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C. R. Arnold

# Identification of Systems in Terms of the Wiener Model

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

# IDENTIFICATION OF SYSTEMS IN TERMS OF THE WIENER MODEL

*C. R. ARNOLD*

*Group 22*

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## ABSTRACT

This report presents briefly a nonlinear model originally proposed by the late Norbert Wiener for the characterization of general systems. Three procedures are then offered for the identification of any given system in terms of the Wiener model. Finally, this report presents the results of a digital computer simulation study (utilizing six somewhat arbitrary systems) which was designed to evaluate the various identification procedures as well as the model itself as a basis for system identification.



Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

# PREFACE

The research reported in this report was conducted during the spring and summer of 1965 while the author was a part-time student at Harvard University under the direction of Professor K. S. Narendra (presently at Yale University). It was presented by Professor Narendra at the Third Annual Allerton Conference on Circuit and System Theory (October 1965) under the title, "On the Use of the Wiener Model for Identification in Adaptive Situations."

The author takes pleasure to acknowledge Lincoln Laboratory for its support in this research. It is also a pleasure to acknowledge Harvard University and Professor Narendra for the intellectual benefits received by virtue of this association. Finally, I wish to thank my former colleague Dr. Harold K. Knudsen and fellow-student Charles P. Neuman for their patient listening and many helpful discussions.

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## I. INTRODUCTION

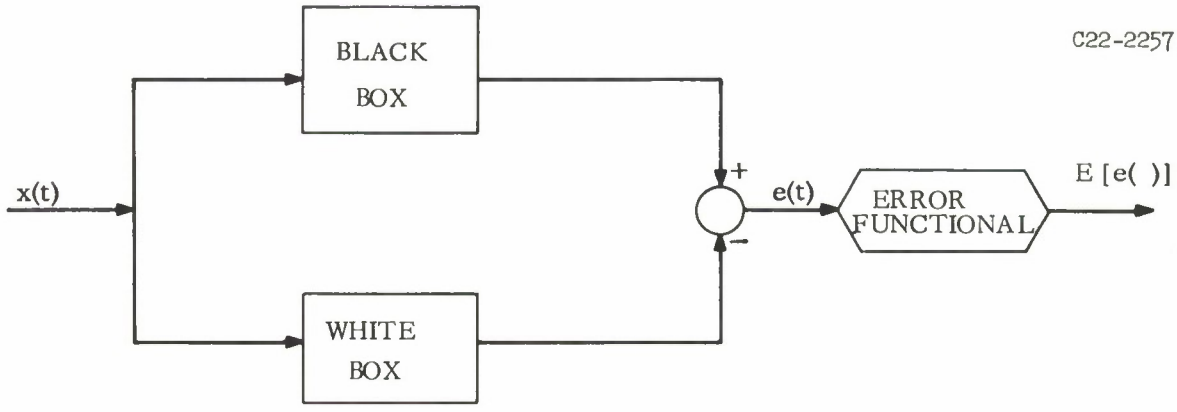
The problem of system identification may be formulated somewhat abstractly as:

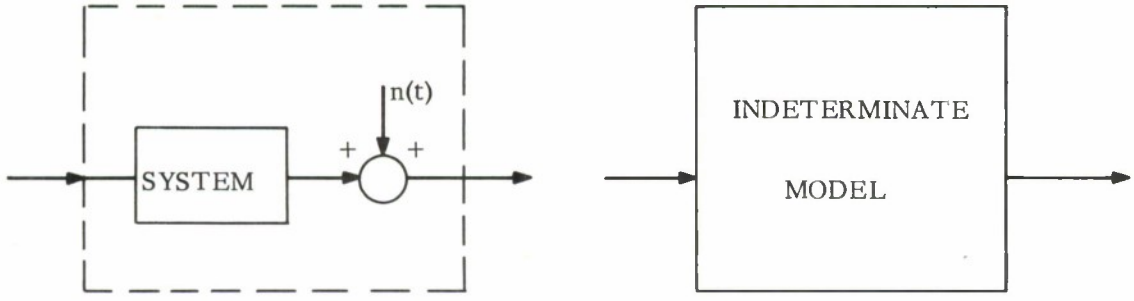
> Given some physical system S and a class C of system models, the identification problem is to determine that specific model M in C which is equivalent (in some sense) to S. The identification is to be accomplished through the observation, often in the presence of noise, of the response of S to various probe functions.

The identification problem may be represented schematically as in Fig. 1 where the black box represents the unknown physical system with, in general, a noisy output. The white box represents an indeterminate model from some class. The problem being to specialize the white box so that the resulting model is equivalent to the black box. The equivalence being in terms of the satisfaction of some criterion by some functional of the error between the system and model for some class of inputs.

In practice the engineer usually chooses, and rightly so, some class of linear models for his white box. Then, he is usually able to determine an adequate model from within some class of models - maybe not his original choice but at least a linear class - and all is well. However, there are times when no linear model can be found which will meet one's adequacy criterion. Then one is forced to consider the identification of systems in terms of nonlinear models.

As to possible nonlinear models, numerous ones have been proposed over the years and they are surveyed in the author's report [1]. One such nonlinear model for the characterization of general systems was proposed by Norbert Wiener [2] in 1949. After a very brief description of the Wiener model in the next section, this report goes on to describe a simulation study (on the IBM-7094 digital computer) which was designed to evaluate the Wiener model as a basis for the identification of real physical systems. As far as this author knows, there has been no attempts to date to actually implement the Wiener model.




b) The black box

c) The white box

Fig. 1. The identification problem.

## II. THE WIENER MODEL

In 1949 Wiener [2] specialized an orthogonal functional decomposition technique of Cameron and Martin [3] to obtain a general model for nonlinear systems. Specifically, as a basis for the generalized Fourier decomposition of [3], Wiener chose the Laguerre functions [4] which are most appropriate for the modeling of physical realizable systems. The resulting model is given schematically in Fig. 2.

Structurally, the Wiener model is a decomposition of a nonlinear system into a linear system with memory followed by a nonlinear no-memory system. The linear system which contains the memory portion of the Wiener model is realized by a Laguerre network [4] which at any instant in time t yields at its output taps the Laguerre coefficient for the infinite past history of the input x ( ):

$$v_k(t) = \int_0^\infty \ell_k(\tau)\, x(t-\tau)\, d\tau \tag{2.1}$$

By the orthogonality of the Laguerre functions $\ell_k(\ )$ and the linearity of the Laguerre network, the outputs $v_k(t)$ will be uncorrelated ( for zero lag) Gaussian variates (and hence statistically independent) when the input probe function is white Gaussian noise. For the zero memory nonlinear functional of the independent Gaussian variates $v_k(t)$, the natural decomposition is a multi-dimensional one in terms of Hermite polynomials. The joint probability density function of the $v_k$'s provides the necessary exponential weighting factor for the orthogonality condition of the Hermite polynomials.

Specifically, the output for the Wiener model is

$$y(t) = \lim_{K\to\infty} \sum_{j_o=o}^{\infty} \sum_{j_1=o}^{\infty} \cdots \sum_{j_K=o}^{\infty} A_{j_o j_1 \cdots j_K} H_{j_o}\left[v_o(t)\right] H_{j_1}\left[v_1(t)\right] \cdots H_{j_K}\left[v_K(t)\right] \tag{2.2}$$

where the $v_k$'s are given by (2.1). The characterizing (or Wiener) coefficients are determined by the time averages

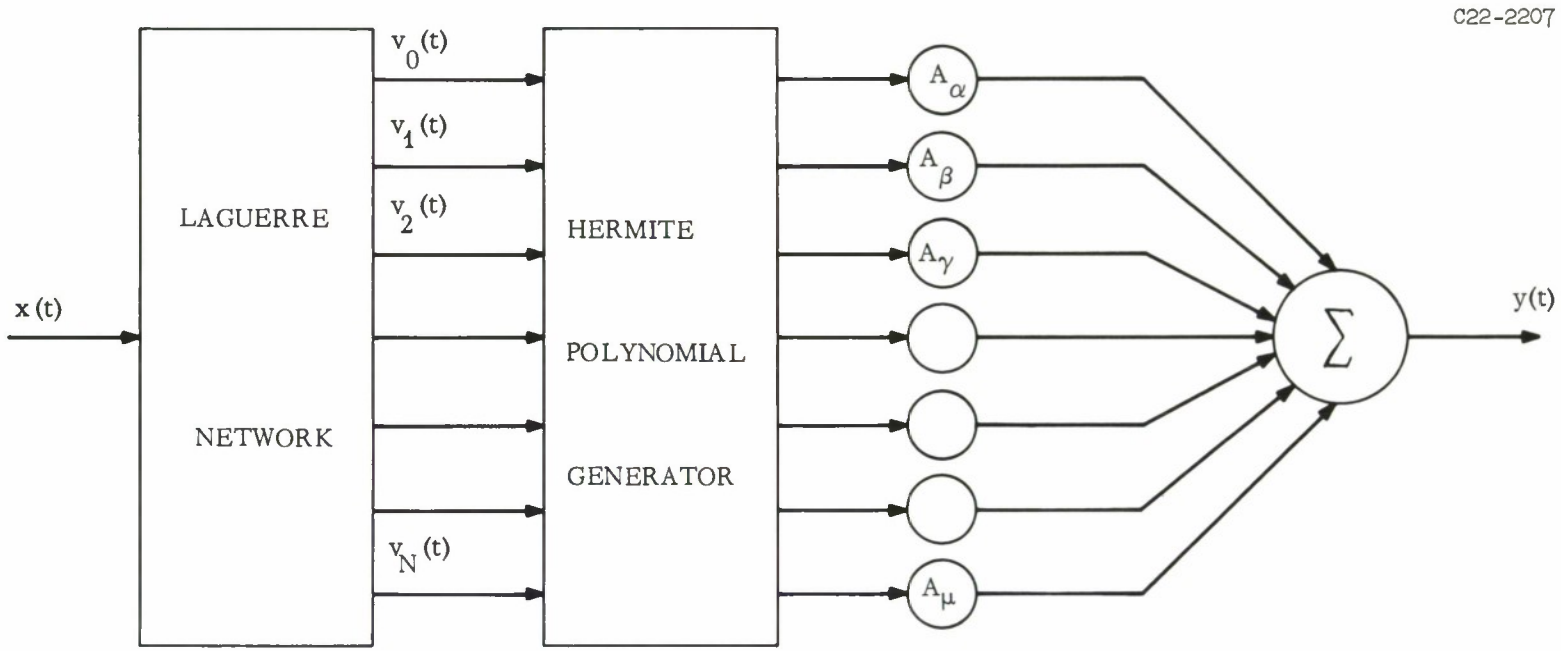



Fig. 2. The Wiener model for non-linear systems.

$$A_{j_o j_1 \ldots j_K} = \overline{y(t)\, H_{j_o}[v_o(t)]\, H_{j_1}[v_1(t)] \ldots H_{j_K}[v_K(t)]} \tag{2.3}$$

where the probe function is white Gaussian noise. (For additional details, see [1]).

On the practical side the Wiener model (2.2) may at first glance appear hopelessly complicated. However, as with all infinite expansion models, the engineer must truncate the model to a finite number of terms. Fig. 3 gives a Wiener model truncated to only the linear and quadratic terms on a three stage Laguerre network. As one can readily see from the figure, the structure is not beyond ordinary engineering synthesis, and yet, the half dozen nonlinear terms may be all that is needed in some situation to obtain an adequate description of the system.

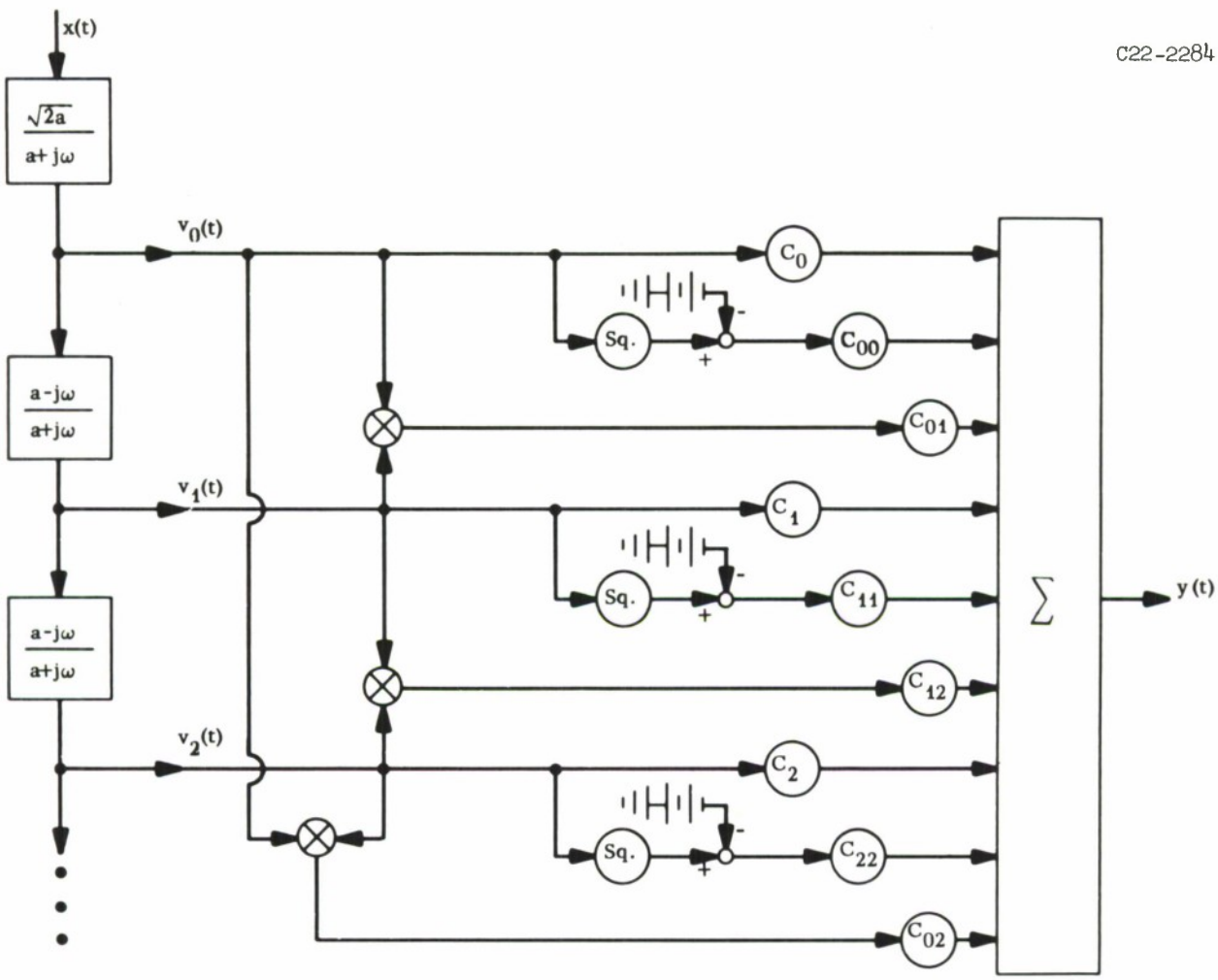



Fig. 3. The truncated Wiener model.

