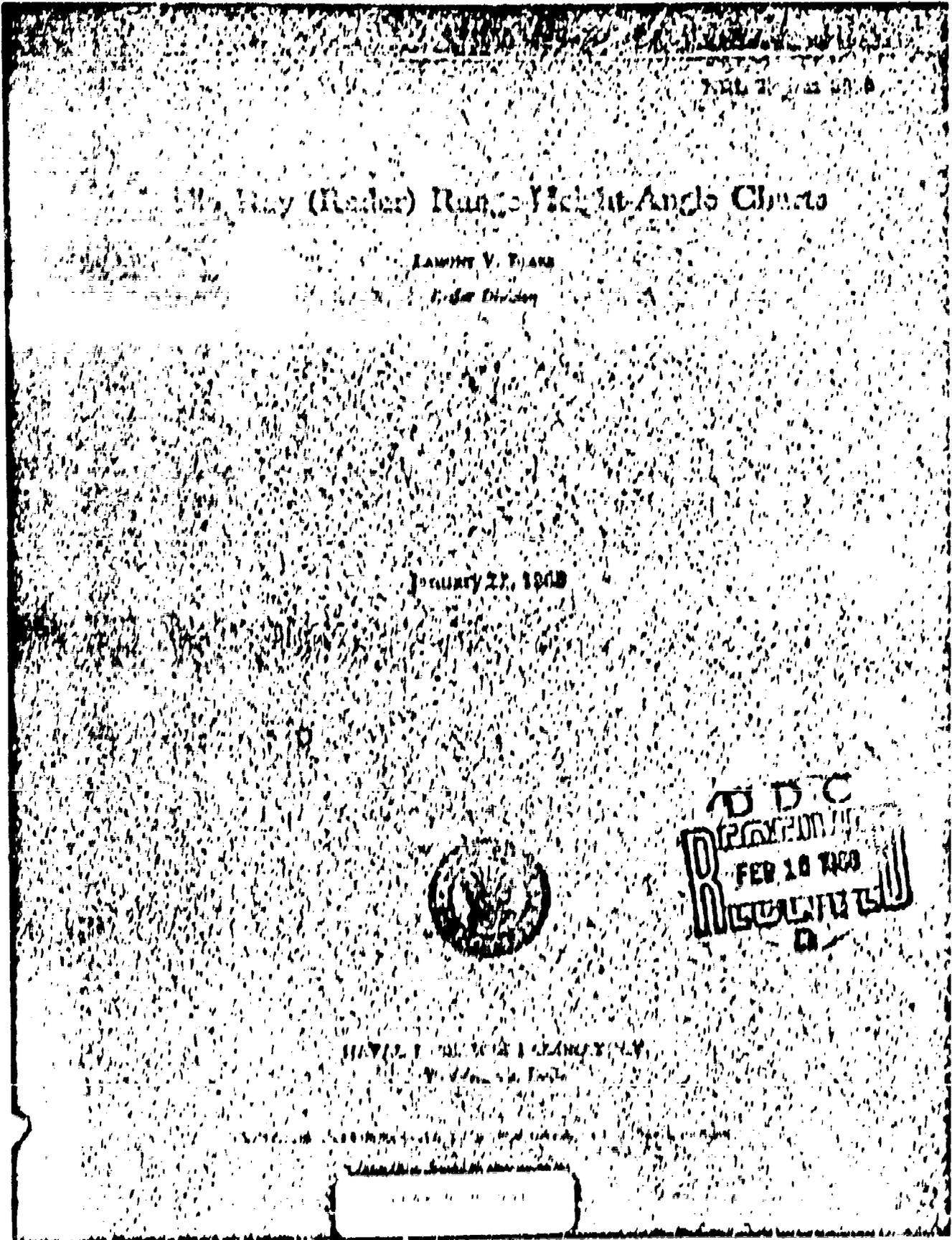


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ABSTRACT

Charts showing the relationship of range, height, and initial elevation angle of the rays from an earth-based antenna have been machine-computed and machine-drawn. The refracted (curved) ray paths are represented on the charts as straight lines, which facilitates plotting radio and radar vertical-plane coverage diagrams. The negative-exponential model of tropospheric refractivity has been assumed, although the method permits any model representable by a piecewise-continuous function to be used. Charts have been made for various systems of range, height, and angle units, with equal and unequal range and height scales, with both linear and nonlinear scales, and for both small and large maximum range and height.

PROBLEM STATUS

This is an interim report on one phase of the problem; work on this and other phases is continuing.

AUTHORIZATION

NRL Problem R05-28
Project RF 008-04-41-4508

Manuscript submitted September 22, 1967.

RADIO RAY (RADAR) RANGE-HEIGHT-ANGLE CHARTS

INTRODUCTION

For plotting earth-based radio or radar system vertical-plane coverage diagrams, a chart with range, height, and angle coordinates is needed. Such a chart should take into account the slight but significant downward bending of radio rays caused by atmospheric refraction. To be most useful, the chart should represent the ray paths as straight lines, to facilitate plotting. The vertical-plane coverage is often described in terms of range and angle rather than range and height.

Such charts were easily constructed for the Schelleng-Burrows-Ferrell (SBF) "effective-earth-radius" model of atmospheric refraction (1), which assumes a linear negative gradient of the index of refraction as a function of height. For this model, the ray lines appear straight in a plot for which the earth's radius is taken to be greater than the true radius by a factor k given by

$$k = \frac{1}{1 + \frac{r_0 (dn/dh)}{n_0}} \quad (1)$$

where r_0 is the earth's radius, n_0 is the index of refraction at zero height, and dn/dh (a negative constant) is the gradient of the refractive index. A standard value for k is $4/3$, corresponding to $(dn/dh) n_0 = -1/r_0$. In the early days of radar (in fact, until the late 1950's) it was customary to plot radar-coverage diagrams using charts constructed on this principle.

Subsequently, the "exponential" model of the atmospheric refractivity was introduced (2,3) as an improvement on the effective-earth-radius model. This model avoids the height errors inherent in the SBF model for ray paths that travel long distances at low angles. Unfortunately, this model permits no simple method of constructing a chart with straight ray lines. A method of construction was devised by the author in 1958,* and samples of the resulting charts are shown as Figs. 1 and 2. They are plotted to show heights to 100,000 ft, and the maximum range is about 350 naut mi. These charts have been widely used.

