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EIGENELEMENTS OF THE FINITE FOURIER TRANSFORM AND THEIR APPLICATION TO ANTENNA PATTERN SYNTHESIS

THOMAS S. FONG

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Project No. 4600
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Work Unit No. 46001001

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Microwave Physics Laboratory

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Antenna Department, Aerospace Group
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for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

ABSTRACT

The eigenvalues and eigenfunctions of the finite Fourier transform corresponding to parameter $c = 10\pi$, 15π, 20π, and 25π are computed and tabulated. The computational details are given and discussed. An antenna pattern synthesis technique that makes use of these eigenelements is illustrated by examples. When the superdirective ratio is constrained to reasonable values, the number of eigenfunctions that contribute significantly to the pattern is found to be approximately equal to twice the number of wavelengths in the aperture. A pattern synthesized with the above eigenfunctions is compared with the pattern obtained by Woodward's method, and considerable improvement is noted over that method.

CONTENTS

1.0 INTRODUCTION 1

2.0 TECHNICAL DISCUSSION 3

2.1 The Differential Equation Satisfied by the Eigenfunctions 3

2.2 Solution to the Differential Equation . . . 6

2.2.1 Power Series Expansion 7

2.2.2 Expansion in Series of Legendre Polynomials 8

2.3 Computational Detail 10

2.4 Examples of Pattern Synthesis 14

3.0 TABLES OF EIGENELEMENTS 23

REFERENCES 51

RELATED CONTRACTS AND PUBLICATIONS . . . 51

ACKNOWLEDGMENTS 52

APPENDIX

A. Program for Computation of the Eigenfunctions and the Eigenvalues 53

B. Program for Determination of k in Equation (9) to a 9-Place Accuracy 54

ILLUSTRATIONS

Figure 1. Typical curve for F(k) 11

Figure 2. Typical eigenfunctions for $c = 10\pi$ and various ψ 13

Figure 3. Effect on eigenfunctions of variations in c . . 14

Figure 4. Magnitude of eigenvalues for $c = 10\pi$ . . . 14

Figure 5. Pattern approximation by eigenelements for $c = 10\pi$ 16

Figure 6. Aperture distributions for $c = 10\pi$ . . . 17

Figure 7. Pattern approximation by eigenelements for $c = 20\pi$ 20

Figure 8. Pattern approximation by Woodward's method for $c = 10\pi$ 20

TABLES

Table 2-1. Generalized Fourier coefficients and eigenvalues 18

Table 3-1. Eigenvalues for $c = 10\pi$ or aperture length = 10λ 24

Table 3-2. Eigenfunctions for $c = 10\pi$ or aperture length = 10λ 25

Table 3-3. Eigenvalues for $c = 15\pi$ or aperture length = 15λ 29

Table 3-4. Eigenfunctions for $c = 15\pi$ or aperture length = 15λ 30

Table 3-5. Eigenvalues for $c = 20\pi$ or aperture length = 20λ 35

Table 3-6. Eigenfunctions for $c = 20\pi$ or aperture length = 20λ 36

Table 3-7. Eigenvalues for $c = 25\pi$ or aperture length = 25λ 42

Table 3-8. Eigenfunctions for $c = 25\pi$ or aperture length = 25λ 43

1.0 INTRODUCTION

Eigenfunctions of the finite Fourier transform, often referred to in the literature as angular prolate spheriodal wave functions of the first kind, have received considerable attention (Flammer, 1957; Stratton, 1956). These functions and the corresponding eigenvalues occupied an important role in a recently studied technique for the synthesis of antenna radiation patterns (Rhodes, 1963; Fong, 1965). The range of tabulated values for both the eigenfunctions and the eigenvalues has been so limited, however, as to restrict any practical application of the synthesis technique until additional values were available. A few usable sets of eigenfunctions and eigenvalues, suitable for antenna aperture sizes of 10, 15, 20, and 25 wavelengths, were computed as a basis for the present study and then applied to the synthesis of patterns from apertures of the same size.

An extensive table of values has not been presented nor has mathematical completeness been emphasized since the objective is the computation of the eigenelements; only the essential steps are included in the discussion. The notation of the parameters used and the physical meanings are consistent with those of the earlier report (Fong, 1965) and are repeated here for convenience.

L = half the aperture length

$c = 2\pi L/\lambda$

$u = \sin\theta$, $\theta = 0$ at broadside

$x = \xi/L$, ξ is the coordinate along the aperture

λ_n = the nth eigenvalue

ψ_n = the nth eigenfunction

The normalization has been taken such that

$$\int_{-1}^{1} \psi_n^2(x)\, dx = 1$$

with $\psi_n(0) > 0$ for n even and $\psi'(0) > 0$ for n odd.

2.0 TECHNICAL DISCUSSION

2.1 THE DIFFERENTIAL EQUATION SATISFIED BY THE EIGENFUNCTIONS

A direct computation of the eigenelements of the finite Fourier transform

$$\int_{-1}^{1} \psi(x)\, e^{jcux}\, dx = \lambda \psi(u)$$

as outlined in an earlier report (Fong, 1965) was initially considered. In this approach the kernel is expressed in the form

$$\sum_{m,\, n=0}^{\infty} A_{mn}\, \varphi_m(x)\, \varphi_n(u)$$

where $\{\varphi_p\}$, $p = 0, 1, \ldots,$ is a set of known basis. For computational purposes the kernel is approximated by a finite sum, and the integral equation reduces to a finite set of simultaneous equations or a matrix equation of the form $Ae = \lambda e$. Because of the essential equality of the 0th, 4th, $8^{th}, \ldots, M^{th}$ eigenvalues, where M depends on the parameter c or the aperture length, the first five digits being the same, the finite sum approximation and the error in computing the coefficients A_{mn} above by integration present considerable uncertainty on the computed eigenvectors $e_0, e_4, \ldots, e_M$ from which the eigenfunctions $\psi_0, \psi_4, \ldots, \psi_M$ are computed. The same difficulty occurs in finding $e_1, e_5, \ldots, e_{M+1}$, etc.

The uncertainty in the eigenvectors may be seen from the following consideration. If it is supposed that λ_0, λ_4, and λ_8 are nearly the same, then $A(\alpha e_0 + \beta e_4 + \delta e_8) \cong \lambda_0(\alpha e_0 + \beta e_4 + \delta e_8)$, where α, β, and δ are constants. This relationship implies that $\alpha e_0 + \beta e_4 + \delta e_8$ is an eigenvector associated with λ_0, approximately. In fact, any linear combination of e_0, e_4, and e_8 would satisfy the equation $Ae = \lambda e$ approximately. Consequently, within the limits of the accuracy of computation,

erroneous solutions can be generated that correspond to arbitrary linear combinations of the correct eigenfunctions. To avoid this difficulty, the number of terms in the finite approximation must be quite large, say at least 100 terms, in addition to the requirement that the coefficients A_{mn} must be accurately determined. The computing time becomes quite long, although the technique is straightforward. In view of this fact, an alternative approach that considered an equivalent Sturm-Liouville differential operator was examined.

For completeness of this discussion, some of the development appearing in the earlier studies, Rhodes (1963) and Fong (1965) is repeated here. The relationship is established between the eigenelements of the integral operator with kernel e^{jcux} above and the eigenelements of the differential operator

$$L_x = \frac{d}{dx}(1 - x^2)\frac{d}{dx} - c^2x^2$$

It is supposed that u and v are two continuous, twice differentiable functions. Then

$$\int_{-1}^{1} (u L_x v - v L_x u)\, dx = \int_{-1}^{1} u \left\{ \frac{d}{dx}\left[(1-x^2)\frac{dv}{dx}\right] - c^2x^2 v \right\} dx$$

$$- \int_{-1}^{1} v \left\{ \frac{d}{dx}\left[(1-x^2)\frac{du}{dx}\right] - c^2x^2 u \right\} dx$$

$$= (1 - x^2)\left(u\frac{dv}{dx} - v\frac{du}{dx}\right)\Bigg|_{-1}^{1}$$

$$= 0 \qquad (1)$$

It follows from straightforward differentiation that

$$L_x e^{jcux} = L_u e^{jcux} \qquad (2)$$

Let $\theta(x)$ be a solution to

$$L_x \theta + k\theta = 0 \tag{3}$$

That is, θ and k are, respectively, an eigenfunction and an eigenvalue of L_x. Denote

$$\int_{-1}^{1} e^{jcux} \theta(x)\, dx$$

by $\psi(u)$. Consider $L_u \psi(u)$.

$$L_u \psi(u) = \int_{-1}^{1} L_u e^{jcux} \theta(x)\, dx$$

As a consequence of Equations (1), (2), and (3) above, it develops that

$$\begin{aligned} L_u \psi(u) &= \int_{-1}^{1} \theta(x)\, L_x e^{jcux}\, dx \\ &= \int_{-1}^{1} e^{jcux} L_x \theta(x)\, dx \\ &= -k \int_{-1}^{1} e^{jcux} \theta(x)\, dx \\ &= -k\, \psi(u) \end{aligned} \tag{4}$$

In view of these Expressions (4), it may be concluded that $\psi(x)$ and $\theta(x)$ are proportional, or the eigenfunctions of the differential operator and the integral operator are the same. There are only a countable number of values of k for which the differential Equation (3) has a real solution $\psi(x)$ that is bounded for all x. For sufficiently large c, the eigenvalues have an asymptotic expansion (Flammer, 1957):

$$k_n = (2n+1)\,c - (2n^2+2n+3)\,2^{-2} - (2n+1)(n^2+n+3)\,2^{-4}\,c^{-1}$$

$$- 5(n^4 + 2n^3 + 7n + 3)\,2^{-6}\,c^{-2}$$

$$- (66n^5 + 165n^4 + 962n^3 + 1,278n^2 + 1,321n + 453)\,2^{-10}\,c^{-3}$$

$$- (252n^6 + 756n^5 + 5,885n^4 + 10,510n^3 + 18,478n^2 + 13,349n$$

$$+ 4,425)\,2^{-12}\,c^{-4}$$

$$- \Big[527\,(2n+1)^7 + 61,529\,(2n+1)^5 + 1,043,961\,(2n+1)^3$$

$$+ 2,241,599\,(2n+1)\Big]\,2^{-20}\,c^{-5} - \cdots$$

For $c > n\pi$, the expansion gives a good approximation, the accuracy being better for smaller n. The problem of two or more nearly equal eigenvalues does not occur for the differential operator as it does for the integral operator. An exact expression for the determination of the eigenvalues of L_x is developed later in the discussion.

2.2 SOLUTION TO THE DIFFERENTIAL EQUATION

Two approaches have been attempted in obtaining the solutions to the differential equation

$$\frac{d}{dx}\left[(1 - x^2)\,\frac{d\psi_n}{dx}\right] + (k_n - c^2x^2)\,\psi_n = 0 \qquad (5)$$

(i) Power series expansion about various points on [-1, 1].

(ii) Expansion in terms of Legendre polynomials.

2.2.1 Power Series Expansion

From the differential Equation (5), it can be seen that the linearly independent solutions consist of an even function and an odd function. The one that is bounded for all x is the desired solution to the present problem since ψ_n has been required to be an absolutely square integrable function on [-1, 1]. For ψ_n to fulfill this requirement implies that it is bounded through

$$\lambda_n \psi_n(u) = \int_{-1}^{1} e^{jcux} \psi_n(x)\, dx$$

Since ±1 are regular, singular points of the differential equation, a series solution expanded about any point in (-1, 1) will have a convergence problem from the computational point of view for values of x close to ±1. However, for expansion about 1, which is a regular, singular point, the series will converge for all x in (-1, 1]. A similar behavior is exhibited for expansion about -1. In view of the even or odd property of the eigenfunction, expansion about 1 or -1 seems to be most convenient because the region of interest [0, 1] is considerably smaller than the region of convergence of the series.

After the indicial equation is examined, the series solution of the form

$$\psi(x) = \sum_p A_p (1 - x)^p$$

is taken. The recurrence formulas for the coefficients then are

$$A_1 = \frac{1}{2} (c^2 - k) A_0$$

$$A_2 = \frac{1}{8} \left[(c^2 - k + 2) A_1 - 2 c^2 A_0 \right]$$

$$A_p = \frac{1}{2p^2} \left[(p^2 - p + c^2 - k) A_{p-1} - 2 c^2 A_{p-2} + c^2 A_{p-3} \right] \quad p \geq 3$$

The crucial role of k in computation of the coefficients is apparent from these expressions. The error in the coefficient A_p due to inaccuracy in k grows with p especially when k is comparable to c^2. The value of $|A_p|$ increases with p until $p \cong c$ and then decreases asymptotically as $|A_p| \sim \frac{1}{2} |A_{p-1}|$ for large p. The number of terms in the series that are contributing significantly to the sum becomes larger as x gets farther away from 1. Since values of A_p are less accurately determined for large p, the series representation becomes poor for x close to 0. If the accuracy of the representation is not satisfactory in the neighborhood of x = 0, which can be determined by a comparison of the computed values of $\psi(x)$ and $\psi(-x)$ near x = 0, an improvement may be achieved by the expansion of another series solution about 0 with the solutions joined together.

2.2.2 Expansion in Series of Legendre Polynomials*

The differential Equation (5) can be compared with the differential equation satisfied by the Legendre polynomials P_n:

$$\frac{d}{dx}\left[(1 - x^2)\frac{dP_n}{dx}\right] + n(n+1)P_n = 0 \tag{6}$$

This comparison suggests a series of the form

$$\sum_0^\infty A_n P_n(x)$$

for the solution to Equation (5). From the recurrence relation

$$x^2 P_n(x) = \frac{n(n-1)}{4n^2-1} P_{n-2}(x) + \left[\frac{n^2}{4n^2-1} + \frac{(n+1)^2}{(2n+1)(2n+3)}\right] P_n(x)$$
$$+ \frac{(n+1)(n+2)}{(2n+1)(2n+3)} P_{n+2}(x)$$

*Flammer, 1957; Stratton, et al., 1956.

and the substitution of the expression

$$\sum_{0}^{\infty} A_n \, P_n(x)$$

into Equation (5), there results

$$A_{n+2} \, c^2 \, \frac{(n+2)(n+1)}{(2n+3)(2n+5)} + A_n \left[n(n+1) - k + c^2 \, \frac{2n(n+1)-1}{(2n-1)(2n+3)} \right]$$

$$+ c^2 \, \frac{n(n-1)}{(2n-3)(2n-1)} \, A_{n-2} = 0 \qquad (7)$$

and $A_{-1} = A_{-2} = 0$. This expression is a difference equation of second order, and it admits two linearly independent solutions. As n becomes large, the ratio A_n/A_{n-2} of one solution behaves as $-4n^2/c^2$, and the other decreases to zero as $-c^2/4n^2$. The boundedness requirement on ψ implies that the latter should be chosen. Let

$$S(n, k) = \left[n(n+1) - k + c^2 \, \frac{2n(n+1)-1}{(2n-1)(2n+3)} \right] \frac{(2n+3)(2n+5)}{(n+2)(n+1)\, c^2}$$

$$T(n) = \frac{n(n-1)(2n+3)(2n+5)}{(2n-3)(2n-1)\,(n+2)(n+1)}$$

The solution to the difference equation can be written as an infinite continued fraction:

$$\frac{A_{n+2}}{A_n} = - \cfrac{T(n+2)}{S(n+2, k) - \cfrac{T(n+4)}{S(n+4, k) - \cfrac{T(n+6)}{S(n+6, k) - \ddots}}} \qquad (8)$$

But $A_2/A_0 = -S(0,k)$; thus, the condition must exist that

$$S(0,k) = \cfrac{T(2)}{S(2,k) - \cfrac{T(4)}{S(4,k) - \cfrac{T(6)}{S(6,k) - \ddots}}} \tag{9}$$

which determines a set of special values for k, the eigenvalues of the differential operator L_x.

Unlike the case of the coefficient for the power series in which the computation started with $n = 0$ and proceeded outward to larger n, the present case starts with large n and comes inward to 0. In view of the resemblance of Equations (5) and (6), it can be seen that the maximum $|A_n|$, which is the largest magnitude of the projection of ψ_p on P_n, should be in the neighborhood of $n = \sqrt{k}$. Since the higher-index coefficients are decaying very rapidly, the number of terms in the series needed to have five place accuracy in the computed eigenfunction for all $|x| \leq 1$ is very reasonable. Of the two representations of the eigenfunction, the latter seems to be more suitable from the computational viewpoint.

2.3 COMPUTATIONAL DETAIL

The steps in computation of the eigenfunctions and eigenvalues of the finite Fourier transform are given below.

(i) The asymptotic expansion for the eigenvalues of L_x was used to provide the first order approximation, and the "exact" (computed to nine places) values of k were computed by Equation (9) for the first N eigenvalues, where N = twice the number of wavelengths in the aperture. For higher-index eigenvalues, the asymptotic expansion becomes poor and, hence, useless. As the index gets higher, even the step by step hunting (evaluate the expression

$$F(k) = S(0,k) - \cfrac{T(2)}{S(2,k) - \cfrac{T(4)}{S(4,k) - T(6) \ddots}}$$

at a discrete set of points) becomes difficult. The step size must be taken smaller and smaller as the index increases. A typical curve for F(k) appears roughly as shown in Figure 1; the Δ becomes smaller as the index increases. As will be demonstrated later, the eigenelements that have indices higher than N+ 1 play only a minor role in pattern synthesis. Thus, for all practical purposes, only the first N+ 1 eigenelements need to be used.

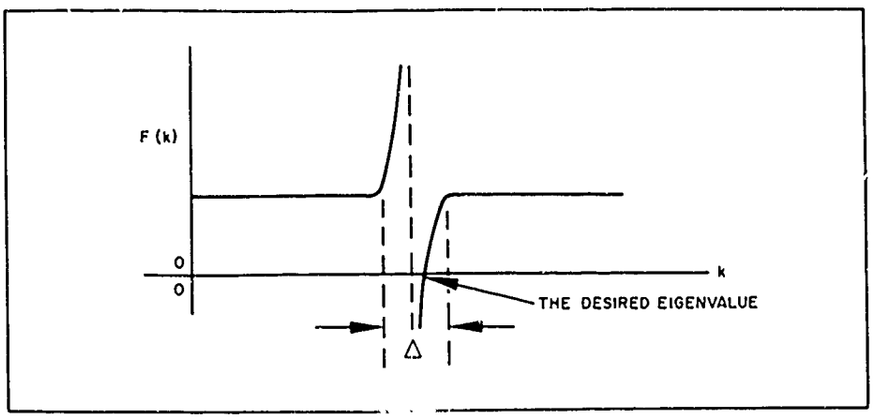

Figure 1. Typical curve for F(k).