## III. IDENTIFICATION PROCEDURES

### A. The Direct Method

In order to identify a given unknown system in terms of the Wiener model, the direct method is to excite both the system and model with a white Gaussian noise process x ( ) of unit intensity and then measure the various time averages indicated by equation (2.3). In theory, each of the functional output taps of the model,

$$\Phi_{ij\ldots\ell} = H_i\left[v_o(t)\right] H_j\left[v_1(t)\right]\ldots H_\ell\left[v_K(t)\right] \tag{3.1}$$

are orthogonal and the determination of each of the characterizing coefficients $A_{ij\ldots\ell}$ is independent of the others. See Fig. 4.

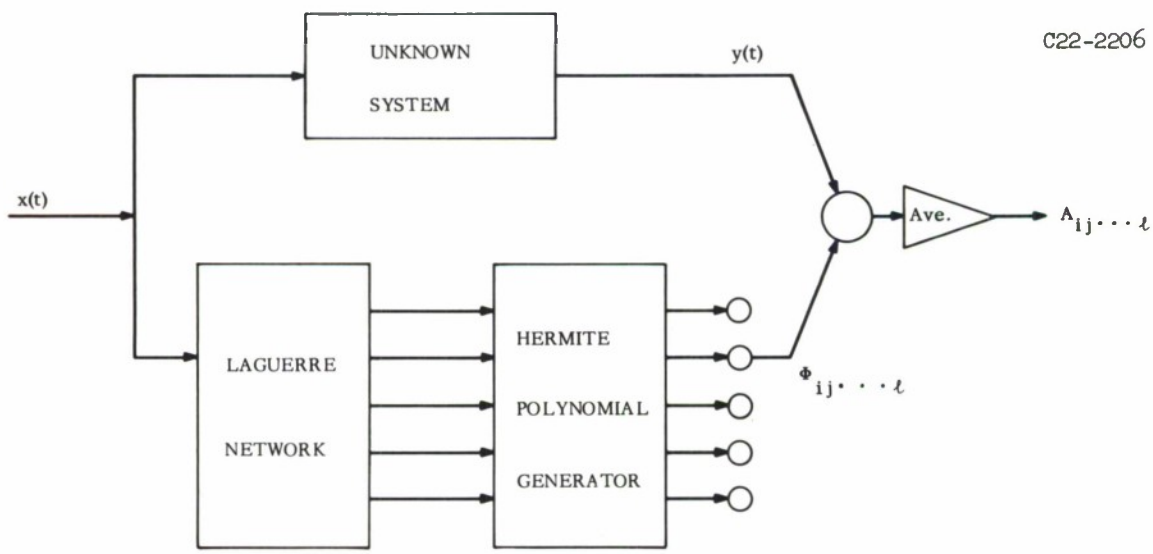



Fig. 4.

In greater detail, for the truncated Wiener model of Fig. 3, the coefficients are determined by the time averages

$$C_k = \overline{y(t)\, v_k(t)}, \tag{3.2a}$$

$$C_{ij} = \overline{y(t) \left\{ v_i(t)\, v_j(t) - \delta_{ij} \right\}}, \tag{3.2b}$$

$$C_{ijk} = \overline{y(t) \left\{ \text{third degree term in } ijk \right\}} \tag{3.2c}$$

$$C_{ijk\ell} = \ldots, \text{ etc.} \tag{3.2d}$$

In practice, however, only finite time samples are used and as noted in [4], for moderately large samples, even the linear functionals $v_k$ ( ) are correlated for zero lag. This non-zero correlation of the outputs from the Laguerre network for finite averages results in a loss of orthogonality for both the linear and the nonlinear terms of the model. The poor results obtained by this technique are illustrated in Section V.

B. A Modified Technique

To mitigate the problem resulting from the sizeable sample correlations, the coefficients of the various degree functionals may be determined in some definite order from a residual error function; rather than from the system's output. Ideally, the various functionals should be determined in the order of their significance. However, this information is not generally available and one must choose some order. A reasonable choice is to assume that the linear functional is most significant. Then the linear coefficients $C_o$, $C_1$, $C_2 \ldots$, $C_K$ of Fig. 3 may be determined by

$$C_k = \overline{y(t)\, v_k(t)} \tag{3.3}$$

Hence, define an error function $e_1(t)$ by

$$e_1(t) = y(t) - \sum_{k=o}^{K} C_k v_k(t), \tag{3.4}$$

Then, assuming that the quadratic functional is the second most significant one, one can determine the quadratic coefficients $C_{ij}$ by the time averages

$$C_{ij} = \overline{e_1(t)\left\{v_i(t)v_j(t) - \delta_{ij}\right\}}. \tag{3.5}$$

Having now the linear and quadratic coefficients, define a second error function

$$e_2(t) = y(t) - \sum_{k=o}^{K} C_k v_k(t) - \sum_{i=o}^{K}\sum_{j=o}^{K} C_{ij}\left\{v_i(t)v_j(t) - \delta_{ij}\right\} \tag{3.6}$$

and with the assumption the third order functional is next most important, determine

$$C_{ijk} = \overline{e_2(t)\left\{v_i(t)\, v_j(t)\, v_k(t) - \delta_{ij}v_k(t) - \delta_{jk}v_i(t) - \delta_{ki}v_j(t)\right\}} \tag{3.7}$$

By now, the technique should be quite clear. Also, it should be realized that the technique can be segmented even further. That is, each coefficient can be determined from a residual error function resulting from the best estimate (model) available before its determination. In fact, for illustration of the technique, the sequence used in the examples of Section V is:

First, the mean value $\overline{y}$ of the output of the system under white Gaussian noise excitation (of unit intensity) is determined. Then, an initial error function is

$$e_o(t) = y(t) - \overline{y} \tag{3.8}$$

Next, all the linear coefficients $C_k$ are determined from

$$C_k = \overline{e_o(t)\, v_k(t)}, \tag{3.9}$$

The residual error after the dc. plus linear approximation is thus

$$e_1(t) = y(t) - y - \sum_{k=o}^{K} C_k v_k(t). \tag{3.10}$$

Next, only the square terms of the quadratic set are determined from

$$C_{kk} = \overline{e_1(t)\left[v_k^2(t) - 1\right]} \tag{3.11}$$

Finally, the cross-term coefficients $C_{ij}$ $(i \neq j)$ are determined by

$$C_{ij} = \overline{e_*(t)\, v_k(t)\, v_j(t)} \qquad i \neq j \tag{3.12}$$

where

$$e_*(t) = y(t) - y - \sum_{k=o}^{K} C_k v_k(t) - \sum_{k=o}^{K} C_{kk}\left[v_k^2(t) - 1\right] \tag{3.13}$$

As will be seen in the examples of Section V, this technique does lead to a stagewise monotone decrease in the mean square error between system and model. It also represents a considerable improvement over the direct method when the actual order of significance of the various functionals agrees with the order of determination selected.

C. The Gradient Technique

The ability to identify an unknown system in terms of a Wiener model by either the direct or modified technique depends upon the Gaussianness of the input probe function and the resulting (in theory) orthogonality of the linear and various

nonlinear functionals [1] . In an adaptive situation, however, one desires a continuous real-time procedure that uses the existing inputs which generally are non-Gaussian. Initially, the author settled upon a gradient technique which circumvents the major objection of the previous techniques (i.e., the Gaussian requirement) and which also provides a rather simple, yet continuous real-time procedure. Moreover, the gradient technique has produced a bit of serendipity in that it overcomes the correlation due to finite sample size even for Gaussian inputs. As we shall see in Section V, the gradient technique can, starting from all coefficients equal to zero, adaptively identify a better model in some time span than the direct technique which averages over the same but entire time span.

Briefly, the technique is to define an error functional as

$$E_T(t) = \int_{t-T}^{t} e^2(\tau)\, d\tau \tag{3.14}$$

where e ( ) is the usual error between system and model,

$$e(t) = y(t) - \sum_{\alpha} A_\alpha \Phi_\alpha \tag{3.15}$$

Then to employ a gradient technique, one requires that [5]

$$\frac{d}{dt}\left\{A_\alpha\right\} = -\beta \frac{\partial}{\partial A_\alpha}\left\{E_T(t)\right\}. \tag{3.16}$$

Since the coefficients appear linearly in the model, from the above equations, it follows that

$$\frac{d}{dt}\left\{A_\alpha\right\} = 2\beta \int_{t-T}^{t} e(\tau)\, \Phi_\alpha\left[x(\ );\ \tau\right] d\tau \tag{3.17}$$

By formally integrating (3.17), one has

$$A_\alpha(t) = A_\alpha(t_o) - 2\beta \int_{t_o}^{t} \int_{t-T}^{t} e(\xi)\Phi_\alpha \left[ x(\ ); \xi \right] d\xi d \tag{3.18}$$

If one adjusts the coefficients $A_\alpha$ in a continuous fashion as indicated by (3.18), the error functional (3.14) for a time-invariant system will be minimized and the final value $A_\alpha(\infty)$ will yield the required characterizing coefficients. In an adaptive situation for a stochastically time-varying system, (3.18) can be implemented on a digital computer by

$$A_\alpha(n+1) = A_\alpha(n) - \beta z_\alpha(n+1) \tag{3.19}$$

where the correction $z_\alpha(n+1)$ is equivalent to the inner integral of (3.18) and may be implemented by a driving simple low-pass (recursive) digital filter with the product of the error and the specific functional $\Phi_\alpha$, e.g.

$$z_\alpha(n+1) = \lambda z_\alpha(n) + \mu e(n+1)\, \Phi_\alpha \left[ n+1 \right] \tag{3.20}$$

for some $\lambda$, $\mu$. For $\lambda$ almost equal to one, the time constant of the low pass filter is quite long and effectively evaluates the integral in (3.18). The choice of $\mu$ can be absorbed into the choice of $\beta$ which does offer some problem. Ideally, initially $\beta$ should be large for rapid convergence but as the minimum is approached, one would like a smaller $\beta$ just to hold one's solution. However, the author has worked only with a fixed $\beta$ who's selection has required at most three attempts.

## IV. THE UNKNOWN SYSTEMS

Figure 5 represents the six systems which were chosen for the evaluation of the Wiener model. For the purpose of identification, however, the selected systems are treated as unknown. In this section, these systems are described briefly and some of the considerations which lead to their selection are discussed.

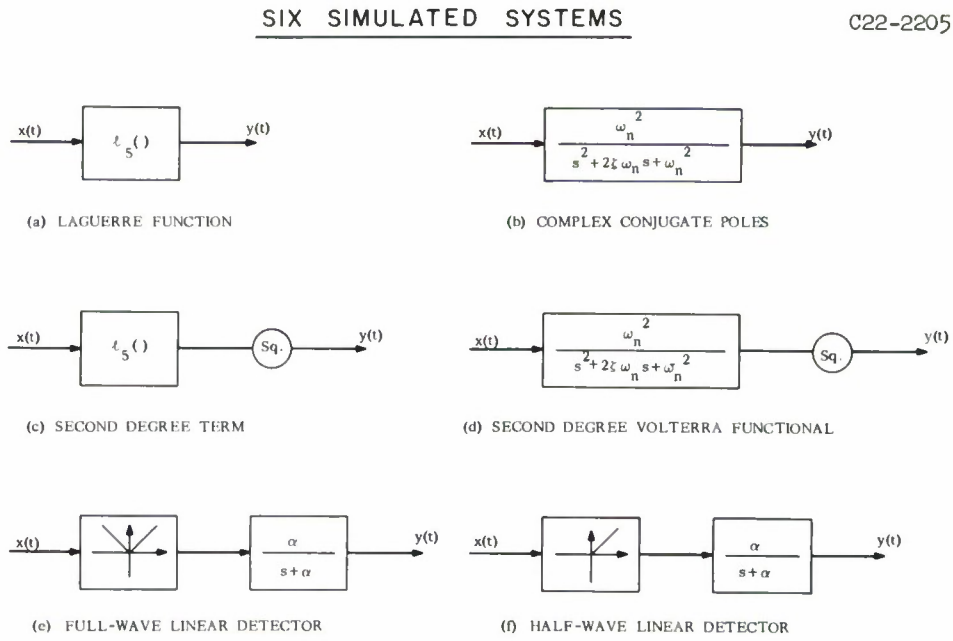



Fig. 5.

The first system selected to play the role of the unknown system was the linear system whose impulse response was the fifth order Laguerre function $\ell_5(\ )$. It was selected because it is a single constituent (linear) functional of the Wiener model and thus permits one to assess to some degree the correlation between the various linear and nonlinear terms of the model.

The second unknown system was also a linear system and consisted of a pair of complex conjugate poles.

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2} \tag{4.1}$$

The damping ratio $\xi$ was taken to be 0.1 and hence (4.1) represents a rather narrow band system. This is the same system considered in [4] and its further consideration here is justified in that it provides a non-trivial approximation problem for the Laguerre functions for even a linear model.

The third system consisted of an isolated second degree term of the Wiener model. Namely, the linear system with impulse response $\ell_5(\ )$ followed by a square-law device. Its choice was also motivated (as in the case of the first system) by a desire to assess the correlation between the various linear and nonlinear terms of the model.

The fourth unknown system consisted of the second system above followed by a square-law device. It represents an isolated second degree Volterra functional [1] whose Wiener coefficients may be calculated exactly with some effort. As will be seen in the next section, however, it affords a rather difficult system for approximation by the Wiener model.

The last two selected systems were the full-wave and half-wave linear detectors. They provide non-trivial systems for approximation by the Wiener model in that the first (full-wave) contains Volterra functions of all even order (0, 2, 4, ...) and the latter (half-wave) contains functionals of all order in its representation.

In all cases, the unknown systems were independently simulated on the digital computer and driven by the same sample of white Gaussian noise. A typical set of input/output series (for the half-wave linear detector) is given in Fig. 6. The parameters of all systems were also all adjusted so that the spectral content of the systems output were approximately equal and hence required a single fixed bandwidth for the Laguerre network portion of the Wiener model.

