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Biased Velocity Estimates Due to Phase-Center Motion

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

BIASED VELOCITY ESTIMATES DUE TO PHASE-CENTER MOTION

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Group 41

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ABSTRACT

The errors induced by scattering-phase variations on estimates of radial velocity and acceleration from radar signals are discussed. For small changes in target orientation during the measurement, the scattering phase introduces bias in the estimates. Using scattering-phase data for a cylinder, we indicate the velocity bias one might observe.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

I. Introduction

The use of a coherent radar for tracking a moving object has long been established as an important technique. By measuring the Doppler-phase, the velocity and acceleration of a target may be determined according to some procedure such as matched-filter estimation. Such estimations usually contain the implicit assumption that the target is a point scatterer. However, most targets are extended and exhibit complex patterns of scattering amplitude and phase as a function of viewing (aspect) angle. Thus, questions often arise as to the effect of the scattering phase of a tumbling or oscillating target on the estimations of velocity and acceleration.

In discussing the effects of scattering phase on velocity and acceleration estimates there are two possible viewpoints. One might be performing an experiment where specific knowledge of the scattering phase versus target orientation was pre-measured and target orientation versus time is determined. In this case, one can subtract the scattering-phase effect and thus obtain better trajectory estimates. On the other hand, one may simply wish to establish a reasonable bound for the effect. It is then only necessary to combine an estimate of the rate of tumbling or oscillation with some knowledge of the scattering phase. It is the second viewpoint which we shall pursue.

In Section II we establish a specialized model for the phase of the received signal. In Section III we briefly discuss the estimation procedure for a point target. By assuming the angular rotation of the target is sufficiently small during the measurement, we may show that the scattering phase introduces bias in the estimates of velocity and acceleration. The results are derived for the procedure of fitting a parabola to the Doppler phase. Finally,

in Section IV we show an example of measured scattering-phase data and indicate the velocity error one might encounter if the target were not stabilized.

II. Scattering Phase Model

The phase of a received signal is (for transmitted and received polarizations i and j respectively)

$$\psi_{ij}(t) = -\omega_o[t - \Delta(t)] - \xi_{ij}[t - \frac{1}{2}\Delta(t)], \tag{2.1}$$

where $\Delta(t)$ is the round-trip delay to the target and ξ_{ij} is the scattering phase evaluated at the time the signal was scattered. The delay $\Delta(t)$ is related to the position of the target by

$$c\Delta(t) = 2\left|\vec{r}[t - \frac{1}{2}\Delta(t)]\right| . \tag{2.2}$$

The time dependence of ξ_{ij} is implicit through its dependence on target orientation. This orientation is described by three Euler angles in the radar coordinate system (line-of-sight, $\hat{k}$, and two orthogonal linear polarization vectors, $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$). For the special case of an axially symmetric target only two angles are necessary and it is convenient to choose them as

$$\cos\theta = \hat{k} \cdot \hat{s} \tag{2.3a}$$

$$\sin\theta\cos\varphi = \hat{\epsilon}_1 \cdot \hat{s} \tag{2.3b}$$

where $\hat{s}$ is a unit vector along the symmetry axis. It can further be shown[1] that the scattering matrix for arbitrary φ can be calculated from the special cases of $\varphi = 0$ and $\varphi = \pi/2$ (i.e. $\hat{\epsilon}_1$ in or orthogonal to the plane of incidence defined by $\hat{k}$ and $\hat{s}$). We may then specialize Eq. (2.1) to the case for which ξ_{ij} depends on time through only one variable, $\theta[t - \frac{1}{2}\Delta(t)]$.

III. Estimation of Velocity and Acceleration

For a point target the Doppler-phase is determined by the range delay $\Delta(t)$. Equation (2.2) indicates that $\Delta(t)$ contains a measure of the target trajectory; in particular, the radial velocity and acceleration are to be estimated. In order to simplify the analysis let us consider the special case for which the trajectory is coincident with the radar line of sight. (In the general case it is necessary to deal with radar coordinates including angles and angle rates.)