(ii) Two even (or odd) consecutive coefficients are computed from Equation (8) as the initial point in the recurrence process in Equation (7). For the range of c of interest, the number of significant terms in the summation is less than 150, so that the ratio A_{152}/A_{150} (or A_{151}/A_{149}) appears to be a suitable place to start. A_{150} (or A_{149}) was taken to be 10^{-40} so that the lower-index coefficient would not be too large and cause overflow on the machine. For $c = 10\pi$, the number of terms used in the summation $\Sigma A_n P_n(x)$ was 15 for the low index eigenfunctions and 30 for the higher ones. For $c = 25\pi$, the number of significant terms ranged from 25 to 55.

(iii) The eigenfunctions are computed with the values of k determined in step (i) and the two even (or odd) consecutive coefficients obtained in step (ii) as input data in a new computer program.

(iv) As a by-product from this program, the eigenvalues of the finite Fourier transform are obtained through the relationship

$$\lambda_n = \frac{1}{\psi_n(u)} \int_{-1}^{1} \begin{Bmatrix} \cos cux \\ j \sin cux \end{Bmatrix} \psi_n(x)\, dx \qquad n = \begin{matrix} 0, 2, 4, \ldots \\ 1, 3, 5, \ldots \end{matrix} \tag{10}$$

by selecting a specific value for u on the right-hand side (of course, not such a value that $\psi_n(u) = 0$).

As a check on the program and the accuracy in determination of the parameters k and A_{152} (or A_{151}) from earlier computer programs for each n, hence each eigenfunction, various values of u were chosen in computing the eigenvalue from Equation (10). Since the left-hand side is independent of u, the results from various selections of u should be invariant. The first N eigenvalues determined by Equation (10) are accurate at least in the first four places after the decimal point. The increment in x was 0.005 in the integration. For <u>higher-index</u> eigenvalues, the accuracy in the result is believed to be within ±5 to ±10 percent, being less certain for higher index. The reason for the increasing uncertainty lies in the very rapid rise in value near 1 of the eigenfunctions; the slope becomes steeper as the index increases. The main contribution to the value of the integral, or the eigenvalue, is from the region near 1. The increment in x in [0.9, 1] was taken as 0.0002 for the computation.

The graphs of some eigenfunctions for $c = 10\pi$ are shown in Figure 2. To show the variations in the eigenfunctions as c takes on various values, ψ_0 for $c = 10\pi$, 15π, 20π, 25π is shown in Figure 3. A graph of the absolute value of the eigenvalues versus n for $c = 10\pi$ is shown in Figure 4. The absolute value of the eigenvalues remains essentially constant for the first N - 4 eigenvalues and then decreases very rapidly after n = N. The eigenfunctions are tabulated in Section 3 for the argument in [0, 1] in 0.01 increments. All computations were performed on the GE 265 machine using BASIC. The essential computer programs are given in the Appendix.

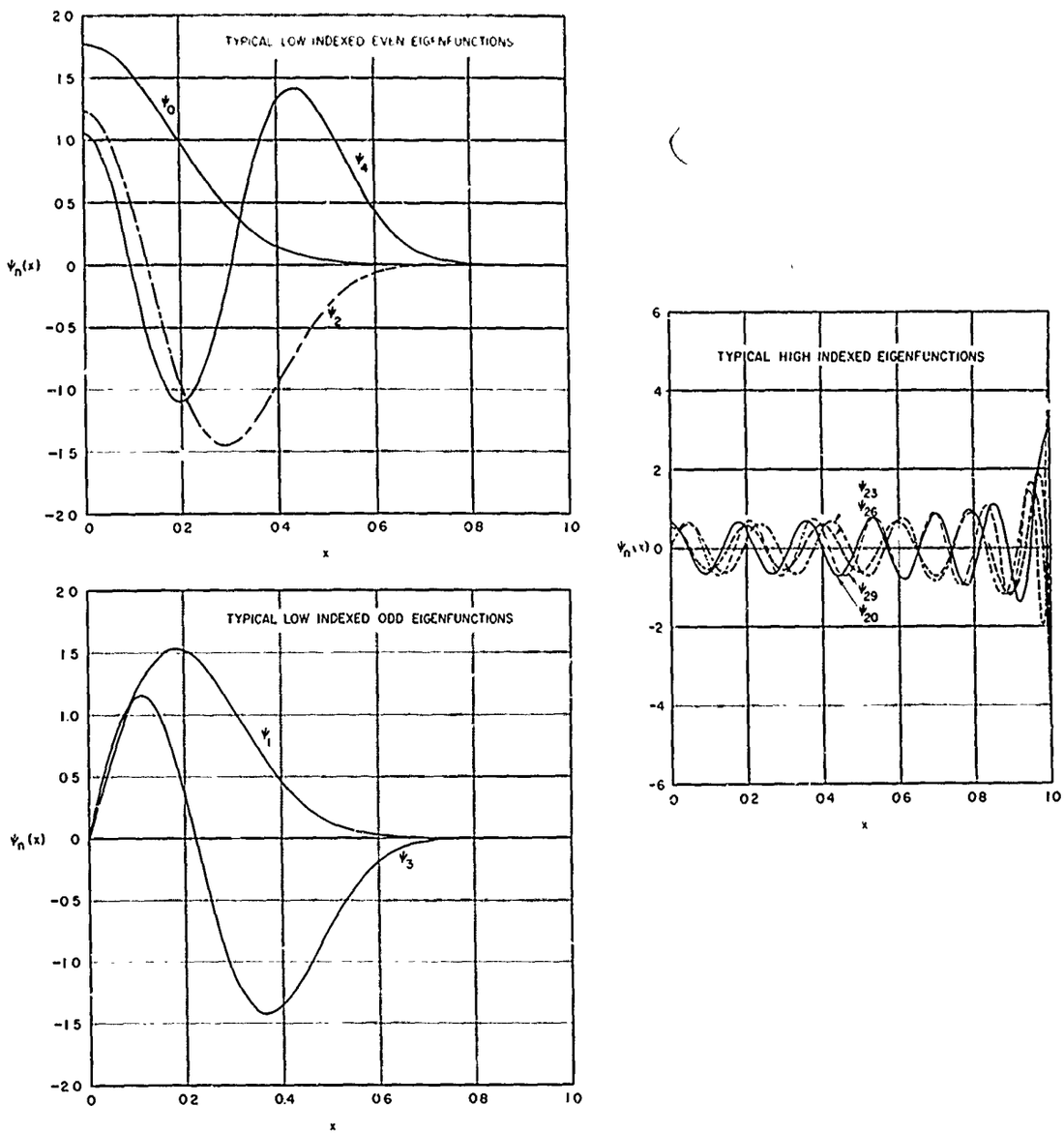

Figure 2. Typical eigenfunctions for c = 10π and various ψ.

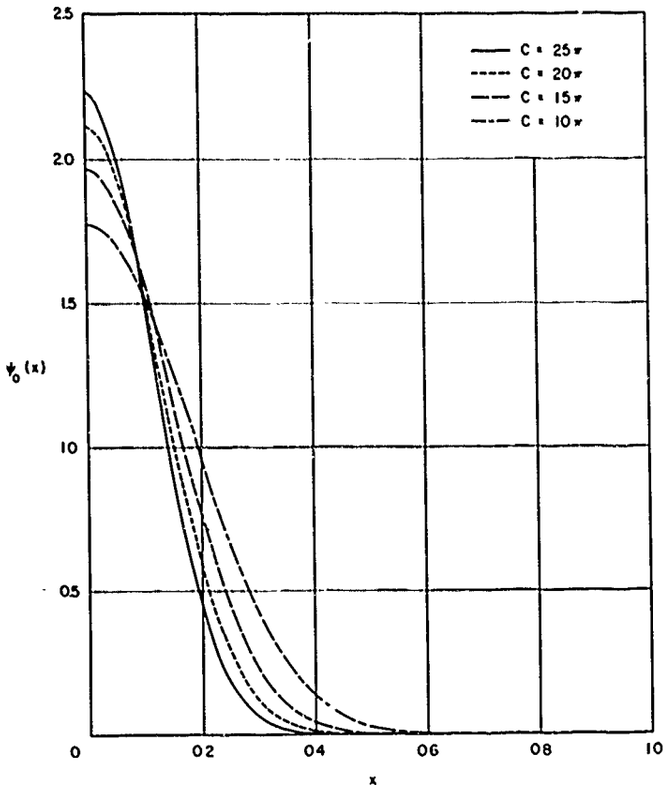

Figure 3. Effect on eigenfunctions of variations in c.

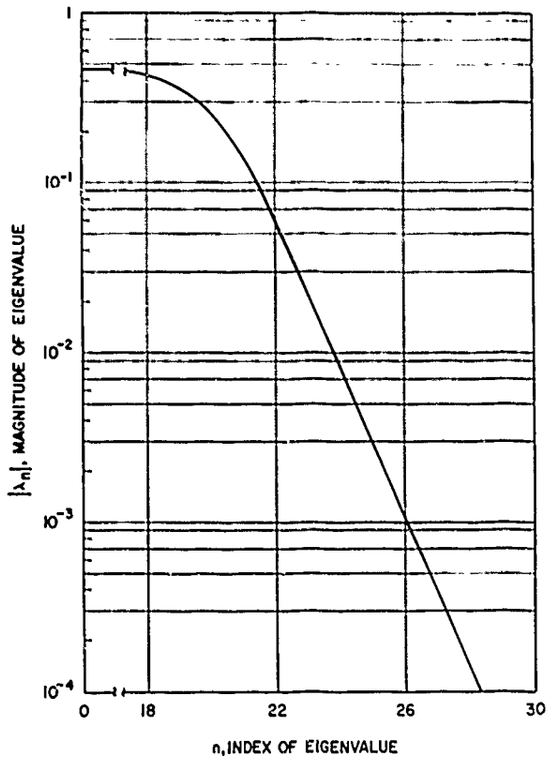

Figure 4. Magnitude of eigenvalues for $c = 10\pi$.

2.4 EXAMPLES OF PATTERN SYNTHESIS

An application of the synthesis technique that utilized the eigenelements of the finite Fourier transform was formulated. An antenna was considered that had a radiation pattern

$$g(u) = \begin{cases} 1 - u & 0 \le u \le \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$$

and an aperture length equal to ten wavelengths. First the generalized Fourier coefficients for g will be obtained via the relation

$$a_n = \int_{-1}^{1} g(u)\, \psi_n(u)\, du \qquad n = 0, 1, \ldots$$

For a fixed superdirective ratio, γ, the Lagrange multiplier μ is computed from (Fong, 1965)

$$\sum_{n=0}^{29} \frac{a_n^2 \left(\frac{2\pi}{c|\lambda_n|^2} - \gamma \right)}{1 + \mu \left(\frac{2\pi}{c|\lambda_n|^2} - \gamma \right)^2} = 0$$

for $\mu \in \left[0, \frac{1}{\gamma - \frac{2\pi}{c\lambda_o^2}} \right]$. The summation goes to 29 only since there are only 30 eigenfunctions available. With the value of μ obtained above, the coefficients $\hat{a}_n$, which provide the best approximation to g subject to the superdirective ratio constraint, are computed by

$$\hat{a}_n = \frac{a_n}{1 + \mu \left(\frac{2\pi}{|\lambda_n|^2} - \gamma \right)} \qquad n = 0, 1, \ldots, 29$$

The optimum aperture distribution is given by

$$\hat{f}(x) = \sum \frac{\hat{a}_n}{\lambda_n} \psi_n(x)$$

while the optimum approximating pattern is

$$\hat{g}(u) = \sum \hat{a}_n \psi_n(u)$$

The graphs of the approximating patterns (corresponding to different superdirective ratios of 1.2 and 10) and the aperture distributions are shown in Figure 5 and Figure 6, respectively.

The values of a_n, $\hat{a}_n$, and $a_n - \hat{a}_n$ corresponding to $\gamma = 1.2$ and 10 are listed in Table 2-1.

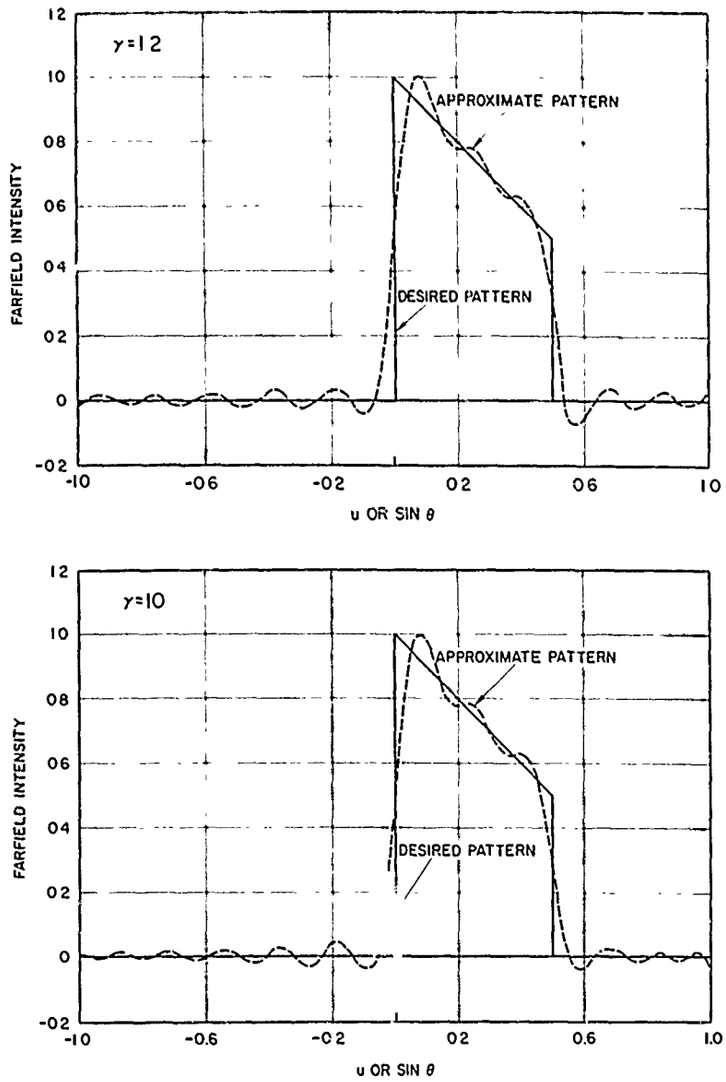

Figure 5. Pattern approximation by eigenelements for c = 10π.

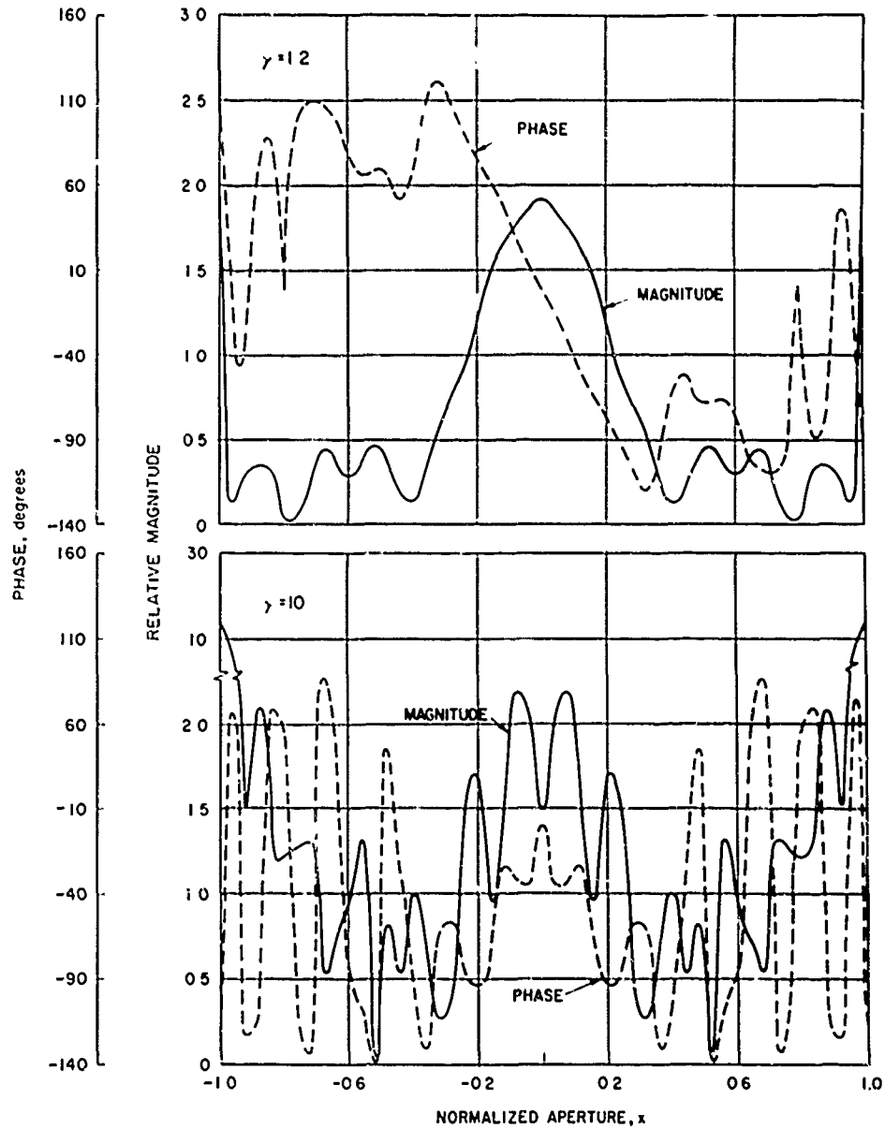

Figure 6. Aperture distributions for $c = 10\pi$.

n	a_n	$\gamma = 1.2$		$\gamma = 10$	
		$\hat{a}_n$	$(a_n-\hat{a}_n)\times 10^5$	$\hat{a}_n$	$(a_n-\hat{a}_n)\times 10^5$
0	0.34425	0.34446	-21	0.34425	*
1	0.34367	0.34388	-21	0.34367	*
2	-0.14729	-0.14738	9	-0.14729	*
3	-0.03768	-0.03770	2	-0.03768	*
4	0.07564	0.07569	-5	0.07564	*
5	0.08009	0.08014	-5	0.08009	*
6	0.01558	0.01559	-1	0.01558	*
7	0.07356	0.07363	-7	0.07356	*
8	-0.02376	-0.02377	1	-0.02376	*
9	0.02850	0.02852	-2	0.02850	*
10	-0.01101	-0.01102	1	-0.01101	*
11	0.01044	0.01045	-1	0.01044	*
12	0.00995	0.00996	-1	0.00995	*
13	0.01588	0.01589	-1	0.01588	*
14	0.01760	0.01761	-1	0.01760	*
15	0.02581	0.02583	-2	0.02581	*
16	0.01353	0.01354	-1	0.01353	*
17	0.02982	0.02984	-2	0.02982	*
18	0.00594	0.00594	0	0.00594	*
19	0.02764	0.02761	3	0.02764	*
20	0.00024	0.00024	0	0.00024	*
21	0.02682	0.02607	75	0.02681	1
22	-0.00488	-0.00418	-70	-0.00487	-1
23	0.02373	0.01209	1164	0.02348	25
24	-0.00816	-0.00085	-731	-0.00746	-70
25	0.01672	0.00022	1650	0.00919	753
26	-0.00646	-0.00001	-645	-0.00089	-557
27	0.01023	*	1023	0.00021	1002
28	0.00097	*	97	*	97
29	0.00848	*	848	*	848

*Absolute value smaller than 10^{-6}.

