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**Linear Programming Models  
for Construction Planning Using the  
Engineer Functional Components System**



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Engineer Functional Components System**

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by  
Jerome Bracken  
Albert D. Tholen

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## **FOREWORD**

The linear programming models for theater construction planning described in this paper have been developed within the scope of efforts on the Theater Construction Model of the Simulation and Gaming Methods for Analysis of Logistics (SIGMALOG) system (see F. L. Bartholomew et al, "A Logistic Gaming and Simulation System: General Concept," RAC-TP-179, Jan 66). These optimization models are expected to play an important part in the more general construction model required by the system. The models can be used as now formulated for detailed analysis at an item level of construction planning problems.

**Lee S. Stoneback**  
Head, Logistics Department

## **ACKNOWLEDGMENTS**

We would like to acknowledge the help of Mr. Robert E. Pugh in designing and programming routines for extracting information from Engineer Functional Components System files and producing data suitable for direct input to the linear programming routines. LTC Richard W. Boberg of Engineer Strategic Studies Group reviewed the models and made very helpful comments on their worth and potential usefulness. A RAC review board consisting of Dr. G. S. Pettee (Chairman), Dr. Harold Fassberg, and Mr. Lazarus H. Todd gave helpful criticism. The Project Advisory Group with LTC Richard L. Moody as chairman also reviewed the document and gave useful suggestions.

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**Linear Programming Models  
for Construction Planning Using the  
Engineer Functional Components System**

## **ABBREVIATIONS**

|               |                                                          |
|---------------|----------------------------------------------------------|
| <b>DA</b>     | <b>Department of the Army</b>                            |
| <b>DA-SLS</b> | <b>Department of the Army Strategic Logistic Studies</b> |
| <b>DCSLOG</b> | <b>Deputy Chief of Staff for Logistics</b>               |
| <b>USACDC</b> | <b>United States Army Combat Developments Command</b>    |
| <b>USAMC</b>  | <b>United States Army Materiel Command</b>               |

### NOTATION USED <sup>†</sup>

- $a_{ij}$  = the number of units of resource  $i$  needed in the construction of one facility  $j$ ,  
where  $i = 1, \dots, m$  and  $j = 1, \dots, n$
- $a'_{jk}$  = the number of units of facility  $j$  needed in the construction of one unit of installation  $k$ , where  $j = 1, \dots, n$  and  $k = 1, \dots, p$
- $b_{ik}$  = the number of units of resource  $i$  needed in the construction of one unit of installation  $k$ , where  $i = 1, \dots, m$  and  $k = 1, \dots, p$
- $c_j$  = a weighting factor expressing the relative worths of facilities to be constructed
- $d_i$  = a limitation on the total number of units of resource  $i$  to be available during the period
- $\bar{d}_i$  = the units of resource  $i$  required to satisfy all the facility requirements
- $d_i^*$  = the units of resource  $i$  required to execute the optimal solution
- $c_k$  = a weighting factor expressing the relative worths of installations to be constructed
- $x_j$  = the unknown number of facilities of type  $j$  to be constructed during the period,  
with lower bound  $x_j^{(l)}$  and upper bound  $x_j^{(u)}$
- $\bar{x}_j$  = the specified number of units of facility  $j$  to be constructed
- $x_j^*$  = the number of facilities constructed in the optimal solution
- $y_k$  = the unknown number of installations of type  $k$  to be constructed during the period,  
with lower bound  $y_k^{(l)}$  and upper bound  $y_k^{(u)}$
- $\bar{y}_k$  = the specified number of units of installation  $k$  to be constructed
- $y_k^*$  = the number of installations constructed in the optimal solution

<sup>†</sup>Superscript  $t$  is used with the same variables in the multiperiod models beginning in the section "Multiperiod Use of One-Period Models."

## ABSTRACT

The Engineer Functional Components System is used by the Army in planning construction of facilities and installations. This operational system employs numerous and extensive tape files containing bills of materials for several hundred types of military installations comprising thousands of items of materiel. The linear programming models discussed herein use this system in formulating linear constraints on construction capabilities due to resource limitations, and linear criterion functions in terms of the various facilities and/or installations to be constructed. The models provide a capability to rapidly consider alternative construction programs. Single-period and multiperiod models are given, including modifications of the basic models to make them applicable to a wider range of problems.

