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CALIFORNIA INSTITUTE OF TECHNOLOGY

Pasadena, California

ANTENNA LABORATORY

Technical Report No. 41

POTENTIAL OF A CHARGED CYLINDER

by

John Lam

Electronics Division
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ABSTRACT

A method, based on complex variable theory, is presented for solving potential problems involving conducting solids of revolution. Attention is paid exclusively to the right circular cylinder, both because this problem possesses a high degree of symmetry which brings about certain simplifications, and because comparison can be made with previous results. The generalization from the cylinder to other axially symmetric geometries should become obvious. This method depends on the establishment of an integral representation for the potential function, which leads to the formulation of the problem in terms of a pair of coupled Fredholm integral equations of the first kind. As an illustrative example these equations are solved numerically in the lowest approximation and the capacitance of the cylinder is calculated.

1. Introduction

In this work we present a method for solving the electrostatic problem of a charged, right circular, conducting cylinder. This problem has been treated previously by Smythe[1,2], using a method which depends on the following very important property of an axially symmetric potential function, namely, the value of the potential everywhere is known once its value on the symmetry axis is determined. Our method also depends on this property, but in quite a different way. Here we use it to establish an integral representation for the potential function which has a very simple form. The problem is then formulated in terms of a pair of integral equations which are well suited for numerical integration. Inasmuch as Smythe has published numerical results of high accuracy, it is not our primary intention to reproduce these solutions in great detail. No heavy computation is therefore attempted. Our interest is mainly methodological. It is hoped that this method can be applied with equal ease to other solids of revolution.

2. Whittaker's Solution

Denote the radius of the cylinder by a and its length $2b$. Let us set up a cylindrical coordinate system such that the origin and the z-axis coincide with the center and the symmetry axis of the cylinder. Thus the cylinder is defined by the intersecting surfaces $\rho = a$ and $z = \pm b$. The potential function we seek is a solution of the axially symmetric Laplace equation

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{\partial^2}{\partial z^2}\right) V(\rho,z) = 0 \tag{2.1}$$

with the condition that it vanishes at infinity and that it reduces to a constant value V_o on the surface of the cylinder.

For $r = \sqrt{\rho^2 + z^2} > b$, let us expand the axially symmetric potential function $V(\rho,z)$ in terms of the spherical harmonics:

$$V(\rho,z) = \sum_{n=0}^{\infty} a_n r^{-n-1} P_n(\cos \theta) \tag{2.2}$$

where the a_n's are real constants. On the upper z-axis $\theta = 0$ and $r = z$. Therefore

$$V(0,z) = \sum_{n=0}^{\infty} a_n z^{-n-1} \quad , \qquad z > b \tag{2.3}$$

The spherical harmonics have the integral representation

$$r^{-n-1} P_n(\cos \theta) = \frac{1}{\pi}\int_0^{\pi} (z + i\rho\cos\alpha)^{-n-1}\, d\alpha \tag{2.4}$$

Multiplying (2.4) by a_n and summing over n we get

$$V(\rho,z) = \frac{1}{\pi}\int_0^{\pi} \sum_{n=0}^{\infty} a_n (z + i\rho\cos\alpha)^{-n-1}\, d\alpha \tag{2.5}$$

Assuming that $V(0,z)$ is an analytic function, we can regard the integrand as the analytic continuation of $V(0,z)$ to complex values of its argument. Hence

$$V(\rho,z) = \frac{1}{\pi}\int_0^{\pi} V(0, z + i\rho\cos\alpha)\, d\alpha \tag{2.6}$$

This is a special case of Whittaker's solution of Laplace's equation[3]. It gives a prescription for obtaining the off-axis value of the potential in terms of the value on the upper axis provided, of course, that the latter is an analytic function. We note that for $-b < z < b$, $V(0,z)$ is undefined. Its value there must be obtained by analytic continuation.

