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RECIPROCAL AND NONRECIPROCAL MICROSTRIP PERIODIC NETWORKS

by

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ABSTRACT

This report presents results of an investigation of a microstrip filter which was designed for the study of nonreciprocal phase in S-band (~3 GHz). Data are presented showing scattering, absorption, and nonreciprocity. The device is a low-pass "comb" filter. When matched to less than 1.1/1 VSWR over a 13% band, its insertion loss is 0.8 - 1.2 db. Nonreciprocal differential phase is frequency-sensitive, reaching 30 degrees near the cutoff frequency of 3.5 GHz. An analysis is presented of the theory of periodic networks with sufficient generality to include the nonreciprocal and nonconservative cases. Comparison with the performance of a reciprocal filter shows good agreement. The agreement with non-reciprocal effects is qualitatively correct, although a full treatment of nonreciprocity must await a detailed field theory of propagation on microstrip with a magnetic substrate. The investigation shows that substantial amounts of nonreciprocal phase can be produced by means of a simple, compact, efficient microstrip structure with low loss and good match over an ample band, suitable for control in the "latching" (remanent) mode of operation.

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RECIPROCAL AND NONRECIPROCAL MICROSTRIP PERIODIC NETWORKS

1. Introduction.

This report presents a summary of the work performed under Subcontract No. 351 during the contract period September 15, 1966 -- June 15, 1967, on the investigation of propagation in nonreciprocal miniature microwave transmission structures.

As discussed in the Proposal of August 12, 1966, the objectives were:

i. design and construction of a miniature microstrip nonreciprocal filter consisting of a ferrite substrate to which a strip transmission-line periodic network is applied by metal deposition and photoetch techniques;

ii. microwave investigation of the match, loss, and dispersion of the filter, including nonreciprocity and its dependence on the magnetic state of the substrate;

iii. theoretical investigation of the transmission properties of nonreciprocal periodic networks in general, and comparison of the theory with the performance of the experimental model.

The experiments were performed in S-band. The filter to be described is of the low-pass "comb" type, composed of four sections ("teeth"), designed for operation in the band centered at 3 GHz, with cutoff at 3.5 GHz.

The ferrite substrate is magnetized by incorporating it as one leg of a closed magnetic circuit. The magnetization is directed transversely with respect to the direction of propagation, in the plane of the substrate, and may be pulsed to its remanent state or excited by a sinusoidal current at, e.g., 60 Hz. Using a swept S-band signal generator as source, phase and other scattering characteristics are observed by means of phase bridge and reflectometer circuits.

The filter was observed to produce nonreciprocal differential phase which increases monotonically with frequency, reaching 30 degrees (difference between phases of transmission for the two opposite states of remanent magnetization) close to cutoff. With the addition of simple end sections for matching to 50-ohm transmission line, the VSWR is less than 1.1/1 over the band from 2.76 to 3.14 GHz; in this band the insertion loss is 0.8 -- 1.2 db, and the differential phase is in the range from 3 to 9 degrees. Investigation of this filter is still in progress. We should like to perform the matching at other points in the passband; in particular, we anticipate that an ample band of low reflection can be created in regions where larger amounts of nonreciprocal phase are available. The rather steep frequency-dependence of nonreciprocal phase in this structure is not understood as yet.

The dispersive properties of the filter (input match and reciprocal transmission phase as functions of frequency) conform reasonably well to prediction, indicating that the correlation between mechanical layout and electrical performance of the network is good. Although no comprehensive theory of nonreciprocal effects in structures of this type exists as yet, the performance of this device indicates that substantial amounts of nonreciprocity can be produced in this way; that is, by means of a simple, compact, efficient microstrip structure, with low loss, good match over an ample band, which can be readily adapted for digital control in the "latching" (remanent) mode of operation.

2. Theory of Nonreciprocal Periodic Networks.

The following theory is formulated in a scattering-matrix representation; the results are, of course, related by a bilinear transformation to those of the impedance representation which is more familiar to many workers. It may be pointed out that, in the presence of nonreciprocity, the impedance formulation loses its main advantage, in that it no longer lends itself to interpretation in terms of lumped-element equivalent circuits composed of simple elements. On the other hand, the gyromagnetic interaction which must ultimately account for the nonreciprocity seems to be more naturally expressible in the scattering picture.

A. General Theory of Scattering in a Periodic Network.

We consider a two-port junction characterized by the scattering matrix S,

$$S = \begin{bmatrix} r & s' \\ s & r' \end{bmatrix} \qquad (1)$$

Our objective is to analyze propagation in a periodic network composed of identical elements (sections) in cascade, each characterized by S. For the present we assume that the network extends infinitely in both directions. The matrix (1) connects the vectors whose components are the incident wave amplitudes $\left[E_1^I,\ E_2^I\right]$ and the scattered wave amplitudes $\left[E_1^S,\ E_2^S\right]$, according to

$$\begin{bmatrix} E_1^S \\ E_2^S \end{bmatrix} = S \begin{bmatrix} E_1^I \\ E_2^I \end{bmatrix} \tag{2}$$

For the purpose of iteration, we transform to the R-matrix representation in which the vector $\left[E_1^I,\ E_1^S\right]$, whose components are the forward and backward (say, toward the left and toward the right) wave amplitudes at port 1, is connected with the corresponding vector of wave amplitudes $\left[E_2^S,\ E_2^I\right]$ at port 2. We have

$$\begin{bmatrix} E_2^I \\ E_2^S \end{bmatrix} = R \begin{bmatrix} E_1^S \\ E_1^I \end{bmatrix} \tag{3}$$

Solving the system (2) to put the relations in the form (3), we find for R,

$$R = \frac{1}{s'} \begin{bmatrix} -\Delta & r' \\ -r & 1 \end{bmatrix} \tag{4}$$

where Δ denotes the determinant of S:

$$\Delta = \det S = r'r - s's \tag{5}$$

In the periodic network, the R matrix acts as a transfer operator connecting the wave amplitudes at the j-th interelement reference plane with those at the (j + 1)-th, applying equally to each pair of reference planes in the entire range $-\infty < j < +\infty$. Hence it is appropriate to modify the notation in (3) to read

$$\begin{bmatrix} E_f^{j+1} \\ E_b^{j+1} \end{bmatrix} = R \begin{bmatrix} E_f^j \\ E_b^j \end{bmatrix} \tag{6}$$

where f and b refer to the forward and backward directions of propagation, respectively.

Now, in the terminology of the theory of group representations, we observe that in the infinite periodic network the boundary value problem of which the E's are solutions (that is, the wave equation together with boundary conditions) is invariant with respect to translation through any integral number of sections. Hence there must exist solutions which "transform according to the irreducible representations of the one-dimensional discrete translation group." The meaning of this statement is developed in treatises on applications of group theory[1]; for the present purpose we state the conclusion, as follows. There must exist wave amplitudes $[E_f^j, E_b^j]$ having the property that under the translation $j \rightarrow j+1$ the only effect is the multiplication of the two waves by the factor $e^{-\chi}$, in which χ is a constant (in general complex), called a characteristic exponent[2]. This consequence of the symmetry, when applied to the present case of a one-dimensional structure composed of discrete sections, is also known as Floquet's theorem[2]. In any case, the conclusion may be expressed as

$$\begin{bmatrix} E_f^{j+1} \\ \\ E_b^{j+1} \end{bmatrix} = e^{-\chi} \begin{bmatrix} 1 & \\ \\ & 1 \end{bmatrix} \begin{bmatrix} E_f^j \\ \\ E_b^j \end{bmatrix} \tag{7}$$

The problem becomes that of determining χ and the wave amplitude $\left[E_f^j,\ E_b^j\right]$ such that both (6) and (7) are satisfied.

We shall see that under ordinary circumstances, when the sections are conservative (reactive), χ is either pure real or pure imaginary. The domains of imaginary χ are the passbands of the network, and the value of χ is the iterative phase in that case. The corresponding pair of wave amplitudes characterizing the normal mode of propagation may be expressed alternatively through an iterative impedance Z according to

$$\frac{Z}{Z_a} = \frac{1 + R}{1 - R} \tag{8}$$

where $R = E_b^j / E_f^j$ and Z_a is the characteristic impedance of the transmission line at the planes at which the E's are defined. For the normal modes, R is, of course, independent of j (eq. 7). The domains of real χ are the stopbands, and the value of χ is the attenuation per section in that case.

