
&a_1 + a_2 + \varphi_2 - 2\varphi_3 \geq 0 \\
&a_1 + a_4 + \varphi_4 - 2\varphi_3 - 1 \leq 2(1 - \beta)
\end{aligned} \tag{3.67}$$

leg 4 touch down and lift off

$$x_1 = a_1 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \qquad a_1 + a_2 + \varphi_2 - 2\varphi_4 \geqslant 0$$
$$x_2 = a_2 + \varphi_2 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \tag{3.68}$$
$$x_3 = a_3 + \varphi_3 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \qquad a_2 + a_3 + \varphi_2 + \varphi_3 - 2\varphi_4 \leqslant 2(1-\beta)$$

Combining equations (3.63) and (3.66), the following equation is obtained,

$$\varphi_4 \geqslant \beta \tag{3.69}$$

But this violates the basic inequality for this gait. namely

$$0 \leqslant \varphi_2 + (1 - \beta \leqslant \varphi_3 + (1 - \beta) \leqslant \varphi_4 + (1 - \beta) \leqslant 1 \tag{3.70}$$

Thus, this gait can not provide constant velocity ideal locomotion.

Analysis of gait 6 (1243) follows:

Again the first two sets of equations are identical as in the relations (3.65) and (3.66). However, when leg 3 and leg 4 are in the air their relationships are,

leg 4 touch down and lift off

$$x_1 = a_1 + \varphi_4 + \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \qquad a_1 + a_2 + \varphi_2 - 2\varphi_4 \geqslant 0$$
$$x_2 = a_2 + \varphi_2 - \varphi_4 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \tag{3.71}$$
$$x_3 = a_3 + \varphi_3 - \varphi_4 - 1 - \left|\begin{matrix}(1-\beta)\\ 0\end{matrix}\right| \qquad a_2 + a_3 + \varphi_3 + \varphi_2 - 2\varphi_4 - 1 \leqslant 2(1-\beta)$$

leg 3 touch down and lift off

$$x_1 = a_1 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix} \qquad a_1 + a_2 + \varphi_2 - 2\varphi_3 \geq 0$$

$$x_2 = a_2 + \varphi_2 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix} \tag{3.72}$$

$$x_4 = a_4 + \varphi_4 - \varphi_3 - \begin{Bmatrix}(1-\beta)\\ 0\end{Bmatrix} \qquad a_1 + a_4 + \varphi_4 - 2\varphi_3 \leq 2(1-\beta)$$

Combining equations (3.65) and (3.71) the following condition is obtained:

$$\varphi_4 \geq \beta - 1/2 \tag{3.73}$$

Combining equations (3.66) and (3.72) the following is obtained:

$$\varphi_2 \leq \varphi_3 + (\beta - 1/2) \tag{3.74}$$

The basic inequality for this gait is

$$0 \leq \varphi_2 + (1-\beta) \leq \varphi_4 + (1-\beta) \leq \varphi_3 + (1-\beta) \leq 1 \tag{3.75}$$

If the relationships (3.65), (3.66), (3.71), (3.72), (3.73), (3.74) and (3.75) are satisfied, then this gait will be able to provide stable constant velocity ideal locomotion.

Parameters which satisfy these relations are $\varphi_2 = .20$, $\varphi_3 = .90$, $\varphi_4 = .60$, $\beta = .9167$, $S = 1.00$, $a_1 = a_2 = 2.0$ and $a_3 = a_4 = -1.0$.

This completes the proof of Theorem 2.

Notice that in the entire analysis above, only the phase variables and duty factors were considered. It is obvious that the initial position and stride also have to do with stability. Thus there must exist optimum values for the variables when the gait is stable. This investigation will not be explored further.

It should be noted that this theorem is applicable to an ideal locomotion machine as defined above. A slightly modified machine might include a torque about the longitudinal axis of the legs. This provides four variables for each leg. Thus two legs on the ground would have eight variables with six equations. An infinite number of solutions will exist if they exist at all. However, these added variables do not change the situation as analyzed. Thus, four legs will still be the minimum.

Definition XIII. A reaction force is the inertial force acting at the center of gravity resulting from accelerations caused by applied forces to the body of an ideal locomotion machine performing ideal locomotion.

Definition XIV. If the total reaction force vector which is caused by gravity plus the horizontal acceleration is extended until it intersects the support plane, it traces a trajectory as a function of time in the plane of motion. This trajectory will be called the reaction force trajectory.

Theorem 3.

Ideal locomotion with an ideal locomotion machine is possible if and only if the reaction force trajectory is contained in the support zone for all time as the machine propagates.

Proof.

Ideal locomotion implies equation (3.21) must now be satisfied with the (a) vector equal to

$$(a) = \begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ mg \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (3.76)$$

where

$$0 < F_{zi}$$

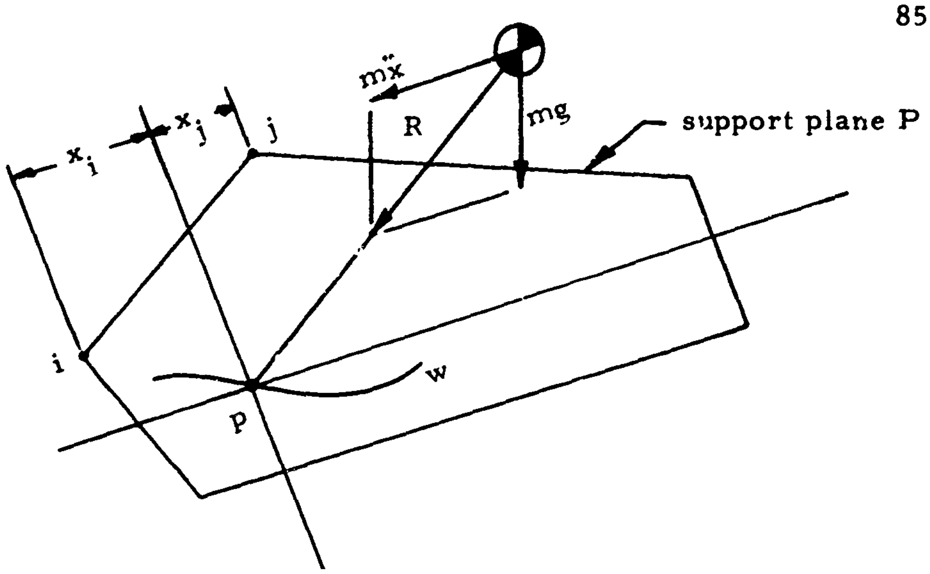
The quantities $m\ddot{x}$ and $m\ddot{y}$ are unspecified except by the magnitude of the reaction force. The magnitude of the reaction force R is equal to $m\sqrt{(\ddot{x})^2 + (\ddot{y})^2 + g^2}$. The direction is dependent upon direction cosines x/R and y/R.

The requirement that all moments applied sum to zero in any plane is still necessary. Thus, if the reaction force is contained by the support polygon at some time, it is to be shown that ideal locomotion is possible with all moments summed to zero. Consider a given support polygon and a given reaction force R which pierces the interior of polygon P at point p, see Figure 3.23. Suppose the intersection of the plane determined by the horizontal reaction force and the vertical force of gravity and the support plane is considered. Since all moments about the body must sum to zero, then moments normal to this plane also must be zero. Pick an axis of rotation normal to this plane through the p .nt p. With ideal legs, it is necessary that all moments about this line sum to zero, and all vertical forces be negative upward from the ground. Suppose the distance to each foot on the left side of the line is x_i and the distances on the right side x_j (see Figure 3.24). The moment equation is

$$\sum_{i=1}^{k} F_{zi} x_i = \sum_{j=1}^{\ell} F_{zj} x_j \qquad (3.77)$$

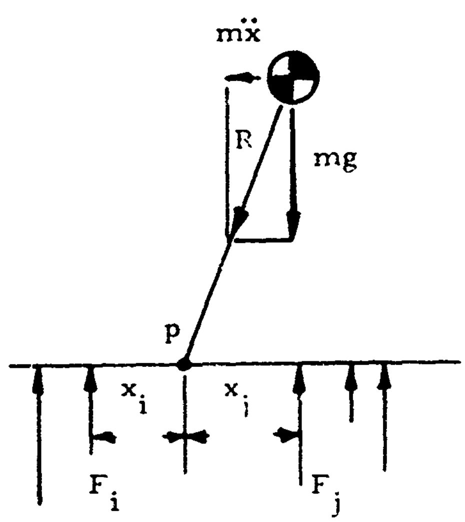
This equation can be satisfied easily with $F_{zi} > 0$ and $F_{zj} > 0$ for every i and j. This must be the case since $x_i > 0$ and $x_j > 0$ for every i and j. Thus ideal locomotion is possible if the reaction force is contained by the support zone.

If the reaction force is not contained and ideal locomotion is to exist, the moments about any line must be zero, with the vertical component of all forces applied to the feet directed upward. The reaction force not being contained implies the point p lies outside the support polygon. Since the support polygon is convex, then the point



A support plane containing the reaction force trajectory w at some time t

Figure 3.23



Moment plane normal to reaction force R

Figure 3.24

p must be adjacent to a side (a, b), or a foot, or corner. Construct a vertical plane normal to the side passing through p, or if p is adjacent to a foot, pass the plane through this foot. Consider moments about this side or moments about a line normal to the constructed vertical plane passing through the foot, or feet. Since all moments must balance to zero about any axis consider axis (a, b) or the single foot (Figure 3.25). The projected reaction force into this plane is R'. The moment produced by this force R' is

$$m = +rR'$$

To balance this requires that

$$\Sigma F_{zi} x_i = -M$$

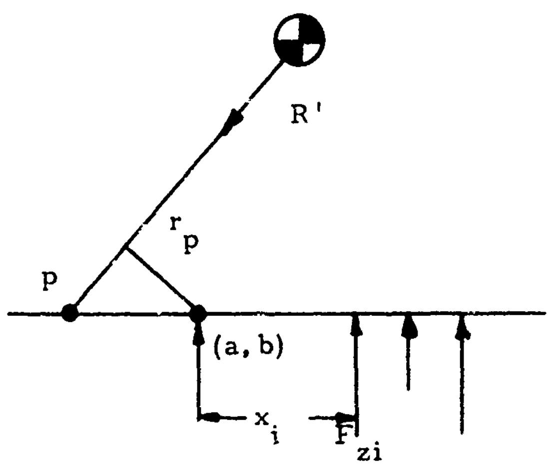
which implies that

$$\Sigma F_{zi} = -M/x_i$$

This is possible only if at least one F_{zi} is negative. This, of course, violates the basic premise. Thus it is not possible to have ideal locomotion with the reaction force outside the support zone. Hence the theorem is proved.

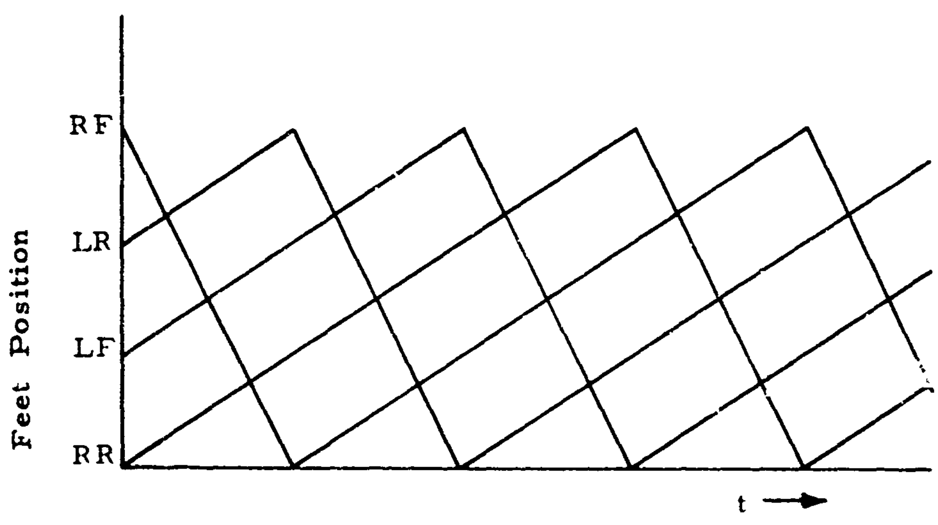
<u>Theorem 4.</u>

There exists automatic control schemes which will allow locomotion (non-ideal) of an ideal locomotion machine. These schemes do not require inertial sensors.



The Projection of R Into A Plane Normal To Line (a, b)

Figure 3.25



Ideal Feet Position - Time Trajectory For A Ouadruped

Figure 3.26

Proof.

The equations of motion derived above, were simulated for a trot gait of a quadruped. This gait has only the two diagonal legs on the ground at any time. Thus the support zone is merely a line between the two diagonal feet. The center of gravity is supported for an instant only. Such a gait was successfully controlled by a simple automatic control law for each leg, namely,

$$T = C_{\alpha}(\alpha - \alpha_c) + C_{\dot{\alpha}}(\dot{\alpha} - \dot{\alpha}_c)$$

where

α = angle made by the leg projection and the vertical body axis in the longitudinal plane

C_{α} = the error proportionality constant

α_c = the commanded angle and

$$\alpha_c = \dot{\alpha}_0 t + \alpha_0 \qquad \text{for } \alpha_{\text{initial}} < \alpha_c < \alpha_{\text{final}}$$

α = the leg angular rate in the α direction

$C_{\dot{\alpha}}$ = the error proportionality constant

$\dot{\alpha}_c$ = the commanded angular rate $\dot{\alpha}_c = \dot{\alpha}_0$

The results can be seen in Figure 3.27. The computer program is shown in Figure 9 of the Appendix.

The values of the spring and damping constants for the simulation are:

Constant	Value
leg longitudinal angular C_α	3.6×10^4
leg longitudinal angular rate $C_{\dot\alpha}$	-6.6×10^4
leg lateral angular C_β	1.5×10^5
leg lateral angular rate $C_{\dot\beta}$	7×10^4
leg longitudinal C_ℓ	5×10^4
leg longitudinal rate $C_{\dot\ell}$	1.5×10^3

3.2.4 Methods of Implementation

Some ideas about designing a machine to satisfy the theorems of locomotion have been described. The question now is, can a computer be designed which satisfies these theorems? According to the last section, if the legs of a quadruped are cycled according to gait numbers 1, 3 or 6 with the appropriate choice of parameters, locomotion can be attained. To implement such gaits and parameters there is the need for a computer as pointed out in Chapter 1. The simplest method would be to design a mechanically geared system to cycle the legs in a desired gait with the desired parameters. The choice of gait and parameters can be made so that the stability margin, S_M, is maximized. As pointed out in the previous section,

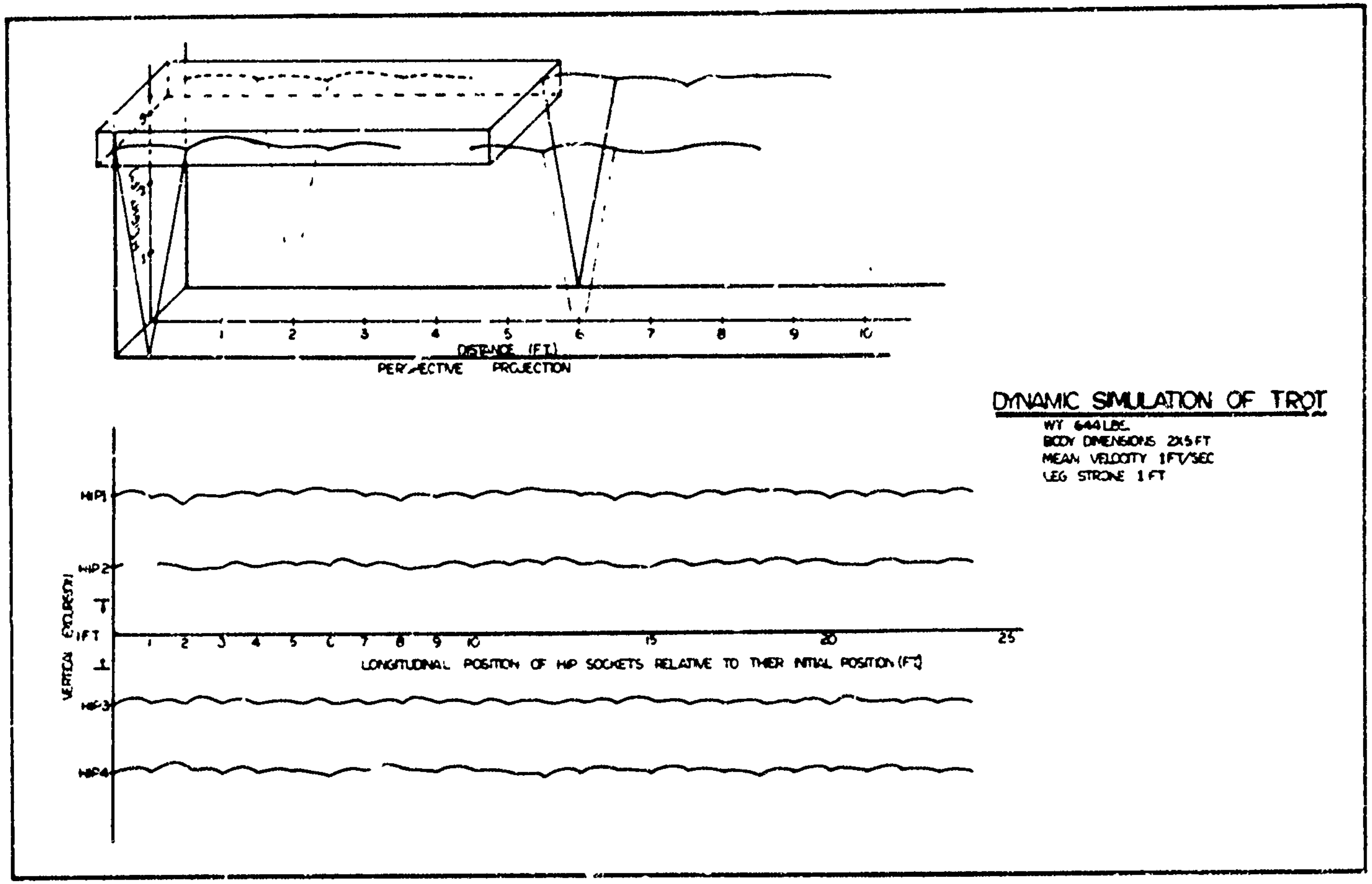

DYNAMIC SIMULATION OF TROT
WT 644 LBS.
BODY DIMENSIONS 2X5 FT
MEAN VELOCITY 1 FT/SEC
LEG STROKE 1 FT
DISTANCE (FT)
PERSPECTIVE PROJECTION
HIP1
HIP2
HIP3
HIP4
1 FT
VERTICAL EXCURSION
LONGITUDINAL POSITION OF HIP SOCKETS RELATIVE TO THEIR INITIAL POSITION (FT)

Figure 3.27

a large stability margin implies the machine is adaptable to a wide range of perturbations or external influences. At low speeds these perturbations can be interpreted as variations in terrain. A gear computer can be quite adequate for a multi-legged machine. The capabilities of this machine are merely a function of the physical dimensions. The problem with such a design, however, is that the machine is necessarily restricted to operating only in this gait. For example, efficiency and stability may be a function of velocity, then if velocity changes it may be desirable to change the gait and its parameters accordingly. To do this independent leg control must be considered. This can be accomplished in two ways. The first is to have continuous control of leg position for each leg and the second is to use a discrete control. For a continuous control the computer can generate ramps with the proper phasing and size to cycle the legs. The curves in Figure 3.26 illustrate such a set of ramp functions required by a quadruped implementing gait number 1. It is clear that all the parameters of the leg and the gait can be varied except of course, the basic body dimensions. It is a rather trivial problem to build such a computer since it involves only ramp generators and switching. It can be done either synchronously or asynchronously.