3. Analytic Properties of $V(0,z)$

The potential function $V(\rho,z)$ has another integral representation in the form of the particular solution of Poisson's equation:

$$V(\rho,z) = \frac{1}{4\pi\varepsilon_o}\int \frac{\sigma}{R}\,dS \tag{3.1}$$

where σ is the surface charge density, R the distance from a point on the cylinder to an observation point, and the integral is taken over the surface of the cylinder. For points on the axis

$$V(0,z) = \frac{1}{2\varepsilon_o}\int_0^a \frac{\sigma(\rho',b)\,\rho' d\rho'}{\sqrt{\rho'^2 + (z-b)^2}} + \frac{1}{2\varepsilon_o}\int_0^a \frac{\sigma(\rho',-b)\,\rho' d\rho'}{\sqrt{\rho'^2 + (z+b)^2}}$$

$$+ \frac{1}{2\varepsilon_o}\int_{-b}^{b} \frac{\sigma(a,z')\,a dz'}{\sqrt{a^2 + (z-z')^2}} \tag{3.2}$$

The three integrals represent contributions from charges on the top, the bottom and the side of the cylinder respectively. From symmetry we have

$$\sigma(\rho',b) = \sigma(\rho',-b)$$

$$\sigma(a,z') = \sigma(a,-z') \tag{3.3}$$

The σ's are continuous and bounded except at the edges $\rho = a$, $z = \pm b$, where, according to the edge condition[4], they diverge only like $\delta^{-1/3}$, δ being the distance from the edge. Thus (3.2) defines $V(0,z)$ as an analytic function of z. Clearly $V(0,z)$ is real for z real and tends to zero like z^{-1} as $|z| \to \infty$.

The singularities of $V(0,z)$ can be read off from (3.2). The first integral has a branch cut from $b-ia$ to $b+ia$; the second integral has a cut from $-b-ia$ to $-b+ia$; and the third integral has two cuts from $-b\pm ia$ to $b\pm ia$ respectively. These branch cuts of $V(0,z)$ from a rectangle in the z-plane which is identical to the section of the cylinder by a plane containing the symmetry axis. It can be shown that this property of $V(0,z)$ is true of all solids of revolution.

The symmetry of the problem in the physical space implies a certain symmetry in the z-plane. We readily deduce from (3.2) and (3.3) that the absolute values of $V(0,z)$ and $V(0,-z)$ are equal. If we analytically continue $V(0,z)$ from the point z to the point $-z$, we must choose a path which avoids the branch cuts near the origin. Then during the process all the square roots in (3.2) change sign. We conclude that whereas the value of the potential on the symmetry axis is an even function of z in the physical space, it is an odd function in the complex z-plane:

$$V(0,-z) = -V(0,z) \qquad (3.4)$$

In general, the value of $V(0,z)$ on the positive real axis of the z-plane coincides with the physical value of the potential on the upper symmetry axis, while the value on the negative real axis differs in sign from the physical value on the lower symmetry axis. This is easily seen from (2.2) and (2.3). In the physical space the switch from the upper axis to the lower axis is done by changing θ from 0 to π. This brings in a factor $(-1)^n$ in (2.2). On the z-plane the switch from the positive real axis to the negative real axis is done by replacing z by $-z$. This brings in a factor $(-1)^{n+1}$ in (2.3).

We now use the above analytic properties to establish an integral representation for $V(0,z)$ which is simpler than (3.2). We represent the first integral in (3.2) by an open contour integral of the Cauchy type along its branch cut:

$$\frac{1}{2\varepsilon_c}\int_0^a \frac{\sigma(\rho',b)\,\rho' d\rho'}{\sqrt{\rho'^2 + (z-b)^2}} = -\frac{V_o}{\pi i}\int_{b-ia}^{b+ia} \frac{f(\xi)\,d\xi}{\xi - z}$$

$$= -\frac{V_o}{\pi}\int_{-a}^{a} \frac{f(y)\,dy}{b + iy - z} \qquad (3.5)$$

where the unknown density function $f(y)$ is taken to be real. It is not difficult to show that both sides of (3.5) have the same singularities and the same asymptotic behavior. The left-hand side being real for z real, $f(y)$ must be an even function:

$$f(-y) = f(y) \qquad (3.6)$$

Let z approach the contour of integration from the right-hand side, i.e., $z \to b + \varepsilon + i\eta$, $\varepsilon > 0$, $-a < \eta < a$. Then, by the Plemelj formula,

$$-\frac{V_o}{\pi i}\int_{b-ia}^{b+ia} \frac{f(\xi)\,d\xi}{\xi - (b+i\eta)} = V_o f(\eta) - \frac{V_o}{\pi i} P\int_{-a}^{a} \frac{f(y)\,dy}{y - \eta} \qquad (3.7)$$

Equating the real parts of (3.5) and (3.7), we get

$$V_o f(y) = \mathrm{Re}\ \frac{1}{2\varepsilon_o} \int_0^a \frac{\sigma(\rho',b)\ \rho'\ d\rho'}{\sqrt{\rho'^2 - y^2}}$$

$$= \frac{1}{2\varepsilon_o} \int_{|y|}^a \frac{\sigma(\rho',b)\ \rho'\ d\rho'}{\sqrt{\rho'^2 - y^2}} \tag{3.8}$$

From this result we see that $f(y)$ is continuous in the interval $-a \leq y \leq a$.

A similar integral representation for the second integral in (3.2) is obtained by putting $-b$ for b in (3.5):

$$\frac{1}{2\varepsilon_o} \int_0^a \frac{\sigma(\rho',-b)\ \rho'\ d\rho'}{\sqrt{\rho'^2 + (z+b)^2}} = -\frac{V_o}{\pi} \int_{-a}^a \frac{f(y)\ dy}{-b + iy - z} \tag{3.9}$$

The third integral in (3.2) has two branch cuts and is represented as follows:

$$\frac{1}{2\varepsilon_o} \int_{-b}^b \frac{\sigma(a,z')\ a\ dz'}{\sqrt{a^2 + (z - z')^2}} = -\frac{V_o}{\pi} \int_{-b+ia}^{b+ia} \frac{g_1(\xi)\ d\xi}{\xi - z} - \frac{V_o}{\pi} \int_{-b-ia}^{b-ia} \frac{g_2(\xi)\ d\xi}{\xi - z} \tag{3.10}$$

where $g_1(\xi)$, $g_2(\xi)$ are real density functions. The left-hand side is real for z real; hence

$$g_1(\xi) = g_2(\xi) = g(\xi) \tag{3.11}$$

and we have

$$\frac{1}{2\varepsilon_o}\int_{-b}^{b}\frac{\sigma(a,z')\,a dz'}{\sqrt{a^2+(z-z')^2}} = -\frac{V_o}{\pi}\int_{-b}^{b}\frac{g(x)\,dx}{x+ia-z}-\frac{V_o}{\pi}\int_{-b}^{b}\frac{g(x)\,dx}{x-ia-z} \qquad (3.12)$$

By (3.4), g(x) is an even function

$$g(-x) = g(x) \qquad (3.13)$$

It can also be shown that g(x) is continuous in the interval $-b \leq x \leq b$.

Collecting all the results we have the following integral representation for V(0,z);

$$V(0,z) = -\frac{V_o}{\pi}\int_{-a}^{a}\frac{f(y)\,dy}{b+iy-z}-\frac{V_o}{\pi}\int_{-a}^{a}\frac{f(y)\,dy}{-b+iy-z}$$
$$-\frac{V_o}{\pi}\int_{-b}^{b}\frac{g(x)\,dx}{x+ia-z}-\frac{V_o}{\pi}\int_{-b}^{b}\frac{g(x)\,dx}{x-ia-z} \qquad (3.14)$$

where f(y) and g(x) are a pair of real, continuous, even functions.

