3,13,27

MDC-TR-67-148

AIR FORCE MISSILE DEVELOPMENT CENTER

TECHNICAL REPORT

GYROSCOPE

STANDARD GYROCOMPASSING ACCURACY TEST

Distribution of this document is unlimited. Qualified users may obtain copies from the Defense Documentation Center.

PREPARED BY

HAYNE A. THOMPSON, JR., CAPTAIN, USAF

and

JUDITH KOESTLER, Physicist

" CENTRAL INERTIAL GUIDANCE TEST FACILITY "

AF MISSILE DEVELOPMENT CENTER

HOLLOMAN AIR FORCE BASE

NEW MEXICO

DECEMBER 1967

41

GYROSCOPE

STANDARD GYROCOMPASSING ACCURACY TEST

PREPARED BY

HAYNE A. THOMPSON, JR., CAPTAIN, USAF

and

JUDITH KOESTLER, Physicist

DIRECTORATE OF GUIDANCE TEST

AIR FORCE MISSILE DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HOLLOMAN AIR FORCE BASE, NEW MEXICO

DECEMBER 1967

FOREWORD

The authors wish to express their thanks to Mr. C. A. Bodwell for his assistance in the preparation of the data analysis section of this report.

This technical report has been reviewed and is approved.

Robert B. Savage

ROBERT B. SAVAGE, Colonel, USAF
Director, CIGTF

ABSTRACT

The Central Inertial Guidance Test Facility (CIGTF) Standard Gyrocompassing Accuracy Test and associated data analysis procedures for single-degree-of-freedom gyros are presented.

The gyrocompassing error, the error in the north direction as determined by the gyroscope, is obtained by orienting the gyro in four specific orientations and measuring the output of the gyro while in the torque-to-balance mode. This procedure is repeated 24 times for a total of 25 north determinations during which the gyro is not shut down. From these data a determination is made of the average gyrocompassing error and of the operating instability of the gyrocompassing error. This "operating" phase is followed by a "shutdown" phase consisting of 10 north determinations, each separated by a two-hour gyro shutdown and two-hour warmup. From these data the shutdown instability of the gyrocompassing error is determined. Although the test and data analysis procedures described in this report are written specifically for single-degree-of-freedom gyros, the theory can easily be applied to two-degree-of-freedom gyros.

This report also contains a description of the CIGTF gyroscope laboratory's two axis automated gyrocompassing test station and a test error analysis.

TABLE OF CONTENTS

Table of Contents (Continued)

LIST OF FIGURES

DEFINITIONS AND SYMBOLS

a	acceleration component (2.3)
a	misalignment angle of the vertical fixture axis coordinate frame with respect to the earth fixed coordinate frame measured about X_E (Appendix II)
b	misalignment angle of the vertical fixture axis coordinate frame with respect to the earth fixed coordinate frame measured about Y_E (Appendix II)
c	misalignment angle of the vertical fixture axis coordinate frame with respect to the earth fixed coordinate frame measured about Z_E (Appendix II)
D	drift coefficient (2.3)
d	misalignment angle of the horizontal fixture axis coordinate frame with respect to the vertical fixture axis coordinate frame measured about X_V (Appendix II)
°	degree (2.3)
E	(subscript) earth fixed coordinate frame (Appendix II)
e	misalignment angle of the horizontal fixture axis coordinate frame with respect to the vertical fixture axis coordinate frame measured about Y_V (Appendix II)
F	(subscript) insensitive to acceleration (2.3)
f	misalignment angle of the horizontal fixture axis coordinate frame with respect to the vertical fixture axis coordinate frame measured about Z_V (Appendix II)
G	(subscript) gyro axes coordinate frame (Appendix II)
g	local acceleration of gravity, defined positive upward (2.3)
GSF	gyrocompassing scale factor (4.1)
H	(subscript) horizontal fixture axis coordinate frame (Appendix II)
h	misalignment angle of the gyro axes with respect to the horizontal fixture axis coordinate frame measured about X_H (Appendix II)

Definitions and Symbols (Continued)

hr	sidereal hour (2.3)
I	(subscript) input axis (Appendix I)
I	input axis (2.1)
$\bar{I}$	electronically averaged value of the torquer current over a ten-second period (4.1)
$\bar{\bar{I}}$	average of 10 ten-second electronically averaged values of the torquer current (2.3)
i	(subscript) running index for runs (5.2)
i	current flow through the torquer (Appendix I)
i	misalignment angle of the gyro axes with respect to the horizontal fixture axis coordinate frame measured about Y_H (Appendix II)
j	(subscript) running index for phases (5.2)
j	misalignment angle of the gyro axes with respect to the horizontal fixture axis coordinate frame measured about Z_H (Appendix II)
k	(subscript) running index for orientations (5.1)
l	(subscript) running index for the 10 ten-second torque-to-balance current samples (5.1)
ma	milliampere (2.3)
N	astronomic north, defined by the intersection of the meridian plane with the horizontal plane, where the meridian plane is defined by the local gravity vector and a line parallel to the rotation axis of the earth and the horizontal plane is defined as a plane perpendicular to the local gravity vector (2.2)
N'	the north direction as determined by the gyro (2.2)
O	(subscript) output axis (2.3)
O	output axis (2.1)

Definitions and Symbols (Continued)

Q	transformation matrix (Appendix II)
S	(subscript) spin axis (Appendix I)
S	spin axis (2.1)
S_T	torquer scale factor (2.3)
s	best estimate of the run standard error for a phase (5.3)
V	(subscript) vertical fixture axis coordinate frame (Appendix II)
X	axis of coordinate frame (Appendix II)
Y	axis of coordinate frame (Appendix II)
Z	axis of coordinate frame (Appendix II)
β	error in 180° rotation from position 2 to position 3 about fixture horizontal axis (Appendix II)
Δ	shutdown instability of gyrocompassing error (5.3)
δ	operating instability of gyrocompassing error (5.3)
ε	error in 180° rotation from position 1 to position 2 about fixture vertical axis (Appendix II)
η	error in 180° rotation from position 3 to position 4 about fixture vertical axis (Appendix II)
θ	the gyrocompassing error: the small angle measured from astronomic north to the north direction as determined by the gyroscope, positive as defined by a rotation about an axis directed vertically up (2.2)
λ	local astronomic latitude angle (Appendix II)
$\sum$	summation (5.1)
σ	standard error (5.2)
ϕ	the acute angle measured from a north-south line to the spin axis, positive as defined by a rotation about an axis directed vertically up (2.2)

Definitions and Symbols (Continued)

ϕ'	the angle from north as determined by the gyro to the spin axis where $\phi' = \phi - \theta$ (2.2)
ω	rotation rate (Appendix II)
ω_E	earth rotation rate (Appendix II)
ω_{EH}	horizontal component of earth rotation rate (2.3)

1. INTRODUCTION

One of the functions of a gyroscope is to provide an accurate north reference for aircraft and missile guidance systems. It follows that, in order to provide a complete evaluation of the gyroscope, a method for determining the gyrocompassing error, the bias error in the gyroscope's determination of the north direction, is necessary. This report describes the method used by the Central Inertial Guidance Test Facility (CIGTF) for evaluating the gyrocompassing error, its operating instability, and its shutdown instability.