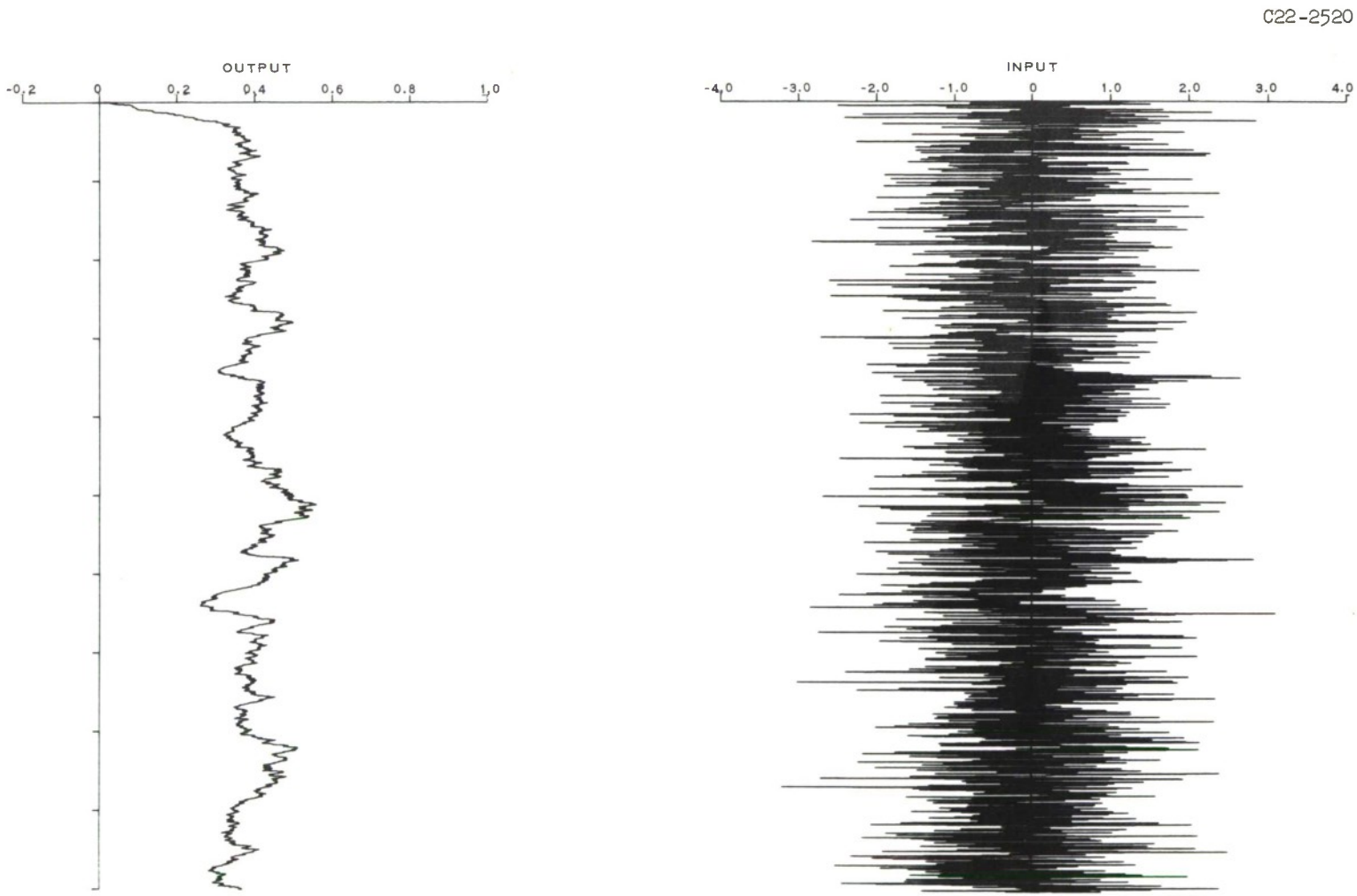



Fig. 6. The input/output for a half-wave linear detector.

## V. RESULTS FROM THE COMPUTER SIMULATION

### A. The Direct and Modified Technique

For each of the systems of Section IV, a Wiener model consisting of an eleven stage Laguerre network with all linear and quadratic terms has been determined by both the direct and the modified technique. The model is, therefore, that of Fig. 3 except for additional linear and quadratic terms. In all, there were eleven linear, eleven square, and fifty-five cross terms in the models.

Since several of the systems contain functionals beyond the capability of the model that contribute a sizeable dc. component to the system's output, it was found necessary to first estimate this dc. component and then remove it before it was possible to obtain even a linear approximation. For the results to be shown in the following figures (Figs. 7-18), this estimated dc. component is plotted on top of the system's output given by the upper curve of each figure.

After each model had been determined by either technique, the approximation resulting from each of the following was evaluated:

1. dc. + linear terms
2. dc. + linear + square terms (5.1)
3. dc. + linear + all quadratic terms

The resulting responses of each of the three approximations are also plotted as successive curves in the following figures.

The mean square error resulting for each of the approximations (5.1) has also been calculated and normalized by the respective system's output variance. That is, the estimated dc. component yields a model whose normalized mean square error equals one (1.0). All other approximations, hopefully, would be less than one. However, the resulting normalized errors obtained for each of the systems via both the direct and the modified technique are given in Table 1. For the table, the legend for the system agrees with that of Fig. 5 and the various approximations are summarized in (5.1). Note that for the first approximation

APPROXIMATION

| SYSTEM | 1 | 2 | 3 |
|---|---|---|---|
| a | 0.101 | (D) 0.257<br>(M) 0.096 | (D) 2.674<br>(M) 0.094 |
| b | 0.176 | (D) 0.242<br>(M) 0.169 | (D) 0.415<br>(M) 0.158 |
| c | 0.959 | (D) 0.064<br>(M) 0.058 | (D) 0.747<br>(M) 0.060 |
| d | 0.961 | (D) 0.895<br>(M) 0.888 | (D) 1.071<br>(M) 0.525 |
| e | 0.976 | (D) 0.935<br>(M) 0.924 | (D) 0.817<br>(M) 0.784 |
| f | 0.302 | (D) 0.395<br>(M) 0.276 | (D) 0.753<br>(M) 0.228 |

D = Direct Method  M = Modified Technique

TABLE 1 - NORMALIZED MEAN SQUARE ERRORS

(dc. + linear), both techniques yield the same model and hence only one value is given.

On the following twelve pages, the responses of each system and of each of its resulting approximations are given. The results for the direct method and the modified technique are given on facing pages for comparisons.

In all cases, the curves represent 7000 values in time with a $\Delta t$ whose corresponding folding frequency $\left(f_{Ny} = \frac{1}{2\Delta t}\right)$ was approximately one hundred times the band-width of the systems considered.

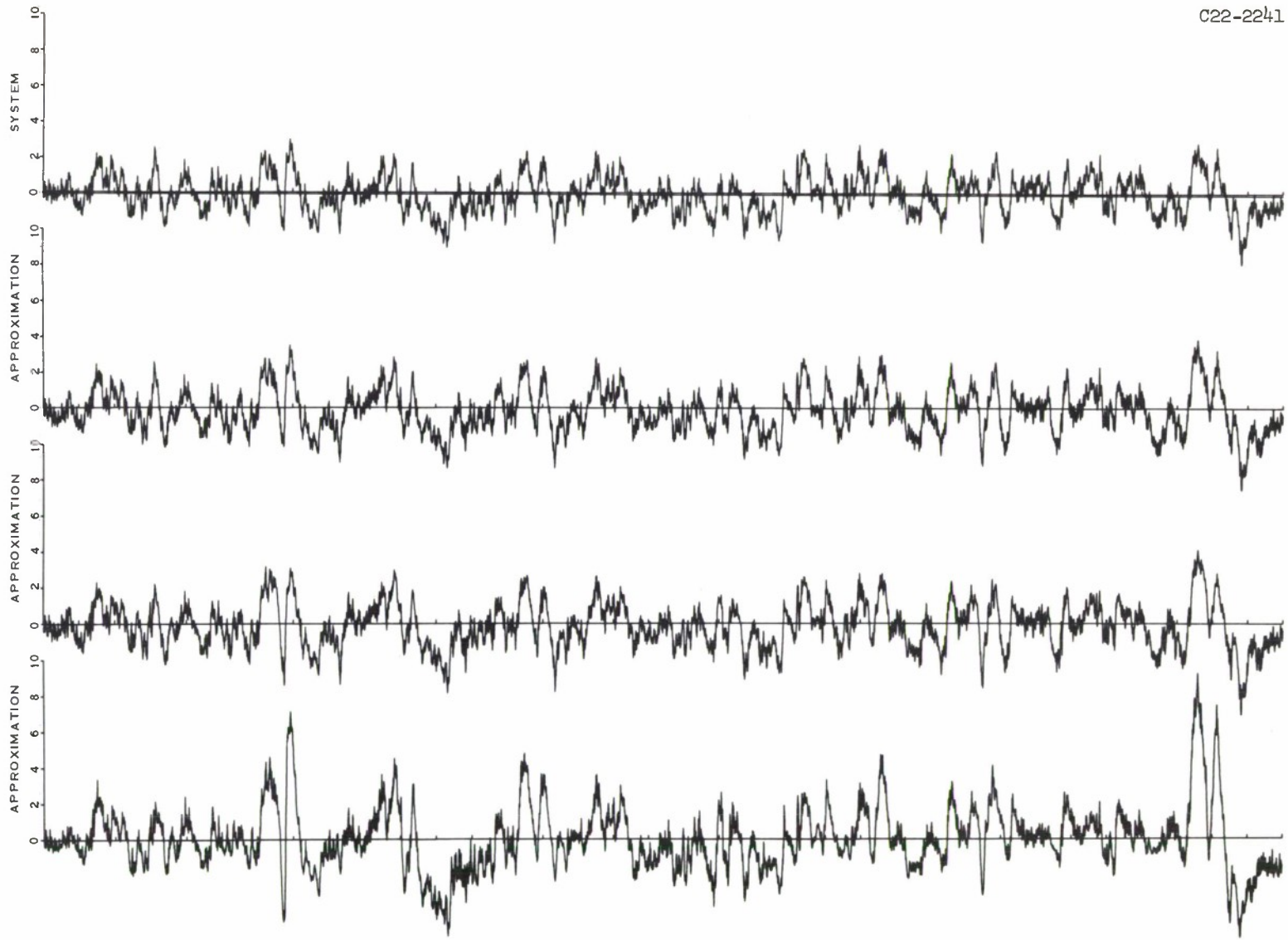



Fig. 7. The response of system a with approximations by the direct method.

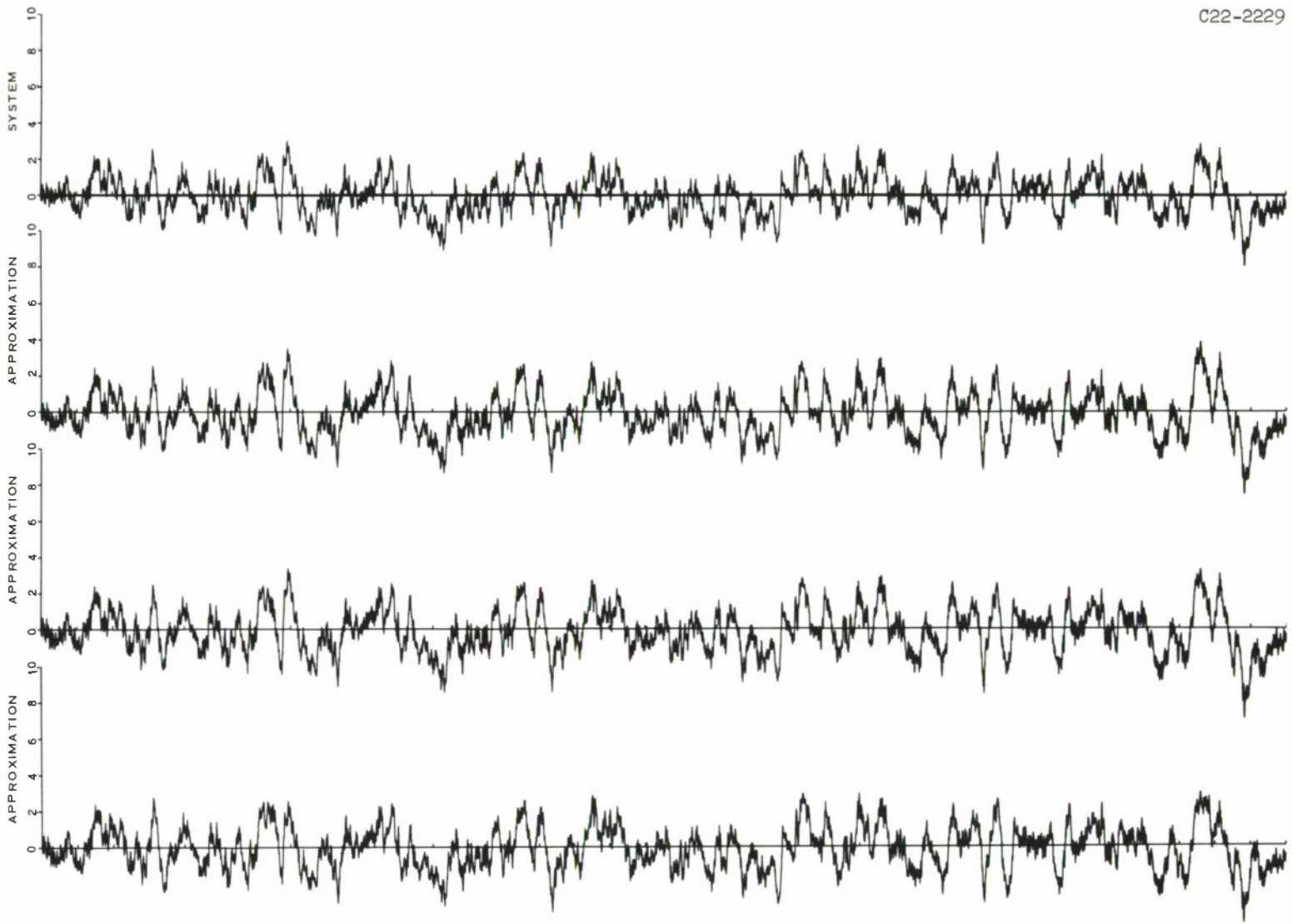



Fig. 8. The response of system a with approximations by the modified technique.

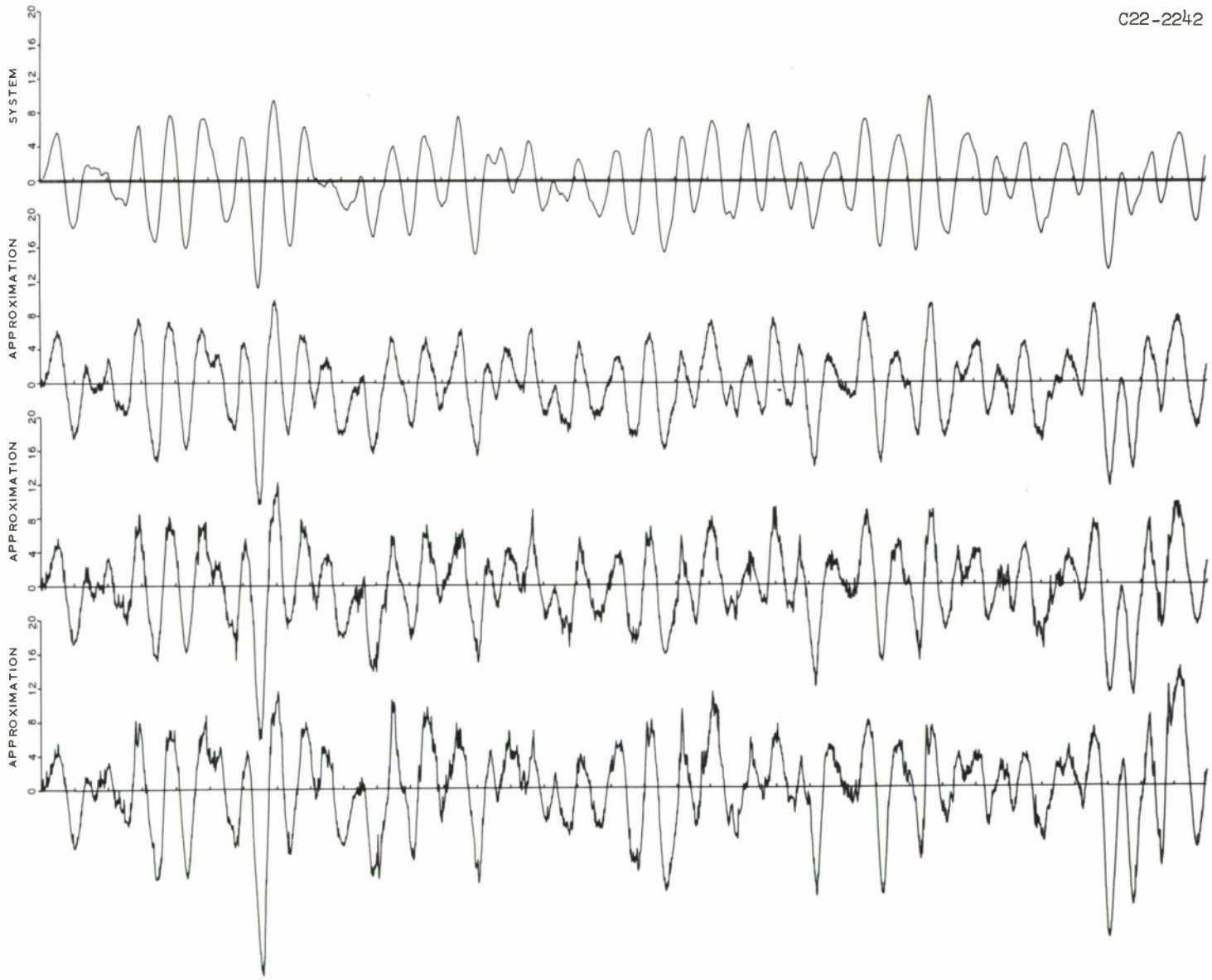



Fig. 9. The response of system b with approximations by the direct method.