Table 2-1. Generalized Fourier coefficients and eigenvalues.

It is interesting to note that there is very little improvement in the approximating radiation patterns when the superdirective ratio increases by greater than 8 times. This fact is apparent from an examination of Table 2-1 in which it can be seen that the significant contribution to the sum is from the first N terms. The high-index coefficients are essentially ignored because of the constraint. As might have been expected, when γ is near 1, greater emphasis is placed on the lower-index coefficients. While the improvement is small in changing the value of γ, the aperture distribution has altered tremendously. The rapid change in phase when $\gamma = 10$ contributes to high reactive power and, hence, the aperture is a high Q aperture in addition to being more difficult to construct.

Another approximating pattern was computed for an aperture of 20 wavelengths with $\gamma = 1.02$, and it is shown in Figure 7. As expected, the approximation improves considerably over that of the earlier cases.

Woodward's method was applied to the same problem; the pattern obtained is shown in Figure 8 for comparison. γ for this case is approximately 1.0003. It should be noted that the value of γ does not enter into this synthesis technique. For this particular value, the pattern obtained by using the eigenelements is essentially the same as either of the ones shown in Figure 5.

A summary of errors in the various approximations, in the mean square sense, is shown below. The error of approximating g by $\hat{g}$ is given by

$$||g - \hat{g}|| = \left\{ \int_{-1}^{1} |g(u) - \hat{g}(u)|^2 \, du \right\}^{1/2}$$

$$= \left\{ \sum_{0}^{\infty} |a_n - \hat{a}_n|^2 \right\}^{1/2}$$

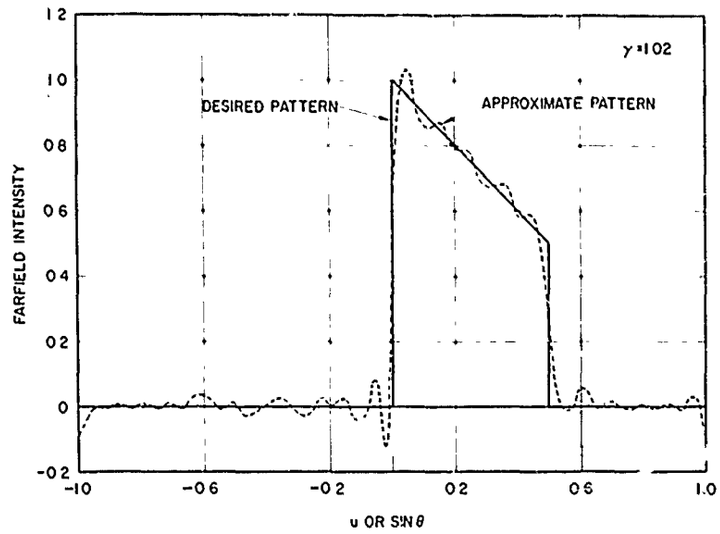

Figure 7. Pattern approximation by eigenelements for c = 20π.

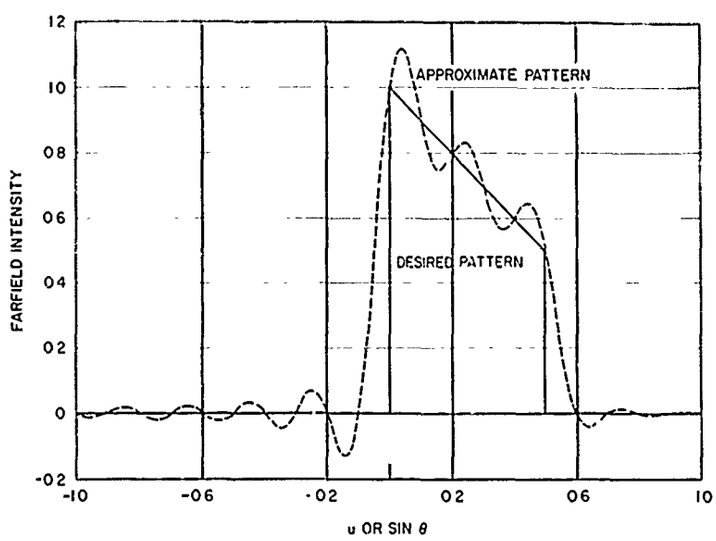

Figure 8. Pattern approximation by Woodward's method for c = 10π.

	$\gamma = 1.2$ 10 λ aperture	$\gamma = 10$ 10 λ aperture	$\gamma = 1.02$ 20 λ aperture	$\gamma = 1.0003$* 10 λ aperture	Woodward $\gamma = 1.0003$ 10 λ aperture
Error	0.0261	0.0162	0.0108	0.1005	0.2004

The norm of g is

$$||g|| = \left\{ \int_{-1}^{1} |g(u)|^2 \, du \right\}^{1/2}$$

$$= \left\{ \int_{0}^{1/2} (1 - u)^2 \, du \right\}^{1/2}$$

$$\doteq 0.5401$$

*This error is estimated; the actual error, being the optimum, is less than this figure, since the estimation was made by using the proper subspace spanned by the first 19 eigenfunctions. γ associated with elements in this subspace is less than 1.0003.

3.0 TABLES OF EIGENELEMENTS

The tables of eigenvalues and eigenfunctions for $c = 10\pi$, 15π, 20π, and 25π are presented in this section.

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Table 3-1. Eigenvalues for $c=10\pi$ or aperture length = 10λ

n	λ_n	n	λ_n
0	.44721	15	-j.44710
1	j.44721	16	.44650
2	-.44721	17	j.44306
3	-j.44721	18	-.42565
4	.44721	19	-j.36716
5	j.44721	20	25497
6	-.44721	21	j.13733
7	-j.44721	22	-.05993
8	44721	23	-j.02524
9	j.44721	24	.00848
10	-.44720	25	j.00288
11	-j.44720	26	-.00104
12	.44720	27	-j.00038
13	j.44720	28	.00014
14	-.44719	29	j.00005

Table 3-2. Eigenfunctions for $c = 10\pi$ or aperture length $= 10\lambda$

$$\psi_n(x) = F_n(x) \times 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	1.77283		0		1.23759		0		1.05663		0		.949254		0		0
.01	1.77011		.138582		1.22812		.165703		1.04232		.180512		.931026		.189502		.01
.02	1.76199		.275913		1.19986		.327919		.999771		.355138		.877039		.370727		.02
.03	1.74853		.41076		1.15327		.483239		.930142		.51819		.789361		.535765		.03
.04	1.72985		.541926		1.08908		.628413		.835324		.664364		.671348		.677414		.04
.05	1.70613		.668268		1.00851		.760421		.717886		.788921		.527509		.789503		.05
.06	1.67756		.788712		.912234		.876539		.580998		.887836		.363334		.867195		.06
.07	1.64439		.902267		.802344		.974401		.428343		.957934		.185075		.907002		.07
.08	1.6069		1.00804		.680339		1.05205		.264008		.99699		-4.95587	4	.907323		.08
.09	1.56542		1.10525		.54808		1.10796		9.23682	2	1.00379		-.18635		.868121		.09
.1	1.52029		1.19322		.407555		1.14111		-8.20381	2	.978179		-.365472		.791107		.1
.11	1.47187		1.27141		.260841		1.15094		-.254636		.921035		-.531131		.679624		.11
.12	1.42055		1.33942		.110065		1.13738		-.420939		.834249		-.677132		.53849		.12
.13	1.36673		1.39696		-4.26355	2	1.10088		-.576673		.720642		-.798046		.37378		.13
.14	1.31082		1.44387		-.195154		1.0423		-.717888		.583867		-.889414		.192558		.14
.15	1.25324		1.48015		-.32545		.962981		-.841059		.428278		-.947904		2.56013	3	.15
.16	1.19439		1.50589		-.491581		.864626		-.943174		.258777		-.97142		-.188136		.16
.17	1.13469		1.52133		-.631742		.74929		-1.0218		8.06541	2	-.959173		-.371466		.17
.18	1.07453		1.52679		-.764283		.619314		-1.07515		-.100594		-.91169		-.539719		.18
.19	1.01429		1.52271		-.88774		.477262		-1.10208		-.27924		-.830776		-.685862		.19
.2	.95433		1.50963		-1.00085		.325858		-1.10216		-.450501		-.715434		-.803832		.2
.21	.894998		1.48815		-1.10257		.167922		-1.07558		-.608703		-.581729		-.888778		.21
.22	.836609		1.45895		-1.19207		6.30366	3	-1.02323		-.749436		-.422629		-.93725		.22
.23	.779455		1.42274		-1.26877		-.156184		-.946569		-.868684		-.247802		-.947322		.23
.24	.723797		1.3803		-1.33229		-.31682		-.847595		-.963132		-6.34048	2	-.918656		.24
.25	.66987		1.33241		-1.3825		-.473034		-.728788		-1.03025		.124153		-.852487		.25
.26	.617874		1.27987		-1.41945		-.622455		-.593011		-1.06835		.308441		-.751549		.26
.27	.56798		1.2235		-1.44342		-.762947		-.443427		-1.07659		.483232		-.61994		.27
.28	.52033		1.16408		-1.45484		-.892651		-.2834		-1.05502		.642721		-.462926		.28
.29	.475032		1.10238		-1.45454		-1.01		-.116408		-1.00449		.781713		-.286717		.29
.3	.432167		1.03914		-1.44266		-1.11376		9.40593	2	-.926622		.895786		-9.81906	2	.3
.31	.391789		.975068		-1.42066		-1.20298		.224582		-.823755		.98142		9.53393	2	.31
.32	.353921		.910803		-1.3893		-1.27708		.391896		-.698804		1.03609		.286668		.32
.33	.318567		.846948		-1.34962		-1.33576		.552966		-.555177		1.05832		.468488		.33
.34	.285706		.784046		-1.30269		-1.37904		.705047		-.396645		1.04768		.634161		.34
.35	.255294		.722576		-1.24961		-1.4072		.845737		-.227218		1.00479		.777712		.35
.36	.227274		.66296		-1.19148		-1.42078		.973016		-.051017		.931233		.894082		.36
.37	.20157		.605553		-1.12938		-1.42056		1.08527		.127847		.829473		.979296		.37
.38	.178094		.550652		-1.06436		-1.40748		1.18129		.305392		.702738		1.03057		.38
.39	.156747		.498492		-.997417		-1.38266		1.26032		.477868		.554876		1.04637		.39
.4	.13742		.449249		-.929476		-1.34732		1.32198		.641849		.390206		1.02643		.4
.41	.12		.403048		-.861397		-1.30278		1.36629		.794307		.213352		.971721		.41
.42	.104368		.359962		-.793954		-1.2504		1.39364		.932671		2.90819	2	.884331		.42
.43	9.04035	2	.320016		-.727833		-1.19155		1.40471		1.05487		-.157848		.767374		.43
.44	7.79836	2	.283197		-.663628		-1.1276		1.4005		1.15935		-.342846		.624819		.44
.45	6.69877	2	.249453		-.601841		-1.05987		1.3822		1.24509		-.521613		.461309		.45
.46	5.72967	2	.218702		-.542884		-.989607		1.35121		1.31156		-.690261		.281965		.46
.47	4.87952	2	.190837		-.487078		-.917995		1.30905		1.35874		-.845403		9.21825	2	.47
.48	4.13716	2	.165725		-.434663		-.846107		1.25734		1.38706		-.984226		-.102572		.48
.49	3.49198	2	.143222		-.385795		-.774903		1.19771		1.39733		-1.10453		-.29695		.49
.5	2.93392	2	.123168		-.340574		-.705231		1.13183		1.39073		-1.20475		-.4859		.5
.51	2.45354	2	.105395		-.299011		-.637813		1.06128		1.3687		-1.28397		-.664816		.51
.52	2.04204	2	8.97308	2	-.261079		-.573246		.987599		1.3329		-1.34186		-.829663		.52
.53	.016913		7.60031	2	-.226694		-.512011		.912212		1.28517		-1.3787		-.977076		.53
.54	1.39385	2	6.40397	2	-.195735		-.454469		.83642		1.2274		-1.39524		-1.10442		.54
.55	1.14288	2	5.36728	2	-.168045		-.400874		.761384		1.16154		-1.39272		-1.20982		.55
.56	9.32239	3	4.47406	2	-.143443		-.35138		.688113		1.08952		-1.37272		-1.29216		.56
.57	7.56378	3	.037089		-.121728		-.306049		.61746		1.01318		-1.33713		-1.35108		.57
.58	6.10350	3	3.05727	2	-.102688		-.264866		.55012		.934267		-1.28802		-1.3869		.58
.59	4.89765	3	2.50561	2	-8.61042	2	-.227747		.486658		.854368		-1.22762		-1.40054		.59
.6	3.90751	3	2.04138	2	-7.17549	2	-.194553		.42741		.774916		-1.15817		-1.39347		.6
.61	3.09919	3	1.65313	2	-5.94222	2	-.165097		.372698		.697156		-1.08191		-1.3676		.61
.62	2.44320	3	1.33043	2	-4.88943	2	-.139156		.32264		.622139		-1.00101		-1.32515		.62
.63	1.91405	3	1.06392	2	-3.99683	2	-.116493		.277265		.550722		-.917461		-1.26857		.63
.64	1.48988	3	8.45238	3	-3.24528	2	-9.68394	2	.236511		.483568		-.833118		-1.20046		.64
.65	1.15203	3	6.66994	3	-2.61693	2	-7.99285	2	.200234		.421159		-.749607		-1.12342		.65
.66	8.84690	4	5.22694	3	-2.09534	2	-6.54905	2	.168229		.363804		-.668332		-1.04		.66
.67	6.74584	4	4.06686	3	-1.65553	2	-5.32603	2	.140241		.311664		-.590456		-.952634		.67
.68	5.10607	4	3.14090	3	-1.31398	2	-4.29825	2	.115982		.264759		-.516905		-.863559		.68
.69	3.83553	4	2.40724	3	-1.02863	2	-.034415		9.51411	2	.222999		-.448371		-.774774		.69
.7	2.85841	4	1.83035	3	-7.98816	3	-2.73318	2	7.73954	2	.186196		-.385325		-.688013		.7
.71	2.11273	4	1.38028	3	-6.15223	3	-2.15248	2	.062421		.154089		-.328041		-.604721		.71
.72	1.54824	4	1.03199	3	-4.69764	3	-1.68049	2	4.99002	2	.12636		-.276612		-.526052		.72
.73	1.12445	4	7.64721	4	-3.55501	3	-1.30023	2	.039528		.102655		-.23098		-.452871		.73
.74	8.09023	5	5.61406	4	-2.66534	3	-9.96641	3	3.10169	2	.082595		-.190957		-.385772		.74
.75	5.76398	5	4.08143	4	-1.97896	3	-7.56528	3	2.41007	2	6.57955	2	-.156257		-.325099		.75
.76	4.06451	5	2.93699	4	-1.45446	3	-5.68452	3	1.85363	2	.051874		-.126517		-.270972		.76
.77	2.83516	5	2.09083	4	-1.05760	3	-4.22608	3	1.41054	2	.040461		-.101324		-.223322		.77
.78	1.95501	5	1.47166	4	-7.60433	4	-3.10691	3	1.06147	2	3.12075	2	-8.02324	2	-.181923		.78
.79	1.33188	5	1.02352	4	-5.40319	4	-2.25741	3	7.89475	3	2.37899	2	-.062785		-.146425		.79
.8	8.95751	6	7.02859	5	-3.79131	4	-1.61993	3	5.79980	3	1.79135	2	-4.85286	2	-.116588		.8
.81	5.94126	6	4.76166	5	-2.62506	4	-1.14727	3	4.20555	3	1.33148	2	-3.70261	2	-9.13115	2	.81
.82	3.88293	6	3.17975	5	-1.79192	4	-8.01207	4	3.00759	3	9.76182	3	-2.78665	2	-7.06627	2	.82
.83	2.49773	6	2.09070	5	-1.20471	4	-5.51216	4	2.11936	3	7.05322	3	-2.06712	2	-5.38986	2	.83
.84	1.57950	6	1.35196	5	-7.96796	5	-3.73180	4	1.47001	3	5.01734	3	-1.50992	2	-4.04664	2	.84
.85	9.79625	7	8.58600	6	-5.17752	5	-2.48303	4	1.00240	3	3.50983	3	-1.08486	2	-2.99189	2	.85
.86	5.96835	7	5.34653	6	-3.30022	5	-1.62129	4	6.71041	4	2.41123	3	-7.65709	3	-2.17252	2	.86
.87	3.54990	7	3.25872	6	-2.05973	5	-1.03705	4	4.40268	4	1.62419	3	-5.30120	3	-1.54792	2	.87
.88	2.05603	7	1.93327	6	-1.25595	5	-6.48477	5	2.82540	4	1.07067	3	-3.59351	3	-1.08035	2	.88
.89	1.15010	7	1.12374	6	-7.46273	6	-3.95413	5	1.76928	4	6.89134	4	-2.37987	3	-7.37076	3	.89
.9			6.31495	7	-4.30020	6	-2.34398	5	1.07790	4	4.31868	4	-1.53573	3	-4.90339	3	.9
.91			3.42802	7	-2.40657	6	-1.34577	5	6.36601	5	2.62593	4	-9.62392	4	-3.17063	3	.91
.92			1.79096	7	-1.29178	6	-7.44334	6	3.62715	5	1.54221	4	-5.83201	4	-1.98476	3	.92
.93					-6.62602	7	-3.94288	6	1.98173	5	8.69737	5	-3.35852	4	-1.19644	3	.93
.94					-3.23986	7	-1.39257	6	1.02956	5	4.67237	5	-1.89011	4	-6.89580	4	.94
.95					-1.46985	7	-9.34054	7	5.07534	6	2.36459	5	-9.92610	5	-3.76194	4	.95
.96							-4.05361	7	2.26427	6	1.10794	5	-4.84400	5	-1.91337	4	.96
.97							-1.56248	7	9.13867	7	4.67710	6	-2.14049	5	-8.85270	5	.97
.98									3.12032	7	1.68525	6	-8.15732	6	-3.56041	5	.98
.99	(Where no values are listed, the absolute value is less than 10^{-7})										4.55147	7	-2.39281	6	-1.12054	5	.99
1													-3.38030	7	-1.81495	6	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-2. (continued)

