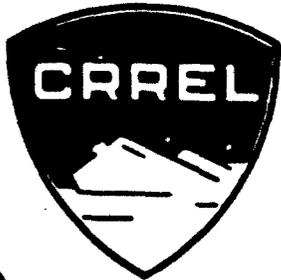


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Special Report 115

DENSITY, TEMPERATURE AND THE UNCONFINED COMPRESSIVE STRENGTH OF POLAR SNOW

by

Austin Kovacs

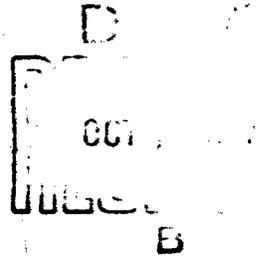
JULY 1967

U.S. ARMY MATERIEL COMMAND
COLD REGIONS RESEARCH & ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE

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PREFACE

This study was performed by Mr. Austin Kovacs under U. S. Army Cold Regions Research and Engineering Laboratory (USA CRREL) task, Experimental Engineering in Snow, Ice and Frozen Ground.

Information reported herein was initiated by USA CRREL in the Construction Engineering Branch, Mr. Edward F. Lobacz, Chief, of the Experimental Engineering Division, Mr. Kenneth A. Linell, Chief.

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SUMMARY

The relationships between several empirical and theoretical methods for determining the unconfined compressive strength of polar snow from depth-density and temperature profiles are discussed and graphically compared. Two unconfined compressive strength equations are proposed for snow at -10C:

$$\sigma_c = 1719 (\gamma - 0.422)$$

and

$$\sigma_c = 968 - 6640\gamma + 13520\gamma^2 - 7235\gamma^3.$$

These equations apply to snow densities from 0.50 to 0.72 g/cm³ and 0.36 to 0.72 g/cm³, respectively.

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INTRODUCTION

A number of methods have been presented for determining the unconfined compressive strength of polar snow when density and temperature are known. Of particular interest to those concerned with snow's resistance to pile driving are the methods devised by Ballard and Feldt (1965), Ballard and McGaw (1965), Butkovich (1956), Mellor and J.H. Smith (1966) and J. L. Smith (1965). The unconfined compressive strengths obtained by these methods, however, were found to deviate considerably from one another with both density and temperature.

This report covers a study made to determine why the anomalies exist. Where possible, inconsistencies in test procedures and data analysis used to develop existing unconfined compressive strength formulas are pointed out. Two equations for determining the unconfined compressive strength of snow are proposed. The formulas take into consideration the decided changes in slope of the Young's and shear modulus curves at a density of 0.5 g/cm^3 for Greenland snow. The slope changes signify that at this density a structural and, therefore, a strength change occur. Analysis of existing test data indicates this reasoning to be valid.

EXISTING METHODS FOR DETERMINING THE UNCONFINED COMPRESSIVE STRENGTH OF SNOW

From a comparison of horizontal snow samples tested at Site II, Greenland, Butkovich (1956) developed an empirical relationship between unconfined compressive strength and density for snow at -10C :

$$\sigma_c = 1418 (\gamma - 0.39) \quad (1)$$

where:

σ_c = unconfined compressive strength, psi.

γ = snow density, g/cm^3 .

Butkovich broke his samples with a Carver hand-actuated hydraulic press. This type of press is not considered ideal for unconfined compression testing because it produces undesirable pulsed loading and head speeds with each pump of the hydraulic jack. The speed at which the press was operated is not known but the average load rate was reported to be 7.5 psi (0.5 kg/cm^2) per second. Jellinek (1957) has found that in testing the tensile strength of ice at -4.5C the magnitude of the results is no longer dependent on the loading rate above 0.5 kg/cm^2 per second. It cannot be assumed, however, that the same is true for unconfined compression tests performed on snow at -10C . Butkovich (1958) and Mellor and J.H. Smith (1966) have shown that the unconfined compressive strength of snow is dependent upon temperature and load rate. The load rate required to mask the plastic effects of deformation is in turn dependent upon the temperature and density of the snow. Therefore, it cannot be assumed that the strain rate for these tests was sufficient to subject the samples to brittle failure over the entire density range.

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Butkovich's samples consisted of cores 3 in. (7.62 cm) in diameter with a length-to-diameter ratio of 2.2 to 1. This ratio was perhaps too low to eliminate the effect of end constraint (Butkovich, 1958).

At Camp Century, Greenland, J. L. Smith (1965) found a relationship similar to Butkovich's for vertical snow samples at -10C:

$$\sigma_c = 1542 (\gamma - 0.40). \quad (2)$$

A constant-velocity motorized press having a head speed of 1 in. (2.54 cm) per minute was used in these tests. The samples were uniformly trimmed to 2.00 in. (5.08 cm) in diameter with a special shaver and had a length-to-diameter ratio of 2.5 to 1.

Ballard and McGaw (1965) presented a theory that attempted to explain snow failure or unconfined compressive strength at any temperature in terms of the crushing strength of snow ice* and the porosity of snow when $\gamma > 0.46$ g/cm³:

$$\sigma_f = \sigma_i \left(1 - \frac{n}{n_L}\right) \quad (3)$$

where:

σ_f = failure strength, psi

σ_i = ultimate strength of fine-grained, randomly oriented polycrystalline ice, psi

n = volumetric porosity of the given snow density

n_L = limiting porosity, i. e. volumetric porosity at which σ_f is zero (extrapolated).

Ballard and Feldt (1965) calculated the effective porosity (n_f)† as a function of porosity over the entire porosity range and developed the following equation:

$$\sigma_f = \sigma_i e^{-2(n/1-n)}. \quad (4)$$

Mellor and J. H. Smith (1966) presented an unconfined compressive strength equation for snow based upon the crushing strength of ice** and the void ratio:

$$\sigma_c = \sigma_i e^{-br^2} \quad (5)$$

*Snow ice in this report refers to consolidated snow with zero permeability.

†The effective porosity (n_f) is defined by Ballard and Feldt as the porosity of the snow along the failure surface. Their geometrical considerations show n_f to be approximately twice n .

**Mellor and J. H. Smith suggest using the strength of clear lake ice for σ_i as they believe this would represent an optimum value for snow ice at a density of 0.917 g/cm³.

where:

b = a dimensionless constant

r = void ratio of the snow.

To develop this equation Mellor and J. H. Smith tested samples prepared from snow ground through a 1-mm sieve. The snow was compacted in tubes (5.72 cm diam, 18.9 cm long) and allowed to sinter for approximately 3 weeks at -10C. At the end of this period the samples were exposed to the test temperature (0 to -50C) for 6 hours before being broken. The samples were crushed under a motorized press with a head speed of 3.64 in. (9.25 cm) or 5.77 in. (14.65 cm) per minute.

COMPARISON OF THE FIVE UNCONFINED COMPRESSIVE STRENGTH EQUATIONS FOR SNOW AT -10C

To use the theoretical expression developed by Ballard and McGaw (eq 3) the limiting porosity and ultimate strength of snow ice have to be determined. To apply this equation to Camp Century snow the necessary values are obtained from the experimental data of J. L. Smith. From eq 2, $\sigma_c = 0$ at a density of 0.40 g/cm³ and n_L becomes $1 - 1.09\gamma = 0.564$. Assuming a density of 0.917 g/cm³ for snow ice and extrapolating σ_c to σ_i , a value of 800 psi (56.1 kg/cm²) is obtained from eq 2.

Since the Ballard and McGaw (1965) equation is a straight line, the unconfined compressive strength when plotted against porosity for each test temperature results in a family of straight lines which pass through zero strength at the limiting porosity and maximum strength at zero porosity. It should, therefore, be obvious that using 800 psi for σ_i along with 0.564 for n_L in eq 3 gives strengths directly obtainable from the simple J. L. Smith equation. Equation 3 is merely a method for expressing any linear unconfined compressive strength vs density relationship in terms of porosity.

It should be pointed out that some investigators have used the temperature dependence of clear lake ice (Fig. 1) to obtain σ_i (Abele et al., 1966; Ramseier, 1966). There is no justification for doing this unless it is desired to obtain σ_c values which are low and, therefore, provide a factor of safety for engineering purposes.