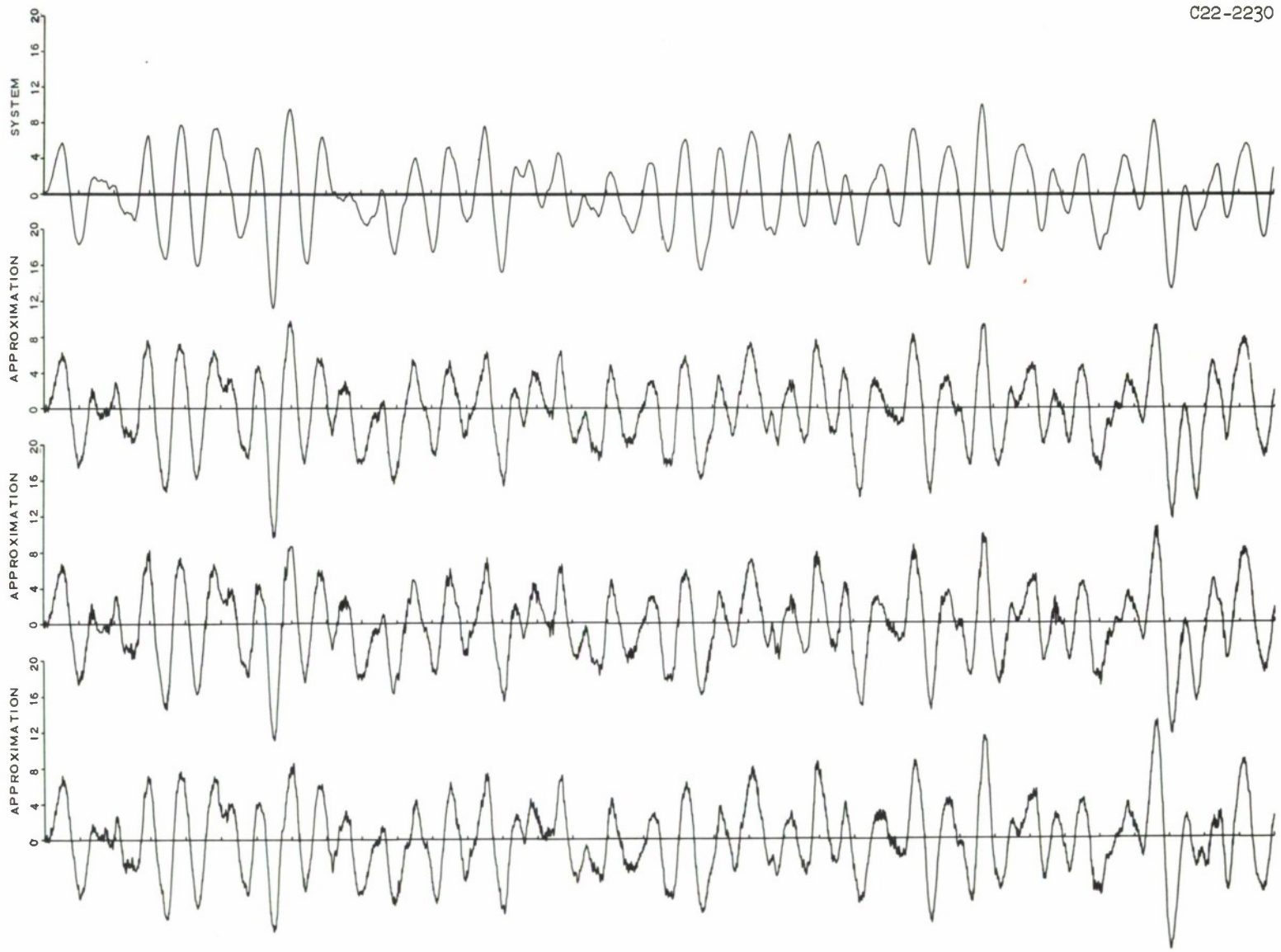



Fig. 10. The response of system b with approximations by the modified technique.

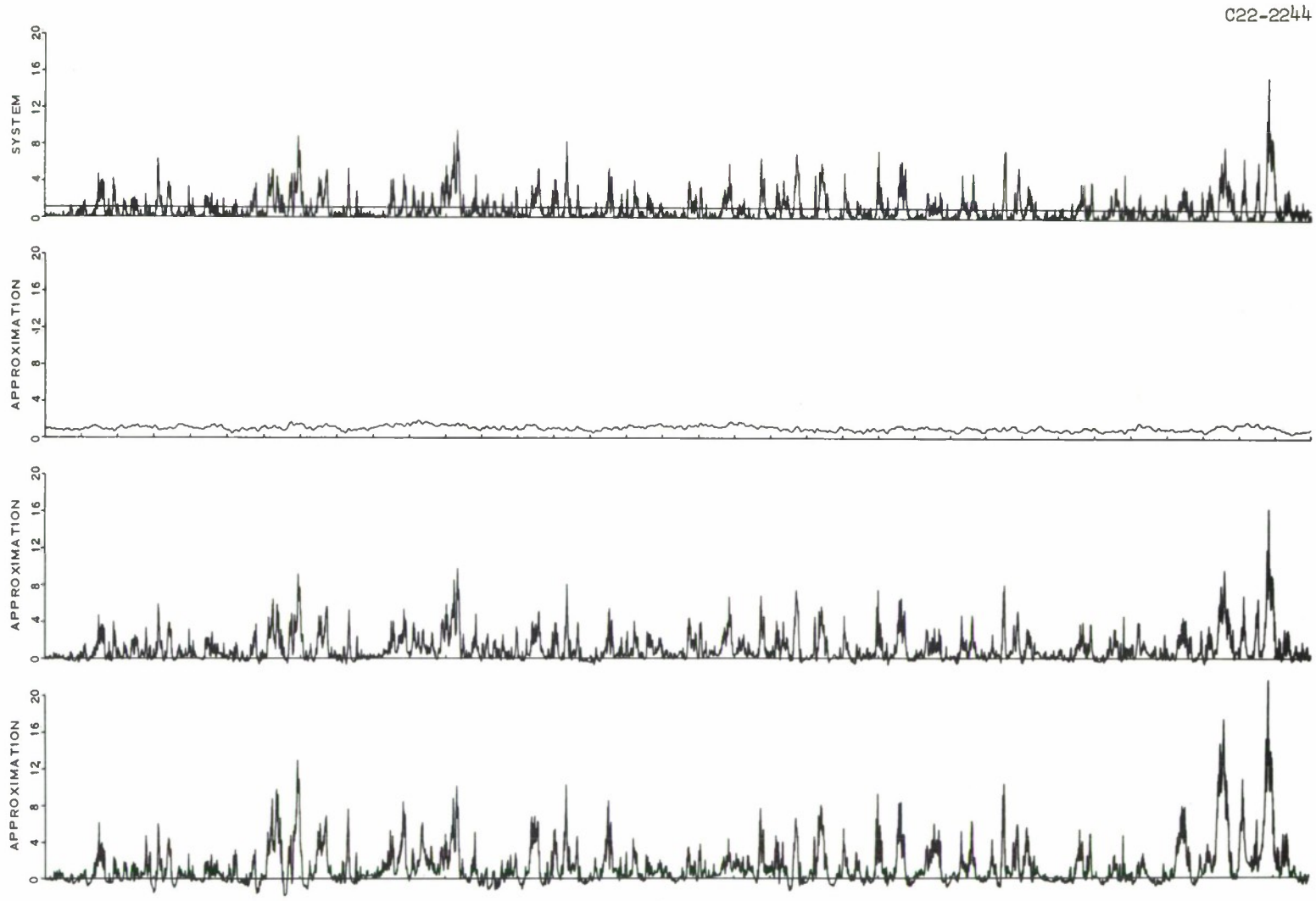



Fig. 11. The response of system c with approximations by the direct method.

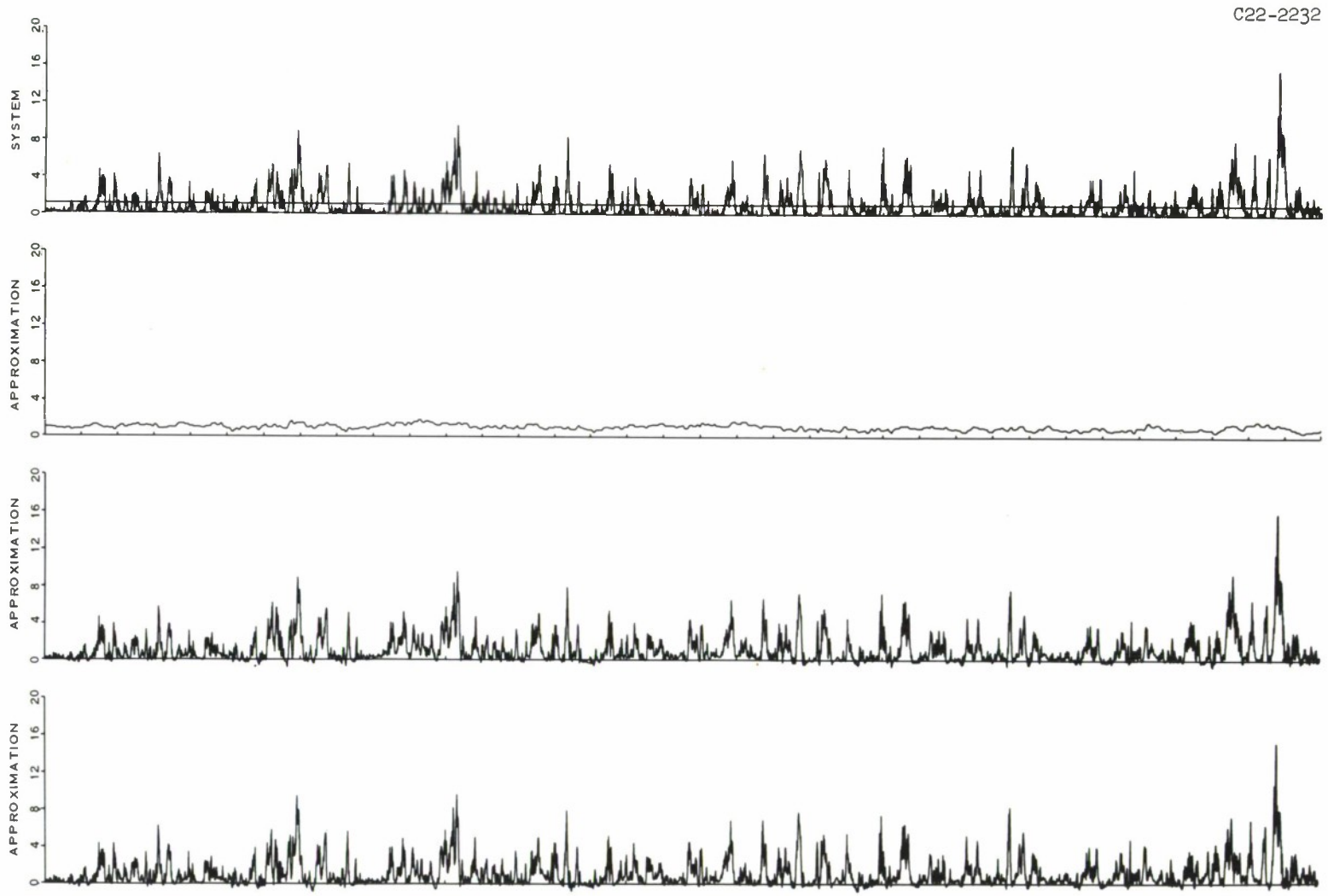



Fig. 12. The response of system c with approximations by the modified technique.

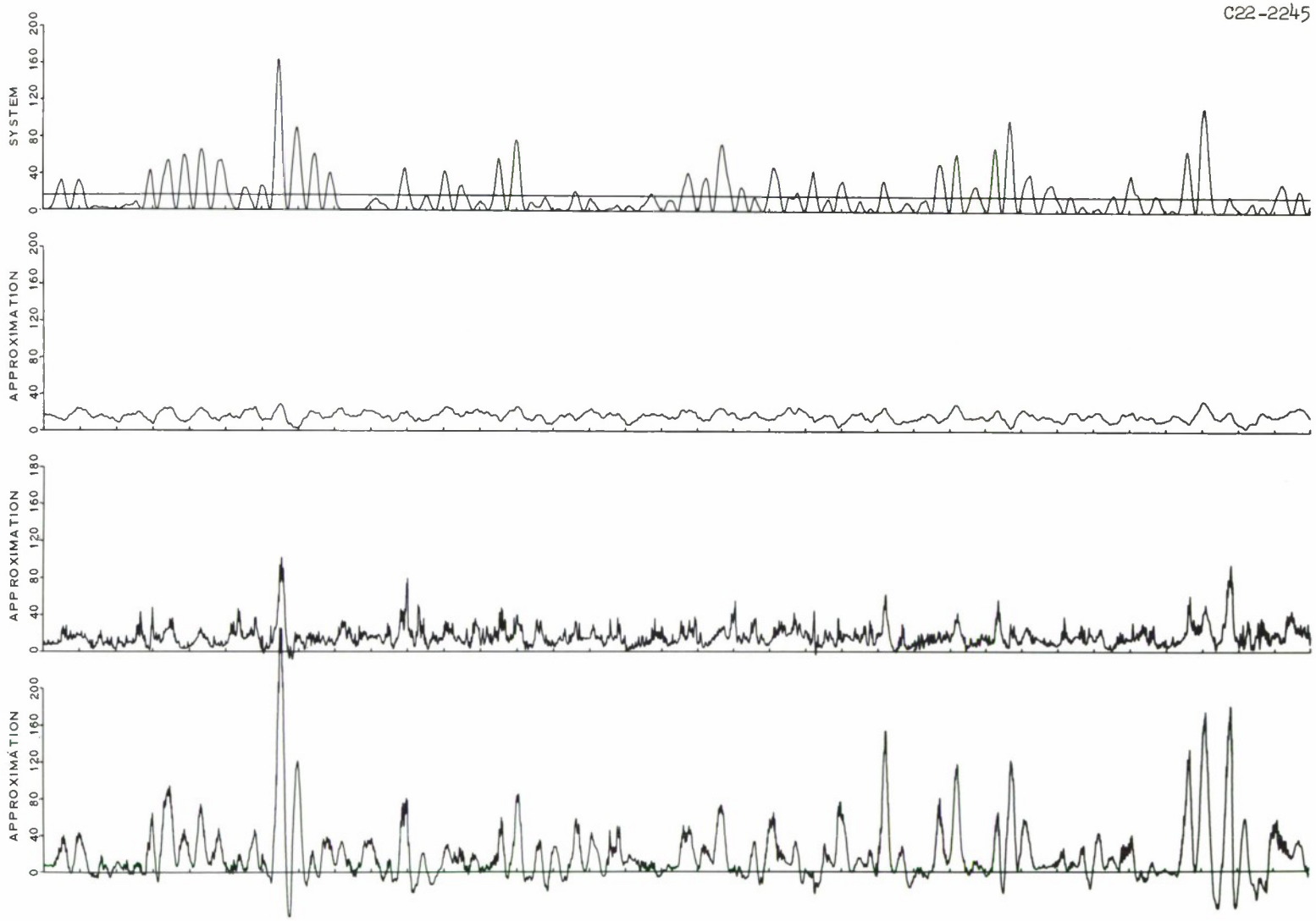



Fig. 13. The response of system d with approximations by the direct method.