To proceed, we eliminate $\left[E_f^{j+1},\ E_b^{j+1}\right]$ between (6) and (7), obtaining the homogeneous system

$$\begin{bmatrix} -\Delta - s'e^{-\chi} & r' \\ \\ -r & 1 - s'e^{-\chi} \end{bmatrix} \begin{bmatrix} E_f^j \\ \\ E_b^j \end{bmatrix} = 0 \tag{9}$$

Equation (9) possesses a nontrivial solution if (and only if) the determinant of coefficients is zero. This condition yields the characteristic equation

$$s'^2 e^{-2\chi} + s' e^{-\chi} (\Delta - 1) + (r'r - \Delta) = 0 \qquad (10)$$

Solving,

$$e^{-\chi} = \frac{1}{2s'} \left[(1 - \Delta) \pm \sqrt{(1 + \Delta)^2 - 4r'r} \right] \qquad (11)$$

Equation (11) specifies the characteristic values of χ corresponding to the normal modes of propagation. With χ thus determined in terms of the scattering coefficients of the individual sections, the ratio $E_b^j/E_f^j = R$ can be evaluated from (9):

$$R = \frac{\Delta + s'e^{-\chi}}{r'} = \frac{r}{1 - s'e^{-\chi}} \qquad (12)$$

This result is completely general, in that no assumptions have been imposed regarding either reciprocity or conservation of energy. To illustrate its meaning, let us apply it to the special case in which the sections are both conservative and reciprocal. A simple and familiar filter-theory result emerges.

For a lossless network, the scattering matrix S is unitary:

$$S^{\dagger} S = I \qquad (13)$$

where $\dagger$ means Hermitean conjugate and I is the unit matrix. When (13) is applied to the matrix (1), we obtain the following constraints on its components:

$$|r|^2 + |s|^2 = |r'|^2 + |s'|^2 = 1 \tag{14}$$

$$|r'| = |r| \quad , \quad |s'| = |s| \tag{15}$$

$$\arg r'r - \arg s's = \pi \tag{16}$$

from which it follows that

$$|\Delta| = |\det S| = |r'r - s's| = 1 \tag{17}$$

For a reciprocal network, S is symmetric. We write s and s' as

$$s = s' = |s| \, e^{-i\varphi} \tag{18}$$

Then, with the conditions due to losslessness (14--17) we have

$$\Delta = -e^{-2i\varphi} \tag{19}$$

The solution (11) of the characteristic equation reduces to

$$e^{-\chi} = \frac{1}{|s|} \cos \varphi \pm \sqrt{\frac{1}{|s|^2} \cos^2 \varphi - 1} \tag{20}$$

or, more simply

$$\cosh \chi = \frac{1}{|s|} \cos \varphi \tag{21}$$

Since the right side of (21) is real, χ must be either pure imaginary or pure real, according as the magnitude of the right side is or is not less than one. As a simple illustration, suppose that the transmission phase φ increases monotonically in some simple way with increasing frequency and that $|s|$ is frequency-insensitive. Let

$$\chi = u + i v \tag{22}$$

Then the passbands occur in the frequency ranges in which $|\cos \varphi| < |s|$, and the iterative phase is

$$v = \arccos\left(\frac{1}{|s|}\cos\varphi\right) \tag{23}$$

The stopbands occur where $|\cos \varphi| \geq |s|$, and the attenuation per section is

$$u = \cosh^{-1}\left(\frac{1}{|s|}|\cos\varphi|\right) \tag{24}$$

B. The Reciprocal Comb Filter -- Normal Modes.

Continuing with the reciprocal, conservative case, we calculate the overall scattering coefficients for a comb filter. The treatment in Secs. 2B and 2C is a reformulation in the scattering representation of the theory of conventional periodic networks[3]. It is included here for reference and by way of background for the discussion of the nonreciprocal case to be presented in Sec. 2D.

According to the prescription (23), the iterative phase can be determined in terms of the phase φ and magnitude $|s|$ of the transmission coefficient of the individual sections. To simplify the calculation, we make the customary assumption that the transverse dimensions of the sections of transmission line used in the comb structure are negligible at the frequencies of interest.

The filter structure under consideration is shown in Fig. 1; Fig. 2 shows a single section, indicating the dimensions used in the calculation. Fig. 3 is a copy of the photoetch positive master, showing the actual size of the structure. To determine the transmission coefficient s it is convenient to begin with a lumped-element analogy. Consider a stub ("tooth") of characteristic impedance Z_b, length ℓ_b, terminated in an open circuit: the impedance at the root of the tooth is

$$Z_p = -i\, Z_b \cot(\beta \ell)_b \tag{25}$$

where β_b is the propagation constant for transmission line of characteristic impedance Z_b. (On microstrip, the propagation constants of all modes, including the fundamental mode, are functions of the characteristic impedance and the dielectric constant of the substrate, due to the partially-dielectric-filled construction of the line.) For brevity, we denote $\beta_b \ell_b$ by $(\beta \ell)_b$. This tooth is in shunt with the transmission line, and may be regarded as the shunt element of a T network composed of shunt Z_p between two equal series Z_s. For such a network, the impedance matrix Z is

$$Z = \begin{bmatrix} Z_p + Z_s & Z_p \\ Z_p & Z_p + Z_s \end{bmatrix} \tag{26}$$

In general, the scattering and impedance matrices are related by

$$S = (z + I)^{-1}(z - I) \tag{27}$$

where $z = (1/Z_a)Z$, Z_a being the characteristic impedance of the transmission line in which the T network is inserted. Denoting Z_p/Z_a and Z_s/Z_a by z_p and z_s, respectively, and applying (27) to (26), we have

$$S = \frac{1}{(2z_p + z_s + 1)(z_s + 1)} \begin{bmatrix} z_s(2z_p + z_s)-1 & 2z_p \\ 2z_p & z_s(2z_p + z_s)-1 \end{bmatrix} \qquad (28)$$

For the case of the shunt tooth we have $z_s = 0$, so

$$S = \frac{1}{2z_p + 1} \begin{bmatrix} -1 & 2z_p \\ 2z_p & -1 \end{bmatrix} \qquad (29)$$

The complete filter section is composed of the shunt tooth placed between two equal lengths ℓ_a of transmission line, of characteristic impedance Z_a. Transforming the reference planes of S to the ends of these elements, we have finally

$$S = \frac{e^{-2i(\beta\ell)_a}}{2z_p + 1} \begin{bmatrix} -1 & 2z_p \\ 2z_p & -1 \end{bmatrix} \qquad (30)$$

from which we may read the transmission coefficient s: using (25),

$$s = e^{-2i(\beta\ell)_a} \frac{2i\zeta\cot(\beta\ell)_b}{2i\zeta\cot(\beta\ell)_b - 1} \qquad (31)$$

where $\zeta = Z_b/Z_a$.

To solve (21) for χ, we require the combination (using eq. 31)

$$\frac{1}{|s|}\cos\varphi = \frac{1}{|s|^2}\,\mathrm{Re}(s) = \cos 2(\beta\ell)_a - \frac{1}{2\zeta}\tan(\beta\ell)_b \sin 2(\beta\ell)_a \qquad (32)$$

Thus,

$$\cosh\chi = \cos 2(\beta\ell)_a - \frac{1}{2\zeta}\tan(\beta\ell)_b \sin 2(\beta\ell)_a \qquad (33)$$

A computer program was composed[4] for the evaluation of χ according to (33) as a function of frequency (as represented by the electrical length $(\beta l)_b$) for various values of ζ and $2(\beta l)_a/(\beta l)_b$. Fig. 4 shows the results for the case $\zeta = 1/3$, $2(\beta l)_a/(\beta l)_b = 0.1$. The iterative phase v (eq. 23) is plotted in the passbands, and the attenuation per section u (eq. 24) in the stopbands.

The solutions of the characteristic equation occur in pairs χ_+ and χ_-, corresponding to net propagation in the forward and backward directions. In the reciprocal, conservative case they are the negatives of each other $\pm\chi$ (note that both satisfy eq. 21). In general they are not; the discrepancy represents the presence of nonreciprocity in gain, loss, or phase.

C. The Reciprocal Comb Filter -- Finite Structure

We have the iterative phases χ_+ and χ_- for the normal modes of propagation in the reciprocal, conservative comb filter, as given by (33). When such a filter, composed of n sections, is inserted into a transmission line of characteristic impedance Z_c, in general both modes are excited, their relative phases and amplitudes being determined by the boundary conditions at the terminal planes $j = 0$ and $j = n$. These relations in turn determine the overall scattering coefficients of the structure. Let S_n denote the scattering matrix representing the obstacle at $j = n$:

$$S_n = \begin{pmatrix} r_n & s_n' \\ s_n & r_n' \end{pmatrix} \qquad (34)$$

Assuming the filter is connected to a matched termination, the reflection on the source side of the obstacle is r_n. This condition must be fulfilled by a superposition of the two modes. Let

$$R_+ = \frac{E_{b+}^j}{E_{f+}^j} , \qquad R_- = \frac{E_{b-}^j}{E_{f-}^j} \tag{35}$$

where + and - refer to the modes associated with χ_+ and χ_-, respectively. The reflection on the source side of the obstacle at the plane $j = n$ is given by

$$\begin{aligned} R_n &= \frac{E_{b+}^n + E_{b-}^n}{E_{f+}^n + E_{f-}^n} \\ &= \frac{R_+ + R_-(E_{f-}^n/E_{f+}^n)}{1 + (E_{f-}^n/E_{f+}^n)} \end{aligned} \tag{36}$$

Solving (36) for the ratio of amplitudes,

$$\frac{E_{f-}^n}{E_{f+}^n} = \frac{R_n - R_+}{R_- - R_n} \tag{37}$$

The presence of the obstacle at the output plane $j = n$ furnishes the requirement

$$R_n = r_n \tag{38}$$

With the relative amplitudes thus determined according to (37), we now seek the resultant reflection coefficient at the input plane $j = 0$. Now, in the passbands th^r iterative phases of the normal modes are v_+ and v_-.