The exponential parameters of eq 5 can be changed to coincide with those of eq 4. The constant b in eq 5 has been tentatively interpreted as an index of grain structure. Using J. L. Smith's test results and eq 5, Mellor found b to be 1.8 for Camp Century snow. With the void ratio being directly related to porosity ($r = n/1-n$) and the constant b known, eq 5 can be expressed as follows:

$$\sigma_c = \sigma_i e^{-1.8(n/1-n)^2} \quad (6)$$

A graphic comparison relating unconfined compressive strength vs density as derived from all the equations is presented in Figure 2. To show the effect of using σ_i for lake ice (700 psi or 49.2 kg/cm² at -10C) upon the unconfined compressive strength vs density relationship, this value is used in eq 3 and 4 for the related curves in Figure 2. Equation 4 is also graphically shown when an ultimate strength of 800 psi (56.1 kg/cm²) is used for σ_i as extrapolated from eq 2.

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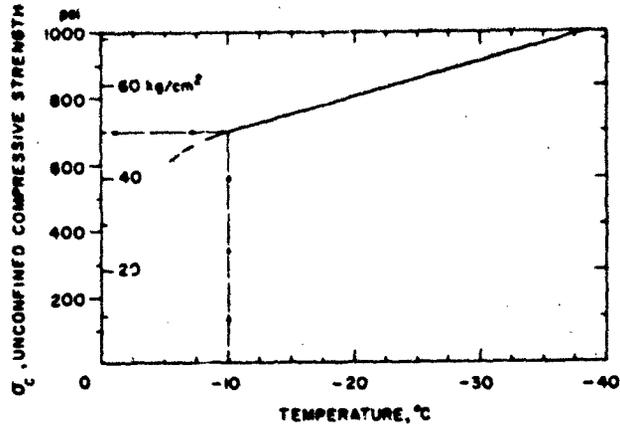


Figure 1. Unconfined compressive strength vs temperature for clear lake ice (from Butkovich, 1964).

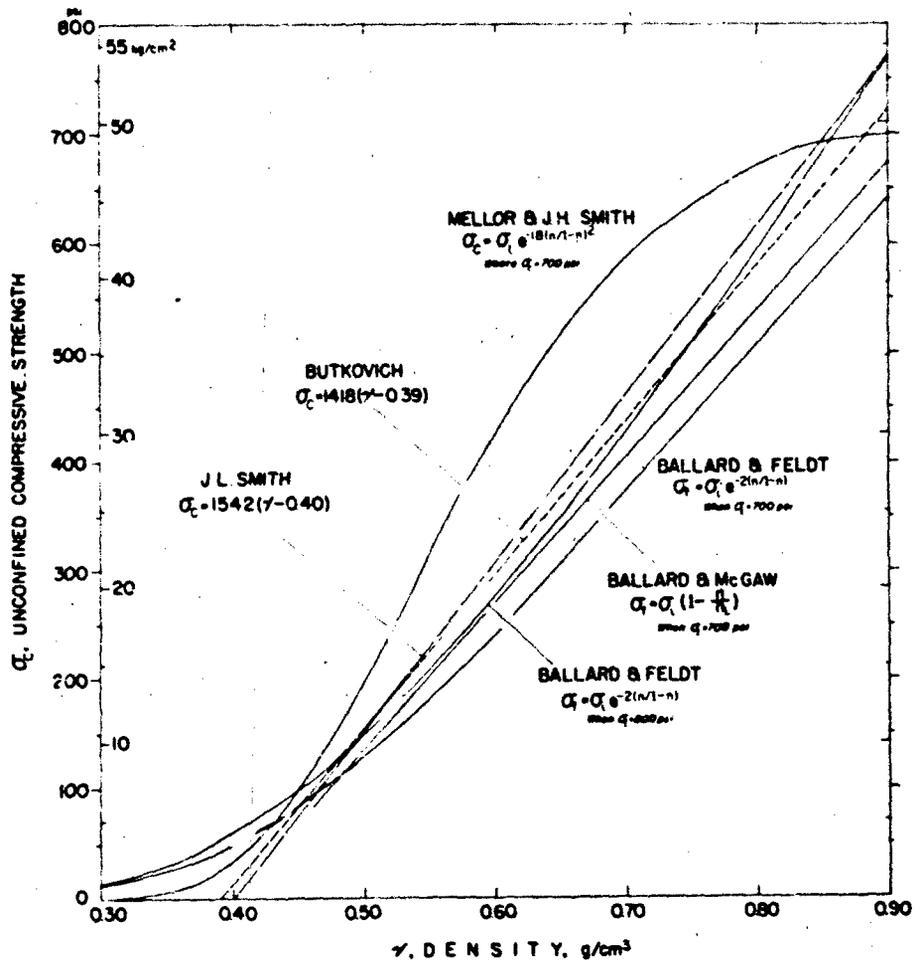


Figure 2. Comparison of existing unconfined compressive strength vs density relationships for polar snow at -10°C.

From an inspection of Figure 2, it is apparent that considerable discrepancy exists between the strengths obtained from the different unconfined compressive strength vs density relationships as presently used. It is believed that the development of the Mellor-J. H. Smith equation could have been adversely affected by the snows tested. Although sintered particle size might have been comparable to that of polar snow of similar density, the samples might not have adequately represented the sintered structure or strength of natural snows of comparable density. Mellor recently has suggested that the large divergence from the other unconfined compressive strength values was due to the temperature and strain rate at which his samples were broken. The strain rate was apparently below that necessary to cause brittle failure in the higher temperature snows. If this did occur, the results would be influenced by creep effects and would be more applicable to snow subjected to creep failure than to the structural collapse of snow associated with brittle failure.

The difference in σ_f between the Ballard-McGaw and Ballard-Feldt curves is related to the considerations upon which their equations were based and the different values of σ_i used.

The slight disagreement between the Butkovich and J. L. Smith rectilinear strength vs density values may have resulted from the different loading characteristics of the test apparatuses and the different length to diameter ratios of the test samples. Although the relationships of Butkovich and Smith are useful for a number of purposes, they do not adequately relate unconfined compressive strength to snow density. This is especially true for snow densities in the pronounced transition range between low-density open-structured snow of about 0.30 g/cm³ and high-density snows of about 0.52 g/cm³ where a sintered transition (stable bond structure) is established within thermally stable in-situ snows (Ramseier, 1963; Gow, 1966).

Although previously mentioned test inconsistencies are believed to have affected the test results, the linearization of the unconfined compressive strength vs snow density relationship into a single equation definitely resulted in less agreement in the correlation of unconfined compressive strength and density. To show this, the Butkovich and J. L. Smith data (Appendix A, Tables AI and AII) were replotted (Fig. 3). Unconfined compressive strengths obtained at temperatures other than -10C were not included to eliminate any strength error associated with the use of a temperature correction factor.

An inspection of the replotted data showed a decided change in unconfined compressive strength vs density at a density of about 0.50 g/cm³. At a similar density, changes in the Young's and shear modulus curves occur (Nakaya, 1959; J. L. Smith, 1965). This indicates a structural change associated with the transition between low-density open-structured snow and closely packed high-density snow in which a sintered transition has been established. Another indication of a structural change existing in in-situ snow at a density of 0.50 g/cm³ is the bend in the P-wave velocity curve near this density depth at Camp Century (Clark, 1966). Because this density appears to represent an area where significant changes in both physical and mechanical measurements occur, it is tentatively referred to as the "transition density."