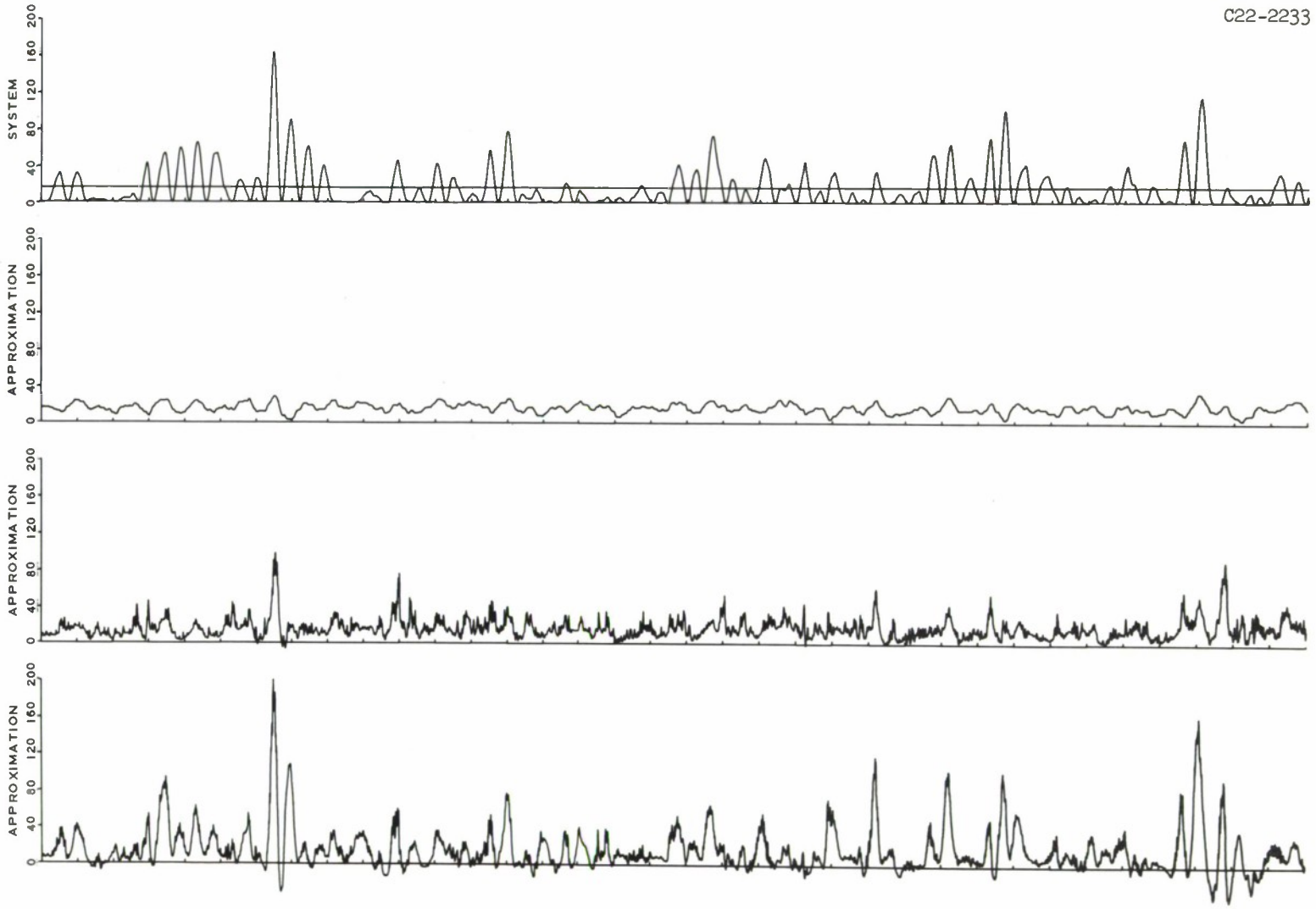



Fig. 14. The response of system d with approximations by the modified technique.

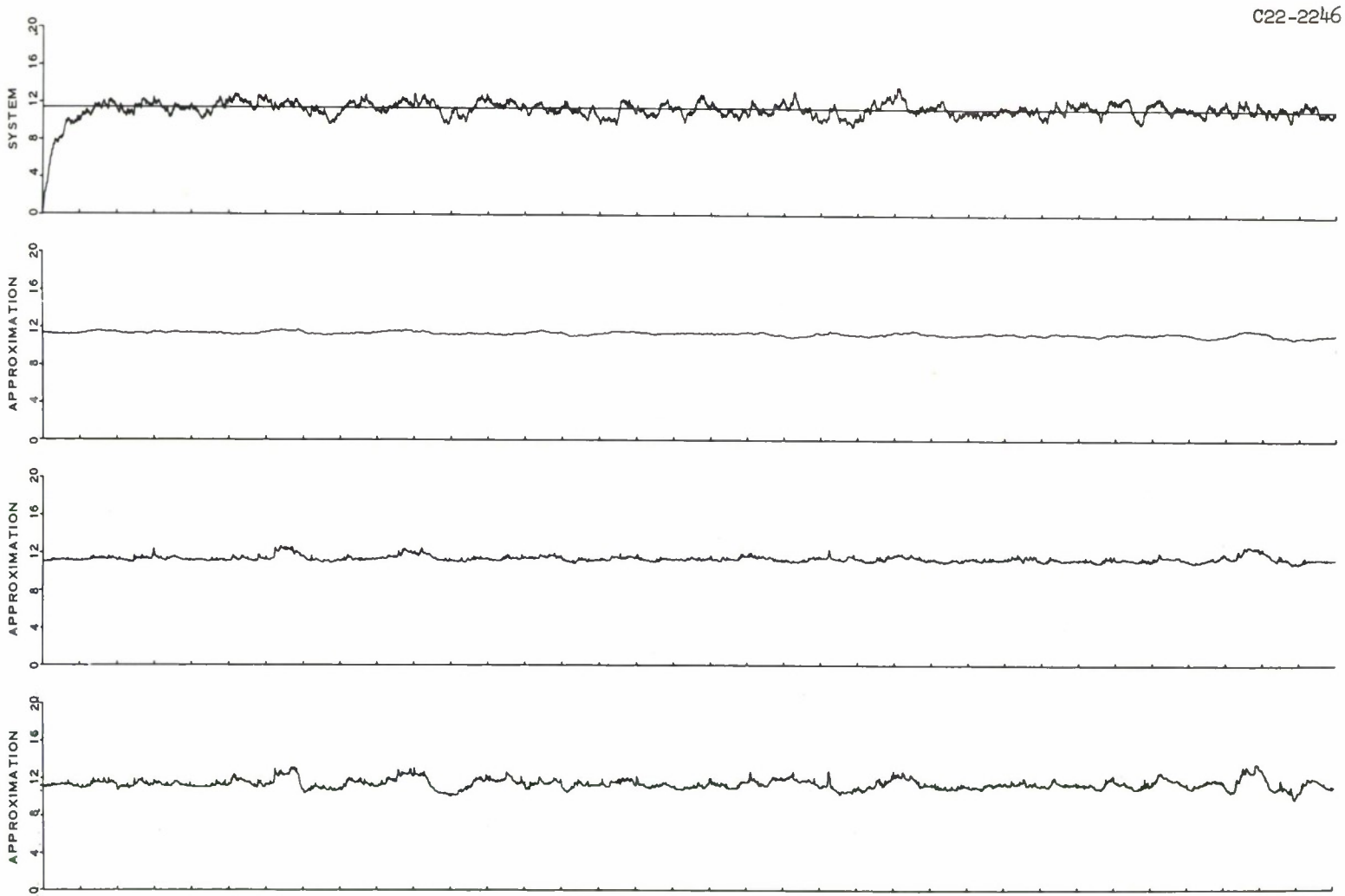



Fig. 15. The response of system e with approximations by the direct method.

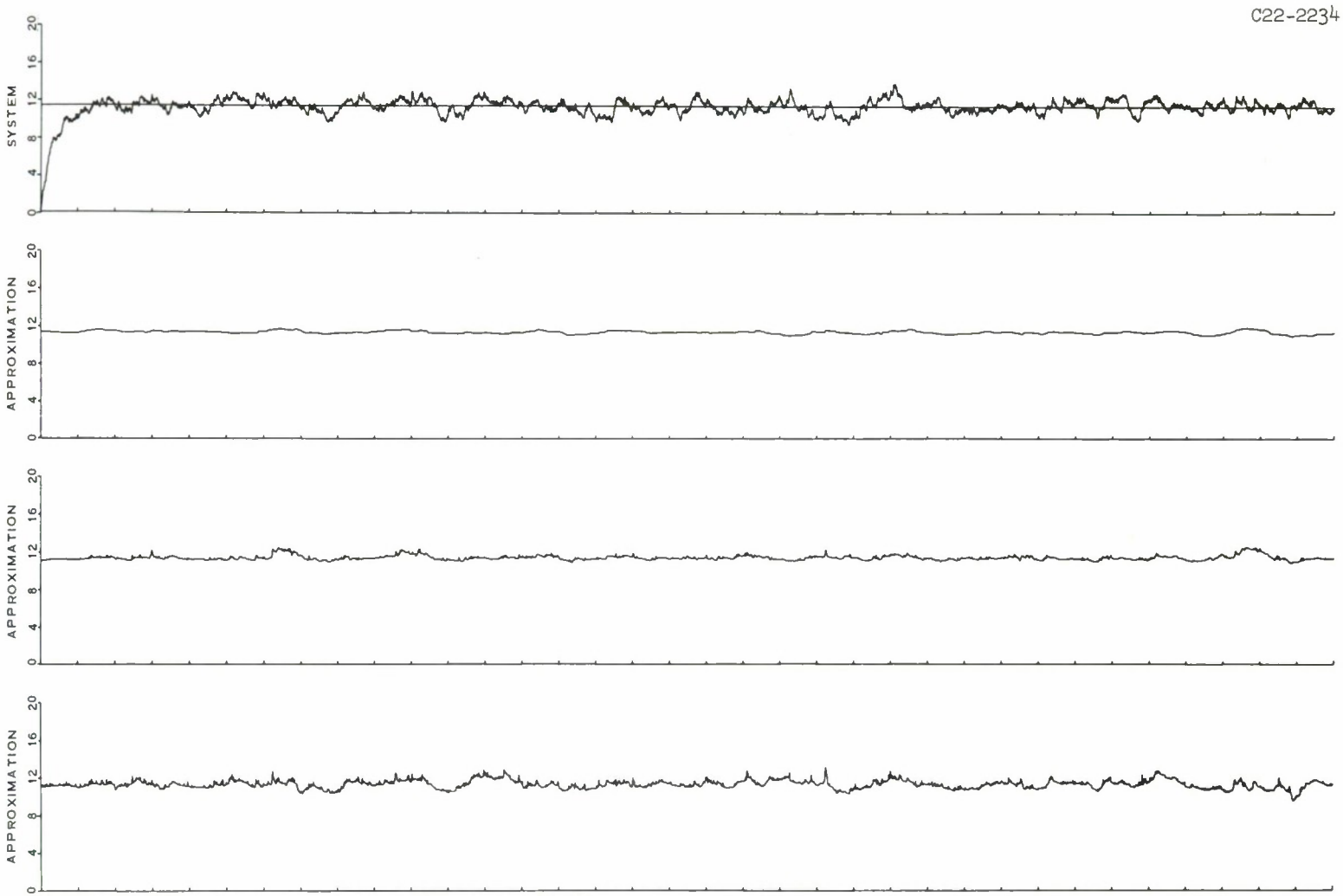



Fig. 16. The response of system e with approximations by the modified technique.

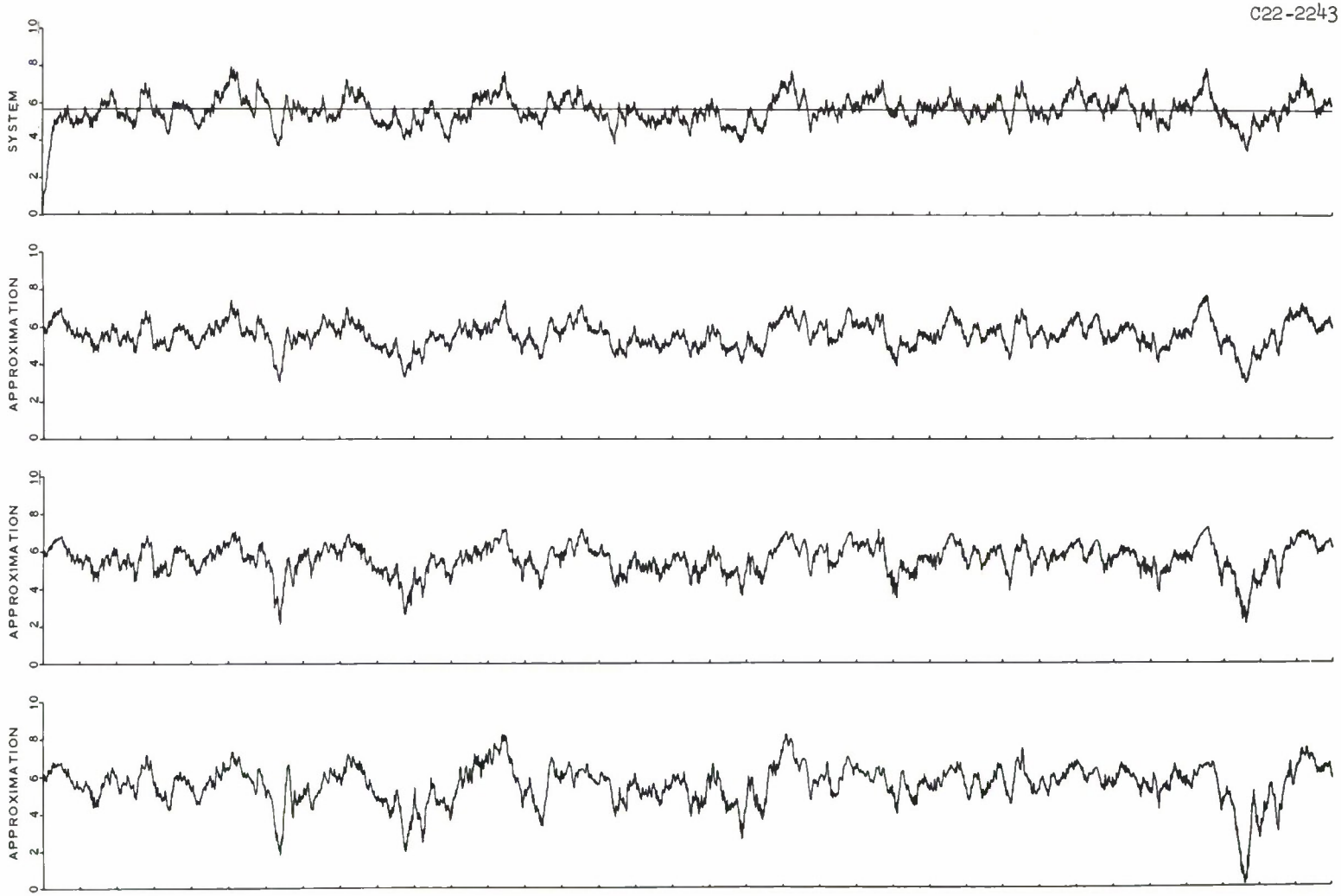



Fig. 17. The response of system f with approximations by the direct method.