## **INTRODUCTION**

The purpose of this paper is to present models developed for use in choosing combinations of facilities and/or installations to be constructed in a theater of operations, subject to resource limitations. The models incorporate an existing Army computer-assisted planning capability—the Engineer Functional Components System, which has been in use for several years to provide bills of materials required for construction of overseas military facilities. The models handle one-period or multiperiod construction problems. They contain constraints on resources of materiel by item, equipment by type, and manpower by skill category and constraints on numbers of facilities and/or installations to be constructed. Within the constraints the models choose construction tasks to maximize a linear function of the facilities and/or installations constructed.

### **Description of the Engineer Functional Components System**

The Engineer Functional Components System is a tool developed by the Army for planning construction projects. Documentation of the system is contained in three manuals: TM 5-301,<sup>1</sup> TM 5-302,<sup>2</sup> and TM 5-303.<sup>3</sup>

The basic elements of the system are items of materiel or equipment and man-hours of effort. Selected groups of these elements constitute facilities, defined in TM 5-301 as "a grouping of items and/or sets consisting primarily of construction material in the necessary quantities required to provide a specified service, such as a building, a mile of road, etc."<sup>1</sup> Groups of facilities constitute installations, defined in TM 5-301 as "a balanced grouping of facilities designed to be located in the same vicinity, such as a 100-bed hospital."<sup>1</sup> The largest planning component is therefore the installation, which when constructed and staffed is an operational entity. Figure 1 is an example of the staff table for an installation designed to be employed for filling drums and cans with petroleum. This installation is composed of 15 distinct types of facilities, the

**DRUM AND CAN LOADING, POL, CAPACITY OF 5,000 DRUMS OR 50,000 CANS PER DAY (Site Dwg. 10-37)**

OE-1.036

Fill 5,000 Drums or 50,000 Cans per day for package.  
POL Operations tanks and lines for 4 products of less.

| FACILITY                                                                                                                                                                                                                                                                                                            | SIZE OR UNIT                                                                                                                                                                                                                     | BASIS                                                                                                                 | FACILITY NUMBER | NO. REQ'D                                                          | MATERIAL COST |        | EFFORT MH |    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|-----------------|--------------------------------------------------------------------|---------------|--------|-----------|----|
|                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                  |                                                                                                                       |                 |                                                                    | ST            | MT     |           |    |
| 1. FUEL STORAGE<br>Tank, POL, w/6" Line                                                                                                                                                                                                                                                                             | 24000 Bbl                                                                                                                                                                                                                        | 2 Tanks / Product                                                                                                     | 415108          | 8                                                                  | 160           | 32     | 4880      |    |
|                                                                                                                                                                                                                                                                                                                     | 3000 Bbl                                                                                                                                                                                                                         |                                                                                                                       |                 |                                                                    | 160           | 32     | 4880      |    |
| 2. FUEL TRANSFER<br>Drum & Can Loading Bldg<br>w/OM equip<br>Manifold, Distribution, Drum 4<br>Can loading<br>Pump & Manifold, Floor<br>Pump & Manifold, Loading<br>Pump Station Fuel Supply<br>Tank, POL, w/4" Line<br>Pipe & Acc. API Grooved, 6" Stl<br>Pipe & Acc., Tubing 6" Stl<br>Pipe & Acc., Tubing 4" Stl | 48"x96'                                                                                                                                                                                                                          | Provide pipelines from tank farm, pumps, & manifolds Drum & Can loading bldg. provided by Engineers. Equipment by OM. | 441123          | 1                                                                  | 60            | 667    | 8380      |    |
|                                                                                                                                                                                                                                                                                                                     | 6"                                                                                                                                                                                                                               |                                                                                                                       | 124301          | 1                                                                  | 1             | 1      | 60        |    |
|                                                                                                                                                                                                                                                                                                                     | 785 Bph                                                                                                                                                                                                                          |                                                                                                                       | 414101          | 1                                                                  | 7             | 12     | 280       |    |
|                                                                                                                                                                                                                                                                                                                     | 700-1400Bph                                                                                                                                                                                                                      |                                                                                                                       | 124101          | 4                                                                  | 28            | 44     | 480       |    |
|                                                                                                                                                                                                                                                                                                                     | 2 Unit                                                                                                                                                                                                                           |                                                                                                                       | 124201          | 3                                                                  | 3             | 3      | 180       |    |
|                                                                                                                                                                                                                                                                                                                     | 250 Bbl                                                                                                                                                                                                                          |                                                                                                                       | 415102          | 3                                                                  | 18            | 6      | 510       |    |
|                                                                                                                                                                                                                                                                                                                     | 1000 Ft                                                                                                                                                                                                                          |                                                                                                                       | 125117          | 2                                                                  | 10            | 18     | 380       |    |
|                                                                                                                                                                                                                                                                                                                     | 1-Mile                                                                                                                                                                                                                           |                                                                                                                       | 125107          | 8                                                                  | 232           | 504    | 4560      |    |
|                                                                                                                                                                                                                                                                                                                     | 1000 Ft                                                                                                                                                                                                                          |                                                                                                                       | 125101          | 1                                                                  | 4             | 7      | 120       |    |
|                                                                                                                                                                                                                                                                                                                     | 3. OTHER CONSTRUCTION<br>Admin & Operations Bldg.<br>Fence, Security, Type Y<br>Fence, Security, Vehicle & Man Gate<br>Tank, Water, w/4" Line<br>Road, 1-Lane, 4" Stabilized Earth or Stone<br><br>Totals for Erection (1,2, &3) |                                                                                                                       | 20"x40'         | Provide operations bldg. security fencing, roads and water storage | 611311        | 1      | 39        | 44 |
| 1000 Ft                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                  | 872103                                                                                                                | 2.73            |                                                                    | 19            | 25     | 1333      |    |
| Each                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                  |                                                                                                                       | 872104          | 2                                                                  | 4             | 4      | 360       |    |
| 1000 Bbl                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                  |                                                                                                                       | 415106          | 1                                                                  | 6             | 2      | 530       |    |
| 1-Mile                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                  | 851202                                                                                                                | 0.50            | 3                                                                  | 2             | 3390   |           |    |
| Totals for Erection (1,2, &3)                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                  |                                                                                                                       |                 | 562                                                                | 743           | 193026 | 19153     |    |
| *185 c. y. fine aggregate and 250 c. y. coarse aggregate not included.                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                  |                                                                                                                       |                 |                                                                    |               |        |           |    |