Although the test and data analysis procedures described in this report are written specifically for single-degree-of-freedom gyros, the theory can easily be applied to two-degree-of-freedom gyros. Also, this method can be used to determine the effect of parameter variation (e.g., gyro rotor voltage variation) and the effect of gyro input axis azimuth variation on the gyrocompassing accuracy of a gyroscope.

2. THEORY

2.1 Ideal Case

Determining the gyrocompassing accuracy of a single-degree-of-freedom gyroscope involves orienting the gyroscope so that the input axis (I) is constrained to the local horizontal plane and the output axis (O) is parallel to the local gravity vector. The gyro is connected in the torque-to-balance mode, i.e., the output of the pickoff is conditioned and applied to the torquer with the result that all gyro precessions are "nulled out". This torque-to-balance current provides a measure of the total nulled precession. Rotating the gyro input axis in the horizontal plane (about the output axis) until minimum "precession" occurs would, for the ideal single-degree-of-freedom gyro, provide a means of determining when the input axis is along an east-west line. When so oriented, the input axis would sense no component of earth rotation rate, and as shown in Figure 1, the spin axis (S) would necessarily be in a north-south orientation and parallel to the horizontal component of earth rate (ω_{EH}).

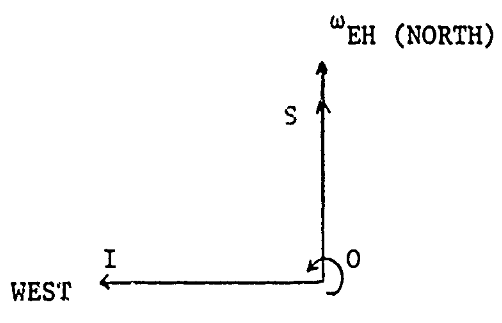


FIGURE 1
MINIMUM PRECESSION (Ideal Case)

2.2 General Case

In practice, due to misalignment of the gyro with respect to the local gravity vector and because of electrical and mechanical imperfections of the gyro, minimum "precession" of the gyro will not necessarily indicate that the spin axis is north. Even if the gyro were perfectly aligned with respect to the gravity vector and all internally caused drift were compensated for based upon previous measurements, minimum "precession" still would not necessarily indicate that the spin axis is precisely north because of gyro drift coefficient instabilities and coefficient variations with time. (See Appendix I for definitions of drift coefficients.) The angle between the spin axis and astronomic north* is the angle ϕ shown in Figure 2.

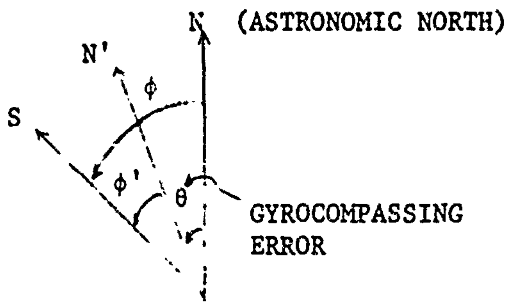


FIGURE 2
MINIMUM PRECESSION (General Case)

Physically ϕ is the sum of three misalignment angles:

$$\phi = a + d + h$$

These three angles a, d, and h are discussed in Appendix II. The difference between true ϕ and a computed ϕ (ϕ') is the gyrocompassing error. Thus, it is necessary to define an angle ϕ' such that

$$\phi' = \phi - \theta$$

---

* Astronomic north is defined by the intersection of the meridian plane with the horizontal plane. The meridian plane is defined by the local gravity vector and a line parallel to the rotation axis of the earth. The horizontal plane is defined as a plane perpendicular to the local gravity vector.

where θ is the gyrocompassing error (the angle between north and N', the north direction as determined by the gyroscope).

To evaluate θ in the laboratory the gyro is mounted on the gyrocompassing fixture (section 3) with the spin axis approximately parallel to the horizontal axis of the test fixture and initially oriented as in Figure 3.

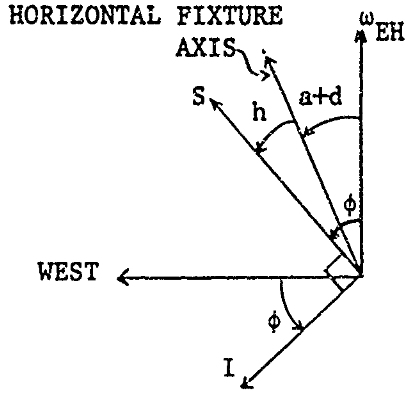


FIGURE 3
LABORATORY ORIENTATION OF THE GYROSCOPE

By summing the gyro outputs from four orientations (section 2.3) the effect of an alignment error (angle h in Figure 3) between the spin axis and the horizontal axis of the fixture is removed. Furthermore, since in the laboratory the horizontal axis of the fixture is accurately aligned to north by optical methods, the angle a+d is constrained to be zero degrees. Therefore a solution for ϕ' in the laboratory is actually a solution for the gyrocompassing error θ.

2.3 Gyroscope Orientations

In Figure 4 the four orientations used for the test are presented. These four orientations are necessary to evaluate θ and to eliminate the effects of misalignments (e.g., spin axis not parallel to test fixture horizontal axis and input axis not horizontal). A discussion of these misalignments and their effects is contained in Appendix II. If ϕ is assumed small, and misalignments (including h) are neglected, the equations (from Appendix I) relating the torque-to-balance current to ϕ in each of the four orientations are