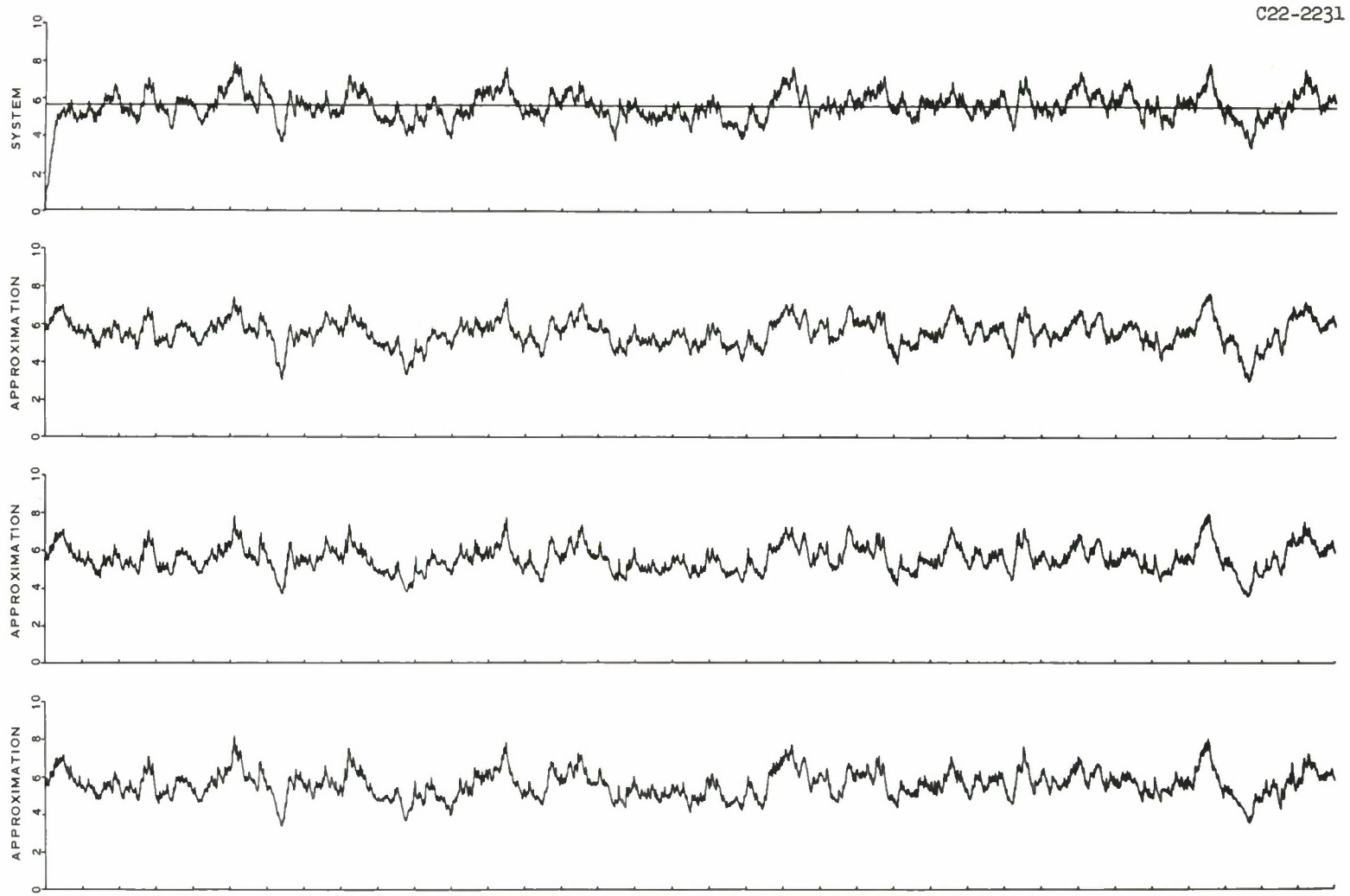



Fig. 18. The response of system f with approximations by the modified technique.

B. The Gradient Technique

On the following pages, Figs. 19-24 give the results of an identification program employing a gradient technique to identify only a linear model plus bias [equivalent to approximation 1 of (5.1)] for each of the systems under consideration. In all cases, the model was limited to the dc. bias plus the linear terms on an eleven stage Laguerre network. Initially, all coefficients of the model were set equal to zero.

In the figures following, as before, the upper curve gives the system's response. The center curve is the response to the linear model (plus bias) and the lower curve is the instantaneous error (between system and model) plotted to twice the scale of the system. The convergence of the model to the system is quite remarkable in several cases. The plotted responses represent 7000 values (in time) in all cases.

Since the gradient technique is adaptive in nature, one cannot calculate a residual mean square error until after the model has converged. In order to circumvent this considerable waiting time with its greatly increased computational requirement, and yet still provide for some comparison of this technique with previous ones, the following calculations have been implemented.

The square of the instantaneous error was driven into a low-pass (digital) filter whose time constant was twice that used in the averaging for the gradient technique. The resulting filter's output gives a fair measure of the time varying average square error. For comparison of the technique, the residual error in the previous techniques was also driven through the same squarer/low-pass filter combination. After 7000 values in time, this technique yielded the relative mean square errors of Table II for the linear models to the linear systems.

| | Technique | |
|---|---|---|
| | Direct/Modified | Gradient |
| System a | 0.188 | 0.075 |
| System b | 3.53 | 2.80 |

TABLE II - RELATIVE MEAN SQUARE ERRORS

Table II demonstrates the superiority of the gradient technique in even the case of linear models. That is, the gradient technique can adaptively identify (starting from all coefficients equal to zero) a better model in 7000 values than the direct averaging technique can determine by averaging over the full 7000 values.

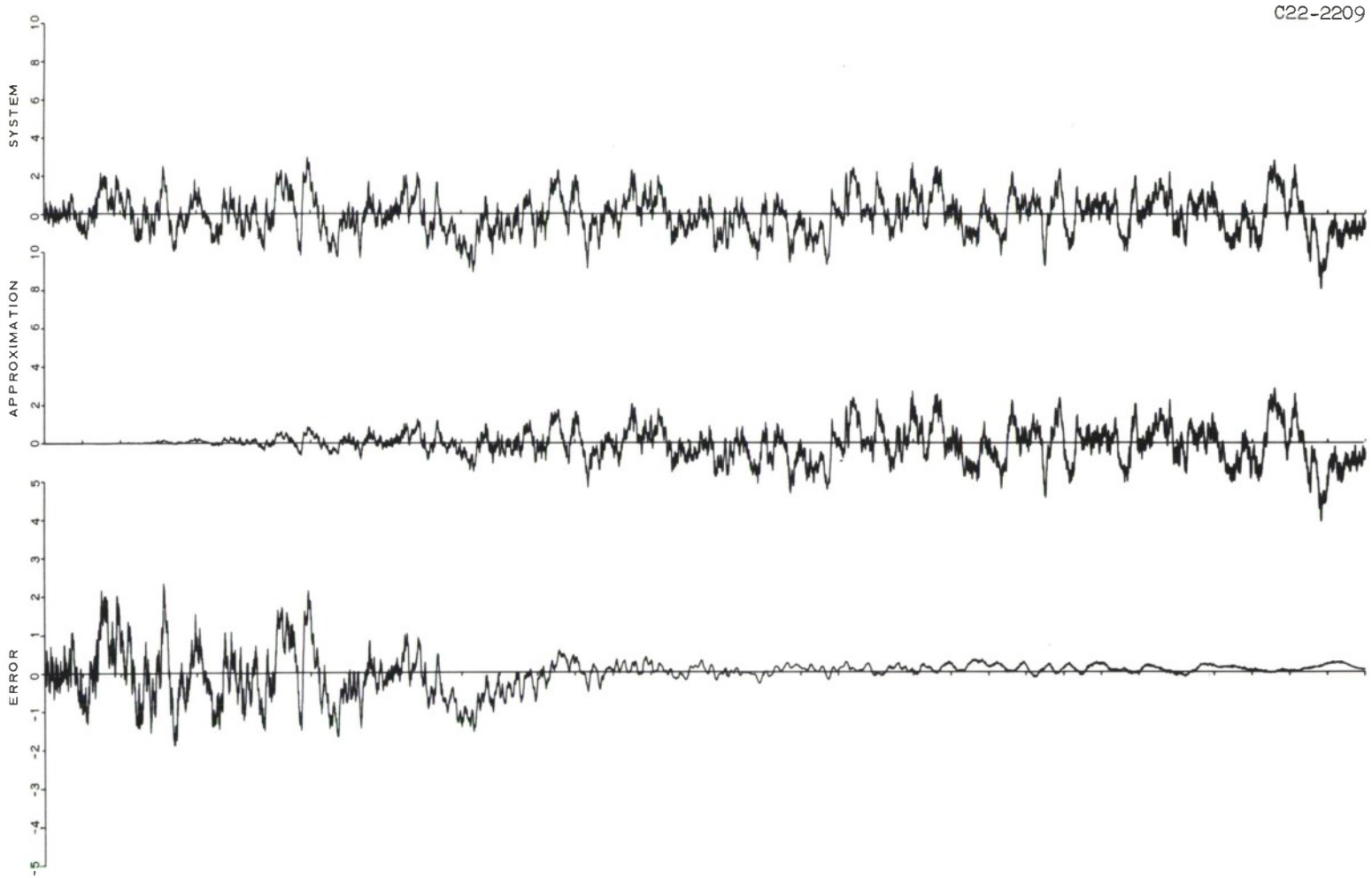



Fig. 19. The response of system a with linear approximation determined by the gradient technique.

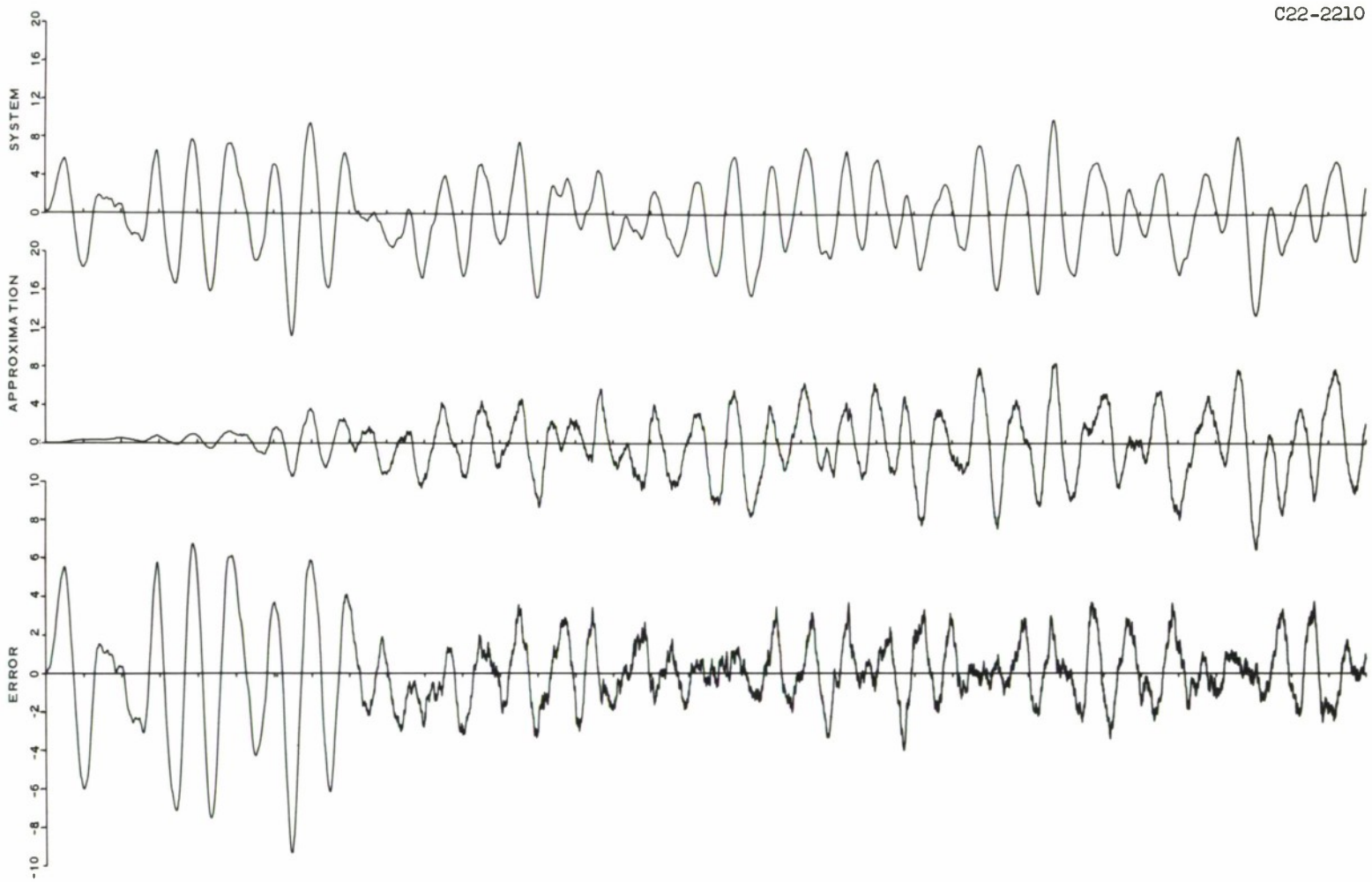



Fig. 20. The response of system b with linear approximation determined by the gradient technique.