Fig. 1—Sample Staff Table for a Drum- and Can-Loading Facility

quantity of each type being specified in units to three significant figures. The first facility listed is 415108, which is a 3000-bbl storage tank with ancillary fittings. Figure 2 is the item listing for this facility. Note also that the construction effort required to assemble the facility is 610 man-hours.

FACILITY NUMBER 415108

Tank, POL, 3000 Barrel, W/6 in pipe & fittings  
To Tank Berm & Berm Drain Assembly

Short Tons 20                      Meas. Tons 4                      MN HRS 610

Stock Number                      U1 Quantity

Sec 71 Pipe & Accessories

Engineer Items

|               |                                   |    |     |
|---------------|-----------------------------------|----|-----|
| 3835-641-8431 | Valve Sect Gate 6IN X 3FT         | EA | 1   |
| 3835-663-7340 | Valve Sect Gate 6IN X 3FT         | EA | 1   |
| 3835-693-4508 | Valve Assy Pressure Relief 1/2 IN | EA | 2   |
| 4710-202-9705 | Tube Stl Grv 6 X 5 TO 25 FT LG    | FT | 100 |
| 4730-273-8359 | Elbow Pipe MI GRVD 90DEG 6        | EA | 4   |
| 4730-273-8610 | CAP Pipe MI GRV 6                 | EA | 1   |
| 4730-277-9721 | Coupling Clamp Pipe MI GRVD 6     | EA | 13  |
| 4730-293-7110 | Tee Pipe MI Str GRV 6             | EA | 1   |

Sec 73 Tanks

Engineer Item

|               |                                  |    |   |
|---------------|----------------------------------|----|---|
| 5430-263-6075 | Tank Petr-Water Vert KD 126M Gal | EA | 1 |
|---------------|----------------------------------|----|---|

Transportation Items

|               |                                   |    |    |
|---------------|-----------------------------------|----|----|
| 4010-132-8050 | Chain Assy 1 LEG 1/4IN X 8 FT     | EA | 1  |
| 4010-274-6825 | Wire Rope STL 6X19 3100LB 3/16IN  | FT | 75 |
| 4030-233-9567 | Clip Wire Rope U-Bolt Type 3/16IN | EA | 4  |

Fig. 2--Item Listing for Facility 415108

Uses of the Engineer Functional Components System

The Engineer Functional Components System has been operational for several years and has been used by the Office of the Chief of Engineers in many planning situations. Normally the system is used in a straightforward manner, whereby from an input list of facilities or installations are produced

a detailed bill of materials and summaries of effort and tonnages. A primary application of the system has been in support of Department of the Army (DA) budgetary planning by the Deputy Chief of Staff for Logistics (DCSLOG). These plans are made to determine support troop unit and materiel requirements before formulation of budgets and are based on postulated war scenarios. The construction unit and materiel requirements (which can account for 70 to 80 percent of nonorganic procurement costs) are determined by using the Engineer Functional Components System. More recently the same system has been used to review or develop the construction portions of contingency plans.

