Contract AF 61(052)-794

FINAL SCIENTIFIC REPORT

"Reflector Arrays"

1 April 1964 - 30 June 1967

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July 1st, 1967

S 127 R 59

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This research has been sponsored in part by the Cambridge Research Laboratories, OAR, through the European Office of Aerospace Research, OAR, United States Air Force under Contract No. AF61(052)-794.
The Van Atta reflector was first described in a patent by Dr. L.C. Van Atta in 1959. The advantage of this passive reflector type should be that the reradiated field has a maximum back in the direction of arrival of the primary plane wave. Since this retrodirective effect of the reflector might be of great importance if used as a navigational aid in the air or at sea, it was worth while to carry out a theoretical investigation of such reflectors, especially since only experimental investigations had been made before this contract was initiated.

The work performed under the contract deals mainly with theoretical and numerical investigations of Van Atta reflectors consisting of dipoles. A survey of the literature concerning active or passive Van Atta reflectors has been made. Both a linear and a two-dimensional plane Van Atta reflector has been investigated numerically and a theory for arbitrary Van Atta reflectors has been developed. An experimental investigation of a linear Van Atta reflector was carried out and the results compared with the theoretical results.
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1. INTRODUCTION

The contract AF 61(052)-794 was running from April 1, 1964 to June 30, 1967.

The objectives of the contract were given as follows:

Carry out a theoretical investigation of the performance of a Van Atta reflector and, by electronic computer calculations, to find the radiation patterns of a number of such reflectors with various characteristics. It is expected that this would lead to a procedure for the design of a Van Atta reflector with a prescribed radiation pattern. It is the intention first to treat the linear and two-dimensional array, later the method of investigation may be extended to include Van Atta reflectors placed on circular cylinders and spheres. Further efforts will be made to supplement the theoretical investigation by experiment.

In this final report a summary of all the work performed under the contract is given. Some related work carried out at this laboratory but not paid by the contract is described, too.

First an analytical investigation of arbitrary Van Atta reflectors was carried out. Each pair of antenna elements with connecting transmission lines was described by an equivalent circuit. Reradiation from the elements and mutual interaction were taken into account. Next a theoretical investigation of a linear Van Atta reflector consisting of four parallel half-wave dipoles was performed. The dipole elements were equispaced and mutual impedances between the dipoles were taken into account. An expression for the reflected field was set up by superposition of the three fields involved. Further an analysis of the general shape of the reradiation pattern was made neglecting the mutual impedances. This analysis showed that for some combinations of the parameters involved the reflector does not act as stated in the patent description of Van Atta reflectors.

An experimental investigation of the four element linear Van Atta reflector was carried out in a radio-anechoic chamber. In agreement with the theoretical investigation, the experiments showed that the reflector only to a limited extent has the retrodirective effect stated in the patent description and that it has a mirror effect to the same extent as it has a retrodirective effect.

Numerical investigations of a four element Van Atta reflector and an optimization of the reflector with respect to the parameters involved was carried out using an electronic computer. The reradiation pattern of the reflector was optimized with the criterion that the minimum values of the
back-scattered field intensities for all angles of incidence should be as large as possible.

Finally a square Van Atta reflector consisting of half-wave dipoles was investigated theoretically and numerically. The effect of mounting the dipoles above and parallel to a conducting plate was also examined. The mutual impedance between the dipole elements and the reradiation both from the elements and from the plate was taken into account. All parameters involved have been varied in a numerical analysis of the reflector. The parameters are: the number of dipole elements, the length of the dipoles, the length and characteristic impedance of the transmission lines, the distance between the dipoles, and the distance from the dipoles to the conducting plate. The effect of changing the parameters is shown in curves of the back-scattering cross section of the reflector as a function of the variation of each of the parameters. The numerical results have been compared with experimental results obtained by others.

The last work on the contract has been an investigation of the bandwidth properties of a 4 by 4 element square Van Atta reflector consisting of dipoles.
2. SURVEY OF LITERATURE

Since Van Atta \textsuperscript{1)} in 1959 proposed his passive retrodirective reflector, several papers have suggested the use of this reflector type in various communication systems \textsuperscript{2)} - \textsuperscript{23)}. Some of the papers tend to give an analytical treatment of the Van Atta reflector but most of them neglect the scattering by and coupling between the antennas. Many of the papers suggest active components inserted in the transmission lines of the reflector, such as modulated phase shifters \textsuperscript{5)}, amplifiers \textsuperscript{6)} \textsuperscript{10)}, and mechanical modulation by means of cavity resonators \textsuperscript{9)}. It has been suggested to use Van Atta reflectors for satellite communication and both passive, semipassive and active systems have been proposed \textsuperscript{6)} \textsuperscript{15)} \textsuperscript{21)} \textsuperscript{22)}. Other applications are for navigational aids, for example used to enhance the reflection from radartargets, from lifeboats, and from aircraft \textsuperscript{3)} \textsuperscript{5)} \textsuperscript{7)} \textsuperscript{10)}. A more detailed discussion of the literature concerning Van Atta reflectors and their applications is given in Scientific Report No. 1 and by Appel-Hansen \textsuperscript{34)} \textsuperscript{35)}. 

3. THEORETICAL INVESTIGATION OF ARBITRARY VAN ATTA ARRAYS

A general treatment which may be used for a number of different investigations is described in Scientific Report No. 1 of the contract. The configuration investigated is shown in fig. 1. The elements are supposed to be dipoles, but the theory could easily be extended to other antenna types. The dipoles are placed on and parallel to an imaginary smooth surface which may be e.g. a plane, a cylinder or a sphere. The field incident on the reflector is a plane wave.

The open circuit voltage induced at the terminals of each antenna element by the primary plane wave is calculated. Using these voltages a system of linear equations is developed for calculating the currents in each antenna taking into account the mutual impedances, the characteristics of the interconnecting transmission lines, and the induced voltage at the element itself (giving the scattered field) and at its mate (giving the retrodirective field).

When the currents are determined the reradiated field may be calculated. For a reflector with all elements parallel this is done using the theory of antenna arrays. Finally the properties of the reflector array are described by calculation of the differential scattering cross section.

The mutual impedances of the dipoles are calculated using the induced EMF method as given in Jordan's book 36). Algol procedures for computing the mutual impedances between linear dipoles with sinusoidal current distributions and for arbitrary wire-antennas with a known current distribution has been developed at this laboratory.

The transmission lines are represented by equivalent circuits of the general x-circuit type. This type of equivalent circuit has been chosen because it has the advantage of being valid for all lengths of the transmission lines. It is assumed to be symmetrical and lossless.

Since both the induced voltages and the mutual impedances are complex quantities the real matrix equation to be solved will be of the 2Nth order where N is the total number of antenna elements. This means that it will be almost necessary to use an electronic computer for solving the matrix equation numerically when reflectors with more than two elements are treated.

A simple numerical example of a four element Van Atta array with equispaced half-wave dipoles is given in SR 1 in order to illustrate the theory developed. The numerical results indicate that a retrodirective effect as
stated by Van Atta is obtained to some degree. However the results also show that the mirror effect of the reflector is of the same order of magnitude as the retrodirective effect. The influence of the mutual impedances and of a mismatch between the antenna elements and the transmission lines may be utilized to change the reradiation pattern to compare better with a prescribed form.
4. A LINEAR VAN ATTA REFLECTOR
CONSISTING OF FOUR HALF-WAVE DIPOLES

A detailed investigation of a four element linear Van Atta reflector consisting of half-wave dipoles was carried out and is described in Scientific Report Nos. 2, 3, and 4 of the contract.

First an expression for the reflected field is developed considering each dipole as a receiving and transmitting antenna matched to its transmission line. When a plane wave is incident on the reflector a current distribution consisting of three different parts will be generated in the antenna elements. The first part is due to the energy transmitted to the antenna through the transmission line from its mate. The second part is the current induced in the antenna by the incident plane wave and the third part is induced in the antenna due to its mutual interaction with the other antennas of the reflector. Only the first part of the current distribution is considered in Van Atta's patent description, while the last two parts are neglected.

In fig. 2 is shown the retrodirective effect of the reflector due to the first part of the current distribution mentioned above. The second part of the current distribution creates a mirror effect of the reflector which when the dipoles are matched to the transmission lines is of the same order of magnitude as the retrodirective effect. This mirror effect which is shown in fig. 3 is not mentioned in the patent description of the reflector.

It is shown that the length of the transmission lines and the angle of incidence has a great influence on the reradiation, and for some values of these two parameters the first and second part of the current in the antennas are of opposite phase and they cancel each other so that the only reflected energy is the small part due to the mutual interaction between the elements. The length of the transmission lines at which the behaviour of the reflector is as much as possible in accordance with the patent description has been found but even this reflector has the mirror effect mentioned above.

