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MATRIX FORMULATION OF BISTATIC ELECTROMAGNETIC SCATTERING

1 of 2

MAY 1968

P. C. Waterman

Work Performed for

ADVANCED RESEARCH PROJECTS AGENCY

Contract Administered by

DEVELOPMENT ENGINEERING DIVISION
DIRECTORATE OF PLANNING AND TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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Project 8051
Prepared by
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FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for Advanced Research Projects Agency; the contract was monitored by the Directorate of Planning and Technology, Electronic Systems Division, of the Air Force Systems Command under Contract AF 19(628)-5165.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

A. P. TRUNFIO
Project Officer
Development Engineering Division
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ABSTRACT

The purpose of this discussion is to relate the previously developed matrix analysis of electromagnetic scattering explicitly to the experimentally measurable bistatic polarization matrix. In addition, in order to perform the numerical computations more conveniently, a new matrix analysis is described, in which the constraints of reciprocity and energy conservation are employed in the course of obtaining a solution.

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SECTION I

INTRODUCTION

The purpose of this discussion is as follows:

(1) To relate the previously developed matrix analysis of electromagnetic scattering explicitly to the experimentally measurable bistatic polarization matrix.

(2) To describe a new matrix analysis, suitable for numerical computation, in which the constraints of reciprocity and energy conservation are employed in the course of obtaining a solution.

Description of the measurable far field quantities requires mapping a vector, the polarization of the incident plane wave, into another vector, giving the polarization of the scattered field; this operation is conveniently described by the dyadic amplitude $\underline{\underline{F}}$. In Section II the spatial electromagnetic fields are first set up in terms of a theoretical quantity, the transition matrix T, then the dyadic amplitude is defined, and by specifying bases, e.g., linear-linear, the experimentally measurable bistatic polarization matrix P is obtained. Finally, the determination of the dyadic amplitude from the transition matrix is described.

Section III gives the matrix analysis, in which contraints of reciprocity and energy conservation are incorporated into the theoretical/numerical determination of the transition matrix T.

In practice, this approach may make feasible the employment of the existing MITRE computer program for a wider range of target sizes and shapes than heretofore possible.

It should perhaps be pointed out that the discussion throughout is expected to be applicable to general radar targets with no energy dissipation in otherwise free space, although to date the requisite Q matrix of Section III has only been discussed for general conducting bodies and has only been implemented on the computer for conducting bodies having an axis of rotational symmetry.

SECTION II

THE BISTATIC POLARIZATION MATRIX

We work in the matrix formalism described earlier [1] using the recent modifications. A more detailed description of some of the quantities discussed below may be found in the references.

An arbitrary monochromatic incident electromagnetic field (time dependence exp $(-i\omega t)$ suppressed) with no singularities in the immediate vicinity of a conducting target may be described by the electric field vector (underline denotes vector quantity)

$$\underline{E}^i(\underline{r}) = \sum_{\sigma mn} \left| \frac{\varepsilon_m (2n+1)(n-m)!}{n(n+1)(n+m)!} \right|^{-1/2} \left| a_{\sigma mn} \underline{M}_{\sigma mn}(\underline{r}) + b_{\sigma mn} \underline{N}_{\sigma mn}(\underline{r}) \right| \tag{1}$$

at any field point $\underline{r}$ measured from an origin lying in the interior of the target. The $a_{\sigma mn}$ and $b_{\sigma mn}$ are complex expansion coefficients, ε_m is the Neumann factor $\varepsilon_o = 1$, $\varepsilon_m = 2$ otherwise, and the spherical partial waves are given explicitly by [2]

$$\underline{M}_{\sigma mn} = \underline{M}_{\substack{e\\o}mn} = \nabla \times \left| \underline{r} \begin{matrix}\cos\\ \sin\end{matrix} m\phi \, P^m_n (\cos\theta) \, j_n(kr) \right|$$

$$= [n(n+1)]^{1/2} \, \underline{C}_{\substack{e\\o}mn} (\theta,\phi) \, j_n(kr)$$

$$\underline{N}_{\sigma mn} = \underline{N}_{\substack{e\\o}mn} = (1/k)\ \nabla \times \underline{M}_{\substack{e\\o}mn}$$

$$= n(n+1)\ \underline{P}_{\substack{e\\o}mn}(\theta,\phi)\ j_n(kr)/(kr) \tag{2}$$

$$+ [n(n+1)]^{1/2}\ \underline{B}_{\substack{e\\o}mn}(\theta,\phi)\ (1/kr)\ d[rj_n(kr)]/dr$$