The system as developed up to the present time is very comprehensive and includes tables for most of the installations and facilities that might be required by the Army in the communications zone of a theater of operations. It is constantly being updated and expanded—its files are very large. With little effort the computer programs now operational can be used for support of many types of Army planning efforts and logistic studies. Beyond the applications to budgeting and contingency planning activities, broader potential uses are possible in the areas of materiel management and construction-unit composition and balance within support forces. The former, materiel management, falls within the mission of the US Army Materiel Command (USAMC). The latter, construction-unit composition and balance, falls within the mission of the US Army Combat Developments Command (USACDC).

#### Theater Construction Planning Research at RAC

For several years, RAC has conducted research for DCSLOG in the area of planning Army construction in theaters of operations. Several computer programs were developed for use in DA Strategic Logistic Studies (DASLS).<sup>4,5</sup> These programs were designed around a modified functional-components file containing selected facilities and installations. Several programs were developed for determining the requirements for construction in terms of installations and facilities. These requirements were stated in terms of standard designations as well as numeric codes. The list of coded requirements was then used as input to the modified components program.

Under present efforts at RAC, computer models are being developed to simulate logistic management and operations—initially in overseas theaters of operations. Concepts of the models are contained in RAC-TP-179.<sup>6</sup> One of the models under development is the Theater Construction Model. It has been within the scope of efforts on this model that the linear programming models described in this paper were developed.

#### EXPERIENCE WITH THE MODEL

A computer program has been written to read as parameters the designations of facilities to be considered and to extract from an Engineer Functional Components System master tape the list of items contained in the facility, in a format suitable for direct input to the LP/44 linear programming package for the IBM 7044. For a simple problem involving only facilities and selected items the user specifies upper and lower bounds and weights on the facilities, constraints on selected items, and options of the LP/44 code to be used.

Several problems have been analyzed using this extract program and the LP/44 package, and the results have been interesting. The extensive analysis that is possible leaves numerous possibilities open for use of the models in examining alternative construction programs and/or policies. Great detail of analysis is made possible by the system, which requires a considerable handling effort in terms of planning personnel. Manual processing of such detailed information was previously out of the question, but the planner can employ this tool if the detail of analysis is desirable.

It should be noted that facilities and installations are integer-valued. The usual problems of rounding integer-valued variables hold in this model. When the numbers are large, this does not pose a problem in practice. When the numbers are small (such as if the number of installations is one or two) a problem is created that may require extensive analysis of integer linear programming problems. At this stage of development of the models, no solutions to this type of problem are offered, but the reader is alerted that the problem exists.

## MODEL LIMITATIONS

The models proposed here are not considered to be the final answer to the problem concerning selection of construction tasks within resource constraints. Rather they offer a method to address the problem that will probably be developed further. It is recognized that current hardware capacity introduces limitations on the size of problem that can be handled at the present time. Such limitations are recognized as currently inhibiting operational applications but not affecting the value of experimenting and testing the logic involved, knowing that such hardware constraints will be overcome in time with the advent of both higher-capacity computers and new techniques.

## COMPUTATION OF RESOURCE REQUIREMENTS FOR GIVEN FACILITY OR INSTALLATION SPECIFICATIONS

### Resource Requirements for Given Facility Specifications

Define<sup>†</sup> as follows:

$a_{ij}$  = the number of units of resource  $i$  needed in the construction of one unit of facility  $j$ , where  $i = 1, \dots, m$  and  $j = 1, \dots, n$

$\bar{x}_j$  = the specified number of units of facility  $j$  to be constructed<sup>‡</sup>

$\bar{d}_i$  = the units of resource  $i$  required to satisfy all the facility construction specifications

The resource requirements for all resources  $\bar{d}_i$  ( $i = 1, \dots, m$ ) to satisfy all facility construction specifications  $\bar{x}_j$  ( $j = 1, \dots, n$ ) may be computed by

$$\sum_{j=1}^n a_{ij} \bar{x}_j = \bar{d}_i, \quad i = 1, \dots, m.$$

Here we use the term "resource" interchangeably with "items and/or sets" defined previously and given as examples in Fig. 2.