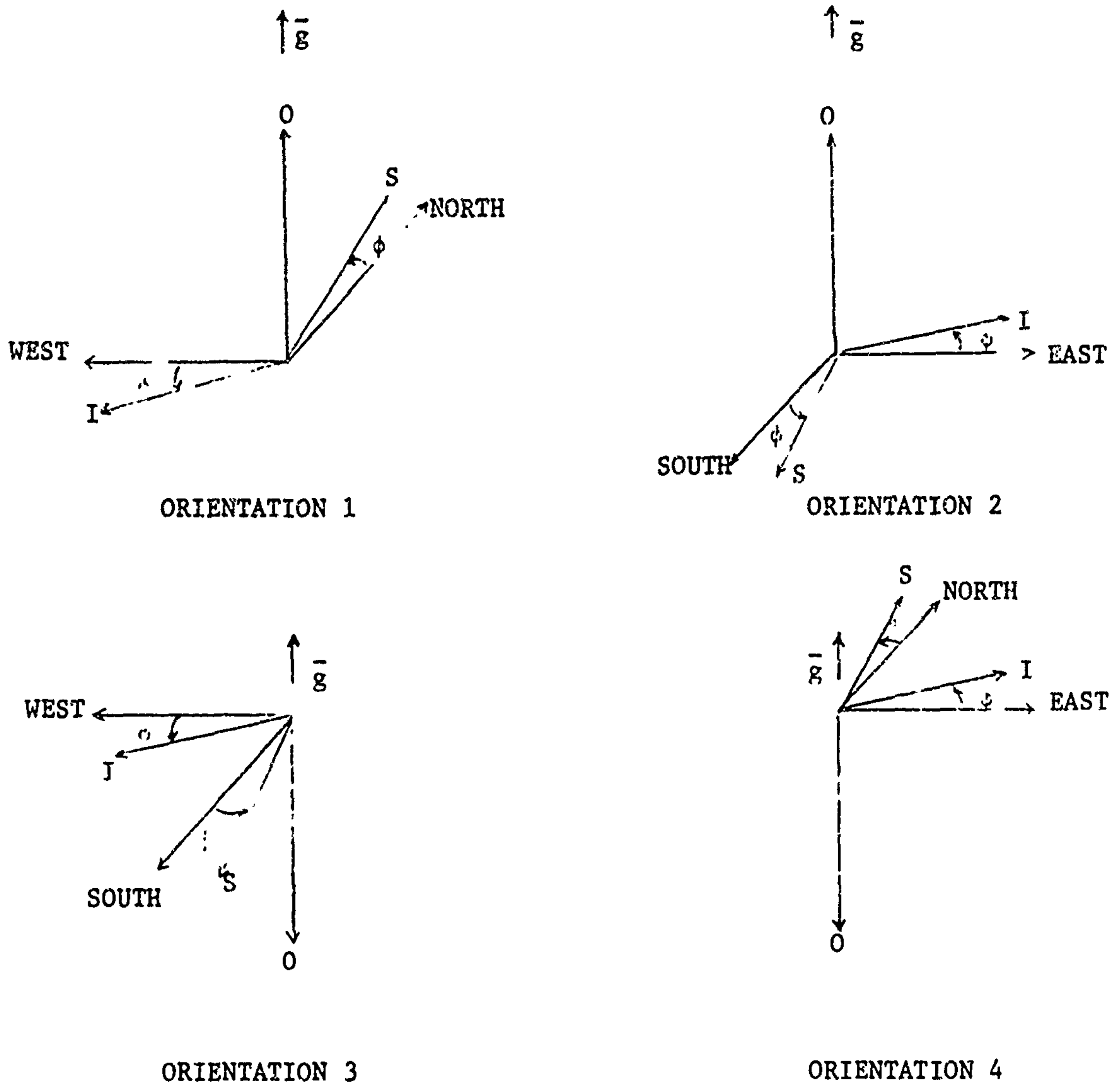


FIGURE 4
GYROCOMPASSING ORIENTATIONS

FOR THE GYROCOMPASSING ACCURACY TEST

$$S_T \bar{\bar{I}}_1 = -\ \phi\omega_{EH} + D_F + D_O a_O \quad (1)$$

$$S_T \bar{\bar{I}}_2 = \phi\omega_{EH} + D_F + D_O a_O \quad (2)$$

$$S_T \bar{\bar{I}}_3 = -\ \phi\omega_{EH} + D_F + D_O a_O \quad (3)$$

$$S_T \bar{\bar{I}}_4 = \phi\omega_{EH} + D_F + D_O a_O \quad (4)$$

where

S_T	gyro torquer scale factor (°/sidereal hour/ma)
$\bar{\bar{I}}_1, \bar{\bar{I}}_2, \bar{\bar{I}}_3, \bar{\bar{I}}_4$	average torque-to-balance current over 10 ten-second intervals in each of the four orientations (ma)
ω_{EH}	horizontal component of earth rotation rate at the Holloman AFB latitude (12.59562°/sidereal hr)
D_F	gyro drift which is insensitive to acceleration (°/hr)
$D_O a_O$	gyro drift (°/hr) attributable to acceleration along the output axis

2.4 Solution for θ

If the drift coefficients in equations 1 through 4 are assumed to be constant during the period of the test, these equations can be combined to solve for θ.

Changing the signs of equations 1 and 3 and adding the results to equations 2 and 4 gives

$$S_T (\bar{\bar{I}}_2 + \bar{\bar{I}}_4 - \bar{\bar{I}}_1 - \bar{\bar{I}}_3) = 4\phi\omega_{EH} \quad (5)$$

Equation 5 is independent of gyro drift and may be rearranged as follows to solve for ϕ' (the computed value of ϕ):

$$\phi'_{radians} = \frac{S_T}{4\omega_{EH}} (\bar{\bar{I}}_2 + \bar{\bar{I}}_4 - \bar{\bar{I}}_1 - \bar{\bar{I}}_3) \qquad (6)$$

Since $\phi' = \phi - \theta$, and ϕ is constrained to be zero in the laboratory, $-\theta$ can be substituted for ϕ'. Then converting from radians to arc seconds and substituting the magnitude of ω_{EH} at Holloman AFB into equation 6, the gyrocompassing error becomes:

$$\theta_{\widehat{sec}} = 4093.979\ S_T (\bar{\bar{I}}_1 + \bar{\bar{I}}_3 - \bar{\bar{I}}_2 - \bar{\bar{I}}_4) \qquad (7)$$

2.5 Gyrocompassing Scale Factor

The quantity 4093.979 S_T which has units of arc seconds/ma (or arc seconds/mvolt if the torque-to-balance current is measured by inserting a precision resister in the feedback loop) is a constant for any one gyro and will be called the gyrocompassing scale factor (GSF). The method used to determine the GSF is described in section 4. Equation 7 may now be written

$$\theta_{\widehat{sec}} = GSF\ (\bar{\bar{I}}_1 + \bar{\bar{I}}_3 - \bar{\bar{I}}_2 - \bar{\bar{I}}_4) \qquad (8)$$

3. TEST FIXTURE

For the gyrocompassing accuracy test the gyro is placed in a mount which in turn is mounted on the gyrocompassing test fixture. A photograph of the CIGTF test station is shown in Figure 5. The gyrocompassing test fixture is a two-axis test device manufactured by Goerz Optical Company. It is capable of providing automatic rotation about either axis or about both axes simultaneously. The two axes, one vertical and the other horizontal, are perpendicular to within five arc seconds. The mounting surface, which is on the end of the horizontal axis, is capable of supporting 40 foot-pounds without deflecting the horizontal axis more than 0.2 $\widehat{sec}$ and the vertical axis more than 0.5 $\widehat{sec}$. The position transducer on each axis is an inductosyn having 360 stable nulls at one degree intervals for 360 degrees of rotation. The fixture can thus be positioned about either axis in one degree increments. The positioning accuracy is better than 0.5 $\widehat{sec}$ with a repeatability of 0.2 $\widehat{sec}$. Positioning of the table is accomplished either manually or by means of an automatic programmer.