x	$F_8(x)$	p	$F_9(x)$	p	$F_{10}(x)$	p	$F_{11}(x)$	p	$F_{12}(x)$	p	$F_{13}(x)$	p	$F_{14}(x)$	p	$F_{15}(x)$	p	x
0	.871835		0		.809538		0		.755149		0		.703694		0		0
.01	.850381		.194718		.785461		.196955		.729038		.196384		.676198		.19249		.01
.02	.787071		.378879		.714657		.381267		.652505		.378324		.59585		.369165		.02
.03	.685006		.542497		.601317		.5411		.530816		.532438		.4689		.515506		.03
.04	.549186		.676705		.45215		.666191		.372349		.647382		.305228		.619476		.04
.05	.38626		.774227		.275985		.748501		.188012		.714685		.117571		.672509		.05
.06	.204198		.829777		8.32446	2	.782732		-9.50492	3	.729372		-7.94609	2	.670212		.06
.07	1.18935	2	.840342		-.114673		.766663		-.206605		.690322		-.270523		.612722		.07
.08	-.181266		.805339		-.306067		.701294		-.389715		.600359		-.440727		.504703		.08
.09	-.363869		.726644		-.479627		.590773		-.546223		.466036		-.576792		.354963		.09
.1	-.532933		.608483		-.6251		.442134		-.665337		.297163		-.668082		.175737		.1
.11	-.674344		.457203		-.733892		.264841		-.738831		.106085		-.70743		-1.83121	2	.11
.12	-.783254		.280919		-.799572		7.01868	2	-.761601		-9.32214	2	-.691691		-.211289		.12
.13	-.854395		8.90747	2	-.818245		-.129436		-.732016		-.286166		-.621994		-.387364		.13
.14	-.884339		-.108069		-.788775		-.321319		-.652026		-.458613		-.503651		-.532067		.14
.15	-.871648		-.299285		-.712842		-.493249		-.527029		-.59791		-.345753		-.633472		.15
.16	-.816936		-.476447		-.59484		-.634279		-.365493		-.693812		-.160464		-.683167		.16
.17	-.722819		-.62807		-.44161		-.735424		-.178384		-.739225		3.79142	2	-.676953		.17
.18	-.593787		-.746805		-.262031		-.79022		2.15889	2	-.730724		.234045		-.615185		.18
.19	-.435964		-.826363		-6.65016	2	-.795129		.220836		-.668799		.41274		-.502749		.19
.2	-.256813		-.86253		.133673		-.749755		.405812		-.55782		.560131		-.348678		.2
.21	-6.47661	2	-.853383		.326936		-.656859		.56333		-.405719		.664737		-.165412		.21
.22	.131182		-.799374		.502149		-.522178		.684415		-.223414		.718352		3.21952	2	.22
.23	.321902		-.703287		.649227		-.354059		.759027		-2.40231	2	.716672		.228089		.23
.24	.498558		-.57008		.759714		-.162926		.782612		.178086		.659634		.406309		.24
.25	.653018		-.406609		.827258		3.93723	2	.75344		.36833		.551423		.552282		.25
.26	.77822		-.221255		.847957		.240305		.673319		.532958		.400151		.654009		.26
.27	.868485		-2.34839	2	.820568		.427437		.547466		.660036		.217248		.703036		.27
.28	.919763		.176653		.746559		.589195		.384147		.740306		.016583		.695146		.28
.29	.929787		.369034		.630007		.715576		.194126		.767837		-.186581		.630708		.29
.3	.898156		.543984		.477352		.798752		-1.00612	2	.740454		-.376753		.514658		.3
.31	.826321		.692754		.297016		.833535		-.214937		.659883		-.539387		.356108		.31
.32	.717488		.807953		9.89236	2	.817685		-.406977		.53163		-.661986		.167628		.32
.33	.576447		.883892		-.106066		.752021		-.573496		.364584		-.735045		-3.57617	2	.33
.34	.409329		.916852		-.30676		.640356		-.703479		.170388		-.752777		-.23778		.34
.35	.223311		.905231		-.492248		.489249		-.788295		-3.73868	2	-.713544		-.42219		.35
.36	2.62836	2	.849593		-.652494		.307596		-.822245		-.244192		-.619987		-.574094		.36
.37	-.173499		.752606		-.778868		.106078		-.802924		-.435525		-.478831		-.681124		.37
.38	-.367797		.618876		-.864592		-.103479		-.731355		-.597938		-.300383		-.734432		.38
.39	-.548712		.454692		-.905078		-.308815		-.611915		-.71997		-9.77631	2	-.729401		.39
.4	-.70902		.267695		-.898142		-.497952		-.45203		-.792938		.114063		-.666022		.4
.41	-.842453		6.64919	2	-.844086		-.659894		-.26169		-.811527		.319401		-.548906		.41
.42	-.94392		-.139769		-.745641		-.785248		-5.27977	2	-.77415		.502979		-.386932		.42
.43	-1.00968		-.341833		-.607786		-.866749		.161591		-.683041		.651079		-.192559		.43
.44	-1.03746		-.530761		-.457456		-.89965		.368096		-.544094		.752543		1.91413	2	.44
.45	-1.02645		-.698333		-.243149		-.881961		.553847		-.366451		.799585		.231659		.45
.46	-.977335		-.837392		-3.44669	2	-.814519		.707277		-.16187		.788361		.428332		.46
.47	-.892163		-.942132		.178389		-.700909		.818823		5.60744	2	.719242		.593638		.47
.48	-.774222		-1.00831		.385136		-.547219		.881496		.272906		.59679		.714407		.48
.49	-.627854		-1.03337		.575923		-.361666		.891272		.474213		.429419		.780843		.49
.5	-.458235		-1.01652		.741791		-.15411		.847311		.646601		.22879		.787285		.5
.51	-.271134		-.958642		.875086		6.45061	2	.751962		.778567		8.97502	3	.732653		.51
.52	-7.26678	2	-.862246		.96978		.282765		.510592		.861236		-.214549		.62054		.52
.53	.130955		-.731251		1.02171		.489391		.431221		.888915		-.425995		.458944		.53
.54	.333667		-.570772		1.0287		.673836		.224017		.85944		-.610374		.259673		.54
.55	.529758		-.386845		.990605		.826808		6.75305	4	.774277		-.754542		3.74556	2	.55
.56	.714061		-.186133		.909209		.940718		-.22628		.638396		-.848102		-.191163		.56
.57	.882107		2.45833	2	.788094		1.01001		-.444225		.459922		-.884116		-.409055		.57
.58	1.03023		.237704		.632379		1.03138		-.641145		.249591		-.859555		-.599788		.58
.59	1.15566		.447092		.448423		1.00387		-.80629		2.00507	2	-.77549		-.748842		.59
.6	1.25651		.646327		.243481		.928808		-.930737		-.214941		-.636993		-.844677		.6
.61	1.33182		.829916		2.53365	2	.809674		-1.00783		-.441365		-.452766		-.879576		.61
.62	1.38148		.993257		-.198071		.651827		-1.03346		-.645792		-.234535		-.850208		.62
.63	1.40619		1.13276		-.418995		.462168		-1.00622		-.816169		3.74878	3	-.757856		.63
.64	1.40733		1.24589		-.630208		.248732		-.927394		-.942494		.246846		-.608319		.64
.65	1.38685		1.33121		-.825274		2.02484	2	-.800753		-1.01736		.479227		-.411483		.65
.66	1.34717		1.38833		-.998769		-.214304		-.632293		-1.0363		.686072		-.18061		.66
.67	1.29104		1.4178		-1.14643		-.446135		-.429817		-.997986		.854198		6.86074	2	.67
.68	1.22139		1.42107		-1.26527		-.667032		-.20245		-.904159		.972855		.319151		.68
.69	1.14124		1.40028		-1.35354		-.86971		3.98869	2	-.759468		1.03435		.553857		.69
.7	1.05357		1.35816		-1.41079		-1.04807		.28702		-.571083		1.03444		.75658		.7
.71	.961216		1.29784		-1.43769		-1.19741		.529047		-.348196		.972527		.913278		.71
.72	.866798		1.22272		-1.43595		-1.3145		.756815		-.101435		.851555		1.01292		.72
.73	.772665		1.13626		-1.40812		-1.39766		.962322		.15779		.677755		1.04815		.73
.74	.680844		1.0419		-1.35743		-1.44683		1.13904		.417936		.460147		1.01574		.74
.75	.59301		.942914		-1.28756		-1.46255		1.28211		.667959		.209903		.916667		.75
.76	.510486		.842287		-1.20248		-1.44771		1.38849		.857869		-6.03904	2	.75599		.76
.77	.434245		.742674		-1.10622		-1.40536		1.45692		1.0992		-.337533		.542385		.77
.78	.364928		.646329		-1.0027		-1.33948		1.48785		1.26536		-.6085		.287503		.78
.79	.302875		.555078		-.895624		-1.25452		1.48327		1.39183		-.861182		5.12915	3	.79
.8	.248162		.470317		-.788312		-1.15515		1.44649		1.47626		-1.08503		-.289757		.8
.81	.200641		.393017		-.683644		-1.04603		1.38181		1.51842		-1.27155		-.581963		.81
.82	.159983		.323761		-.583994		-.931612		1.29425		1.52005		-1.41469		-.857029		.82
.83	.12572		.262778		-.491209		-.815967		1.18924		1.48456		-1.51095		-1.10209		.83
.84	9.72917	2	.209995		-.406608		-.702653		1.07276		1.41674		-1.55946		-1.30657		.84
.85	7.40765	2	.165093		-.331014		-.594631		.948653		1.32236		-1.56181		-1.46272		.85
.86	5.54293	2	.127561		-.264793		-.494216		.823321		1.20775		-1.52172		-1.5659		.86
.87	4.07081	2	9.67505	2	-.207923		-.403068		.700573		1.07946		-1.44471		-1.61467		.87
.88	2.92965	2	7.19789	2	-.160054		-.322222		.583997		.943865		-1.33757		-1.61064		.88
.89	2.06208	2	5.23231	2	-.120987		-.252137		.476377		.806844		-1.20791		-1.55818		.89
.9	1.41619	2	3.71532	2	-8.87403	2	-.192771		.379683		.673555		-1.06358		-1.46392		.9
.91	9.46171	3	2.56933	2	-6.36259	2	-.143673		.295092		.548245		-.912261		-1.33614		.91
.92	6.12640	3	1.72351	2	-4.43004	2	-.104075		.223058		.434152		-.76103		-1.18415		.92
.93	3.82532	3	1.11642	2	-2.98247	2	-.072987		.163409		.333477		-.616007		-1.0176		.93
.94	2.28788	3	6.93954	3	-1.93021	2	-4.92894	2	.115465		.247414		-.482138		-.845878		.94
.95	1.29831	3	4.10268	3	-1.19103	2	-3.18126	2	7.81603	2	.176247		-.363058		-.677515		.95
.96	6.89174	4	2.27657	3	-6.92162	3	-1.54051	2	5.01769	2	.119483		-.261052		-.519782		.96
.97	3.34410	4	1.16056	3	-3.71440	3	-1.09876	2	.030062		7.60084	2	-.177117		-.378356		.97
.98	1.42171	4	5.28654	4	-1.77588	3	-5.59160	3	1.63549	2	4.42675	2	-.111093		-.257155		.98
.99	4.80571	5	1.90316		-6.98578	4	-2.38381	3	7.57587	3	2.24351	2	-6.18556	2	-.158312		.99
1	3.88893	6	[illegible]		-1.71525	4	-6.76809	4	2.49327	3	8.57927	3	-2.75473	2	-8.22656	2	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-2. (continued)

x	$F_{16}(x)$	p	$F_{17}(x)$	p	$F_{18}(x)$	p	$F_{19}(x)$	p	$F_{20}(x)$	p	$F_{21}(x)$	p	$F_{22}(x)$	p	$F_{23}(x)$	p	x
0	.648877		0		.578864		0		-.545372		0		-.53833		0		0
.01	.520869		.18291		.551834		.168147		-.518299		.179983		-.566628		.202596		.01
.02	.539253		.349389		.473259		.32021		-.439757		.341581		-.475094		.383029		.02
.03	.411045		.484475		.350446		.441406		-.317515		.468281		-.333203		.52155		.03
.04	.247268		.576017		.194824		.520106		-.163665		.547117		-.15597		.602977		.04
.05	6.20103	2	.615761		2.08765	2	.548708		6.56380	3	.570001		3.79396	2	.618355		.05
.06	-.128788		.600095		-.155196		.524412		.176311		.53454		.227971		.565934		.06
.07	-.308697		.53057		-.316981		.4495		.526747		.444292		.394128		.451372		.07
.08	-.46221		.412786		-.449378		.3311		.448734		.308408		.518788		.287124		.08
.09	-.576072		.257841		-.54		.180602		.524322		.14071		.588705		9.11011	2	.09
.1	-.640419		7.93977	2	-.580331		1.23855	2	.547938		-4.17107	2	.596389		-.115278		.1
.11	-.64965		-.106556		-.566518		-.157367		.517139		-.220225		.540897		-.309415		.11
.12	-.60281		-.283332		-.499741		-.312311		.334861		-.376564		.427355		-.469995		.12
.13	-.505874		-.435033		-.386105		-.437494		.309148		-.494675		.26537		-.579318		.13
.14	-.361221		-.547979		-.236033		-.520784		.15636		-.562371		8.17396	2	-.625246		.14
.15	-.187015		-.611932		-6.35921	2	-.554048		-2.00287	2	-.572577		-.114983		-.602543		.15
.16	3.84798	3	-.621017		.115381		-.533934		-.190955		-.52407		-.320169		-.513478		.16
.17	.195008		-.57426		.294163		-.462213		-.343446		-.421623		-.4541		-.367601		.17
.18	.370043		-.475684		.426992		-.345621		-.4623		-.275538		-.56035		-.180731		.18
.19	.513871		-.333961		.530467		-.195227		-.535592		-.100617		-.607469		2.67469	2	.19
.2	.614043		-.161652		.584803		-2.53909	2	-.555662		8.53277	2	-.590216		.232094		.2
.21	.66181		2.59087	2	.584748		.147606		-.520886		.263282		-.51014		.412697		.21
.22	.652884		.211968		.530084		.307113		-.433891		.414963		-.37545		.548546		.22
.23	.587809		.379849		.425669		.437705		-.303284		.524688		-.200183		.624445		.23
.24	.471929		.514436		.281003		.526667		-.141834		.580991		-2.75096	3	.631709		.24
.25	.314946		.603527		.109365		.565225		3.45702	2	.577814		.195981		.569155		.25
.26	.130103		.638934		-.073399		.549406		.208473		.515159		.374887		.443286		.26
.27	-6.69283	2	.617222		-.250337		.480441		.362572		.399121		.514815		.267625		.27
.28	-.25938		.540037		-.404951		.364673		.481428		.241301		.600629		6.12798	2	.28
.29	-.430903		.413977		-.52273		.21298		.553008		5.76591	2	.62284		-.153117		.29
.3	-.566468		.250027		-.592502		3.97531	2	.569892		-.133091		.578654		-.351875		.3
.31	-.65462		6.26021	2	-.607484		-.13846		.530042		-.311379		.472317		-.512851		.31
.32	-.68748		-.131718		-.565932		-.304526		.437033		-.45877		.314724		-.617898		.32
.33	-.661926		-.315648		-.471334		-.442368		.299744		-.559864		.122314		-.654915		.33
.34	-.579765		-.472729		-.33211		-.538508		.131513		-.603896		-8.46275	2	-.619233		.34
.35	-.447606		-.58879		-.160871		-.583376		-.051148		-.585876		-.284031		-.514207		.35
.36	-.276316		-.653212		2.67233	2	-.572255		-.230163		-.50715		-.454668		-.350925		.36
.37	-8.01244	2	-.659886		.213389		-.505767		-.387671		-.375318		-.577835		-.147061		.37
.38	.124559		-.607781		.381802		-.389854		-.507793		-.203512		-.639955		7.50136	2	.38
.39	.320517		-.50106		.515196		-.23525		-.578227		-9.10049	3	-.633785		.290659		.39
.4	.491161		-.348743		.60382		-5.64931	2	-.591492		.188049		-.559313		.475682		.4
.41	.621925		-.163944		.636143		.129431		-.54572		.367587		-.423846		.609032		.41
.42	.701494		3.72597	2	.609665		.304696		-.44489		.510766		-.241309		.675183		.42
.43	.722758		.237221		.526276		.452329		-.298497		.602384		-.03084		.665944		.43
.44	.68343		.418262		.393132		.55784		-.120663		.63239		.165192		.581469		.44
.45	.586255		.564226		.222037		.61063		7.12075	2	.596973		.383544		.43036		.45
.46	.438809		.661898		2.83987	2	.605044		.25812		.499038		.542574		.228828		.46
.47	.252894		.702185		-.170148		.540958		.421352		.347985		.64459		-1.00924	3	.47
.48	4.35867	2	.680937		-.355339		.423851		.544309		.158831		.677802		-.233514		.48
.49	-.171989		.599348		-.509245		.26434		.614201		-4.92645	2	.637681		-.442359		.49
.5	-.37606		.463888		-.619357		7.72517	2	.62335		-.254915		.527559		-.603473		.5
.51	-.551655		.285787		-.672344		-.119847		.570027		-.43668		.358392		-.697804		.51
.52	-.684		8.00977	2	-.665064		-.308091		.458694		-.57528		.147712		-.713556		.52
.53	-.751732		-.135578		-.595631		-.469292		.299656		-.655617		-8.21579	2	-.647666		.53
.54	-.777845		-.342587		-.47018		-.587578		.10812		-.668407		-.306426		-.506322		.54
.55	-.730269		-.52282		-.29943		-.650958		-9.72371	2	-.611231		-.500473		-.304447		.55
.56	-.622087		-.660279		-9.83644	2	-.652506		-.296068		-.48891		-.64252		-.06419		.56
.57	-.46129		-.742464		.115071		-.591097		-.468342		-.313131		-.71605		.187446		.57
.58	-.260175		-.761473		.32156		-.47159		-.596337		-.101367		-.711712		.421577		.58
.59	-3.43842	2	-.714722		.502138		-.304462		-.666433		.124823		-.628489		.6107		.59
.6	.198326		-.605212		.639913		-.1049		-.670537		.34195		-.473999		.731889		.6
.61	.419475		-.441325		.721606		.108564		-.606982		.526993		-.263862		.769548		.61
.62	.611314		-.236171		.738793		.315744		-.480818		.6598		-2.01839	2	.717407		.62
.63	.758222		-6.52538	3	.688728		.496684		-.30344		.725245		.230699		.579505		.63
.64	.847932		.22854		.574664		.633542		-9.15906	2	.714921		.461001		.370024		.64
.65	.872494		.44923		.405652		.712305		.134184		.628175		.644499		.111942		.65
.66	.628911		.636668		.195821		.724183		.351485		.47237		.759465		-.16537		.66
.67	.71938		.774467		-3.68009	2	.666528		.538233		.262327		.791196		-.429378		.67
.68	.551141		.850108		-.271815		.543211		.67486		1.89817	2	.733852		-.648095		.68
.69	.335942		.856002		-.488215		.364385		.746303		-.232632		.591394		-.79387		.69
.7	8.91696	2	.790161		-.666219		.145676		.743604		-.465876		.377466		-.846828		.7
.71	-.1713		.656409		-.78902		-9.31557	2	.664961		-.655244		.114222		-.79753		.71
.72	-.426411		.46413		-.844295		-.329961		.516093		-.779067		-.169836		-.648533		.72
.73	-.657324		.297567		-.825353		-.542196		.3099		-.821918		-.442795		-.414608		.73
.74	-.846777		-3.52698	2	-.731804		-.708996		6.54013	2	-.776437		-.672779		-.121517		.74
.75	-.98031		-.303991		-.569705		-.813138		-.193942		-.644413		-.831528		.196564		.75
.76	-1.04727		-.557401		-.351156		-.842718		-.442383		-.436935		-.897746		.500775		.76
.77	-1.04151		-.775136		-9.33836	2	-.79237		-.654336		-.17363		-.859844		.752059		.77
.78	-.961797		-.939219		.182618		-.663941		-.806879		.119076		-.717732		.915951		.78
.79	-.811763		-1.03544		.453765		-.46652		-.882089		.410387		-.483443		.967107		.79
.8	-.599614		-1.0544		.696754		-.21583		-.868969		.668186		-.180437		.893046		.8
.81	-.337454		-.992222		.889964		6.69732	2	-.764798		.862339		.158398		.696595		.81
.82	-4.03692	2	-.850805		1.01522		.357018		-.57574		.967959		.493928		.396652		.82
.83	.274663		-.637623		1.05928		.627774		-.316642		.968314		.784851		2.70669	2	.83
.84	.589971		-.365133		1.01496		.853308		-1.00288	2	.857063		.552359		-.366426		.84
.85	.888591		-.049801		.881704		1.01051		.315654		.63958		1.08458		-.731006		.85
.86	1.15438		.289149		.665696		1.08115		.628396		.333203		1.04138		-1.01293		.86
.87	1.37496		.631163		.379366		1.05347		.895382		-3.36869	2	.857233		-1.16447		.87
.88	1.54049		.955858		4.04485	2	.923377		1.08591		-.423695		.543641		-1.15096		.88
.89	1.64508		1.24446		-.3294		.694883		1.17425		-.793426		.129582		-.957165		.89
.9	1.68682		1.48106		-.706264		.379945		1.14209		-1.09724		-.339704		-.592163		.9
.91	1.66763		1.65369		-1.06581		-2.38782	3	.980467		-1.2914		-.805706		-9.21723	2	.91
.92	1.59292		1.75496		-1.3851		-.427406		.690896		-1.33833		-1.20121		.47929		.92
.93	1.47039		1.78242		-1.64432		-.866657		.285653		-1.21048		-1.45614		1.03415		.93
.94	1.31227		1.73848		-1.82818		-1.29017		-.212889		-.893465		-1.50413		1.46545		.94
.95	1.12846		1.63005		-1.92698		-1.66882		-.733789		-.388296		-1.28935		1.65569		.95
.96	.931677		1.46775		-1.93714		-1.9766		-1.35983		.287796		-.772889		1.48724		.96
.97	.733579		1.26499		-1.86135		-2.19265		-1.93041		1.10183		.061985		.854008		.97
.98	.544634		1.03683		-1.70813		-2.30288		-2.44472		2.00699		1.20607		-.32658		.98
.99	.373518		.798806		-1.49098		-2.30097		-2.86507		2.94576		2.6226		-2.1025		.99
1	.226704		.565764		-1.22721		-2.18882		-3.15993		3.85423		4.24795		-4.47798		1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-2. (continued)