If the scattered signal is first received at $t = T$, we expand $\Delta(t)$ and $r(t)$ in series about this time. For the special trajectory being considered, these expansions are probably valid for a long interval of time; however, for non-zero angle rates, there may be a more limited interval of convergence We assume that the series are certainly valid over the interval of time necessary for any specific method of data processing (typically on the order of .05 to .5 second for good velocity resolution). Assuming the expansion

$$\Delta(t) = T - \beta(t - T) - \alpha(t - T)^2 + \ldots , \tag{3.1}$$

1. S. L. Borison, "Diagonal Representation of the Radar Scattering Matrix for an Axially Symmetric Body", IEEE Trans. on Ant. and Prop., Vol AP-13, No. 1, p. 176, January 1965 (DDC 613887).

we use Eq. (2.2) to relate T, β, and α to range, radial velocity and acceleration. The results are[2]

$$T = \frac{2}{c} r(T/2) \tag{3.2a}$$

$$\beta = \frac{2v/c}{1 + v/c} \tag{3.2b}$$

$$\alpha = \frac{a/c}{(1 + v/c)^3} , \tag{3.2c}$$

where

$$v = \frac{dr(t)}{dt} \Big|_{t = T/2} \tag{3.3a}$$

$$a = \frac{d^2 r(t)}{dt^2} \Big|_{t = T/2} . \tag{3.3b}$$

Thus, estimates of T, β, and α are equivalent to estimates of r, v, and a at the time $T/2$.

The specific details of an estimation procedure will depend on the duration-bandwidth product, TB, of the signal.[2] For small TB it may be sufficient to estimate the velocity and derive acceleration as the time derivative of v; for large TB, it may be necessary to simultaneously estimate v and a. Whatever the scheme we shall find the estimates $\hat{\beta}$ and $\hat{\alpha}$ with tolerances (or errors) $\delta\beta$ and $\delta\alpha$ respectively. (Neglecting terms in

2. E. J. Kelly and R. P. Wishner, "Matched-Filter Theory for High-Velocity, Accelerating Targets", IEEE Trans. on Military Electronics, Vol MIL-9, No. 1, pp. 56 - 69, January 1965. (DDC 616440).

v/c, Eq. (3.2b) implies that $\hat{v} = \frac{c}{2}\hat{\beta}$ and $\delta v = \frac{c}{2}\delta\hat{\beta}$). We now wish to determine the additional errors incurred due to phase-center motion.

The method for calculating the effects of phase-center variations will depend on the duration-frequency product, TF [F is essentially the instantaneous value of $d\theta(t)/dt$]. For $TF \gg 1$, the phase,* $\xi[\theta(t)]$, will undergo many repetitive cycles; calculation of the errors induced in $\hat{v}$ and $\hat{a}$ will tend toward zero, independent of the estimation scheme due to long-time averaging. For $TF \sim 1$, the errors induced in $\hat{v}$ and $\hat{a}$ will depend on the initial value of ξ and on the particular method of data processing. For $TF \ll 1$ the errors induced in $\hat{v}$ and $\hat{a}$ depend only on the local values of ξ and its time derivatives; furthermore, these errors may be determined for the procedure of fitting a parabola to the Doppler phase.

The method to be followed is straightforward. For $TF \ll 1$, we assume ξ may also be expanded in a series about $t = T$. The terms in $(t - T)^n$ may be associated with corresponding terms from the expansion of $\Delta(t)$. It is obvious that the estimates of $\hat{\beta}$ and $\hat{\alpha}$ are simply biased by the changes in ξ; these biases are independent of the estimation procedure, and the error estimates $\delta\hat{\beta}$ and $\delta\hat{\alpha}$ are unchanged. The results of expanding ξ in a series (neglecting terms of order v/c) is

$$\xi[\theta\{t - \tfrac{1}{2}\Delta(t)\}] = \xi[\theta(T/2)] + (t - T)\frac{d\xi}{d\theta}\Big|_{\theta(T/2)}\frac{d\theta}{dt}\Big|_{T/2} + \tag{3.4}$$
$$+ \frac{1}{2}(t - T)^2\left[\frac{d^2\xi}{d\theta^2}\Big|_{\theta(T/2)}\left(\frac{d\theta}{dt}\right)^2_{(T/2)} + \frac{d\xi}{d\theta}\Big|_{\theta(T/2)}\left(\frac{d^2\theta}{dt^2} + \frac{a}{c}\frac{d\theta}{dt}\right)_{T/2}\right] + \ldots$$

* Hereafter we shall drop the polarization subscripts.