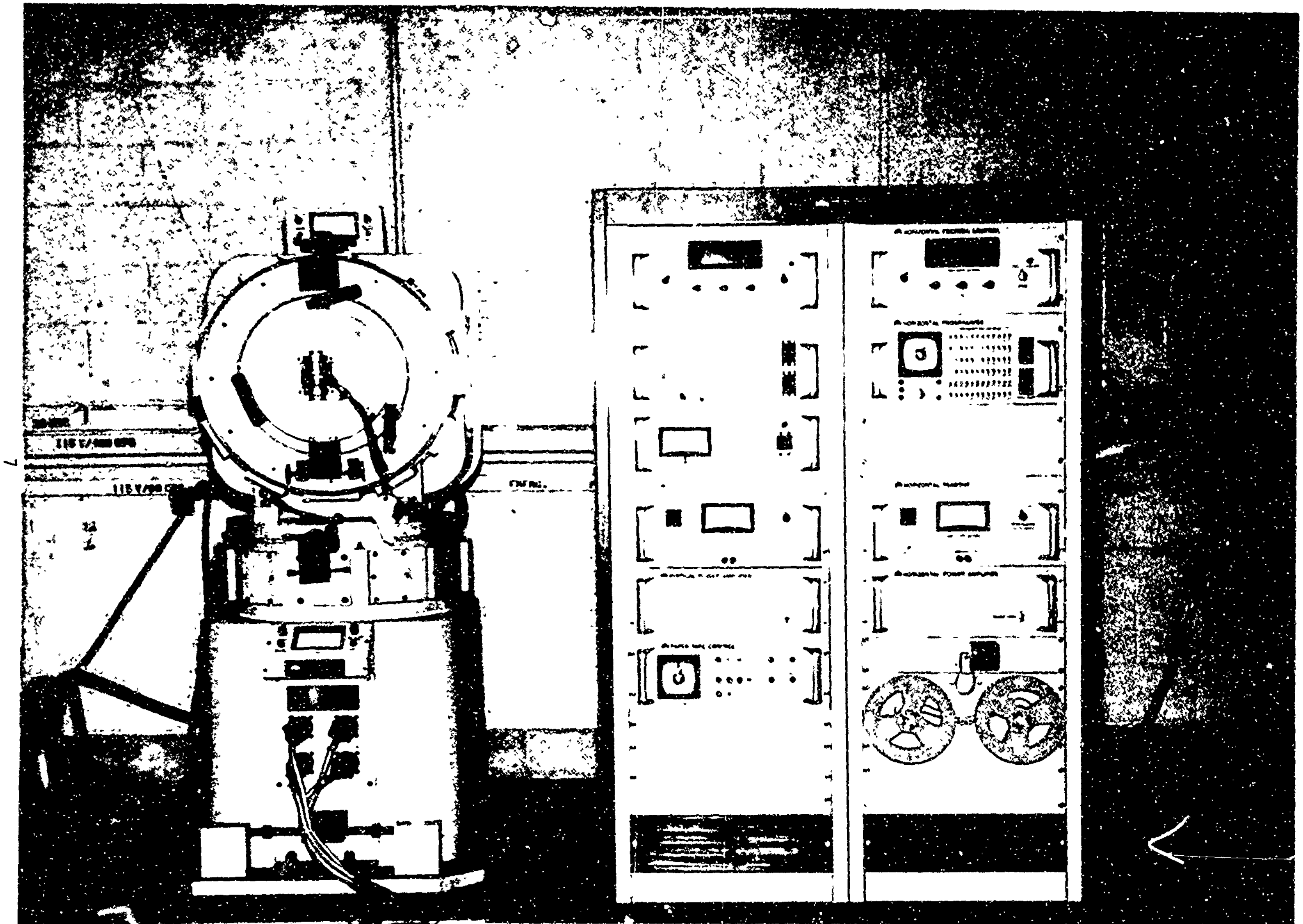


FIGURE 5
GYROCOMPASSING TEST STATION

4. TEST PROCEDURE

4.1 Determination of the Gyrocompassing Scale Factor

Prior to the start of the test, the vertical test fixture axis is aligned parallel to the local gravity vector by means of bubble levels mounted on the vertical axis mounting surface. In addition, the horizontal axis is aligned parallel to the horizontal component of earth rotation rate by optical means. Digital displays located in the gyrocompassing fixture electronics racks, which indicate axis rotational position, should both register zero degrees. The gyro, which is in the torque-to-balance mode, is positioned with the output axis vertical up and the input axis approximately horizontal west (Orientation 1 in Figure 4). After the gyro has reached steady state conditions in this orientation, 10 ten-second electronically averaged values of the gyro torquer current, $\bar{I}_1$ are recorded. The torque-to-balance current equation for this orientation is

$$S_T\bar{\bar{I}}_1 = -\phi\omega_{EH} + D_F + D_O a_O \tag{9}$$

The test fixture is next rotated one degree in a positive direction about the vertical (up) axis. After the gyro reaches steady state conditions in this orientation, 10 ten-second electronically averaged values of the gyro torquer current, $\bar{I}_1'$, are recorded. In this orientation the torque-to-balance current equation is

$$S_T\bar{\bar{I}}_1' = -(\phi + .01745)\,\omega_{EH} + D_F + D_O a_O \tag{10}$$

where

$$1° = .01745 \text{ radians}$$

and $\bar{\bar{I}}_1'$ is the average torque-to-balance current of the ten $\bar{I}_1'$.