Thus,

$$E_{f+}^{0} = e^{i\,nv_+}E_{f+}^{n}\ , \qquad E_{f-}^{0} = e^{i\,nv_-}E_{f-}^{n} \tag{39}$$

Hence

$$\frac{E_{f-}^{0}}{E_{f+}^{0}} = e^{i\,n(v_- - v_+)}\frac{E_{f-}^{n}}{E_{f+}^{n}}$$

$$= e^{i\,n(v_- - v_+)}\frac{R_n - R_+}{R_- - R_n} \tag{40}$$

The reflection coefficient R_0 at the plane $j = 0$ is

$$R_0 = \frac{E_{b+}^{0} + E_{b-}^{0}}{E_{f+}^{0} + E_{f-}^{0}} \tag{41}$$

$$\frac{R_+ + R_-(E_{f-}^{0}/E_{f+}^{0})}{1 + (E_{f-}^{0}/E_{f+}^{0})} \tag{42}$$

With (40), this is

$$R_0 = \frac{R_+(R_- - R_n) + R_-(R_n - R_+)e^{i\,n(v_- - v_+)}}{(R_- - R_n) + (R_n - R_+)e^{i\,n(v_- - v_+)}} \tag{43}$$

To take account of the obstacle at $j = 0$, we introduce the scattering maxtrix S_0:

$$S_0 = \begin{bmatrix} r_0 & s_0' \\ s_0 & r_0' \end{bmatrix} \tag{44}$$

Since the incident and scattered amplitudes on the filter side of this obstacle are $E_{b+}^0 + E_{b-}^0$ and $E_{f+}^0 + E_{f-}^0$, respectively, we have

$$\begin{bmatrix} E_R \\ E_{f+}^0 + E_{f-}^0 \end{bmatrix} = \begin{bmatrix} r_0 & s_0' \\ s_0 & r_0' \end{bmatrix} \begin{bmatrix} 1 \\ E_{b+}^0 + E_{b-}^0 \end{bmatrix} \tag{45}$$

where the 1 signifies the incident signal and E_R is the overall reflection at the source side of the obstacle. To solve (45) for E_R, we make use of (41) and also the condition on S_0 which holds for a conservative junction,

$$\det S_0 = r_0' r_0 - s_0' s_0 = -1$$

This follows from (17) provided the reference planes of the obstacle are suitably chosen. Then

$$E_R = \frac{r_0 + R_0}{1 - r_0' R_0} \tag{46}$$

with R_0 given by (43). Solving (45) for $E_{f+}^0 + E_{f-}^0$,

$$E_{f+}^0 + E_{f-}^0 = \frac{s_0}{1 - r_0' R_0} \tag{47}$$

Then with (40) we obtain

$$E_{f+}^0 = \frac{s_0}{1 - r_0' R_0} \; \frac{R_- - R_n}{(R_- - R_n) + e^{in(v_- - v_+)}(R_n - R_+)} \tag{48}$$

Transforming to the plane $j = n$,

$$E_{f+}^n = \frac{s_0}{1 - r_0' R_0} \; \frac{(R_- - R_n)e^{-inv_+}}{(R_- - R_n) + e^{in(v_- - v_+)}(R_n - R_+)} \tag{49}$$

Similarly, for the - mode we have from (37)

$$E_{f-}^{n} = \frac{s_0}{1 - r_0'R_0} \frac{(R_n - R_+)e^{-inv_+}}{(R_- - R_n) + e^{in(v_- - v_+)}(R_n - R_+)} \tag{50}$$

Thus, the amplitude incident on the obstacle at the plane j = n is

$$E_{f+}^{n} + E_{f-}^{n} = \frac{s_0}{1 - r_0'R_0} \frac{(R_- - R_+)e^{-inv_+}}{(R_- - R_n) + e^{in(v_- - v_+)}(R_n - R_+)} \tag{51}$$

Finally, the overall transmission coefficient E_T of the filter is s_n times this:

$$E_T = \hat{s}_n(E_{f+}^{n} + E_{f-}^{n})$$

$$= \frac{s_0 s_N}{1 - r_0'R_0} \frac{(R_- - R_+)e^{-inv_+}}{(R_- - R_n) + e^{in(v_- - v_+)}(R_n - R_+)} \tag{52}$$

Equations (46) and (52), with (43), specify the reflection and transmission of the filter, in terms of the normal mode phases v_+, v_- and iterative reflection coefficients R_+, R_-, together with the scattering matrices S_0 and S_n of the obstacles at the ends of the structure.

For use in the subsequent discussion of nonreciprocity we have retained the generality in the distinction between χ_+ and χ_-. In the passband of a reciprocal, conservative filter we have the simple relation $\chi_\pm = \pm v$; it follows from this (see eq. 12) that $R_+ = 1/R_- = R$, so (43) and (50) reduce somewhat, to

$$R_0 = \frac{R_n(Re^{inv} - 1/Re^{inv}) - 2i\sin nv}{(Re^{-inv} - 1/Re^{-inv}) + 2iR_n \sin nv} \tag{53}$$

$$E_T = \frac{s_0 s_n}{1 - r_0' R_0} \; \frac{R - 1/R}{(Re^{-inv} - 1/Re^{-inv}) + 2iR_n \sin nv} \tag{54}$$

The form of E_R remains the same as in (46):

$$E_R = \frac{r_0 + R_0}{1 - r_0' R_0}$$

In the case of the filter shown in Fig. 1, the obstacles at $j = 0$ and $j = n$ are the steps from Z_c to Z_a and back again. The scattering coefficients for these steps are

$$r_n = r_0' = -r_0 = \frac{\rho_0 - 1}{\rho_0 + 1} \tag{55}$$

and

$$s_n = \rho_0 s_0 = \frac{2\rho_0}{\rho_0 + 1} \tag{56}$$

where $\rho_0 = Z_c/Z_a$. This problem has been solved as part of the computer program[4], for the case

$$Z_a = 75 \text{ ohms}$$
$$Z_b = 25 \text{ ohms}$$
$$Z_c = 50 \text{ ohms}$$
$$2(\beta l)_a/(\beta l)_b = 0.25$$

The magnitude $|E_T|$ of the transmission coefficient is shown in Fig. 5. For comparison, Fig. 6 shows a recorder tracing of the observed transmission. The value of $2(\beta l)_a/(\beta l)_b$ is uncertain in the experimental filter, as indicated in Fig. 2, due to fringing of the radiation at the closely-spaced joints of the structure. The relative positions of the transmission maxima in the case computed in Fig. 5 with $2(\beta l)_a/(\beta l)_b = 0.25$ agree reasonably well with those of the experimental model.

D. The Reciprocal Comb Filter -- Matching.

A familiar problem in filter theory is that of matching to within given specifications over a specified band. Conditions for match are implied in the formulation of Sec. 2C; specifically, in the characterization by S_n and S_0, equations (34) and (44), of the obstacles at the planes $j = n$ and $j = 0$. In the calculation of Sec. 2C we used the scattering coefficients (55) and (56) appropriate for a simple step to and from transmission line of standard characteristic impedance Z_c. Substitution of stubs or other dispersive end sections in place of these alters the overall reflection, and may be made so as to widen the matched regions and place them at a desired position in the passband.

To illustrate this procedure in the case of the filter of Fig. 3, matching stubs were incorporated[5] so as to create a match over a band centered at 2.95 GHz. The complete structure is shown in Fig. 7, and its observed performance is shown in the recorder tracings, Figs. 8 and 9. The reflection, Fig. 8, shows that the VSWR is less than 1.1/1 from 2.76 to 3.14 GHz (13% band) and less than 1.3/1 from 2.65 to 3.17 GHz (18% band). The insertion loss, Fig. 9, is in the range 0.8--1.2 db over the well-matched region.