Numerical analysis have been made with a spacing between the dipole elements of half a wavelength. It turns out that in most cases maximum reflection is not back in the direction of incidence of the primary plane wave, and that the mutual interaction between the dipoles causes asymmetries in the reradiation pattern. This means that it is possible to increase the back-scattered energy by choosing a proper combination of the distance between the elements and the length of the transmission lines. The above
mentioned investigations are described in Scientific Report No. 2.

In order to verify the theoretical results obtained, an experimental investigation of a four element linear Van Atta reflector was carried out in a radioanechoic box at the laboratory. This investigation is treated in Scientific Report No. 3. The four half-wave dipoles of the experimental reflector were slot fed dipoles with open-ended terminations. The length of the connecting transmission lines could be changed by means of line-stretchers thus examining the influence of the length of the lines on the reflecting properties of the reflector. The measurements were performed at 3.21 GHz as this frequency gives the best matching between each dipole and the connected transmission line.

The Van Atta reflector was placed on a moveable pedestal in the center of the anechoic box, and the measurements were based upon the principle of interference between the signal reflected by the reflector and a reference signal. Radiation patterns were measured in the plane normal to the axis of the dipoles for discrete values of the angle of incidence of the primary plane wave.

A good agreement between the experimental and theoretical results was found. The measured results confirmed the theoretical results of Scientific Report No. 2 and showed (1) that maximum reradiation is not always back in the direction of incidence, (2) that the reflector has a mirror effect to the same degree as it has a retrodirective effect, (3) that the reradiation depends strongly upon the length of the transmission lines, and (4) that the mutual impedances causes asymmetries in the radiation patterns.

As an extension of the theoretical analysis of the four element linear Van Atta reflector an optimization of this reflector has been carried out, the results of which is given in Scientific Report No. 4.

First the original expression for the reradiated field as derived in Scientific Report No. 2 was changed to a more general form which makes it possible to study the influence of asymmetries in the location of the dipoles, unequal lengths of the transmission lines, and a mismatch between the dipoles and the transmission lines. Further, a method was developed for computing a quantity which may be used as a measure of the deviation from the retrodirective effect of the reflector. This quantity is shown to be just as useful as the reradiation pattern itself when two different Van Atta reflectors are to be compared.

A perfectly working retrodirective Van Atta reflector has not been found. However, when mutual coupling is neglected, a condition for the smallest deviation from retrodirectivity has been derived. It is shown that the mutual coupling between the dipoles usually causes the reradiation of
the reflector to decrease and the deviation from retrodirectivity to increase. However for certain values of the parameters involved it turns out that coupling may increase the back-scattering up to 50% for some angles of incidence.

The numerical optimization of the reradiation pattern of the four element Van Atta reflector was carried out using a computational technique starting with an a priori reasonable set of parameters selected by examining 1600 different reflectors. The parameters were then perturbed about their initial values, the effect on the reradiation pattern was observed, and a new set of parameters giving an improved result was selected. The success of this method depends on the correctness of the original set of parameters and the computer program for perturbing the parameter values.

An attempt was made to fulfill the following two criteria:

1. The minimum value of the back-scattered field intensity, as a function of the angle of incidence, should be as large as possible and the deviation from Van Atta effect as small as possible.
2. The minimum value of the back-scattered field intensity, as a function of the angle of incidence, should be above various prescribed levels and the deviation from Van Atta effect as small as possible.

For both optimization processes it turned out that the optimum value of the spacing was close to 1.5 wavelengths. Further it was found that, due to coupling, the minimum value of the back-scattered field intensity may be increased and the deviation from the retrodirective effect decreased if the transmission lines are permitted to be of unequal lengths and asymmetries are permitted in the location of the dipoles around the center of the reflector. However, the improvements are small and asymmetries often cause the opposite effect.
5. SQUARE VAN ATTA REFLECTORS

WITH OR WITHOUT A CONDUCTING PLATE

Another configuration of the reflector array is the plane, square Van Atta reflector consisting of parallel dipoles. The investigation of this reflector is described in Scientific Report No. 5.

Theoretical investigation of this reflector has already been described in Scientific Report No. 1. However, in order to be able to compare the theory with experimental results obtained by Sharp, the effect of mounting the dipoles above and parallel to a conducting plate has to be taken into account. The system investigated is shown in fig. 4.

The reflecting properties of the plate are supposed not to be influenced by the presence of the dipoles. The reflected field is found using the method of physical optics as described, for example, in Kerr's book for a plate the dimensions of which are not small compared to the wavelength.

The reradiating properties of the dipoles when the plate is present, is calculated as if the plate was infinite in extent, using the theory of images. The system of dipoles may then be treated along the same lines as in Scientific Report No. 1, but the induced voltage, the mutual impedance, and the determination of the field reradiated from the dipoles have to be changed because of the image.

The induced voltage is still found as described for an arbitrary reflector in Scientific Report No. 1, but now the electromagnetic field vector is changed in such a way that the distance from the dipoles to the plate is involved, according to ordinary reflection theory.

The new values of self- and mutual impedances are found using the method of images, too.

By using the values of the induced voltages and the values of the self- and mutual impedances thus found, the system of equations (23) of Scientific Report No. 1 will give the new values of the currents on the antenna elements when the presence of the plate is taken into account.

After that the reradiation pattern from the dipole reflector itself mounted in a distance h above the plate is calculated with the above-mentioned currents on the dipoles. The final reradiation pattern of the dipoles is found using the theory of antenna arrays on the array consisting of two parallel Van Atta reflectors in free space with the distance 2h, where h is the
distance between the dipoles and the plate.

The total field reradiated from the reflector system is found by adding the field reradiated from the dipoles and the field reflected from the conducting plate.

Using the above-mentioned theory a computer program has been developed and the numerical results have been compared with results obtained by Sharp from experimental investigations of a 16 element square Van Atta reflector. The computed back-scattering cross section shows a good agreement with the results measured for the experimental reflector as shown in fig. 5.

Furthermore, a series of computations has been performed in order to examine the changes in the back-scattering cross section due to changing of the parameters of the reflector. The parameters are the number (N) of elements, the length (a) and characteristic impedance ($Z_0$) of the transmission lines, the distance (d) between adjacent dipole elements, and the distance (h) from the elements to the plate.

The most important results obtained by this numerical investigation is:

(i) that the back-scattering cross section becomes larger if a distance of 0.35 wavelengths from the dipoles to the plate is used instead of 0.25 wavelengths as used by Sharp,

(ii) that the shape of the curves of back-scattering becomes more irregular when more elements are used in the reflector but the level of back-scattering is increased,

(iii) that the shape of the curves of back-scattering becomes more smooth when a mismatch between the elements and the transmission lines is introduced in such a way that the characteristic impedance of the line is larger than the self-impedance of the dipole. However, the magnitude of the back-scattered energy is then decreasing more rapidly for oblique directions of incidence,

(iv) that for certain lengths of the transmission lines the back-scattering in the direction normal to the reflector tends to zero. This is in accordance with the results obtained for the four element linear reflector mentioned in section 4.

In Scientific Report No. 5 a great number of numerical results obtained by the parameter variation is given as curves of the back-scattering cross section.
6. BANDWIDTH PROPERTIES OF THE SQUARE VAN ATTA REFLECTOR

The final work on the contract deals with a computation of the bandwidth properties of a 16 element square Van Atta reflector similar to the experimental model used by Sharp.

Part of this work is an investigation of the influence on the back-scattering properties of the reflector of a change in the length of the dipole elements. In the previous investigations on this contract only reflectors consisting of half-wave dipoles have been investigated. The examination of the influence of the length of dipoles is carried out along the same lines as the examination of the influence of the other parameters of the reflector as described in Scientific Report No. 5. This means that the length of the dipoles is changed while keeping all other parameters of the reflector fixed with values corresponding to the dimensions of the experimental reflector used by Sharp.

The results of this investigation is shown in fig. 6 of this report. From this it turns out as expected that the back-scattered energy decreases both when the dipole length is less than and greater than half a wavelength. This is due to the fact that the matched half-wave dipole has optimal reradiating properties. However, when the dipoles are less than half a wavelength the mutual coupling between the dipoles decrease and a better retrodirective effect of the reflector is obtained. This effect is further strengthened because of the mismatch between the dipole and the transmission line, the self resistance $R_A$ of the dipole being less than the characteristic impedance $Z_0$ of the line. This is in accordance with the results of the investigation of the four element linear reflector where it was found that the deviation from retrodirective effect decreases when the factor $R_A/Z_0$ decreases.