CHARTS WITH NONLINEAR RANGE-HEIGHT SCALES

There has subsequently been some demand for charts that go to greater ranges and heights, and also for charts that give an expanded presentation of the lesser ranges and heights. The idea of constructing a chart with nonlinear range and height scales, which could result in achieving both of these goals in a single chart, was appealing. A logarithmic scale is naturally considered for such a purpose. However, it cannot be used, because it has no "origin" (zero-zero point); the origin is at minus infinity on a log scale. This point must be locatable in order to plot ray lines that emanate from it.

*A 1958 report, co-authored with Frank D. Clarke, described the procedure. This report, NRL Memorandum Report 879, is now out of print. The method is therefore re-described here, in a more general form to allow for the use of nonlinear range and height scales.

L. V. BLAKE

Plotted to present refracted ray paths as straight lines for cool-weather atmosphere. May be used for gear-round atmosphere with small error, except below 2 degree elevation in warm, humid weather, or when "ducting" occurs.

Constructed on the basis of analysis and computations by M. I. T. Lincoln Laboratory as presented in Technical Report No. 186 dated 1 August 1950. See also NRL Memorandum Report 879, December 1950, for details of construction.

"Slant range" here means "distance along the ray path."

Antenna height assumed 80 feet. May be used for antenna heights (h) up to 100 feet if height readings are corrected by adding (h-80) feet.

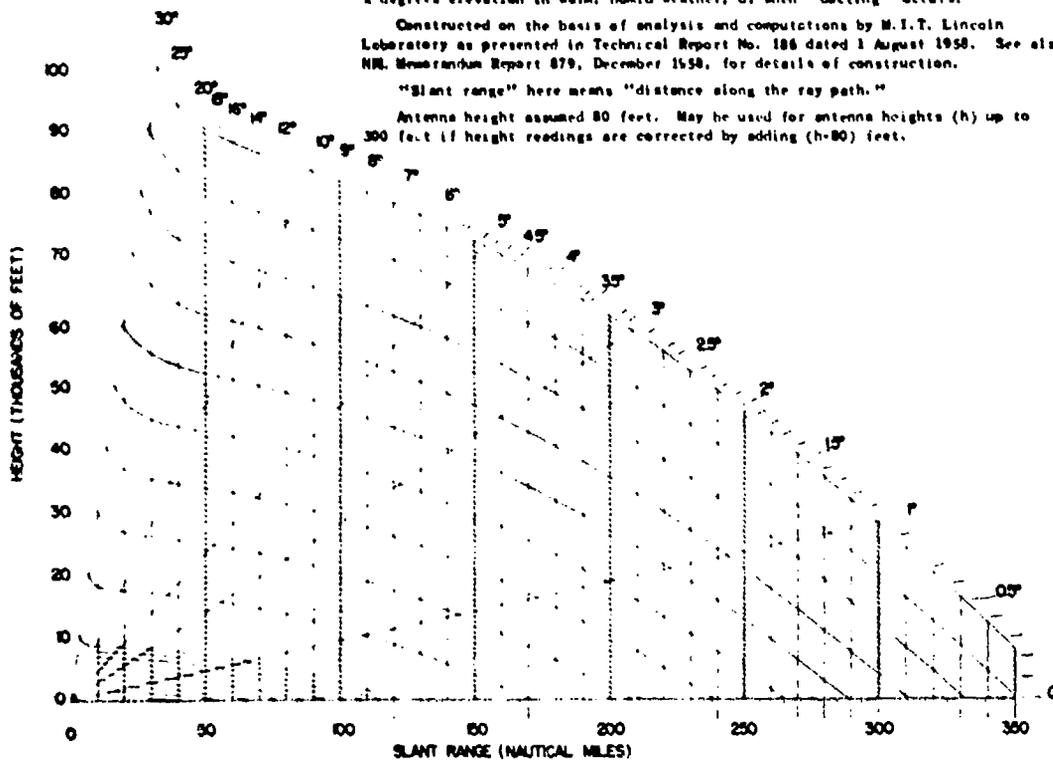


Fig. 1 - Hand-drawn linear-scale 100,000-ft-height chart with straight vertical constant-range lines

The problem has been solved by using fractional-power-law scales. For example, the coordinate r corresponding to a range R at zero elevation angle is given by

$$r = r_{\max} \cdot (R/R_{\max})^p \quad (2)$$

where p is a positive number between zero and one, r_{\max} is the maximum value of r , and R_{\max} is the maximum value of R . As examination of some of the resulting charts shows, such scales have properties approximating those of logarithmic scales. For example, successive decades of the scale, though not of equal length as they would be in a log scale, differ by a factor of less than 2 for $p = 0.25$. Moreover, unlike a log scale, a power-law scale has an $r = 0$ point, as required. For the linear-scale charts, Figs. 3 and 4, p is of course equal to one.

If the range and height scales are made the same — that is, if the horizontal distance on the zero-elevation-angle line corresponding to a range R is the same as the distance on the 90-degree-elevation-angle line corresponding to a height $h = R$ — then angles on the chart correspond to the actual initial elevation angles of the rays, and the loci of constant range are circular arcs. The loci of constant height are then plotted in this coordinate system by computing range as a function of angle, for fixed values of height. These height curves are downward-curving lines that are not in general orthogonal to either the range lines or the ray lines. The technique for obtaining a straight-ray-line chart is thus simply to plot the ray lines as straight lines, plot the constant-range loci, and constrain the height lines to fit this system of coordinates.