E. Nonreciprocity.

We now inquire how the results of the scattering theory are affected if we relax the requirement of reciprocity while retaining that of losslessness. The appropriate modification in the scattering matrix S of the individual sections (equation 1) is that it is no longer symmetric, but still unitary (equations 13--17). Note that the transmission coefficients s and s' may differ in phase only (equation 15). Let

$$s = |s| \; e^{-i\varphi_+} \; , \qquad s' = |s| \; e^{-i\varphi_-} \tag{57}$$

The equality of $|s|$ and $|s'|$ is the content of the so-called "reactive isolator theorem;" namely, that no conservative (reactive) two-port junction can transmit (or reflect) with different amplitudes in the two directions -- in particular, a "reactive isolator" for which $r = 0$, $s' = 0$ is not physically realizable.

If we let

$$\frac{1}{2}(\varphi_+ + \varphi_-) = \varphi \; , \qquad \frac{1}{2}(\varphi_+ - \varphi_-) = \psi \tag{58}$$

then the solution (11) of the characteristic equation (10) becomes

$$e^{-\chi} = e^{-i\psi}\left[\frac{1}{|s|}\cos\varphi \pm \sqrt{\frac{1}{|s|^2}\cos^2\varphi - 1}\right] \tag{59}$$

or

$$\cosh(\chi - i\psi) = \frac{1}{|s|}\cos\varphi \tag{60}$$

We note that if χ_+ denotes one of the solutions of (60), then

χ_- given by

$$\chi_- - i\psi = -(\chi_+ - i\psi) \qquad (61)$$

is another; hence,

$$\chi_+ - \chi_- = 2i\psi = i(\varphi_+ - \varphi_-) \qquad (62)$$

Thus the differential phase for n sections is just n times the differential phase per section. This holds irrespective of mismatches or any other dispersive effects; in fact, it holds in the stopbands as well as in the passbands. It is noteworthy that in the presence of nonreciprocity χ can never be pure real -- since by (60) we have that $\chi - i\psi$ is real. That is, propagation persists into the stopbands. We note also that in the stopbands where the real part u of $\chi = u + i\,v$ is different from zero (60) gives $u_+ = u_-$; that is, the attentuation is the same in both directions. This is simply a manifestation of the "reactive isolator theorem" for periodic networks.

F. Data on Nonreciprocal Phase.

An embodiment of the comb filter was constructed using a substrate of thickness 0.040 in., made of the polycrystalline yttrium-iron garnet, saturation magnetization 680 gauss. Other materials, in the range 400--600 gauss, were also used. Magnetization in the plane of the substrate, transverse to the direction of propagation (i.e., parallel to the teeth) was accomplished by attaching a U-shaped nickel-ferrite yoke, 1.5 in. long, suitably wound with magnet wire. The yoke was placed in contact with the underside (ground-plane side) of the substrate. This arrangement is preferable to placing the yoke on the upper surface, in that it avoids spurious interaction with the radiation; examination of the magnetization curve showed that the remanent magnetization is not appreciably affected by the "air" gap due to the presence of the ground plane. Remanent magnetization is approximately $4\pi M_r = 300$ gauss.

Differential phase (the quantity $2n\psi$ in the notation of Sec. 2E, equation 58) was measured by pulsing the magnetic circuit successively to its remanent states in the two directions and observing the phase change by means of a phase bridge. The data for this model are shown in Fig. 10. Differential phase is strongly frequency-dependent, reversing sign at 2.5 GHz and rising to about 20^o at the cutoff frequency of the filter. It continues to rise without interruption in the stopband, in agreement with the theory as discussed in Sec. 2E. The negative differential phase occurring at the low end of the band is not fully understood, but appears to originate in small imperfections in the structure.

3. Conclusion.

The investigation clearly demonstrates the potential value of microstrip periodic structures as a means for performing the function of nonreciprocal, digital, phase control, as well as the more conventional functions of reciprocal filtering. It has been verified that microstrip network design presents no special difficulties in the range of structural arrangements treated in this program. (Certain circuit elements, such as those involving tightly coupled pairs of strips, appear to present special problems due to the composite dielectric-air nature of microstrip; these have not been considered in the present program.) The next substantial objective in the general study of microstrip transmission on magnetic substrates will be that of placing nonreciprocal effects on a firm theoretical footing. In particular, it should be noted that guidance is not yet available for the determination of circuit arrangements which optimize nonreciprocity while fulfilling other device requirements (match, bandwidth, loss, peak and average power capability). Programs such as the present one, together with investigations by other groups active in this field[6] will extend our experience in this area, but theoretical support will also be required in the form of analytical and computational investigations of propagation in the heavily-loaded inhomogeneous, magnetically anisotropic, microstrip transmission-line structure. Application of network synthesis methods, based on scattering analysis such as the one presented in this report or equivalent treatment of nonreciprocal networks, will also be needed.

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For motivation and continuing interest in this work, I am grateful to D. H. Temme and C. Blake of Lincoln Laboratory. I am also indebted to the following persons for their assistance. Microstrip circuits were prepared by T. Cronin and H. Ehlers. Experimental measurements were performed by E. R. Straka and G. Schlueter. The computer program for calculation of filter characteristics was composed by D. N. Klauke and adapted for the IBM 7094 by T. Bryant.

FOOTNOTES

1. Morton Hamermesh: Group Theory and its Applications to Physical Problems; Addision-Wesley Publishing Co., Reading, Mass., 1962.

2. Norman W. McLachlan: Theory and Application of Mathieu Functions; Oxford, Clarendon Press, 1947. E. L. Ince: Ordinary Differential Equations; 4th ed., Dover Publications, 1953.

3. Ernst A. Guillemin: Communication Networks, Vol. II; John Wiley & Sons, Inc., New York, 1935.

4. The program for evaluation of χ and determination of the overall scattering coefficients of the reciprocal comb filter was prepared by Douglas N. Klauke, Worcester Polytechnic Institute, for operation on the IBM 1620, and was adapted for operation on the IBM 7094 by Thomas Bryant, University of Maine. The Fortran list of the program is presented in Fig. 11.

5. The experimental matching procedure was performed by G. Schlueter, Lincoln Laboratory Summer Staff.

6. In particular, see H. Hair et al., Syracuse University Research Corporation reports under Contract AF 33(615)-3332, 1966.67.

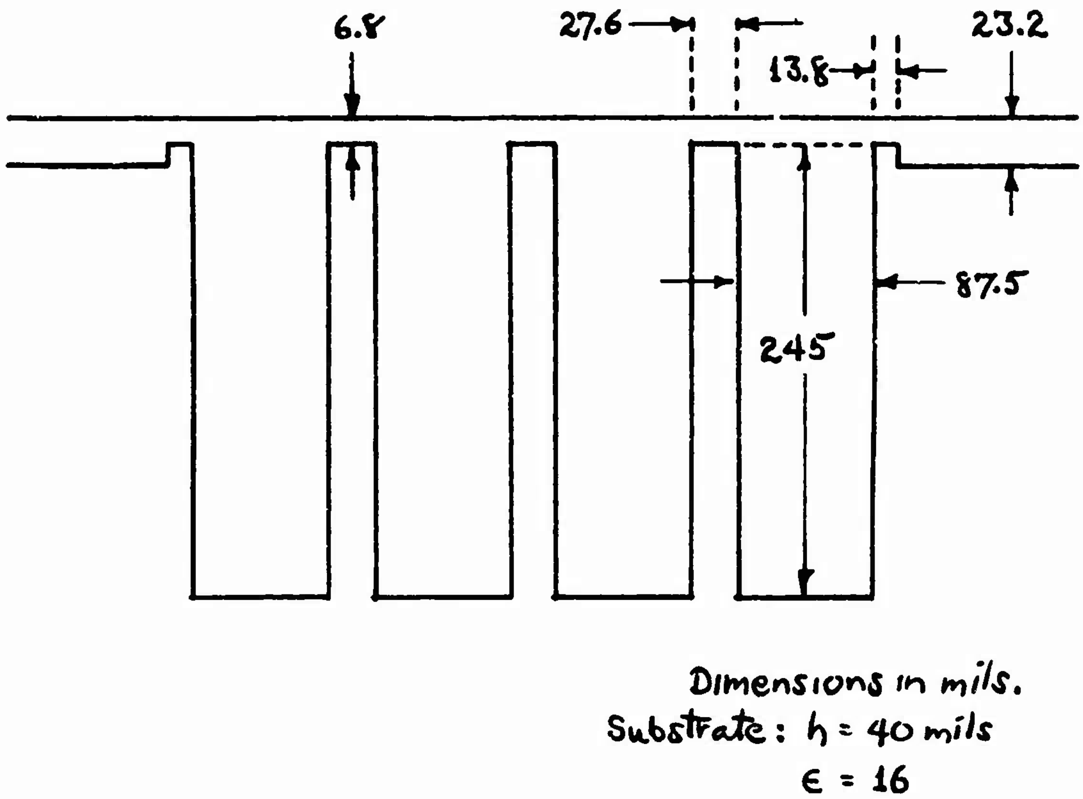

Figure 1. Four-Section Microstrip Comb Filter.