x	$F_{24}(x)$	p	$F_{25}(x)$	p	$F_{26}(x)$	p	$F_{27}(x)$	p	$F_{28}(x)$	p	$F_{29}(x)$	p	x
0	.636265		0		.657344		0		.672186		0		0
.01	.640385		.211649		.61773		.234185		.627507		.247408		.01
.02	.645644		.413386		.601237		.458488		.439392		.460632		.02
.03	.334779		.558342		.323535		.586825		.311536		.610192		.03
.04	.135727		.637384		.10553		.660225		6.95985	2	.679376		.04
.05	-7.[illegible]	2	.641127		-.125393		.649252		-.17506		.647085		.05
.06	-.284215		.56905		-.341128		.555215		-.396648		.529135		.06
.07	-.458522		.423567		-.515158		.390014		-.56567		.337716		.07
.08	-.580254		.239054		-.616604		.174653		-.659558		.099207		.08
.09	-.636073		1.99356	2	-.66003		-.063423		-.665673		.103445		.09
.1	-.619517		-.201941		-.617791		-.233802		-.583012		-.385253		.1
.11	-.532195		-.403338		-.489947		-.486976		-.422373		-.564015		.11
.12	-.384173		-.551722		-.306426		-.618103		-.204966		-.664773		.12
.13	-.191857		-.638041		-8.46983	2	-.670193		4.03701	2	-.673298		.13
.14	2.28671	2	-.648878		.138144		-.636335		.280966		-.588103		.14
.15	.235592		-.58270A		.36531		-.520567		.484653		-.420709		.15
.16	.422049		-.4471		.534226		-.337464		.62405		-.194116		.16
.17	.560865		-.257898		.639615		-.110305		.680235		6.03239	2	.17
.18	.636005		-5.73545	2	.666185		.131864		.645322		.307219		.18
.19	.638621		.188456		.610305		.357918		.523579		.51203		.19
.2	.568036		.392701		.478513		.538653		.330934		.685893		.2
.21	.43216		.550967		.286769		.650457		9.29303	2	.689679		.21
.22	.246055		.644169		5.85479	2	.678573		-.158635		.636746		.22
.23	3.08435	2	.660848		-.17801		.618876		-.389922		.433965		.23
.24	-.188257		.598586		-.393546		.478583		-.569585		.280874		.24
.25	-.388148		.464358		-.561107		.275392		-.673004		2.70536	2	.25
.26	-.543751		.273774		-.659513		3.52862	2	-.685669		-.231937		.26
.27	-.637546		4.92869	2	-.676027		-.210753		-.605232		-.459506		.27
.28	-.658395		-.182426		-.60802		-.430717		-.441944		-.623172		.28
.29	-.603576		-.393611		-.463389		-.595714		-.217398		-.699204		.29
.3	-.478291		-.558743		-.259673		-.683754		3.82741	2	-.676074		.3
.31	-.297057		-.657605		-2.19695	2	-.68269		.290402		-.556242		.31
.32	-8.01772	2	-.677759		.220129		-.591912		.504443		-.355375		.32
.33	.147546		-.616118		.436185		-.422562		.650695		-.103216		.33
.34	.359828		-.479424		.598724		-.196219		.708418		.166255		.34
.35	.531917		-.283541		.686721		5.77977	2	.6688		.413823		.35
.36	.643466		-5.16549	2	.698373		.30618		.536331		.603544		.36
.37	.680934		.188435		.602467		.515962		.328379		.707348		.37
.38	.639235		.407616		.459156		.656875		.072989		.70922		.38
.39	.522428		.57897		.218352		.715158		-.194828		.60771		.39
.4	.343368		.681067		-3.24098	2	.676309		-.437839		.416396		.4
.41	.122323		.700642		-.281405		.546399		-.621725		.162176		.41
.42	-.115277		.634339		-.496691		.341737		-.719925		-.118411		.42
.43	-.341735		.489261		-.650181		8.89049	2	-.717487		-.384354		.43
.44	-.530284		.282279		-.721305		-.178618		-.613394		-.5961		.44
.45	-.658233		3.81212	2	-.699776		-.424824		-.42093		-.721426		.45
.46	-.709687		-.213522		-.587103		-.615932		-.166339		-.740561		.46
.47	-.67753		-.441564		-.396623		-.725241		.115098		-.64843		.47
.48	-.564406		-.617324		-.152009		-.73642		.383399		-.457684		.48
.49	-.382587		-.718122		.11559		-.646437		.599731		-.195289		.49
.5	-.1527		-.730202		.371435		-.46603		.731951		.100182		.5
.51	9.85552	2	-.650633		.581657		-.218611		.759404		.384281		.51
.52	.341448		-.487897		.717707		6.26524	2	.6762		.613282		.52
.53	.54669		-.261091		.76023		.339165		.492468		.750873		.53
.54	.688932		2.22311	3	.701852		.572117		.233319		.773943		.54
.55	.749881		.26928		.548435		.727885		-6.43359	2	.67655		.55
.56	.720653		.506111		.318758		.782884		-.357172		.471382		.56
.57	.603067		.681814		4.21713	2	.727178		-.601084		.188389		.57
.58	.40969		.772557		-.244831		.566308		-.758314		-.129333		.58
.59	.1626		.764818		-.503406		.320995		-.803028		-.431878		.59
.6	-.109031		.657415		-.697519		2.46931	2	-.725691		-.670222		.6
.61	-.372286		.462031		-.798932		-.280777		-.535108		-.804106		.61
.62	-.592408		.202118		-.791321		-.55096		-.257785		-.80888		.62
.63	-.746816		-8.97561	2	-.672941		-.745132		6.54424	2	-.680159		.63
.64	-.808748		-.375834		-.457409		-.8324		.385136		-.435474		.64
.65	-.77008		-.617924		-.172402		-.796658		.6506		-.112277		.65
.66	-.632955		-.782411		.143639		-.639609		.817792		.237251		.66
.67	-.411952		-.844879		.446458		-.381318		.856635		.554299		.67
.68	-.135739		-.793742		.692001		-5.80477	2	.756523		.783058		.68
.69	.170709		-.632345		.842699		.282442		.529051		.880474		.69
.7	.460059		-.37919		.973193		.587621		.207339		.824444		.7
.71	.697291		-6.61465	2	.774624		.808049		-.158187		.61897		.71
.72	.843563		.265237		.456819		.905277		-.507155		.295103		.72
.73	.893681		.568939		.247899		.858573		-.778797		-9.28383	2	.73
.74	.819579		.800607		-.108817		.669366		-.922297		-.475532		.74
.75	.632337		.923857		-.460371		.362494		-.906288		-.780369		.75
.76	.352369		.915908		-.751572		-1.61807	2	-.725823		-.94484		.76
.77	1.36464	2	.771718		-.933327		-.405793		-.405355		-.929323		.77
.78	-.339355		.505948		-.97375		-.739435		3.19304	3	-.726818		.78
.79	-.659875		.152285		-.849799		-.955357		.427517		-.367416		.79
.8	-.899109		-.240018		-.581307		-1.00756		.786526		8.40477	2	.8
.81	-1.01875		-.612176		-.201493		-.876056		1.00495		.537642		.81
.82	-.994051		-.903907		.231537		-.572867		1.02869		.894457		.82
.83	-.819107		-1.06254		.645253		-.143601		.889154		1.05694		.83
.84	-.509535		-1.05204		.963508		.337537		.456715		.999933		.84
.85	-.102452		-.860661		1.11944		.777263		-4.82392	2	.688093		.85
.86	.346181		-.505837		1.06872		1.07997		-.573258		.18504		.86
.87	.76898		-3.53455	2	.801083		1.16632		-.996088		-.399592		.87
.88	1.09266		.476223		.347838		.992958		-1.20074		-.917557		.88
.89	1.2515		.935917		-.[illegible]16721		.56895		-1.10701		-1.2136		.89
.9	1.19717		1.2456		-.791531		-3.49057	2	-.699033		-1.16766		.9
.91	.910[illegible]		1.31897		-1.21431		-.687928		-4.61837	2	-.740189		.91
.92	.40931		1.10102		-1.3243		-1.21589		.691748		-9.49777	3	.92
.93	-.241475		.527962		-1.19[illegible]3		-1.43401		1.28286		.813683		.93
.94	-.931274		.154815		-.517918		-1.19693		1.47193		1.41092		.94
.95	-1.5037		-.979003		.2[illegible]431		-.464364		1.06432		1.43723		.95
.96	-1.765		-1.64692		1.2267		.622897		4.62877	2	.673637		.96
.97	-1.19693		-1.83077		1.8[illegible]		1.66122		-1.26147		-.738311		.97
.98	-.[illegible]		-1.12346		[illegible]1018		1.37724		-2.02556		-1.9773		.98
.99	1.51[illegible]07		.913938		-.[illegible]31219		.014943		-.710612		-1.14515		.99
1	[illegible].63206		4.77709		-4.89541		-5.00193		5.10184		5.18895		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-3. Eigenvalues for $c = 15\pi$ or aperture length = 15λ

n	λ_n	n	λ_n
0	.36515	18	-.36515
1	j.36515	19	-j.36515
2	-.36515	20	.36515
3	-j.36515	21	j.36514
4	.36515	22	-.36514
5	j.36515	23	-j.36513
6	-.36515	24	.36509
7	-j.36515	25	j.36493
8	.36515	26	-.36421
9	j.36515	27	-j.36054
10	-.36515	28	.34488
11	-j.36515	29	j.29765
12	.36515	30	-.21139
13	j.36515	31	-j.11750
14	-.36515	32	.04961
15	-j.36515	33	j.01863
16	.36515	34	-.00702
17	j.36515	35	