The bandwidth properties of the 16 element square Van Atta reflector similar to the experimental reflector used by Sharp with or without the conducting plate has been computed and the results are shown in figs. 7a and 8a. The curves show the back-scattered energy for different angles of incidence as a function of $\lambda/\lambda_0$, where $\lambda_0$ is the wavelength corresponding to the center frequency.

For increasing values of $\lambda/\lambda_0$ larger than 1.0 the back-scattered energy from the reflector without a conducting plate decreases in a regular manner for all angles of incidence.
For values of $\lambda/\lambda_0 < 1.0$ the shape of the curves is very irregular and gives no information at all. When the conducting plate is taken into account the curves for $\lambda/\lambda_0 > 1.0$ are almost as regular as when the plate is not present but the level of back-scattered energy is higher for directions of incidence near normal incidence (0°) and lower for directions of incidence near broadside (90°). For $\lambda/\lambda_0 < 1.0$ the curves are just as irregular as in the case where the plate is not present.

However, from the results of this investigation it turned out that the retrodirective effect of this 16 element reflector is essentially improved when $\lambda/\lambda_0 = 1.3$. This corresponds to a 16 element square Van Atta reflector with the following parameter values:

- length of dipoles: 0.305 $\lambda$
- radius of dipoles: 0.0115 $\lambda$
- distance between dipoles: 0.462 $\lambda$
- distance from dipoles to plate: 0.192 $\lambda$
- length of transmission lines: 0.315 $\lambda$
- characteristic impedance of transmission lines: 73.0 ohms

A measure of retrodirective effect is given in figs. 7b and 8b as

$$N = \frac{\text{number of examined angles of incidence giving retrodirective effect}}{\text{total number of angles of incidence examined}} \times 100\%$$

For the reflector without a plate $N$ obtains its maximum 90% for the above-mentioned wavelength $\lambda = 1.3\lambda_0$ of the incident plane wave. When the conducting plate is present the optimum value of $N$ is 50% and this value is obtained in the whole range from $\lambda = 1.1\lambda_0$ to $\lambda = 1.5\lambda_0$. The reason for the decrement of $N$ in the second case is that the retrodirective effect is reduced when the direction of incidence turns towards broadside because of interference with the field scattered from the conducting plate.

However it is obvious in both cases that a Van Atta reflector with a better retrodirective effect than the effect measured by Sharp may be obtained from the same physical reflector at the expense of the reradiated energy if the reflector is used at lower frequencies than the center-frequency.
7. CONCLUSION

In this report a survey of the investigations performed under Contract AF 61(052)-794 "Reflector Array" has been given including the results of the final work on the contract which have not been described in any previous report.

By comparing the results obtained with the objectives of the contract as stated in section 1 of this report it is seen that the first period of the text may be covered by Scientific Reports Nos. 1, 2, 4, and 5. The second period is in part covered by Scientific Report No. 4. The linear and two-dimensional array mentioned in the third period has been investigated while the other configurations mentioned in this period have not been dealt with. The fourth and last period is covered in Scientific Report No. 3 for a linear array.

Further an investigation of the bandwidth properties of a square reflector has been carried out.

Using the theory explained in the reports issued under this contract it should be possible to investigate other types of Van Atta reflectors. This might be as well reflectors consisting of dipoles mounted in two- or three-dimensional arrays for example over conducting cylinders or spheres, as reflectors consisting of other types of antenna elements such as horns, paraboloid antennas, crossed dipoles or monopoles.

Probably results which compare better with the experimental results measured by Sharp may be obtained using another theory for the field reflected from the conducting plate than the physical optics theory used in Scientific Report No. 5. This theory may be Keller's geometrical theory of optics which, in contrast to the theory used, will take into account the scattering of the incident field about the edges of the conducting plate.
8. COMPUTER PROGRAM

In Appendix I a copy of the computer program developed for the numerical investigation of square Van Atta reflectors with or without a conducting plate is printed.

The program is written in FORTRAN IV and the computer used in an IBM 7090 run by the Northern Europe University Computing Center, Technical University of Denmark.
9. REFERENCES

Papers issued under this contract are marked with an asterisk (*).


31. Tove Larsen and E. Draeg Nielsen, Square Van Atta reflector with or without a conducting plate, Scientific Report No. 5, Contract No. AF 61(052)-794, Laboratory of Electromagnetic Theory, Technical University of Denmark, August 1966.


APPENDIX 1

AUTHOR'S CHECK

ATTACH - EFN SOURCE STATEMENT - IFN(S) -

CALCULATION OF RERADIATION PATTERN OF SQUARE VAN ATTACH REFLECTOR
WITH AND WITHOUT A CONDUCTING PLATE

THE INPUT PARAMETERS ARE

COMMON INPUT

I = NUMBER OF ELEMENTS IN EACH ROW
J = 1 IF CONDUCTING PLATE IS TAKEN INTO ACCOUNT; ELSE 0
KPQ = ANGLE OF INCIDENCE WILL BE CHOSEN AS KFC+FADC (IN DEGREES)
MCB = 1 IF RESULTS IN DECIBELS, 1 IF RESULTS IN DIRECT NUMERICAL VALUES AND 2 IF BOTH CASES ARE WANTED
C = DISTANCE BETWEEN ELEMENTS (IN WAVELENGTHS)
R = DISTANCE FROM ELEMENTS TO PLATE (IN WAVELENGTHS)
ZD = CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINES (IN OHMS)
RAD = DIPOLE RADII (IN WAVELENGTHS)
CLE = LENGTH OF DIPOLES (IN WAVELENGTHS)
W = ANGLE OF POLARIZATION OF INCIDENT WAVE (IN DEGREES)
FII = ANGLE OF INCIDENCE OF INCOMING PLANE WAVE (IN DEGREES)
LFI = ANGLE OF REFLECTION (IN DEGREES)
MA = 1 IF NEW CALCULATIONS ARE WANTED, ELSE MA = 1
PW = 1 IF BANDWIDTH CALCULATIONS ARE WANTED, ELSE PW = 0
C = IF PARAMETER VARIATION IS WANTED

IF BW = 1 (BANDWIDTH CALCULATION) USE FOLLOWING INPUT

MCIP = 1 IF THE PHYSICAL LENGTH OF DIPOLES HAS TO BE UNCHANGED
ELSE MCIP = 1
MD = 1 IF THE PHYSICAL DISTANCE D=LAMBDAC HAS TO BE UNCHANGED
ELSE MD = 1
MR = 1 IF THE PHYSICAL DISTANCE R=LAMBDAC HAS TO BE UNCHANGED
ELSE MR = 1
MD = 1 IF THE PHYSICAL LENGTH B=LAMBDAC HAS TO BE UNCHANGED
ELSE MD = 1
DIFF = HALF THE RANGE OVER WHICH THE FACTOR LAMBDAC/LAMBDAC
IS VARIED
RATIO = THE STEPS IN WHICH THE FACTOR LAMBDAC/LAMBDAC IS VARIED
(LAMBDAC = FREQUENCY, LAMBDAC = CENTER FREQUENCY)
IF BW = C (PARAMETER VARIATION) USE THE FOLLOWING INPUT

DI = STEPS IN CHANGING I DURING PARAMETER VARIATION
CC = STEPS IN CHANGING D DURING PARAMETER VARIATION
CR = STEPS IN CHANGING R DURING PARAMETER VARIATION
CDB = STEPS IN CHANGING B DURING PARAMETER VARIATION
CZ = STEPS IN CHANGING ZC DURING PARAMETER VARIATION
CCLE = STEPS IN CHANGING DLE DURING PARAMETER VARIATION

IMX = MAXIMUM VALUE OF I IN PARAMETER VARIATION
CMX = MAXIMUM VALUE OF D IN PARAMETER VARIATION
RMX = MAXIMUM VALUE OF R IN PARAMETER VARIATION
BMX = MAXIMUM VALUE OF B IN PARAMETER VARIATION
ZMX = MAXIMUM VALUE OF ZC IN PARAMETER VARIATION
CLMX = MAXIMUM VALUE OF DLE IN PARAMETER VARIATION

CIMENSION A(72,72),C(72),X(72),L(72,10)
CIMENSION H(36,6),Q(2,1C),S(5),V(36,10),IB(6)
CIMENSION G(19,10),TEI(1C),PHI(10),V(10),I(19),F(19)
INTEGER BW,DI,PAGINA