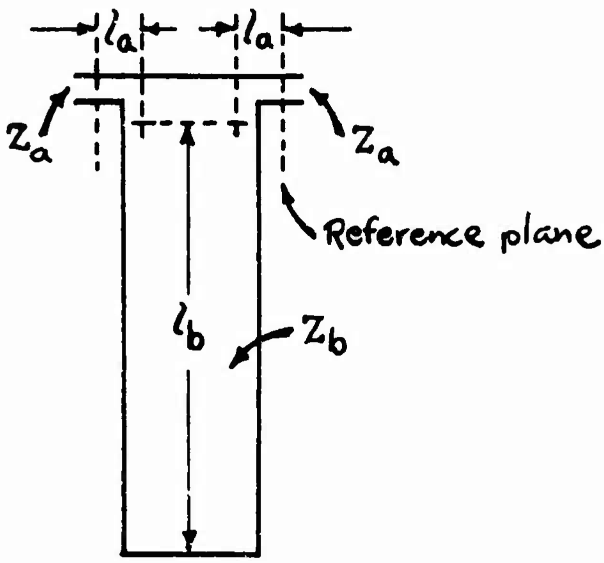

Figure 2. Single Section of the Comb Filter.

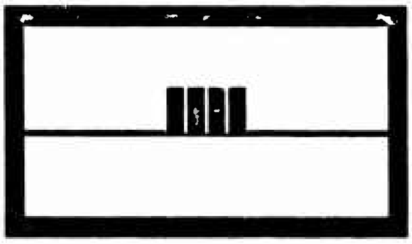

Figure 3. Microstrip Comb Filter -- Photo-Etch Positive Master, Actual Size.

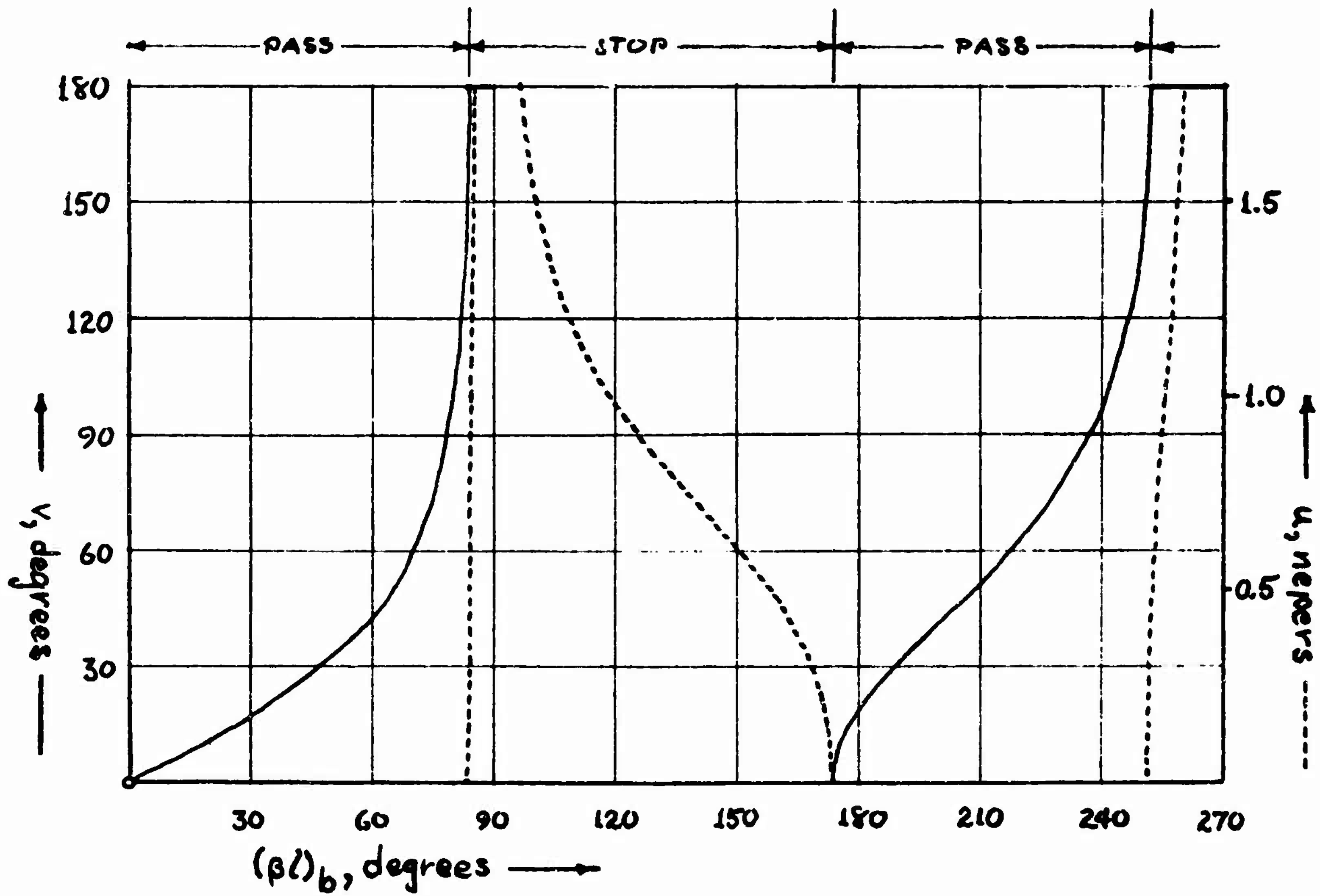

Figure 4. Characteristic Exponent $\chi = u + iv$ for the Normal Modes of the Comb Filter; $\varsigma = 1/3$, $2(\beta l)_a/(\beta l)_b = 0.1$.

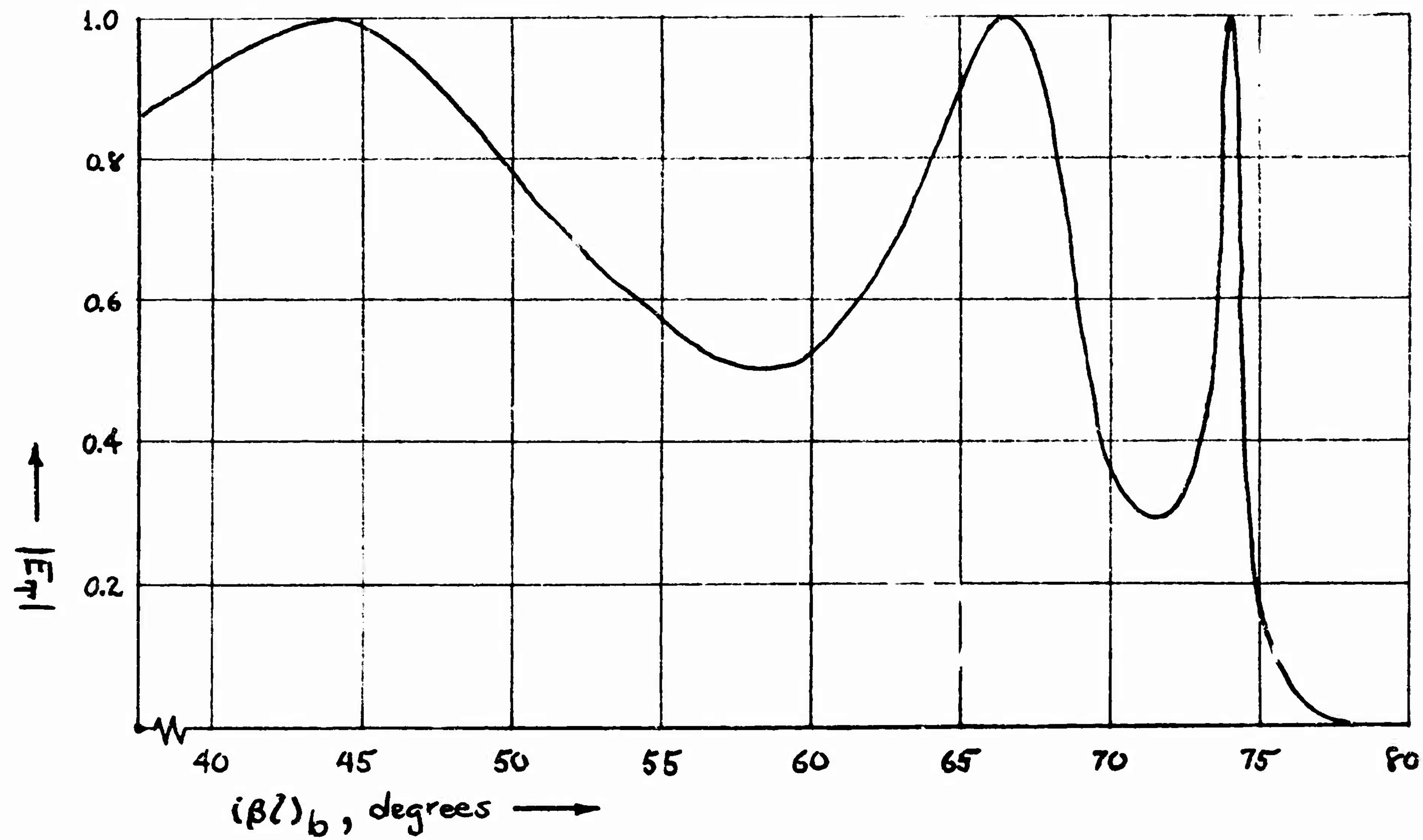

Figure 5. Amplitude of Transmission $|E_T|$ of a Four-Section Comb Filter; $Z_a = 75\Omega$, $Z_b = 25\Omega$, $Z_o = 50\Omega$, $\zeta = 1/3$, $2(\beta l)_a/(\beta l)_b = 0.25$.