The second way is more interesting. The idea is to take advantage of the fact that the stability margin, $S_M > 0$, then design a discrete computer which quantizes the leg commands. The positions of the legs may be continuous but the force or torque commands can be discrete. There is evidence that physiological mechanisms work in this manner, with commands arriving at muscles in a discrete fashion [25]. The dynamics of the muscle and body then integrate or smooth these pulses or steps to provide a reasonable continuous output. It is possible to compare this type of control to that of a single motor unit in physiology. Two ways to make the machine operate more smoothly are to increase the number of motors or motor units per leg and to increase the number of pulses per unit time. Physiological systems use a large number of motor units for smooth action and increase the frequency to increase the magnitude of force. The advantage of using a discrete system is that digital signals can be processed by the computer, thus eliminating the requirement for continuous systems and their implications on component tolerances, energy requirements, etc.

With digitally controlled mechanisms there is a requirement for feedback as there is no information about the state of the legs. Again, in physiology there exists proprioceptive feedback [26]. The feedback information to a digital computer is discrete, thus an encoder is required. The muscle spindle is the corresponding feedback element in physiological systems.

The leg position wave-form for a quadruped operating with such a controller depends upon the surface conditions, the machine design, etc. Chapter 4 describes such a machine. A typical wave-form is presented in Figure 4.11.

3.2.5 Control with Respect to the Locomotion Theorem

The basic requirement of the machine is to provide locomotion along with stability. Once stability is obtained one can postulate the size of disturbances that can be tolerated.

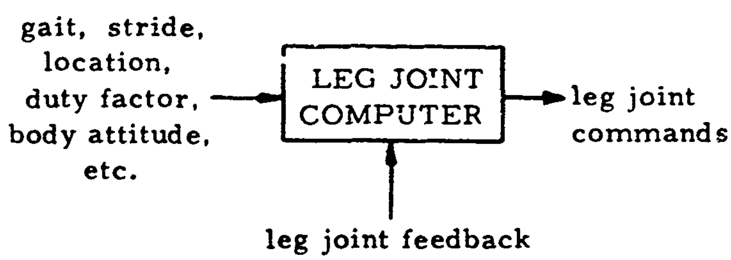
The directional control of a locomotion machine is essentially one of introducing disturbances to the stable locomotion. Two ways to accomplish this control are: to treat steering control as a perturbation to a steady state, or to consider it as a separate function. The gait during such a maneuver may or may not be broken depending upon the magnitude of the change. It is basically a transient problem. It may be interesting to study the maneuverability of crawling, or low speed, gaits. This has not been done but it is clear from the analysis in 3.2.4 how to proceed; for example, one way is to shorten the stride on one side of the machine.

Another equally important problem for all machines is that of starting and stopping. It is clear, if the accelerations of starting and stopping are low it is no problem to start or stop a machine which has a finite stability margin in the proper directions. .is can also be accomplished by changing some parameters of the gait to account for the acceleration, such as the initial conditions of the legs relative to the center of gravity (a's). This is clear for machines which satisfy either Theorem 1 or Theorem 3. To start and stop a machine such as a biped, is a far more difficult problem. In fact, bipeds can not maintain a constant velocity below a certain threshold while maintaining the same gait parameters such as stride, etc. Therefore, to start such a machine it is necessary to accelerate up to the minimum threshold velocity in one cycle or less. To stop the machine the same requirement must be satisfied. This requirement makes the design of such machines difficult.

3.3 Computer Design Theory

3.3.1 Generalized Computer Requirements

The computer for the systems previously mentioned must be one with enough flexibility to allow changes in the machine's parameters. The parameters requiring adjustment may be the stride, the gait, location of stride relative to center of gravity, the phase relations of the gait, the duty factor, and body attitude. These parameters may be the input to a generalized computer. The outputs are clearly commands to the various leg actuators. A leg command can be the position of each joint as a function of time, or a torque command to each joint. If torque is commanded, feedback is required to provide the computer with knowledge of the positions of the various joints and legs. If position is commanded, force or torque is dependent upon a function of the error in position, position rate, etc. The general computer can be represented as in the following figure:



The Leg Joint Force Command Computer
Figure 3.28

The input can change according to the machine's operator or decision element. Leg joint feedback is, in general, required but could be eliminated if position commands were the output of the computer, or the machine were designed for a very specialized task. Such an open loop controller was designed and tested on a quadruped. The results are presented in Chapter 4.

Another possible requirement is that of error detection. The computer should be capable of sensing the loss of synchronism at any of its joints. When this happens some alternative program is required to prevent the machine from falling. If enough mechanical redundancy were available so that the loss of synchronism at any one leg would not mean disaster, as occurs on a centiped, a spider, or a machine with fewer legs but larger feet, an alternative program would not be necessary. Thus it becomes a trade off as to whether redundancy is constructed into a sophisticated computer with error detection capabilities, etc., or into a mechanical design with more legs and larger, or more sophisticated, feet.

There are two types of synchronization required for a machine if legs are not allowed to slip. The first is synchronization of the time of touch down of each leg; the second is synchronization of the return stroke so that all legs have essentially the same return time. These two forms of synchronization are necessary if a consistent stability margin is to be maintained from step to step. If slippage of the legs is anticipated, then there is an additional requirement for synchronization when the legs are on the ground. In all cases, if the locomotion is stable, synchronization has a tolerance dependent upon the allowable stability margin. The design of a practical computer, as presented in Chapter 4, aids in visualization of these factors.

3.3.2 Analog Control Schemes

The computer can be mechanized by continuous elements. The switching may be accomplished by reversing drive motors, by non-linear devices such as a cam for a mechanical computer, or a function generator for an electronic computer. This could be done by indexing the cams, or synchronizing the function generators. The rate at which these cams, or function generators, move may be determined by a clock (synchronous), or they may be sequentially determined (asynchronous). By asynchronous it is meant that the cams, or function generators, operate in a specific sequence with appropriate delays and the sequence is repeatable. The velocity of a clock system is dependent on the clock frequency. The maximum velocity is reached when the actuators are providing maximum effort. The velocity of an asynchronous system, on the other hand, depends upon the cam, or function generator, rate and delays. An asynchronous system could have its speed determined by the maximum effort of the actuators. To control velocity for such a system it is necessary to limit the power to the actuators. There is evidence that some physiological mechanisms use this scheme for control above a certain power level. That is, muscles act in a maximum power mode when producing a maximum effort locomotion. The command to each leg joint is dependent only on a sequence of events. To obtain a lower power output, or finer control, antagonist muscles are activated below a certain power level and locomotion switches to another control scheme [27].

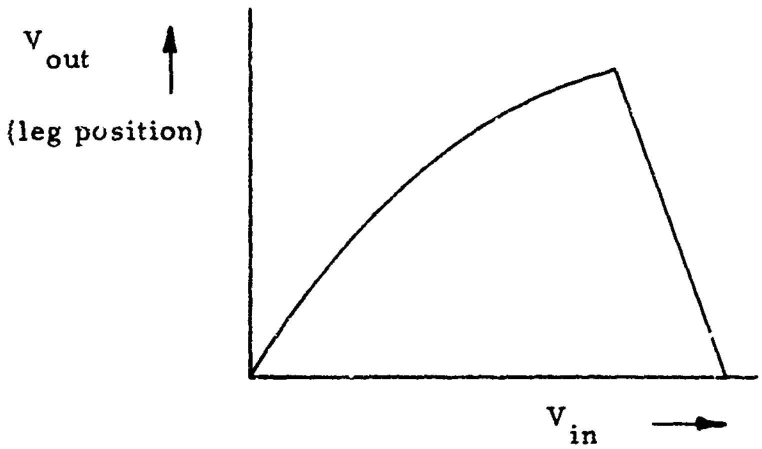
3.3.3 Digital Control Schemes

Using a digital computer in place of the analog elements provides the machine with a wide range of flexibility. Leg control functions can be stored and called upon at will. Thus, for each primary program, alternatives may exist in the event of some hazard. The requirements and functions of the computer are the same as discussed in Section 3.3.2 with the difference being in how the signals are processed and how they are used by the actuators to provide a continuous output. One way to provide an output from the computer is to send out pulses. The pulses arriving at the actuators command power to the various joints according to the pulse duration and frequency. This is similar to a single motor unit of a muscle processing the signal from a motor nerve. A machine so designed amazingly resembles primative leg locomotion in some basic animal form such as crustaceans and insects [28]. The information required by the computer to provide this output is the input from the decision element and the position or velocity of the various joints. The input information can also be in the form of pulses of varying width and duration. For example, if one were to design a completely flexible computer, it must be instaneously reprogrammable from commands emitted from the higher decision center. One way to do this is to store all programs and by use of a logic command inhibit the undesirable ones.

Feedback information from the various joints may also be in the form of pulses. Two ways to process these pulses are either by a parallel code or a serial code. Parallel coded information can be treated much more quickly than serial coded information but it has the disadvantage of requiring parallel paths for information. Serial processing requires more time and complexity. It is possible to provide some of each type in a particular design. Again physiological systems seem to exhibit information processing both serially and in parallel.

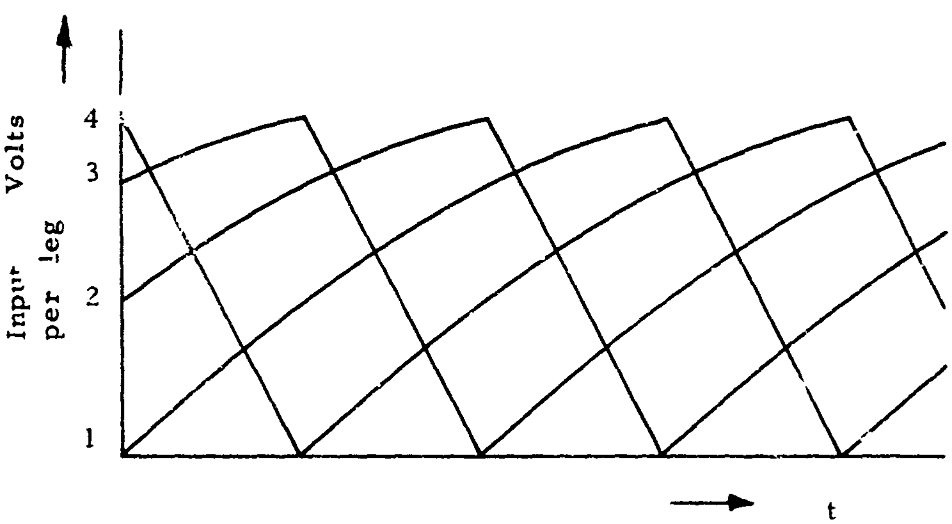
3.3.4 Algorithms for the Design of Analog Systems

Some specific algorithms which govern the design of a computer are considered. Suppose it is desired to generate the leg trajectories of Figure 3.26 for a quadruped. Also, assume the parameters are φ_i, β, S_i, a_i, and T for i = 1, 2, 3, 4. To generate these trajectories it is possible to have four function generators with the functions shown in Figure 3.29. If v_{in} were generated by a clock, then the output would be a function of time. If the output is the position command of the hip joint actuator, then the desired



Leg Command Function

Figure 3.29



Coordinated Leg Commands

Figure 3.30

trajectory for one cycle of one leg could be obtained. To repeat the cycle it would be necessary to recycle v_{in} to zero. To synchronize four such systems simply requires the proper delay in generating the input voltages to the function generators. Thus the input voltage can have the form shown in Figure 3.30. The slope represents the clock for this system. To adjust the various parameters such as stride, duty factor, position relative to the center of gravity, etc., the function size and shape can be modified. One such curve is required for each joint of the leg.

Other algorithms can be derived from this basic scheme. The clock can never be completely removed, even though the synchronization signals may be obtained asynchronously from a computation depending upon the desired gait and the positions of all other joints of the system.

3.3.5 Finite State Algorithms for Leg Sequencing

The simplest algorithm for leg sequencing was that algorithm proposed by Tomovic and McGhee [29]. This scheme considered the leg as having two states, 0 and 1, where

$0 \Longrightarrow$ on the ground and driving the body forward

$1 \Longrightarrow$ up in the air and returning

This scheme is useful in the design of some simple computer systems which have carefully controlied parameters. Or. it may be used as the [illegible] ion logic for one of the schemes previously mentioned. The states (0, 1) can be commanded by a clock running a shift register. The commands to the legs can be tapped off the shift register at a point which corresponds to the proper phase. This will be a fully synchronous system. Another way to design such a simple digital controller is to let the states (0, 1) be triggered by a logical network whose inputs are the states (0, 1) of the other joints of the system [30]. This is an asynchronous computer. A suitable combination of the two systems may also be possible. The entire concept may be considered as a digital to analog converter with crude quantization. This conclusion is reached as no phase information is available between pulses.

To refine this basic algorithm one could quantize a joint into a number of bits depending on the quality desired [29]. The command to the actuators would then correspond to the states 0, 1 and 2 where

$0 \Rightarrow$ no power

$1 \Rightarrow$ full drive power

$2 \Rightarrow$ reverse power

The average drive power would be the time average of the pulses (1). The algorithm can be implemented either synchronously or asynchronously. All joints need not have a high degree of quantization, some may be rather crude, such as having only two states, while others may have multiple states. For example, a two joint leg, in which the knee joint is only required to perform the duty of ground clearance and all the power and driving force comes from a hip joint, does not require accurate knowledge of knee joint position. Here it is only required to know whether the knee is straight or bent. The hip position, on the other hand, must be synchronized with the positions of the other hips. The accuracy of synchronization is dependent upon the joint quantization. The accuracy of synchronization required by the machine to maintain the proper stability margin is dependent upon the machine des ner.

Other digital algorithms certainly exist for different types of machines. The choice is dependent upon factors such as complexity, sophistication and economics.

3.4 Summary

In this chapter the dynamics of the general legged locomotion machine were investigated. The complexity of the problem was discussed. The concept of ideal locomotion was introduced to simplify the dynamics into a tractable form. This concept provides important insight into the requirements for locomotion of legged machines.

Theorems were developed from the concept of an ideal machine indicating necessary and sufficient conditions for particular modes of locomotion. The conditions, though not exactly applicable to non-ideal machines, can be used as the first approximation in the design of such machines.

Quadruped crawling gaits are studied and it is determined that three possible crawling gaits exist. The design of quadrupeds can be accomplished directly from these results.

An example of a control law required to provide non-ideal locomotion is presented. The example illustrates that the dynamics of locomotion can be solved satisfactorily without the use of inertial sensors. This was of course, for an ideal machine operating on an ideal surface. When the surface is altered by a random variable, the results should be more informative.

The design of computers to satisfy the theorems and definitions of ideal locomotion are discussed. The concept of digitally controlled legged locomotion machines are discussed. Some finite state algorithms were presented along with a discussion of the range of applicability of such algorithms.

Chapter 4

CONSTRUCTION OF A PRACTICAL MACHINE

4.1 Objective

The objective of this chapter is to describe the construction of a machine using the results obtained in Chapter 3. This particular machine is a quadruped. A quadruped machine was chosen as it represents a rather general class of locomotion machines in the sense that it has a non-trivial stability problem and also because it appears in many forms of nature.

4.2 The Mechanical Design and Alternative Designs

As mentioned in Chapter 2, there are many ways to construct a practical machine. Considerations such as economy and simplicity were omitted from the discussion in Chapter 2. As is usual in engineering practice, economy and simplicity are often at the opposite end of the scale from efficiency and sophistication. With economy and simplicity the principal criteria, the reasoning behind the machine design selected is presented in the following paragraph.