Table 3-4. Eigenfunctions for $c = 15\pi$ or aperture length = 15λ

$$\psi_n(x) = F_n(x) \times 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	[illegible]		0		1.37722		0		1.18213		0		1.35886		0		0
.01	1.35745		.188684		1.36129		[illegible]		1.1578		.249441		1.35749		.26441		.01
.02	1.35587		.37418		1.31389		.447128		1.08782		.486465		.945204		.511032		.02
.03	1.32344		.555[illegible]		1.23617		.653133		.963187		[illegible]		.79744		.743486		.03
.04	1.29245		.72[illegible]		1.1509[illegible]		.43745		.81273		.872585		.602868		.887412		.04
.05	1.25333		.89[illegible]		.997332		[illegible]		.602308		1.01138		.372884		.991951		.05
.06	1.2066		1.04471		.843[illegible]		1.1[illegible]48		.407116		1.0[illegible]52		1.0[illegible]1		1.03018		.06
.07	1.75[illegible]		1.18795		.669342		1.2140[illegible]		.133536		1.12[illegible]9		-.138408		.999651		.07
.08	1.69277		1.3061		.480362		1.26769		-5.47470	[illegible]	1.1[illegible]209		-.39008		[illegible]		.08
.09	1.6[illegible]715		1.4128		.210571		1.28[illegible]12		-.301657		1.02315		-.613654		.7453[illegible]3		.09
.1	1.55669		1.50275		.074357		1.[illegible]573		-.527555		.694313		-.81406		.958526		.1
.11	1.48232		1.57488		-.133861		1.19416		-.733084		.721993		-.962357		.29991		.11
.12	1.40485		1.62[illegible]		-.339784		1.0[illegible]		10919		.514647		-1.05634		3.33[illegible]61	[illegible]	.12
.13	1.3[illegible]513		1.6[illegible]		-.539341		.963691		1.05326		.282361		-1.09095		[illegible]		.13
.14	1.24401		1.6[illegible]538		-.72879		.803877		-1.15624		3.60169	2	-1.06456		-.48319		.14
.15	1.1623		1.69839		-.904807		.620762		-1.21601		-.212673		-.378933		-.703942		.15
.16	1.08075		1.67543		-1.06455		.41[illegible]		-1.23087		-.462336		-.839123		-.880421		.16
.17	1.0001		1.64318		-1.20573		.20641		-1.[illegible]085		-.672187		-.653059		-1.00168		.17
.18	9.0992		1.60952		-1.3266		-1.30739	2	-1.1[illegible]763		-.662579		-.531059		-1.06041		.18
.19	.844023		1.55[illegible]		-1.42601		-.233964		-1.01442		-1.01543		-.185188		-1.0933		.19
.2	.769309		1.49724		-1.50338		-.448082		-.869733		-1.12402		7.14527	2	-.981166		.2
.21	.698439		1.42544		-1.55868		-.653098		-.687[illegible]		-1.18515		.315474		-.348839		.21
.22	.630725		1.3528		-1.59239		-.84517		-.485211		-1.19719		.563926		-.664997		.22
.23	.5667		1.27222		-1.60943		-1.01604		-.266677		-1.15909		.774973		-.439851		.23
.24	.506621		1.19326		-1.59913		-1.1[illegible]		-3.87038	2	-1.07373		.948478		-.138069		.24
.25	.450625		1.10258		-1.57514		-1.29503		.191603		-.945413		1.07646		7.39628	2	.25
.26	.398778		1.01594		-1.53537		-1.39807		.417594		-.779722		1.1534		.337096		.26
.27	.351087		.9[illegible]144		-1.48189		-1.47576		.633108		-.583652		1.17643		.580724		.27
.28	.3075		.8460[illegible]		-1.41689		-1.52824		.832759		-.354386		1.14532		.790416		.28
.29	.267918		.7640[illegible]		-1.34259		-1.55633		1.01205		-.131996		1.06237		.964616		.29
.3	.232902		.[illegible]567		-1.26119		-1.561[illegible]		1.16749		.107147		.932115		1.08519		.3
.31	.200178		.61[illegible]708		-1.17479		-1.54538		1.2966		.344321		.761024		1.14976		.31
.32	.171643		.543245		-1.08539		-1.51045		1.39795		.572174		.557013		1.1[illegible]99		.32
.33	.146377		.478606		-.994786		-1.45914		1.47107		.784182		.32899		1.10424		.33
.34	.1[illegible]4146		.41[illegible]958		-.904614		-1.39411		1.51643		.974884		8.63581	2	.998697		.34
.35	.104707		.3[illegible]447		-.816283		-1.31809		1.53528		1.14002		-.161461		.844099		.35
.36	.087817		.312031		-.730981		-1.23376		1.52555		1.27057		-.405471		.649587		.36
.37	.073233		.2[illegible]961		-.649673		-1.14372		1.50169		1.36281		-.637474		.424[illegible]		.37
.38	6.07199	2	.3389		-.573105		-1.05035		1.45456		1.45818		-.850383		.179088		.38
.39	5.00518	2	.19575		-.501815		-.955833		1.39122		1.50324		-1.03843		-7.61037	2	.39
.4	4.10145	2	.164883		-.436152		-.862095		1.31488		1.51754		-1.19732		-.330679		.4
.41	3.34079	2	.137[illegible]		-.37629		-.770769		1.22871		1.50932		-1.32424		-.57509		.41
.42	2.70469	2	.114712		-.322256		-.683197		1.13576		1.47256		-1.41787		-.800976		.42
.43	2.17622	2	9.47294	[illegible]	-.27395		-.600436		1.03887		1.4216		-1.47825		-1.00146		.43
.44	1.74006	2	7.77025	[illegible]	-.231165		-.523263		.940608		1.35103		-1.50664		-1.17136		.44
.45	1.38248	2	6.33534	[illegible]	-.193618		-.452202		.843208		1.26751		-1.50533		-1.30721		.45
.46	1.09129	2	5.12182	[illegible]	-.160961		-.387542		.748561		1.17469		-1.47742		-1.4073		.46
.47	8.55775	3	.041152		-.132808		-.329374		.658139		1.07934		-1.42656		-1.47154		.47
.48	6.66607	3	.032631		-.108752		-.277616		.573293		.974187		-1.35676		-1.50126		.48
.49	5.15723	3	2.60[illegible]	2	-8.83738	2	-.232045		.494684		.872437		-1.27217		-1.49902		.49
.5	3.96228	3	2.34489	2	-.071261		-.192344		.4229		.772919		-1.17689		-1.46848		.5
.51	3.02272	3	1.59612	[illegible]	-5.70142	2	-.158099		.358193		.677931		-1.07478		-1.41317		.51
.52	2.28937	3	1.23649	[illegible]	-4.52556	2	-.128855		.300634		.587744		-.9694		-1.33823		.52
.53	1.72122	3	9.50575	3	-.035635		-.104129		.249946		.504616		-.863358		-1.24809		.53
.54	1.28437	3	7.35088	3	-2.78318	2	-8.34248	2	.205937		.428827		-.760788		-1.14731		.54
.55	9.51077	4	5.48703	3	-.021558		-6.62576	2	.168053		.360723		-.662314		-1.04016		.55
.56	6.98722	4	4.11860	3	-1.65584	2	-5.21607	2	.135877		.30036		-.570055		-.930498		.56
.57	5.09326	4	3.06595	3	-1.26096	2	-4.06972	2	.108828		.247562		-.485147		-.821604		.57
.58	3.68223	4	2.26306	3	-9.51895	3	-3.14657	2	8.63331	2	.20197		-.40829		-.716387		.58
.59	2.64009	4	1.65597	3	-7.12198	3	-2.41044	2	6.70286	2	.165088		-.339797		-.616828		.59
.6	1.87696	4	.001201		-5.28022	3	-1.82923	2	.05277		.130334		-.279657		-.52465		.6
.61	1.32305	4	5.63120	4	-3.87840	3	-.013749		4.06474	2	.103073		-.227602		-.440803		.61
.62	9.24590	5	6.14311	4	-2.82166	3	-1.02333	2	3.09939	2	3.08544	[illegible]	-.183165		-.365975		.62
.63	6.40564	5	4.33316	4	-2.03283	3	-7.54061	3	2.33901	2	6.24372	2	-.145743		-.300011		.63
.64	4.40042	5	3.02939	4	-1.44987	3	-5.49971	3	1.74665	2	4.78093	2	-.114646		-.24302		.64
.65	2.99815	5	2.09044	4	-1.02345	3	-3.96917	3	1.23033	2	3.62034	2	-8.91438	2	-.194454		.65
.66	2.02726	5	1.42944	4	-7.14786	4	-2.83376	3	9.42750	3	2.71053	2	-6.85021	2	-.155678		.66
.67	1.36191	5	9.67224	5	-4.93762	4	-2.00076	3	6.81041	3	2.00593	2	-5.20194	2	-.11994		.67
.68	9.10700	6	6.46478	5	-3.37238	4	-1.39692	3	4.88287	3	1.46694	2	-3.90116	2	-9.24258	[illegible]	.68
.69	6.07857	6	4.27733	5	-2.27649	4	-9.63299	4	3.43088	3	1.05976	2	-2.88965	2	-7.03071	[illegible]	.69
.7	4.06802	6	2.73374	5	-1.51815	4	-5.56392	4	2.39083	3	7.56047	3	-2.11312	2	-5.27798	[illegible]	.7
.71	2.74722	6	1.80194	5	-9.99746	5	-4.41632	4	1.64490	3	5.32435	3	1.52503	2	-.03909		.71
.72	1.88909	6	1.14723	5	-6.49817	5	-2.93258	4	1.11682	3	3.69978	3	-1.08576	2	-2.85263	2	.72
.73	1.33648	6	7.20428	5	-4.16631	5	-1.92087	4	7.47927	4	2.51555	3	-7.62265	3	-2.05299	2	.73
.74	9.83018	7	4.45933	5	-2.63352	5	-1.24038	4	4.93776	4	1.71288	3	-5.27440	3	-1.45384	2	.74
.75	7.59899	7	2.72004	5	-1.64001	5	-7.89147	5	3.21169	4	1.13996	3	-3.59504	3	-1.01948	2	.75
.76	6.19338	7	1.63252	5	-1.00560	5	-4.94311	5	2.05671	4	7.46927	4	-2.41229	3	-7.01279	3	.76
.77	5.29 56	7	9.35869	7	-6.06614	6	-3.04603	5	1.29577	4	4.81485	4	-1.59240	3	-4.74525	3	.77
.78	4.71902	7	5.36842	7	-3.59475	6	-1.84493	5	8.02478	5	3.05109	4	-1.03334	3	-3.15624	3	.78
.79	4.3503[illegible]	7	3.16435	7	-2.09236	6	-1.09733	5	4.88077	5	1.89694	4	-6.58609	4	-2.06189	3	.79
.8	4.08449	7	1.76957	7	-1.19561	6	-6.40287	6	2.91243	5	1.15958	4	-4.11892	4	-1.32174	3	.8
.81	3.89403	7	—	—	-6.68030	7	-3.69971	6	1.70288	5	6.93978	5	-2.52486	4	-8.30924	4	.81
.82	3.75197	7			-3.66410	7	-2.04660	6	9.74381	6	4.06507	5	-1.51513	4	-5.10934	4	.82
.83	3.63080	7			-1.94861	7	-1.11733	6	5.44378	6	2.30728	5	-8.88802	5	-3.07313	4	.83
.84	3.52073	7			-1.02898	7	-5.94581	7	2.96325	6	1.55998	5	-5.08846	5	-1.86442	4	.84
.85	3.42458	7			—		-3.07501	7	1.57544	6	7.07118	6	-2.83790	5	-1.03254	4	.85
.86	3.35718	7					-1.51640	7	8.10428	7	3.73989	6	-1.53841	5	-5.74259	5	.86
.87	3.26360	7					—		4.02158	7	1.91153	6	-8.08459	6	-3.69830	5	.87
.88	3.18859	7							1.91837	7	9.43793	7	-4.10579	6	-1.61604	5	.88
.89	3.10998	7							—		4.46436	7	-2.00921	6	-8.12568	6	.89
.9	3.03206	7									[illegible]	7	-9.40348	7	-3.91867	6	.9
.91	2.96182	7	(Where no values are listed the absolute value is less than 10^{-7})										-4.23422	7	-1.80591	6	.91
.92	2.89231	7											-1.79176	7	-7.88376	7	.92
.93	2.83689	7													-3.25268	7	.93
.94	2.79193	7													-1.23300	7	.94
.95	2.71251	7															.95
.96	2.66634	7															.96
.97	2.58513	7															.97
.98	[illegible]	7															.98
.99	[illegible]	7															.99
1	[illegible]	7															1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-4. (continued)

x	$F_8(x)$	p	$F_9(x)$	p	$F_{10}(x)$	p	$F_{11}(x)$	p	$F_{12}(x)$	p	$F_{13}(x)$	p	$F_{14}(x)$	p	$F_{15}(x)$	p	x
0	.92356		0		.925298		0		.871901		0		.5345		0		0
.01	.952076		.274954		.885459		.28639		.830363		.287415		.702865		.2[illegible]0409		.01
.02	.842462		.526843		.76089		.536497		.692886		.54152		.63434		.54[illegible]757		.02
.03	.669013		.734552		.566067		.730875		.480329		.73287		.407264		.72[illegible]73		.03
.04	.424845		.880688		.318927		.863475		.215655		.839296		.12967		.8[illegible]0302		.04
.05	.186881		.953045		4.22027	2	.903[illegible]		-7.25627	2	.848469		-.164153		.79041[illegible]		.05
.06	-8.54467	2	.945015		-.238689		.833383		-.353232		.759317		-.438047		.66[illegible]		.06
.07	-.351677		.85967		-.497793		.717723		-.59611		.58213		-.658156		.455625		.07
.08	-.59188		.70068		-.711986		.510201		-.775056		.337353		-.797365		.134737		.08
.09	-.78815		.483069		-.861147		.25148		-.870841		5.32009	2	-.838573		-.110763		.09
.1	-.925949		.22608		-.931325		-3.27395	2	-.873193		-.237614		-.776622		-.39657[illegible]		.1
.11	-.995153		-5.07634	2	-.917945		-.314596		-.781863		-.50166		-.619084		-.625627		.11
.12	-.990805		-.32405		-.820545		-.56537		-.606615		-.708631		-.385214		-.77357[illegible]		.12
.13	-.913372		-.571375		-.6486		-.761238		-.366136		-.834811		-.10357		-.823873		.13
.14	-.768702		-.772591		-.417664		-.882687		-8.60005	2	-.86576		.191496		-.76691[illegible]		.14
.15	-.567503		-.911426		-.148511		-.917869		.204089		-.797924		.464053		-.610332		.15
.16	-.324493		-.97678		.134781		-.86341		.473471		-.638987		.680963		-.374243		.16
.17	-.057268		-.963558		.407016		-.724666		.693774		-.406939		.815892		-8.90345	2	.17
.18	.215018		-.873008		.644136		-.515132		.841885		-.121982		.852469		.208671		.18
.19	.473063		-.712527		.825343		-.255054		.902339		.166479		.786211		.480674		.19
.2	.698799		-.49498		.934889		5.05859	2	.868867		.445[illegible]01		.625003		.692128		.2
.21	.876651		-.237591		.963595		.314519		.744973		.671818		.388084		.815967		.21
.22	.994582		3.94945	2	.908574		.56982		.543476		.826284		.103677		.836319		.22
.23	1.04485		.31484		.775326		.77[illegible]457		.2851		.889657		-.194429		.750463		.23
.24	1.02445		.567406		.575197		.903524		-3.74451	3	.854927		-.470926		.569111		.24
.25	.985165		.778167		.325277		.9[illegible]0938		-.293712		.725894		-.693176		.314975		.25
.26	.783361		.931517		4.66509	2	.910468		-.555542		.516646		-.835027		1.98437	2	.26
.27	.579577		1.01637		-.237457		.785997		-.763005		.249918		-.87983		-.279451		.27
.28	.336731		1.02687		-.503663		.589[illegible]52		-.895494		-4.54706	2	-.822295		-.549661		.28
.29	7.11174	2	.9627		-.730365		.337515		-.939992		-.337798		-.669002		-.745749		.29
.3	-.200689		.828915		-.899471		5.40[illegible]4	2	-.892245		-.595876		-.437522		-.854957		.3
.31	-.461958		.635435		-.997778		-.23608[illegible]		-.757032		-.792367		-.154286		-.859687		.31
.32	-.697059		.356161		-1.0173		-.507505		-.547548		-.906623		.148528		-.759258		.32
.33	-.892393		.127845		-.958714		-.736704		-.283967		-.926755		.436719		-.565683		.33
.34	-1.03714		-.151181		-.825215		-.904177		8.62339	3	-.85074		.67794		-.302185		.34
.35	-1.12378		-.422545		-.627975		-.995984		.302709		-.68646		.845295		-2.67445	4	.35
.36	-1.14837		-.668109		-.382121		-1.00478		.570989		-.450721		.920255		.304142		.36
.37	-1.11056		-.873101		-.106028		-.930264		.788913		-.167372		.894579		.575013		.37
.38	-1.01344		-1.02504		.180182		-.778949		.93688		.135237		.771047		.780469		.38
.39	-.865092		-1.11536		.456153		-.563434		1.00189		.427192		.562981		.896462		.39
.4	-.668061		-1.13958		.702985		-.301152		.97852		.679999		.292613		.909467		.4
.41	-.438705		-1.09736		.904132		-1.27878	2	.869212		.869349		-1.14506	2	.817878		.41
.42	-.186465		-.992213		1.04684		.279483		.68377		.977359		-.317382		.632011		.42
.43	7.68431	2	-.831092		1.12262		.553793		.38278		.994112		-.593536		.372727		.43
.44	.339705		-.623687		1.12767		.790245		.153491		.918364		-.811718		6.88955	2	.44
.45	.591503		-.381651		1.0628		.972325		-.147098		.757392		-.949988		-.245969		.45
.46	.822993		-.11775		.933035		1.08795		-.439327		.526042		-.99472		-.537505		.46
.47	1.02666		.154982		.74729		1.13012		-.700374		.245113		-.941731		-.774263		.47
.48	1.19694		.424018		.516942		1.09711		-.910532		-6.07293	2	-.796412		-.931042		.48
.49	1.33028		.678012		.255458		.992256		-1.05462		-.36529		-.572899		-.991411		.49
.5	1.42511		.907326		-2.28926	2	.823374		-1.1229		-.643184		-.292415		-.949212		.5
.51	1.4817		1.1044		-.30394		.601905		-1.11154		-.871969		1.90005	2	-.808904		.51
.52	1.50191		1.263[illegible]		-.574478		.341848		-1.02252		-1.03389		.333108		-.584775		.52
.53	1.4889		1.38296		-.822966		5.85141	2	-.863045		-1.11709		.622155		-.299177		.53
.54	1.44683		1.46069		-1.04006		-.232121		-.644721		-1.11624		.861359		1.99968	2	.54
.55	1.38048		1.49833		-1.21891		-.515463		-.382317		-1.03254		1.03096		.342261		.55
.56	1.29497		1.49875		-1.35532		-.778183		-9.24715	2	-.873222		1.11769		.637592		.56
.57	1.19547		1.46612		-1.44764		-1.0094		.207636		-.650506		1.11552		.879259		.57
.58	1.08691		1.40549		-1.49658		-1.20099		.501529		-.380321		1.02572		1.04616		.58
.59	.973836		1.32245		-1.50484		-1.34781		.774493		-8.07792	2	.856277		1.12446		.59
.6	.860226		1.22274		-1.47674		-1.44766		1.01439		.229352		.620753		1.10844		.6
.61	.749421		1.11194		-1.41769		-1.50101		1.21218		.53209		.336748		1.00046		.61
.62	.644075		.995219		-1.33382		-1.51069		1.36218		.811456		.024179		.81024		.62
.63	.546165		.871149		-1.23145		-1.48136		1.462		1.05441		-.296487		.553488		.63
.64	.457021		.761577		-1.1168		-1.41899		1.51233		1.25143		-.605864		.250082		.64
.65	.377395		.651572		-.995636		-1.33033		1.51644		1.39679		-.887012		-7.79252	2	.65
.66	.307542		.549416		-.873032		-1.22259		1.47567		1.48849		-1.12645		-.408318		.66
.67	.247307		.456643		-.753256		-1.10199		1.40875		1.52787		-1.31478		-.720536		.67
.68	.196223		.374114		-.639673		-.975438		1.31122		1.51913		-1.44687		-.997145		.68
.69	.155596		.302115		-.534753		-.848239		1.1948		1.46861		-1.52169		-1.22488		.69
.7	.118588		.240458		-.44011		-.724939		1.06696		1.38408		-1.54189		-1.39519		.7
.71	.090287		.1886		-.3566		-.605056		.934425		1.27398		-1.51304		-1.50433		.71
.72	6.77655	2	.145743		-.28443		-.503082		.802994		1.1468		-1.44287		-1.55303		.72
.73	5.01233	2	.110932		-.223293		-.403559		.677317		1.01054		-1.34038		-1.54582		.73
.74	3.65212	2	8.31401	2	-.172498		-.326189		.560867		.872235		-1.21501		-1.49018		.74
.75	2.62014	2	6.13299	2	-.131086		-.259931		.455963		.737755		-1.07596		-1.39551		.75
.76	1.84991	2	4.45083	2	-9.79567	2	-.197402		.363886		.611619		-.931575		-1.27216		.76
.77	1.28458	2	3.17601	2	-7.19472	2	-.149335		.285019		.497004		-.788971		-1.13052		.77
.78	8.76718	3	2.22701	2	-5.19103	2	-.11122		.219037		.395821		-.653781		-.980272		.78
.79	5.87642	3	1.53338	2	-3.67679	2	-8.11758	2	.165083		.308876		-.530083		-.829835		.79
.8	3.86488	3	1.03587	2	-2.55464	2	-5.31005	2	.121951		.236069		-.420462		-.685958		.8
.81	2.49169	3	6.85925	3	-1.73962	2	-4.07468	2	8.82383	2	.176613		-.326157		-.553776		.81
.82	1.57285	3	4.44724	3	-1.15983	2	-3.02797[illegible]		6.24816	2	.129247		-.247294		-.436431		.82
.83	9.70833	4	2.81967	3	-7.56192	3	-1.67794	2	.043254		.092436		-.183133		-.335624		.83
.84	5.85065	4	1.74569	3	-4.81465	3	-1.23113	2	2.92384	2	6.45372	2	-.132335		-.251667		.84
.85	3.43634	4	1.05355	3	-2.98872	3	-7.36955	3	.019271		4.39286	2	-9.32022	2	-.183825		.85
.86	1.96299	4	6.18580	4	-1.80535	3	-4.39586	3	1.23631	2	2.91042	2	-6.38821	2	-.130625		.86
.87	1.08795	4	3.52504	4	-1.05878	3	-2.99739	3	7.70421	3	1.87322	2	-4.25357	2	-9.01561	2	.87
.88	5.83296	5	1.94412	4	-6.01218	4	-1.73107	3	4.65177	3	1.16848	2	-2.74527	2	-6.05133	2	.88
.89	3.01467	5	1.03416	4	-3.29462	4	-9.78140	4	2.71317	3	7.04353	3	-1.71272	2	-.03901		.89
.9	1.49530	5	5.28332	5	-1.73521	4	-9.31564	4	1.52288	3	4.08824	3	-1.02937	2	-2.43159	2	.9
.91	7.07971	6	2.57849	5	-8.73897	5	-4.76470	4	8.18718	4	2.27453	3	-5.93434	3	-1.45473	2	.91
.92	3.17589	6	1.19387	5	-4.18126	5	-1.55759	4	4.19029	4	1.20595	3	-3.26342	3	-8.30926	3	.92
.93	1.33900	6	5.19834	6	-1.88449	5	-6.38122	5	2.02547	4	6.04675	4	-1.69335	3	-4.49978	3	.93
.94	5.22324	7	2.09876	6	-7.90500	6	-2.77438	5	9.14621	5	2.83744	4	-8.29588	4	-2.28838	3	.94
.95	1.84943	7	7.72634	7	-3.03690	6	-1.11063	5	3.79949	5	1.22773	4	-3.74314	4	-1.07812	3	.95
.96			2.50481	7	-1.04366	6	-3.98[illegible]48	6	1.41861	5	4.79053	5	-1.52777	4	-4.61006	4	.96
.97					-3.10668	7	-1.24662	6	4.58084	6	1.62474	5	-5.44511	5	-1.72989	4	.97
.98							-5.06[illegible]	7	1.18933	6	4.47488	6	-1.58585	5	-5.34709	5	.98
.99	(Where no values are listed, the absolute value is less than 10^{-7})								2.10174	7	8.51331	7	-3.21989	6	-1.17034	5	.99
1													-2.14400	7	-8.76311	7	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-4. (continued)