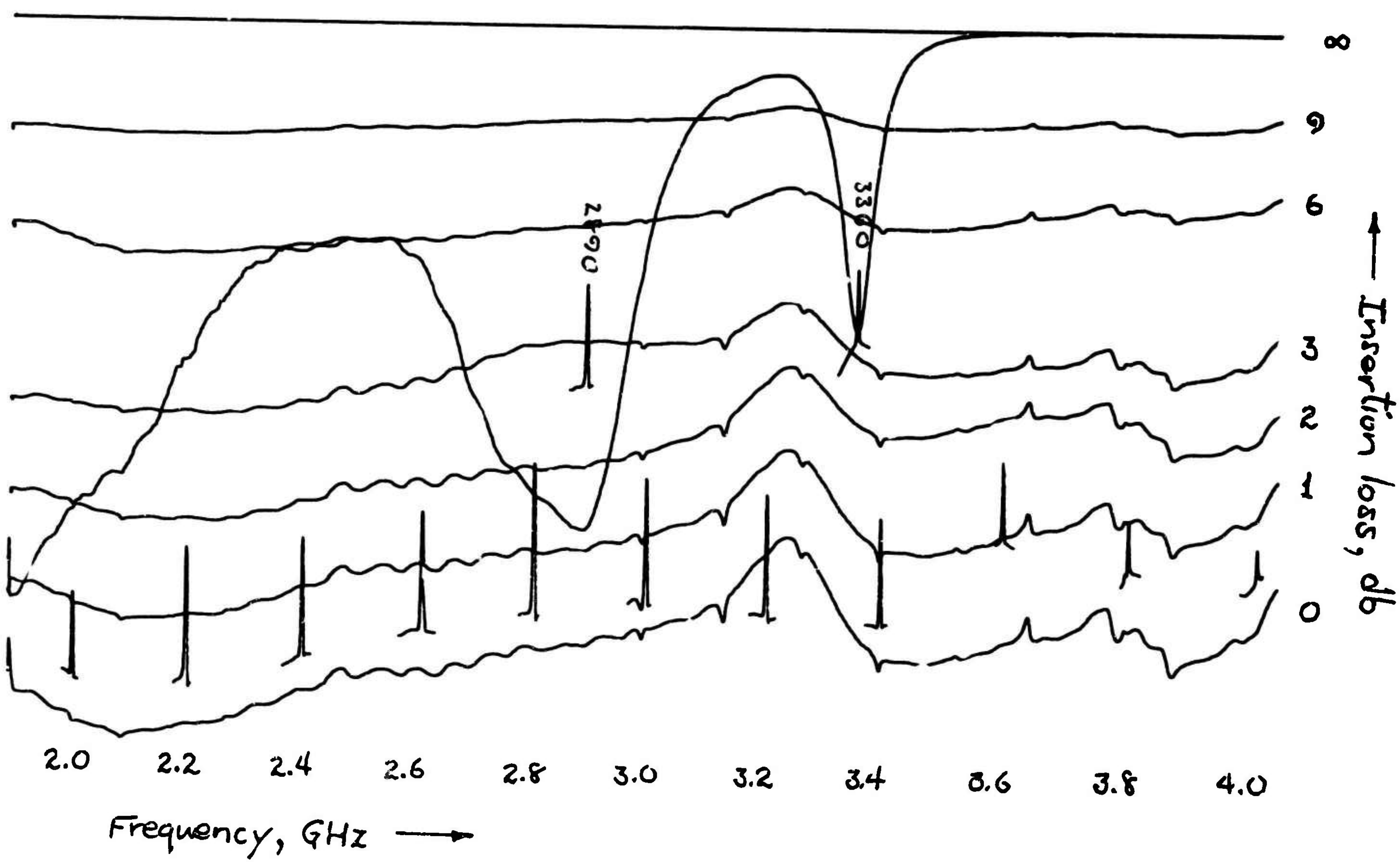

Figure 6. Observed Insertion Loss of a Four-Section Microstrip Comb Filter.

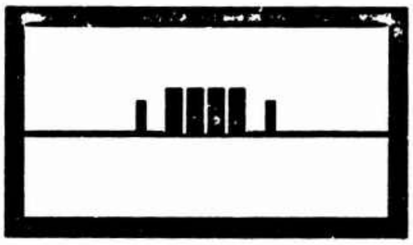

Figure 7. Filter with Matching End Stubs -- Photo-Etch Positive Master, Actual Size.

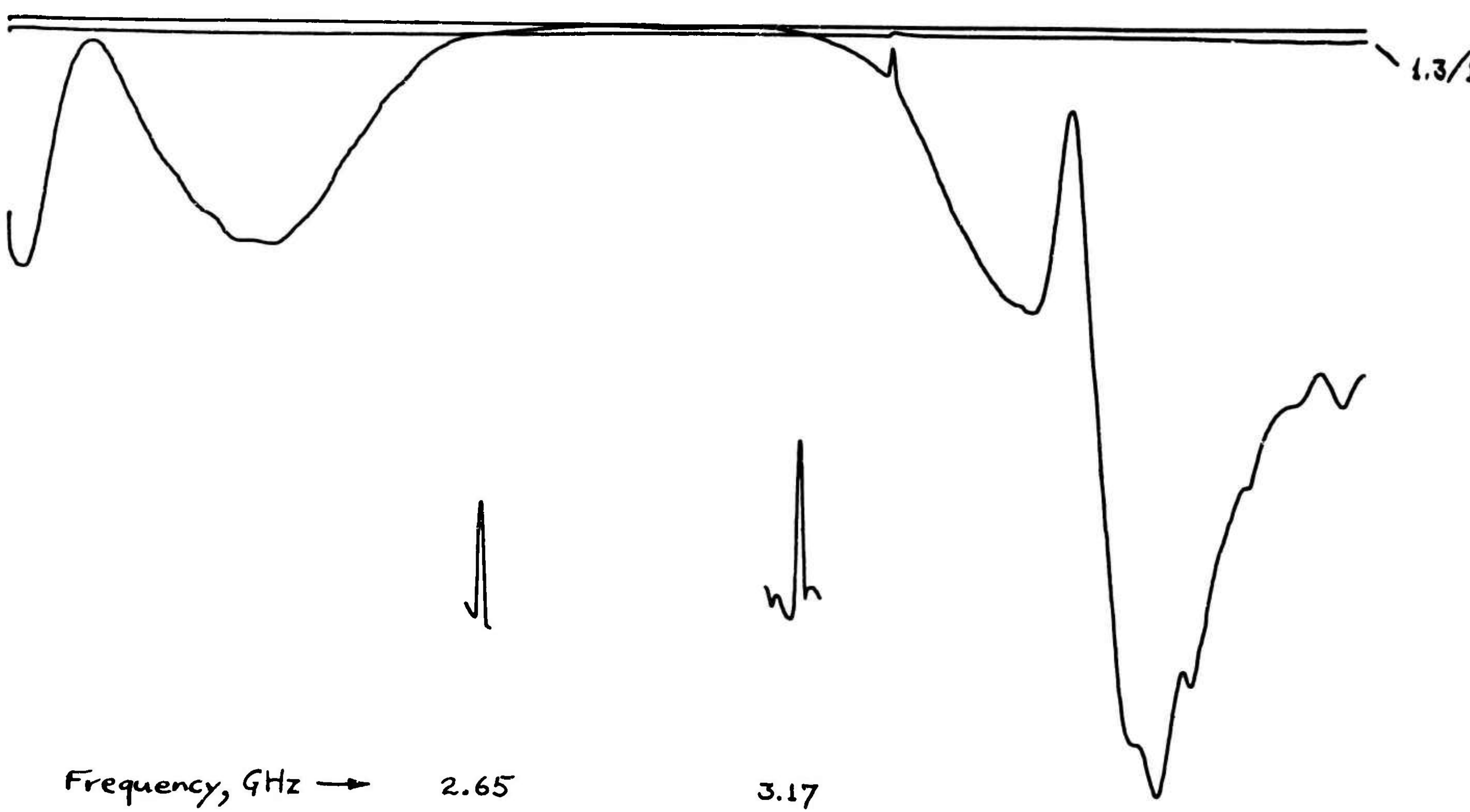

Figure 8. Reflection, Filter of Fig. 7. Substrate: yttrium-iron garnet, $4\pi M_s = 550$; $\epsilon \cong 16$, $h = 0.040$ in.