Then

$$S_T(\bar{\bar{I}}_1 - \bar{\bar{I}}_1') = .01745\,\omega_{EH} \tag{11}$$

When both sides of the equation are divided by $4\omega_{EH}$ $(\bar{\bar{I}}_1 - \bar{\bar{I}}_1')$, the result is

$$\frac{S_T}{4\omega_{EH}} = \frac{.00436}{(\bar{\bar{I}}_1 - \bar{\bar{I}}_1')} \text{ radians/ma} \tag{12}$$

Converting radians to $\widehat{\sec}$, and substituting the value for the horizontal component of earth rotation rate, yields

$$4093.979\ S_T = \frac{900}{(\bar{\bar{I}}_1 - \bar{\bar{I}}_1')} \ \widehat{\sec}/\text{ma} \tag{13}$$

Thus the gyrocompassing scale factor GSF is given by

$$\text{GSF} = \frac{900}{(\bar{\bar{I}}_1 - \bar{\bar{I}}_1')} \ \widehat{\sec}/\text{ma} \tag{14}$$

4.2 Four Orientation Test Sequence

The gyrocompassing test sequence which has been programmed on paper tape is initiated by the test fixture automatic programmer. The sequence of operations that are produced by the programmer are as follows:

4.2.1 With the spin axis parallel to the horizontal fixture axis, the gyro is rotated about the fixture horizontal and/or vertical axes until the gyro is in Orientation 1 (O vertical up, I horizontal west). This orientation is maintained until the gyro has settled and 10 ten-second electronically averaged values of the gyro torquer current are recorded. The values are automatically printed and punched on paper tape.

4.2.2 The gyro is rotated 180° about the fixture vertical axis to place the gyro in Orientation 2 (O vertical up, I horizontal east). This orientation is maintained until the gyro has settled and 10 ten-second electronically averaged values of the gyro torquer current are recorded.

4.2.3 Next the gyro is rotated 180° about the fixture horizontal axis to place the gyro in Orientation 3 (O vertical down, I horizontal west). This orientation is maintained until the gyro has settled and 10 ten-second electronically averaged values of the gyro torquer current are recorded.

4.2.4 The gyro is rotated 180° about the fixture vertical axis to place the gyro in Orientation 4 (O vertical down, I horizontal east). This orientation is maintained until the gyro has settled and 10 ten-second electronically averaged values of the gyro torquer current are recorded.

4.3 Operating Phase

The sequence of commands and the resulting data obtained from the gyro in the four orientations described in section 4.2 are necessary for one north determination or one "run". This procedure is repeated 24 times for a total of 25 runs during which the gyro is not shut down. This comprises the "operating phase" of the gyrocompassing accuracy test.

4.4 Shutdown Phase

The operating phase is followed by a gyro shutdown of approximately two hours. After the two hour shutdown the gyro is warmed up for two hours and one north determination is made. This procedure of shutdown, warmup, and north determination is repeated nine times. These ten north determinations, each separated by a gyro shutdown and warmup period, comprise the "shutdown phase" of the gyrocompassing accuracy test.

5. DATA ANALYSIS

5.1 Orientation Values

5.1.1 The mean value of the 10 ten-second torque-to-balance current samples in orientation k is:

$$\bar{\bar{I}}_k = \frac{\sum_{\ell=1}^{10} \bar{I}_{k\ell}}{10} \tag{15}$$

where

k = 1, 2, 3, and 4

5.1.2 The variance of the 10 current samples in orientation k is:

$$\sigma_{\bar{I}_k}^2 = \frac{\sum_{\ell=1}^{10} (\bar{I}_{k\ell} - \bar{\bar{I}}_k)^2}{9} \tag{16}$$

5.1.3 The variance of the mean of the 10 current samples in orientation k is:

$$\sigma_{\bar{\bar{I}}_k}^2 = \frac{\sigma_{\bar{I}_k}^2}{10} \tag{17}$$

5.2 Run Values

5.2.1 The gyrocompassing error in arc seconds for run i of phase j is:

$$\theta_{ij} = GSF\ (\bar{\bar{I}}_1 + \bar{\bar{I}}_3 - \bar{\bar{I}}_2 - \bar{\bar{I}}_4) \tag{18}$$

where

j=1 for the operating phase and j=2 for the shutdown phase.

5.2.2 The best estimate of the standard error of θ for run i of phase j is:

$$\sigma_{\theta_{ij}} = GSF \sqrt{\sigma_{\bar{\bar{I}}_1}^2 + \sigma_{\bar{\bar{I}}_3}^2 + \sigma_{\bar{\bar{I}}_2}^2 + \sigma_{\bar{\bar{I}}_4}^2} \tag{19}$$

5.3 Phase Values

5.3.1 The gyrocompassing error for phase j is:

$$\theta_j = \frac{\sum_{i=1}^{n} \theta_{ij}}{n} \tag{20}$$

where n is the number of runs in the phase.

5.3.2 The best estimate of the standard error of θ_j is:

$$\sigma_{\theta_j} = \frac{1}{n} \sqrt{\sum_{i=1}^{n} \sigma_{\theta_{ij}}^2} \tag{21}$$

except for the case described in paragraph 5.4.2.

5.3.3 The best overall estimate (RMS) of the run standard error for the n runs constituting phase j is:

$$s_j = \sqrt{\frac{\sum_{i=1}^{n} \sigma_{\theta_{ij}}^2}{n}} \tag{22}$$

which by equation (21) becomes

$$s_j = \sqrt{n}\ \sigma_{\theta_j} \tag{23}$$

5.3.4 The operating instability is the best estimate of the standard deviation of the θ_{ij} from the operating phase, i.e.,

$$\delta = \sqrt{\frac{\sum_{i=1}^{25} (\theta_{ij} - \theta_j)^2}{24}} \qquad (24)$$

where j=1.

5.3.5 The shutdown instability is the best estimate of the standard deviation of the θ_{ij} from the shutdown phase, i.e.,

$$\Delta = \sqrt{\frac{\sum_{i=1}^{10} (\theta_{ij} - \theta_j)^2}{9}} \qquad (25)$$

where j=2.

5.4 Significance Tests

5.4.1 A Student "t" test is performed to determine if θ_j is significantly different from zero at the 5% significance level. For a large number of degrees of freedom

$$\left|\frac{\theta_j}{\sigma_{\theta_j}}\right| \geq 1.96$$

implies that the average gyrocompassing error for phase j is significant.

5.4.2 A Snedecor F Test is performed to determine if the operating instability is significantly greater than the RMS of the run standard errors at the 5% significance level. With 24 degrees of freedom for the numerator and 900 degrees of freedom for the denominator,

$$\frac{\delta^2}{s_1^2} \geq 1.53$$

implies that the variation introduced by repositioning the gyro is significantly greater than that attributable to noise, and the best estimate of the standard error of θ_j is then $\frac{\delta}{\sqrt{n}}$ rather than σ_{θ_j}.
In this case the "t" test described in 5.4.1 is corrected accordingly.

5.4.3 A Snedecor F Test is performed to determine if the shutdown instability is significantly greater than the RMS of the run standard errors of the shutdown phase at the 5% significance level. With 9 degrees of freedom for the numerator and 360 degrees of freedom for the denominator, then

$$\frac{\Delta^2}{s_2^2} \geq 1.91$$

implies that the variation introduced by shutdown and by repositioning the gyro is significantly greater than that attributable to noise.