4. An Integral Representation for V(ρ,z)

We replace z by $z + i\rho\cos\alpha$ in (3.14), substitute the result in (2.6), and interchange the order of integration. The integrals over α have the form

$$\frac{1}{\pi}\int_{0}^{\pi}\frac{d\alpha}{A + B\cos\alpha} \qquad (4.1)$$

where A and B are complex numbers. By an identity of Jacobi (4.1) is equal to

$$\frac{1}{\sqrt{A^2 - B^2}} \qquad (4.2)$$

where the sign of the square root is chosen to satisfy the inequality

$$|A - \sqrt{A^2 - B^2}| < |B| \tag{4.3}$$

We therefore obtain the integral representation

$$V(\rho,z) = \frac{V_o}{\pi} \int_{-a}^{a} \frac{f(y)\, dy}{\sqrt{(z-b-iy)^2 + \rho^2}} + \frac{V_o}{\pi} \int_{-a}^{a} \frac{f(y)\, dy}{\sqrt{(z+b-iy)^2 + \rho^2}}$$

$$+ \frac{V_o}{\pi} \int_{-b}^{b} \frac{g(x)\, dx}{\sqrt{(z-x-ia)^2 + \rho^2}} + \frac{V_o}{\pi} \int_{-b}^{b} \frac{g(x)\, dx}{\sqrt{(z-x+ia)^2 + \rho^2}} \tag{4.4}$$

For ρ, z large and positive, the signs of the square roots are chosen to make the real parts positive. For other ranges of value the signs are determined by continuation. It can be shown that the real parts are taken to be positive throughout. Using the evenness and reality of $f(y)$ and $g(x)$, we can also write (4.4) in the form

$$V(\rho,z) = \frac{2V_o}{\pi} \text{Re} \int_{0}^{a} \frac{f(y)\, dy}{\sqrt{(z-b-iy)^2 + \rho^2}} + \frac{2V_o}{\pi} \text{Re} \int_{0}^{a} \frac{f(y)\, dy}{\sqrt{(z+b-iy)^2 + \rho^2}}$$

$$+ \frac{2V_o}{\pi} \text{Re} \int_{-b}^{b} \frac{g(x)\, dx}{\sqrt{(z-x-ia)^2 + \rho^2}} \tag{4.5}$$

The capacitance is given by

$$C = 16\varepsilon_o \left[\int_0^a f(y)\, dy + \int_0^b g(x)\, dx \right] \tag{4.6}$$

The boundary conditions for $V(\rho,z)$ are

$$V(\rho,\pm b) = V_o \quad , \quad 0 < \rho < a \tag{4.7}$$

$$V(a,z) = V_o \ , \qquad -b < z < b \tag{4.8}$$

Combining (4.5), (4.7) and (4.8) we have

$$\frac{2}{\pi}\int_0^{\rho} \frac{f(y)\,dy}{\sqrt{\rho^2 - y^2}} + \frac{2}{\pi}\,\mathrm{Re}\int_0^{a} \frac{f(y)\,dy}{\sqrt{(2b - iy)^2 + \rho^2}}$$

$$+ \frac{2}{\pi}\,\mathrm{Re}\int_{-b}^{b} \frac{g(x)\,dx}{\sqrt{(x - b + ia)^2 + \rho^2}} = 1 \ , \qquad 0 < \rho < a \tag{4.9}$$

$$\frac{2}{\pi}\,\mathrm{Re}\int_0^{a} \frac{f(y)\,dy}{\sqrt{(b - z + iy)^2 + a^2}} + \frac{2}{\pi}\,\mathrm{Re}\int_0^{a} \frac{f(y)\,dy}{\sqrt{(b + z - iy)^2 + a^2}}$$

$$+ \frac{2}{\pi}\,\mathrm{Re}\int_{-b}^{b} \frac{g(x)\,dx}{\sqrt{(x - z + ia)^2 + a^2}} = 1 \ , \quad -b < z < b \tag{4.10}$$

These are a pair of coupled Fredholm integral equations of the first kind which summarize the formulation of the problem. We note that in the usual integral equation formulation of the problem we start with the integral representation (3.1) and impose the boundary conditions (4.7) and (4.8). In this way we also obtain a pair of coupled integral equations--in $\sigma(\rho',b)$ and $\sigma(a,z')$. But here the kernels are elliptic integrals. Moreover the surface charge densities are known to diverge at the edges.