FORMAT(315)
FORM (415)
FORMAT(4FS,3)
FORMAT(2FS,3)
FORMAT(3FS,3)
FORMAT(6FS,3)
FORMAT(15,5F9.3)
READ(5,3C1) I,J,KPQ,KDB
READ(5,3C5) D,R,B,ZO,RAC,DLE
READ(5,3C3) W,FII
READ(5,3C6) LFI,MA,BW

IF(BW.EQ.1) READ(5,3C1) MDIP,KD,MR,MW
IF(BW.EQ.1) READ(5,3C6) DIFF

IF(BW.EQ.C) READ(5,3C6) DI,CD,DR,CDB,DZ,CCLE
IF(BW.EQ.C) READ(5,3C6) IMX,CMX,RMX,BMX,ZMX,CLMX

IF(BW.EQ.C) DIFF = 0.
IF(BW.EQ.C) MDIP = 1
K=IC
LTETA=10
FAKTOR=1.C-DIFF
PAGINA = C
IST=I
CST=C
RST=R
BST=B
ZST=Z0
CLST=CLE

IF(BW.EQ.-1) GO TO 502
IF(BW.EQ.C) GO TO 503

501 I=1 +DI
GO TO 515
C=C +CD
GO TO 515
B=B +CCB
GO TO 515
ZD=ZD+DZ
GO TO 515
R=R +CR
GO TO 515
CLE=CLE+CCLE
GO TO 515
IF(UW.EQ.C) GO TO 5C3
IF(MC.EQ.C) C=C/FAKTCR
IF(MR.EQ.C) R=R/FAKTCR
IF(MB.EQ.C) B=B/FAKTCR
5C3 C=C*6.2831853
R=R*6.2831853
B=B*6.2831853
PAGINA = PAGINA + 1
N=I**2
M=I/2
IF(M*2-I) 11,12,11
11 LE=1
GO TO 2
12 LE=I
LL=I-LE+I
M=I-LE-1
IF(LE) 14,15,14
14 L5=(I-1)/2+(M-1)*I+1
H(L5,1)=C.C
L6=(I*(I-1))/2+M
H(L6,2)=C.C
CONTINUE
11=1
FAKT1 = 1.CCCCC
IF(MCIP.EQ.0) FAKT1 = FAKTCR
CALL BETA(IJ,ZC,R,H,FAKT1,RAD,CLE)
TAL=0.
OPG=FLOAT(KPQ)/1C.
TAL=TAL+1.
CLE=CLE/FAKTCR
GO 24 M=1,K
YM = W
VVV(M)=W
IFN(M) = FII
XM = YM = OPQ
XK = K - 1
TETA(M) = (XM * SC_) / XK
IF(ABS(TETA(M)) < SC_) THEN 22, 21, 22
21 IF(ABS(21, 25, 22)
22 TETI = TETA(M) * CI74532525
Z = 0.0174532525
OP = COS(3.14159265 * DLE * SIN(TETI) * SIN(FII^2)) * CCS(3.14159265 * CLE))
1 (SIN(w2) * CCS(FII^2) - CCS(k^2) * SIN(FII^2) * CCS(TETI)) / (SQRAT(1. -
2 SIN(TETI)^2) * CCS(FII^2) * CCS(TETI))

Q(1, M) = P * FLOAT(J) * CCS(2. * R * CCS(TETI))
Q(2, M) = P * FLOAT(J) * CCS(2. * R * CCS(TETI))
DO 23 L = 1, N
23 CV(L, M) = P(L, 1) * CCS(FII^2) * SIN(TETI)
1 -H(L, 2) * CCS(FII^2) * SIN(TETI)
24 CONTINUE
GO TO 26
25 K = K - 1
26 CALL MAT1(LE, 1, N, J, B, A, H)
IF(LE = 1) THEN 11C, 41, 1IC
41 CALL MAT2(LE, 1, N, J, B, A, H)
110 CONTINUE

DO 50 L = 1, K
IF(LE = 1) THEN 45, 48, 49
48 N1 = (N + 1) / 2
N2 = (3 * N + 1) / 2
C(N1) = Q(1, L) * CCS(V(N1, L)) - C(2, L) * CCS(V(N1, L)) * CCS(B)
C(N2) = Q(1, L) * CCS(V(N1, L)) + C(2, L) * CCS(V(N1, L)) * CCS(B)
49 MM = (N - LE) / 2
NL = 2 * N + L
DO 80 CC M = 1, MM
NM1 = N + 1 - M
MN = N + M
M1 = M + (N + LE) / 2
M2 = M + (3 * N + LE) / 2
RVM = Q(1, L) * CCS(V(M, L)) - C(2, L) * CCS(V(M, L))
CVM = Q(1, L) * CCS(V(M, L)) + C(2, L) * CCS(V(M, L))
RVM1 = Q(1, L) * CCS(V(NM1, L)) - C(2, L) * CCS(V(NM1, L))
CVM1 = Q(1, L) * CCS(V(NM1, L)) + C(2, L) * CCS(V(NM1, L))
C(M) = (RVM - RVM1) * CCS(B * C.5)
C(M1) = (RVM + RVM1) * CCS(B * C.5)
C(MN) = (CVM - CVM1) * CCS(B * C.5)
C(M2) = (CVM + CVM1) * CCS(B * C.5)
800 CONTINUE
NOW THE MATRIX IS FILLED UP AND THE SOLVING OF THE
EQUATIONS WILL START

IF(LE = 1) THEN 7CC, 6CC, 7CC
500 CALL SOLVE(2 * N, A, C, 1, 0, 01, 5, X, IT)
GO TO 52
700 CALL SOLVE(2*N,A,C,2,C.01,5,X,IT)
52 N2= 2*N
DO 51 KM=1,N2
51 UKM,L)=X(KM)
50 CONTINUE

NOW THE RERRADIATION PATTERN WILL BE CALCULATED

C
C IF(IT) 94,55,55
55 KLM = 0
210 IF(KLM) 230,220,230
220 FI=LFI+180
KT= 90
GO TO 240
230 FI=LFI
KT=1CC
240 OCALL PATT(KLM,KT,K,1,N,J,D,R,W,ZC,FII,TETAE,F1,FETA,HEU,G,T,F,CLE
1)
KLM=KLM+1
IF(KLM-1) 281,210,281
281 WRITE (6,CC)
WRITE (6,71) N,PAGINA
WRITE (6,72)
WRITE (6,74)
IF(J.EQ.1) WRITE(6,76)
IF(BW.EQ.C) GO TO 310
WRITE(6,2CC) FAKTOR
IF(MDIP.EQ.0) WRITE(6,333)
IF(MR.EQ.C) WRITE(6,555)
IF(MB.EQ.0) WRITE(6,666)
310 IF(MDB.NE.C) GO TO 120
DO 111 MK = I,19
DO 111 KK = IvK
111 G(MK,KK) = DB(G(MK,KK))
120 AA = B/6.2316531
DPI=D/6.2316531
77 WRITE(6,177) DPI
WRITE(6,777) RAD
WRITE(6,688) DLE
WRITE(6,175) A
HH=R/6.2318531
IF(J-1) 180,178,180
178 WRITE (6,175) HH
180 WRITE(6,181) ZO
IF(MDB.NE.C) WRITE(6,CC)
WRITE (6,78) (TETA(K),M=1,K)
WRITE (6,79) (PHII(M),M=1,K)
WRITE (6,80) (VV(M),M=1,K)
WRITE (6,81)
C
DO 84 MK=I,15
84 WRITE (6,9C) T(MK),F(MK),(G(MK,P),P=1,K)
IF(MDB.NE.2) GO TO 96
WRITE(6,3C)
C 014C02
ATTAS - EFN SRCRCE STATEMENT - IFN(S) -