The requirements for this legged machine are as follows:

1) legs with the ability to support the structure, supply

drive power and clear the terrain on return

2) a suspension system coupling the legs opposite each other (left to right) to allow the legs to travel and maintain a driving force

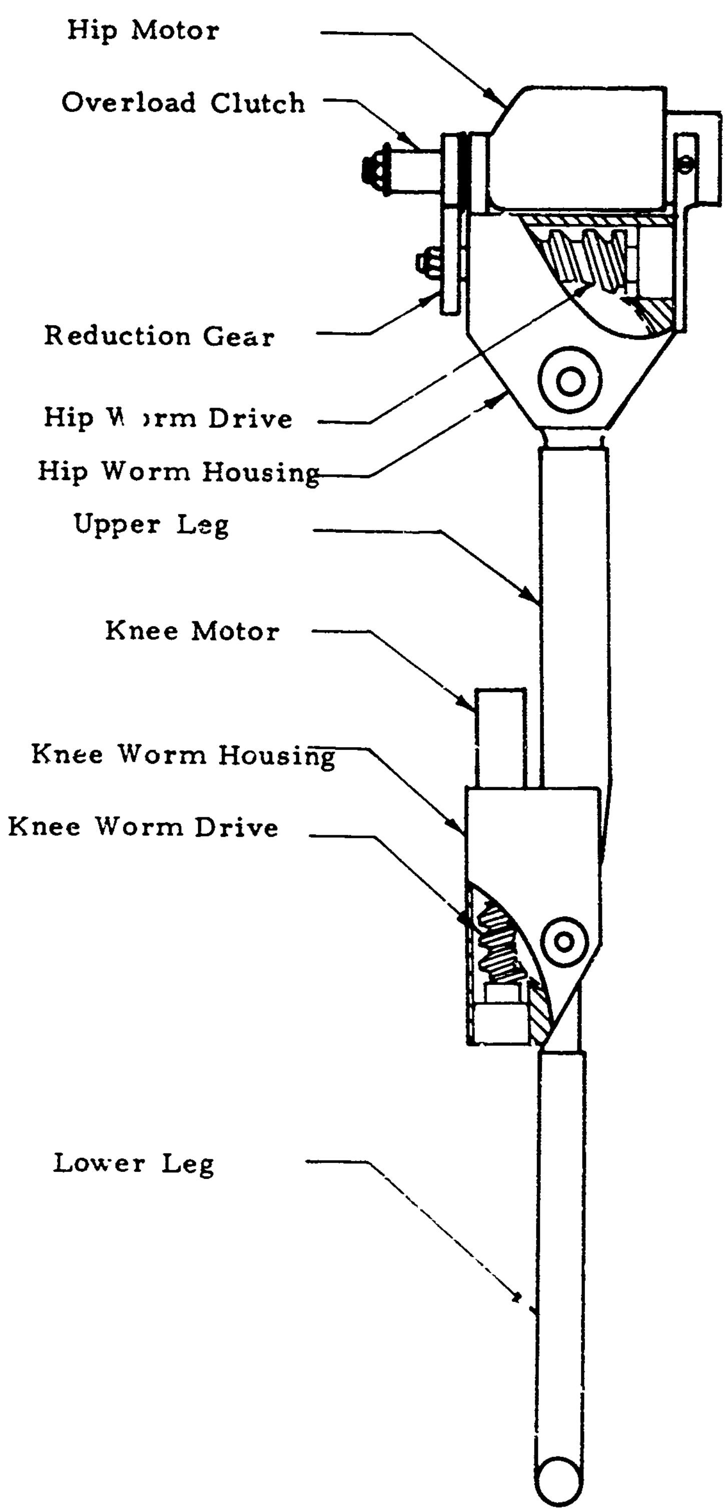
3) a rigid body to connect the four legs and suspension systems

4) power elements

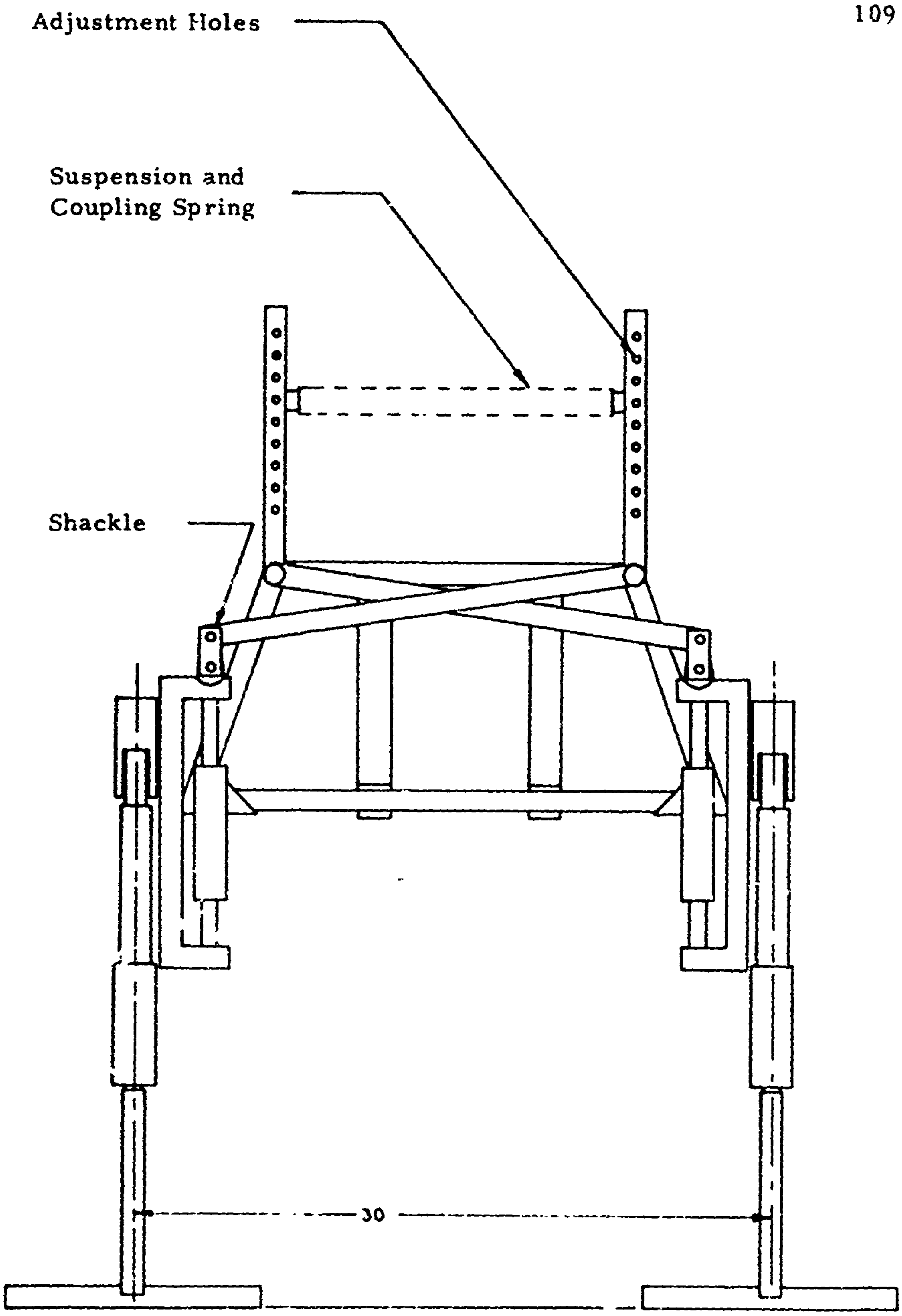
5) energy storage elements

The first requirement dictates the design of a leg subsystem. Suppose an additional constraint is imposed on the subsystem, that is, the leg system must be able to support the body regardless of the position of the leg with respect to the body. This enables the machine to stop at any position. There are many mechanical devices which will satisfy this requirement, for example, a hydraulic system. For economy, a worm gear system was chosen. To supply power, d-c electric motors are used. The advantage of electric motors is ease of control and system simplicity; the disadvantage is a low power to weight ratio. The drive power is supplied at the leg-to-body, or hip, joint. For terrain clearance on leg return, a bending knee was chosen. This configuration allows simplicity and a choice of compatible components. Thus the knee is constructed with an electric motor (reversible) and a worm gear. This implies a bending knee similar to the physiological knee. Energy storage at the joints is not realizable with worm gears, therefore the motors are required to supply power for every motion of the legs. The use of worm gears makes the overall machine efficiency rather low, but efficiency is of secondary importance to simplicity for this machine. A schematic of the leg system is presented in Figure 4.1.

Requirement 2 can be solved in many ways. One solution is to restrict the legs to moving up and down, then to interconnect the legs by a spring and shackle arrangement. A mechanism designed for this purpose is shown in Figure 4.2. This design has the advantage of adjustability by relocation of the spring. Thus, both the effective spring constant and static position of the body can be easily adjusted. The springs serve two purposes; the first is to support the body and the second to couple the left leg to the right. This mechanism is used for the front and back set of legs. While this choice leads to a simple system, in general it is not desirable to use a single passive device to balance both front and rear legs of a quadruped. The problem which arises with two identical passive suspension systems is that not only does the desired left to right coupling occur but also a left front to right rear coupling and visa versa. This latter coupling is highly undesirable. This undesirable feature can be minimized if the stride, or leg stroke, is reduced. Again, for reasons of



Leg Mechanism
Figure 4. 1

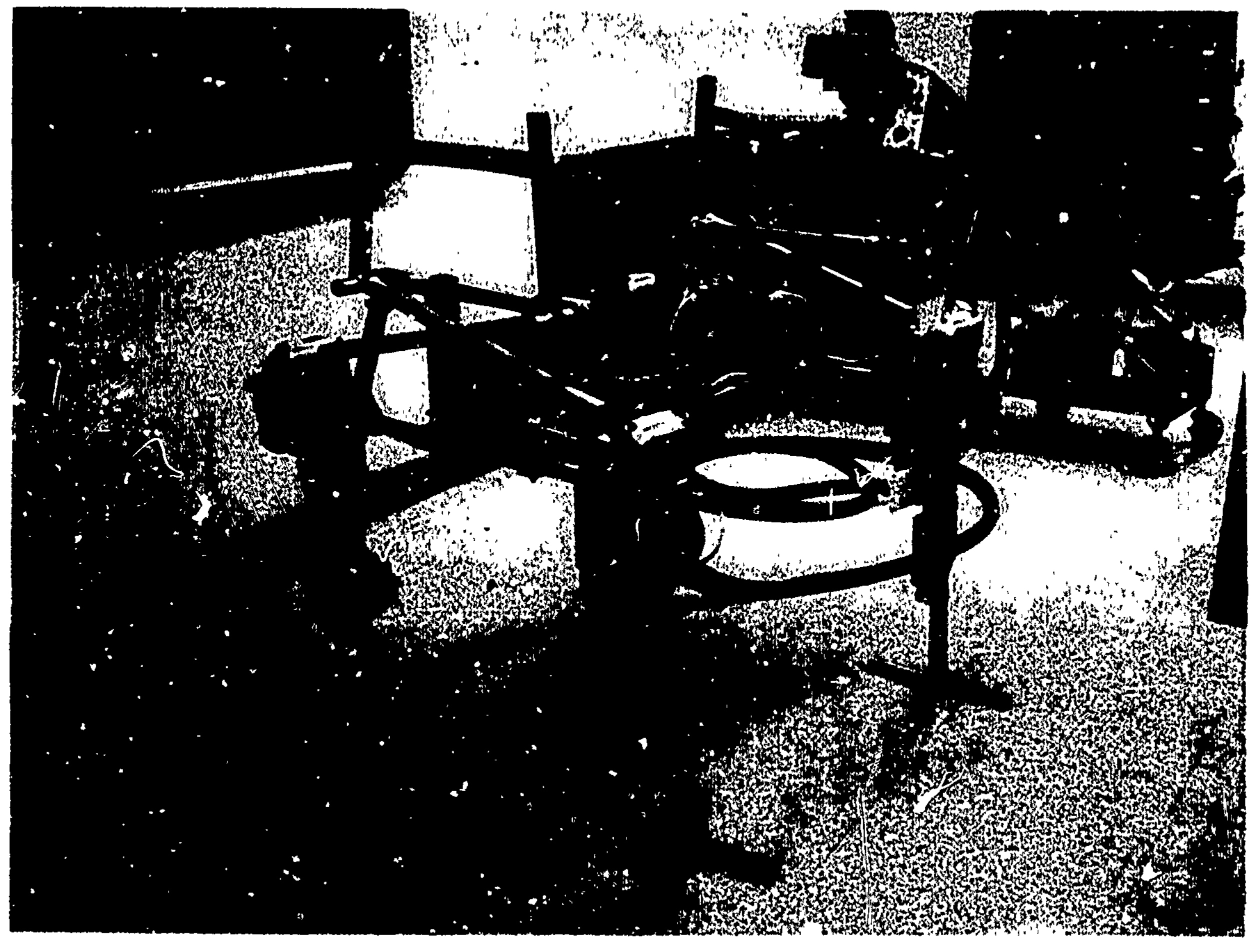


Leg-Pelvic Arrangement for the Designed Quadruped
Figure 4.2

simplicity, this was done instead of designing an active device similar to the pelvic and shoulder structures of living quadrupeds.

The body amounts to simply a rigid structure consisting of tubes which properly locate the legs with respect to each other. In biological systems the body weight is, in general, much larger than the leg weight. This gives rise to certain desirable dynamics. In particular the unspring weight to body weight ratio is low, implying little shift of the center of gravity as the legs move, low impact energy, etc. The present machine is designed with the opposite condition. This design is acceptable due to the gaits and dynamics chosen for the machine. Figure 4.3 is a photograph of the completely assembled machine.

To power a general machine some device which can transfer potential energy to kinetic energy and visa versa would be most efficient. Thus ideal leg actuators may be pnuematic. This is true since the working fluid is compressable in such a case. In addition to energy exchange in the oscillating leg, energy exchange is desirable in the overall machine. The potential energy of the rising body should be transferred to kinetic energy as the body falls. The kinetic energy should be stored as potential energy as a foot strikes the ground and released again as kinetic energy as the foot comes off the ground. The machine designed has none of these energy features



The Complete Quadruped Machine
Figure 4.3

due to the worm gear and leg construction.

4.3 Computer Design

4.3.1 Basic Requirements

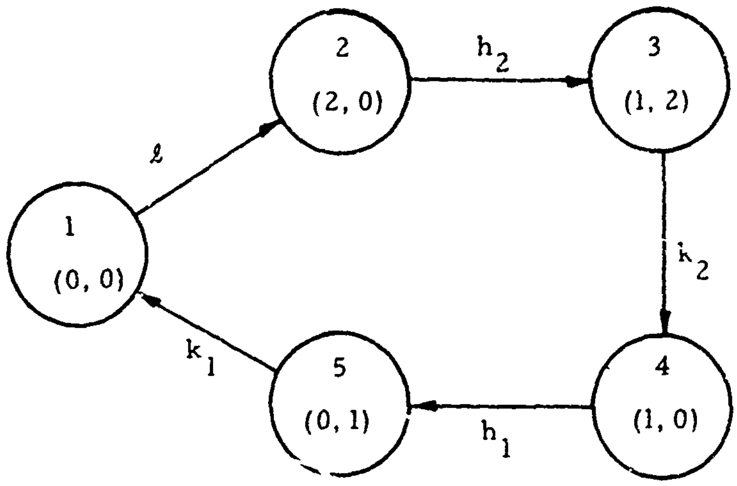
The machine is designed to provide locomotion on level, smooth terrain. It is controlled by digital signals to the leg actuators. The actuators have three states: forward, stop and reverse. The power control is regulated by the dynamics of the machine and the chosen gait. The legs are locked immediately when stopped. Given these basic requirements, the design of a computer to perform two simple gaits, namely the trot and the crawl, follows.

4.3.2 Computer Designs

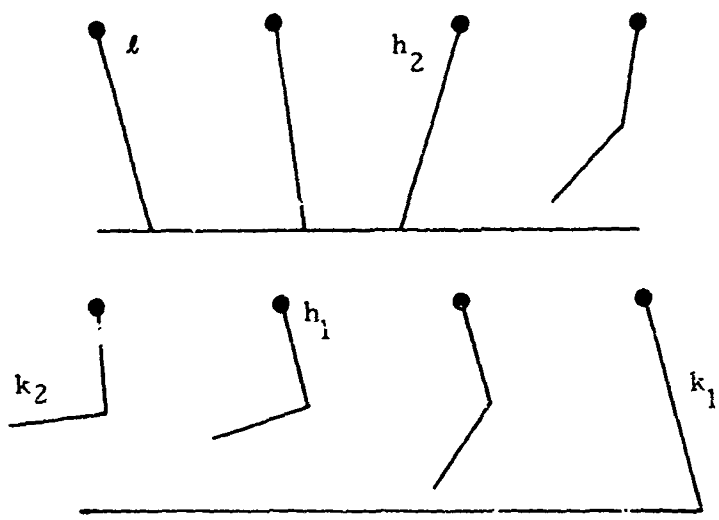
4.3.2.1 Trot

The first gait implemented on this particular machine was the trot. In this gait a rest or stop position was chosen for each leg. The first leg logic attempted was a monostable sequence such that once the logic was initiated it would continue until it reached the rest or stop state. The state diagram for this subsequence, along with the corresponding leg behavior, is shown in Figure 4.4

A relay logic was used to implement this state diagram. The circuit diagram is shown in Figure 1 of the Appendix. This circuit.



(a) State Diagram For Monostable Leg Sequence



(b) Leg Positions Resulting From State Diagram

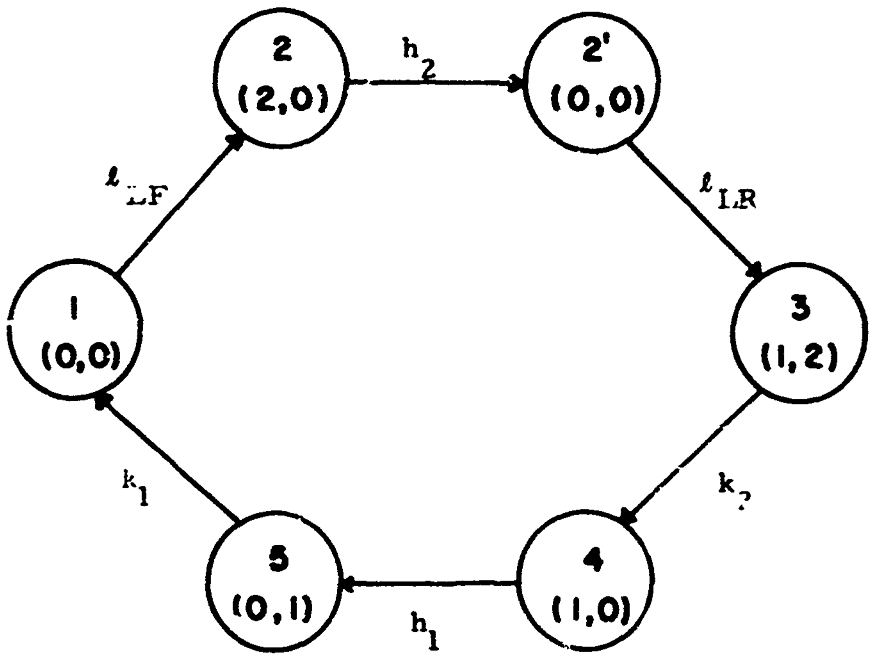
Figure 4.4

being monostable, causes the leg cycle to continue until a stable, fully extended forward state is reached unless a motor is stalled so that one of the limits becomes unreachable. To implement the trot, using this leg logic, the following sequence for triggering each leg logic circuit is used:

Table 4.1: Trot Sequence

Legs	1	2	3	4
	LF	RF	LR	RR
1 = trigger	1	0	0	1
	0	1	1	0

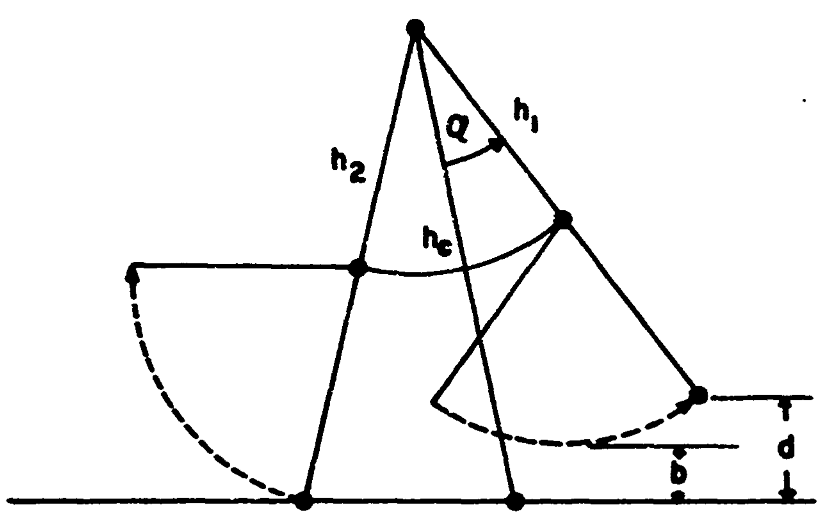
This logic can be commanded by a clock. However, in order to protect the machine against possible starting transients and improper clock frequency, a simple safety latch circuit was incorporated so that a rest state would also exist at h_2, thus producing a bi-stable leg. The new modified state diagram for the synchronous trot is shown in Figure 4.5. The trigger to start from this new rest state is derived from the start signal of the opposite pair of legs. This coupling insures that a leg can not bend and begin its return until a command for the opposite diagonal pair of legs is given. This circuit however, is still not adequate for a trot due to the fact that the leg must kick forward in order to gain ground clearance. If the pair of legs at the rear limit, or state 2', are allowed to bend the instant the



where:
ℓ_{LF} = left front trigger
ℓ_{LR} = left rear trigger

Left Front Leg Control For A Trot

Figure 4.5

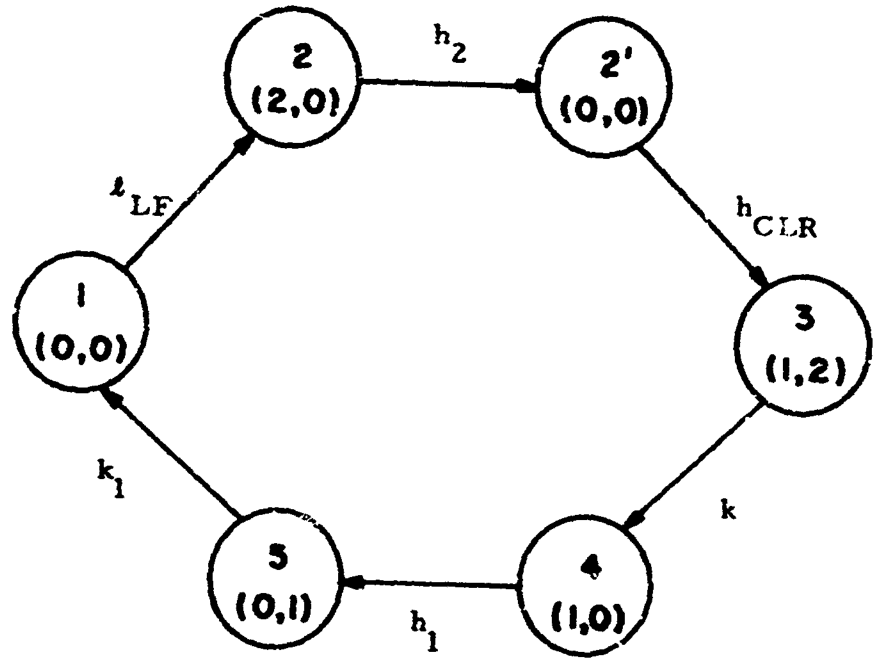


Leg Clearance Diagram

Figure 4.6

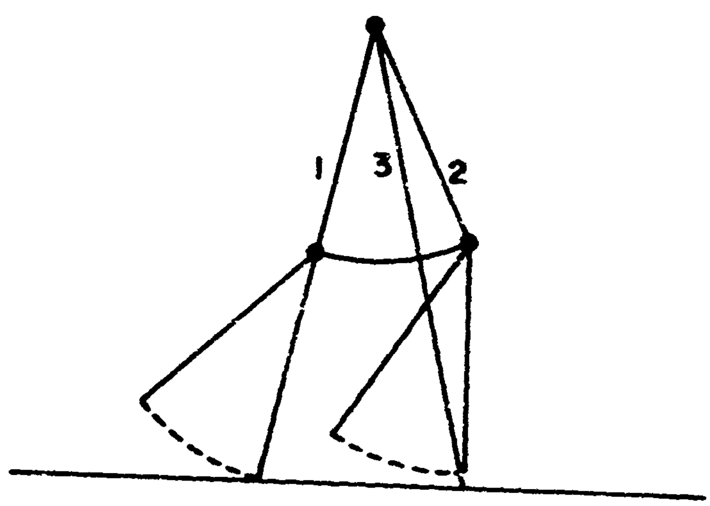
command for the other pair arrives, the machine would take a large drop, d. This phenomenon can be explained by the leg clearance diagram shown in Figure 4.6. In order to gain clearance b for straightening the leg, a prekick angle α had to be introduced. An additional input to the state diagram had to be added, namely h_c, to insure that the legs stopped at the rear limit will not be lifted until the other pair of legs are in a position to support the structure. Finally the successful state diagram is shown in Figure 4.7. The final modification to the circuit diagram, made in order to incorporate the interlock, is shown in Figure 2 of the Appendix.

To summarize, the legs have been cross-coupled in pairs to account for the leg prekick angle α and to synchronize the leg return cycle of one pair with touch down of the opposite pair. Because of the cross-coupling, the machine can be made either synchronous or asynchronous. In order to make this machine asynchronous merely replace ℓ_{LF} by h_{2LR}. This was not done, however. It would perhaps be possible to find a frequency such that the state diagram of Figure 4.4 would be able to drive the machine in a trot. The problem is that the difference in characteristics of the leg motors, and even the slightest change in terrain, can drastically change the required frequency. Therefore, without some form of synchronization it would be nearly impossible to find the proper frequency for



Trot Leg Control State Diagram

Figure 4.7



Ideal Leg Cycle

Figure 4.8

this gait. As a result of these considerations, it is concluded that:

1) A monostable leg can work in only the most ideal situations; a bi-stable leg can be made to operate under varying external conditions.

2) Synchronization is easily accomplished with a set of bi-stable legs; synchronization would be virtually impossible with monostable legs due to differences in motor characteristics.

3) Three positions of the hip must be known in order to implement the trot with this machine. If a knee and hip position controller were available so that the knee hip trajectory could follow the form shown in Figure 4.8, the prekick could be essentially eliminated even with such an elementary number of joints. The leg proceeds from position 1 to 2 to 3. Note that the hip joint moves forward of the desired angle but the foot never goes beyond the desired contact point. Mechanically, to implement this geometry, a free condition of the knee would be desirable. This is not possible with worm gears, resulting in the requirement of the undesirable prekick.

4) The trot was successful in this machine because of the

wide feet. If smaller or point feet were used it would be much more critical to find, not only the proper frequency, but the proper amount of power required for the machine to successfully trot. The simulation in Chapter 3 could have been used to aid in this determination. A support diagram for the particular trot executed by this machine is presented in Figure 3 of the Appendix. Since the wide feet of this machine enable it to continuously satisfy Theorem 1 in a trot, the machine can be made to operate at any arbitrary speed up to a maximum.

5) There is no need for a shoulder to pelvis arrangement in the trot since the legs directly opposite each other are never on the ground at the same time.

4.3.2.2 The Crawl

The crawl foot-fall sequence is given in the following table:

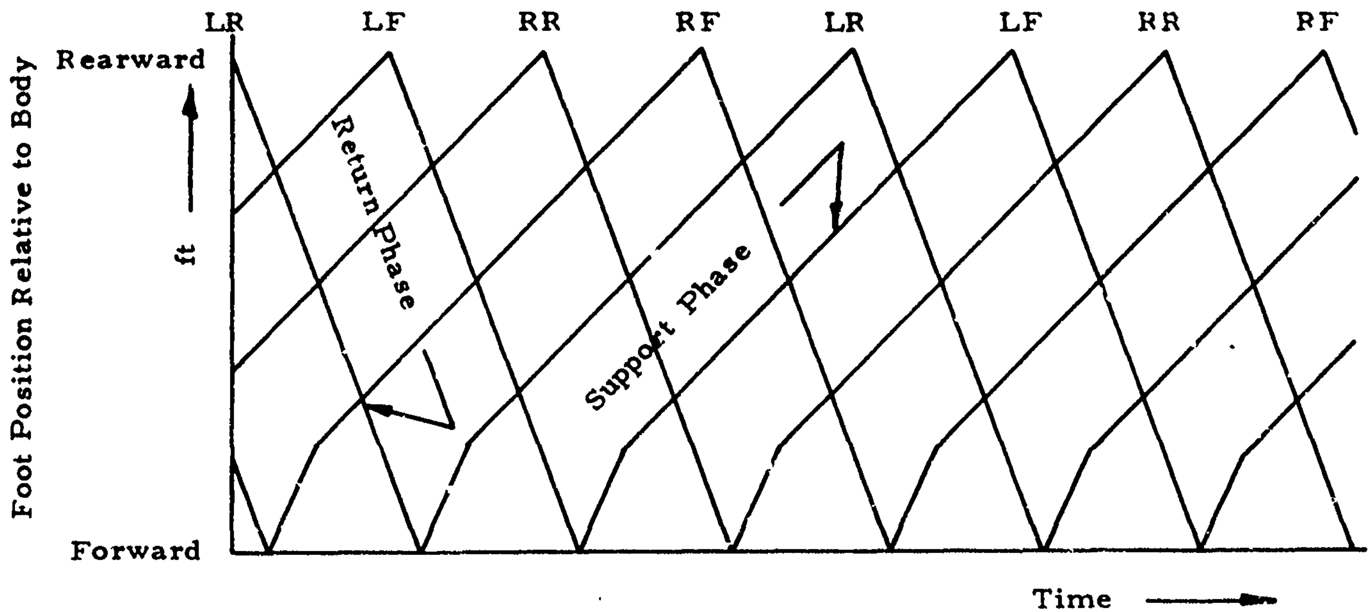
Table 4.2: Foot-fall Sequence for the Crawl

Legs	1 LF	2 RF	3 LR	4 RR
Foot-falls	0	1	1	1
0 = in air	1	1	1	0
1 = on ground	1	0	1	1
	1	1	0	1

The crawl for this particular machine is a more difficult gait to implement than the trot. The problem is that there are at least three legs on the ground at any one time. This implies that the shoulder arrangement must work, and that the three legs must have a non-zero load greater than some minimum amount for a given surface, otherwise one of the three legs is [illegible] to slip. Note that any set of diagonal legs properly placed can support the machine. This implies the third leg at that moment is redundant. It was noted in Chapter 3 that redundant legs sometimes cycle at a higher frequency than do the supporting legs of a spider [28]. This same basic concept holds here, that is, the redunant leg tends to slip. Further, since the machine has wide feet, a non-reversible drive, and different position and dimensional tolerances for each leg, some slippage must take place. Noting these points, the design of the sequence controller is developed in the following paragraphs.

First it is necessary to investigate the leg positions as a function of time. The leg positions and associated support patterns for the particular crawl chosen are illustrated by Figure 4 in the Appendix. Notice that the stability margin in the longitudinal direction can be increased by increasing the stroke, widening the feet, and by changing the duty factor. Figure 4 of the Appendix can be translated into a foot position trajectory diagram as shown in

Figure 4.9. This shows the ideal trajectory desired. The objective is to achieve this by the simplest logical network possible. The complexity of a logical network is determined by the number of decisions required to produce the desired output. The methods for impleme ng these trajectories are dependent upon the types and numbers of inputs given the computer. The information to a digital computer must be limited to discrete positions of the hip and knee. The hip position can be measured with respect to the body and the knee with respect to the upper leg. The set of possible computer outputs is, of course, the same for each motor, i.e. forward, stop, and reverse. The simplest form of control would be to use two discrete positions on the knee and two on the hip. Two basic problems exist with such a simple control scheme. The first is that the motors and legs are not identical; the second is that slippage will occur whenever a leg becomes redundant. Experience with the quadruped machine described earlier in this chapter has shown that these two problems can result in the legs getting out of phase by such a large amount that the gait changes. Since the attempt is to implement a crawling gait, this can not be tolerated. The next step, therefore, is to include more discrete points on the hip so that those legs on the ground can not get too far out of phase. This is allowable since, as pointed out in Chapter 3, the phase variables

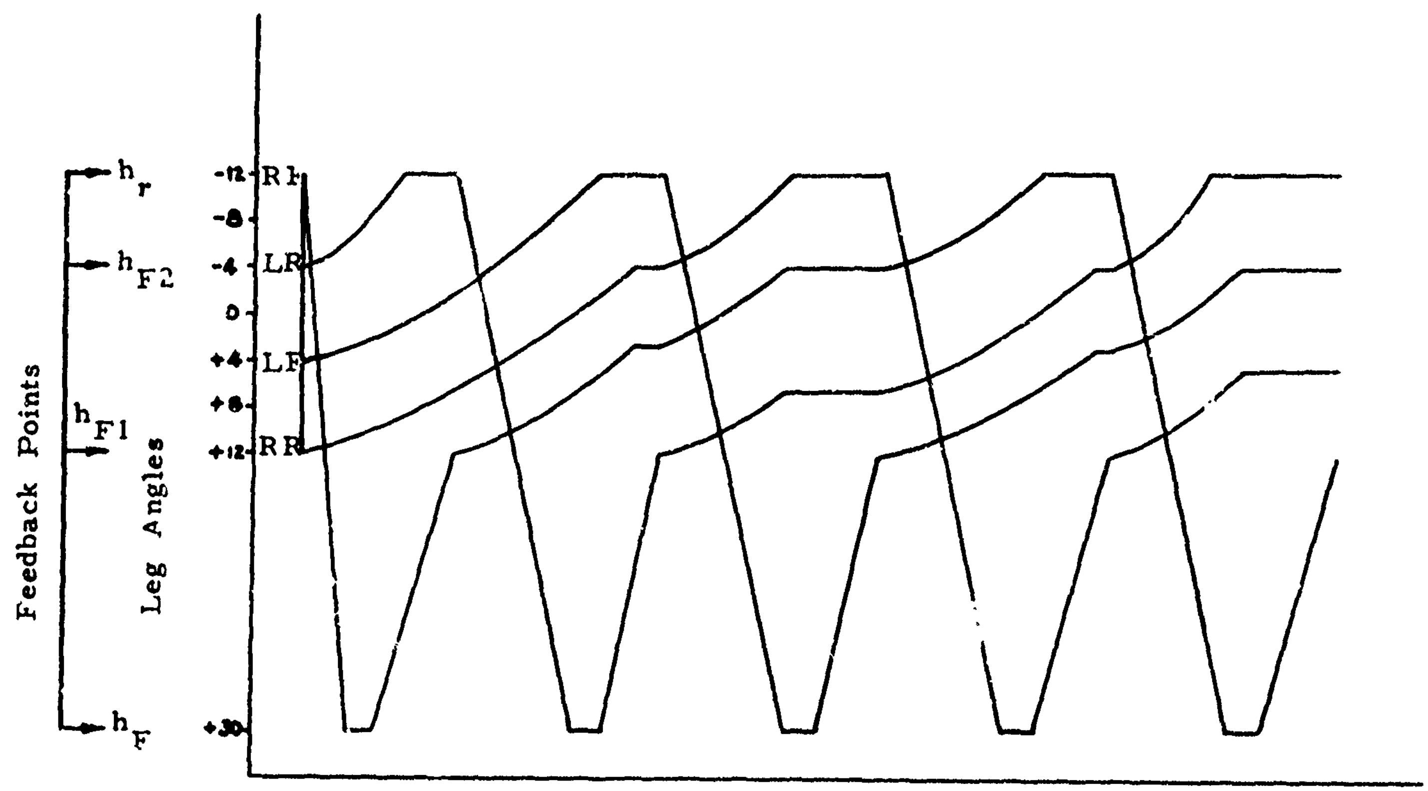


Ideal Foot Position Time Relationship For a 1423 Crawl
Figure 4.9

have a rather wide tolerance. With this machine, the tolerance is even wider due to the configuration of the feet. In the final form of the crawl controller, four discrete feedback points are used on each hip joint. The additional feedback points are used to stop the motion of legs which have moved too far relative to the other legs during the support phase of their motion. This results in trajectories similar to those shown in Figure 4.10. Note that this is only a possible trajectory as the exact trajectory would depend on the surface conditions and the parameters of the machine. Figure 4.11 shows the actual recorded results of the four legs while the machine is crawling.

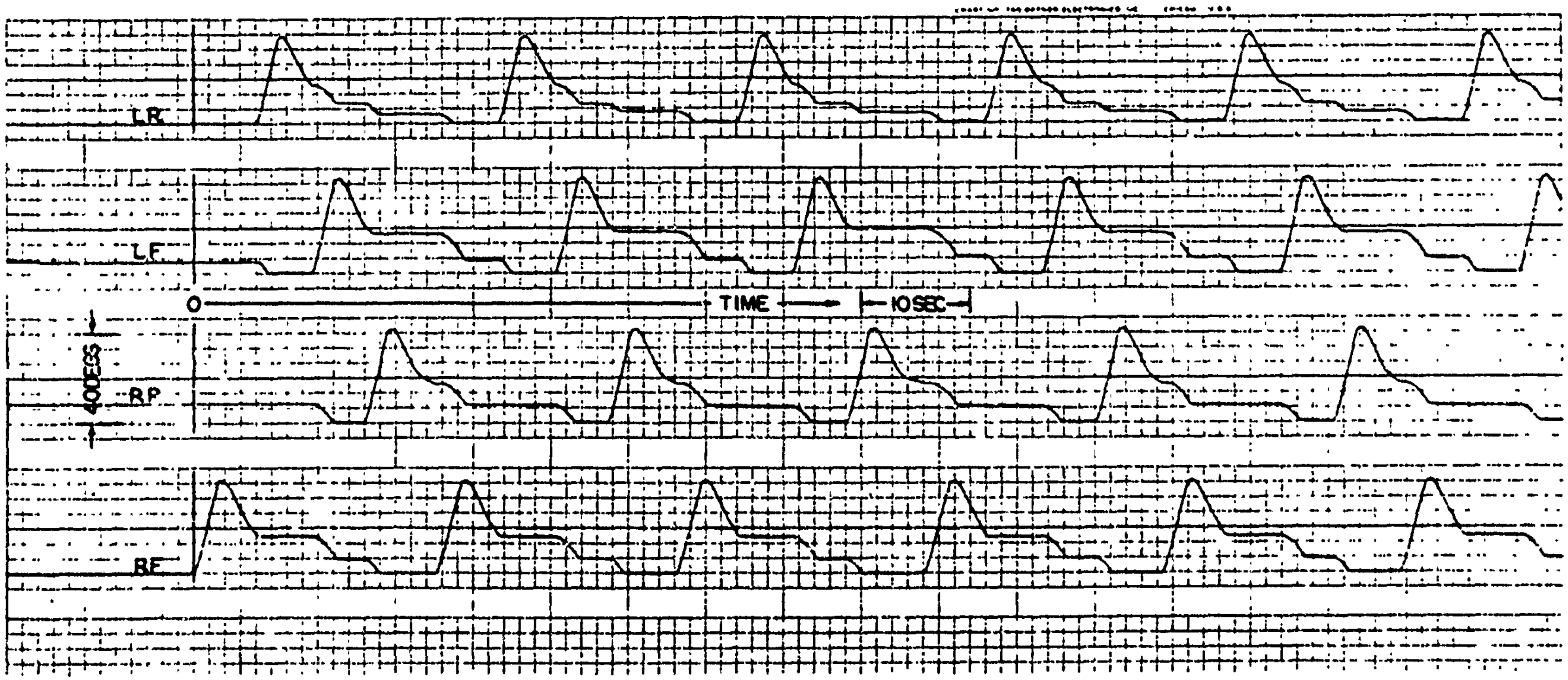
The motors of the machine have three states as in the trot mechanization; i.e. forward, reverse and stop. To accomplish this, each motor will have two flip-flops associated with it. Thus, if both flip-flops are reset (0, 0) the motor is stopped. If forward drive is desired the state is (1, 0), reverse is (0, 1). The state (1, 1) is not used. There will be four flip-flops per leg for the two motors, the hip and the knee. The flip-flops will be used to drive the power relays A, B, C, and D for each leg as shown in Figure 1 of the Appendix.