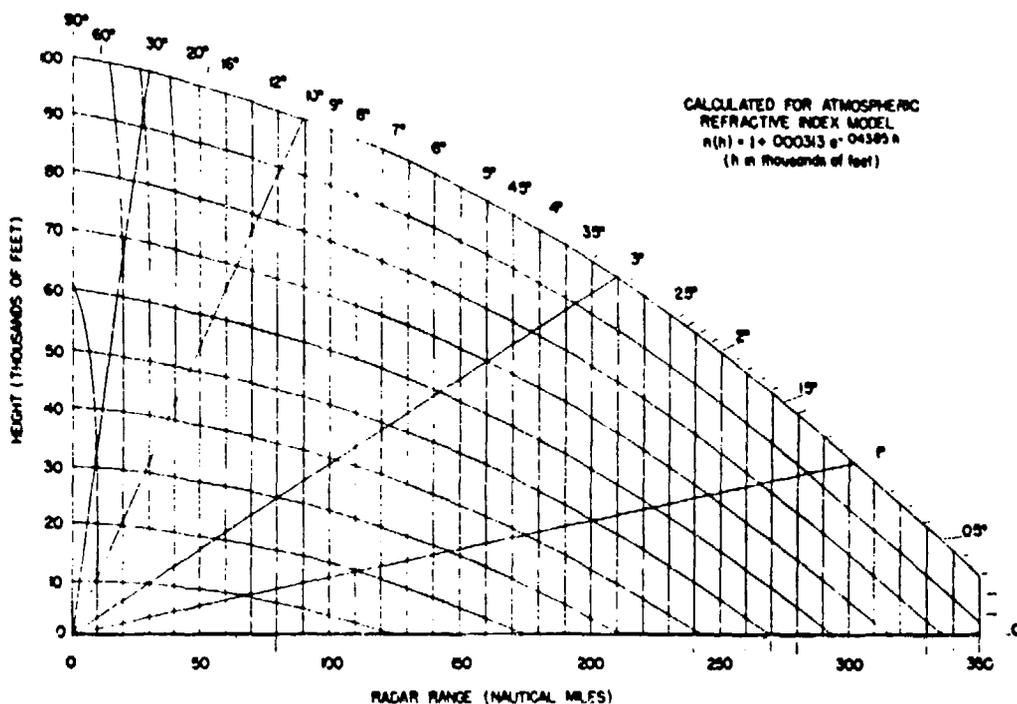


Fig. 2 - Hand-drawn linear-scale 100,000-ft-height chart with elliptical constant-range lines

If the range and height scales are not the same (in the sense defined previously), then the loci of constant range are segments of ellipses, and the angle scale is nonlinear—that is, the angle on the paper corresponding to a given angle in space is greater at the low angles and less at the high angles. This feature may be advantageous when the low-angle region is of primary interest.

Moreover, as shown in Fig. 1, the constant-range loci can be vertical straight lines if the chart is to be restricted to angles below 90 degrees. With this type of chart, the lines of constant height become asymptotically vertical at 90-degree elevation angle, whereas they become horizontal when the range lines are circles or ellipses, as shown by Fig. 2.

MACHINE PLOTTING

A serious problem in the construction of the charts shown as Figs. 1 and 2 was the draftsmanship. Many tedious manhours were spent constructing them. Figure 1 was done by a professional draftsman, while Fig. 2 was done by the author. The precision of Fig. 1 is excellent, while that of Fig. 2 is only fair. This problem is even more severe if manual drafting methods are used for charts with nonlinear scales. However, equipment is now available at the Naval Research Laboratory with which these charts can be

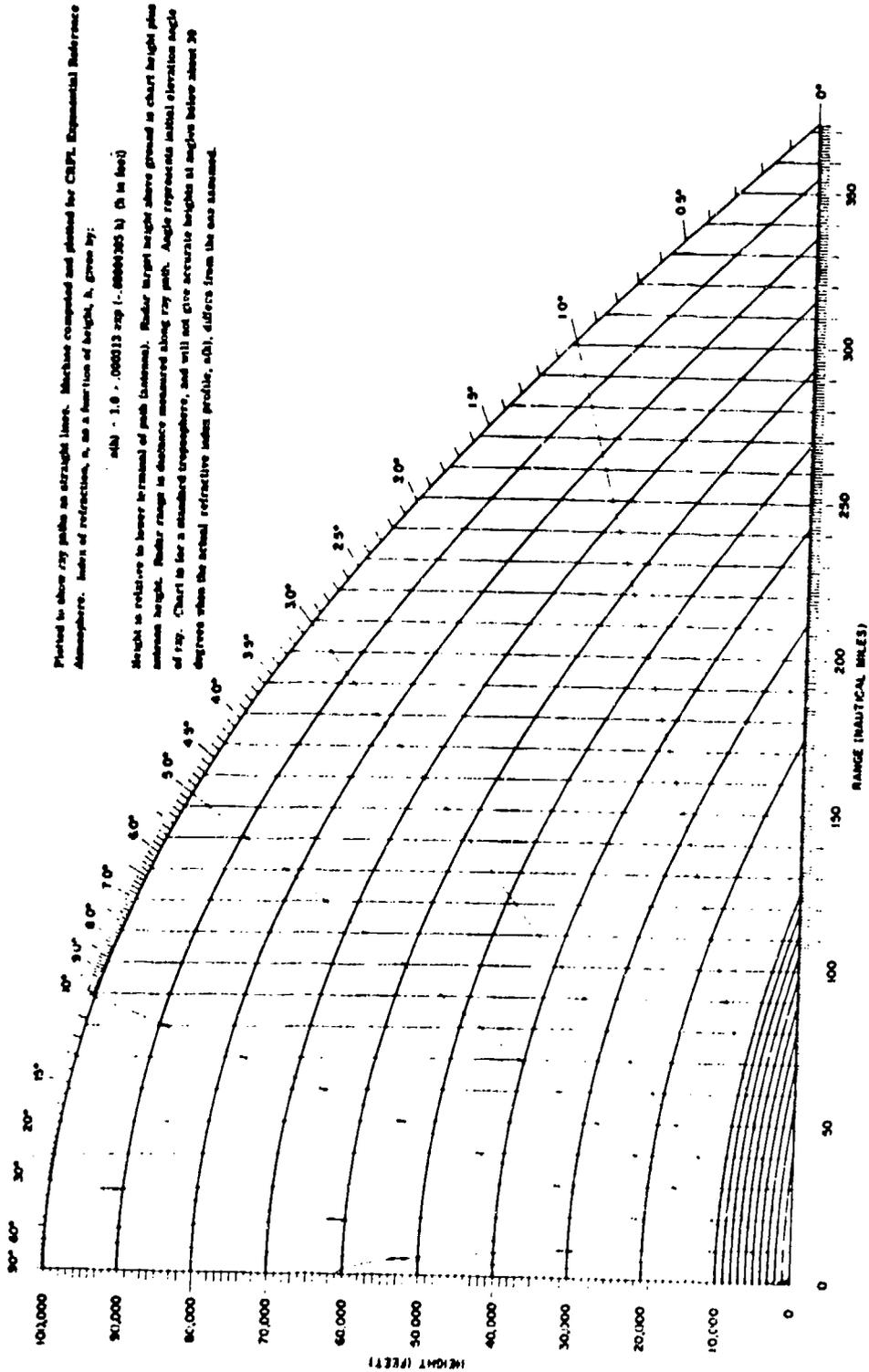


Fig. 3 - Machine-drawn linear-scale 100,000-ft.-height chart with elliptical constant-range lines

Plotted to show ray paths as straight lines. Machine computed and plotted for CRPL Exponential Reference Atmosphere. Index of refraction, n , as a function of height, h , given by:

$$n(h) = 1.0 - .000313 \exp(-.000041933 h) \quad (h \text{ in feet})$$

Height is relative to lower terminal of path (antenna). Radar target height above ground is chart height plus antenna height. Radar range is distance measured along ray path. Angle represents initial elevation angle of ray. Chart is for a standard troposphere, and will not give accurate heights at angles below about 30 degrees when the actual refractive index profile, $n(h)$, differs from the one assumed.