Using the foregoing observations as a guide, an arithmetic least squares analysis of the Butkovich and J. L. Smith data was made between the transition density and the highest test density of 0.72 g/cm³. An analysis of the combined data in this density range was also made. Because of insufficient

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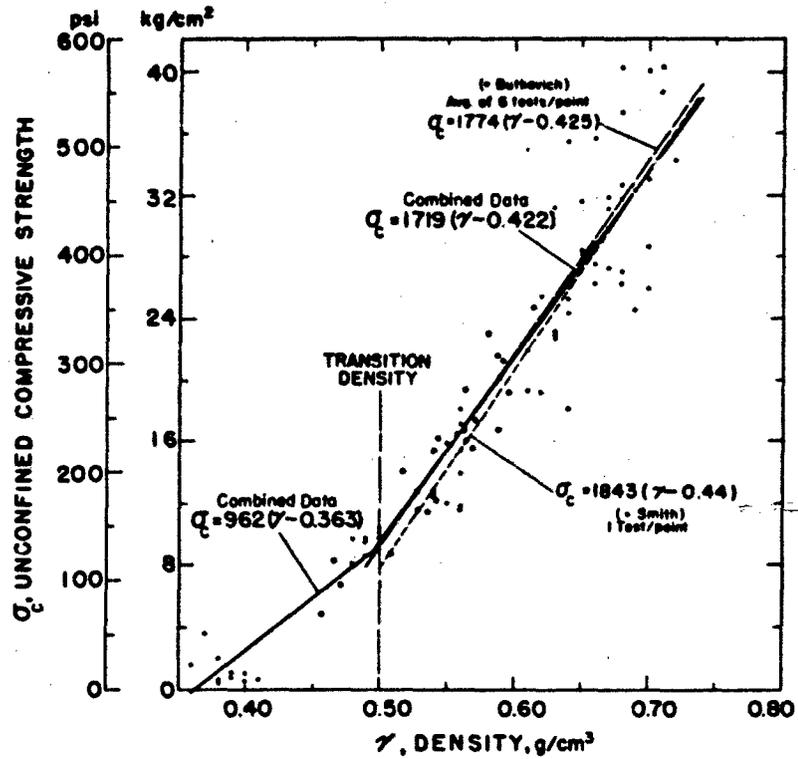


Figure 3. Linear relationships of unconfined compressive strength vs density for data of Butkovich and J. L. Smith above and below the transition density of 0.50 g/cm³. Snow temperature -10C.

Table I. Statistical data related to curves in Figure 3.

Symbols	Butkovich		J. L. Smith	
	γ=0.50 to 0.62 g/cm ³	γ=0.50 to 0.72 g/cm ³	γ=0.50 to 0.72 g/cm ³	γ=0.36 to 0.49 g/cm ³
Slope m, psi/(g/cm ³)	1.774 × 10 ³	1.843 × 10 ³	1.719 × 10 ³	9.625 × 10 ²
Intercept a, psi	-7.534 × 10 ²	-8.108 × 10 ²	-7.251 × 10 ²	3.494 × 10 ²
Std error of est S _{yc} , psi	2.418 × 10	5.727 × 10	3.719 × 10	1.939 × 10
Simple corr coef R	0.912	0.874	0.925	0.907

test data between the transition density and the lowest test density (0.36 g/cm³) to permit the establishment of individual unconfined compressive strength vs density relationships, only a combined data analysis was made. For snow in the 0.50 g/cm³ to 0.72 g/cm³ density range, the analyses gave the following unconfined compressive strength relationships:

For Butkovich's data (138 tests):

$$\sigma_c = 1774 (\gamma - 0.425). \quad (7)$$

For J. L. Smith's data (54 tests):

$$\sigma_c = 1843 (\gamma - 0.440). \quad (8)$$

For the combined data (192 tests):

$$\sigma_c = 1719 (\gamma - 0.422). \quad (9)$$

The 35 tests below the density of 0.50 g/cm³ rendered an unconfined compressive strength vs density relationship of:

$$\sigma_c = 962 (\gamma - 0.360). \quad (10)$$

Immediately apparent from Figure 3 is the reasonably close agreement between the rectilinear curves passing through the combined J. L. Smith and Butkovich data.

J. L. Smith (1965) has shown that plotting the Young's and shear moduli vs density results in a rectilinear curve above the transition density of 0.50 g/cm³. Below this density the curves bow toward the left, intercepting the x-axis at some density below 0.30 g/cm³. To determine whether or not the Butkovich-J. L. Smith test results would fit a comparable trend with good statistical data - curve correlation, a computer evaluation (Mock, 1960) was made. The combined data were fitted with arithmetic, exponential, power and 2nd-5th degree polynomial regression equations. Statistically, the following 3rd degree polynomial regression curve fitted the data best:

$$\sigma_c = 988 - 6646\gamma + 13520\gamma^2 - 7235\gamma^3. \quad (11)$$

This equation is only valid for natural snow between the densities of 0.36 and 0.72 g/cm³. Above or below this range the formula gives erroneous unconfined compressive strengths.

Equation 11, along with the previously presented linear expressions for the combined data above and below the transition density, is shown graphically in Figure 4. Here it is seen that the polynomial not only becomes quasi-linear above the density of 0.52 g/cm³ but follows the linear relationships for the combined data above the transition density.

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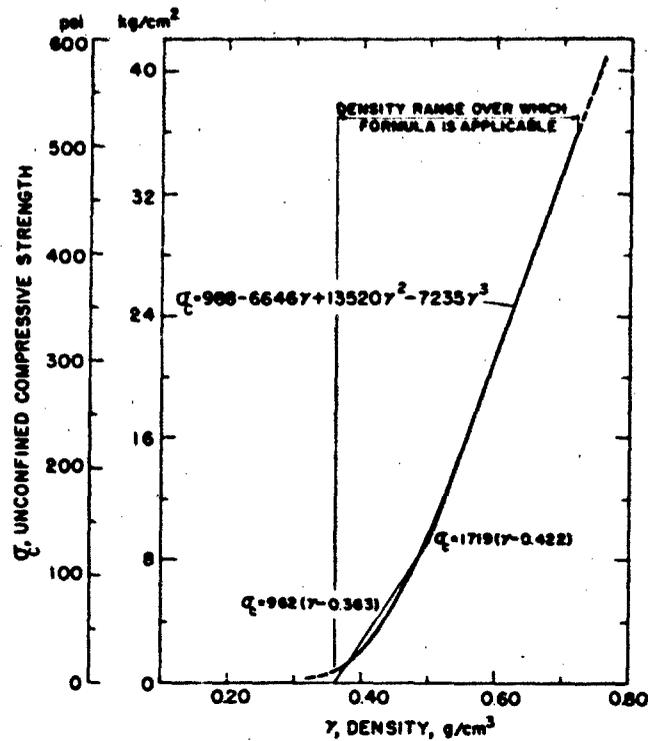


Figure 4. Linear unconfined compressive strength vs density relationships for the combined Butkovich and J. L. Smith data above and below the transition density of 0.50 g/cm³ in relation to the polynomial relationship for the entire data range. Snow temperature -10C.

Table II. Statistical data related to polynomial curve in Figure 4.

Symbols	Butkovich and Smith at -10C. $\gamma=0.36$ to 0.72 g/cm ³	
Slopes m, psi/(g/cm ³)	B ₁	-6.646×10^3
	B ₂	1.352×10^4
	B ₃	-7.235×10^3
Intercept a, psi		9.883×10^2
Std error of est S _{ye} , psi		3.492×10
Multiple corr coef R		0.953

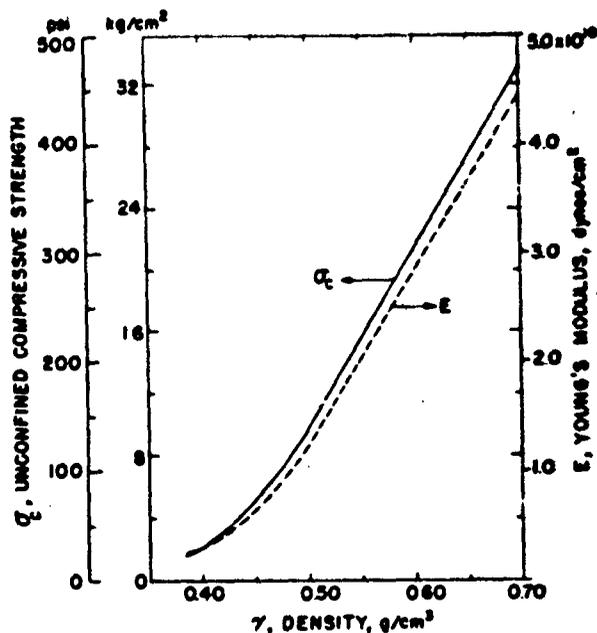


Figure 5. Curve trend comparison between equation 11's unconfined compressive strength relationship and J. L. Smith's dynamic Young's modulus vs density relationship. Snow temperature -10C.

In Figure 5 the curve of equation 11 is shown graphically in relation to J. L. Smith's dynamic Young's modulus vs density curve.* It is readily apparent from this figure that the unconfined compressive strength and dynamic Young's modulus vs density curves do assume similar trends. Finally eq 11 is graphically shown in comparison with the earlier unconfined compressive strength vs density relationships in Figure 6.