5. Two Special Cases

Before proceeding to discuss the solution of (4.9) and (4.10) we shall first examine a few special cases.

Case I: A Circular Disk

Letting $b = 0$ in (4.9) we obtain the limiting case of a charged circular disk:

$$\frac{4}{\pi}\int_0^{\rho} \frac{f(y)\,dy}{\sqrt{\rho^2 - y^2}} = 1 \quad , \qquad 0 < \rho < a \tag{5.1}$$

This is a special case of the Abel integral equation

$$\int_0^{\rho} \frac{f(y)\,dy}{\sqrt{\rho^2 - y^2}} = F(\rho) \quad , \qquad 0 < \rho < a \tag{5.2}$$

The solution is

$$f(y) = \frac{2}{\pi}\frac{d}{dy}\int_o^{y} \frac{F(\rho)\,\rho\,d\rho}{\sqrt{y^2 - \rho^2}} \quad , \qquad 0 < y < a \tag{5.3}$$

Therefore we have

$$f(y) = \frac{1}{2} \tag{5.4}$$

and

$$V(\rho,z) = \frac{V_o}{\pi}\int_{-a}^{a} \frac{dy}{\sqrt{(z-iy)^2 + \rho^2}}$$

$$= \frac{V_o}{\pi}\, i \,\ell n \,\frac{z - ia + \sqrt{(z - ia)^2 + \rho^2}}{z + ia + \sqrt{(z + ia)^2 + \rho^2}}$$

$$= \frac{2V_o}{\pi}\tan^{-1}\left[\frac{2a^2}{\rho^2 + z^2 - a^2 + \sqrt{(\rho^2 + z^2 - a^2)^2 + 4a^2 z^2}}\right]^{1/2} \tag{5.5}$$

Our method of solution of this problem is closely related to that of Heins and MacCamy[5].

Case II: Two Coaxial Circular Disks

Letting $g(x) = 0$ in (4.9) we have

$$\frac{2}{\pi}\int_0^\rho \frac{f(y)\,dy}{\sqrt{\rho^2 - y^2}} = \mp \frac{1}{\pi}\int_{-a}^{a} \frac{f(y)\,dy}{\sqrt{(2b - iy)^2 + \rho^2}} + 1\,, \qquad 0 < \rho < a \tag{5.6}$$

where the upper and lower signs correspond to the cases of equally and oppositely charged disks respectively. If we treat the right-hand side of (5.6) formally as a known function of ρ, this is an Abel integral equation. Upon applying (5.3) we get

$$f(y) \pm \frac{2b}{\pi}\int_{-a}^{a} \frac{f(y')\,dy'}{4b^2 + (y' - y)^2} = 1 \tag{5.7}$$

which is a Fredholm integral equation of the second kind. This equation was first derived by Love[6], who also obtained, by a different method, an integral representation similar to (4.4).

6. Numerical Solution of the Integral Equations

We are unable to solve the equations (4.9) and (4.10) exactly. This should become quite clear in the last section where we could not even obtain an exact analytic solution for the special case of two coaxial disks. However, the form of these equations is simple enough for them to be integrated numerically. A numerical solution usually gives $f(y)$ and $g(x)$ as polynomials of y and x respectively.

Alternatively we can approximate $f(y)$ and $g(x)$ by polynomials from the start:

$$f(y) = \sum_{n=0}^{N} A_{2n}\left(\frac{y}{a}\right)^{2n}$$

$$g(x) = \sum_{m=0}^{M} B_{2m}\left(\frac{x}{b}\right)^{2m} \tag{6.1}$$

Substituting these expressions into (4.9) and (4.10) and carrying out the integrations, we obtain two functional equations involving the $N + M + 2$ coefficients A_{2n} and B_{2m}. In these equations we set ρ successively equal to $N+1$ values in the range $0 < \rho < a$ and z equal to $M+1$ values in the range $0 < z < b$. In this way we obtain $N + M + 2$ linear algebraic equations for the determination of the unknown coefficients.