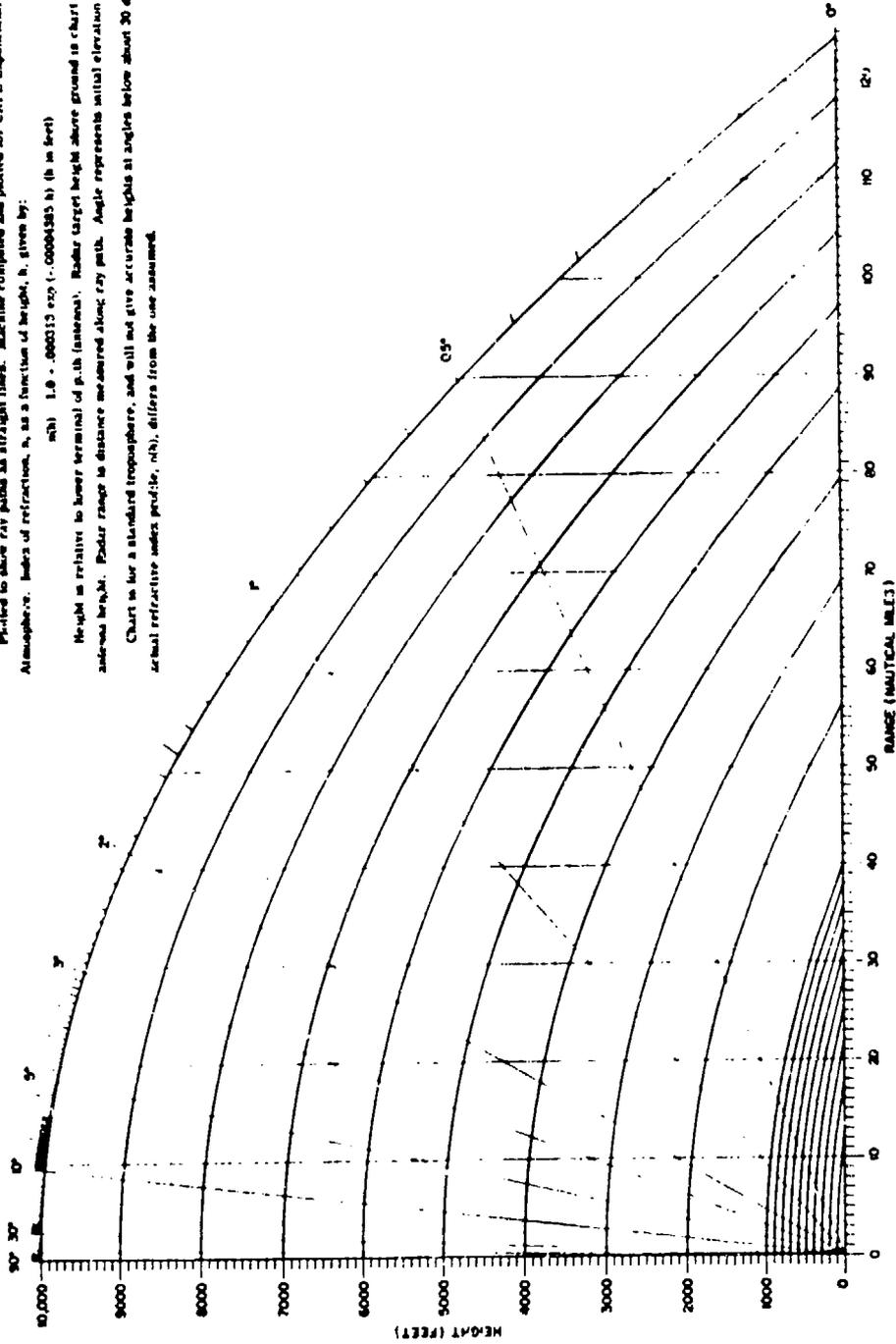


Fig. 4 - Machine-drawn linear-scale 10,000-ft-height chart with elliptical constant-range lines

machine plotted. The machine* is driven by a punched paper tape which provides xy coordinates of the curves and lines to be plotted. With this machine, highly precise charts can be produced with minimal labor and time.

DESCRIPTION OF CHARTS PLOTTED

Partly as an exercise, a chart with a format almost identical to that of Fig. 2 was computed and machine drawn. It is shown as Fig. 3. Although the constant-range lines at large ranges appear to be vertical straight lines, they are actually segments of ellipses. As can be seen, the agreement between the two charts is excellent, but the machine-drawn chart is far more precise. Also, additional height lines, height-scale tick marks, and angle tick marks have been provided. Then, a similar chart with smaller maximum height and maximum range, Fig. 4, was drawn. This chart is suitable for coverage diagrams that are restricted to the height region below 10,000 ft.

Then a chart with 1/4-power range and height scales ($p = 0.25$) was plotted (Fig. 5). It has a maximum range of about 1000 naut mi and a maximum height of 1,000,000 ft (about 164 naut mi). These scales are suitable for some aerospace applications not covered by Figs. 1 through 4. On this chart the height and range scales are equivalent; therefore the range lines are circular arcs, and the angle scale is true.

Next, a similar chart bounded in both height and range by the 1500-naut-mi range circle was drawn (Fig. 6). This chart is suitable for showing the coverage of radar or radio systems that exceed the limits of Fig. 5. Figure 7 is a metric-system version of the same chart, to 3000 km (1620 naut mi). Finally, Fig. 8 is a chart with 1/2-power ($p = 0.5$) range and height scales, but otherwise similar to Fig. 3. The elliptical shape of the constant-range lines is clearly evident here. This single chart combines the advantages of Figs. 3 and 4 — i.e., expansion of the close-in region, and fairly large maximum range and height.