x	$F_{16}(x)$	p	$F_{17}(x)$	p	$F_{18}(x)$	p	$F_{19}(x)$	p	$F_{20}(x)$	p	$F_{21}(x)$	p	$F_{22}(x)$	p	$F_{23}(x)$	p	x
0	.79971		0		.75956		0		.725583		0		.69148		0		0
.01	.74[illegible]		.221563		.701786		.290915		.665209		.28834		.629503		.283442		.01
.02	.698748		.540568		.536424		.5324[illegible]		.494101		.526869		.45466		.514395		.02
.03	.644[illegible]		.711038		.288398		.694497		.24069		.674369		.198236		.650075		.03
.04	.5[illegible]144	2	.777848		-2.74912	3	.746745		-5.29315	2	.705326		-9.38826	2	.665315		.04
.05	-.237083		.751556		-.204313		.672416		-.337983		.61433		-.369407		.557224		.05
.06	-.[illegible]248		.518218		-.[illegible]41211		.494628		-.367089		.417017		-.578692		.345724		.06
.07	-.[illegible]		.340701		-.705762		.[illegible]37731		-.302145		.14737		-.685061		6.98832	2	.07
.08	-.739435		5.32532	[illegible]	-.[illegible]6282		-5.74932	2	-.726634		-.148131		-.668521		-.2193		.08
.09	-.77[illegible]		-.24241		-.703598		-.344029		-.619365		-.418517		-.532185		-.468314		.09
.1	-.[illegible]0397		-.503382		-.536998		-.576224		-.412001		-.617109		-.300284		-.631017		.1
.11	-.45[illegible]		-.691783		-.288247		-.717157		-.141299		-.709556		-1.41441	2	-.67717		.11
.12	-.177483		-.780241		4.94317	3	-.744215		.15647		-.679756		.215163		-.598025		.12
.13	.120362		-.755836		.2[illegible]8104		-.6[illegible].59		.429019		-.532632		.515962		-.407988		.13
.14	.402[illegible]		-.621952		.54676		-.457663		.631183		-.293293		.665128		-.141941		.14
.15	.630075		-.397765		.713156		-.189169		.729382		-2.73519	3	.695839		.15128		.15
.16	.772151		-.115456		.77197		.110185		.707193		.289154		.60234		.417159		.16
.17	.860117		.164436		.714117		.393076		.564999		.53234		.400994		.606973		.17
.18	.7357[illegible]		.458651		.548161		.614759		.234492		.684837		.127392		.685335		.18
.19	.561[illegible]		.668389		.298728		.740107		4.49151	2	.72029		-.16935		.637465		.19
.2	.311103		.782932		3.34064	3	.749132		-.293219		.632291		-.438197		.471783		.2
.21	1.6[illegible]81	2	.782932		-.293798		.640117		-.510927		.435538		-.629537		.218415		.21
.22	-.280735		.67673		-.548142		.429879		-.685784		.163333		-.709811		-.076324		.22
.23	-.542		.470614		-.721528		.151117		-.7[illegible]8872		-.13808		-.664259		-.358384		.23
.24	-.731669		.196604		-.787856		-.15[illegible]719		-.69[illegible]504		-.417384		-.500636		-.575885		.24
.25	-.824424		-.106689		-.736953		-.434215		-.516974		-.626916		-.247349		-.688632		.25
.26	-.80713		-.396546		-.570051		-.64939		-.259032		-.730762		5.11185	2	-.675497		.26
.27	-.682437		-.632155		-.328682		-.764529		4.26218	2	-.710864		.342223		-.538325		.27
.28	-.466434		-.780333		-3.11507	2	-.761404		.339115		-.570085		.574562		-.301653		.28
.29	-.187424		-.82013		.2728		-.640074		.582321		-.331749		.706922		-8.28252	3	.29
.3	.110067		-.749704		.938931		-.418854		.732643		-3.57115	2	.715532		.288479		.3
.31	.410126		-.567057		.726951		-.131463		.7654		.268337		.598267		.534498		.31
.32	.650679		-.308588		.810264		.178178		.674791		.529226		.375053		.684665		.32
.33	.80842		-5.65222	3	.775961		.462693		.474822		.702284		8.44241	2	.711104		.33
.34	.862805		.300345		.628585		.67849		.197052		.759734		-.22317		.608285		.34
.35	.80651		.567637		.389006		.792393		-.114467		.689638		-.494133		.394082		.35
.36	.646743		.759779		.091355		.786648		-.410211		.503601		-.681042		.106612		.36
.37	.403239		.850546		-.221928		.66157		-.643061		.231955		-.750879		-.202281		.37
.38	.106606		.827395		-.506191		.435449		-.775758		-8.06683	2	-.690788		-.477798		.38
.39	-.206114		.693042		-.720929		.141746		-.786782		-.382712		-.510322		-.66917		.39
.4	-.49604		.464982		-.835484		-.175954		-.673797		-.6242		-.239844		-.741648		.4
.41	-.727293		.173049		-.833311		-.470465		-.453558		-.754958		7.46593	2	-.681328		.41
.42	-.871378		-.144624		-.714201		-.698006		-.160118		-.781201		.379443		-.498606		.42
.43	-.91057		-.446671		-.494201		-.82469[illegible]		.161361		-.669425		.622184		-.223509		.43
.44	-.839912		-.693899		-.203281		-.831293		.461199		-.446972		.760911		9.39054	2	.44
.45	-.667623		-.854325		.118903		-.716184		.692955		-.149211		.771161		.398066		.45
.46	-.41333		-.907182		.428509		-.495465		.820561		.176191		.650158		.63482		.46
.47	-.108433		-.845421		.683502		-.200359		.823797		.476953		.417313		.761611		.47
.48	.213484		-.676413		.84931		.127231		.701318		.704574		.110981		.755021		.48
.49	.514886		-.420819		.903374		.44073		.470784		.822071		-.21755		.61495		.49
.5	.761548		-.109826		.83801		.69581		.166123		.809846		-.51456		.364711		.5
.51	.929778		.218852		.661248		.356111		-.16762		.668759		-.729041		4.70293	2	.51
.52	.989358		.525674		.395573		.39874		-.48104		.419947		-.824965		-.28333		.52
.53	.945326		.774054		7.48109	2	.817077		-.727692		.101471		-.785306		-.569033		.53
.54	.798421		.93465		-.260379		.621559		-.870897		-.237659		-.615249		-.760074		.54
.55	.564228		.988642		-.567783		.338072		-.889011		-.545103		-.34134		-.822412		.55
.56	.267171		.929664		-.808966		4.25982	3	-.778441		-.773258		-7.16217	3	-.74389		.56
.57	-6.23464	[illegible]	.764203		-.953973		-.335714		-.554008		-.886509		.333755		-.536176		.57
.58	-.391302		.510499		-.984828		-.637037		-.246664		-.866577		.626463		-.233214		.58
.59	-.687506		.196156		-.897444		-.860222		.101043		-.71517		.823578		.114847		.59
.6	-.922779		-.145206		-.701763		-.976095		.441047		-.453605		.392512		.449864		.6
.61	-1.07551		-.477791		-.420203		-.969343		.726478		-.119506		.821162		.715462		.61
.62	-1.1324		-.767517		-8.46764	2	-.840159		.918064		.258863		.618769		.866409		.62
.63	-1.08923		-.985494		.26737		-.603636		.989324		.569717		.31535		.876109		.63
.64	-.95069		-1.11074		.597378		-.288591		.929863		.82525		-4.35125	2	.740976		.64
.65	-.729358		-1.13183		.869989		6.84176	2	.746554		.968408		-.403636		.481009		.65
.66	-.443957		-1.04763		1.05676		.42561		.461976		.977967		-.716503		.13654		.66
.67	-.117219		-.866493		1.13887		.742175		.111521		.851246		-.917452		-.238283		.67
.68	.226447		-.605034		1.1086		.982631		-.261715		.604161		-.992529		-.584153		.68
.69	.563214		-.285841		.969451		1.12073		-.612412		.268696		-.923029		-.845982		.69
.7	.871807		6.50931	2	.735122		1.14187		-.898667		-.111726		-.717086		-.981493		.7
.71	1.13502		.421048		.427332		1.04411		-1.08691		-.488251		-.402214		-.967682		.71
.72	1.34059		.757057		7.31502	2	.637617		-1.1555		-.813079		-2.10816	2	-.80419		.72
.73	1.48207		1.05195		-.298125		.54277		-1.00867		-1.04551		.374727		-.513149		.73
.74	1.55776		1.28322		-.657964		.187317		-.916671		-1.15679		.731982		-.135617		.74
.75	1.57101		1.46076		-.981223		-.19707		-.634185		-1.1331		1.00303		.27472		.75
.76	1.5289		1.56102		-1.24314		-.578349		-.277451		-.976554		1.15192		.659341		.76
.77	1.44118		1.59251		-1.44554		-.927349		.119536		-.703941		1.15866		.964296		.77
.78	1.31508		1.56184		-1.56727		-1.22036		.52115		-.344048		1.02108		1.14576		.78
.79	1.17424		1.47912		-1.61384		-1.44088		.894056		6.66819	2	.754247		1.17762		.79
.8	1.01773		1.35654		-1.59147		-1.58032		1.21648		.486756		.367589		1.05342		.8
.81	.859329		1.20708		-1.51074		-1.6379		1.45021		.884239		-3.57869	2	.786812		.81
.82	.70702		1.04329		-1.38498		-1.61963		1.60325		1.22189		-.483552		.4082		.82
.83	.566756		.876389		-1.22867		-1.53683		1.6668		1.4775		-.921716		-3.55673	2	.83
.84	.442481		.715588		-1.05601		-1.40423		1.64732		1.6383		-1.25754		-.507735		.84
.85	.336157		.567781		-.87574		-1.2381		1.55726		1.70167		-1.52424		-.948415		.85
.86	.248341		.437478		-.710326		-1.05448		1.41321		1.67442		-1.68681		-1.32004		.86
.87	.178594		.326979		-.555548		-.867748		1.23359		1.57074		-1.74246		-1.59239		.87
.88	.123768		.236705		-.42[illegible]401		-.689659		1.03556		1.40967		-1.69938		-1.74856		.88
.89	8.31593	[illegible]	.165625		-.357255		-.528774		.838247		1.21232		-1.5744		-1.78759		.89
.9	.05386		.111713		-.216461		-.390387		.651777		.999301		-1.38982		-1.71888		.9
.91	3.34975	2	7.23792	[illegible]	-.146473		-.276781		.486189		.788703		-1.17007		-1.56344		.91
.92	1.99302	2	4.48367	2	-9.48075	2	-.187734		.346842		.594656		-.938662		-1.34792		.92
.93	1.12285	2	[illegible].03916	[illegible]	-5.83617	2	-.121181		.235541		.426671		-.715668		-1.10096		.93
.94	5.92762	3	1.46534	2	-3.78355	2	-7.38931	2	.151255		.289645		-.516156		-.849385		.94
.95	2.93121	[illegible]	7.55318	3	-1.83613	2	-4.21131	2	9.09506	2	.184404		-.349486		-.615284		.95
.96	1.31688	3	3.56138	3	-7.11748	3	-2.20718	2	5.64497	2	.108616		-.21949		-.414633		.96
.97	[illegible].21076	4	1.48865	3	-4.03354	3	-1.05392	2	2.51878	2	5.78753	2	-.12531		-.254677		.97
.98	1.71203	4	5.[illegible]1211	4	-1.50778	3	-4.14414	3	1.03147	2	2.67607	2	-6.26589	2	-.138415		.98
.99	1.05255	5	1.33666	4	-4.20308	4	-1.25927	3	3.52478	3	9.76969	3	-2.52449	2	-6.19024	2	.99
1	3.53214	6	1.36728	5	-[illegible].7576		-1.79848	4	6.39686	4	1.97378	3	-6.11282	3	-1.80279	2	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-4. (continued)

x	$F_{24}(x)$	p	$F_{25}(x)$	p	$F_{26}(x)$	p	$F_{27}(x)$	p	$F_{28}(x)$	p	$F_{29}(x)$	p	$F_{30}(x)$	p	$F_{31}(x)$	p	x
0	.695575		0		.613438		0		.556237		0		.529451		0		0
.01	.592884		.479158		.59141		.260206		.497551		.240249		.471227		.262952		.01
.02	.416779		.496191		.377847		.466538		.333067		.458656		.303339		.467147		.02
.03	.160881		.619604		.127785		.576256		9.81898	2	.524547		7.33368	2	.566946		.03
.04	-.125946		.621074		-.148268		.566595		-.157648		.507181		-.168258		.540019		.04
.05	-.388912		.500235		-.394555		.459475		-.380274		.380227		-.37503		.392347		.05
.06	-.577751		.280766		-.561274		.221123		-.525557		.170992		-.306605		.136906		.06
.07	-.65631		5.73334	3	-.614551		-4.33346	2	-.554272		-7.54433	2	-.52284		-.113671		.07
.08	-.609455		-.270813		-.543762		-.299181		-.468571		-.305923		-.424013		-.35883		.08
.09	-.445975		-.494482		-.362759		-.495413		-.283426		-.470657		-.231699		-.52361		.09
.1	-.196922		-.621197		-.108046		-.585701		-3.78738	2	-.533966		1.19802	2	-.570934		.1
.11	9.02851	2	-.609873		.165029		-.556744		.216203		-.481374		.253514		-.489926		.11
.12	.360878		-.507357		.412553		-.412295		.42502		-.325678		.43976		-.298397		.12
.13	.563171		-.288696		.573225		-.182015		.344255		-.098591		.529605		-3.89269	2	.13
.14	.65841		-1.26378	2	.618401		8.66097	2	.548438		.150415		.503041		.230592		.14
.15	.628171		.266697		.538663		.33805		.436374		.367608		.36558		.449855		.15
.16	.477901		.494432		.349773		.520188		.231461		.505376		.147151		.569631		.16
.17	.235919		.622687		8.95557	2	.555099		-2.31243	2	.535375		-.104421		.562801		.17
.18	-5.19276	2	.634352		-.189682		.546928		-.273826		.449085		-.333863		.430508		.18
.19	-.330949		.518441		-.43163		.385225		-.467346		.265339		-.490571		.201999		.19
.2	-.547989		.300165		-.58733		.143051		-.562558		[illegible]	2	-.559753		-7.18095	2	.2
.21	-.661528		2.20166	2	-.625045		-.129778		-.538915		-.224436		-.470134		-.329638		.21
.22	-.649589		-.261685		-.536699		-.376923		-.400926		-.424969		-.296537		-.513519		.22
.23	-.513555		-.495546		-.339585		-.547132		-.17731		-.534435		-.056735		-.581793		.23
.24	-.279893		-.632969		-7.29662	2	-.604845		.084981		-.528796		.196749		-.518517		.24
.25	8.58013	3	-.647175		.209742		-.557592		.3306		-.406703		.408121		-.337165		.25
.26	.296973		-.534609		.451657		-.358664		.507462		-.199522		.530566		-7.77703	2	.26
.27	.530609		-.31668		.603852		-.104481		.57776		5.40096	2	.536602		.201899		.27
.28	.664959		-3.54464	2	.635215		.172816		.526022		.297333		.424225		.439187		.28
.29	.674086		.254408		.538772		.416044		.362455		.477764		.217503		.580566		.29
.3	.555602		.496377		.333194		.574744		.121112		.555959		-3.85557	2	.593702		.3
.31	.331193		.64294		5.91673	2	.615612		-.147532		.5143		-.2878		.47485		.31
.32	4.26163	2	.664942		-.228658		.529459		-.386765		.361014		-.475178		.24989		.32
.33	-.256085		.557322		-.472555		.333229		-.545783		.123469		-.558905		-.031259		.33
.34	-.5087		.340204		-.623264		6.66665	2	-.590366		-.133621		-.519736		-.305673		.34
.35	-.657596		5.51056	2	-.649899		-.215733		-.510249		-.363802		-.365382		-.511445		.35
.36	-.70172		-.243016		-.546227		-.455839		-.321427		-.526006		-.129006		-.601604		.36
.37	-.604375		-.496347		-.332055		-.603824		-6.29693	2	-.570605		.137862		-.554796		.37
.38	-.392678		-.655393		-4.942[illegible]	2	-.628453		.211049		-.492013		.376532		-.380306		.38
.39	-.105427		-.688596		.245642		-.523635		.442837		-.306158		.534023		-.116255		.39
.4	.204241		-.588526		.494237		-.309848		.583035		-5.22706	2	.574783		.178707		.4
.41	.478587		-.373445		.646282		-3.01332	2	.601188		.215369		.486695		.438392		.41
.42	.666486		-8.39351	2	.670561		.258569		.492311		.439908		.293544		.603896		.42
.43	.731905		.224785		.561061		.496979		.278164		.569836		3.13783	2	.63694		.43
.44	.66175		.493386		.338347		.63561		2.30634	3	.578599		-.240523		.528717		.44
.45	.467914		.66919		4.56687	2	.624973		[illegible]		.462099		-.4621		.30221		.45
.46	.185027		.719196		-.259448		.52181		-.459708		.244152		-.583749		7.37994	3	.46
.47	-.135459		.630855		-.516487		.28994		-.620424		-2.92586	2	-.577372		-.28977		.47
.48	-.434825		.420276		-.673896		-4.31044	3	-.611499		-.29965		-.442837		-.32189		.48
.49	-.657804		.12645		-.69935		-.301379		-.473269		-.5085		-.208343		-.635463		.49
.5	-.76267		-.199418		-.586294		-.540485		-.233283		-.609859		7.54435	2	-.603079		.5
.51	-.728966		-.48428		-.355453		-.612006		5.91885	2	-.530402		.346284		-.429989		.51
.52	-.561319		-.684784		-5.09968	2	-.667692		.343232		-.424733		.543952		-.153387		.52
.53	-.288736		-.757789		.267935		-.52664		.558932		-.174766		.623563		.165526		.53
.54	4.05092	2	-.687878		.538697		-.275774		.659938		.116802		.56571		.455021		.54
.55	.36749		-.486442		.70738		3.53141	2	.623336		.387531		.381129		.648914		.55
.56	.653166		-.189743		.73944		.344151		.454765		.57855		.108797		.701649		.56
.57	.788967		.147681		.626632		.587885		.187595		.647279		-.192238		.598977		.57
.58	.805524		.463076		.388887		.715989		-.123642		.576833		-.455574		.361742		.58
.59	.677996		.697091		7.07279	2	.700617		-.414349		.380066		-.622012		.041796		.59
.6	.42706		.804843		-.267101		.542514		-.623194		9.72853	2	-.652776		-.289186		.6
.61	9.53876	2	.764343		-.559218		.271235		-.704941		-.211949		-.538464		-.555603		.61
.62	-.259734		.580747		-.748169		-6.03962	2	-.640128		-.481289		-.301686		-.695037		.62
.63	-.57635		.285629		-.795582		-.386564		-.439526		-.651727		7.31058	3	-.672743		.63
.64	-.798587		-6.85735	2	-.689756		-.641455		-.142411		-.684404		.321193		-.490164		.64
.65	-.886345		-.417958		-.448209		-.772409		.191106		-.569339		.570093		-.185437		.65
.66	-.822296		-.69866		-.114693		-.750617		.492162		-.328145		.697015		.17444		.66
.67	-.614736		-.858402		.248813		-.577227		.697268		-1.00483	2	.670754		.508184		.67
.68	-.29664		-.865982		.573613		-.283585		.761477		.318065		.493419		.738441		.68
.69	8.02094	2	-.717025		.797173		7.44703	2	.668081		.58558		.200784		.809581		.69
.7	.453647		-.43497		.87495		.42722		.432596		.733176		-.144772		.701049		.7
.71	.761465		-6.71354	2	.788945		.704541		.100242		.725778		-.46738		.433095		.71
.72	.951634		.325415		.551396		.84965		-.262692		.560652		-.69412		6.33799	2	.72
.73	.950774		.668749		.202897		.83043		-.581961		.268822		-.771265		-.325078		.73
.74	.869491		.907779		-.194711		.646134		-.79034		-.090635		-.676774		-.642144		.74
.75	.603792		.996762		-.569378		.328113		-.841492		-.442386		-.426178		-.811063		.75
.76	.232435		.916797		-.851803		-6.56152	2	-.71989		-.710378		-7.03774	2	-.786642		.76
.77	-.189266		.677081		-.987952		-.460879		-.444737		-.833963		.31441		-.566891		.77
.78	-.598074		.313432		-.948891		-.731661		-6.70041	2	-.781227		.642624		-.195234		.78
.79	-.932468		-.117593		-.736178		-.963509		.339957		-.556684		.837953		.247608		.79
.8	-1.14174		-.547892		-.381895		-.966842		.69446		-.201575		.850607		.660635		.8
.81	-1.19312		-.908739		5.66493	2	-.784338		.922493		.213487		.66945		.945043		.81
.82	-1.07601		-1.14158		.505551		-.445013		.372551		.660224		.326186		1.0213		.82
.83	-.802644		-1.20691		.69154		7.53682	2	.826517		.830097		-.109635		.877607		.83
.84	-.405471		-1.08393		1.14464		.467609		.504328		.981961		-.544072		.519397		.84
.85	6.83266	2	-.60211		1.21911		.875481		6.07396	2	.876369		-.878185		2.71173	2	.85
.86	.564075		-.373619		1.09575		1.14749		-.423286		.573547		-1.02948		-.487752		.86
.87	1.02743		.127743		.78558		1.22862		-.855502		.125258		-.050966		-.900089		.87
.88	1.4116		.659002		.32744		1.0945		-1.1491		-.383648		4361		-1.09928		.88
.89	1.68284		1.14137		-.218816		.755476		-1.23872		-.850503		.15944		-1.01622		.89
.9	1.82372		1.53394		-.782246		.25463		-1.0927		-1.1741		.405931		-.643104		.9
.91	1.83369		1.73553		-1.25206		-.335748		-.71924		-1.27426		.930186		-4.13142	2	.91
.92	1.72732		1.80846		-1.68827		-.945795		-.165384		-1.10793		1.28758		.665226		.92
.93	1.53653		1.87534		-1.93		-1.48164		.491177		-.678557		1.37417		1.3107		.93
.94	1.2758		1.71656		-2.00021		-1.83847		1.15507		-3.71503	2	1.12996		1.71869		.94
.95	.936986		1.46551		-1.90639		-2.0569		1.72999		.72611		.55344		1.73824		.95
.96	.724742		1.16238		-1.67728		-2.10294		2.13485		1.49687		-.294544		1.[illegible]7752		.96
.97	.483042		.847607		-1.3564[illegible]		-1.92866		2.31707		2.15677		-1.29892		.328152		.97
.98	.287224		.59624		-.994465		-1.50795		2.2602		2.60418		-2.30908		-1.0254		.98
.99	.143614		.315765		-.640114		-1.19817		1.98956		2.77133		-3.16498		-2.6141		.99
1	[illegible]	2	.134083		-.333575		-.763484		1.[illegible]4681		2.63878		-3.72572		-4.20951		1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-4. (continued)