If eq 9 is used to determine unconfined compressive strengths above a density of 0.72 g/cm³, an ultimate strength for snow ice of 850 psi (59.77 kg/cm²) is obtained:

$$\sigma_c = 1719 (\gamma - 0.422) = 850 \text{ psi when } \gamma = 0.917 \text{ g/cm}^3.$$

This is 150 psi (10.65 kg/cm²) more than for clear lake ice at -10C. The value may nevertheless represent the optimum strength of snow ice (σ_i) which is required in the use of eq 3 and 4.

*J. L. Smith's dynamic Young's and shear modulus data are listed in Table AV of Appendix A. Statistically it was found that a 3rd degree polynomial regression equation also fitted these data best. See Appendix B for equations and statistical results. Appendix C gives the linear relationships between the unconfined compressive strength of polar snow and its dynamic Young's and shear moduli at -10C over the entire σ_c range.

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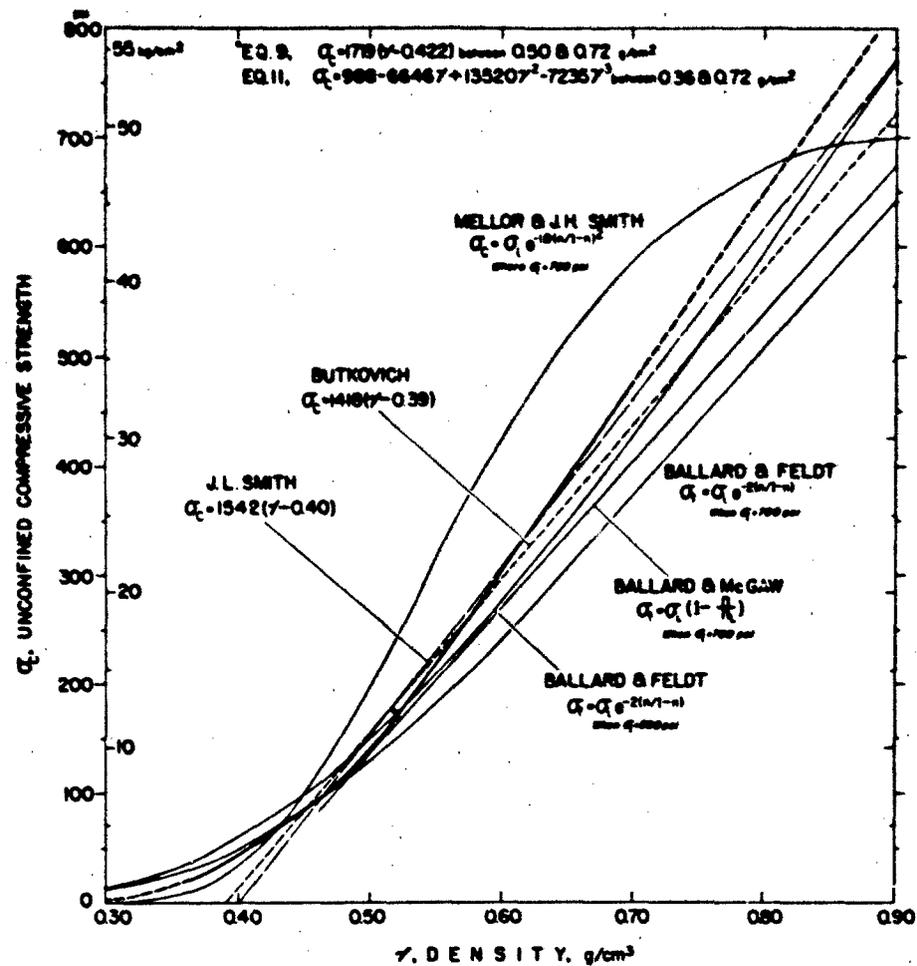


Figure 6. Comparison of the unconfined compressive strength vs density relationship of eq 11 with earlier relationships. Snow temperature -10C.

Using 850 psi for σ_1 in eq 4 results in the unconfined compressive strength vs density relationship shown in Figure 7. When this strength vs density relationship is compared with that of the polynomial curve of eq 11 in the same figure, it is apparent that a considerable lack of agreement still exists between the theorized and the empirical expressions in the low-density range.

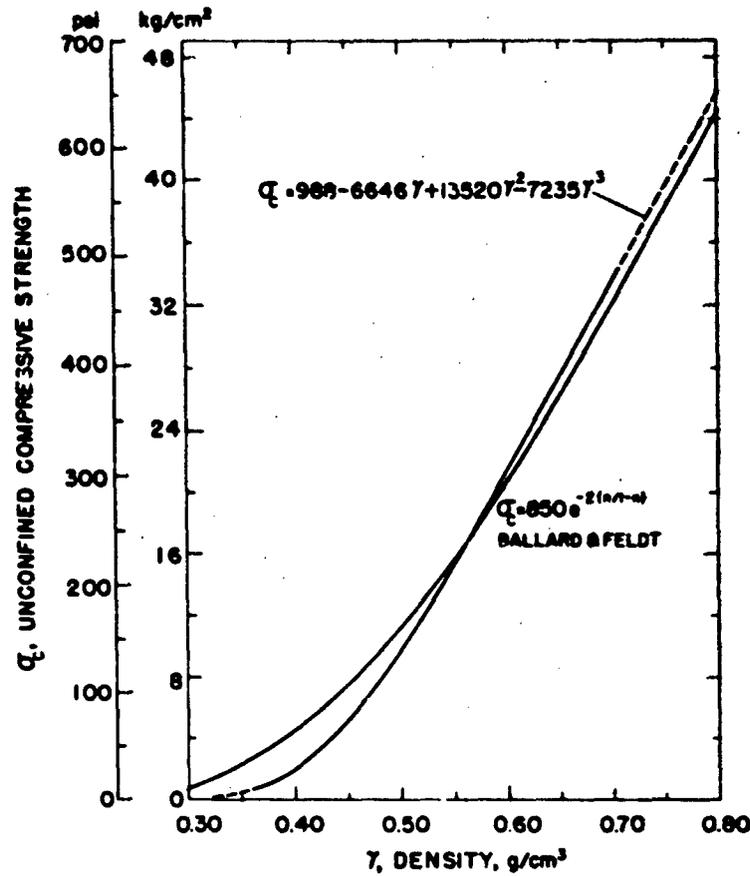


Figure 7. Comparison between the Ballard-Feldt and eq 11 unconfined compressive strength vs density relationships. Snow temperature -10C.

TEMPERATURE CORRECTION FACTORS

The Ballard-Feldt, Ballard-McGaw and Mellor-J. H. Smith equations give unconfined compressive strength directly in terms of the associated snow temperature. To relate strength values obtained from the formulas of Butkovich, J. L. Smith, Ballard-McGaw (modified) or the author to a temperature other than -10C, a temperature correction must be made. Bender (1957) gives an empirical relationship between unconfined compressive strength and temperature:

$$\log \frac{\sigma_2}{\sigma_1} = 0.16 \log \frac{\theta_2}{\theta_1} \quad (12)$$

where:

- σ_1 = unconfined compressive strength at temp θ_1
- σ_2 = unconfined compressive strength at temp θ_2 .

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Mellor and J. H. Smith also show unconfined compressive strength as a function of temperature. Their equation gives unconfined compressive strength at any temperature as related to -10°C :

$$\frac{\sigma_{\theta} - \sigma_0}{\sigma_{-10}} = 1.73 e^{-4.76/\theta} \quad (13)$$

where:

σ_0 = unconfined compressive strength at $\theta = 0^{\circ}\text{C}$.

The curve of the Mellor and J. H. Smith equation was found to be inconsistent with the above equation ($\sigma_0/\sigma_{-10} \neq 0.41$ as they indicate but is -0.075 when $\sigma_0 = \sigma_{-10}$). Mellor has, therefore, suggested using the curve (Fig. 8) in preference to the equation.

If polar snows are structurally similar, the straight-line portion of their unconfined compressive strength vs density relationships should, if extended to the abscissa, pass through a "common intercept density." This would be true regardless of temperature and test, provided the test subjected the specimens to similar structural failures, e.g. failure occurring through brittle fracture.