By associating terms from Eqs. (3.1) and (3.4) in Eq. (2.1), it is clear that the $\hat{\beta}$ estimate will be

$$\hat{\beta} = \frac{2v}{c} + \frac{1}{\omega_o} \frac{d\xi}{d\theta} \Big|_{\theta(T/2)} \frac{d\theta}{dt} \Big|_{T/2} .$$

Thus the bias in the velocity estimate is

$$\hat{v}_{bias} = \frac{c}{2\omega_o} \frac{d\xi}{d\theta} \Big|_{\theta(T/2)} \frac{d\theta}{dt} \Big|_{T/2} . \tag{3.5}$$

Similarly Eqs. (3.1) and (3.4) lead to an evaluation of the instantaneous acceleration bias.

To use Eq. (3.5) it is necessary to have some knowledge of the scattering phase and of the body dynamics. If we assume a reasonable value for $d\theta/dt = 2\pi F(t)$, where $F(t)$ has only small variations, we may calculate

$$\hat{v}_{bias} = \frac{\lambda F}{2} \frac{d\xi}{d\theta} \tag{3.6}$$

for any aspect angle, θ. For the acceleration bias we find

$$\hat{a}_{bias} = \pi\lambda F^2 \left[d^2\xi/d\theta^2 + \frac{1}{2\pi F} \left(\frac{a}{c} + \frac{d\ell nF}{dt} \right) d\xi/d\theta \right] . \tag{3.7}$$

(Note that $\hat{a}_{bias}$ depends on the true value of a.) In some cases $\xi(\theta)$ may be available by static-range measurements; if not, some reasonable theoretical predictions can be made.

As a final point in this section, let us note that a consistency check is available for using Eqs. (3. 6) and (3. 7). For the observation (integration) time τ, $\Delta\theta \simeq \tau F$. If the function $\xi(\theta)$ can be well represented by only linear and quadratic terms over any interval of length $\Delta\theta$, then Eqs. (3. 6) and (3. 7) are valid. If $\Delta\theta$ (i. e. τF) just encompasses a region of rapid linear (or quadratic) change in ξ , the estimate of $\hat{v}_{bias}$ (or $\hat{a}_{bias}$) will be nearly maximized. If $\Delta\theta$ is too large (i. e. τF too large) Eqs. (3. 6) and (3. 7) will tend to overestimate by assuming large local rates of change in $\xi(\theta)$.

IV. An Example

As an example of data for use in Eqs. (3. 6) and (3. 7), Fig 1 shows the scattering phase measured at L-band for a metal cylinder (37. 345 inches long and 4. 194 inches diameter).[3] Note that there is no ambiguity in the phase since one can readily identify the fly-back of the pen-recorder. A 360° change in phase corresponds to a deflection from the center to the outer circle in Fig 1. Figure 2 shows the corresponding radar cross section. Phase calibration data are also presented in Ref 3, however, the data is sufficiently linear for our purposes.

A brief inspection of Fig 1 shows that quadratic phase changes might be seen for intervals of data such that $0 < \Delta\theta = \tau F \lesssim 10^{\circ}$. Except near $\theta = 0, \pi/2$, π and $3\pi/2$, if $20^{\circ} \lesssim \Delta\theta < 90^{\circ}$, $\xi(\theta)$ could be fitted by a dominant linear dependence --- though not derivable from local behavior

3. N. Landry, "L-Band Phase Measurement System" TCR No. 49, Radio Corp. of America, Missile and Surface Radar Division, (16 January 1964). (U)

(i.e. $d\xi/d\theta$). What we learn is that Eqs. (3.6) and (3.7) should be valid for $\Delta\theta \lesssim 10^{\circ}$ for any θ. For $10^{\circ} \lesssim \Delta\theta \lesssim 20^{\circ}$, estimates of velocity and acceleration errors depend on the initial value of θ and the details of the data processing. For $20^{\circ} \lesssim \Delta\theta \lesssim 90^{\circ}$ the specific value of θ is important however, application of Eq. (3.6) will probably yield an upper bound for $\hat{v}_{bias}$ because $d\xi/d\theta$ will usually overestimate the linear behavior of $\xi(\theta)$ for this interval. For $\Delta\theta \gg 90^{\circ}$, the phase will be cyclic and errors in $\hat{v}$ and $\hat{a}$ will probably tend toward zero due to smoothing.