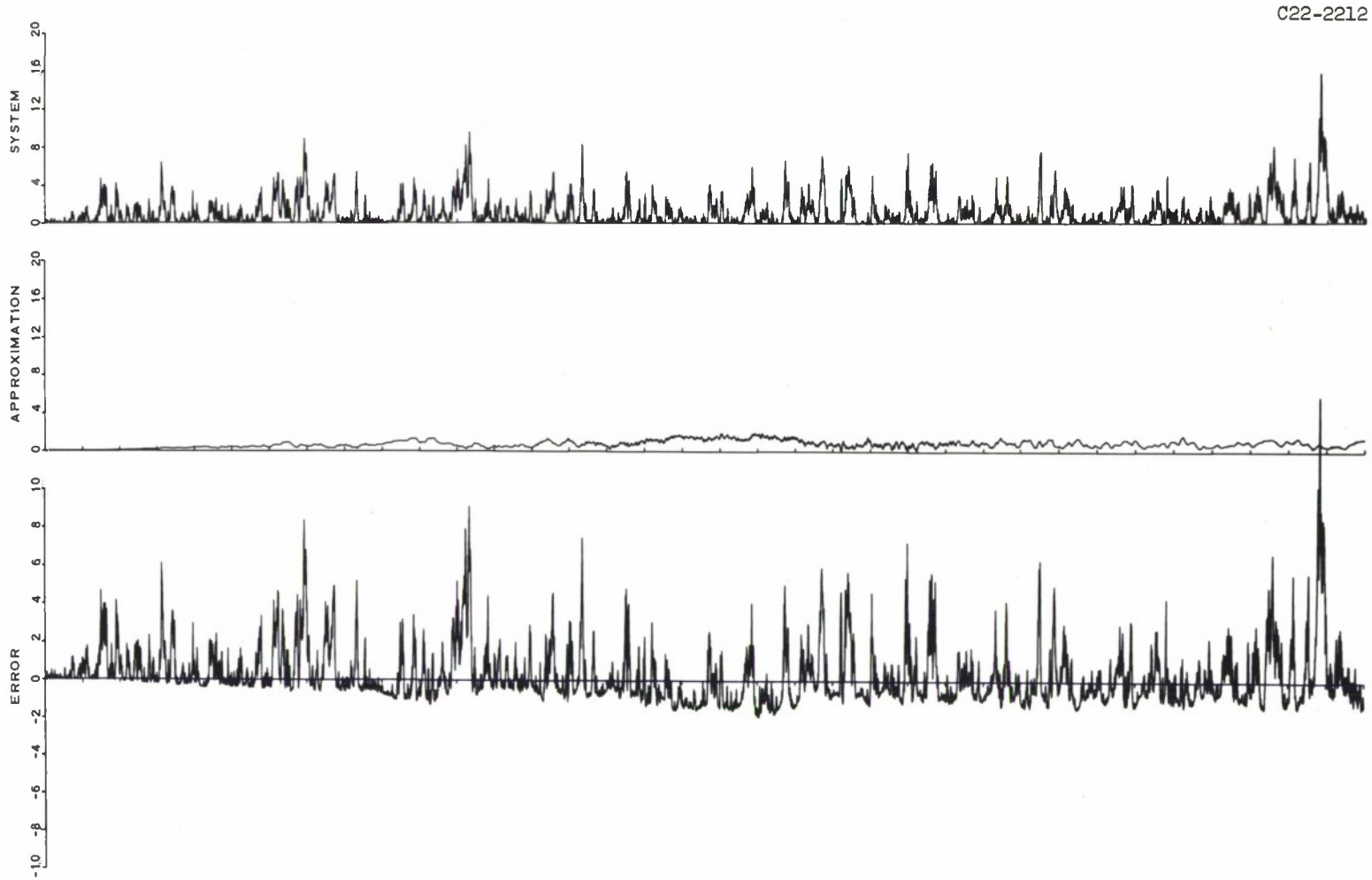



Fig. 21. The response of system c with linear approximation determined by the gradient technique.

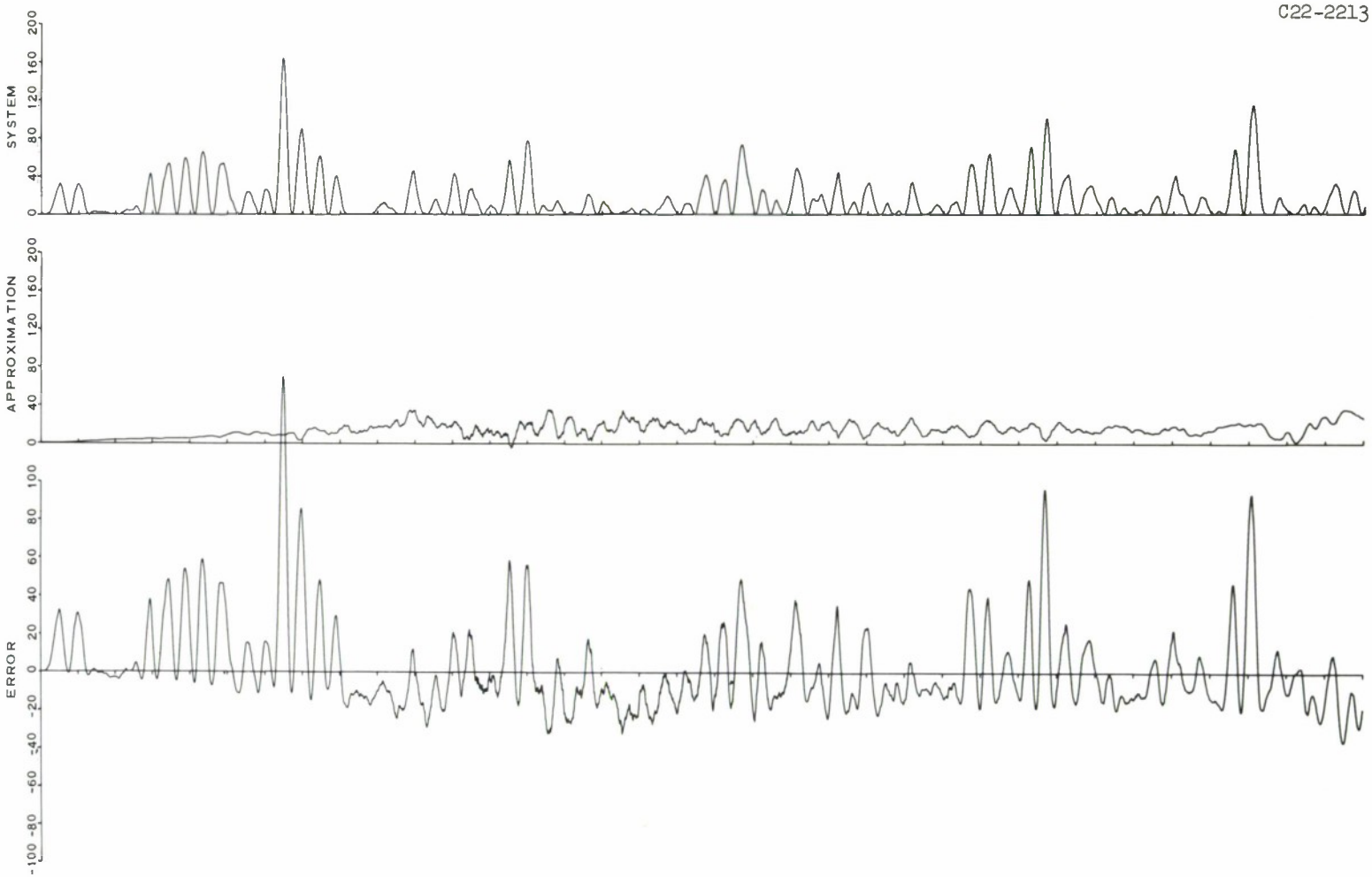



Fig. 22. The response of system d with linear approximation determined by the gradient technique.

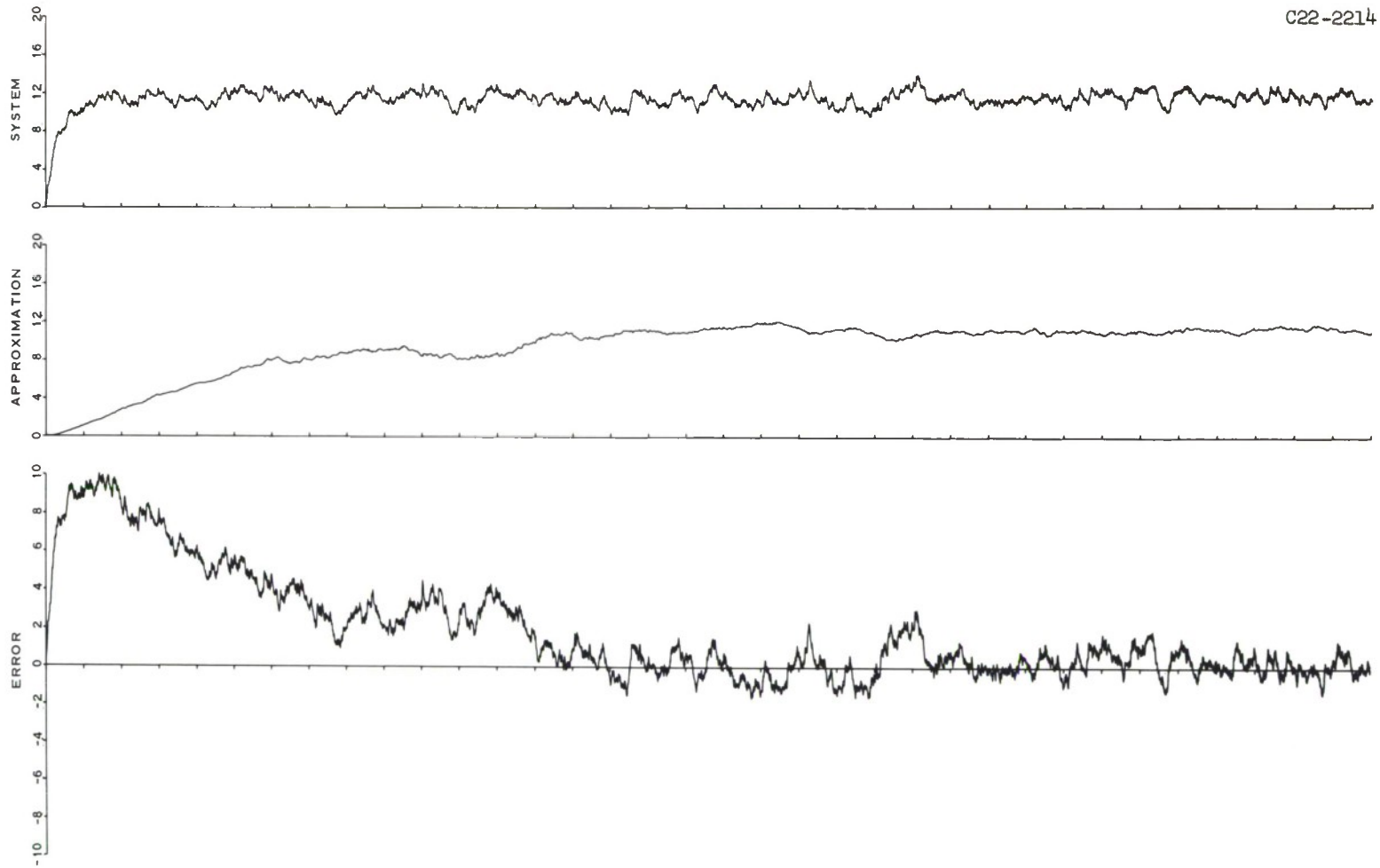



Fig. 23. The response of system e with linear approximation determined by the gradient technique.

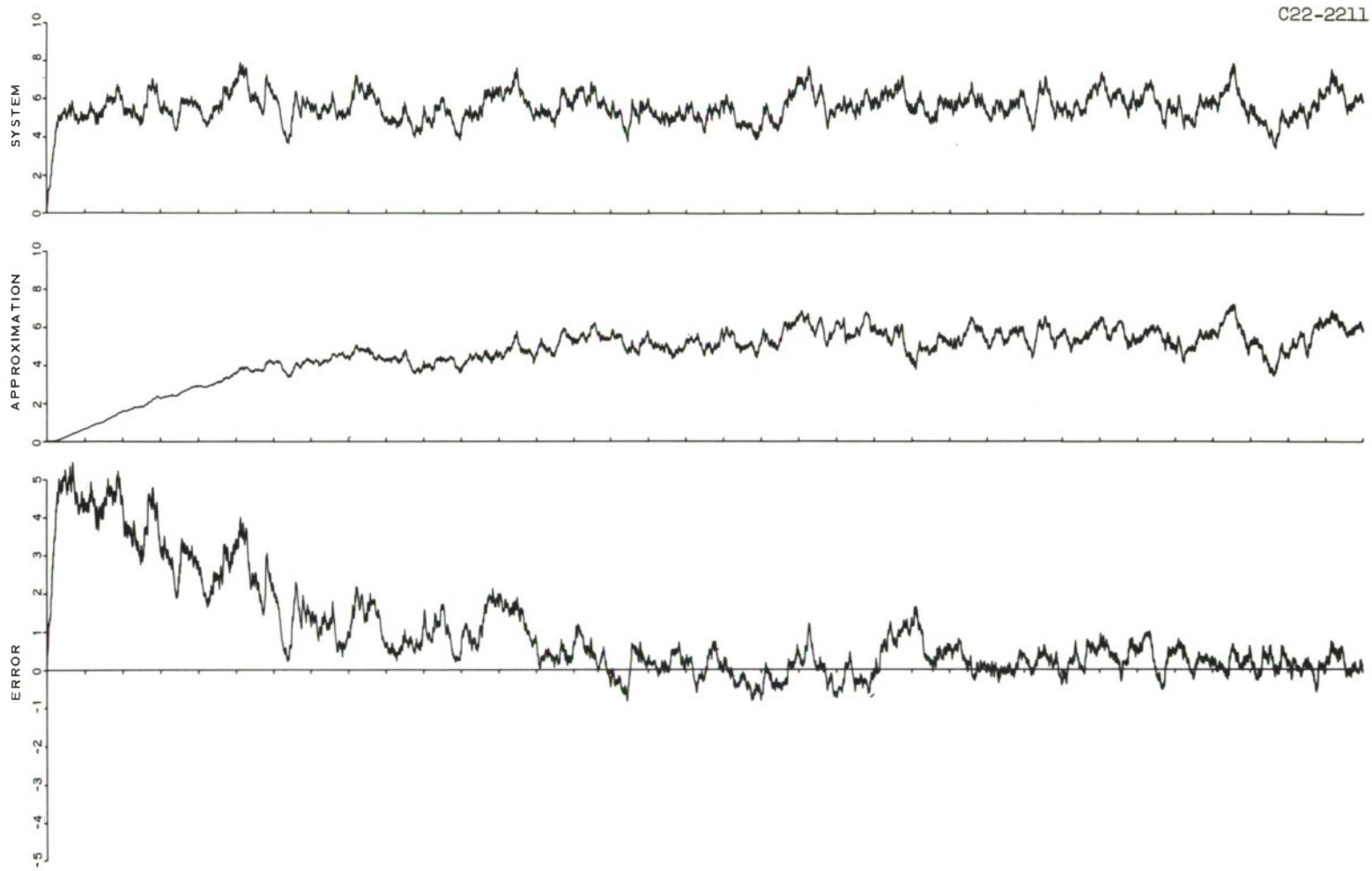



Fig. 24. The response of system f with linear approximation determined by the gradient technique.