REFRACTIVE-INDEX MODEL

These charts are all plotted for the CRPL Exponential Reference Atmosphere (C) with $\lambda = 313$, which means that the index of refraction n as a function of height h is given by

$$n(h) = 1.0 + 0.000111 e^{-0.111h/313} \quad (3)$$

where h is in kilometers. The ray-height values as a function of range and angle, and the corresponding xy coordinates required for the machine plotting, were computed by the NRL CDC-3800 computer from Fortran programs written by the author. The refractive-index model can be changed by changing a single data card of the program, so that similar charts can be readily produced for any other values of the exponential constants, or in fact for any other mathematical model that is representable by a piecewise-continuous function (although in that case a function subprogram would have to be revised, rather than a single data card).

*This digitally controlled drafting machine is Gerber Scientific Co. Model 875. It is capable of plotting to a maximum size of 5 ft by 8 ft with a precision of 0.005 in. The pen travels in a straight line between successive xy coordinate values. A turret head allows selection of various pen widths.

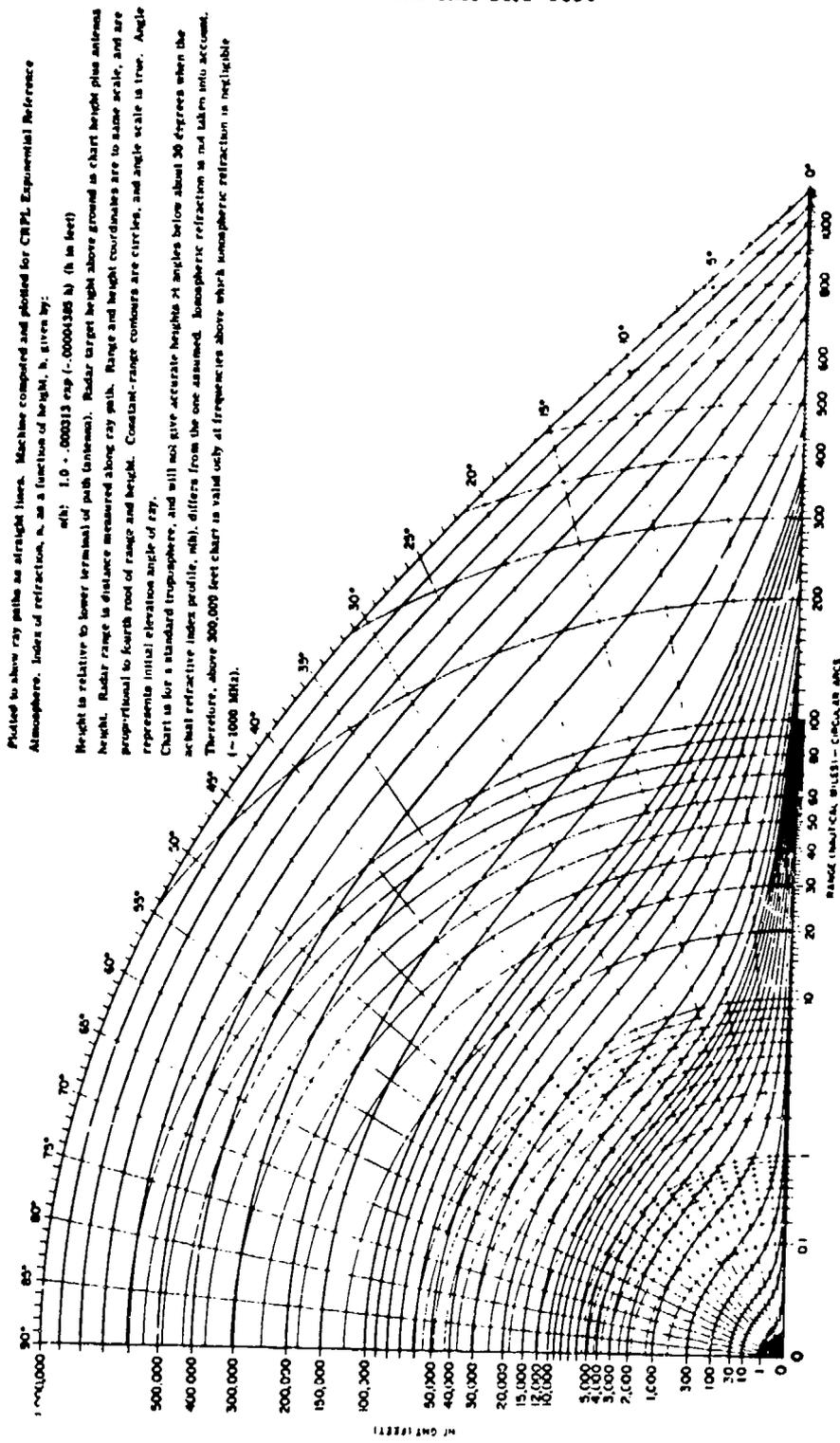


Fig. 5 - Machine-drawn 1/4-root-scale 1,000,000-ft-height chart with circular constant-range lines

L. V. BLAKE

Plotted to show ray paths as straight lines. Machine computed and plotted for CRPL Exponential Reference Atmosphere. Index of refraction, n , as a function of height, h , given by:

$$n(h) = 1.0 - 0.000313 \exp(-.266421 h) \quad (h \text{ in nautical miles})$$

Height is relative to lower terminal of path (antenna). Radar target height above ground is chart height plus antenna height. Radar range is distance measured along ray path. Range and height coordinates are to same scale, and are proportional to fourth root of range and height. Constant-range contours are circles, and angle scale is true. Angle represents initial elevation angle of ray.

Chart is for a standard troposphere, and will not give accurate heights at angles below about 30 degrees when the actual refractive index profile, $n(h)$, differs from the one assumed. Tropospheric refraction is not taken into account. Therefore, above 50 n. mi. chart is valid only at frequencies above which tropospheric refraction is negligible (≈ 1000 MHz).