5.4.4 A Snedecor F Test is performed to determine if the shutdown instability is significantly greater than the operating instability at the 5% significance level. With 9 degrees of freedom for the numerator and 24 degrees of freedom for the denominator, then

$$\frac{\Delta^2}{\delta^2} \geq 2.30$$

implies that the variation introduced by gyro shutdown between runs is significantly greater than that attributable to repositioning the gyro.

APPENDIX I

SINGLE-DEGREE-OF-FREEDOM GYRO PERFORMANCE MODEL

The assumed performance model (Reference 1:2-4) for a single-degree-of-freedom gyro is

$$S_T i = D_F + D_I a_I + D_O a_O + D_S a_S + D_{II} a_I^2 + D_{SS} a_S^2 + D_{IO} a_I a_O$$

$$+ D_{IS} a_I a_S + D_{OS} a_O a_S + \omega_I$$

where

i	current flow through the torquer (ma)
S_T	torquer scale factor (°/hr/ma)
a_I	acceleration, with respect to inertial space, of the gyro case along the input axis (g)
a_O	acceleration, with respect to inertial space, of the gyro case along the output axis (g)
a_S	acceleration, with respect to inertial space, of the gyro case along the spin axis (g).
D_F	gyro drift (°/hr) which is insensitive to acceleration
$D_I a_I$	gyro drift (°/hr) attributable to acceleration along the input axis, where D_I (°/hr/g) is a drift coefficient
$D_O a_O$	gyro drift (°/hr) attributable to acceleration along the output axis, where D_O (°/hr/g) is a drift coefficient
$D_S a_S$	gyro drift (°/hr) attributable to acceleration along the spin axis, where D_S (°/hr/g) is a drift coefficient
$D_{II} a_I^2$	gyro drift (°/hr) attributable to the square of acceleration along the input axis, where D_{II} (°/hr/g^2) is a drift coefficient

$D_{SS}a_S^2$	gyro drift (°/hr) tributable to the square of acceleration along the spin axis, where D_{SS} (°/hr/g^2) is a drift coefficient
$D_{IO}a_Ia_O$	gyro drift (°/hr) attributable to the product of accelerations along the input axis and output axis where D_{IO} (°/hr/g^2) is a drift coefficient
$D_{IS}a_Ia_S$	gyro drift (°/hr) attributable to the product of accelerations along the input axis and spin axis, where D_{IS} (°/hr/g^2) is a drift coefficient
$D_{OS}a_Oa_S$	gyro drift (°/hr) attributable to the product of accelerations along the output axis and spin axis, where D_{OS}(°/hr/g^2) is a drift coefficient
ω_I	angular velocity, with respect to inertial space, of the gyro case about the input axis (°/hr)
hr	sidereal hour
g	local acceleration of gravity, defined positive upward

APPENDIX II

EFFECT OF MISALIGNMENT ANGLES IN THE GYROCOMPASSING ACCURACY TEST

1. INTRODUCTION

In the laboratory gyrocompassing accuracy test the effect of misalignment angles must be considered and, where possible, eliminated. The identification and elimination of the misalignment angles are discussed here.

2. DERIVATION OF DRIFT RATE DUE TO MISALIGNMENTS

2.1 Transformation of Coordinates

In order to derive an expression for all possible misalignment angles, three transformations of coordinates for each of the four test orientations are necessary. The transformations will be accomplished in the following sequence: the earth fixed coordinates (defined by the local gravity vector and astronomic north) will be transformed into the vertical fixture axis coordinate frame; the vertical fixture axis coordinate frame will be transformed into the horizontal fixture axis coordinate frame; and finally the horizontal fixture axis coordinate frame will be transformed into the gyro input, output, and spin axes coordinate frame.

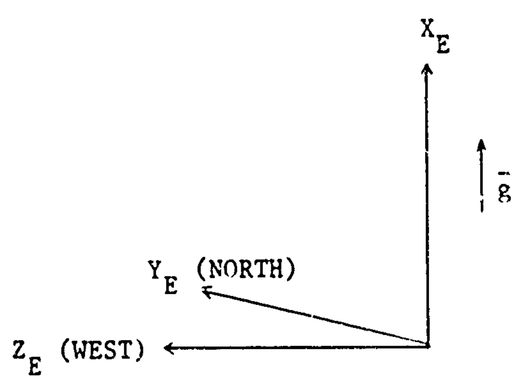
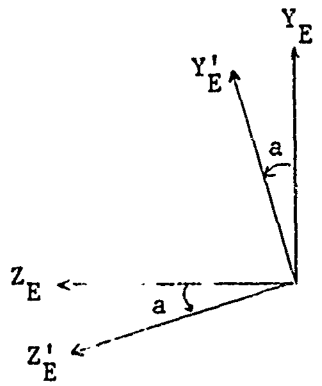


FIGURE II-1

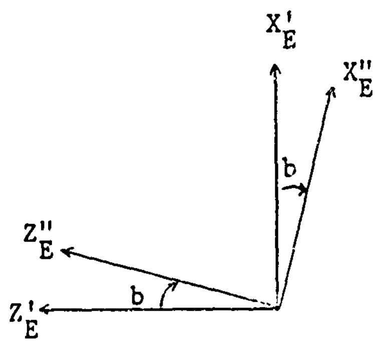
EARTH FIXED COORDINATE FRAME

In the earth fixed coordinate frame shown above, the local gravity vector $\bar{g}$ is parallel to the X_E axis, Y_E is astronomic north, and Z_E is west.

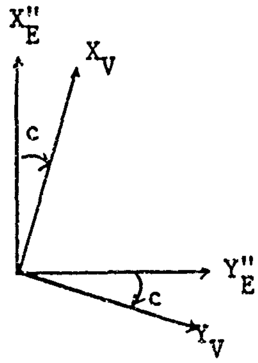
Angles a, b, and c are associated with the earth fixed to vertical fixture axis transformation of coordinates in the following manner:



$$\begin{bmatrix} X'_E \\ Y'_E \\ Z'_E \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & \sin a \\ 0 & -\sin a & \cos a \end{bmatrix} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$$



$$\begin{bmatrix} X''_E \\ Y''_E \\ Z''_E \end{bmatrix} = \begin{bmatrix} \cos b & 0 & -\sin b \\ 0 & 1 & 0 \\ \sin b & 0 & \cos b \end{bmatrix} \begin{bmatrix} X'_E \\ Y'_E \\ Z'_E \end{bmatrix}$$



$$\begin{bmatrix} X_V \\ Y_V \\ Z_V \end{bmatrix} = \begin{bmatrix} \cos c & \sin c & 0 \\ -\sin c & \cos c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X''_E \\ Y''_E \\ Z''_E \end{bmatrix}$$

where X_V is the vertical fixture axis and Y_V and Z_V are in a plane perpendicular to it. Using small angle approximations

$$\sin x = x$$

$$\cos x = 1$$

and neglecting second degree terms, the vertical fixture axis coordinate frame may be written in terms of the earth fixed coordinate frame:

$$\begin{bmatrix} X_V \\ Y_V \\ Z_V \end{bmatrix} = \begin{bmatrix} 1 & c & 0 \\ -c & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -b \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & a \\ 0 & -a & 1 \end{bmatrix} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$$

or

$$\begin{bmatrix} X_V \\ Y_V \\ Z_V \end{bmatrix} = \begin{bmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{bmatrix} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$$