Each leg goes through the state diagram shown in Figure 4.12. State 6 represents the leg on the ground when it is ready to drive.

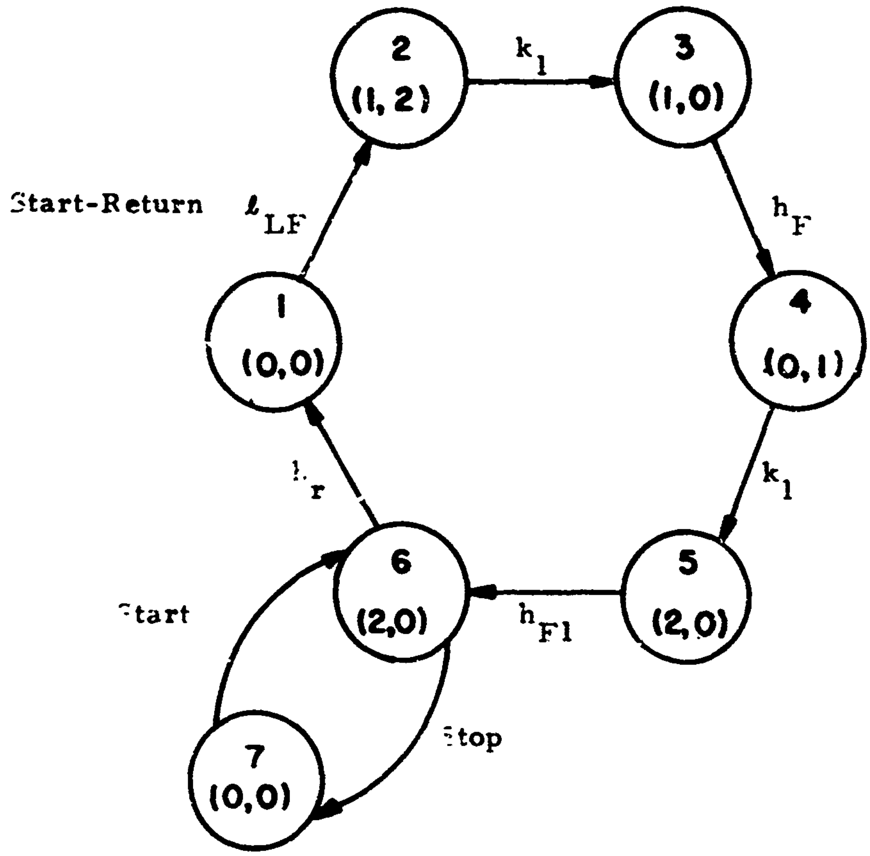


Predicted Foot Fall Trajectories

Figure 4.10



Experimental Leg Trajectories for the Constructed Quadruped
Figure 4.11



State Diagram for Each Leg of the Crawl Sequence
Figure 4.12

The start-stop sequence operates on states 6 and 7. The circuit diagram to implement this sequence is shown in Figure 5 of the Appendix. The flip-flops are indicated on this Figure as h_0 = hip drive, h_R = hip return, k_R = knee bend, and k_F = knee straight.

The start-stop of the drive flip-flop and the start-return signals are derived from the positions and states of all four legs as mentioned above. These signals are tailored specifically for the 1423 crawling gait. The start-stop and start-return signals for the left front leg are accomplished by the logic shown in Table 4.3. The logic is based on a number of assumptions. They are:

1) Each leg's return cycle is long relative to the time required to drive 1/3 of the total stride.
2) Some slippage of the feet on the ground must occur due to the impreciseness of each leg and the fact that locking joints are used.
3) When slippage does occur the slippage is limited to an amount acceptable by the stability margin of the machine.
4) The knee position will be sensed by microswitches indicating fully bent, k_R, and fully straight, k_F, conditions of the knee joint.
5) The logic is developed by consideration of Figure 4.10.

6) The comments of Table 4. 3 provide an explanation of the logical decisions required to implement the gait.

7) The lower case letters of Table 4. 3 and the boolean expression indicates switch states. The upper case letters indicate the flip-flop states. The boolean expressions corresponding to the rows of Table 4. 3 are

$$[LF(h_F\ k_F)]_1 + [RR(h_{F_1}\ H_R\ h_{F_2})\ LF(h_{F_2}\ H_R)]_3 + [RF(h_{F_1}\ H_R\ h_{F_2})\ LF(h_{F_2}\ H_R\ h_R)]_5 = LF\ START \tag{4.1}$$

$$[LR(h_{F_2}\ H_R)\ LF(h_{F_1})\ RR(h_{F_1}\ H_R\ h_{F_2})']_2 + [LF(h_{F_2})\ RF(h_{F_1}\ H_R\ h_{F_2})]_4 + [LFh_R]_6 = LF\ STOP \tag{4.2}$$

The subscripts on the brackets correspond to the comments in Table 4. 3.

The circuit diagram mechanizing the start-stop boolean expressions for all four legs is shown in Figure 6 of the Appendix. These logic gates and flip-flops are produced by Digital Equipment Corporation. The logic code is shown in Figure 9 . The hip positions will be sensed by Fairchild integrated circuit comparators mod. 1 μ710. A potentiometer will indicate the hip position. The

Table 4.3: LEFT FRONT START-STOP AND RETURN CYCLE SEQUENCE OF EVENTS

LEGS / LF POS	INPUTS FROM 4 LEGS TO THE LEFT FRONT															LF HIP MOTOR FF COMMANDS			COMMENT
	RF			LR			RR			LF						LF start stop		return cycle	
degs	h_{F1}	h_{F2}	H_R	h_{F1}	h_{F2}	H_R	h_{F1}	h_{F2}	H_R	k_F	h_F	h_{F1}	h_{F2}	h_R	H_R	S	H_o R	ℓ	
+30		—			—			—		1	1	1	1	1	0	1	0	0	1
+30 → +12		—			—			—		1	0	1	1	1	0	–	0	0	
+12		—		–	1	0		—		1	0	0	1	1	0	–	0	0	
+12 → -4		—		–	0	0	—	—	1	1	0	0	1	1	0	0	1	0	2
+12 → -4		—			—		1	1	0	1	0	0	1	1	0	0	–	0	
+12 → -4		—			—		0	0	0	1	0	0	1	1	0	1	0	C	3
-4		—			—			—		1	0	0	0	1	0	0	1	0	4
-4	1	1	0		—			—		1	0	0	0	1	0	0	–	0	
-4	0	0	0		—			—		1	0	0	0	1	0	1	0	0	5
-4 → -12	0	–	0		—			—		1	0	0	0	1	0	1	0	0	
-12		—			—			—		1	0	0	0	0	0	0	1	0	6
-12		—		1	–	0		—		1	0	0	0	0	0	0	1	0	
-12		—		0	–	0		—		1	0	0	0	0	0	0	1	1	7

Table 4.3 continued

Comments

LF = Left Front Leg

LR = Left Rear Leg

RF = Right Front Leg

RR = Right Rear Leg

(1) Start drive when hip in fully extended position and knee in fully extended position.

(2) Stop when LF has passed h_{F1} and LR reaches h_{F2} to wait for RR to complete return stroke. End stop when RR is in a supporting position.

(3) Start LF when RR is between h_{F1} and h_{F2}. End start when LF h_{F2} is reached to wait for RF return.

(4) Stop when LF h_{F2} is reached. End stop when RF has completed its return and is between h_{F1} and h_{F2}.

(5) Start when RF completes return and reaches h_{F1}. End start when LF reaches rear limit h_R.

(6) Stop when LF h_R is reached.

(7) Supply trigger to return cycle of LF when LR completes return cycle and reaches the point of touch down h_{F1}. (This is not a part of the start-stop sequence.)

comparators then provide discrete position information for the hip position to the computer. The logic used will be 0 = -3 volts when the leg is behind the switch point and 1 = 0 volts when the leg is ahead of the switch point.

The switching algorithm was successful for the stride selected. If the stride is increased the operation may not be as satisfactory as indicated in Figures 4.10 and 4.11 since the quantization would represent a longer leg travel.

The action of the legs was a series of stops and starts during each leg cycle. The number of starts and stops can be increased by adding more bits on the legs. The resulting trajectories ought to be considerably smoother than when only four hip positions are used. Therefore, if more hip positions are used, even better performance can be expected. The minimum number of required discrete positions of the hip is dependent upon the gait, parameters of the gait, and the particular machine. In this case it was found to be four positions. This number was determined by a consideration of the effect of foot slippage on support patterns for the crawl.

4.4 Summary

In this chapter the construction of a quadruped locomotion machine was outlined. A trot and a crawl were mechanized by two different computers for the same mechanical design. The trot was implemented synchronously, the crawl was implemented asynchronously. The trot computer required three bits of feedback information while the crawl required four. These were apparently the minimum numbers. Factors such as tolerances in mechanical dimensions, in floor variations, and in motor characteristics forced the selection of more feedback points than were anticipated. This resulted in computers with some degree of immunity to these tolerances. This implies the computers designed have adaptability to varying conditions. It was further concluded that more feedback points on the hip would result in smoother operation and also in more immunity to varying external conditions.

Chapter 5

CONCLUSIONS AND EXTENSIONS

5.1 The Contributions of this Dissertation

This dissertation attempts to provide some insight into the control problems associated with legged locomotion machines. The problems of machine design are discussed in general terms. A mathematical model for the analysis of such machines is constructed. This model allows the development of fundamental concepts for the analysis for such machines. The mathematical model is an approximation to the realistic situation, it provides, however, a guide to the true picture. The concepts developed from this model are tested by the construction of a quadruped machine. A quadruped was chosen since it does not have a trivial dynamic stability problem and it provides a statically stable base from which to begin. The concept of finite state control of legged locomotion is explored and found to be suitable in certain instances. These are listed. Two finite state controllers are constructed for the quadruped. The controllers were used to implement the trot and the basic crawl. Comparisons between the finite state controller designed and physiological systems are made.

5.2 Extensions

The theory of legged locomotion developed in this dissertation provides a basis for expansion into areas where realistic application of the concepts can be put to use. Mechanical design problems, computer designs and computational techniques are some of these areas. It is hoped that such applications will lead to improved designs in the areas of orthotic and prosthetic devices and to a greater understanding into the actual rehabilitation process.

5.2.1 Mechanical Construction

This area requires extensive research before a reasonably efficient legged machine can be constructed. The specific items for investigation are actuators, energy storage devices, materials for construction and power supplies. No mention has been made of power supplies in this dissertation; however, this is essential to any machine. There are many possibilities for a simple power source. The selection is dependent upon the application. In the field of prosthetics, energy sources must be very light and portable. For powering a legged vehicle a relatively cumbersome source could be allowed.

Actuator requirements were briefly discussed in Chapters 2 and 4, while the design of ideal legged locomotion actuators was omitted. The design problem to be solved is one of proper balance between energy storage, force capability, displacement and efficiency. It may be possible to extract some general requirements for the design applicable to all legged locomotion machines.

Energy storage devices were mentioned as desirable for efficient legged machines. In considering the design of specific devices, a careful investigation must be made if efficient locomotion is to be achieved. One possibility is an incorporation of the energy storage device in the actuator design.

The materials to be used in the construction of prosthetic and orthotic devices are especially important. Low weight to strength ratios are vital. In addition to tensile strength, the leg material must also have ductility since it must, on occasion, absorb an undue stress. This stress could occur when suddenly encountering an obstacle such as a rock or a hole in the ground. Ductility is essential since a brittle material, though of high strength, could break in such a case.

As pointed out in Chapter 3, ideal dynamics result when all of the mass of a locomotion machine is contained in the body. In addition, it is desirable to have large rotational inertias in the body as compared with the legs. The large inertias provide a stable platform for the legs to work from. The manueverability of a machine, however, is inversely proportional to the mass and rotational inertia for a given power in the legs. Consequently there exists a trade-off between a stable platform and manueverability for a given power available in the legs.

Other mechanical problems include leg subsystem design, a ground clearance problem, leg kinematics, to solve ground clearance, and the desirable number of joints per leg.

5.2.2 Computational Problems and Computer Design

The importance of the flexibility of the computer has been demonstrated. It has also been shown that digital computers can offer a highly reliable, small, flexible and adequate solution to the problem, and that quantization can be effectively nullified with a large number of steps. The areas for further investigation include: first, a consideration to what gaits this simple algorithmic control can be extended, i.e., can the simple logical computer used to solve the crawl be used for other locomotion gaits; second, the problems of starting, stopping and traversing terrains where the g field has been rotated; third, the problems of changing gait as speed changes; fourth, the problems of ground clearance and how to design the logic to provide for the proper control of this kinematic problem; fifth, the problem of deriving other possible general leg control algorithms with simplicity as the goal; sixth, determining how far the concept of error correction can be extended; seventh, the requirement for an emergency recovery program, i. e., if the machine were on its side could such a program upright the machine; and eighth, determining what gaits can be mechanized without sensors, or if sensors are required what type should they be.

Returning to area three, the problem of interest is that of designing a logical computer to automatically change gaits when required by velocity and the terrain, if this is possible. The idea to be explored here is to de-emphasize the discrete aspects of gaits and investigate only the phasing of the various legs to provide the required support and driving forces. To exemplify area four, consider a designed quadruped which bends the knee by 90°. This is not required since ground clearance requires only a few degrees for this machine and this stroke. In order to accomplish this, a fine control is required of the knee. Again the quantization can be finite and limited to a few bits for the entire knee. The knee motion then, would have to be coordinated with the hip and the leg control loop would be more complex. If more joints are available the computer algorithm becomes more complex.

5.2.3 Biped, Triped, Multiped Walking Machines

The theorems were developed with a general mulitped locomotion machine in mind. They were tested, however, on a quadruped. The question of their validity to bipeds, tripeds, and other multipeds must be further investigated. The general concepts of controllability, stability, machine efficiency and machine organization can be applied to a specific problem. The application of the concepts may be modified as a function of the number of legs. In addition, for bipeds, more theoretical considerations may have to be developed before machines can be constructed with any level of confidence. Therefore, much work remains to be done in legged locomotion before a reasonably complete theory is developed.

5.2.4 Relation of this Dissertation to the Theoretical Work of the Past and the Future

This dissertation consolidates material from many fields of science. There have been some generalizations proposed to explain the observed facts reported by many authors, notably, Muybridge, Hildebrand, Wilson and Elftman [3], [11], [24], [28], [31]. The design of machines has been proposed to satisfy these generalizations. The construction of controllers for the leg systems has been done using principles introduced by Tomovic [29], [32]. The controllers designed are more sophisticated and flexible than the basic system proposed by Tomovic; however, the fundamental principle is used. The theoretical questions yet to be answered involve stability and controllability when the system is subjected to random variations in terrain. This would lead to the design of computers for leg control which may differ from the basic computer introduced. This problem requires further investigation.

It was pointed out that locomotion machines could be constructed without the use of inertial instruments. When the random terrain is introduced, a question of whether this still holds exists. Further questions remain to be answered; e.g. what conditions are required in order that such machines can be constructed without inertial instrumentation and can inertial instrumentation improve the performance of such machines? The theory developed in this dissertation assumed zero body rotation. What effect does high rotational velocity and acceleration have on this theory? These simple questions give insight into some of the important problems relevant to the construction of locomotion machines.

5.2.5 Optimal Control Problems for Leg Systems

The basic requirements of the legs are to support the body, provide a driving force to the body and to relocate themselves in order to continue the process. The question to be answered is can this relocation process be done in an energy optimal manner, where energy optimal implies minimum work dissipated in heat. Thus, energy storage must be accounted for. Even more interesting is the question of whether this can be done optimally when the system is subjected to a random terrain. The energy optimal trajectory provides the designer parameters to select for the construction of the system. The theory in general involves designing the system to minimize total energy. This can be done by minimizing force, velocity and total excursion since these are the factors in computing friction losses. The clearance required is determined by the randomness of the terrain. The exact trajectory may be determined by the optimal control algorithm. It may be desirable to alter this trajectory according to the expected terrain. Therefore, to be truly optimal the leg system needs to be adaptive to the terrain, implying use of additional sensors.

5.2.6 Control for the Overall Machine

It was shown that legged locomotion machines could be controlled by simple algorithms without the use of inertial instruments. The control was accomplished simply by kinematic considerations. Such a system must, by necessity, be of limited use.

In order to assure stability in the event of a large disturbance additional sensors could be very helpful. Sensors such as a vertical gyro, accelerometer, eye, or force sensor on the legs, can be used to extend the stability margin of such machines. When the disturbance is large, the sensors provide the additional information required by the computer to compute the desirable leg positions for recovery. The concept of control utilizing additional sensory information requires extensive work.

The control problem, as pointed out in chapter 2, can be interpreted as one of minimizing a criterion function. For the mathematical model assumed in this dissertation, the problem is one of the classical forms of optimal control, i.e., given an ideal legged locomotion machine, compute the leg forces and phases so that the cost function is minimized. The problem is that the system is nonlinear, time varying, and has many constraints. This implies it may be difficult, if not impossible, to find a solution in the classical sense.

If random terrain and fluid gusts are introduced to the model the problem becomes one of stochastic optimum control. This can be an intersting study in stochastic nonlinear optimum control.

5.2.7 Applications to the Fields of Prosthetics, Orthotics and Rehabilitation

The theory developed here can be applied to the design of lower limb prosthetic and orthotic appliances. Further study in the direction of biped locomotion is required to provide insight into the design of efficient locomotion devices for bipeds.

Two major problems exist with respect to biped prosthetic devices, a technical problem and a cosmetic one. The cosmetic problem is one of attractiveness to the wearer. The technical problems lie in mechanical efficiency, weight, and control. Many methods of increasing efficiency of these devices were discussed; however, research must still be done to show the validity of these ideas. The problem of weight must be solved through a clever mechanical design. The control problem has many difficult aspects; stability for standing and locomotion at various speeds must be solved. Further a maximum speed must be known and indicated to the wearer or operator. The problem of interface between the human and the machine is another problem of indetermined magnitude. This problem has been researched extensively in the past, however, very few generalizations have been achieved.

The extension of the theory to rehabilitation and single leg prosthetics and orthotics is another area to be further researched. Here the problem becomes one of matching the forces, torques and kinematics required for "normal" locomotion of the one good leg. The problem requires considerable attention.

REFERENCES

1. Kuang-Chung Lo, Romance of Three Kingdoms, Translated by C.H. Brewett-Taylor, C.E. Tuttle Co., 1959.

2. United States Patents
   a) Rygg, L.A., "Mechanical Horse," no. 491927, 1893
   b) Gaskill, W.H., "Mechanical Movement," no. 912108, 1909
   c) Nilson, F.A., "Supporting and Propelling Mechanism for Motor Vehicles," no. 1574679, 1926
   d) Kinch, E.A., "Vehicle Propelling Device," no. 1669906, 1928
   e) Ehrlich, A., "Vehicle Propelled by Steppers," no. 1691233, 1928
   f) Snell, E., "Reciprocating Load Carrier," no. 2430537, 1947
   g) Urschel, W.E., "Walking Tractor," no. 2491064, 1949
   h) Carson, P.E., "Walking Tractor," no. 2822878, 1958
   i) Bair, I.R., "Amphibious Walking Vehicle," no. 2918738, 1959

3. Muybridge, Animal in Motion, Dover Co., New York, 1957.

4. McGhee, R.B. and Tomovic, R., "A Finite State Approach to the Synthesis of Bioengineering Control Systems," IEEE Transactions on Human Factors in Electronics, Vol. HFE-7, No. 2, pp. 65-69, June, 1966.