Comparison of the intercept of the extended unconfined compressive strength vs density relationship for vertically sampled South Pole snow with that of eq 9 showed that the two polar snows do indeed share a quasi-common intercept density (Fig. 9). Their intercepts are 0.414 and 0.422 g/cm^3 respectively for a mean of 0.418 g/cm^3 . The South Pole samples were broken at -49.4°C , or 39.4°C lower than the samples used to develop eq 9. The difference between the slope of the curve passing through the South Pole data [$2562 \text{ psi/(g/cm}^3)$] and that of eq 9 [$1719 \text{ psi/(g/cm}^3)$] is $843 \text{ psi/(g/cm}^3)$. Dividing the change in the strength slopes by the difference in

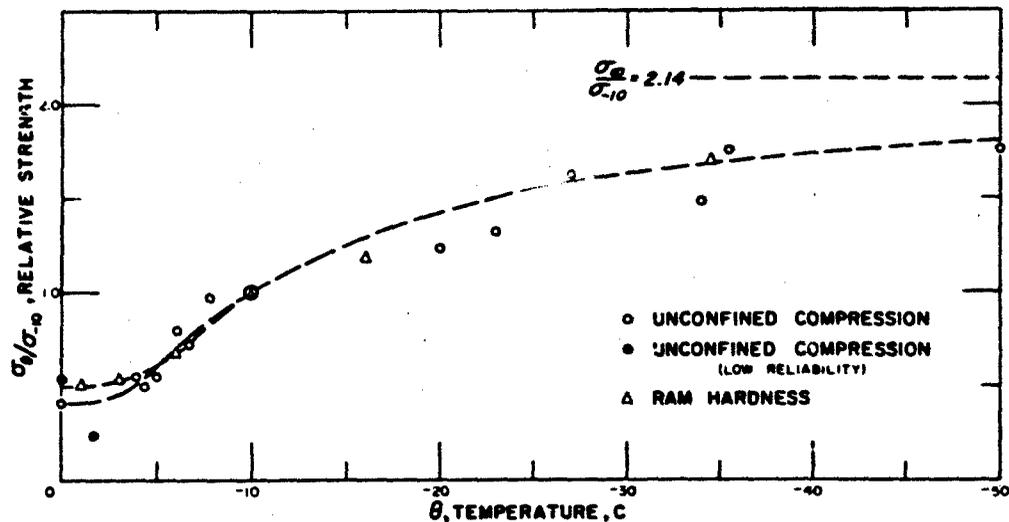


Figure 8. Relative unconfined compressive strength vs temperature (from Mellor-J. H. Smith, 1966).

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Page 13: The units for slope, intercept and standard error of estimate in Table III should read kg cm/g, kg/cm² and kg/cm² respectively for Ramseier's data and dynes cm/g, dynes/cm² and dynes/cm² respectively for J. L. Smith's data.

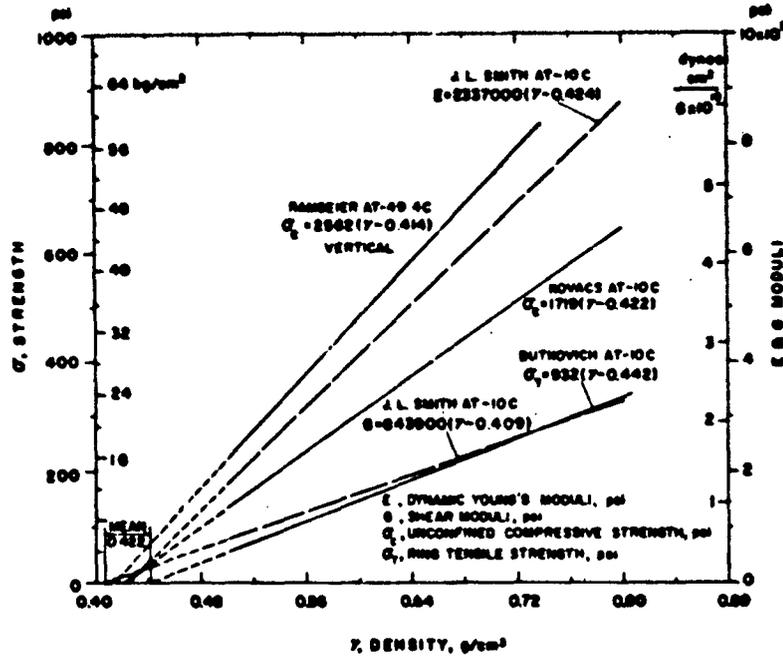


Figure 9. Intercept density relationship between the extended straight-line portions of different tests.

Table III. Statistical data* related to curves in Figure 9.

Symbol	Ramseier unconfined compressive strength	Butkovich ring tensile strength	J. I. Smith Young's modulus (E)	Shear modulus (G)
Slope m. psi/(g/cm ³)	1.809×10^2	9.316×10^2	1.612×10	5.824
Intercept a. psi	7.489×10	-4.117×10^2	-6.832	-2.381
Std error of est S_{y_e} , psi	3.674	-4.044	2.012×10^{-1}	9.061×10^{-2}
Simple corr coef R	0.936	0.998	0.995	0.991

* Test data listed in Appendix A.

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temperature results in a constant for the change in the strength line slope (M) of $\approx 21 \text{ psi}/(\text{g}/\text{cm}^3)$ for each degree C.

Equation 9 can, therefore, be modified to account for unconfined compressive strengths at any temperature below -10C as follows:

$$\sigma_c = (1719 + C)(\gamma - 0.422) \quad (14)$$

where:

$C = \Delta T \times M = \text{change in strength line slope with temperature}$

$\Delta T = \text{change in temperature below } -10\text{C}.$

UNCONFINED COMPRESSIVE STRENGTH OF POLAR SNOW VS DEPTH-DENSITY AND TEMPERATURE

From depth-density and temperature profiles taken at Camp Century, the related snow density and temperature per foot were obtained (Table IV). With these data, the unconfined compressive strengths vs depth were calculated using the methods previously discussed. The calculated strengths from the equations of Butkovich (eq 1), Ballard-Feldt (eq 4) and the author (eq 9) were temperature-corrected to the in-situ temperature using the Bender formula (eq 12). For these calculations, σ_i in eq 4 is 850 psi as extrapolated from eq 9. Strengths calculated using the J. L. Smith equation (eq 2) were temperature-corrected using both the Bender and Mellor-J. H. Smith (eq 13) methods. For comparative purposes, the Ballard-Feldt (eq 4) and Ballard-McGaw (eq 3) equations were used with Butkovich's temperature related strengths of lake ice (Fig. 1) for σ_i . For n_L in eq 3, a value of 0.564 after J. L. Smith was used. A higher σ_i obtained by extrapolating one of the other c_c equations to a density of $0.917 \text{ g}/\text{cm}^3$ was not used in eq 3 for reasons previously discussed. All calculations are listed in Table IV and shown graphically in Figures 10 and 11.

An inspection of the unconfined compressive strength curves in Figure 10 shows that the Ballard-Feldt equation (eq 4) gives the lowest strength values when the temperature related strength of lake ice is used for σ_i . When the same equation is used with the ultimate strength of snow ice extrapolated from eq 9, and temperature-corrected after Bender's formula, σ_c values comparable to those of the Butkovich equation (eq 2) temperature-corrected after Bender are obtained. The Ballard-McGaw equation (eq 3) using Butkovich's temperature-related strength of lake ice for σ_i closely parallels the J. L. Smith (eq 2) values temperature-corrected after Bender (eq 12). By comparing the calculations in Table IV, it is found that the two curves are within a constant $23 \pm 3 \text{ psi}$ ($1.62 \pm 0.21 \text{ kg}/\text{cm}^2$) of one another. When the Bender equation is not used to correct σ_c strengths formulated by the J. L. Smith equation, the parallelism does not exist. This is apparent in Figure 2 where no temperature correction is necessary at -10C and in Figure 10 for the strength curve corrected by the Mellor-J. H. Smith method (eq 13). In addition, as is readily apparent in Figure 10, the high values are again obtained using Mellor's strength equation (eq 5).

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Table IV. Unconfined compressive strength relationships.