CO 112 MK = 1,19
CO 112 KK = K
112 G(MK,KK) = DB{G(MK,KK)}
113 WRITE(6,9C) T(MK),F(MK),(G(MK,H),H=1,K)
GO TO 96
94 WRITE(6,95)
96 D=CPI
B=AA
R=AF
IF(BW.EQ.1) GO TO 504
IF(ID,NE,C.AND.I.LT.IMX) GC TC 501
I=O
I=IST
IF(IDC,NE.O..AND.D.LT.DMX) GC TO 510
DD=O
C=OC
C=OC
IF(IDC,NE.O..AND.C.LT.CMX) GC TC 512
CDB=O.
B=BST
IF(DZ,NE.C..AND.Z0.LT.ZMX) GC TC 513
CZ=O.
Z0=ZST
IF(DR,NE.C..AND.R.LT.RMX) GC TO 511
CR=O.
R=RST
IF(IDD,NE.O..AND.DLE.LT.DLX) GC TO 514
DLE=O.
DLE=DLST
C 504 IF(BW.EQ.O) GO TO 505
C
IF(MD.EQ.C) D=D*FAKTOR
IF(MR.EQ.C) R=R*FAKTOR
IF(MB.EQ.O) B=B*FAKTOR
DLE=DLE*FAKTOR
FAKTOR=FAKTOR*FAKTOR
IF(FAKTOR.LT.1.C+0.1) GC 10 502
505 IF(MA) 93,100,93
93 CONTINUE
300 FORMAT (1H1)
71 OFORMAT(3IH SQUARE VAN ATTA REFLECTOR WITH,13,10H ELEMENTS,50X,4P
1AGE,14)
72 OFORMAT(102H-CALCULATED REFRACTION PATTERN *G* IN THE DIRECTION (I
VEN BY THE ANGLES TETA AND F1 FCR VARICUS CASES)
74 OFORMAT(67H OF INCIDENCE AND PCLARIZATION -GIVEN BY THE ANGLES TETA
II,FI1 AND V)
76 FORMAT(42HCHERE THE CONDLCING PLATE IS TAKEN INTO ACCOUNT)
200 OFORMAT(45HOBANDWIDTH CALCULATIONS FCR LAMBDA/LAMBDAO =,F8.3,4P AN
1F)
333 OFORMAT(62H NO VARIATION CF PHYSICAL DIPCLE LENGTH (LENGTH= 0.5* LA
IMBDA1)
444 FORMAT(5CH NO VARIATION CF PHYSICAL DISTANCE BETWEEN DIPCLES)
555 FORMAT(43H NO VARIATION CF PHYSICAL HEIGHT EVER PLATE)
666 FORMAT(41H NO VARIATION CF PHYSICAL LENGTH (DE LINES)
177 FORMAT (27HODISTANCE BETWEEN ELEMENTS=,F10.3H WAVELENGTH+S)
014CO2
ATTA - EFN SRCRCE STATEMENT - IFN(S) -

777 FORMAT(27H DIPOLE RADIUS = $F11.3$, 12H WAVELENGTHS)
888 FORMAT(27H DIPOLE LENGTH = $F11.3$, 12H WAVELENGTHS)
175 FORMAT(29H LENGTH OF TRANSMISSION LINES =, F8.2, 13H WAVELENGTHS)
179 FORMAT(33H DISTANCE FROM ELEMENTS TO PLATE =, F4.2, 13H WAVELENGTHS)
181 FORMAT*(48H CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINES =, F5.2, 6H 1 OHMS)
30 FORMAT(48H RADIATION PATTERN VALUES MEASURED IN DECIBELS)
78 FORMAT(9H TETA = , F2X, 10F9.1)
79 FORMAT(9H FII = , F2X, 10F5.1)
80 FORMAT(9H V = , F2X, 10F5.1)
81 FORMAT(11H TETA = FI)
90 FORMAT(1H, F4.1, F6.0, 10F9.2)
95 FORMAT(42H THE SYSTEM OF EQUATIONS IS SINGULAR)
STOP
END

$IBFTC DEC1 DECK

REAL FUNCTION DB(X)

* DB = CONVERTS THE RADIATED ENERGY INTO DECIBELS

DB = -99.9
XI = 1COCCCCC.$X
KK = IFIX(XI)
IF(KK .NE. 0) GO TO 1
RETURN
1 CB = 10. * ALOGIC(X)
RETURN
END
SUBROUTINE MATI(LE,I,N,J,B,IB,A,H)

COMPUTES THE FIRST PART OF THE MATRIX ECUATICA

DIMENSION IB(I),A(72,72),H(36,6)
MM=(N-LE)/2
CD 32 M=1,MM
CD GO L=1,1
MX=N-LE+1
IF(140=MX-5) 27,30,30
27 IF(140=(MX-LE)+50*I-5) 28,29,29
28 LL=N-2*M+2
GO TO 31
29 LL=3*M+N-2*L+1+2
GO TO 31
30 CONTINUE

Y=(1,3)-FLOAT(J)*H(1,15)-H(LL,3)+FLOAT(J)*H(LL,5)*CCS(B=0.5)
A(M,2*M-1)= Y
MN=M+N
A(MN,2*M)= Y
NM1=2*(N-M)+1
NM2=2*(N-M)+2
ML1=M+(N+LE)/2
ML2=M+(3*N+LE)/2
A(M,NM1)= -Y
A(MN,NM2)= -Y

OY=(H(1,4)-FLCAT(J)*H(1,6)-H(LL,4)+FLCAT(J)*H(LL,6))*CCS(B=0.5)
1 1+SIN(B=0.5)
A(M,2*M)= Y
A(MN,NM1)= Y
A(M,NM2)= -Y
A(MN,2*M-1)= -Y

Y=(H(1,3)-FLOAT(J)*H(1,15)+H(LL,3)-FLCAT(J)*H(LL,5))*SIN(B=0.5)
A(ML1,2*M-1)= Y
A(ML1,NM1)= Y
A(ML2,2*M)= Y
A(ML2,NM2)= Y

OY=(H(1,4)-FLOAT(J)*H(1,6)+H(LL,4)-FLCAT(J)*H(LL,6))*SIN(B=0.5)
1 -COS(B=0.5)
A(ML1,2*M)= Y
A(ML1,NM2)= Y
A(ML2,2*M-1)= -Y
A(ML2,NM1)= -Y
CONTINUE

QMM=(N-3*LE)/2
LL=(N-LE)/2
CO 40 M=1,MM
ML=M+1
IF(LL-ML) 555,666,666

CO 4C L=ML,LL
K1=C
L2= L-M
CO 35 M1=M,L,L2
K1=K1+1
CO 34 LL=1,LL
MX=M1-L)*1
IF(12+MX-1) 19,24,34

LB(K1)=MX+1-1
K1=K1+1
LB(K1)=LL-1
GO TO 35

CONTINUE

IB(5)*I-1-18(2)
IB(6)=I-1-18(4)
IF(IB(1)-18(3)) 37,37,36

LL1= I*(IB(4)-IB(2))-18(3)+18(1)+1
GO TO 113

LL1= I*(IB(4)-IB(2))+IB(3)-18(1)+1
113 IF(18(1)-18(5)) 39,39,38

LL2= I*(IB(6)-IB(2))-18(5)+18(1)+1
GO TO 114

LL2= 1*(IB(6)-IB(2))+IB(5)-18(1)+1

Y=(H(LL1,3)-FLOAT(J)*H(LL1,5)-H(LL2,3)+FLOT(J)*H(LL2,5))
1 *COS(E*C.5)
A(L,2*H-1)=Y
A(H,2*L-1)=Y
NL=N4L
NM=N+M
NM1= 2*(N-M)+1
NL1=2*(N-L) +1
NM2=NM1+1
NL2=NL1+1
LE1=L+(N+LE)/2
ME1=M+(N+LE)/2
LE2=L+(3*N+LE)/2
ME2=M+(3*N+LE)/2
A(NM,2*L)= Y
A(NL,2*M)= Y
A(L,NM1)= -Y
A(M,NL1)= -Y
A(NL,NM2)= -Y
A(NM,NL2)= -Y

OY=-1*(H(LL1,6)-FLOAT(J)*H(LL1,9)-H(LL2,4)+FLOT(J)*H(LL2,6))
1 *COS(E*C.5)
A(L,2*H)=Y
SUB1 -  EFN  SCLRCE STATEMENT - IFN(S) -

A(M, 2*L) = Y
A(NL, NM1) = Y
A(NM, NL1) = Y
A(L, NM2) = -Y
A(M, NL2) = -Y
A(NL, 2*M-1) = -Y
A(NM, 2*L-1) = -Y

C

OY = (H(LL1, 3) - FLCAT(J) * H(LL1, 5) + H(LL2, 3) - FLCAT(J) * H(LL2, 5))
   * SIN(B*C*S)
A(LE1, 2*M-1) = Y
A(ME1, 2*L-1) = Y
A(LE1, NM1) = Y
A(ME1, NL1) = Y
A(LE2, 2*M) = Y
A(ME2, 2*L) = Y
A(LE2, NM2) = Y
A(ME2, NL2) = Y

C

OY = -(H(LL1, 4) - FLCAT(J) * H(LL1, 6) + H(LL2, 4) - FLCAT(J) * H(LL2, 6))
   * SIN(B*C*S)
A(LE1, 2*M) = Y
A(ME1, 2*L) = Y
A(LE1, NM2) = Y
A(ME1, NL2) = Y
A(LE2, 2*M-1) = -Y
A(ME2, 2*L-1) = -Y
A(LE2, NM1) = -Y
A(ME2, NL1) = -Y