5. "Land Locomotion Laboratory," Technical Report No. 9560, U.S. Army Task Automotive Center, Warren Michigan, November, 1966.

6. Howell, R., Speed in Animals, Univ. of Chicago Press, Chicago, 1944.

7. Manter, J., "Dynamics of Quadrupedal Walking," Journal of Experimental Biology, Vol. 15, No. 4, pp. 522-539, Oct. 1938.

8. Tomovic, R. and Karplus, W.J., "Land Locomotion - Simulation and Control," Third International Analogue Computation Meeting, Press Academiques Europeannes, pp. 385-390.

9. Napier, J., "Antiquity of Human Walking," Scientific American, pp. 56-67, April 1967.

10. Shik and Orlovskii, "Coordination of the Limbs During Running of the Dog," Institute of Biological Physics, U.S.S.R., Academy of Sciences, Moscow, April 1965.

11. Hildebrand, M., "How Animals Run," Scientific American, pp. 148-157, May 1960.

12. Wilson, D.M., "Insect Walking," Animal Review of Entomology, Vol. II, pp. 103-123, 1966.

13. Huxley, T., "Animal Automatism," The Humbolt Library of Science, Vol. No. 53, March 1884.

14. Bekker, M., Theory of Land Locomotion, University of Michigan Press, 1962.

15. Liston, R.A., "Walking Machine," 1965 Annual Meeting of the American Society of Agricultural Engineers, University of Georgia, June 1965.

16. LaSalle, S. and Lefschetz, S., Stability by Liapunov's Direct Method with Applications, Academic Press, 1961.

17. Kalman, R.E., "Mathematical Description of Linear Dynamical Systems," Journal Siam, Series A on Control, Vol. I, No. 2, pp. 152-192, 1963.

18. Tou, J.T., Modern Control Theory, McGraw-Hill Book Co., New York, N.Y., 1964.

19. Kenney, J.D., "Investigation for a Walking Device for High Efficiency Lunar Locomotion," Space General Corp., Glendale, Calif., American Rocket Society Annual Meeting, Philadelphia, Penn., 1965.

20. Bekker, M., "Is the Wheel the Last Word in Land Locomotion?", New Scientist, No. 248, pp. 406-410, Aug. 17, 1961.

21. Shigley, R., "Mechanics of Walking Vehicles," Land Locomotion Laboratory Report No. 7, OTAC, Detroit, Michigan, 1957.

22. Miratori, S., "Project Chin Ma (Golden Horse)," a brochure published by the author, Piazza Cavour 4-3, Chievari, Italy.

23. McGhee, R.B., "Some Finite State Aspects of Legged Locomotion," Journal of Mathematical Biosciences, April 1968.

24. Hildebrand, M., "Symmetrical Gaits of Horses," Science, Vol. 150, pp. 701-708, Nov. 5, 1965.

25. Bekey, G.A. and Angel, E.S., "Asynchronous Finite State Models of Manual Control Systems," University of Southern California, Electronic Sciences Laboratory Report USCEE 160.

26. Guyton, A.S., Textbook of Medical Physiology, W.B. Saunders Co., Philadelphia, Pa., 1966.

27. Young, L.R. and Stark, L., "Biological Control Systems - A Critical Review and Evaluation," NASA CR 190, March 1965.

28. Wilson, D.M., "Stepping Patterns in Tarantula Spiders," Journal of Experimental Biology, Great Britain, Number 47, pp. 133-157, March 1967.

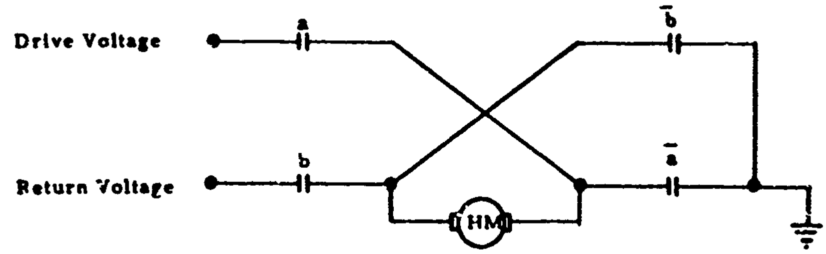
29. McGhee, R.B., "Finite State Control of Quadruped Locomotion," Simulation, September 1967.

30. McGhee, R.B., "Abstract Quadruped Locomotion Automata," University of Southern California, Electronic Sciences Laboratory Report USCEE 130, April 1965.

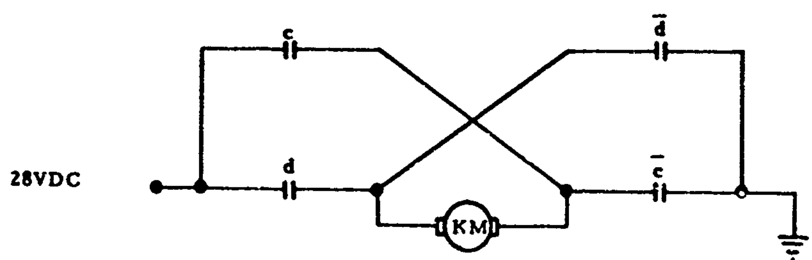
31. Elftman, H., "Basic Function of the Lower Limbs," Biomedical Engineering, pp. 342-345, August 1967.

32. Tomovic, R., "A General Theoretical Model for Creeping Displacements," Cybernetica, Vol. IV, No. 2, pp. 98-107, 1961.