<sup>†</sup>All definitions used in the paper are repeated in a list of notation on page 3.

<sup>‡</sup>A bar is used over certain symbols in this section for quantities that become decision variables and resource limitations in the next section.

### Facility Requirements for Given Installation Specifications

Define as follows:

$a'_{jk}$  = the number of units of facility  $j$  needed in the construction of one unit of installation  $k$ , where  $j = 1, \dots, n$  and  $k = 1, \dots, p$

$\bar{y}_k$  = the specified number of units of installation  $k$  to be constructed

To construct specified installations  $\bar{y}_k$  ( $k = 1, \dots, p$ ) the number of units of facilities  $\bar{x}_j$  ( $j = 1, \dots, n$ ) may be computed by

$$\sum_{k=1}^p a'_{jk} \bar{y}_k = \bar{x}_j, \quad j = 1, \dots, n.$$

This may serve as an input to the calculations of resource requirements  $\bar{d}_i$  ( $i = 1, \dots, m$ ) described in the previous paragraph.

### Resource Requirements for Given Installation Specifications

Define as follows:

$b_{ik}$  = the number of units of resource  $i$  needed in the construction of one unit of installation  $k$ , where  $i = 1, \dots, m$  and  $k = 1, \dots, p$ . This quantity may be computed by

$$\sum_{j=1}^n a_{ij} a'_{jk} = b_{ik}, \quad \begin{matrix} i = 1, \dots, m, \\ k = 1, \dots, p. \end{matrix}$$

To construct specified installations  $\bar{y}_k$  ( $k = 1, \dots, p$ ) the required number of units of resources  $\bar{d}_i$  ( $i = 1, \dots, m$ ) may be computed by

$$\sum_{k=1}^p b_{ik} \bar{y}_k = \bar{d}_i, \quad i = 1, \dots, m.$$

### Discussion

It should be noted that data for  $a_{ij}$ 's are available directly from DA TM 5-303,<sup>3</sup> which gives the bill of materials by item for given facility construction requirements. Data for  $a'_{jk}$ 's are available directly from DA manual TM 5-301,<sup>1</sup> the facilities contained in given installations. Computation of  $b_{ik}$ 's would enable direct computation of resource requirements for installation specifications, rather than indirect two-step computation of, first, facility requirements and then resource requirements.

## ONE-PERIOD LINEAR PROGRAMMING MODELS FOR CHOOSING FACILITIES OR INSTALLATIONS TO BE CONSTRUCTED

### One-Period Model for Choosing Facilities

Define as follows:

- $d_i$  = a limitation on the total number of units of resource  $i$  to be available during the period
- $x_j$  = the unknown number of facilities of type  $j$  to be constructed during the period, with lower bound  $x_j^{(l)}$  and upper bound  $x_j^{(u)}$
- $c_j$  = a weighting factor expressing the relative worths of facilities to be constructed, to be used in developing the weighting function to be maximized<sup>†</sup>

The constraints on the linear programming model are that the resource limitations not be exceeded

$$\sum_{j=1}^n a_{ij} x_j \leq d_i, \quad i = 1, \dots, m.$$

and that the lower and upper bounds in the facilities to be constructed are satisfied

$$x_j^{(l)} \leq x_j \leq x_j^{(u)}, \quad j = 1, \dots, n.$$

Subject to the constraints, the linear programming problem is to choose facilities  $x_j$  ( $j = 1, \dots, n$ ) to be constructed during the period to maximize the weighting function

$$\sum_{j=1}^n c_j x_j.$$

The resources  $d_i^*$  ( $i = 1, \dots, m$ ) required to execute the optimal solution  $x_j^*$  ( $j = 1, \dots, n$ ) may be computed by

$$\sum_{j=1}^n a_{ij} x_j^* = d_i^*, \quad i = 1, \dots, m.$$

### One-Period Model for Choosing Installations

Define as follows:

- $y_k$  = the unknown number of installations of type  $k$  to be constructed during

<sup>†</sup>In many applications the program also provides valuable results if task requirements are treated as being of equal importance.

the period, with lower bound  $y_k^{(l)}$  and upper bound  $y_k^{(u)}$   
 $c_k$  = a weighting factor expressing the relative worths of installations to be  
 constructed, to be used in the weighting function to be maximized