To be specific let us determine the maximum velocity bias as a function of τF for this target. For $\tau F \lesssim 10^{\circ}$ we simply locate the maximum value of $d\xi/d\theta$. One can see that large values of $d\xi/d\theta$ occur four times for each 90° change in θ. The maximum value of $(\Delta\xi/\Delta\theta)$ is on the order of $50^{\circ}/1^{\circ} = 50$. Thus, by using Eq. (3.6) regardless of the value of τF, we would estimate ($\lambda \sim 1$ ft at L-band)

$$\max |\hat{v}_{bias}| \simeq 25\ F \ \text{ft/sec}$$

for F in $\sec^{-1}$. In fact, for a realistic case of $\tau = .05$ sec and $F = .5\ \sec^{-1}$, $\tau F < 10^{\circ}$, and we might well observe a velocity bias on the order of 25 ft/sec. This value would be considerably larger than the velocity resolution of the signal ($\lambda/2\tau \simeq 10$ ft/sec).

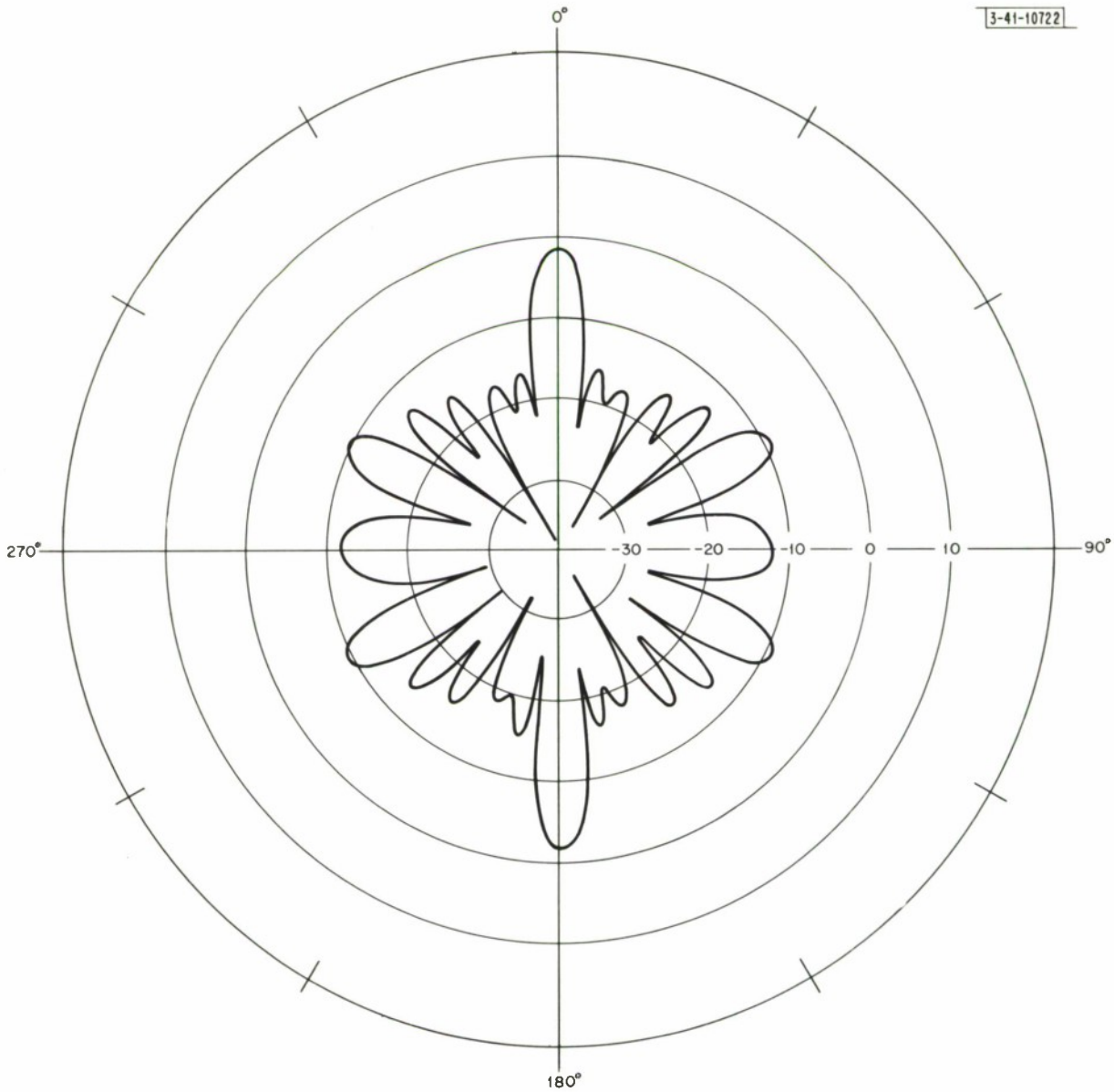

Figure 1. Cylinder (phase) 1300 MCS, VV Polarization.