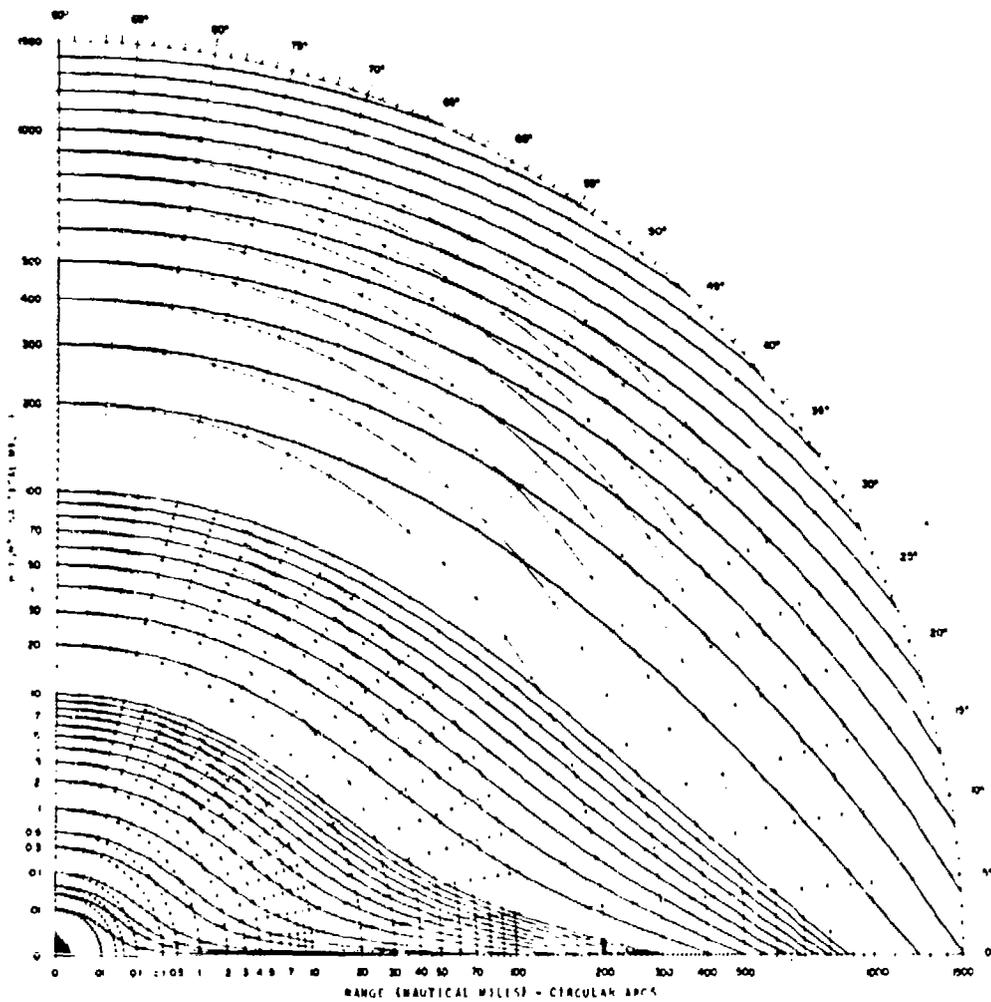


Fig. 6 - Machine-drawn 1/4-root-scale 1500-naut-mi-range chart with circular constant-range lines

Plotted to show ray paths as straight lines. Machine computed and plotted for CRPL Exponential Reference Atmosphere. Index of refraction, n , as a function of height, h , given by:

$$n(h) = 1.0 - .000313 \exp(-.142857 h) \quad (h \text{ in kilometers})$$

Height is relative to lower terminal of path (antenna). Radar target height above ground is chart height plus antenna height. Radar range is distance measured along ray path. Range and height coordinates are to same scale, and are proportional to fourth root of range and height. Constant-range contours are circled, and angle scale is true. Angle represents initial elevation angle of ray.

Chart is for a standard troposphere, and will not give accurate heights at angles below about 20 degrees when the actual refractive index profile, $n(h)$, differs from the one assumed. Ionospheric refraction is not taken into account. Therefore, above 100 km chart is valid only at frequencies above which ionospheric refraction is negligible (~ 4000 MHz).

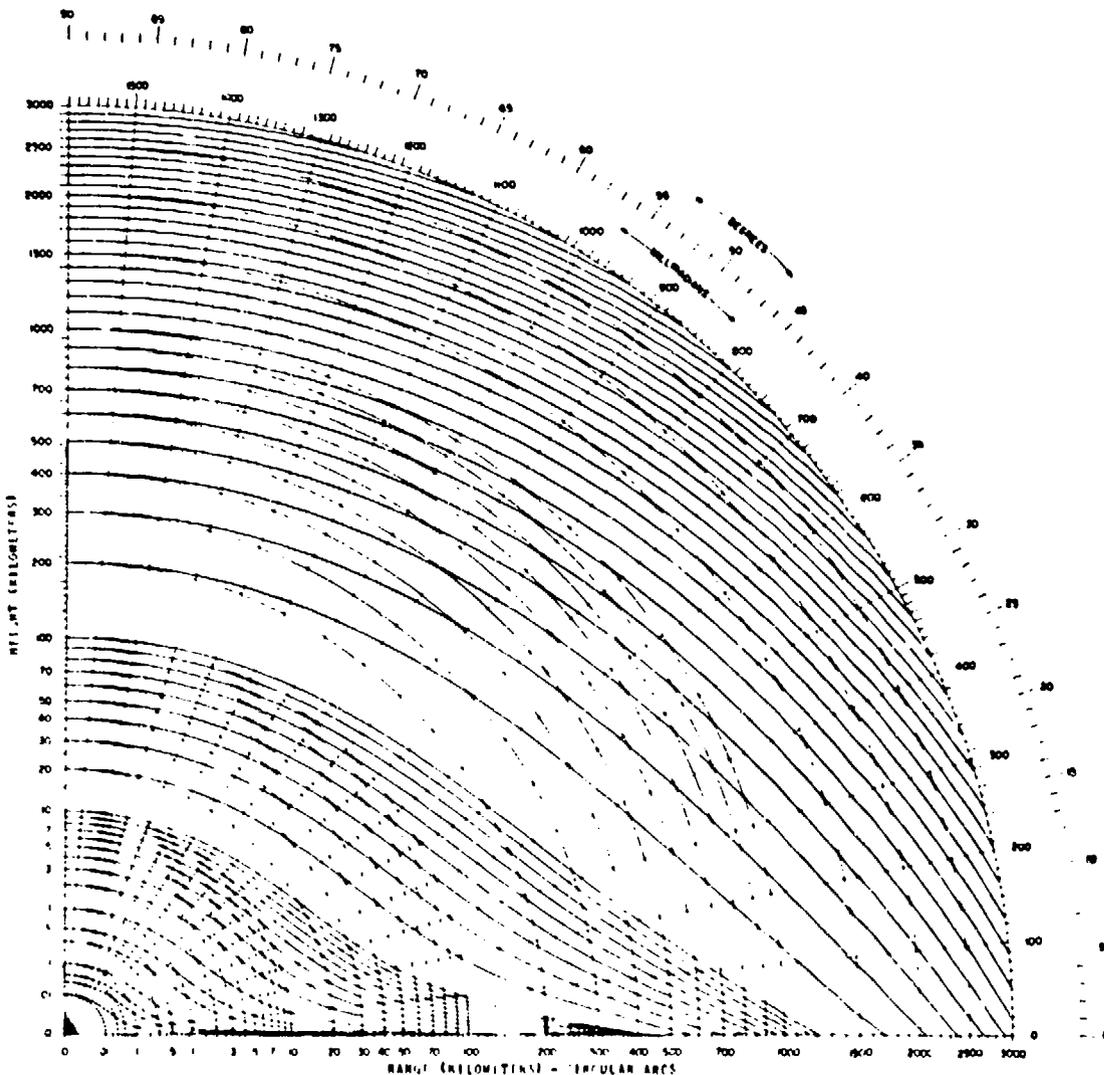


Fig. 7 - Machine-drawn 1/4-root-scale 3000-km-range chart with circular constant-range lines