The three indices run through values $\sigma = e, o$ (even or odd), $m = 0, 1, \ldots, n$, and $n = 1, 2, \ldots$. The P^m_n are associated Legendre functions, and the $j_n(kr)$ are spherical Bessel functions. The vector spherical harmonics are defined by[3][note that $\hat{r} = \hat{r}(\theta,\phi)$]

$$\underline{P}_{\substack{e\\o}mn}(\hat{r}) = \hat{r} \begin{matrix}\cos\\ \sin\end{matrix} m\phi\ P^m_n(\cos\theta)$$

$$\underline{B}_{\substack{e\\o}mn}(\hat{r}) = [n(n+1)]^{-1/2}\ r\ \nabla[\begin{matrix}\cos\\ \sin\end{matrix} m\phi P^m_n(\cos\theta)] = \hat{r} \times \underline{C}_{\substack{e\\o}mn} \tag{3}$$

$$\underline{C}_{\substack{e\\o}mn}(\hat{r}) = [n(n+1)]^{-1/2}\ \nabla \times [\underline{r} \begin{matrix}\cos\\ \sin\end{matrix} m\phi\ P^m_n(\cos\theta)] = -\hat{r} \times \underline{B}_{\substack{e\\o}mn}\ .$$

In similar fashion, the most general scattered wave satisfying the radiation condition at infinity may be written

$$\underline{E}^s(\underline{r}) = \sum_{\sigma mn} i \left| \frac{\varepsilon_m(2n+1)(n-m)!}{n(n+1)(n+m)!} \right|^{1/2} \left| f_{\sigma mn}\ \underline{M}^{(3)}_{\sigma mn}(\underline{r}) + g_{\sigma mn}\ \underline{N}^{(3)}_{\sigma mn}(\underline{r}) \right| \tag{4}$$

in terms of the *outgoing* spherical vector partial waves, obtained by replacing $j_n(kr)$ by $h_n(kr)$ (the spherical Hankel function of the first kind) everywhere in Equation (2).

Because of the linearity of the problem, the coefficients $f_{\sigma mn}$, $g_{\sigma mn}$ of the scattered wave must be obtainable by a linear operation on those of the incident wave: one writes

$$f_{\sigma mn} = \sum_{\sigma' m' n'} [T^1{}_{\sigma mn\sigma' m' n'}\, a_{\sigma' m' n'} + T^2{}_{\sigma mn\sigma' m' n'}\, b_{\sigma' m' n'}]$$

$$g_{\sigma mn} = \sum_{\sigma' m' n'} [T^3{}_{\sigma mn\sigma' m' n'}\, a_{\sigma' m' n'} + T^4{}_{\sigma mn\sigma' m' n'}\, b_{\sigma' m' n'}] \tag{5a}$$

or, in an obvious matrix notation

$$\begin{vmatrix} f \\ g \end{vmatrix} = \begin{vmatrix} T^1 & T^2 \\ T^3 & T^4 \end{vmatrix} \begin{vmatrix} a \\ b \end{vmatrix} \tag{5b}$$

The matrices having elements $T^1{}_{11}$, $T^1{}_{12}$, and so forth, where the indices σ, m, n have been reordered into a single index, or more briefly

$$T \equiv \begin{vmatrix} T^1 & T^2 \\ T^3 & T^4 \end{vmatrix} \tag{5c}$$

will be called *transition matrix* . Once obtained for a given target, it enables one to compute the scattered wave corresponding to any given incident wave (1), by applying Equations (4) and (5).

In radar work one is primarily interested in the scattered far field due to excitation by a plane wave having specified direction of incidence and specified polarization. Because we are interested in mapping a vector (the incident polarization) into another vector (the far scattered $\underline{E}$ field) it is convenient to introduce the *dyadic* [4,5] far field amplitude $\underline{\underline{F}}(\hat{k}_s, \hat{k}_i)$ describing the scattered electric vector far from the target (i.e., $kr >> 1$). The directions of incidence and scattering (i.e., observation) are given by $\hat{k}_i = \hat{k}_i(\theta_i, \phi_i)$ and $\hat{k}_s = \hat{k}_s(\theta_s, \phi_s)$, respectively, and $\underline{e}_i$ denotes the electric polarization vector associated with the incident wave. One then can write

$$\underline{E}_s(\underline{r}) \sim (1/kr) \exp(ikr) \underline{\underline{F}} (\hat{k}_s,\hat{k}_i) \cdot \underline{e}_i \text{ as } kr \to \infty . \qquad (6)$$