40 CONTINUE
999 CONTINUE
RETURN
ENC
SUBROUTINE MAT2(LE, I, N, J, B, A, H)
C
*MAT2* COMPUTES THE SECOND PART OF THE MATRIX EQUATION

DIMENSION A(72,72), H(36,6)
Y=H(1,3)-FLOAT(J)*H(1,5)*CCS(B)
C
NE=(N+LE)/2
NE1=(3*N+LE)/2
A(NE,N)= Y
A(NE1,N+1)= Y
A(NE,N+1)= -(H(1,4)-FLOAT(J)*H(1,6))*CCS(B)+SIN(B)
A(NE1,N)= -A(NE,N+1)
C
MM=(N-LE)/2
DO 47 M=1,MM
LL=(N-LE)/2
DO 43 L=1,LL
MX=M+L-1
IF(2*MX-1) 42,43,43
42 NX=MX+I-1
NY=I-1
GO TO 44
43 CONTINUE
44 IF(2*NX-I) 46,46,45
45 LL3= I*(1-1/2-NY)+3-1/2+NY
GO TO 115
46 LL3= I*(1-1/2-NY)+(1+1)/2-NY
C
115 Y=(H(LL3,3)-FLOAT(J)*H(LL3,5))*CCS(B)
NM=NM+M
N1=(N+1)/2
N2=(3*N+1)/2
NM1=2*(N-M)+1
NM2= NM1+1
M2= (3*N+1)/2+M
M1= (N+1)/2+M
A(N1,2*M-1)= Y
A(N1,NM1)= Y
A(N2,NM2)= Y
A(N2,2*M)= Y
C
Y=-H(LL3,4)-FLOAT(J)*H(LL3,6)*CCS(B)
A(N1,2*M)= -Y
A(N1,NM2)= Y
A(N2,2*M-1)= -Y
A(N2,NM2)= -Y
Y = 2. * (H(LL3, 3) - FLOAT(J) * H(LL3, 5)) * SIN(B * 0.5)
A(M2, N+1) = Y
A(M1, N) = Y
A(M1, N+1) = -2. * (H(LL3, 4) - FLOAT(J) * H(LL3, 6)) * SIN(B * 0.5)
A(M2, N) = -A(M1, N+1)
A(M, N) = 0. C
A(M, N+1) = C * C
A(NM, N) = C * C
RETURN
ENC
SUB3 - EFN SOURCE STATEMENT - IFN(S) -


*PATT COMPUTES THE RERACIATION PATTERN OF THE REFLECTOR

C DIMENSION TETAI(10), H(36, 6), L(72, 10), G(19, 10), T(19), F(19)
PO(X)= SIN(0.5*X*XI*D)/(C.5*X*XI*D)
MKM=0
C0 91 NTETA=IC*KLTETA
MKM=MKM+1
MK = KLM+ 9*MKM
NX = (MKM-11)*(KLM-1)+ MKM + KLM
IF(KLM) 26C, 25C, 260
250 MTETA=1C-NTETA
GO TO 27C
260 MTETA=NTETA - 1C
270 DO 89 M=1,K
GR=0.
GI =0.
Y=3.14159265/180.
TETA=MTETA
TETA = TETAI(NX)
T(MK)=TETA
F(MK)=FI
OXY1=120.3.14159265*(COS(3.14159265*DLE)*SIN(TETA*Y)*SIN(F1*Y))
1 2*(SQR(1-(SIN(TETA*Y)*SIN(F1*Y)))*2)
2 SIN(3.14159265*DLE))
N2=2*N
C0 84 L=1,N2,2
L1=(L+1)/2
CL=H(L1,1)*SIN(TETA*Y)*CCS(F1*Y)+H(L1,2)*SIN(TETA*Y)*SIN(F1*Y)
AA= U(L,M)
NL=L+1
BB= U(NL,M)
GR=GR+AA*COS(C1)+BB*SIN(C1)
GI*GI-AA*SIN(C1)+B/COS(C1)
84 CONTINUE
IF(J-1) 87, 85, 87
85 C0 86 L=1,N2,2
L1=(L+1)/2
OCL=H(L1,1)*SIN(TETA*Y)*CCS(F1*Y)+H(L1,2)*SIN(TETA*Y)*SIN(F1*Y)
1 2*R*COS(TETA*Y)
AA= U(L,M)
NL=L+1
BB= U(NL,M)
AN=N
C14CC2

SUB3 - CEFN SCLRCE STATE=ETAT - IFN(S) -

GR=GR-AA*COS(CI)-BR*SIN(CI)
GI=CI+AA*SIN(CI)-BB*CSC(CI)

86 CONTINUE
XI=I
AR=COS(R*(COS(TETA(I)*Y)+CSC(TETA*Y)))
AI=SIN(R*(COS(TETA(I)*Y)+CSC(TETA*Y)))
AL=SIN(TETA(I)*Y)*CSC(FI*I*Y)+SIN(TETA*Y)*CSC(FI*I*Y)
BF=SIN(TETA(I)*Y)*SIN(FI*I*Y)+SIN(TETA*Y)*SIN(FI*I*Y)
BL=AL*ICCCC.
REI=RE=1CCCCC.

IF(IFIX(AL)) 29G,291,23C

291 SAL=1.
GO TO 292
290 SAL = PC(AL)
292 IF(IFIX(BE1)) 294,293,294
293 SBE=1.
GO TO 295
294 SBE = PO(BE)
295 XK2= C.5*SAL*SBE=AN=D**2
TP=COS(W*Y)*COS(FI*I*Y)+SIN(W*Y)*CSC(TETA*Y)*SIN(FI*I*Y)
TT=-COS(W*Y)*SIN(FI*I*Y)+SIN(W*Y)*CSC(TETA*Y)*CSC(FI*I*Y)
FT=-TP*CSC(FI*I*Y)*CSC(TETA*Y)+TT*SIN(FI*I*Y)*CSC(TETA*Y)
FP=TP*SIN(FI*I*Y)+TT*CSC(FI*I*Y)
OG(WK,**)= ((XK1*GR*SIN(FI*I*Y)*CSC(TETA*Y)+(XK2*AR*FT)**2
1 + (XK1*GR*SIN(FI*I*Y)*CSC(TETA*Y)+(XK2*AI*FT)**2
2 + (XK1*GR*CSC(FI*I*Y)+(XK2*AR*FP)**2
3 + (XK1*GR*CSC(FI*I*Y)+(XK2*AI*FP)**2)/(3.14159265)**2)

87 IF(J-1) 66,65,6E
88 CG(WK,**)=XK1*2*(GR*2*G**)2*((SIN(FI*I*Y))**2*(CSC(TETA*Y))**2
1 + (CSC(FI*I*Y))**2)/(3.14159265**3)
89 CONTINUE
91 CONTINUE
RETURN
END
SUBROUTINE BETA (II, J, ZC, R, H, FAKTOR, RAD, DLE)