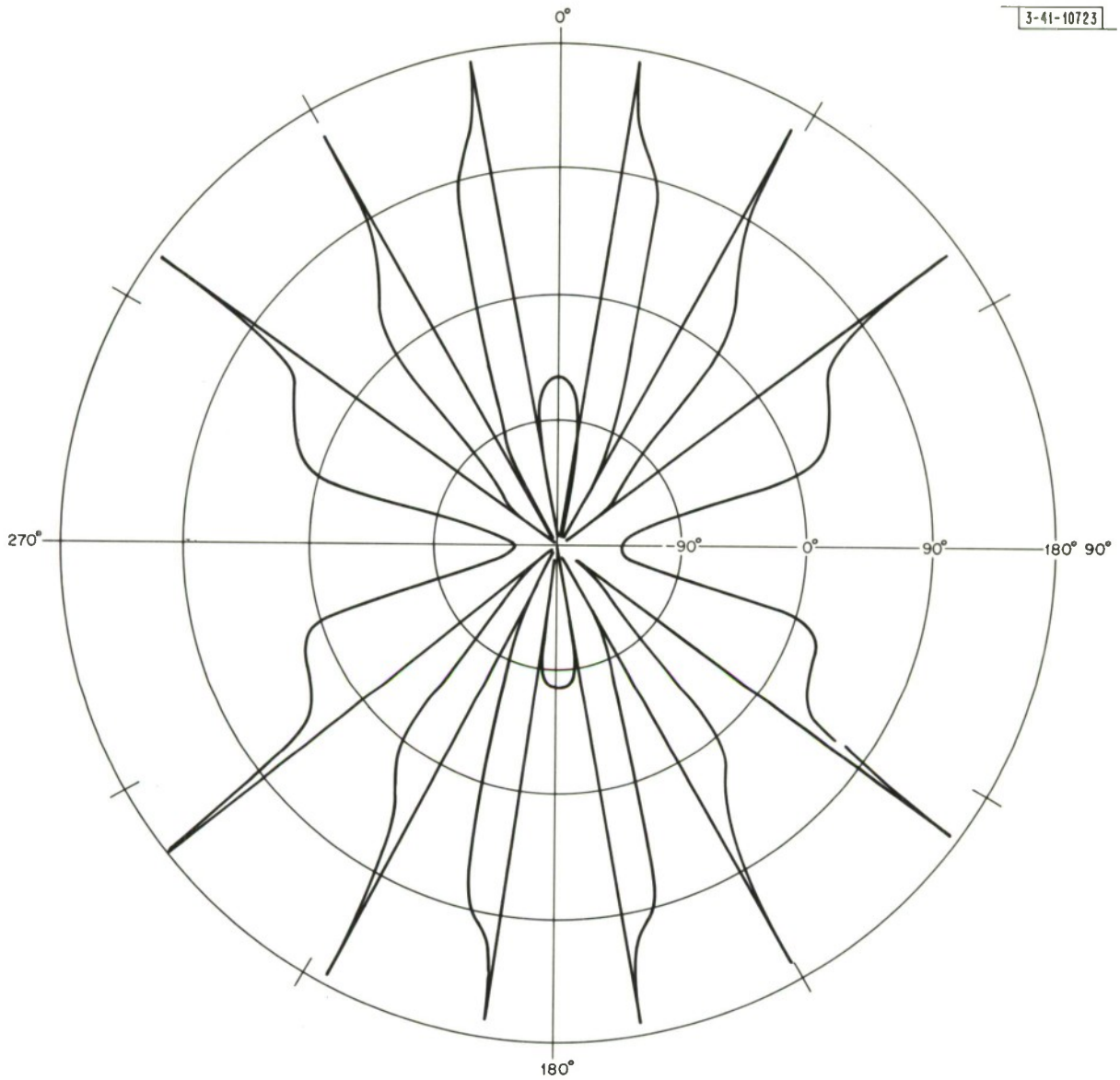

Figure 2. Cylinder (cross-section) 1300 MCS, VV Polarization.

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14. KEY WORDS

velocity, scattering, phase-center motion, Doppler, point target, signals