Because both the incident wave and the far field $\underline{E}_s$ are transverse to their directions of propagation, the dyadic amplitude may be constrained to satisfy [5]

$$\underline{\underline{F}}(\hat{k}_s,\hat{k}_i) \cdot \hat{k}_i = \hat{k}_s \cdot \underline{\underline{F}}(\hat{k}_s,\hat{k}_i) = 0 \qquad (7)$$

thus having only four independent components. In addition, the reciprocity theorem, relating to interchange of source and observation

point, requires that [5]

$$\underline{\underline{F}}(\hat{k}_s,\hat{k}_i) = \underline{\underline{F}}^T(-\hat{k}_i, -\hat{k}_s) \tag{8}$$

where the superscript T designates the dyadic transpose. It may also be shown, as a consequence of energy conservation, that [6]

$$\int d\Omega_s \underline{\underline{F}}^{T*}(\hat{k}_s,\hat{k}_i) \cdot \underline{\underline{F}}(\hat{k}_s,\hat{k}_i) = -2\pi i \, [\underline{\underline{F}}(\hat{k}_i,\hat{k}'_i) - \underline{\underline{F}}^{T*}(\hat{k}'_i,\hat{k}_i)] \tag{9}$$

where the integration is over the unit sphere of directions of $\hat{k}_s$.

The relationship between the dyadic amplitude and its constraints, on the one hand, and the transition matrix, on the other hand, will be made apparent shortly. It is convenient first, however, to examine the (2 x 2 complex) bistatic polarization matrix. One can define the vector scattering amplitude $\underline{e}_s$ to be just $\underline{E}_s(\underline{r})$ exclusive of the radial factor $(1/kr)\exp(ikr)$, so that from Equation (6) there results

$$\underline{e}_s = \underline{\underline{F}}(\hat{k}_s,\hat{k}_i) \cdot \underline{e}_i \ . \tag{10}$$

The incident polarization may be written

$$\underline{e}_i = e_{\theta_i} \hat{\theta}_i + e_{\phi_i} \hat{\phi}_i \tag{11}$$

where if both components are real (or if both are complex, but with equal phase) the incident wave is linearly polarized. Otherwise, the incident wave is elliptically polarized.

In similar fashion the scattered polarization vector may be written

$$\underline{e}_s = e_{\theta_s} \hat{\theta}_s + e_{\phi_s} \hat{\phi}_s \tag{12}$$

and this of course will in general represent an elliptic state of polarization, even though the incident wave may be linearly polarized. The base vectors of Equations (11) and (12) appear the natural choice for a linear-linear basis, in terms of which the 2 x 2 bistatic polarization matrix P enables one to compute

$$\begin{vmatrix} e_{\theta_s} \\ e_{\phi_s} \end{vmatrix} = \begin{vmatrix} P_{\theta_s \theta_i} & P_{\theta_s \varphi_i} \\ P_{\phi_s \theta_i} & P_{\phi_s \varphi_i} \end{vmatrix} \begin{vmatrix} e_{\theta_i} \\ e_{\phi_i} \end{vmatrix} \tag{13a}$$

in which

$$\begin{aligned} P_{\theta_s \theta_i} &\equiv \hat{\theta}_s \cdot \underline{\underline{F}} \cdot \hat{\theta}_i \\ P_{\theta_s \varphi_i} &\equiv \hat{\theta}_s \cdot \underline{\underline{F}} \cdot \hat{\phi}_i \\ P_{\phi_s \theta_i} &\equiv \hat{\phi}_s \cdot \underline{\underline{F}} \cdot \hat{\theta}_i \\ P_{\phi_s \varphi_i} &\equiv \hat{\phi}_s \cdot \underline{\underline{F}} \cdot \hat{\phi}_i \end{aligned} \tag{13b}$$

Two observations should be noted in the system of Equations (10 - 13). First, the dyadic amplitude $\underline{\underline{F}}$ is in a sense more general than, say, the P of Equation (13), in that $\underline{\underline{F}}$ provides a full description of the problem without having specified the bases, e.g., linear-linear, circular-circular, linear-circular. The other observation is that, in considering the bistatic problem, one is led quite naturally to choose two distinct coordinate systems for the description of incident and scattered polarization states. This has the notable consequence that when one specializes P to the monostatic case, choosing $\hat{k}_s = -\hat{k}_i$ (for which one easily verifies that $\hat{\theta}_s = \hat{\theta}_i$, $\hat{\phi}_s = -\hat{\phi}_i$) the linear-linear polarization matrix, say for a conducting sphere, becomes ($|a|^2$ = radar cross section)

$$P = \begin{vmatrix} a & 0 \\ 0 & -a \end{vmatrix} \tag{14}$$

This result differs from the accepted (single coordinate system) description of monostatic scattering, in which the minus sign is absent. Regardless of accepted convention, however, the two-coordinate system description giving rise to Equation (14) appears almost imperative wherever bistatic as well as monostatic measurements are being considered.