* * BETA * BUILDS UP AN ARRAY OF SELF- AND MUTUAL IMPEDANCES *

DIMENSION F(36,6)
CALL SELF (CLE/FAKTOR, RAD, RS, XS)
GO TO M = 1, II
IF (M = 1) 17, 16, 17
F (1, 3) = RS/ZC
F (1, 4) = - XS/ZC
GO TO 18
16 OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       ABS (C.1591549* (H(1,1) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 3) = Z/ZC
OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       ABS (C.1591549* (H(1,1) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 4) = Z/ZC
17 OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       0.1591549* SQRT ((H(1,1) - H(1,2))* 2 + (R**2)),
1       ABS (C.1591549* (H(1,2) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 5) = Z/ZC
OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       0.1591549* SQRT ((H(1,1) - H(1,2))* 2 + (R**2)),
1       ABS (C.1591549* (H(1,2) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 6) = Z/ZC
18 IF (J) 19, 2, 15
19 OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       0.1591549* SQRT ((H(1,1) - H(1,2))* 2 + (R**2)),
1       ABS (C.1591549* (H(1,2) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 5) = Z/ZC
OZ = AMPE (CLE/FAKTOR, DLE/FAKTCR,
1       C.1591549* SQRT ((H(1,1) - H(1,2))* 2 + (R**2)),
1       ABS (C.1591549* (H(1,2) - H(1,2))) , 0.0, 0.0, 0.01, 10)
T (M, 6) = Z/ZC
18 CONTINUE
RETURN
SUBROUTINE SELF(H, A, R, X)
C
C *SELF* COMPUTES THE SELF IMPEDANCE OF THE CYCLES
C
C REAL CSINT, CINSI
PI=3.14159265
AA=SQRT(2.)*2.*PI*A
HH=PI*I
CSINT=CINSI(2.*HH)
CI2= C.577215665*ALOG(2.*HHI-REAL(CSINT))
S2= REAL(CSINT)
SI2= AIMAG(CSINT)
CSINT= CINSI(4.*HH)
CI4= 0.577215665*ALOG(4.*HHI-REAL(CSINT))
S4= REAL(CSINT)
SI4= AIMAG(CSINT)
X=-29.997925*(SIN(2.*HHI)-C.577215665*ALOG(HH/(AA**2)))+2.*CI2
C -CI4 - COS(2.*HHI*(2.*SI2-SI4)-2.*S12)
R= 29.997925*((2.*PI*CCS:2.*HHI)*S2-CCS(2.*HHI)*S4)
C -2.*SIN(2.*HHI)*SI2+SIN(2.*HHI)*SI4)
X=X/(SIN(HH)**2)
R=R/(SIN(HH)**2)
12 CONTINUE
13 CONTINUE
RETURN
END
COMPLEX FUNCTION CINSI(X)
C CINSI COMPUTES AS ITS REAL PART THE MODIFIED COSINE INTEGRAL AND AS C ITS IMAGINARY PART THE SINE INTEGRAL.
IF (X.GT.1.0) GO TO 20
C=1.0
S=1.0
TC=1.0
TS=1.0
Y=X*X
I=0
10 I=I+1
TC=-Y*TC/FLOAT(2*I*(2*I-1))
TS=-Y*TS/FLOAT(2*I*(2*I+1))
TERM=TC/FLOAT(2*I)
TERMS=TS/FLOAT(2*I+1)
C=C+TERM
S=S+TERMS
EC=ABS(TERM/C)
ERROR=1.0-EC
IF (EC.GT.ERROR) GO TO 10
CIN=1.0-C
SI=X*S
CINSI=CMPLX(CINSI)
RETURN
20 Y=X*X
F=(Y*(Y*(Y*(Y*(38.027264)+265.187033)+335.677320)+157.102495))
C (X*(Y*(Y*(Y*(Y*(40.021433)+322.624911)+57C.236280)+157.105422))
G=(Y*(Y*(Y*(Y*(42.242e55)+352.018498)+211.821899))
C (Y*(Y*(Y*(Y*(45.59558)+449.690326))
S=SIN(X)
C=COS(X)
CIN=0.577215665+ALOG(X)-F*S+G*C
SI=1.57079633-F*C-G*S
CINSI=CMPLX(CINSI)
RETURN
END
C14CC2

$IBFTC ZCAL  CECK
ZCAL      -      EFN      SOURCE      STATEMENT      -      IFN(S) -

REAL FUNCTION AMPE(H1,H2,YC,ZN,TETA,FI,NC,DELTA,WRC)

*AMPE* COMPLETES THE MTLAL IMPEDANCES BETWEEN THE CYCLES BY AN
INTEGRATION USING RROMBERG'S METHOD

THE REAL PART OF THE MTLAL IMPEDANCE FOR NC=1 AND
THE IMAGINARY PART FOR NC=C

DIMENSION TRAP(11)
STEP=+2
X=-H2/2.
CI=RX(H1,H2,YC,ZN,TETA,FI,NC,X)
X=H2/2.
FC=RX(H1,H2,YC,ZN,TETA,FI,NC,X)
TRAP(11)=(C1+FC)*STEP/2.
C1=C.
CO 2C3 K=1,NC=0
SUM=C.
ERRCR=C.
STEP=STEP/2.
M=3**K
XX=H2
CO 2C4 L=1,XX=2
LL=M-L
XL=FLOAT(LL)
XM=FLOAT(X)
X=XL/XM
X=X*(-1/2.*)*(1.-X)*H2/2.
FD=RX(H1,H2,YC,ZN,TETA,FI,NC,X)
C2=SUM+FC
IF(ABS(FC)-ABS(SLM))=2C6,2C6,2C5
205 ERRCR=ERRCR+SLM-(C2-FC)
CO TO 207
206 ERRCR=ERRCR+FC-(C2-SLM)
207 SUM=C2
204 CONTINUE
TRAP(K+1)=TRAP(K)/2.+(C2+ERRCR)*STEP
P=1.
KK=1C-K
CO 2C8 LL=KK,S,1
L=-(LL-1C)
P=P*4.
208 TRAP(L)=(TRAP(L+1)*P-TRAP(L))/(P-1.)
C2= TRAP(1)
IF(K-1) 2C5,2C5,211
211 IF(ABS(C2-C1)-DELTA*ABS(C2))=2C5,210,2C9
209 C1=C2
C14C02
ZCAL - EFN SCLRCE STATEMENT - IFN(S) -

203 CONTINUE
MORC=K-1
210 Q=25.97/25/SIN(3.14159265*H1)/SIN(3.14159265*H2)
IF(NGO-1) ZC1,ZC,C,ZC1
200 AMP= -Q*C2
GOTC 202
201 AMP= Q*C2
202 CONTINUE
RETURN
ENC
REAL FUNCTION RX(H1,H2,YC,ZC,TETA,FI,KC,X)

C
C RCUTINE AMPE
C
H1=H1/2,
B2=-2/2,
A=COS(TETA)
3=SIN(TETA)
C=B*3IN(FI)
D=B*COS(FI)
E=2.0*COS(3.14159265*H1)
F=Z0+B1
G=Z0-B1
SX=X*C
SY=X*C
SZ=X*A
RD2=SX**2+(YC+SY)**2
P=Z0+SZ
R=SQRT(RD2+H**2)
H=F+SZ
P=G+SZ
R1=SQRT(RD2+H**2)
R2=SQRT(RD2+P**2)
IF(ONO-1) 214,211,214
211 AK=SIN(6.28318531*R1)/R1
AL=SIN(6.28318531*R2)/R2
AM=SIN(6.28318531*R)/R
GO TO 212
214 AK=COS(6.28318531*R1)/R1
AL=COS(6.28318531*R2)/R2
AM=COS(6.28318531*R)/R
GO TO 212
212 CONTINUE
IF(Y0) 216,215,218
215 IF(SX) 218,216,218
216 IF(SY) 218,217,218
217 RX=(E*AM-AK-AL)*A*SIN(6.28318531*(B2-ABS(X)))
GO TO 219
218 ORX=((AK+H*AL*P-E*AM*H)*(X*C**2)+Y0*C+X*(C**2))/RC2
1+(E*AM-AK-AL)*A*SIN(6.28318531*(B2-ABS(X)))
219 CONTINUE
RETURN
ENC
SUBROUTINE SOLVE(NN,A,B,IN, EPS, ITMAX, X, IT)

CSOLVE LINEAR EQUATION SOLVER WITH ITERATIVE IMPROVEMENT VERSION IV
CSOLVES AX = B WHERE A IS AXN MATRIX AND B IS AX1 VECTOR
C
IN=
C 1 FOR FIRST ENTRY
C 2 FOR SUBSEQUENT ENTRIES WITH NEW B
C 3 TO RESTORE A AND B
C EPS AND ITMAX ARE PARAMETERS IN THE ITERATION
C IT=
C -1 IF A IS SINGULAR
C 0 IF NOT CONVERGENT
C NUMBER OF ITERATIONS IF CONVERGENT
C CALLS MAP SUBROUTINES ILCG2,DCT,SDCT AND CAD
C
C TO MODIFY DIMENSIONS, CHANGE THE NEXT 3 (ACT 2 BUT 3) CARCS.
C
OCIMENSION A(172,72), B(72), X(72), DX(72), R(72), Z(72), RAM(72), IRP(72),
IAA(72,72)
MA=72
C
MA MUST = DECLARED DIMENSION OF SYSTEM
EQUIVALENCER(DX)
GO TO (ICCC,2CC,03CCC)IN
1000 N=NN
NM1=N-1
NP1=N+1
C
C EQUILIBRATION
C
CO 51C I=1,N
KTOP=ILOG2(A(I,1))
DO 503 J=2,N
503 KTOP=MAX(KTOP, ILOG2(A(I,J)))
RM(I)=2.**(-KTOP)
DO 509 J=1,N
509 A(I,J)=A(I,J)*RM(I)
510 CONTINUE
C
C SAVE EQUILIBRATED DATA
C
DO 548 I=1,N
DO 548 J=1,N
548 AA(I,J)=A(I,J)
C
C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
C
C FO 99 M=1,NM1
SOLV - EFNSCLRGE STATEMENT - IFAS -