Depth (ft)	Density (g/cm ³)	Temp (-°C)	Ballard-McGaw* (psi) (kg/cm ²)	Ballard-Feldt* (psi) (kg/cm ²)	Ballard-Feldt† (psi) (kg/cm ²)	Mellor-J.M. Smith* (psi) (kg/cm ²)	J.L. Smith** (psi) (kg/cm ²)					
1	.520	14.0	175	12.31	161	11.32	194.5	13.7	259	18.21	195	13.71
2	.520	16.8	180	12.66	167	11.74	200.3	15.1	269	18.92	201	14.14
3	.520	17.7	183	12.87	169	11.88	201.9	14.2	273	19.20	203	14.28
4	.528	18.2	197	13.85	180	12.66	214.0	15.0	297	20.87	217	15.26
5	.535	19.0	208	14.83	190	13.16	225.5	15.9	317	22.29	230	16.17
6	.550	19.0	233	16.38	210	14.77	247.6	17.4	358	25.18	257	19.07
7	.560	20.7	255	17.93	228	16.03	266.4	18.7	392	27.57	276	19.41
8	.558	21.2	252	17.72	226	15.89	264.3	18.6	390	27.43	275	19.34
9	.560	21.4	257	18.07	230	16.17	267.9	18.8	396	27.85	278	19.55
10	.560	21.7	257	18.07	231	16.24	268.4	18.9	397	27.92	278	19.55
11	.558	22.0	255	17.93	229	16.10	265.9	18.7	394	27.71	277	19.48
12	.548	22.5	240	16.88	217	15.26	251.3	17.7	368	25.88	260	18.28
13	.554	22.8	250	17.58	225	15.82	261.1	18.4	384	27.00	272	19.13
14	.564	23.0	268	18.85	239	16.81	277.3	19.5	414	29.11	290	20.19
15	.575	23.0	287	20.18	254	17.86	295.1	20.8	445	31.29	309	21.73
16	.584	24.0	305	21.45	271	18.86	312.1	22.0	474	33.33	327	23.00
17	.592	24.0	325	22.86	291	20.46	336.0	23.6	507	35.79	351	24.68
18	.598	24.0	325	22.86	291	20.46	336.0	23.6	507	35.79	351	24.68
19	.594	24.0	320	22.50	287	20.18	329.1	23.1	499	35.09	344	24.19
20	.590	24.1	313	22.51	280	19.69	322.5	22.7	488	34.32	337	23.70
21	.585	24.1	306	21.92	273	19.20	314.0	22.1	477	33.54	328	23.07
22	.586	24.2	308	21.66	275	19.33	315.9	22.2	480	33.76	329	23.14
23	.590	24.2	314	22.08	280	19.69	322.7	22.7	492	34.60	337	23.70
24	.588	24.2	311	21.87	277	19.48	319.3	22.5	480	33.76	334	23.49
25	.596	24.2	324	22.78	290	20.39	333.0	23.4	506	35.58	346	24.33
26	.599	24.2	329	23.14	294	20.68	338.1	23.8	511	35.94	354	24.89
27	.596	24.2	324	22.78	290	20.39	333.0	23.4	506	35.58	346	24.33
28	.600	24.4	331	23.18	296	20.82	340.0	23.9	516	36.29	356	25.04
29	.607	24.4	342	24.05	307	21.59	352.5	24.8	532	37.41	368	25.88
30	.615	24.4	356	25.03	319	22.43	366.7	25.8	553	38.89	382	26.86

Depth (ft)	Density (g/cm ³)	Temp (-°C)	J.L. Smith†† (psi) (kg/cm ²)	Burkovich** (psi) (kg/cm ²)	Equation 9** (psi) (kg/cm ²)	Equation 14 (psi) (kg/cm ²)				
1	.520	14.0	226	15.89	194	13.64	177	12.44	177	12.44
2	.520	16.8	244	17.16	200	14.06	182	12.80	182	12.80
3	.520	17.7	250	17.58	202	14.21	185	13.08	184	12.94
4	.524	18.2	268	18.85	216	15.19	200	14.06	200	14.06
5	.535	19.0	286	20.11	228	16.03	215	15.12	216	15.19
6	.550	19.0	319	22.43	252	17.72	244	17.16	244	17.16
7	.560	20.7	352	24.75	270	18.99	265	18.64	268	18.85
8	.558	21.2	351	24.68	268	18.85	265	18.64	266	18.71
9	.560	21.4	357	25.11	272	19.13	268	18.85	270	18.99
10	.560	21.7	360	25.32	273	19.20	268	18.85	270	18.99
11	.558	22.0	360	25.32	270	18.99	265	18.64	268	18.85
12	.548	22.5	340	23.91	255	17.93	248	17.44	250	17.58
13	.554	22.8	356	25.04	265	18.64	260	18.28	262	18.42
14	.564	23.0	384	27.00	282	18.42	279	19.62	283	19.90
15	.575	23.0	408	28.69	299	21.01	301	21.17	305	21.45
16	.584	24.0	437	30.73	316	22.22	320	22.50	326	22.93
17	.598	24.0	470	33.05	339	23.84	347	24.40	354	24.89
18	.598	24.0	470	33.05	339	23.84	347	24.40	354	24.89
19	.594	24.0	460	32.35	332	23.35	340	23.91	346	24.33
20	.590	24.1	453	31.86	327	23.00	332	23.35	339	23.84
21	.585	24.1	441	31.01	318	22.36	323	22.71	328	23.07
22	.586	24.2	441	31.15	320	22.50	325	22.86	331	23.28
23	.590	24.2	454	31.93	327	23.00	333	23.42	339	23.84
24	.588	24.2	450	31.64	324	22.78	330	23.21	335	23.56
25	.596	24.2	468	32.91	336	23.63	344	24.19	351	24.68
26	.599	24.2	476	33.47	341	23.98	350	24.61	357	25.11
27	.596	24.2	468	32.91	336	23.63	344	24.19	351	24.68
28	.600	24.4	480	33.76	344	24.19	352	24.82	360	25.32
29	.607	24.4	498	35.02	356	25.04	367	25.81	374	26.30
30	.615	24.4	516	36.29	368	25.88	383	26.93	390	27.43

*When σ_c = strength of clear lake ice.

†When σ_c = 850 psi (extrapolated from eq 9, then temperature-corrected using Bender's eq 12).

**Temperature-corrected after Bender.

††Temperature-corrected after Mellor and J. L. Smith.

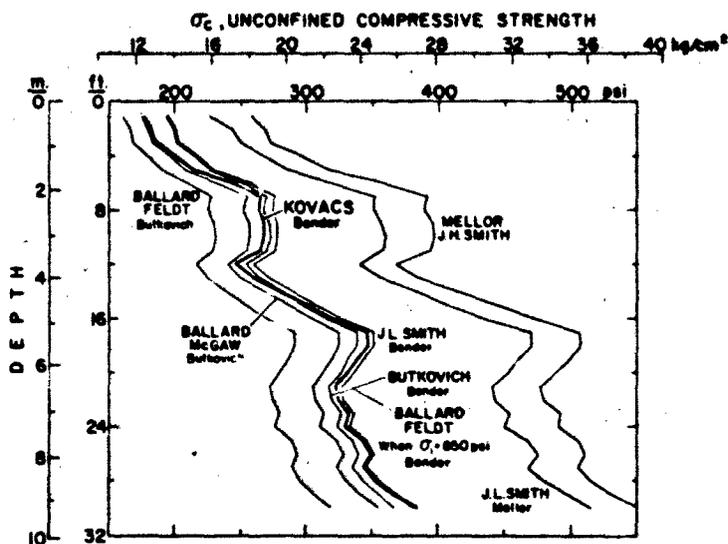


Figure 10. Relationship between values obtained from the different unconfined compressive strength equations when they are used to determine strengths from depth-density and temperature profiles. The name of the investigator who developed the equation is shown in capital letters; other names indicate type of temperature correction used.