So far we have seen how the dyadic amplitude $\underline{\underline{F}}$ $(\hat{k}_s, \hat{k}_i)$, which might be regarded as the basic quantity, relates to the bistatic polarization matrix P and hence experimental measurement. It

only remains to relate $\underline{\underline{F}}$ to the theoretical formulation of the problem, in particular to the transition matrix T. The desired result is obtained almost immediately, by 1) introducing the appropriate plane wave expansion coefficients [2] in the incident wave of Equation (1 , 2) employing the asymptotic form of the spherical Hankel functions valid at large distances from the origin [6] in the scattered wave of Equations (4), and finally, (3) comparing the result with Equation (6) to get

$$\underline{\underline{F}}(\hat{k}_s , \hat{k}_i) = \sum_{\substack{\sigma mn \\ \sigma' m' n'}} (-i)^n \left| \frac{\varepsilon_m (2n + 1)(n - m)!}{(n + m)!} \right|^{1/2} \tag{15}$$

$$\left\{ [\underline{C}(\hat{k}_s), \quad i\underline{B}(\hat{k}_s)] \begin{vmatrix} T^1 & T^2 \\ T^3 & T^4 \end{vmatrix} \begin{vmatrix} \underline{C}(\hat{k}_i) \\ -i\underline{B}(\hat{k}_i) \end{vmatrix} \right\} (i)^{n'} \left| \frac{\varepsilon_{m'} (2n' + 1)(n' - m')!}{(n' + m')!} \right|^{1/2}$$

where summation indices have been suppressed in the expression in curly brackets for clarity. This expression when written out consists of the sum of dyads of four types, the first of which is given by

$$\underline{C}_{\sigma mn}(\hat{k}_s) \; T^1_{\sigma mn \sigma' m' n'} \; \underline{C}_{\sigma' m' n'}(\hat{k}_i)$$

and so forth.

Because the spherical vector harmonics $\underline{C}(\hat{k}_s)$ and $\underline{B}(\hat{k}_s)$ have no components in the observation direction $\hat{k}_s$, the constraints of Equation (7) are seen to be satisfied by inspection. Comparison of

Equation (15) with the reciprocity theorem Equation (8), however, in view of orthogonality of the spherical vector harmonics [3], leads to the requirement that the transition matrix be symmetric, i.e.,

$$T = T' \tag{16}$$

or, in more detail, $(T^1)' = T^1$, $(T^2)' = T^3$, $(T^3)' = T^2$, $(T^4)' = T^4$. Finally, the energy conservation requirement (9) gives rise to the constraints that T be unitary-related, i.e., that

$$T'^*T = \mathrm{Re}\ T\ . \tag{17}$$

SECTION III

SOLUTION OF THE SYSTEM OF MATRIX EQUATIONS

Rather than deal with the transition matrix T, the procedure to be described is slightly more transparent if one employs the S-matrix defined by

$$S = 1 - 2T \quad . \tag{18}$$

The derivation of a matrix equation for the determination of S has been discussed elsewhere. Specifically, one has

$$QS = -Q^* \tag{19}$$

where the matrix elements of Q are explicitly known. To this equation may be adjoined the constraints due to Equations (16) and (17), which are readily seen to be

$$S = S' \tag{20}$$

and

$$S'^*S = 1 \tag{21}$$

i.e., S is symmetric and unitary, respectively. Two extremes of view with regard to the system of Equations (19), (20) and (21) are as follows: First, one might truncate the infinite matrix Equation (19),

solve numerically by digital computer, then compare the resulting solution with Equations (20) and (21), the latter thus being employed as consistency checks. On the other hand, one might attempt to treat all three equations from a unified point of view from the onset, obtaining a solution in some sense of Equation (19) subject to the constraints (20) and (21). The first approach has been employed up to now on the computer for bodies of rotational symmetry, and works quite satisfactorily for a restricted range of body shapes and sizes. The second approach is proposed in an effort to extend the range of bodies that can be handled, in view of the fact that the constraints essentially determine three quarters of the solution [i.e., of the $2N^2$ real parameters appearing in the $N \times N$ (truncated) complex matrix S, it can be shown that only $N(N + 1)/2$ are independent, if S satisfies (20 and 21)].