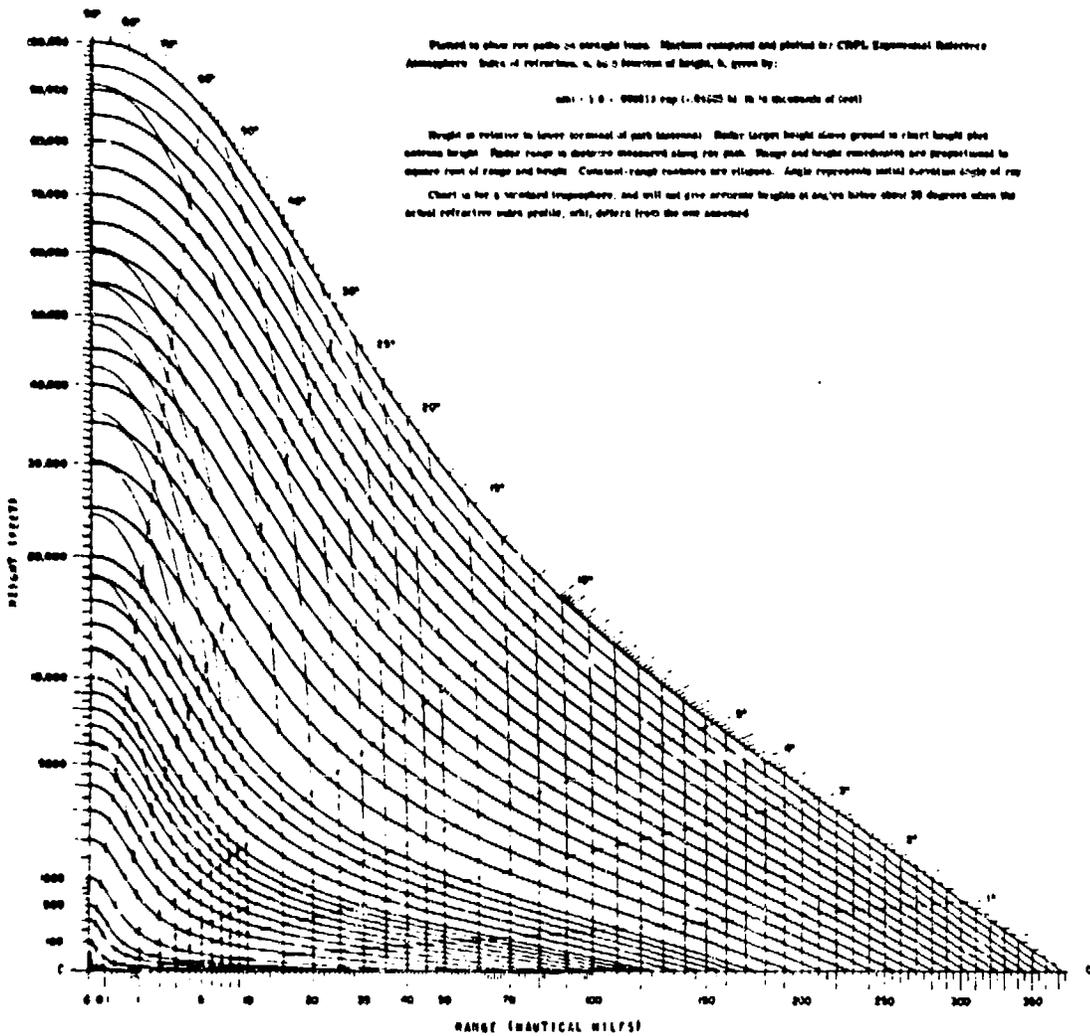


Fig. 8 - Machine-drawn square-root-scale 100,000-ft-height chart with elliptical constant-range lines

Some of these charts go to heights that are well into or above the ionosphere, which starts at roughly 100 km, or about 300,000 ft. Since ionospheric refraction has not been taken into account, the charts should be used above these heights only for frequencies above those at which significant ionospheric refraction occurs. This requirement is met at frequencies above 1000 MHz under daytime ionospheric conditions, and to somewhat lower frequencies at night. It would be possible to make a chart that does include the effect of ionospheric refraction; however, such a chart would be valid only over a small region of frequency.

The charts are also valid only for tropospheric conditions that conform to Eq. (3). Thus for accurate radar height-finding or similar purposes, a chart for the specific atmospheric index variation actually encountered should be used. The charts in this report are merely intended to represent a "standard" exponential atmosphere, in the same sense that the 4/3-earth-radius SBF model has been used as a standard linear atmosphere. The exponential model gives much more accurate range-height-angle information, at long ranges, low angles, and great heights, than does the SBF model, although the latter is adequate for low-altitude, short-range work. Incidentally, an SBF model conforming to the zero-altitude gradient of the exponential model of Eq. (3) would be one with $k \approx 1.4$, as defined by Eq. (1), or about 7/5, instead of 4/3. In other words, the $k = 4/3$ SBF model represents somewhat less than average refraction for the United States; it corresponds more nearly to cool-weather conditions (2).

The most suitable exponential model for use as a convention is in fact a matter on which not all agree. Some qualified workers feel that a cool-weather model would be a better choice, arguing that the CRPL model of Eq. (3) is an average of conditions that include some abnormally high summertime values when ducting conditions exist, so that the resulting average is unrealistic as an average *exponential* model. However this may be, the primary purpose here is not to argue for adoption of a particular refractive-index model, but rather to describe the procedure used to produce useful charts. On the other hand, the CRPL model is not wholly unsuitable as a convention, since it does represent refraction conditions lying somewhere between the maximum and minimum that will be encountered in practice. If or when some authoritative standards organization adopts a different standard exponential model, it will be a simple matter to produce charts of the type described here for that model.