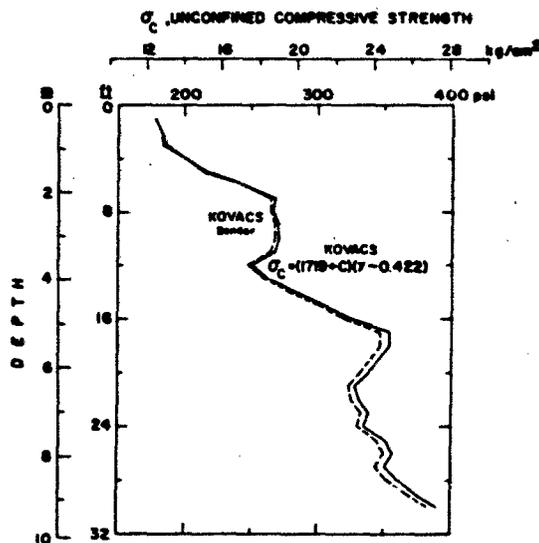


Figure 11. Comparison between the values obtained from eq 9, temperature-corrected after Bender (eq 12), and from eq 14 when they are used to determine the unconfined compressive strength of polar snow from depth-density and temperature profiles.

Because of space limitation, σ_c values obtained using eq 14 are not plotted in Figure 10. They are instead presented in Figure 11 along with eq 9 strength values temperature-corrected after Bender (eq 12). See Kovacs-Bender curve, Figure 10. In Table IV it is seen that at a density of 0.520 g/cm^3 and temperature of -14C the two strength curves converge. At a density of 0.615 g/cm^3 and a temperature of -24.4C they are 7 psi (0.49 kg/cm^2) apart. This results from the fact that the Bender temperature correction is not a constant but decreases with decreasing temperature. Nevertheless the agreement between the two temperature-corrected σ_c values is exceedingly good.

DISCUSSION

It is not surprising to find empirical unconfined compressive strength vs density relationships in disagreement when one considers that no standard test procedure exists. Mellor-J. H. Smith (1966) and Yosida *et al.* (1955) have shown the effect of strain rate and temperature upon unconfined strength. Butkovich (1956) points out the effects of end constraint (when the sample diameter-to-length ratio becomes too small) and shape upon crushing strength values. Wuori (1966) mentions the increased strength values and scatter associated with improper press-sample interface surfaces, i. e. using a nonlubricated interface. He also points out the problem of resiliency in the testing machine which permits absorption of energy which is later released rapidly when sample failure is initiated. The applied stress is, therefore, quasi-dynamically applied and the sample fails at a lower stress level than would occur under stable loading conditions. Unless these areas in the unconfined compression test are standardized, better agreement between the results of different investigators will not be possible.

There are strong indications that all polar snows of comparable density are structurally similar (Ramseier, 1963; Gow, 1966). If this is true, the straight-line portion of nondestructive elastic and shear modulus and destructive strength vs density relationships, if extended to the abscissa, should join at a "common intercept density." The strength difference between similar tests would be directly related to temperature. The closely related intercept densities of different tests performed upon South Pole and Greenland snows, shown in Figure 9, tend to support this conclusion. (See Appendix A for associated test data.) With the establishment of standard destructive test procedures which insure sample failure through brittle fracture, test results will become more comparable and the pin pointing of the "common intercept density" related to all polar snows will be possible.

To avoid the possible unfavorable effect of using a temperature correction factor, only strength data obtained at -10C were used to develop eq 9 and 11. Although eq 9 represents the unconfined compressive strength of natural snow in the 0.50 to 0.72 g/cm^3 density range of the data, it may accurately express strengths for higher density snow and snow ice. For snow ice above a density of 0.83 g/cm^3 the equation gives σ_c strengths higher than tests performed on clear lake ice indicate. Ballard and Feldt (1965) point out that because the resistance along the plane of failure migration in clear lake ice is lower than through snow ice, σ_c values for snow ice can be expected to be higher than those for clear ice. It therefore seems unrealistic to set the σ_i strength of clear lake ice as an ultimate for snow ice as Mellor and J. H. Smith do or to use the temperature related σ_i strength of clear lake ice for that of snow ice in eq 3 and 4 as some investigators (Abele *et al.*, 1966; Ramseier, 1966) have done.

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The Mellor-J. H. Smith unconfined compressive strength equation (eq 5) has been shown to give values in considerable disagreement with other empirical relationships. The same is true for their temperature-correction equation (eq 13). It is believed that this discrepancy is related to the laboratory-prepared snow tested and the speed at which the samples in the higher temperature range were broken.

No advantage can be seen in using the Ballard and McGaw equation (eq 3) rather than linear unconfined compressive strength vs density relationships. Equation 3 requires the extrapolation of values from experimental results, which are used simply to express the σ_c vs density relationship in terms of porosity. The additional computations needed to obtain results directly obtainable from the simpler equation for the same line do not seem justified.

The Ballard and Feldt equation (eq 4) must also use for σ_c a value extrapolated from an empirical relationship. When eq 4 is compared with eq 11 in Figure 7, it is seen that good agreement exists between the two equations above the "transition density." Below this density eq 4 gives values which are higher than experimental results presently indicate. This is particularly true for snow with a density of less than 0.30 g/cm^3 in which a completely unstable structure exists. It is, therefore, recommended that eq 4 not be used to obtain unconfined compressive strengths below the "transition density" until new test data indicate that such strengths actually exist.

CONCLUSIONS

Equation 9 as modified to eq 14 is a most simple method for determining the unconfined compressive strength of natural snow at any temperature below -10C . The equation has been shown to be in good agreement with test results for snow above the "transition density" of 0.50 g/cm^3 and its temperature correction factor has been shown to be in excellent agreement with Bender's (eq 12).

The unconfined compressive strength vs density trend established by eq 11 for 0.36 to 0.72 g/cm^3 density snow has been found to agree with both physical and mechanical properties of polar snow. Because the equation also fits the strength data with a high multiple correlation coefficient of 0.95 and standard error of estimate of 35 , it seems to provide a more realistic unconfined compressive strength vs density relationship for snow in the low density range than the earlier methods discussed.

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APPENDIX A. TEST DATA

Table AI. Butkovich's unconfined compressive strength data at -10C.

Mean* density (g/cm ³)	Max (psi)	Crushing strength		Mean (psi)	Mean (kg/cm ²)
		Min (psi)	Mean (psi)		
0.543	245.7	217.3		232.9	16.35
0.457	85.2	52.5		68.2	4.80
0.509	140.6	119.3		125.0	8.80
0.472	112.2	75.3		96.6	6.78
0.516	174.4	157.6		164.7	11.58
0.466	133.5	105.1		117.9	8.28
0.530	207.3	164.7		181.2	12.88
0.517	207.3	193.1		201.6	14.15
0.564	288.3	274.1		276.9	19.45
0.532	191.7	164.7		174.7	12.29
0.510	235.7	193.1		220.1	14.21
0.528	207.3	146.3		181.2	12.91
0.558	249.9	207.3		235.7	16.55
0.527	180.3	133.5		166.1	11.68
0.540	203.1	139.2		178.9	12.59
0.571	299.6	217.3		248.5	17.50
0.562	265.5	207.3		238.5	16.80
0.587	33.7	296.8		303.9	29.35
0.564	252.8	224.4		234.4	19.16
0.580	340.8	305.3		328.0	26.85
0.591	349.3	248.5		303.9	21.35
0.578	275.5	188.9		240.0	16.85
0.596	305.1	234.3		274.1	19.30
0.589	305.1	267.0		288.3	20.15
0.569	261.3	187.4		222.9	15.67
0.614	377.7	319.5		347.9	24.45

* Mean of six tests.

Table AII. J. L. Smith's unconfined compressive strength data at -10C.

Density (g/cm ³)	Crushing strength (psi)	Density (g/cm ³)	Crushing strength (psi)	Density (g/cm ³)	Crushing strength (psi)
0.36	22.3	1.57	0.55	227.3	15.98
0.37	50.7	1.57	0.56	200.0	14.05
0.38	27.6	1.94	0.55	172.1	12.10
0.40	15.2	1.05	0.56	259.7	18.25
0.38	7.1	0.50	0.56	165.6	13.54
0.38	7.3	0.51	0.56	168.8	11.96
0.40	7.1	0.50	0.64	259.7	18.25
0.37	11.6	0.82	0.61	313.0	22.01
0.41	8.8	0.62	0.62	363.6	25.55
0.37	16.7	1.17	0.63	324.7	22.83
0.50	144.4	10.15	0.63	445.0	31.29
0.50	139.6	9.81	0.67	454.5	31.96
0.49	123.4	8.40	0.67	425.3	29.91
0.49	139.6	9.81	0.68	386.4	27.17
0.48	138.0	9.73	0.66	375.0	26.37
0.49	136.4	9.60	0.63	327.9	23.06
0.50	149.3	10.50	0.64	360.4	25.34
0.49	123.4	8.70	0.66	405.8	28.54
0.49	136.4	9.60	0.71	574.7	40.41
0.48	116.9	8.24	0.68	452.9	31.85
0.56	246.7	17.35	0.64	347.4	24.43
0.55	172.1	12.10	0.65	402.6	28.31
0.67	389.6	27.40	0.63	324.7	22.83
0.62	274.7	19.32	0.61	276.0	19.41

Table AIII. Butkovich's ring tensile strength at -10C. Site II.