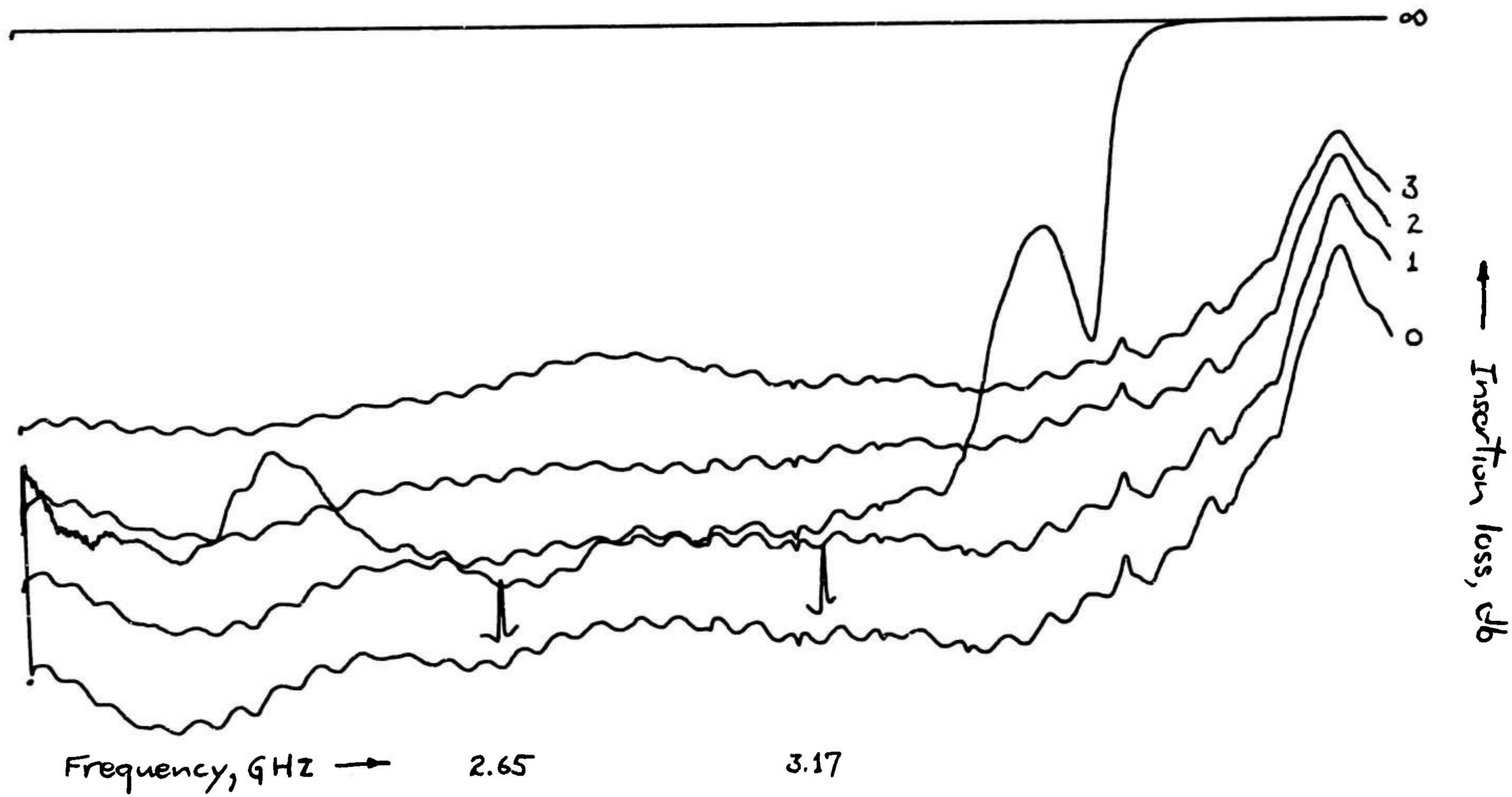

Figure 9. Transmission, Filter of Fig. 7. Substrate: yttrium-iron garnet, $4\pi M_s = 550$; $\epsilon \cong 16$, $h = 0.040$ in.

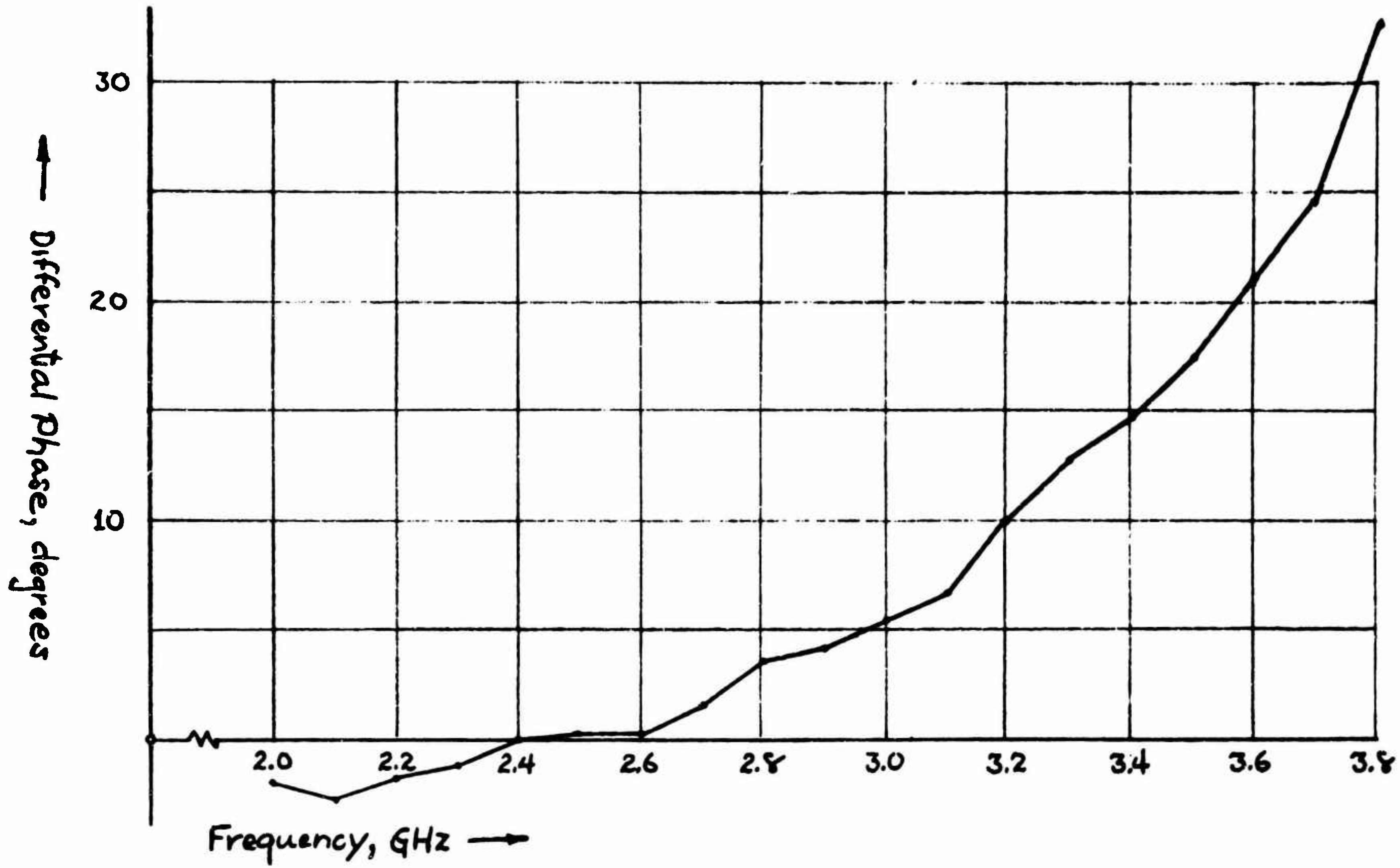

Figure 10. Nonreciprocal Differential Phase of the Comb Filter between Remanent States. Substrate: a yttrium-iron garnet, $4\pi M_s$ = 680 gauss.