The constraints in the mathematical programming model are that the re-  
 source limitations are not exceeded

$$\sum_{k=1}^p b_{ik} y_k \leq d_i, \quad i = 1, \dots, m,$$

and that the lower and upper bounds on the installations to be constructed are  
 satisfied

$$y_k^{(l)} \leq y_k \leq y_k^{(u)}, \quad k = 1, \dots, p.$$

Subject to the constraints, the linear programming problem is to choose instal-  
 lations  $y_k$  ( $k = 1, \dots, p$ ) to be constructed during the period, to maximize the  
 weighting function

$$\sum_{k=1}^p c_k y_k.$$

The resources  $d_i^*$  ( $i = 1, \dots, m$ ) required to execute the optimal solu-  
 tion  $y_k^*$  ( $k = 1, \dots, p$ ) may be computed by

$$\sum_{k=1}^p b_{ik} y_k^* = d_i^*, \quad i = 1, \dots, m.$$

#### ONE-PERIOD LINEAR PROGRAMMING MODEL FOR CHOOSING COM- BINED FACILITY-INSTALLATION CONSTRUCTION PROGRAMS

It would be desirable in some cases to differentiate between facilities con-  
 structed as a part of installations, and facilities constructed separately. This  
 is particularly useful because a facility might be weighted quite differently in  
 the separate applications.

Using the definitions in the previous section and assuming that facilities  
 $x_j$  ( $j = 1, \dots, n$ ) are not a part of installations  $y_k$  ( $k = 1, \dots, p$ ), constraints  
 can be written that the resource limitations are not exceeded by facilities con-  
 structed separately plus installations constructed

$$\sum_{j=1}^n a_{ij} x_j + \sum_{k=1}^p b_{ik} y_k \leq d_i, \quad i = 1, \dots, m.$$

Lower and upper bounds could be imposed on facilities constructed separately

$$x_j^{(l)} \leq x_j \leq x_j^{(u)}, \quad j = 1, \dots, n.$$

and on installations

$$y_k^{(l)} \leq y_k \leq y_k^{(u)}, \quad k = 1, \dots, p.$$

Subject to the constraints the linear programming problem is to choose facilities  $x_j$  ( $j = 1, \dots, n$ ) and installations  $y_k$  ( $k = 1, \dots, p$ ) to maximize the weighting function

$$\sum_{j=1}^n c_j x_j + \sum_{k=1}^p e_k y_k.$$

The resources  $d_i^*$  ( $i = 1, \dots, m$ ) required to execute the optimal solution  $x_j^*$  ( $j = 1, \dots, n$ ) and  $y_k^*$  ( $k = 1, \dots, p$ ) may be computed by

$$\sum_{j=1}^n a_{ij} x_j^* + \sum_{k=1}^p b_{ik} y_k^* = d_i^*, \quad i = 1, \dots, m.$$

#### MULTIPERIOD USE OF ONE-PERIOD MODELS

Consider the one-period model for choosing facilities described previously. To use this model over a number of time periods, in the first period constraints would be established on the number of units of resources available  $d_i$  ( $i = 1, \dots, m$ ), lower and upper bounds on the facilities to be constructed  $x_j^{(l)}$  and  $x_j^{(u)}$  ( $j = 1, \dots, n$ ), and weights  $c_j$  ( $j = 1, \dots, n$ ). In the second and succeeding periods, the resources  $d_i^*$  ( $i = 1, \dots, m$ ) used during the previous period would be examined, along with the additional resources made available, to obtain a new  $d_i$  ( $i = 1, \dots, m$ ), new lower and upper bounds would be set on the facilities to be constructed depending on the facilities that were constructed, and new weights would be assigned.

Symbolically, let the time period be denoted by a superscript  $t$ . Define as follows:

$d_i^t$  = the limitation on resource  $i$  during the  $t$ th period

$x_j^t$  = the unknown number of facilities of type  $j$  constructed during the  $t$ th period, with lower and upper bounds  $x_j^{t(l)}$  and  $x_j^{t(u)}$

$c_j^t$  = the weighting function on facilities of type  $j$  in the  $t$ th period

$x_j^{*t}$  = the chosen construction of type  $j$  during the  $t$ th period

$d_i^{*t}$  = the required resources of type  $i$  necessitated by the chosen construction

$$x_j^{*t} (j = 1, \dots, n)$$

The multiperiod use of the one-period model would proceed as follows:

(1) Determine initial stocks of resources and specify  $d_i^1$  ( $i = 1, \dots, m$ ).

Specify  $x_j^{1(l)}$ ,  $x_j^{1(u)}$ , and  $c_j^1$  ( $j = 1, \dots, n$ ).