Mean* density (g/cm ³)	Max (psi)	Ring tensile strength		Mean (psi)	Mean (kg/cm ²)
		Min (psi)	Mean (psi)		
0.541	110.0	65.0		89.4	6.3
0.574	145.0	99.0		124.4	8.7
0.613	260.0	126.0		160.2	11.3
0.648	227.0	179.0		199.2	14.0
0.661	272.0	149.0		201.2	14.1
0.681	268.0	194.0		223.0	15.7
0.695	272.0	216.0		238.0	16.7
0.716	292.0	210.0		250.5	17.6

* Mean of 10 tests.

APPENDIX A

Table AIV. Ramometer's unconfined compressive strength data* at -49.4C. South Pole.

Density (g/cm ³)	Crushing strength (psi)	Density (kg/cm ³)	Crushing strength (psi)	Density (g/cm ³)	Crushing strength (psi)						
0.470	112.3	8.5	0.570	420.9	29.6	0.614	612.9	43.1			
0.471	112.3	7.9	0.566	365.5	25.7	0.620	577.3	40.6			
0.474	95.3	6.7	0.571	447.9	31.5	0.616	615.7	43.3			
0.481	119.4	8.4	0.570	479.2	33.7	0.620	496.3	34.9			
0.494	196.2	13.8	0.590	318.5	22.4	0.622	513.3	37.5			
0.498	186.3	13.1	0.587	432.3	30.4	0.627	466.4	32.8			
0.491	174.9	12.3	0.580	487.7	34.3	0.629	594.4	41.8			
0.501	152.2	10.7	0.586	450.8	31.7	0.627	447.8	31.5			
0.508	267.6	18.8	0.586	517.6	36.4	0.630	487.7	34.3			
0.512	278.7	19.6	0.590	470.7	33.1	0.626	516.2	36.3			
0.509	261.6	18.4	0.600	618.6	43.5	0.627	460.7	32.4			
0.517	313.5	20.6	0.592	496.3	34.9	0.628	536.1	37.7			
0.518	268.8	18.9	0.594	428.0	30.1	0.627	543.2	38.2			
0.527	256.0	18.0	0.593	438.0	30.8	0.627	600.1	42.2			
0.522	290.1	20.4	0.603	438.0	30.8	0.631	536.1	37.7			
0.533	317.1	22.3	0.608	540.4	38.0	0.632	571.6	40.2			
0.518	342.7	24.1	0.611	460.7	32.4	0.642	558.8	39.3			
0.540	342.7	24.1	0.607	452.2	31.8	0.640	618.6	43.5			
0.539	354.1	24.9	0.607	445.1	31.3	0.640	506.2	35.6			
0.547	385.4	27.1	0.603	527.6	37.1	0.642	486.3	34.2			
0.547	348.4	24.5	0.612	500.5	35.2	0.646	523.3	36.8			
0.556	351.2	24.7	0.613	453.6	31.9	0.647	550.3	38.7			
0.557	329.9	23.2	0.609	540.4	38.0	0.647	594.4	41.8			
0.556	408.1	28.7	0.619	567.4	39.9	0.648	686.8	48.3			
0.556	382.5	26.9	0.618	573.1	40.3	0.646	632.8	44.5			
0.564	479.2	33.7	0.618	577.3	40.6						

* Vertical samples

Table AV. J. L. Smith's dynamic Young's and shear moduli at -10C.

Density (g/cm ³)	Sonic wave velocity (ft/sec)			Elastic constants (dynes/cm ²)	
	C _L	C _S	C _R	E	G
0.39	3358	1728	1581	2.68 x 10 ¹⁰	1.01 x 10 ¹⁰
0.41	3636	2127	1943	4.19	1.69
0.425	3865	2480	2251	5.61	2.43
0.44	4305	2486	2297	6.40	2.57
0.49	5763	3235	2950	1.21 x 10 ¹⁰	4.75
0.508	5370	3560	3200	1.34	6.05
0.52	6355	3567	3280	1.57	6.19
0.551	7300	4150	3800	2.22	8.80
0.56	7128	3740	3470	1.94	7.44
0.56	7063	3706	3689	2.12	8.42
0.57	7307	4039	3732	2.24	8.79
0.58	7692	4111	3804	2.39	9.23
0.60	8261	4335	3970	2.73	1.04 x 10 ¹⁰
0.615	8882	4660	4330	3.33	1.28
0.617	8100	4610	4260	3.08	1.22
0.62	8991	4835	4462	3.51	1.36
0.668	9100	5250	4810	3.99	1.60
0.683	8820	5020	4650	4.05	1.61
0.725	9975	5330	4820	4.85	1.85
0.728	10100	5500	5100	5.32	2.06
0.735	10010	5050	4630	4.45	1.66
0.744	10030	5360	4940	5.24	2.02
0.756	10570	5430	5000	5.48	2.08
0.774	10640	5500	5050	5.75	2.18
0.790	10940	5440	5070	5.86	2.20
0.834	11500	5690	5220	6.67	2.49
0.860	11650	5750	5290	7.08	2.64
0.894	11930	5800	5310	7.43	2.75
0.894	11700	5790	5310	7.45	2.78
0.914	12900	5850	5390	7.99	2.92

Note: C_L = longitudinal wave velocity.
 C_S = shear wave velocity.
 C_R = Rayleigh wave velocity.
 E = dynamic Young's moduli.
 G = dynamic shear moduli.

APPENDIX B: STATISTICAL DATA FOR POLYNOMIAL EXPRESSIONS
FITTING J. L. SMITH'S DYNAMIC YOUNG'S AND SHEAR MODULUS DATA

Symbols	J. L. Smith's		
		E	G
Slopes m, dynes cm/g	B ₁	-3.158 x 10	-1.209 x 10
	B ₂	6.856 x 10	2.711 x 10
	B ₃	-3.228 x 10	-1.338 x 10
Intercept a, dynes/cm ²		4.038	1.481
Std. error of est. S _{y_e} , dynes/cm ²		1.943 x 10 ⁻¹	8.288 x 10 ⁻²
Multiple correlation coef. R		0.997	0.996

Polynomials: (for $\gamma = 0.390$ to 0.914 g/cm³)

$$E = 4.03 - 31.58\gamma + 68.56\gamma^2 - 32.28\gamma^3$$

$$G = 1.48 - 12.09\gamma + 27.11\gamma^2 - 13.38\gamma^3$$

APPENDIX C: RELATIONSHIP BETWEEN THE UNCONFINED COMPRESSIVE STRENGTH OF POLAR SNOW AND ITS DYNAMIC YOUNG'S AND SHEAR MODULI AT -10C.

Equations:

$$\sigma_c = 0.000738 E - 2.009$$

$$\sigma_c = 0.001977 G - 10.838$$

Statistical Data Related to the Above Equations

<u>Symbols</u>	<u>E</u>	<u>G</u>
Slope m	7.38×10^{-4}	1.977×10^{-3}
Intercept a, psi	-2.01	-1.08 x 10
Std. err. of est. S_{y_e} , psi	4.496	1.92×10
Simple corr. coef. R	1.000	0.998

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13. ABSTRACT <p>The relationships between several empirical and theoretical methods of determining the unconfined compressive strength of polar snow from depth-density and temperature profiles are discussed and graphically compared. Two unconfined compressive strength equations are proposed for snow at -10°C. The formulas take into consideration the decided changes in slope of the Young's and shear modulus curves at a density of 0.5 g/cm³ for Greenland snow. The slope changes signify that at this density a structural and, therefore, a strength change occur. Analysis of existing test data confirms this reasoning.</p>			

16. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Snow--Strength--Mathematical analysis Snow cover--Density Snow cover--Temperature						