Incidentally, Fig. 1 is a chart based on the cool-weather exponential model of Ref. 2:

$$n(h) = 1.0 + 0.000320 e^{-0.0001217h} \quad (3a)$$

where h is in feet. The corresponding decay constant for h in kilometers is 0.1217. Comparison of Figs. 1 and 2 indicates the extent to which the range and height values differ for the two models. Figure 2 is for the model of Eq. (3).

EQUATIONS FOR COMPUTING CHART COORDINATES

The first step for plotting a range-height-angle chart is to compute the range values R corresponding to specified values of height h , and angle θ . The basic equation for this computation is

$$R(h, \theta) = \int_0^h \frac{n(h) dh}{\sqrt{1 - \left[\frac{n(h) + n^2(h)}{n^2(h) + (1 + h^2/r_0^2)} \right]}} \quad (4)$$

where r_e is the true radius of the earth, n_0 is the value of $n(h)$ at $h = 0$, and $n(h)$ is the refractive index given as a function of height, in this case by Eq. (3). Equation (4) is derived from Snell's law, as explained in Ref. 4 and elsewhere.⁶ This equation is solved numerically by the computer using a Simpson's rule subroutine. The conventional value 6370 km was used for r_e .

The x and y cartesian coordinates of a point on the chart corresponding to a given range, height, and angle are found fairly simply when the range and height scales are the same and the angle scale on the paper is "true." Since the constant-range lines are circles, it is known that the xy coordinates corresponding to a range R lie on a circle of radius r given by

$$r^2 = r_e^2 + (R/n_{max})^2 \quad (5)$$

where the quantities are as defined for Eq. (3). In fact, the equation is the same, except that Eq. (5) gives the value of r that lies on the zero-elevation-angle line, which is equal to r_e . The equations for the xy point corresponding to a pair of range R and angle θ values are of course

$$x = r \cos \theta \quad (6)$$

and

$$y = r \sin \theta \quad (7)$$

The procedure for plotting a chart is first to solve Eq. (4) for the range value corresponding to given height and angle, then to compute x and y for this range and angle from Eqs. (5), (6), and (7). A slight programming problem arises because in order to plot a locus of constant height, it is required to generate successive xy values along a line of constant height, taking the elevation angle as the independent variable. The problem is that for computational economy and simplicity it is easier to compute R as a function of increasing θ with h constant. However, the required sequence of xy values along a constant height line can be generated by suitable programming. The program listings are not given in this report.⁷

The computation of the x and y values is slightly more complicated when the range and height scales are different, so that the angle scale is nonlinear, and the constant-range loci are segments of ellipses. It is required first to compute the ellipticity factor λ , which is the ratio of the major semi-axis of the ellipse to the minor semi-axis. For a chart whose maximum x and y coordinate values (e.g., in inches) are x_{max} and y_{max} , and whose maximum range and height values are R_{max} and h_{max} , this factor is given by

$$\lambda = (y_{max}/x_{max}) + (R_{max}/h_{max})^2 \quad (8)$$

where r is defined as in Eq. (5), and R_{max} and h_{max} are expressed in the same units of distance.

The xy values for an ellipse of constant range as a function of the angle $\theta = \tan^{-1}(y/x)$ and the major and minor semi-axes a and b are given by:

⁶The factor $n(h)$ in the numerator is not derived from Snell's law, but is included to make $R(n, \theta)$ correspond to the time of travel of a wave packet over the ray path - i.e., the distance as it would be measured by a radar, rather than that which would be measured by a tape measure.

⁷Program listings will be furnished to qualified requesters. Address inquiries to the Director, Naval Research Laboratory, Washington, D.C. 20370, mentioning NRL Report 6690.

$$x = \frac{a}{\sqrt{1 + (1 - a^2) K^2}} \quad (9)$$

and

$$y = b \sqrt{1 - x^2} \quad (10)$$

The parameters a and b are computed from the equations

$$a = r_{max} \cdot (R \cdot R_{max})^P \quad (11)$$

and

$$b = K \cdot a. \quad (12)$$

An angle θ on the chart (i.e., the angle corresponding to a point xy as defined by $\tan^{-1}(y/x)$) is related to the ray initial-elevation angle θ_0 by the expression

$$\tan \theta = K \tan \theta_0. \quad (13)$$

Consequently, the x and y coordinates of a point on a constant-height line are found from the R and θ_0 values by first finding θ from Eq. (13), next finding a and b from Eqs. (11) and (12), and then applying Eqs. (9) and (10).

NOTES ON CHART CONSTRUCTION AND USE

Not only the height lines, but also the constant range lines, the angle lines, and the range, height, and angle tick marks are machine computed and machine plotted. The only hand work on the charts is the lettering of the legend and the numbering of the coordinate scales. It was a problem to decide how densely to draw the range, height, and angle lines, especially on the nonlinear charts, where accurate interpolation between adjacent lines is difficult. If too many lines are drawn, however, the chart becomes excessively cluttered and hard to use. Ordinarily the charts are useful when data are available in the form of pairs of range-angle values, and the corresponding heights can then be found from the chart. It is assumed, however, that high accuracy will not be needed when the charts are so used. If high accuracy is required, then first of all an actual rather than a standard or conventional model of the refractive-index variation should be used, as has already been mentioned. Second, the height should then be computed directly rather than being read from a chart. Methods of performing these computations are described in Ref. 4.

The height represented on the chart is that relative to the origin of the ray — the antenna phase center. If the antenna is at an appreciable height above sea level, and the ray (e.g., radar target) height above sea level is desired, the antenna height must be added to the height read from the chart. The charts are computed on the assumption, however, that the antenna is not at a great height above sea level (this is inherent in the assumption that the index of refraction is 1.000313 at the ray origin). Therefore, if the antenna height is much more than about 1000 ft above sea level, these charts should not be used. The refractive index model, Eq. (3), is based on the average indexes and gradients observed over the United States, at stations averaging 700 ft above sea level (3). In more general terms, the charts are meant to serve as conventions for shipboard or land-based antenna sites at low or moderate altitude above sea level.

These charts may be reproduced singly or in quantity without explicit permission. Where they are used in publications, mention of their source will be appreciated. Larger size copies, more suitable for reproduction than those included in this report, can be furnished to qualified organizations upon request.

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10 KEY WORDS	LINE A		LINE B		LINE C	
	ROLE	WT	ROLE	WT	ROLE	WT
Radio Radar Range Height Elevation angle Ray charts Refraction						