(2) Solve for  $x_j^{*1}$  ( $j = 1, \dots, n$ ).

(3) Compute  $d_i^{*1}$  ( $i = 1, \dots, m$ ).

(4) For the second period and thereafter, determine the status of resources as  $d_i^t = d_i^{t-1} - d_i^{*t-1} + u_i^t$  ( $i = 1, \dots, m$ ), where  $u_i^t$  is the number of resources added to be available during the  $t$ th period. Specify  $x_j^{t(l)}$ ,  $x_j^{t(u)}$ , and  $c_j^t$  ( $j = 1, \dots, n$ ). Solve for  $x_j^{*t}$  ( $j = 1, \dots, n$ ). Compute  $d_i^{*t}$  ( $i = 1, \dots, m$ ).

The one-period model for choosing facilities and one-period model for choosing combined facility-installation construction programs could be similarly used over a number of time periods.

#### MULTIPERIOD LINEAR PROGRAMMING MODELS

In this section a multiperiod model for choosing facilities is presented. It can be readily extended to the other problems.

##### Model for Choosing Facilities

To facilitate presentation of the models in this section, matrix notation is adopted. Define as follows:

$$A = (a_{ij}) \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, n \end{array}$$

$s = (s_i)$  = the number of units of resource  $i$  on hand before the start of the initial time period, where  $i = 1, \dots, m$

$u^t = (u_i)^t$  = the number of units of resource  $i$  added to be available in time  $t$ , where  $i = 1, \dots, m$  and  $t = 1, \dots, T$

$x^t = (x_j)^t$  = the unknown number of units of facility  $j$  to be constructed in the  $t$ th time period, where  $j = 1, \dots, n$  and lower and upper

bounds  $x^{(l)t} = (x_j^{(l)})^t$  and  $x^{(u)t} = (x_j^{(u)})^t$  are specified for all the time periods

$z^t = (z_i)^t =$  the number of units of resource  $i$  carried over to the next period in time  $t$ , where  $i = 1, \dots, m$

Define the weighting function  $c^t = (c_j)^t =$  the weighting factor expressing the relative worths of facilities to be constructed, where  $j = 1, \dots, n$ .

In the first period a constraint may be written that the resources used plus the resources carried over into the next period equal the total availabilities this period

$$Ax^1 + z^1 = s + u^1.$$

For succeeding time periods the constraint may again be written that the resources used, plus the resources carried over to the next period equal the newly available resources plus the resources carried over from the previous period

$$Ax^t + z^t = u^t + z^{t-1}, \quad t = 1, \dots, T.$$

Another constraint may be written that the facilities constructed fall within the lower and upper bounds

$$x^{(l)t} \leq x^t \leq x^{(u)t}, \quad t = 1, \dots, T.$$

It might be desirable to constrain the total construction of a given type of facility over several time periods within some limitations, rather than one period at a time. One way this could be accomplished would be by setting

$$X^{(l)} \leq \sum_{t=t_n}^{t_N} x^t \leq X^{(u)},$$

where  $t_n$  and  $t_N$  are the first and last time periods considered, and  $X^{(l)}$  and  $X^{(u)}$  are the lower and upper total-facility-construction limitations within these time periods.

Subject to the constraints, the linear programming problem is to choose  $x^t$ ,  $t = 1, \dots, T$  to maximize

$$c_1^1 x_1^1 + c_2^1 x_2^1 + \dots + c_n^1 x_n^1 + c_1^2 x_1^2 + \dots + c_n^T x_n^T.$$

Figure 3 shows a schematic simplex tableau for the multiperiod model for choosing facilities, not including lower and upper bounds on the variables.

| $c^1$ |       | $c^2$ |       | $c^3$ |       | $c^{T-1}$ |           | $c^T$ |       |             |         |
|-------|-------|-------|-------|-------|-------|-----------|-----------|-------|-------|-------------|---------|
| $x^1$ | $z^1$ | $x^2$ | $z^2$ | $x^3$ | $z^3$ | $x^{T-1}$ | $z^{T-1}$ | $x^T$ | $z^T$ |             |         |
| A     | I     |       |       |       |       |           |           |       |       | $= s + u^1$ |         |
|       | -I    | A     | I     |       |       |           |           |       |       | $= u^2$     |         |
|       |       |       | -I    | A     | I     |           |           |       |       | $= u^3$     |         |
|       |       |       |       |       |       |           |           |       |       | .           |         |
|       |       |       |       |       |       |           |           |       |       | .           |         |
|       |       |       |       |       |       |           |           |       |       | .           |         |
|       |       |       |       |       |       | -I        | A         | I     |       | $= u^{T-1}$ |         |
|       |       |       |       |       |       |           |           | -I    | A     | I           | $= u^T$ |

Fig. 3—Schematic Simplex Tableau for Multiperiod Facilities Model

Not including lower and upper bounds on availabilities.