Fig. 11. Fortran List of the Program for Evaluation of χ, E_T and E_R of the Reciprocal Comb Filter.

Page 1 of 3.

```
//E25245 JOB    S360,
//              'WEISS, J.A.  UE09'
//START  EXEC  PROC=DUMMY
//SOURCE EXEC PROC=FORTRANH
//SYSIN DD *
C
C     MICROSTRIP COMB FILTER
C
C     FILTER DESIGN  J.A.WEISS NOTES OF 1/30/67, PAGE 33.
C     THIS PROGRAM WAS COMPOSED BY D.N.KLAUKE, WORCESTER POLYTECHNIC INSTITUTE,
C       FOR THE 1620, AND ADAPTED FOR THE 7094 BY T. BRYANT.
C
C     INPUT DATA  F = 2BETA-LA/BETA-LB, RATIO OF ELECTRICAL LENGTH OF SERIES
C                       CONNECTION 2BETA-LA TO THAT BETA-LB OF SHUNT TOOTH
C                 J = NUMBER OF F VALUES TO BE USED
C                ZA = CHARACTERISTIC IMPEDANCE OF SERIES CONNECTION
C                ZO = CHARACTERISTIC IMPEDANCE OF INPUT AND OUTPUT LINES
C              ZETA = RATIO OF CHARACTERISTIC IMPEDANCE ZB OF SHUNT TOOTH TO
C                       THAT ZA OF SERIES CONNECTION
C                 N = NUMBER OF SECTIONS.
C
C     OUTPUT  COSH X, WHERE X IS THE CHARACTERISTIC EXPONENT X=U+IV
C                 V = ITERATIVE PHASE CONSTANT
C               ES1 = INPUT REFLECTION COEFFICIENT
C               ES2 = FILTER TRANSMISSION COEFFICIENT.
C
    1 FORMAT (2F7.4)
    2 FORMAT(F6.1,F8.4,F8.4,F7.2,F8.4,F7.4,8F8.4)
    3 FORMAT (6HOZETA=,F9.4,15X,17H2BETA-LA/BETA-LB=,F9.4)
    4 FORMAT (7F10.4)
    5 FORMAT (4HOZA=,F7.2,10X,3HZO=,F7.2,10X,2HN=,F6.2)
    7 FORMAT(3X,'B        COSHX      V      VDEG      R       Z     ES1R     ES1
     1I    ES2R     ES2I     ES1M     ES2M    SQUARE')
      READ(5,17)J
   17 FORMAT(I10)
      READ (5,4)EN,ZA,ZO
      N=0
  200 READ (5,1)Y,F
      WRITE (6,3)Y,F
      WRITE (6,5)ZA,ZO,EN
      WRITE (6,7)
      N=N+1
      A=0.0
   10 BLB=A*.017453
      B=A
      B2LA=F*BLB
      IF(A-90.0) 20,30,20
   20 IF (A) 21,30, 21
   21 IF (A-180.0) 22,30,22
   22 IF(A-270.0)40,30,40
   30 WRITE (6,2)A
      GO TO 90
   40 COSHX=COS(B2LA)-(1.0/(2.0*Y))*SIN(BLB)*SIN(B2LA)/COS(BLB)
      IF (COSHX-1.0) 50,50,45
   45 WRITE (6,2)A,COSHX
      GO TO 90
   50 IF(COSHX+1.0)60,85,85
```

Fig. 11. Page 2 of 3.

```
60 B=A-10.0
61 IF (B-90.0)62,75,62
62 IF (B) 63,75,63
63 IF (B-180.0)64,75,64
64 IF (B-270.0)65,75,65
65 BLB =B*.017453
   B2LA=F*BLB
   COSHX=COS(B2LA)-(1.0/(2.0*Y))*SIN(BLB)*SIN(B2LA)/COS(BLB)
   IF (COSHX)69,76,69
69 IF (COSHX+1.0)80,70,70
70 CALL CLEM(COSHX,Y,BLB,B2LA,B,EN,ZA,ZO)
   GO TO 77
75 WRITE (6,2)B
   GO TO 77
76 WRITE (6,2)B,COSHX
77 B=B+0.5
   GO TO 61
80 WRITE (6,2)B,COSHX
   WRITE (6,2)A
   GO TO 90
85 CALL CLEM(COSHX,Y,BLB,B2LA,B,EN,ZA,ZO)
90 A=A+5.0
   IF(A-270.0)10,100,100
100 IF (N-J)200,110,110
110 RETURN
   END
   SUBROUTINE CLEM(COSHX,Y,BLB,B2LA,B,EN,ZA,ZO)
 2 FORMAT(F6.1,F8.4,F8.4,F7.2,F8.4,F7.4,8F8.4)
   V=ATAN((SQRT(1.0-COSHX**2))/COSHX)
   IF (V)410,420,420
410 V=3.14159+V
420 VNEG= -V
   VDEG=V*57.2958
   R=2.0*Y*COS(BLB)*(SIN(V)-SIN(B2LA))/SIN(BLB)-COS(B2LA)
   IF (R-1.0)700,800,700
700 Z=(1.0+R)/(1.0-R)
   RN=(1.-ZA/ZO)/(1.0+ZA/ZO)
   RSUM=(1.+R**2)/R
   RDIF=(1.-R**2)/R
   RONR=RN*COS(EN*V)*RDIF
   RONI=SIN(EN*V)*(RN*RSUM-2.0)
   RODR=COS(EN*V)*RDIF
   RODI=SIN (EN*V)*(2.0*RN-RSUM)
   X=RODR**2+RODI**2
   ROR =(RONR*RODR+RONI*RODI)/X
   ROI =(RONI*RODR-RONR*RODI)/X
   ES1NR=-RN+ROR
   ES1NI=ROI
   ES1DR=1.0-RN*ROR
   ES1DI=RN*ROI
   U=ES1DR**2+ES1DI**2
   ES1R=(ES1NR* ES1DR+ES1NI*ES1DI)/U
   ES1I=(ES1NI*ES1DR-ES1NR*ES1DI)/U
   SPS=4.0*ZO/(ZA*(1.0+ZO/ZA)**2)
   ES2NR=RDIF
   ES2NI=0.0
   ES2DR=RDIF*COS(EN*V)
```

Fig. 11. Page 3 of 3.

```
      ES2DI =-(RSUM-2.0*RN)*SIN(EN*V)
      T=ES2DR**2+ES2DI**2
      ES2R=(ES2NR*ES2DR+ES2NI*ES2DI)/T
      ES2I = (ES2NI*ES2DR-ES2NR*ES2DI)/T
      W=(1.0-RN*ROR)**2+(RN*ROI)**2
      ES2RR =-SPS*(ES2R*(1.0-RN*ROR)-ES2I*RN*ROI)/W
      ES2II =-SPS*(ES2I*(1.0-RN*ROR)+ES2R*RN*ROI)/W
      FFF=ES1R**2+ES1I**2
      EEE=ES2RR**2+ES2II**2
      ES1M=SQRT(FFF)
      ES2M=SQRT(EEE)
      SQUARE= FFF+EEE
  800 WRITE (6,2)B,COSHX,V,VDEG,R,Z,ES1R,ES1I,ES2RR,ES2II,ES1M,ES2M,SQUA
     1RE
      RETURN
      END
/*
//LINK EXEC PROC=LINKSRCE
//EXEC EXEC PROC=EXECUTE
//FT05F001 DD *
          1
    6.0000    75.0000    50.0000
  .3333  .2500
  .3333  .1500
  .3333  .3000
/*
```

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13. ABSTRACT
~~This~~ The report presents results of an investigation of a microstrip filter which was designed for the study of nonreciprocal phase in S-band (~3 GHz). Data are presented showing scattering, absorption, and nonreciprocity. The device is a low-pass "comb" filter. When matched to less than 1.1/1 VSWR over a 13% band, its insertion loss is 0.8 - 1.2 db. Nonreciprocal differential phase is frequency-sensitive, reaching 30 degrees near the cutoff frequency of 3.5 GHz. An analysis is presented of the theory of periodic networks with sufficient generality to include the nonreciprocal and nonconservative cases. Comparison with the performance of a reciprocal filter shows good agreement. The agreement with nonreciprocal effects is qualitatively correct, although a full treatment of nonreciprocity must await a detailed field theory of propagation on microstrip with a magnetic substrate. The investigation shows that substantial amounts of nonreciprocal phase can be produced by means of a simple, compact, efficient microstrip structure with low loss and good match over an ample band, suitable for control in the "latching" (remanent) mode of operation.

14. KEY WORDS
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phasers
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