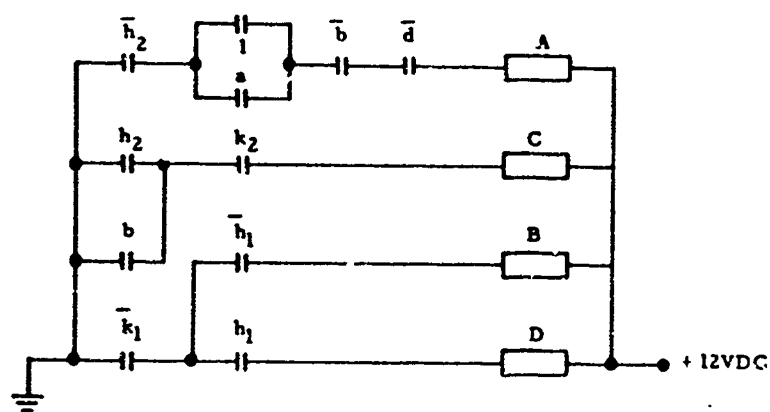
APPENDIX



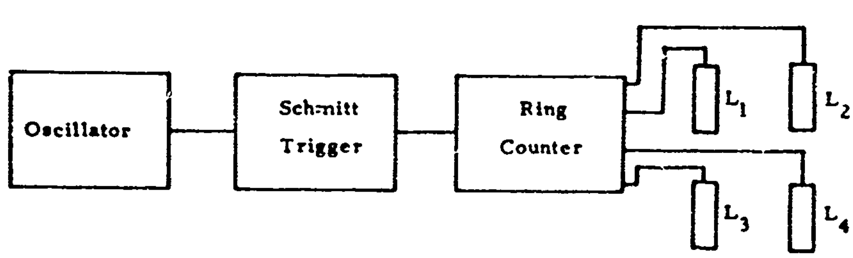
(a) Hip Motor Drive Circuit



(b) Knee Motor Drive Circuit

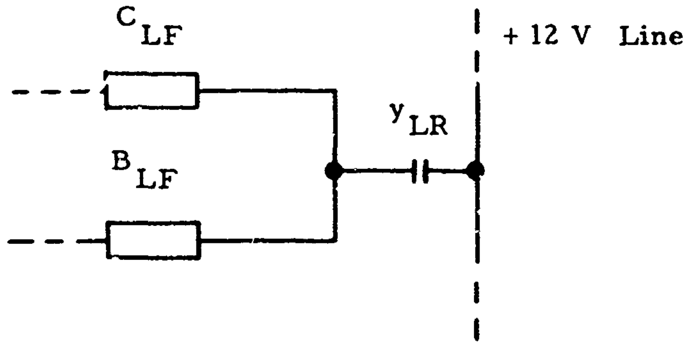


(c) Relay Monostable Leg Logic

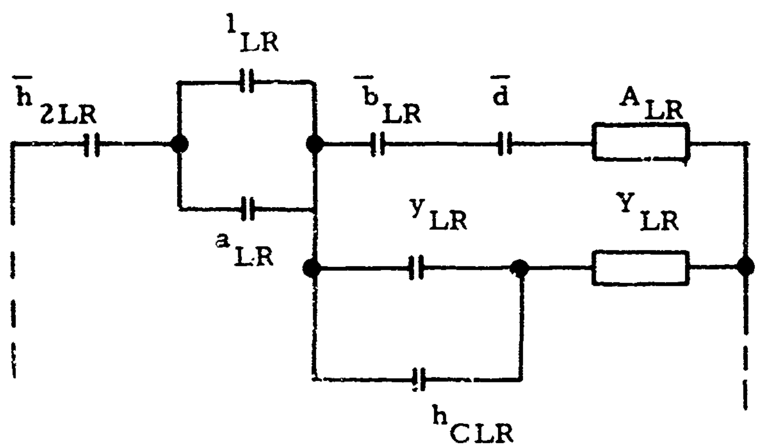


(d) Trigger Logic for Monostable Leg Logic

Figure 1 Trot Circuit

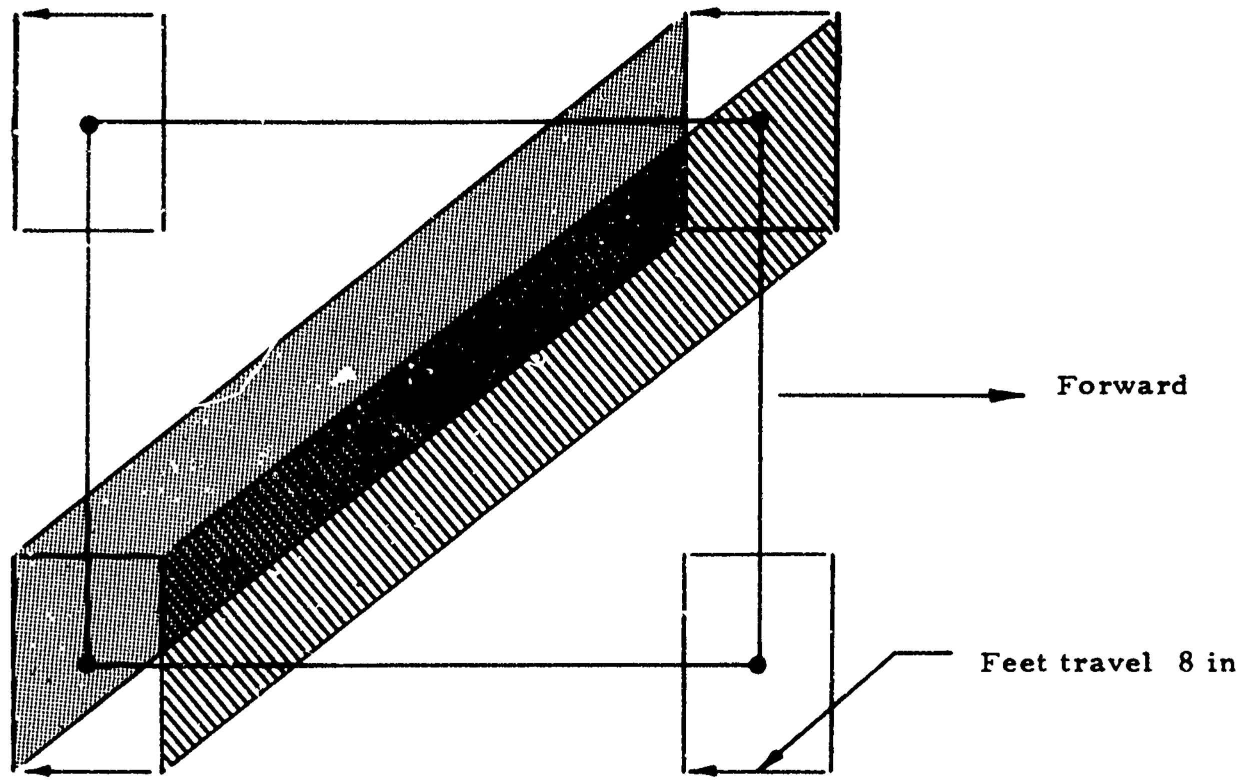


(a) Leg Return Interrupt Circuit

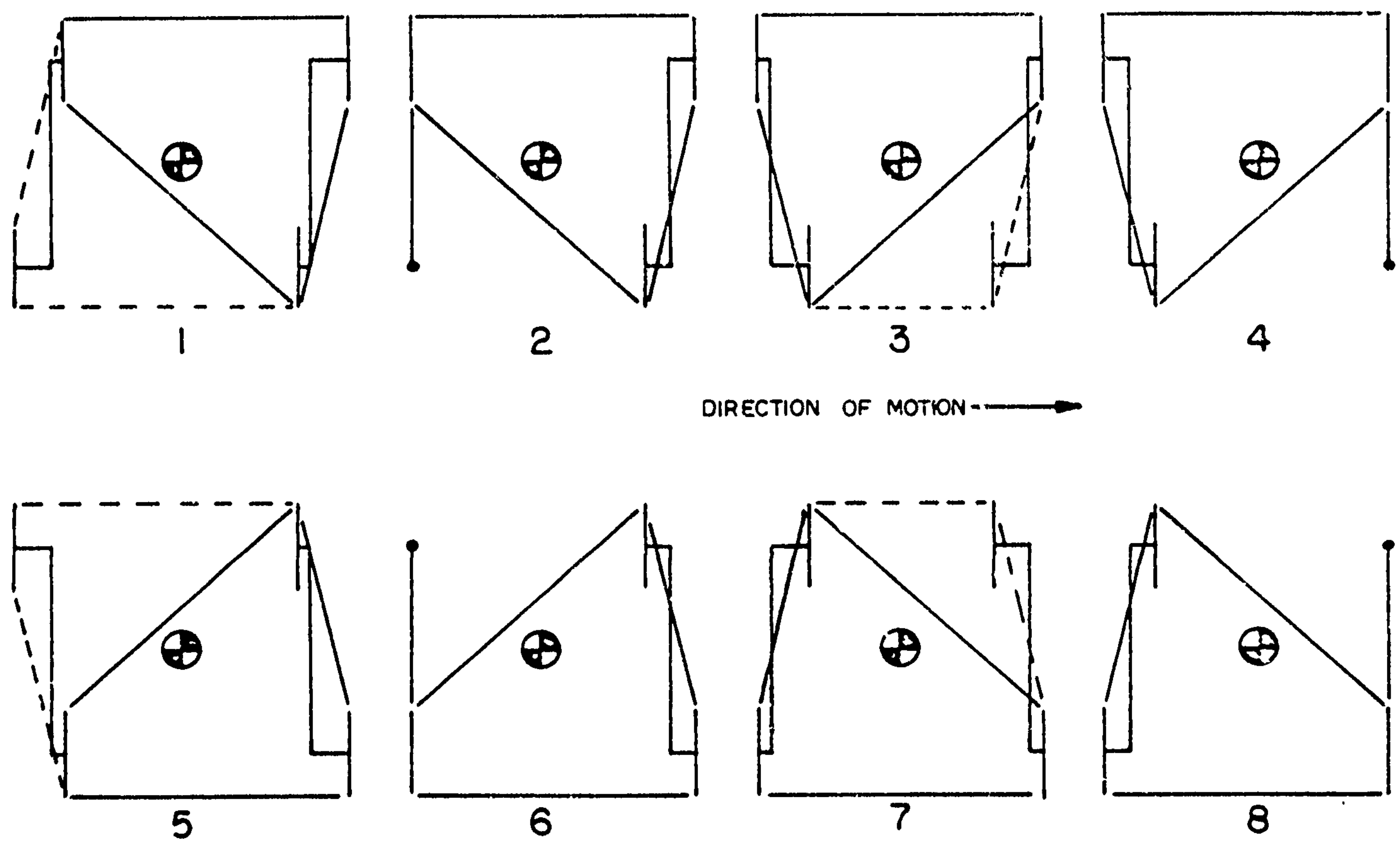


(b) Interrupt and Leg Drive Logic

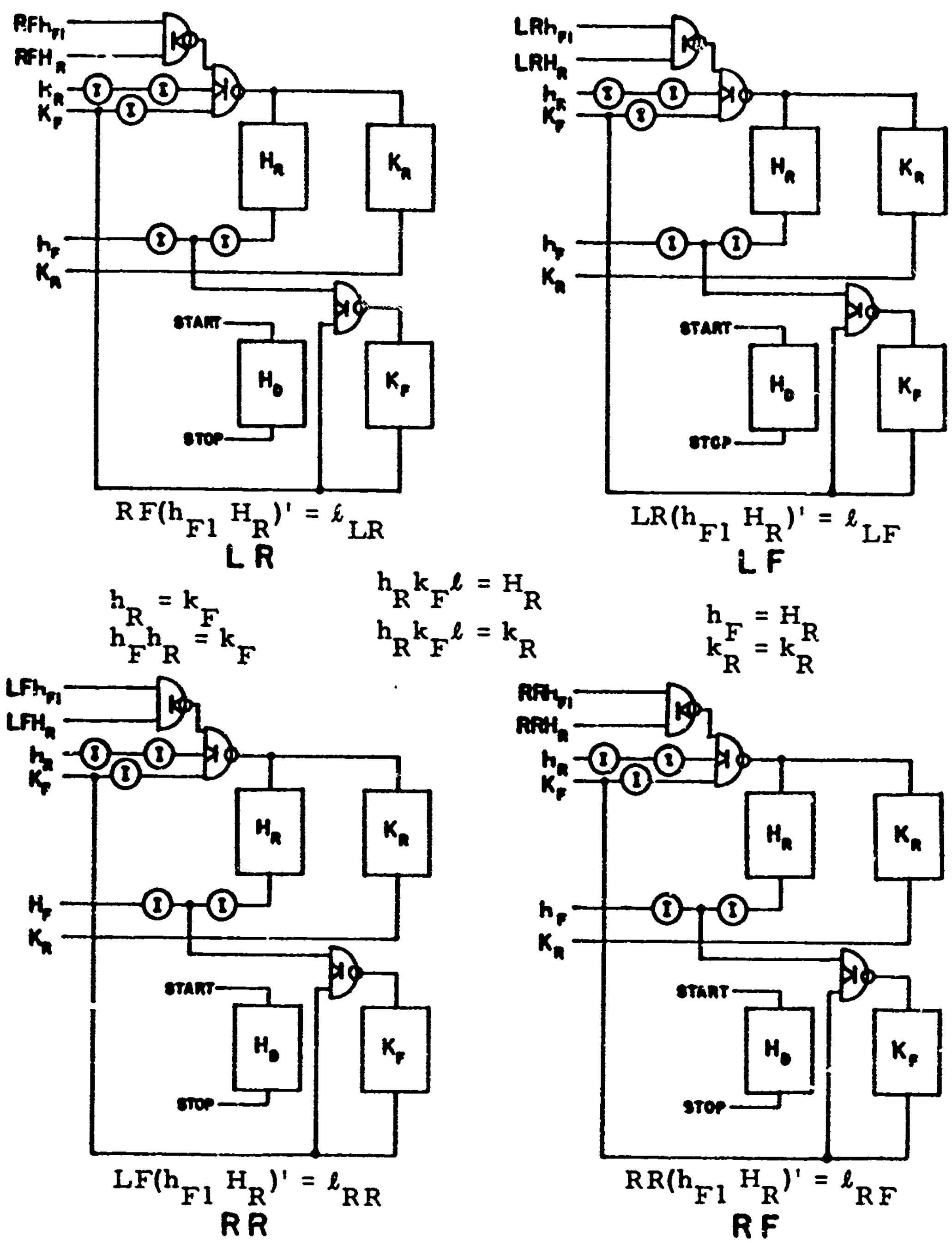
TROT INTERCOUPLING CIRCUIT
Figure 2



Support Pattern - Trot Sequence
Figure 3

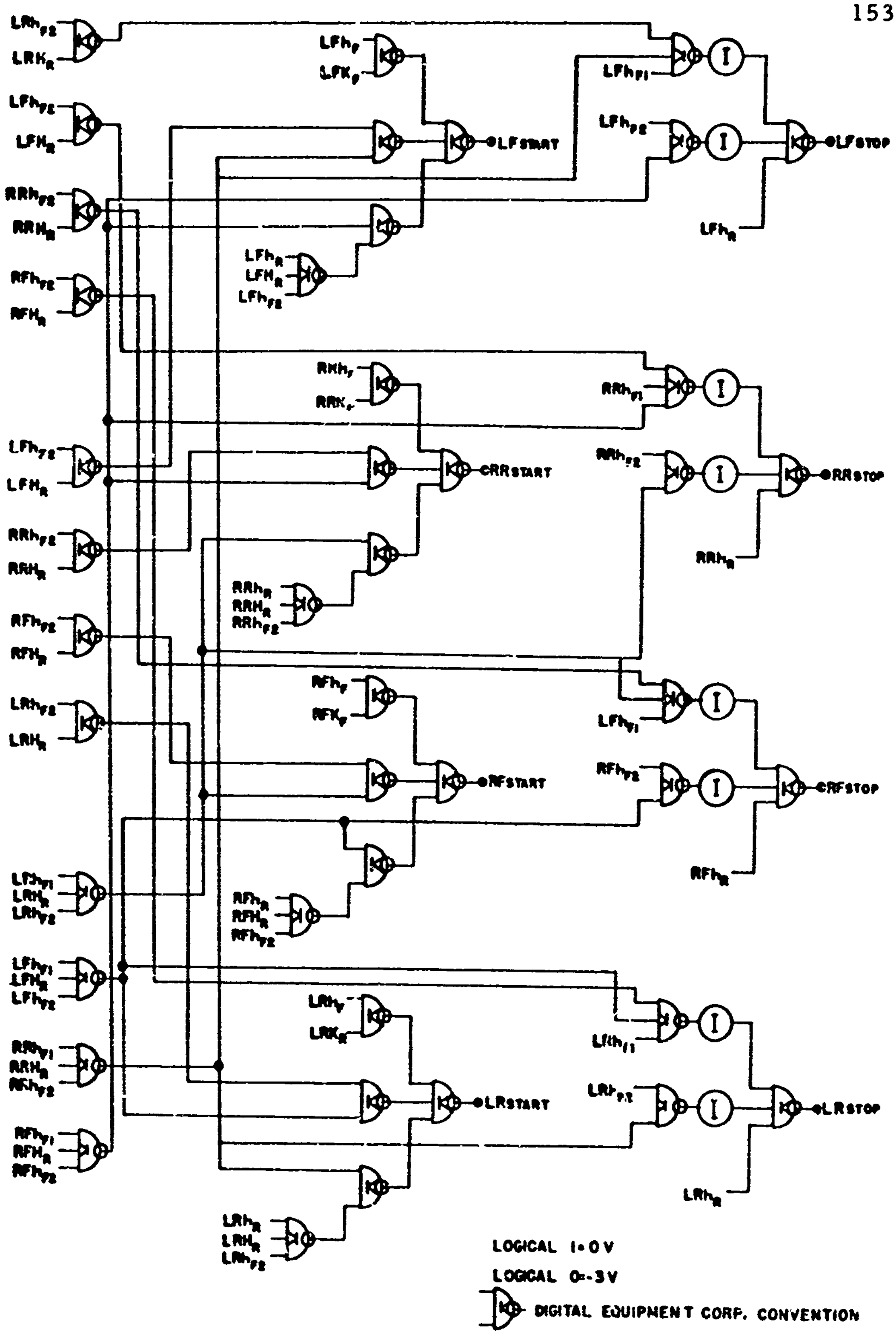


POSITION OF THE CENTER OF GRAVITY
DURING THE 1423 CRAWL OF
THE DESIGNED MACHINE
Figure 4

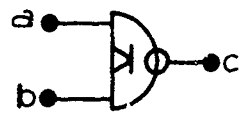


LEG CONTROL CIRCUITS

Figure 5

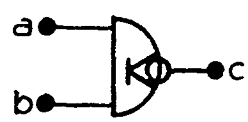


Start - Stop Logic Circuit
Figure 6



volts

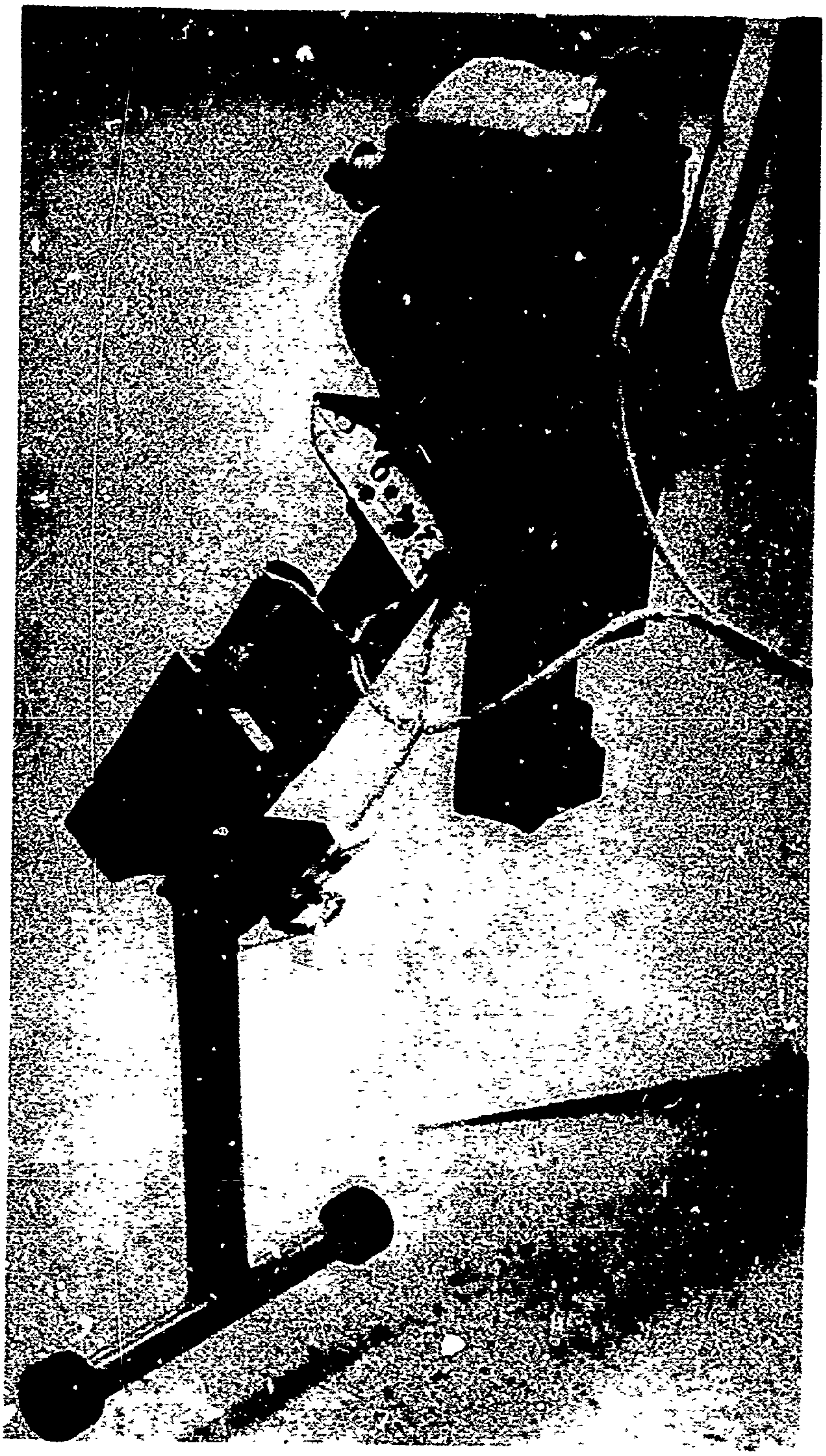
a	b	c
-3	-3	0
-3	0	-3
0	-3	-3
0	0	-3
—	—	-3



volts

a	b	c
0	0	-3
0	-3	0
-3	0	0
-3	-3	0
—	—	0

Voltage truth tables for Digital Equipment Corporation conventions
Figure 7



Photograph of Assembled Leg
Figure 8

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FORTRAN IV    MODEL 44  PS          VERSION 2        DATE  68028                    PAGE  0001

                C HORSE DYN SIM
 0001                 READ(5,100)W,B,CAD,CA,CBD,CB,CL,CLD,HLO,AMASS,ENXX,ENYY,ENZZ,G,DEL
                     1TA1,S,X1E,X2E,X3E,X4E,Y1E,Y2E,Y3E,Y4E,UX,VY,WZ,SXS,SYS,SZS,THET,PH
                     2IS,PSIS,PL,OM,RN,X1B,Y1B,X2B,Y2B,X3B,Y3B,X4B,Y4B,DALF,ALFDO
 0002             100 FORMAT(E12.5)
 0003                 WRITE(6,800)W,B,CAD,CA,CBD,CB,CL,CLD,HLO,AMASS,ENXX,ENYY,ENZZ,G,D
                     1ELTA1,S,X1E,X2E,X3E,X4E,Y1E,Y2E,Y3E,Y4E,UX,VY,WZ,SXS,SYS,SZS,THET
                     2,PHIS,PSIS,PL,OM,RN,X1B,Y1B,X2B,Y2B,X3B,Y3B,X4B,Y4B,DALF,ALFDO
 0004             800 FORMAT(1HO,7E15.5/7E15.5/7E15.5/7E15.5/7E15.5/7E15.5/7E15.5/
                     17E15.5//)
 0005                 TIME=0.0
 0006             50  ALF1C=+0.16515E+00
 0007                 ALF2C=+0.16515E+00
 0008                 ALF3C=+0.16515E+00
 0009                 ALF4C=+0.16515E+00
 0010                 WRITE(6,600)
 0011             600 FORMAT(1H1)
 0012                 DO30 J=1,10
 0013                 DO40 I=1,100
 0014                 X1BF=(X1E-SXS)
 0015                 Y1BF=(Y1E-SYS)
 0016                 X4BF=(X4E-SXS)
 0017                 Y4BF=(Y4E-SYS)
 0018                 X1F=COS(THET)*COS(PSIS)*X1BF+COS(THET)*SIN(PSIS)*Y1BF-SIN(THET)*SZ
                     1S
 0019                 Y1F=(COS(PSIS)*SIN(THET)*SIN(PHIS)-SIN(PSIS)*COS(THET))*X1BF+(COS(
                     1PSIS)*COS(PHIS)+SIN(PSIS)*SIN(THET)*SIN(PHIS))*Y1BF-COS(THET)*SIN(
                     2PHIS)*SZS
 0020                 Z1F=(SIN(PSIS)*SIN(PHIS)+COS(PSIS)*SIN(THET)*COS(PHIS))*X1BF+(SIN(
                     1PSIS)*SIN(THET)*COS(PHIS)-COS(PSIS)*SIN(PHIS))*Y1BF-COS(PHIS)*COS(
                     2THET)*SZS
 0021                 X4F=COS(THET)*COS(PSIS)*X4BF+COS(THET)*SIN(PSIS)*Y4BF-SIN(THET)*SZ
                     1S
 0022                 Y4F=(COS(PSIS)*SIN(THET)*SIN(PHIS)-SIN(PSIS)*COS(THET))*X4BF+(COS(
                     1PSIS)*COS(PHIS)+SIN(PSIS)*SIN(THET)*SIN(PHIS))*Y4BF-COS(THET)*SIN(
                     2PHIS)*SZS
 0023                 Z4F=(SIN(PSIS)*SIN(PHIS)+COS(PSIS)*SIN(THET)*COS(PHIS))*X4BF+(SIN(
                     1PSIS)*SIN(THET)*COS(PHIS)-COS(PSIS)*SIN(PHIS))*Y4BF-COS(PHIS)*COS(
                     2THET)*SZS
 0024                 HL1=SQRT((X1F-X1B)**2+(Y1F-Y1B)**2+Z1F**2)
 0025                 HL4=SQRT((X4F-X4B)**2+(Y4F-Y4B)**2+Z4F**2)
 0026                 ALF1=ATAN((X1F-X1B)/Z1F)
 0027                 ALF4=ATAN((X4F-X4B)/Z4F)
 0028                 BET1=ATAN(-(Y1F-Y1B)/SQRT((X1F-X1B)**2+Z1F**2))
 0029                 BET4=ATAN(-(Y4F-Y4B)/SQRT((X4F-X4B)**2+Z4F**2))
 0030                 X1BD=(UX-RN*Y1B)
 0031                 X4BD=(UX-RN*Y4B)
 0032                 Y1BD=(VY+RN*X1B)
 0033                 Y4BD=(VY+RN*X4B)
```

Figure 9

```
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 0034          Z1BD=(WZ-OM*X1B+PL*Y1B)
 0035          Z4BD=(WZ-OM*X4B+PL*Y4B)
 0036          VTX1=COS(ALF1)*X1BD-SIN(ALF1)*Z1BD
 0037          VTY1=SIN(ALF1)*SIN(BET1)*X1BD+COS(BET1)*Y1BD+COS(ALF1)*SIN(BET1)*Z
              11BD
 0038          VLR1=SIN(ALF1)*COS(BET1)*X1BD-SIN(BET1)*Y1BD+COS(ALF1)*COS(BET1)*Z
              11BD
 0039          HL1D=-VLR1
 0040          VTX4=COS(ALF4)*X4BD-SIN(ALF4)*Z4BD
 0041          VTY4=SIN(ALF4)*SIN(BET4)*X4BD+COS(BET4)*Y4BD+COS(ALF4)*SIN(BET4)*Z
              14BD
 0042          VLR4=SIN(ALF4)*COS(BET4)*X4BD-SIN(BET4)*Y4BD+COS(ALF4)*COS(BET4)*Z
              14BD
 0043          HL4D=-VLR4
 0044          ALF1D=-VTX1/HL1*COS(BET1)
 0045          BET1D=VTY1/HL1
 0046          ALF4D=-VTX4/HL4*COS(BET4)
 0047          BET4D=VTY4/HL4
 0048          FL1=CL*(HL1-HLO)+CLD*(HL1D)
 0049          FL4=CL*(HL4-HLO)+CLD*(HL4D)
 0050          AM1B= CB*BET1+CBD*BET1D
 0051          AM4B= CB*BET4+CBD*BET4D
 0052          ALF1C=ALF1C+DALF
 0053          ALF4C=ALF4C+DALF
 0054          AM1A=CA*(ALF1C-ALF1)-CAD*(ALFDO-ALF1D)
 0055          AM4A=CA*(ALF4C-ALF4)-CAD*(ALFDO-ALF4D)
 0056          FA1=-AM1A/HL1*COS(BET1)
 0057          FA4=-AM4A/HL4*COS(BET4)
 0058          FB1=-AM1B/HL1
 0059          FB4=-AM4B/HL4
 0060          FX1=COS(ALF1)*FA1+SIN(ALF1)*SIN(BET1)*FB1+SIN(ALF1)*COS(BET1)*FL1
 0061          FY1=COS(BET1)*FB1-SIN(BET1)*FL1
 0062          FZ1=-SIN(ALF1)*FA1+COS(ALF1)*SIN(BET1)*FB1+COS(ALF1)*COS(BET1)*FL1
 0063          FX4=COS(ALF4)*FA4+SIN(ALF4)*SIN(BET4)*FB4+SIN(ALF4)*COS(BET4)*FL4
 0064          FY4=COS(BET4)*FB4-SIN(BET4)*FL4
 0065          FZ4=-SIN(ALF4)*FA4+COS(ALF4)*SIN(BET4)*FB4+COS(ALF4)*COS(BET4)*FL4
 0066          PX=FX1+FX4
 0067          PY=FY1+FY4
 0068          PZ=FZ1+FZ4
 0069          AX=0
 0070          AY=0
 0071          AZ=0
 0072          TX=AM1B*COS(ALF1)+AM4B*COS(ALF4)
 0073          TY=AM1A+AM4A
 0074          TZ=-AM1B*SIN(ALF1)-AM4B*SIN(ALF4)
 0075          AL=FZ1*Y1B+FZ4*Y4B
 0076          AM=FY1*X1B+FY4*X4B
 0077          AN=-FX1*Y1B+FY1*X1B-FX4*Y4B+FY4*X4B
 0078          SI = SIN(THET)
```

Figure 9 continued