The gradient technique also has considerable merit over the direct or modified techniques in the case of the full truncated Wiener model, i.e., approximation 3 of (5.1). The following limitation of the gradient technique which must be noted, however, is the following: As the number of degrees of freedom (i.e., the number of coefficients) of the model increases, so does the time required for convergence. Fig. 25 gives the results of the identification of system c employing the gradient technique by a model composed of six stages of a Laguerre network (28 terms) starting from only the dc. component. The relative mean square errors for this model and technique and the model and techniques of part A is:

$$\text{Direct} \rightarrow 4.295 \qquad \text{Modified} \rightarrow 0.192 \qquad \text{Gradient} \rightarrow 0.159 \tag{5.2}$$

Because of the structure of the identification program, it was also a simple matter to iterate the program using the previous iterations estimate of the coefficients as initial values. Moreover, the subsequent iterations can increase the complexity of the model. Figures 26 and 27 illustrate this technique for the program in the case of system f (The Half-Wave Linear Detector). For Fig. 26, the model consisted of all linear and quadratic terms (plus dc.) upon a four stage Laguerre network. Fig. 27 is the result of a second iteration of the program starting from the final values of Fig. 26 and with the addition of two more stages to the network with corresponding coefficient initially equal to zero.

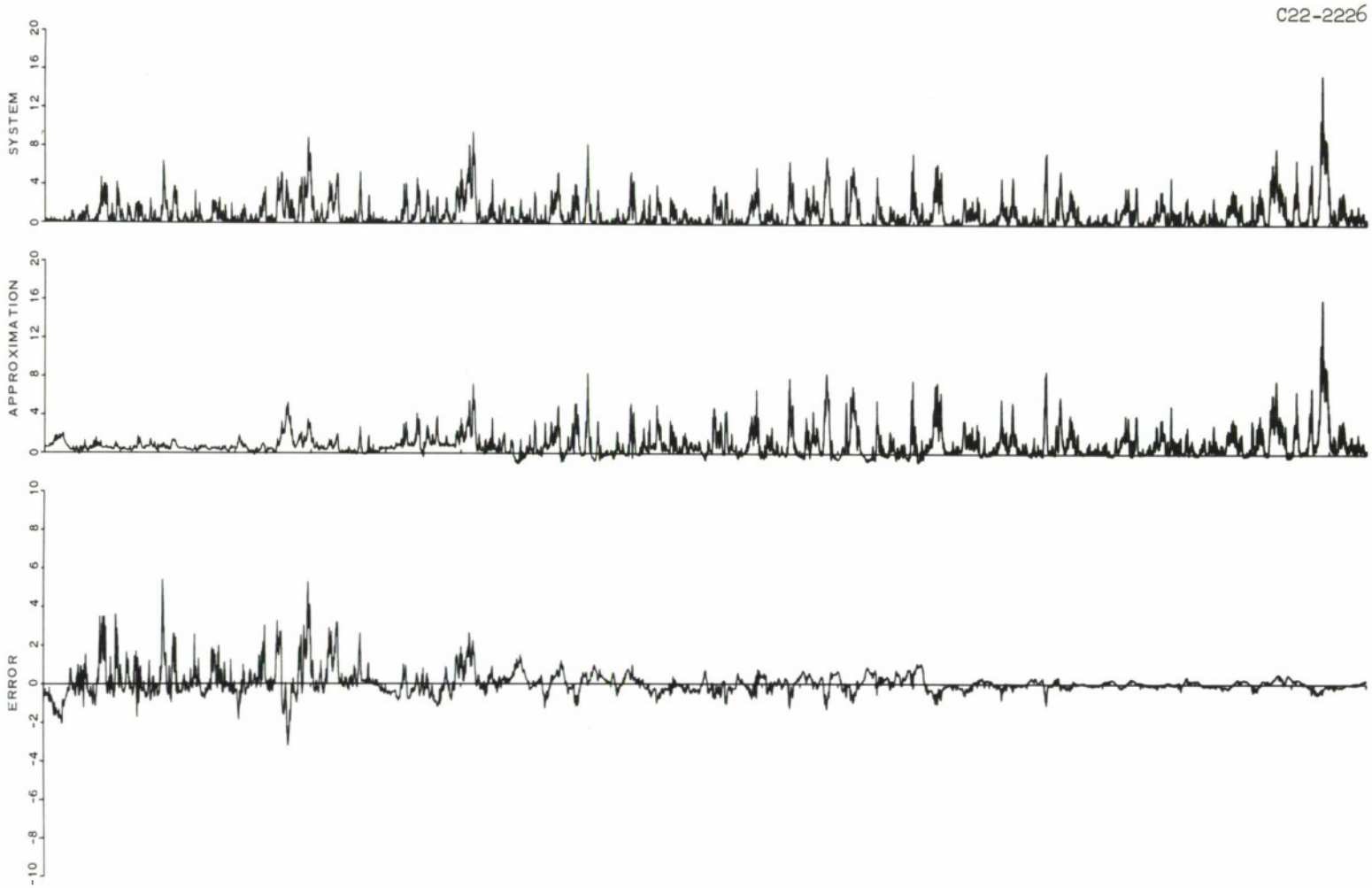



Fig. 25. The response of system c with 28 term approximation determined by the gradient technique.

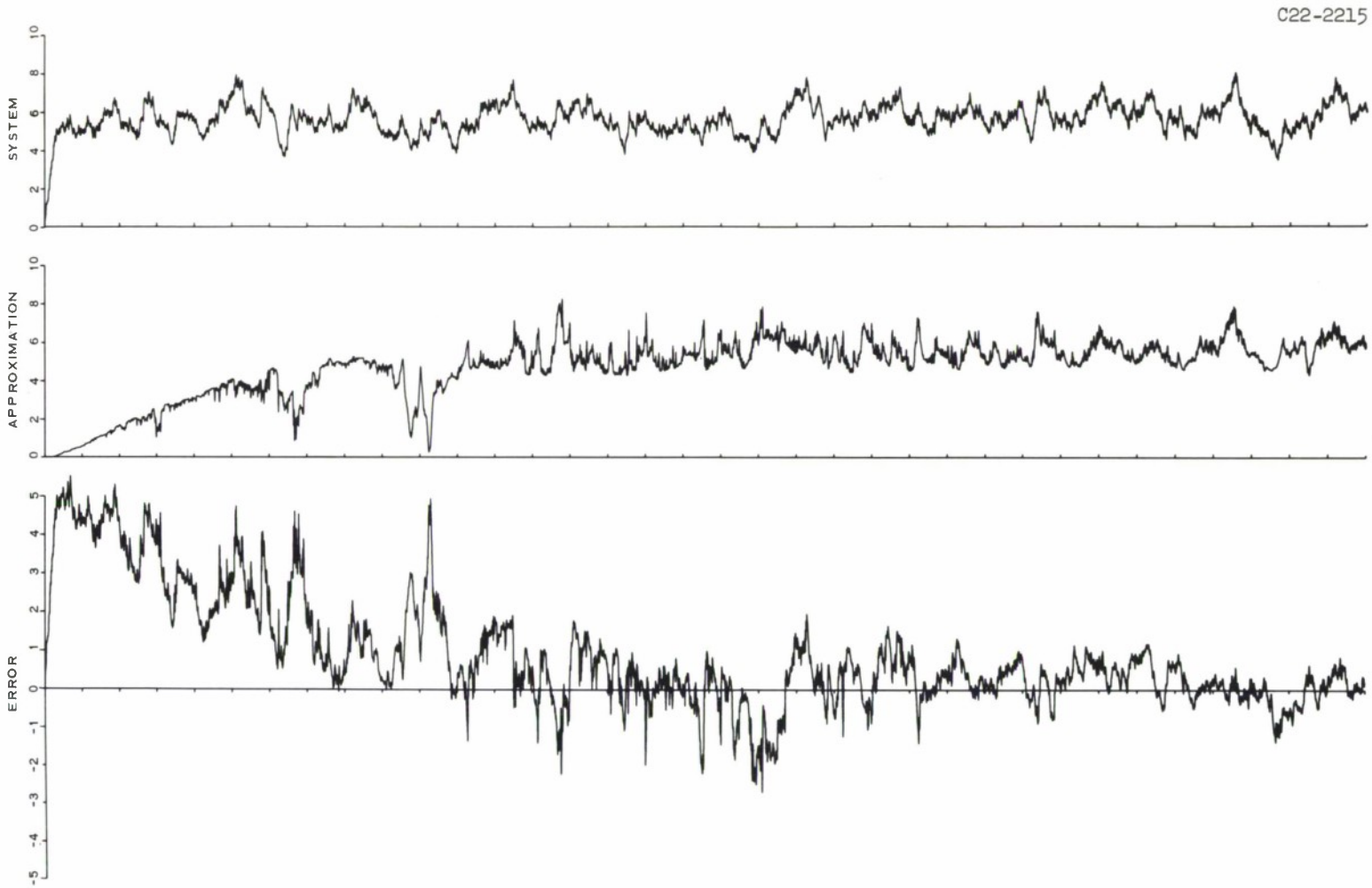



Fig. 26. The response of system f with 15 term approximation determined by the gradient technique.

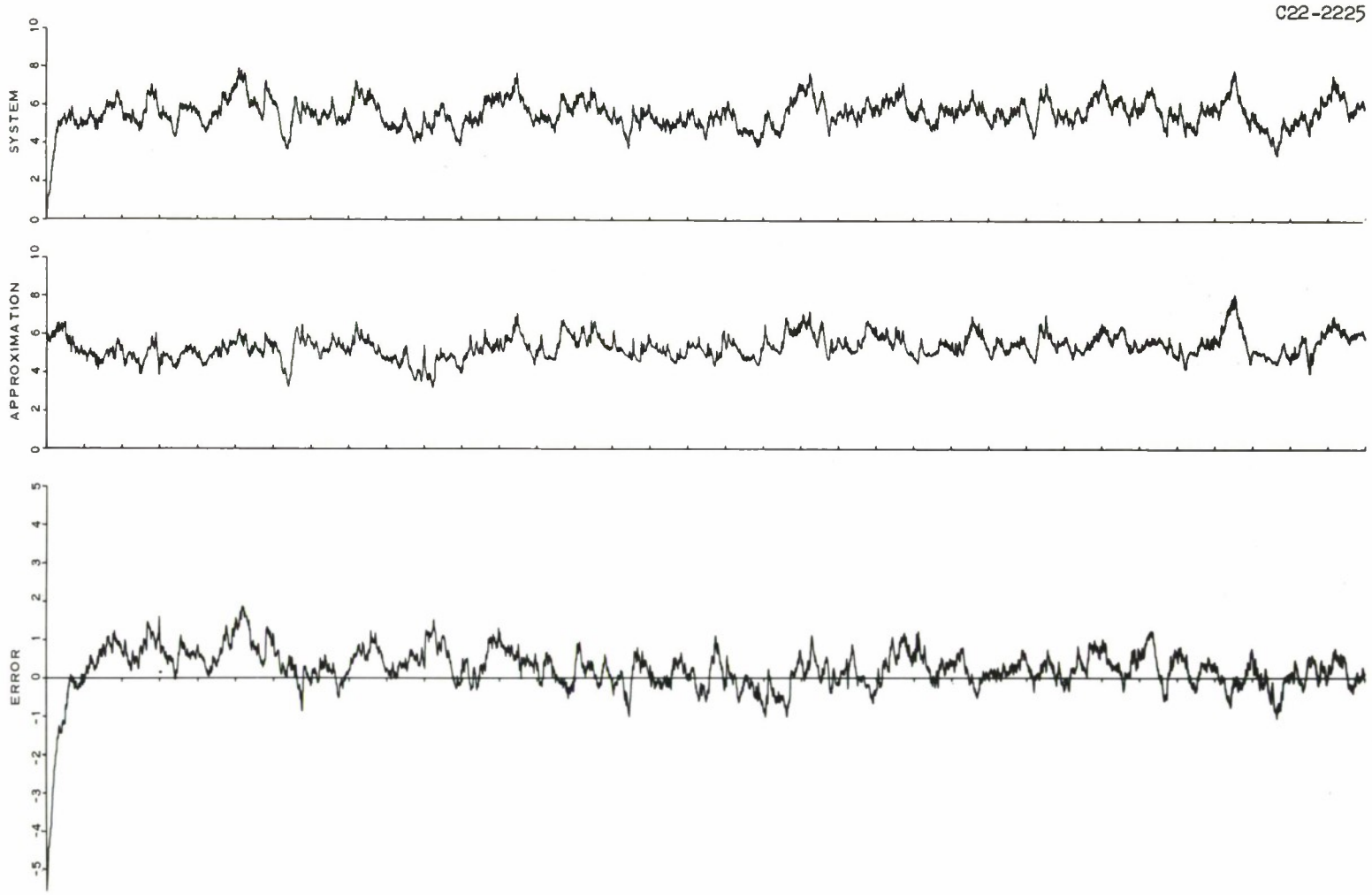



Fig. 27. The response of system f with 28 term approximation determined by a second iteration of the gradient technique.

## VI. SUMMARY AND CONCLUSIONS

The general system identification problem has been presented as a fitting of some mathematical model to a given physical system. The only model considered in this report is a nonlinear model originally proposed by Norbert Wiener in 1949. This so-called Wiener model and its truncated version, which is necessitated by practical implementation, have been described briefly. Then three procedures for the identification of any given system in terms of the Wiener model have been offered. Finally, the report has presented results from a digital computer simulation study (utilizing six somewhat arbitrary systems) which was designed to evaluate the various identification procedures as well as the model itself as a basis for system identification.

Some of the major conclusions which may be drawn from this study are the following:

1) The implementation of a truncated Wiener model is well within the capability of modern digital computers. For systems of modest frequency response, the corresponding Wiener model may well be run in real-time.

2) Of the three identification procedures considered, the gradient technique consistently yielded a better approximation (Wiener model) for the system under identification. This is primarily because the gradient technique does not depend upon the orthogonality of the constituent functionals of the model and hence mitigates the effect of possible non-Gaussian inputs and of the finite averaging times. Moreover, the gradient technique is also ideally suited for use in an adaptive situation because of its ability to use existing inputs and because of the simple recursive nature of the required algorithms.

3) For most of the systems considered for identification, the truncated

Wiener model has been somewhat inappropriate but then again, these arbitrary systems were purposely chosen to be difficult in order to evaluate the Wiener model as a basis for system identification. Other systems could have yielded more impressive results.

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13. ABSTRACT

This report presents briefly a nonlinear model originally proposed by the late Norbert Wiener for the characterization of general systems. Three procedures are then offered for the identification of any given system in terms of the Wiener model. Finally, this report presents the results of a digital computer simulation study (utilizing six somewhat arbitrary systems) which was designed to evaluate the various identification procedures as well as the model itself as a basis for system identification.

14. KEY WORDS

nonlinear systems
Wiener model