Therefore the transformation matrix is defined:

$$Q_{VE} = \begin{bmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{bmatrix}$$

Similarly, angles d, e, and f are associated with the transformation of the gyrocompassing fixture vertical axis coordinates to the gyrocompassing fixture horizontal axis coordinates. Y_H is the horizontal fixture axis with X_H and Z_H perpendicular to it. The corresponding transformation matrix is Q_{HV}, where

$$Q_{HV} = \begin{bmatrix} 1 & f & -e \\ -f & 1 & d \\ e & -d & 1 \end{bmatrix}$$

Angles h, i, and j are associated with the transformation of the gyrocompassing fixture horizontal axis coordinates to the gyro coordinate frame. The transformation matrix is Q_{GH}, where

$$Q_{GH} = \begin{bmatrix} 1 & j & -i \\ -j & 1 & h \\ i & -h & 1 \end{bmatrix}$$

Therefore the transformation matrix for earth fixed to gyro coordinate frame is

$$Q_{GE} = Q_{GH}\, Q_{HV}\, Q_{VE} = \begin{bmatrix} 1 & (c+f+j) & -(b+e+i) \\ -(c+f+j) & 1 & (a+d+h) \\ (b+e+i) & -(a+d+h) & 1 \end{bmatrix}$$

2.2 Orientation One

In orientation one of the gyrocompassing accuracy test the output axis (X_G) is up, the input axis (Z_G) is west, and the spin axis (Y_G) is north. The resulting components of gravity along the three axes in units of g are

$$\begin{bmatrix} a_O \\ a_S \\ a_I \end{bmatrix} = Q_{GE_1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where Q_{GE_1} is the Q_{GE} defined in section 2.1 of this appendix. Therefore

$$a_O = 1$$

$$a_S = -\,(c+f+j)$$

$$a_I = +\,(b+e+i)$$

The components of earth rate along these three axes are

$$\begin{bmatrix} \omega_O \\ \omega_S \\ \omega_I \end{bmatrix} = Q_{GE_1} \begin{bmatrix} \omega_E \sin\lambda \\ \omega_E \cos\lambda \\ 0 \end{bmatrix}$$

where ω_E is earth rotation rate and λ is the astronomic latitude angle. The only component sensed by the gyro is ω_I:

$$\omega_I = (b+e+i)\,\omega_E \sin\lambda - (a+d+h)\,\omega_E \cos\lambda$$

When these quantities are substituted into the assumed performance model, (Appendix I), the resulting drift equation for orientation 1 is:

$$S_T\bar{\bar{I}}_1 = D_F + (b+e+i)\,D_I + D_O - (c+f+j)\,D_S + (b+e+i)\,D_{IO}$$

$$- (c+f+j)\,D_{OS} + (b+e+i)\,\omega_E \sin\lambda - (a+d+h)\,\omega_E \cos\lambda$$

2.3 Orientation Two

To arrive at orientation 2, a rotation of 180° + ε about the gyrocompassing fixture vertical axis X_V is required, where ε is a small error in the 180° rotation. Therefore

$$Q_{GE_2} = Q_{GH}Q_{HV} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -\varepsilon \\ 0 & \varepsilon & -1 \end{bmatrix} Q_{VE}$$

Thus

$$Q_{GE_2} = \begin{bmatrix} 1 & (c-f-j) & (-b+e+i) \\ (c-f-j) & -1 & -(a+d+h+\varepsilon) \\ (-b+e+i) & (a+d+h+\varepsilon) & -1 \end{bmatrix}$$

and the components of acceleration along the three gyro axes are:

$$a_O = 1$$
$$a_S = c-f-j$$
$$a_I = -b+e+i$$

The earth rate sensed by the gyro is

$$\omega_I = (-b+e+i)\ \omega_E \sin\lambda + (a+d+h+\varepsilon)\ \omega_E \cos\lambda$$

The resulting drift equation for orientation 2 is:

$$S_T\bar{\bar{I}}_2 = D_F + (-b+e+i)\ D_I + D_O + (c-f-j)\ D_S + (-b+e+i)\ D_{IO}$$

$$+ (c-f-j)\ D_{OS} + (-b+e+i)\ \omega_E \sin\lambda + (a+d+h+\varepsilon)\omega_E \cos\lambda$$

2.4 Orientation Three

To arrive at orientation 3 it is necessary to rotate 180° + β about the gyrocompassing fixture horizontal axis Y_H, where β is an error in the 180° rotation. Therefore

$$Q_{GE_3} = Q_{GH} \begin{bmatrix} -1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & -1 \end{bmatrix} Q_{HV} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -\varepsilon \\ 0 & \varepsilon & -1 \end{bmatrix} Q_{VE}$$

$$= \begin{bmatrix} -1 & (-c+f-j) & (b-e-i-\beta) \\ (c-f+j) & -1 & (-a-d+h-\varepsilon) \\ (b-e-i-\beta) & (-a-d+h-\varepsilon) & 1 \end{bmatrix}$$

The components of acceleration along the gyro axes are

$$a_O = -1$$

$$a_S = c-f+j$$

$$a_I = b-e-i-\beta$$

and the earth rate sensed by the gyro is

$$\omega_I = (b-e-i-\beta)\ \omega_E \sin\lambda - (a+d-h+\varepsilon)\ \omega_E \cos\lambda$$

The resulting drift equation for orientation 3 is:

$$S_T \bar{\bar{I}}_3 = D_F + (b-e-i-\beta)\ D_I - D_O + (c-f+j)\ D_S - (b-e-i-\beta)\ D_{IO}$$

$$- (c-f+j)\ D_{OS} + (b-e-i-\beta)\omega_E \sin\lambda - (a+d-h+\varepsilon)\ \omega_E \cos\lambda$$