TOP = ABS (A(M,M))
IMAX = M

10 CONTINUE
IF (TOP) 14, 13, 14
IT = -1
C *SINGULAR*
RETURN
14 IRP(N) = IMAX
23 IF (IMAX - N) 25, 29, 24
24 CC 25 J = 1, N
TEM = A(M,J)
A(M,J) = A(IMAX, J)
A(IMAX, J) = TEM
29 MP1 = M + 1
CC 32 I = MP1, N
EM = A(I, M) / A(M, M)
A(I, M) = EM
IF (EM) 31, 33, 31
31 CC 32 J = MP1, N
32 A(I, J) = A(I, J) - A(M, J) * EM
33 CONTINUE
99 CONTINUE
IRP(N) = N
IF (A(M, N)) 128, 113, 120
113 IT = -1
RETURN
120 CONTINUE
C STORAGE FOR A NOW CONTAINS TRIANGULAR L AND U SUCH THAT (L+U)*U = A
C
C DUPLICATE INTERCHANGES IN DATA
C
CO 229 I = 1, N
IP = IRP(I)
IF (I - IP) 221, 229, 221
221 CC 222 J = 1, N
TEM = AA(I, J)
AA(I, J) = AA(IP, J)
AA(IP, J) = TEM
222 CC 229 CONTINUE
C
C PROCESS RIGHT HAND SIDE
C
2000 CONTINUE
CO 601 I = 1, N
601 B(I) = B(I) * RM(I)
CO 609 I = 1, N
IP = IRP(I)
TEMP = B(I)
B(I) = B(IP)
B(IP) = TEMP
609 CONTINUE
C SOLVE FOR FIRST APPROXIMATION TO X
C
199 CO 200 I=1,N
200 Z(I)=-SDOT(I-1,A(I,1),MA,2(I)),1,-B(I))
CO 201 K=1,N
I=NP1-K
201 X(I)=-SDOT(N-I,A(I,I+1),MA,X(I+1),1,-Z(I))/A(I,I)
C
C ITERATIVE IMPROVEMENT
C
IF(ITMAX)370,370,3CO
300 TOP=0.0
DO 303 I=1,N
303 TOP=MAX(TOP,ABS(X(I)))
EPSX=EPS*TOP
DO 369 IT=1,ITMAX
C FIND RESIDUALS
DC 319 I=1,N
319 R(I)=-DOT(N,AA(I,1),MA,X(I),1,-B(I))
C FIND INCREMENT
DO 329 I=1,N
329 Z(I)=-SDOT(I-1,A(I,1),MA,Z(I),1,-R(I))
DO 339 K=1,N
I=NP1-K
339 DX(I)=-SDOT(N-I,A(I,I+1),MA,DX(I+1),1,-Z(I))/A(I,I)
C INCREMENT AND TEST CONVERGENCE
TOP=C.C
DO 342 I=1,N
TEMP=X(I)
X(I)=CAD(X(I),DX(I))
DELX=ABS(X(I)-TEMP)
TOP=AMAX1(TOP,DELX)
342 CONTINUE
IF(TOP-EPSX<381,381,369
369 CONTINUE
370 IT=0
381 RETURN
C
C RESTORE A AND B
C
3000 CONTINUE
CO 709 K=1,N
I=NP1-K
IP=IRP(I)
IF(I-IP)701,709,701
701 TEMP=B(I)
B(IP)=B(IP)
B(IP)=TEMP
DO 702 J=1,N
TEMP=AA(I,J)
AA(I,J)=AA(IP,J)
AA(IP,J)=TEMP
702 CONTINUE
CO 729 I=1,N
B(I)=B(I)/RM(I)
DO 729 J=1,N
A(I,J)=A(I,J)/R(I)

CONTINUE
RETURN
END

INTEGER FUNCTION ILOG2(Z)
C
* ILOG2* ROUTINE FOR USE WITH ROUTINE SCLVE
C
ILOG2=C
IF (Z.NE.C.) GC TO 1
RETURN
1  ILOG2=AINT(3.322C*ALOG10(ABS(Z)))
RETURN
END
DOT AND FRIENDS Routines for Use with Solve

ENTRY DOT (N, A(1), MA, B(1), MB, C) Double Inner Product
ENTRY SDOT (N, A(1), MA, B(1), MB, C) Inner Product
ENTRY TLOG2 (A) Floating Point Exponent
ENTRY DAD (A, B) Add with Round

SNAD MACRN M Store Negative of Address in Decrement
SUB =0100000 Complement if Positive
ALS 18
STD M
ENDM SNAD

DOT SAVE 1,2,4

STZ S
STZ S+1
CLA* 8,4
LDQ C+1
STD C
CLA* 3,4
TZE NONE Skip Loop if N = 0
STD N

CLA 4,4
PAC ,1
CLA* 5,4
SNAD MA
CLA 6,4
PAC ,2
CLA* 7,4
SNAD MB
LXN N,4
LOOP LDQ O,1
FNP O,2
DFAD S
OST S

MA TXI ++1,1,** (X1)=(X1)+MA
MB TXI ++1,2,** (X2)=(X2)+MB
TIX LOOP, 4,1 End Loop

NONE OFAD C
FRN
RETURN DOT

SAVE 1,2,4

STZ  S
CLA* 8,4
STO  C
CLA* 3,4
TZE SNONE
STO N
CLA 4,4
PAC 1
CLA* 5,4
SNAD SMA

CLA 6,4
PAC 2
CLA* 7,4
SNAD SHB
LXA N,4
SLOOP LDQ 0,1
FMP 0,2
FAD S
STO S
SMA TXI ++1,1,**
SMB TXI ++1,2,**
TIX SLOOP 4,1
SNONE FAD C
RETURN SDOT

ILOGZ CAL* 3,4
ANA =03770000000000

SUB =02000000000000
AKS 27
TRA 1,4

DAD CAL* 3,4
FAD* 4,4
FRN
TRA 1,4

EVEN
C PZE
S PZE
N PZE
•LDIR
•LORG
END
Fig 1. Coordinate system for arbitrary Van Atta array.
Fig 2. The Van Atta principle (retrodirective effect).
The paths $ABCD$ and $A'B'C'D'$ are equal.

Fig 3. The specular reflection (mirror effect).
The paths $ABC$ and $A'B'C'$ are equal.
Fig. 4. Square Van Atta reflector with conducting plate.
Fig. 5. Normalized back-scattering cross section of 16 element square Van Atta reflector with conducting plate.

\( a = 0.41\lambda, d = 0.6\lambda, h = 0.25\lambda, Z_0 = 73\, \text{ohms}, \theta_i = 0^\circ, \phi = 90^\circ. \)
Fig. 6. Normalized back-scattering cross section as a function of the length of the dipoles.
N = 16 elements, $a = 0.41\lambda$, $d = 0.6\lambda$, $h = 0.25\lambda$
$Z_o = 73$ ohms, dipole radius = $0.015\lambda$
Fig. 7.16 element square Van Atta reflector without plate.

\[ a = 0.41 \lambda, \quad d = 0.6 \lambda, \quad Z_0 = 73 \text{ohms} \]

dipole length = 0.5\( \lambda \), dipole radius = 0.015\( \lambda \)

\( \phi = 0^\circ, \quad \psi = 90^\circ \)
Measure $M$ of the retrodirective effect.

Fig. 8.16: Element square Van Atta reflector with plate.
- $a = 0.41 \lambda_0$, $d = 0.6 \lambda_0$, $h = 0.25 \lambda_0$, $Z_0 = 73$ ohms
- Dipole length = $0.5 \lambda_0$, Dipole radius = $0.015 \lambda_0$
- $\phi_1 = 0^\circ$, $\nu = 90^\circ$
### REFLECTOR ARRAYS

The Van Atta reflector was first described in a patent by Dr. L.C. Van Atta in 1959. The advantage of this passive reflector type should be that the reradiated field has a maximum back in the direction of arrival of the primary plane wave. Since this retrodirective effect of the reflector might be of great importance if used as a navigational aid in the air or at sea, it seemed worthwhile to carry out a theoretical investigation of such reflectors, especially since only experimental investigations had been made before this contract was initiated.

The work performed under the contract dealt mainly with theoretical and numerical investigations of Van Atta reflectors consisting of dipoles. A survey of the literature concerning active or passive Van Atta reflectors has been made. Both a linear and a two-dimensional plane Van Atta reflector has been investigated numerically and a theory for arbitrary Van Atta reflectors has been developed. An experimental investigation of a linear Van Atta reflector was carried out and the results compared with the theoretical results.
Van Atta reflector
Reflector
Array
Adaptive Antenna System
Dipoles
Mutual Impedance
Image Theory
Conducting Plate
Bandwidth

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