### Extension to Other Cases

A multiperiod model could be built for the installation construction program by defining the installation-construction coefficient matrix

$$B = (b_{ik}), \quad \begin{matrix} i = 1, \dots, m \\ k = 1, \dots, p \end{matrix}$$

and the variable  $y^t = (y_k)^t$  = the unknown number of units of installation  $k$  to be constructed in the  $t$ th period, where  $k = 1, \dots, p$ , with lower and upper bounds  $y^{(l)t}$  and  $y^{(u)t}$  specified for all time periods. The other variables would be the same, and the formulation would be similar.

Also, using  $A$  and  $B$ ,  $r^t$  and  $y^t$ , and the other variables, but separating facilities constructed separately from those in installations, the one-period model for choosing combined facility-installation construction programs could be extended.

### ALTERNATIVE SPECIFICATION OF OBJECTIVE FUNCTIONS

One difficult problem in the use of the models is the specification of lower and upper bounds and weights on the facilities and/or installations to be constructed. The weighting process is for the purpose of specifying the

value of one unit of each facility and/or installation. Within each type of facility or installation, each unit is assumed to be able to be weighted equally.

Specification of lower and upper bounds solves the problem to some extent, making the total number of facilities and/or installations constructed lie within a certain range. However, the weights still are with respect to all units within each type of facility and/or installation.

One improvement in the weighting process assumed throughout this paper, and discussed above, will now be outlined. For purposes of discussion assume that we are considering the one-period linear programming model for choosing combined facility-installation construction programs. The user of the model will be required to specify a ranked list of inputs, each of which contains lower and upper bounds, and an associated weighting function.

Define the  $q$ th set of lower and upper bounds on facilities to be constructed separately

$$x_j^{(l)q} \leq x_j \leq x_j^{(u)q} \quad j = 1, \dots, n$$

and lower and upper bounds on installations

$$y_k^{(l)q} \leq y_k \leq y_k^{(u)q} \quad k = 1, \dots, p,$$

where  $q = 1, \dots, Q$ . Also define the  $q$ th set of weighting factors on facilities constructed separately  $c_j^q$  ( $j = 1, \dots, n$ ) and on installations  $c_k^q$  ( $k = 1, \dots, p$ ). The key element in this formulation is that the  $q$ th set of weighting factors is applicable only to those units constructed above and beyond the lower bounds and below the upper bounds. Thus comparing facilities and/or installations above the lower bounds with those of the same or other types below the lower bounds is avoided.

The set of lower and upper bounds and weights is assumed to be ordered by desirability. The first step is to compute for  $q = 1$

$$\sum_{j=1}^n a_{ij} x_j^{(l)1} + \sum_{k=1}^p b_{ik} y_k^{(l)1} = d_i^{(l)1}, \quad i = 1, \dots, m,$$

the resources required by the first set. If  $d_i^{(l)1} \leq d_i$ ,  $i = 1, \dots, m$ , then the lower bounds are feasible. Assume that they are to be constructed. Then for the constraints

$$\sum_{j=1}^n a_{ij} x_j + \sum_{k=1}^p b_{ik} y_k \leq d_i - d_i^{(l)1}, \quad i = 1, \dots, m$$

and upper bounds

$$x_j \leq x_j^{(u)1} - x_j^{(l)1}, \quad j = 1, \dots, n,$$

$$y_k \leq y_k^{(u)1} - y_k^{(l)1}, \quad k = 1, \dots, p,$$

choose facilities and installations  $x_j$  ( $j = 1, \dots, n$ ) and  $y_k$  ( $k = 1, \dots, p$ ) to maximize

$$\sum_{j=1}^n c_j^1 x_j + \sum_{k=1}^p c_k^1 y_k.$$

If the lower bounds are not feasible for  $q = 1$ , perform the feasibility analysis for  $q = 2$  etc. until a feasible lower bound is encountered. Then perform the steps described for  $q = 1$ . If none of the lower bounds for  $q = 1, \dots, Q$  is feasible, a new set must be formulated or some other analysis performed.

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