As an illustration we use the lowest approximation by taking only the first terms in (6.1):

$$f(y) = A_o , \qquad g(x) = B_o \tag{6.2}$$

Putting $\rho = \frac{1}{2} a$, $z = \frac{1}{2} b$ respectively in (4.9) and (4.10) we get

$$a_{11}A_o + a_{12}B_o = 1$$

$$a_{21}A_o + a_{22}B_o = 1 \tag{6.3}$$

In terms of the ratio $\lambda = b/a$, the coefficients are given by

$$a_{11} = 1 + \frac{2}{\pi} \tan^{-1} \frac{1 + \beta}{2\lambda + \alpha}$$

$$a_{12} = \frac{1}{\pi} \ln \frac{(2\lambda + \alpha)^2 + (1 + \beta)^2}{(1 + \frac{1}{2}\sqrt{3}\,)^2}$$

$$a_{21} = \frac{2}{\pi} \tan^{-1} \frac{1 + \beta'}{\frac{1}{2}\lambda + \alpha'} + \frac{2}{\pi} \tan^{-1} \frac{1 + \beta''}{\frac{3}{2}\lambda + \alpha''}$$

$$a_{22} = \frac{1}{\pi} \ln \frac{(\frac{1}{2}\lambda + \alpha')^2 + (1 + \beta')^2}{(\frac{3}{2}\lambda - \alpha'')^2 + (1 - \beta'')^2} \qquad (6.4)$$

where

$$\binom{\alpha}{\beta} = \left[\sqrt{(2\lambda^2 - \tfrac{3}{8})^2 + 4\lambda^2} \pm (2\lambda^2 - \tfrac{3}{8})\right]^{1/2}$$

$$\binom{\alpha'}{\beta'} = \left[\frac{\lambda}{2} \sqrt{\frac{1}{16}\lambda^2 + 1} \pm \frac{1}{8}\lambda^2\right]^{1/2}$$

$$\binom{\alpha''}{\beta''} = \left[\frac{3}{2}\lambda\sqrt{\frac{9}{16}\lambda^2 + 1} \pm \frac{9}{8}\lambda^2\right]^{1/2} \qquad (6.5)$$

The capacitance is given by

$$C = 16\varepsilon_o(A_o + \lambda B_o)\, a \qquad (6.6)$$

Letting $\lambda = 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 4$ successively, we obtain the following results:

λ	A_o	B_o	C in farads	Smythe's value
0	.500	--	$.708 \times 10^{-10}a$	$.708 \times 10^{-10}a$
1/4	.578	.310	$.928 \times 10^{-10}a$	$.922 \times 10^{-10}a$
1/2	.625	.290	1.0[illegible] $\times 10^{-10}a$	$1.07 \times 10^{-10}a$
1	.634	.330	$1.36 \times 10^{-10}a$	$1.33 \times 10^{-10}a$
2	.588	.341	$1.79 \times 10^{-10}a$	$1.75 \times 10^{-10}a$
4	.531	.312	$2.52 \times 10^{-10}a$	$2.47 \times 10^{-10}a$

In this approximation the capacitance differs from Smythe's value by about 2%.

However, if we use the above solution to compute the surface charge density, we find that our value deviates quite seriously from the result of Smythe. This is not surprising in view of the crudity of our approximation and the fact that the charge density is a local quantity. The capacitance, on the other hand, is a global quantity and is not very sensitive to the number of terms we take in (6.1) to represent $f(y)$ and $g(x)$. Taking more terms will certainly increase the accuracy. Nevertheless, we will not go further into elaborate calculations.

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