To develop a unified analysis, observe first that if S could be constructed in the form

$$S = U' U \tag{22}$$

where U is unitary, then both constraints would be satisfied by inspection. This suggests that, rather than inverting Q directly in Equation (19), it be made unitary. Thus, consider the upper triangular matrix M (i.e., all elements are zero below the main diagonal) which, by premultiplication, makes Q into a unitary matrix Q_{unit},

viz .

$$MQ = Q_{unit} \cdot \tag{23}$$

Premultiplying Equation (19) by M , one can write

$$Q_{unit}S = - MQ^* = -MM^{*-1} Q^*_{unit} \quad . \tag{24}$$

Upon solving for S there now results

$$S = -Q'^*_{unit} (MM^{*-1}) Q^*_{unit} \quad . \tag{25}$$

Substituting this result in Equation (20), the symmetry constraint, it follows without difficulty that the matrix product MM^{*-1} must be symmetric. But each of the matrices appearing in the product is upper triangular, and their product is again upper triangular. Consequently, the product must be a diagonal matrix. Further, the diagonal elements can be written out explicitly, giving

$$MM^{*-1} = \begin{vmatrix} M_{11}/M_{11}^* & 0 & \cdots \\ 0 & M_{22}/M_{22}^* & \\ 0 & 0 & \\ \vdots & & \\ \cdot & & \end{vmatrix}$$

If next we can arrange to choose the diagonal elements of M to be real, then

$$MM^{*-1} = 1 \ . \tag{25}$$

From Equation (24) the S-matrix is now given by

$$S = - Q'^{*}_{unit} \, Q^{*}_{unit} \tag{26}$$

which is of the required form (22). Substituting Equation (26), along with the identity $Q'^{*}_{unit} \, Q_{unit} = 1$ back in Equation (18), the desired transition matrix is finally given by

$$T = Q'^{*}_{unit} \, \mathrm{Re} \, (Q_{unit}) \ . \tag{27}$$

Returning to M for a moment, Equation (25) states simply that M is real. Thus, the process may be summed up in the (formal) theorem: Given the matrix Equation (19), with constraints (20) and (21) on the solution, it follows that the given matrix Q cannot be arbitrary, but must be such as to be factorizable into the product of a <u>real</u> upper triangular matrix and a unitary matrix, namely

$$Q = M^{-1} \, Q_{unit} \ . \tag{28}$$

In order to carry out numerically the above analysis, which is formal in the sense that infinite matrices have been treated without rigorous verification of the consequent limiting processes involved, it is of course necessary to truncate S and Q at a finite number N of rows and columns. Having done this, construction of the $N \times N$ unitary matrix Q_{unit} is done by Schmidt orthogonalization of the N vectors given by the rows of Q, beginning with the bottom row and working up. Thus, writing $\underline{q}_N$ for the N component vector obtained from the bottom row, $\underline{q}_N$ is first normalized to unit length by multiplying by

$$(\underline{q}_N{}^* \cdot \underline{q}_N)^{-1/2} \equiv \left| \sum_{n=1}^{N} Q^*_{Nn} \, Q_{Nn} \right|^{-1/2}$$

This constant, which is real by definition, is just the last diagonal entry M_{NN} of the triangular matrix, whereas the normalized vector constitutes the bottom row of Q_{unit}. Next one chooses a linear combination of $\underline{q}_N$ and the vector $\underline{q}_{N-1}$ obtained from the next to last row, so that the resulting vector is orthogonal to $\underline{q}_N$ and so forth.

A brief numerical investigation of this procedure has been performed in order to ascertain its feasibility and usefulness in the computer program for electromagnetic scattering. Results indicate more rapid convergence, versus truncation size N, than is obtained by straightforward numerical solution of Equation (19) employing matrix inversion.

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The purpose of this discussion is to relate the previously developed matrix analysis of electromagnetic scattering explicitly to the experimentally measurable bistatic polarization matrix. In addition, in order to perform the numerical computations more conveniently, a new matrix analysis is described, in which the constraints of reciprocity and energy conservation are employed in the course of obtaining a solution.

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