2.5 Orientation Fo

To arrive at orientation four, it is necessary to rotate $180°+\eta$ about the gyrocompassing fixture vertical axis, X_V. Then

$$Q_{GE_4} = Q_{GH} \begin{bmatrix} -1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & -1 \end{bmatrix} Q_{HV} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -\eta \\ 0 & \eta & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -\varepsilon \\ 0 & \varepsilon & -1 \end{bmatrix} Q_{VE}$$

$$= \begin{bmatrix} -1 & (-c-f+j) & (b+e+i+\beta) \\ (-c-f+j) & 1 & (a+d-h+\varepsilon+\eta) \\ (-b-e-i-\beta) & (a+d-h+\varepsilon+\eta) & -1 \end{bmatrix}$$

The components of acceleration along the gyro axes are

$$a_O = -1$$

$$a_S = -(c+f-j)$$

$$a_I = -(b+e+i+\beta)$$

and the earth rate sensed by the gyro is

$$\omega_I = -(b+e+i+\beta)\,\omega_E \sin\lambda + (a+d-h+\varepsilon+\eta)\,\omega_E \cos\lambda$$

Therefore the drift equation for orientation four is:

$$S_T\ddot{I}_4 = D_F - (b+e+i+\beta)\,D_I - D_O - (c+f-j)\,D_S + (b+e+i+\beta)\,D_{IO}$$

$$+ (c+f-j)\,D_{OS} - (b+e+i+\beta)\,\omega_E \sin\lambda + (a+d-h+\varepsilon+\eta)\,\omega_E \cos\lambda$$

2.6 Resulting Drift Equation

When the drift equations for orientations 1 and 3 are subtracted from the sum of those for orientations 2 and 4, the following equation results.

$$S_T(\bar{\bar{I}}_2 + \bar{\bar{I}}_4 - \bar{\bar{I}}_1 - \bar{\bar{I}}_3) = -4bD_I + 4cD_{OS} - 4b\omega_E \sin\phi + 4(a+d)\,\omega_E \cos\phi$$

$$+ (3\varepsilon + \eta)\,\omega_E \cos\phi \qquad (26)$$

The angle (a+d) is the angle between north and the horizontal fixture axis, measured positive about an axis directed vertically up. To solve for its computed value (a+d)' equation 26 is rearranged as follows

$$(a+d)' = \frac{S_T(\bar{\bar{I}}_2 + \bar{\bar{I}}_4 - \bar{\bar{I}}_1 - \bar{\bar{I}}_3)}{4\,\omega_E \cos\phi} + \frac{b(\omega_E \sin\phi + D_I)}{\omega_E \cos\phi} - \frac{cD_{OS}}{\omega_E \cos\phi} - \frac{(3\varepsilon+\eta)}{4} \qquad (27)$$

The angles b and c are the misalignment angles between the vertical axis of the gyrocompassing test fixture and the local gravity vector in the east-west plane and the north-south plane respectively. These misalignments are minimized (less than 1 sec) by the use of bubble levels prior to gyrocompassing. The angles ε and η are errors in the 180° rotation between orientations. The positioning accuracy of the gyrocompassing test fixture is 0.5 sec or better. Since these misalignment effects are very small, they are neglected, thus

$$(a+d)' = \frac{S_T(\bar{\bar{I}}_2 + \bar{\bar{I}}_4 - \bar{\bar{I}}_1 - \bar{\bar{I}}_3)}{4\,\omega_E \cos\phi} \qquad (28)$$

From section 2.2

$$\phi' = \phi - \theta \qquad (29)$$

and

$$\phi = a+d+h \qquad (30)$$

Since h is not computed equation 30 may be written

$$\phi' = (a+d)' + h \tag{31}$$

where ϕ' is the computed value of ϕ and $(a+d)'$ is the computed value of $(a+d)$. Substituting equations 30 and 31 into equation 29 and subtracting h from both sides of the equation yields

$$(a+d)' = (a+d) - \theta$$

In the laboratory test the angle a+d is constrained to be zero. Thus a solution for $(a+d)'$ becomes a solution for $-\theta$. From equation 28 the gyrocompassing error then becomes

$$\theta_{radians} = \frac{S_I (\bar{\bar{I}}_1 + \bar{\bar{I}}_3 - \bar{\bar{I}}_2 - \bar{\bar{I}}_4)}{4 \omega_E \cos \lambda}$$

When radians are converted to arc seconds and the magnitude of ω_E cos at Holloman AFB is substituted into this equation, the result is equation 7 of section 2.4.

Therefore, the effects of misalignments which can not be readily estimated or minimized, such as the misalignment of the spin axis with respect to the horizontal fixture axis (angle h) and the misalignment of the gyro output axis with respect to the vertical fixture axis (angles i, j, e, and f), are removed from the determination of the gyrocompassing error by the four-orientation test.

REFERENCES

1. Russell, James F., Gyroscope Standard Torque-to-Balance Test. Technical Report. Holloman Air Force Base, New Mexico: Air Force Missile Development Center, June 1967. Unclassified.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author) AFMDC HOLLOMAN AFB, NEW MEXICO DIRECTORATE OF GUIDANCE TEST	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b GROUP

3 REPORT TITLE

GYROSCOPE STANDARD GYROCOMPASSING ACCURACY TEST

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

5 AUTHOR(S) (First name, middle initial, last name)

Hayne A. Thompson, Jr., Captain, USAF
Judith Koestler, Mrs.

6 REPORT DATE	7a. TOTAL NO OF PAGES	7b NO OF REFS
December 1967	41	1

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
b PROJECT NO 5177-27	MDC-TR-67-148
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	

10 DISTRIBUTION STATEMENT

Distribution of this document is unlimited. Qualified users may obtain copies from the Defense Documentation Center.

11 SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY

13 ABSTRACT

The Central Inertial Guidance Test Facility (CIGTF) Standard Gyrocompassing Accuracy Test and associated data analysis procedures for single-degree-of-freedom gyros are presented.

The gyrocompassing error, the error in the north direction as determined by the gyroscope, is obtained by orienting the gyro in four specific orientations and measuring the output of the gyro while in the torque-to-balance mode. This procedure is repeated 24 times for a total of 25 north determinations during which the gyro is not shut down. From these data a determination is made of the average gyrocompassing error and of the operating instability of the gyrocompassing error. This "operating" phase is followed by a "shutdown" phase consisting of 10 north determinations, each separated by a two-hour gyro shutdown and two-hour warmup. From these data the shutdown instability of the gyrocompassing error is determined. Although the test and data analysis procedures described in this report are written specifically for single-degree-of-freedom gyros, the theory can easily be applied to two-degree-of-freedom gyros.

This report also contains a description of the CIGTF gyroscope laboratory's two axis automated gyrocompassing test station and a test error analysis.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gyroscope Standard Gyrocompassing Accuracy Tests						

AFSC-HOLLOMAN AFB NMEX

UNCLASSIFIED
Security Classification