```
FORTRAN IV     MODEL 44  PS           VERSION 2         DATE  68028                 PAGE 0003

            C1 = COS(THET)
            S2 = SIN(PHIS)
            C2 = COS(PHIS)
            S3 = SIN(PSIS)
            C3 = COS(PSIS)
            USD=VY*RN-WZ*QM + (PX+AX)/AMASS-G*S1
            VSD=WZ*PL-UX*RN + (PY+AY)/AMASS+G*C1*S2
            WSD=UX*QM-VY*PL + (PZ+AZ)/AMASS+G*C1*C2
            PSD=((ENYY-ENZZ)*QM*RN + TX+AL)/ENXX
            QSD=((ENZZ-ENXX)*RN*PL +TY+AM)/ENYY
            RSD=((ENXX-ENYY)*PL*QM +TZ+AN)/ENZZ
            THETD = QM*C2-RN*S2
            PSISD = (QM*S2+RN*C2)/C1
            PHISD = PL +PSISD*S1
            SXSD = UX*(C1*C3)+VY*(-C2*S3+S2*S1*C3)+WZ*(S2*S3+C2*S1*C3)
            SYSD = UX*(C1*S3)+VY*(C2*C3+S2*S1*S3)+WZ*(-S2*C3+C2*S1*S3)
            SZSD =-UX*S1+VY*(C1*S2)+WZ*(C1*C2)
            UX=UX+USD*DELTA1
            VY=VY+VSD*DELTA1
            WZ=WZ+WSD*DELTA1
            PL=PL+PSD*DELTA1
            QM=QM+QSD*DELTA1
            RN=RN+RSD*DELTA1
            X1BE=SXS+X1B*(C1*C3)+Y1B*(-C2*S3+S2*S1*C3)
            X2BE=SXS+X2B*(C1*C3)+Y2B*(-C2*S3+S2*S1*C3)
            X3BE=SXS+X3B*(C1*C3)+Y3B*(-C2*S3+S2*S1*C3)
            X4BE=SXS+X4B*(C1*C3)+Y4B*(-C2*S3+S2*S1*C3)
            Y1BE=SYS+X1B*(C1*S3)+Y1B*(C2*C3*S2*S1*S3)
            Y2BE=SYS+X2B*(C1*S3)+Y2B*(C2*C3*S2*S1*S3)
            Y3BE=SYS+X3B*(C1*S3)+Y3B*(C2*C3*S2*S1*S3)
            Y4BE=SYS+X4B*(C1*S3)+Y4B*(C2*C3*S2*S1*S3)
            Z1BE=SZS-X1B*S1+Y1B*(C1*S2)
            Z2BE=SZS-X2B*S1+Y2B*(C1*S2)
            Z3BE=SZS-X3B*S1+Y3B*(C1*S2)
            Z4BE=SZS-X4B*S1+Y4B*(C1*S2)
            THET=THET+THETD*DELTA1
            PHIS=PHIS+PHISD*DELTA1
            PSIS=PSIS+PSISD*DELTA1
            SXS=SXS+SXSD*DELTA1
            SYS=SYS+SYSD*DELTA1
            SZS=SZS+SZSD*DELTA1
         40 CONTINUE
            TIME=TIME+100.0*DELTA1
            WRITE(6,500) TIME,UX,VY,WZ,SXS,SYS,SZS,THET,PHIS,PSIS,PL,QM,RN,HL1
           1,HL4,ALF1,ALF4,BET1,BET4,FL1,FL4,AM1B,AM4B,AM1A,AM4A,
           2X1BE,Y1BE,Z1BE,X2BE,Y2BE,Z2BE,X3BE,Y3BE,Z3BE,X4BE,Y4BE,Z4BE
        500 FORMAT(1H0,F5.2,6E15.5/E21.5,5E15.5/E21.5,5E15.5/E21.5,5E15.5/
           1E21.5,5E15.5/E21.5,5E15.5//)
         30 CONTINUE
```

Figure 9 continued

```
FORTRAN IV     MODEL 44  PS            VERSION 2          DATE  68079                 PAGE  0004

0125              WRITE(6,700)
0126          700 FORMAT(1H\)
0127              DO10 L=1,10
0128              DO20 K=1,100
0129              X2BF=(X2B-SXS)
0130              Y2BF=(Y2B-SYS)
0131              X3BF=(X3B-SXS)
0132              Y3BF=(Y3B-SYS)
0133              X2F=COS(THET)*COS(PSIS)*X2BF+COS(THET)*SIN(PSIS)*Y2BF-SIN(THET)*SZ
                 1S
0134              Y2F=(COS(PSIS)*SIN(THET)*SIN(PHIS)-SIN(PSIS)*COS(PHIS))*X2BF+(COS(
                 1PSIS)*COS(PHIS)+SIN(PSIS)*SIN(THET)*SIN(PHIS))*Y2BF+COS(THET)*SIN(
                 2PHIS)*SZS
0135              Z2F=(SIN(PSIS)*SIN(PHIS)+COS(PSIS)*SIN(THET)*COS(PHIS))*X2BF+(SIN(
                 1PSIS)*SIN(THET)*COS(PHIS)-COS(PSIS)*SIN(PHIS))*Y2BF-COS(PHIS)*COS(
                 2THET)*SZS
0136              X3F=COS(THET)*COS(PSIS)*X3BF+COS(THET)*SIN(PSIS)*Y3BF-SIN(THET)*SZ
                 1S
0137              Y3F=(COS(PSIS)*SIN(THET)*SIN(PHIS)-SIN(PSIS)*COS(THET))*X3BF+(COS(
                 1PSIS)*COS(PHIS)+SIN(PSIS)*SIN(THET)*SIN(PHIS))*Y3BF-COS(THET)*SIN(
                 2PHIS)*SZS
0138              Z3F=(SIN(PSIS)*SIN(PHIS)+COS(PSIS)*SIN(THET)*COS(PHIS))*X3BF+(SIN(
                 1PSIS)*SIN(THET)*COS(PHIS)-COS(PSIS)*SIN(PHIS))*Y3BF-COS(PHIS)*COS(
                 2THET)*SZS
0139              HL2=SQRT((X2F-X2B)**2+(Y2F-Y2B)**2+Z2F**2)
0140              HL3=SQRT((X3F-X3B)**2+(Y3F-Y3B)**2+Z3F**2)
0141              ALF2=ATAN((X2F-X2B)/Z2F)
0142              ALF3=ATAN((X3F-X3B)/Z3F)
0143              BET2=ATAN(-(Y2F-Y2B)/SQRT((X2F-X2B)**2+Z2F**2))
0144              BET3=ATAN(-(Y3F-Y3B)/SQRT((X3F-X3B)**2+Z3F**2))
0145              X2BD=(UX-RN*Y2B)
0146              X3BD=(UX-RN*Y3B)
0147              Y2BD=(VY+RN*X2B)
0148              Y3BD=(VY+RN*X3B)
0149              Z2BD=(WZ-OM*X2B+PL*Y2B)
0150              Z3BD=(WZ-OM*X3B+PL*Y3B)
0151              VTX2=COS(ALF2)*X2BD-SIN(ALF2)*Z2BD
0152              VTX3=COS(ALF3)*X3BD-SIN(ALF3)*Z3BD
0153              VTY2=SIN(ALF2)*SIN(BET2)*X2BD+COS(BET2)*Y2BD+COS(ALF2)*SIN(BET2)*Z
                 12BD
0154              VTY3=SIN(ALF3)*SIN(BET3)*X3BD+COS(BET3)*Y3BD+COS(ALF3)*SIN(BET3)*Z
                 13BD
0155              VLR2=SIN(ALF2)*COS(BET2)*X2BD-SIN(BET2)*Y2BD+COS(ALF2)*COS(BET2)*Z
                 12BD
0156              VLR3=SIN(ALF3)*COS(BET3)*X3BD-SIN(BET3)*Y3BD+COS(ALF3)*COS(BET3)*Z
                 13BD
0157              HL2D=-VLR2
0158              HL3D=-VLR3
0159              ALF2D=-VTX2/HL2*COS(BET2)
```

Figure 9 continued

```
FORTRAN IV     MODEL 44  PS          VERSION 2          DATE  68028                    PAGE  0005

               ALF3D=-VTX3/HL3*COS(BET3)
               BET2D=VTY2/HL2
               BET3D=VTY3/HL3
               FL2=CL*(HL2-HLO)+CLD*(HL2D)
               FL3=CL*(HL3-HLO)+CLD*(HL3D)
               AM2B= CB*BET2+CBD*BET2D
               AM3B= CB*BET3+CBD*BET3D
               ALF2C=ALF2C+DALF
               ALF3C=ALF3C+DALF
               AM2A=CA*(ALF2C-ALF2)-CAD*(ALFDO-ALF2D)
               AM3A=CA*(ALF3C-ALF3)-CAD*(ALFDO-ALF3D)
               FA2=-AM2A/HL2*COS(BET2)
               FA3=-AM3A/HL3*COS(BET3)
               FB2=-AM2B/HL2
               FB3=-AM3B/HL3
               FX2=COS(ALF2)*FA2+SIN(ALF2)*SIN(BET2)*FB2+SIN(ALF2)*COS(BET2)*FL2
               FX3=COS(ALF3)*FA3+SIN(ALF3)*SIN(BET3)*FB3+SIN(ALF3)*COS(BET3)*FL3
               FY2=COS(BET2)*FB2-SIN(BET2)*FL2
               FY3=COS(BET3)*FB3-SIN(BET3)*FL3
               FZ2=-SIN(ALF2)*FA2+COS(ALF2)*SIN(BET2)*FB2+COS(ALF2)*COS(BET2)*FL2
               FZ3=-SIN(ALF3)*FA3+COS(ALF3)*SIN(BET3)*FB3+COS(ALF3)*COS(BET3)*FL3
               PX=FX2+FX3
               PY=FY2+FY3
               PZ=FZ2+FZ3
               AX=0
               AY=0
               AZ=0
               TX=AM2B*COS(ALF2)+AM3B*COS(ALF3)
               TY=AM2A+AM3A
               TZ=-AM2B*SIN(ALF2)-AM3B*SIN(ALF3)
               AL=FZ2*Y2B+FZ3*Y3B
               AM=FY2*X2B+FY3*X3B
               AN=-FX2*Y2B+FY2*X2B-FX3*Y3B+FY3*X3B
               C2 = COS(PHIS)
               S2 = SIN(PHIS)
               C1 = COS(THET)
               S1 = SIN(THET)
               S3 = SIN(PSIS)
               C3 = COS(PSIS)
               USD=VY*RN-WZ*QM + (PX+AX)/AMASS-G*S1
               VSD=WZ*PL-UX*RN + (PY+AY)/AMASS+G*C1*S2
               WSD=UX*QM-VY*PL + (PZ+AZ)/AMASS+G*C1*C2
               PSD=((ENYY-ENZZ)*QM*RN + TX+AL)/ENXX
               QSD=((ENZZ-ENXX)*RN*PL +TY+AM)/ENYY
               RSD=((ENXX-ENYY)*PL*QM +TZ+AN)/ENZZ
               THETD = QM*C2-RN*S2
               PSISD = (QM*S2+RN*C2)/C1
               PHISD = PL +PSISD*S1
               SXSD = UX*(C1*C3)+VY*(-C2*S3+S2*S1*C3)+WZ*(S2*S3+C2*S1*C3)
```

Figure 9 continued

```
FORTRAN IV    MODEL 44  PS          VERSION 2          DATE  68028                    PAGE  0006

0209            SYSD = UX*(C1*S3)+VY*(C2*C3+S2*S1*S3)+WZ*(-S2*C3+C2*S1*S3)
0210            SZSD =-UX*S1+VY*(C1*S2)+WZ*(C1*C2)
0211            UX=UX+USD*DELTA1
0212            VY=VY+VSD*DELTA1
0213            WZ=WZ+WSD*DELTA1
0214            PL=PL+PSD*DELTA1
0215            QM=QM+QSD*DELTA1
0216            RN=RN+RSD*DELTA1
0217            X1BE=SXS+X1B*(C1*C3)+Y1B*(-C2*S3+S2*S1*C3)
0218            X2BE=SXS+X2B*(C1*C3)+Y2B*(-C2*S3+S2*S1*C3)
0219            X3BE=SXS+X3B*(C1*C3)+Y3B*(-C2*S3+S2*S1*C3)
0220            X4BE=SXS+X4B*(C1*C3)+Y4B*(-C2*S3+S2*S1*C3)
0221            Y1BE=SYS+X1B*(C1*S3)+Y1B*(C2*C3*S2*S1*S3)
0222            Y2BE=SYS+X2B*(C1*S3)+Y2B*(C2*C3*S2*S1*S3)
0223            Y3BE=SYS+X3B*(C1*S3)+Y3B*(C2*C3*S2*S1*S3)
0224            Y4BE=SYS+X4B*(C1*S3)+Y4B*(C2*C3*S2*S1*S3)
0225            Z1BE=SZS-X1B*S1+Y1B*(C1*S2)
0226            Z2BE=SZS-X2B*S1+Y2B*(C1*S2)
0227            Z3BE=SZS-X3B*S1+Y3B*(C1*S2)
0228            Z4BE=SZS-X4B*S1+Y4B*(C1*S2)
0229            THET=THET+THETD*DELTA1
0230            PHIS=PHIS+PHISD*DELTA1
0231            PSIS=PSIS+PSISD*DELTA1
0232            SXS=SXS+SXSD*DELTA1
0233            SYS=SYS+SYSD*DELTA1
0234            SZS=SZS+SZSD*DELTA1
0235      20    CONTINUE
0236            TIME=TIME+100.0*DELTA1
0237            WRITE(6,400) TIME,UX,VY,WZ,SXS,SYS,SZS,THET,PHIS,PSIS,PL,QM,RN,HL2
               1,HL3,ALF2,ALF3,BET2,BET3,FL2,FL3,AM2B,AM3B,AM2A,AM3A,
               2X1BE,Y1BE,Z1BE,X2BE,Y2BE,Z2BE,X3BE,Y3BE,Z3BE,X4BE,Y4BE,Z4BE
0238     400    FORMAT(1H0,F5.2,6E15.5/E21.5,5E15.5/E21.5,5E15.5/E21.5,5E15.5/
               1E21.5,5E15.5/E21.5,5E15.5//)
0239      10    CONTINUE
0240            X1E=X1E+S
0241            X2E=X2E+S
0242            X3E=X3E+S
0243            X4E=X4E+S
0244            Y1E=-W/2.0
0245            Y2E=W/2.0
0246            Y3E=-W/2.0
0247            Y4E=W/2.0
0248            IF(SXS.LT.25.0)GO TO 50
0249            END
```

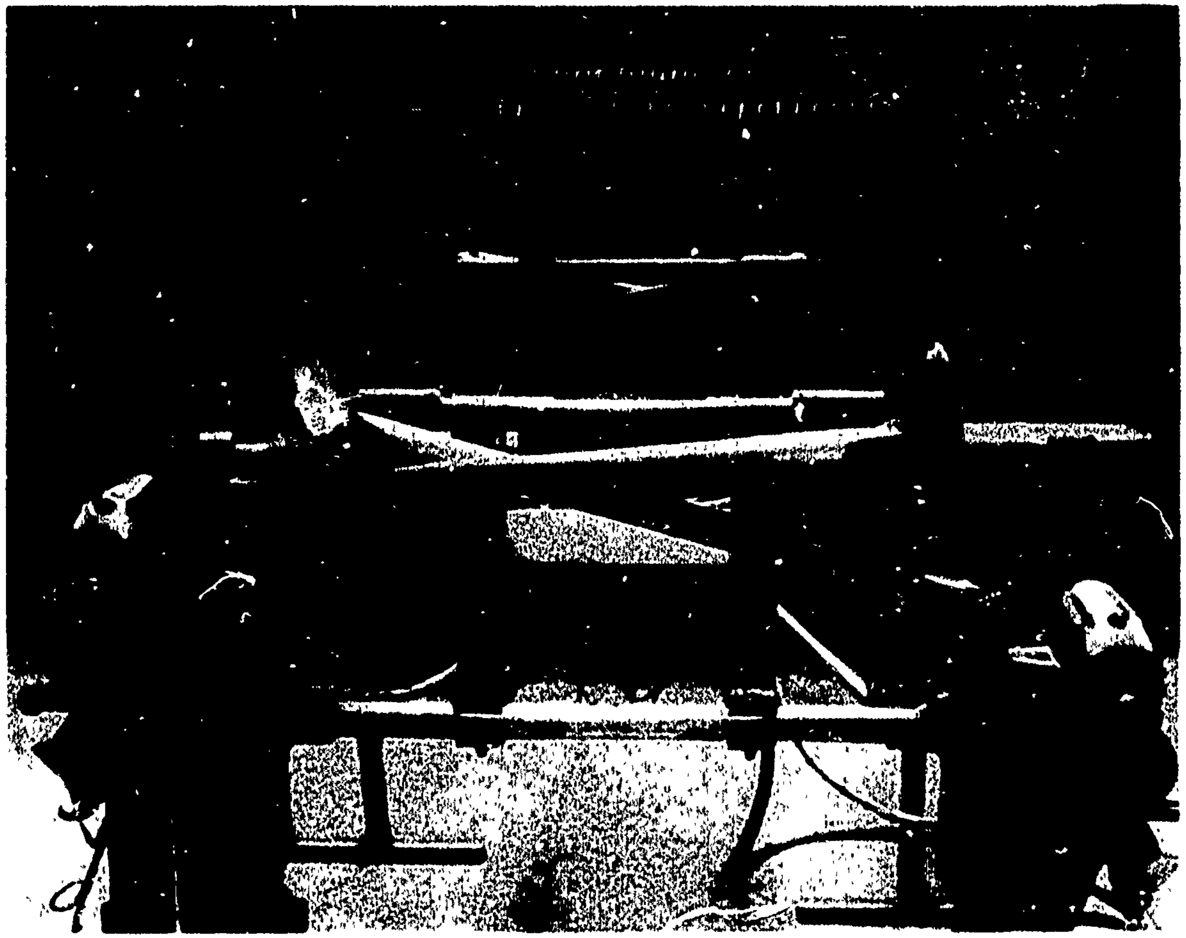
Figure 9 continued

```
68/028                                    FRANK     LINKAGE EDIT

LINKAGE EDITOR HIGHEST SEVERITY WAS  0
// RESET
//           EXEC

  0.20000E 01   0.50000E 01  -0.66000E 05   0.36000E 05   0.70000E 05   0.15000E 06   0.50000E 05
  0.15000E 04   0.30479E 01   0.20000E 02   0.12000E 02   0.66000E 04   0.30000E 04   0.32200E 02
  0.10000E-02   0.20000E 01   0.55000E 01   0.65000E 01   0.15000E 01   0.50000E 00  -0.10000E 01
  0.10000E 01  -0.10000E 01   0.10000E 01   0.10000E 01   0.0          -0.16667E 00   0.25000E 01
  0.0          -0.30000E 01   0.0           0.0           0.0           0.0           0.0
  0.0           0.25000E 01  -0.10000E 01   0.25000E 01   0.10000E 01  -0.25000E 01  -0.10000E 01
 -0.25000E 01   0.10000E 01  -0.33150E-03  -0.33150E 00
```

Figure 9 continued



Leg-Pelvis Photograph

Figure 10

Machine Detail Photograph

Figure 11

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13 ABSTRACT

This report is concerned with the limb coordination control problem for animals and legged vehicles. After a general survey of previous work in the area, a discussion of the mechanical, control and computer aspects of the problem is presented. This, is followed by a development of several theoretical models for legged locomotion systems and a subsequent determination of the properties of these models. In particular, a necessary and sufficient condition for the stability of low speed locomotion is established. It is shown that only three of the 5040 theoretically possible gaits for quadrupeds satisfy this condition. The theoretical results on stability are used as a basis to synthesising a simple finite state control algorithm for two different gaits. The ability of these algorithms to produce stable locomotion is verified by the construction and successful testing of a small artificial quadruped with eight independently powered joints coordinated by a small special purpose digital computer.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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Off-the-Road Locomotion						
Quadruped						
Gaits						
Legged Locomotion Stability						
Legged Locomotion Dynamics						