x	$F_{32}(x)$	p	$F_{33}(x)$	p	$F_{34}(x)$	p	$F_{35}(x)$	p	x
0	.571563		0		.638481		0		0
.01	.505399		.279291		.536112		.296253		.01
.02	.322012		.453314		.355466		.519946		.02
.03	6.32796	2	.592022		.054789		.616261		.03
.04	-.211697		.552272		-.258826		.561533		.04
.05	-.440[illegible]		.383237		-.475146		.369038		.05
.06	-.[illegible]78		.124293		-.587567		8.58766	2	.06
.07	-.[illegible]		-.164135		-.576656		-.218697		.07
.08	-.442430		-.414651		-.417213		-.470006		.08
.09	-.214[illegible]		-.568612		-.157187		-.606319		.09
.1	6.13524	2	-.589837		.141536		-.593972		.1
.11	.322077		-.473103		.406923		-.433675		.11
.12	.507965		-.245434		.570858		-.169945		.12
.13	.576275		4.01264	2	.6[illegible]402		.138237		.13
.14	.51105		.316846		.463613		.413278		.14
.15	.326838		.519861		.261351		.587477		.15
.16	6.54569	2	.60135		-3.40349	2	.617673		.16
.17	-.213433		.541784		-.321826		.495932		.17
.18	-.446005		.354603		-.532964		.291648		.18
.19	-.578756		8.32059	2	-.51634		-5.55119	2	.19
.2	-.580848		-.209086		-.551409		-.350133		.2
.21	-.451251		-.453786		-.353127		-.559524		.21
.22	-.21918		-.593172		-8.65733	2	-.631625		.22
.23	6.27379	2	-.594062		.234179		-.547949		.23
.24	.32984		-.45554		.402312		-.328336		.24
.25	.520746		-.209452		.615732		-2.63077	2	.25
.26	.551161		8.68903	2	.601617		.283918		.26
.27	.5242		.364012		.444[illegible]3		.529608		.27
.28	.334427		.596516		.175876		.638461		.28
.29	6.47251	2	.618484		-.154513		.593646		.29
.3	-.223365		.53442		-.413541		.401194		.3
.31	-.463641		.32312		-.594564		.107835		.31
.32	-.600426		3.35224	2	-.631521		-.214266		.32
.33	-.601441		-.26658		-.515858		-.485189		.33
.34	-.465426		-.506339		-.273294		-.637035		.34
.35	-.222682		-.628634		3.79424	2	-.630933		.35
.36	.071626		-.603493		.342827		-.467013		.36
.37	.349552		-.43562		.567163		-.184789		.37
.38	.547819		-.163532		.655594		.146259		.38
.39	.618715		.149182		.565268		.443694		.39
.4	.545012		.428649		.371759		.63258		.4
.41	.342318		.608096		6.57266	2	.664339		.41
.42	5.62547	2	.643768		-.258996		.529268		.42
.43	-.247751		.525587		-.523128		.259498		.43
.44	-.499411		.279962		-.661236		-7.85054	2	.44
.45	-.33969		-3.60214	2	-.637923		-.400178		.45
.46	-.63471		-.347775		-.456923		-.623913		.46
.47	-.48388		-.580739		-.16076		-.691716		.47
.48	-.220[illegible]		-.678161		.179107		-.584258		.48
.49	9.55501	2	-.614913		.479394		-.32642		.49
.5	.391229		-.403976		.665312		1.82625	2	.5
.51	.597225		-9.39466	2	.689151		.363063		.51
.52	.664085		.242022		.542544		.619741		.52
.53	.574058		.523347		.259249		.721045		.53
.54	.349761		.681242		-9.24142	2	.638347		.54
.55	.030456		.675483		-.42586		.389791		.55
.56	-.299337		.504636		-.657403		3.66372	2	.56
.57	-.566408		.207179		-.727177		-.331559		.57
.58	-.706235		-.146984		-.614617		-.619431		.58
.59	-.685229		-.472771		-.3445		-.75037		.59
.6	-.503624		-.690215		1.81955	2	-.6869		.6
.61	-.201798		-.744011		.383647		-.441476		.61
.62	.151241		-.617559		.659195		-7.45874	2	.62
.63	.472839		-.337985		.772696		.31974		.63
.64	.685863		2.93666	2	.691378		.637693		.64
.65	.737059		.395899		.431316		.792784		.65
.66	.610275		.670861		5.48055	2	.739341		.66
.67	.33129		.78362		-.344116		.486039		.67
.68	-.037118		.7017		-.662768		9.54949	2	.68
.69	-.409022		.440037		-.815979		-.33055		.69
.7	-.695108		5.88584	2	-.758751		-.677167		.7
.71	-.823895		-.349722		-.49946		-.846855		.71
.72	-.759444		-.683531		-9.97882	2	-.786863		.72
.73	-.51125		-.855548		.339207		-.505647		.73
.74	-.133547		-.816395		.701975		-7.34256	2	.74
.75	.295842		-.568407		.838371		.394251		.75
.76	.645345		-.167356		.841043		.766309		.76
.77	.854534		.289604		.56304		.932363		.77
.78	.854741		.684799		.176199		.835064		.78
.79	.636812		.91296		-.372966		.489937		.79
.8	.244879		.905306		-.781575		-1.42536	2	.8
.81	-.231[illegible]		.691608		-.906166		-.536138		.81
.82	-.67629		.406207		-.916959		-.91938		.82
.83	-.375062		-.321842		-.577173		-1.03806		.83
.84	-1.04258		-.793645		-4.81068	2	-.837418		.84
.85	-.[illegible]		-1.07722		.527222		-.337424		.85
.86	-.[illegible]		-1.07405		.97957		.269351		.86
.87	.141.[illegible]		-.768259		1.16042		.849689		.87
.88	.6[illegible]877[illegible]		-.219916		.[illegible]674		1.18368		.88
.89	1.1[illegible]99		.432921		.483406		1.13004		.89
.9	1.[illegible]289		1.00345		-.225133		.663061		.9
.91	.[illegible]216[illegible]		1.30633		-.523716		-9.78529	2	.91
.92	.[illegible]34664		1.[illegible]0793		-1.36401		-.901278		.92
.93	-.337284		.679637		-1.34[illegible]5		-1.42608		.93
.94	-1.12719		-.186641		-.77306		-1.38697		.94
.95	-1.62[illegible]02		-1.13803		.23027		-.664431		.95
.96	-1.77497		-1.32703		1.33579		.570734		.96
.97	-1.1[illegible]		-1.85126		2.30225		1.77303		.97
.98	.160[illegible]		-.839196		1.59223		2.01091		.98
.99	2.23233		1.46368		-.707189		5.04017	2	.99
1	4.[illegible]361		5.10137		-5.3312		-5.46191		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-5. Eigenvalues for $c = 20\pi$ or aperture length $= 20\lambda$

n	λ_n	n	λ_n
0	.31623	23	-j.31623
1	j.31623	24	.31623
2	-.31623	25	j.31623
3	-j.31623	26	-.31623
4	.31623	27	-j.31623
5	j.31623	28	.31623
6	-.31623	29	j.31623
7	-j.31623	30	-.31622
8	.31623	31	-j.31622
9	j.31623	32	.31622
10	-.31623	33	j.31620
11	-j.31623	34	-.31619
12	.31623	35	-j.31593
13	j.31623	36	.31530
14	-.31623	37	j.31142
15	-j.31623	38	-.29795
16	.31623	39	-j.25692
17	j.31623	40	.17559
18	-.31623	41	j.09214
19	-j.31623	42	-.03767
20	.31623	43	-j.01415
21	j.31623	44	.00532
22	-.31623	45	j.00208

Table 3-6. Eigenfunctions for c = 20π or aperture length = 20λ

$$\psi_n(x) = F_n(x) \quad X \quad 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	2.11154		0		1.43384		0		1.27673		0		1.15751		0		0
.01	2.105		.234545		1.42392		.283109		1.24154		.311841		1.11113		.331708		.01
.02	2.08549		.464774		1.39274		.554074		1.13781		.602826		.975016		.633446		.02
.03	2.05336		.68651		1.2417		.801326		.971373		.853541		.767939		.877994		.03
.04	2.0332		.895818		1.13123		1.01441		.751504		1.04736		.496614		1.04338		.04
.05	1.9339		1.08914		.947377		1.18447		.493353		1.17155		.166405		1.11485		.05
.06	1.78911		1.26338		.732532		1.30462		.212221		1.21815		-.133710		1.08619		.06
.07	1.81323		1.416		.497229		1.37026		-.71224	2	1.18443		-.455599		.96022		.07
.08	1.73057		1.54505		.246714		1.37921		-.391741		1.07298		-.733751		.748432		.08
.09	1.64131		1.64923		1.11476		1.33172		-.766[illegible]		.891506		-.957959		.469876		.09
.1	1.5469		1.7279		-.2[illegible]		1.23045		-.4[illegible]		.652147		-1.10933		.149332		.1
.11	1.45877		1.78106		-.52534		1.08017		-1.[illegible]942		.370594		-1.17608		-.184982		.11
.12	1.3499		1.8093		-.759691		.88751		-1.23719		6.49459	2	-1.1561		-.503983		.12
.13	1.24686		1.81377		-.977346		.660552		-1.31214		-.245546		-1.04917		-.780296		.13
.14	1.14576		1.79612		-1.17351		.408393		-1.32345		-.541771		-.864659		-.990616		.14
.15	1.04613		1.75839		-1.34136		.140682		-1.27162		-.805985		-.616642		-1.11763		.15
.16	.949036		1.70291		-1.48136		-.132847		-1.15634		-1.02289		-.323507		-1.15137		.16
.17	.85545		1.63227		-1.59755		-.40281		-.933952		-1.18049		-6.43172	3	-1.08981		.17
.18	.766032		1.54914		-1.66306		-.660565		-.675674		-1.2707		.312265		-.93884		.18
.19	.681611		1.45627		-1.70751		-.898531		-.527419		-1.28961		.610786		-.711495		.19
.2	.602482		1.35633		-1.72241		-1.11044		-.255617		-1.23755		.869391		-.426612		.2
.21	.529059		1.25183		-1.71		-1.29148		2.79331	2	-1.11879		1.07159		-.107078		.21
.22	.461475		1.14527		-1.67315		-1.4384		.311[illegible]36		-.940897		1.20559		.222178		.22
.23	.399859		1.03868		-1.61412		-1.54947		.534259		-.714415		1.26393		.536258		.23
.24	.344146		.933976		-1.53346		-1.62444		.933294		-.451659		1.24463		.812235		.24
.25	.294194		.832751		-1.44482		-1.66436		1.[illegible]391		-.166059		1.15052		1.03084		.25
.26	.24978		.736304		-1.34981		-1.67142		1.25559		.128735		.9868		1.17766		.26
.27	.210615		.64564		-1.24271		-1.64872		1.41025		.419644		.770201		1.24401		.27
.28	.176362		.561482		-1.13233		-1.60002		1.52499		.694836		.508365		1.22713		.28
.29	.145638		.484295		-1.02092		-1.52954		1.59987		.944254		.218065		1.13		.29
.3	.121081		.414307		-.911196		-1.4417		.1[illegible]		1.15999		-6.51665	2	.960704		.3
.31	9.92605	2	.351542		-.805221		-1.34091		5.31736	2	1.33646		-.326434		.731477		.31
.32	8.07672	2	.295852		-.704465		-1.23142		7.32566	2	1.47049		-.672153		.45748		.32
.33	6.42745	2	.24695		-.610723		-1.11716		4.70[illegible]15	2	1.56113		-.950828		.15552		.33
.34	5.23534	2	.204444		-.524292		-1.00162		3.53[illegible]6	2	1.60948		-1.15345		-.157235		.34
.35	4.15784	2	.167864		-.445357		-.887794		2.6[illegible]47	2	1.61838		-1.33376		-.464451		.35
.36	3.29309	2	.136691		-.375609		-.778156		1.33321	2	1.59197		-1.46851		-.751517		.36
.37	2.58215	2	.110383		-.313493		-.674625		1.35461	2	1.53538		-1.55627		-1.00648		.37
.38	2.00912	2	8.85944	2	-.259203		-.578597		3.59973	3	1.45427		-1.59916		-1.22032		.38
.39	1.55107	2	7.01904	2	-.212332		-.490989		6.719[illegible]1	3	1.3545		-1.60042		-1.38724		.39
.4	1.18795	2	5.52626	2	-.172321		-.412285		1.63637		1.2418		-1.56499		-1.50462		.4
.41	5.[illegible]2611	3	4.31372	2	-.138543		-.3426		1.63725		1.12153		-1.49870		-1.57273		.41
.42	6.80213	3	3.33814	2	-.110354		-.281754		1.6[illegible]636		.99845		-1.40822		-1.59435		.42
.43	5.08385	3	2.56063	2	-3.70713	2	-.229328		1.54329		.876647		-1.29936		-1.57419		.43
.44	3.76780	3	1.94686	2	-6.93532	2	-.184739		1.46467		.759422		-1.17995		-1.51838		.44
.45	2.76868	3	1.46697	2	-2.26424	2	-.147289		1.37786		.649298		-1.05422		-1.43587		.45
.46	2.01630	3	1.09536	2	-4.33924	2	-.11622		1.26171		.548052		-.927641		-1.32787		.46
.47	1.45632	3	8.10379	3	-.53367		-9.07567	2	1.14537		.456777		-.804339		-1.20745		.47
.48	1.04214	3	5.93953	3	-2.3[illegible]97	2	-7.01351	2	1.[illegible]261		.375977		-.687525		-1.07915		.48
.49	7.38951	4	4.31209	3	-1.71662	2	-5.36317	2	[illegible]		.305663		-.579522		-.948699		.49
.5	5.1910	4	3.10048	3	-1.26503	2	-4.05789	2	.742938		.24546		-.48[illegible]332		-.820867		.5
.51	3.61206	4	2.20751	3	-9.22345	3	-3.03757	2	.63426		.194712		-.395[illegible]		-.6994		.51
.52	2.48909	4	1.55609	3	-6.66251	3	-2.24932	2	.43574		.152577		-.319826		-.587018		.52
.53	1.69832	4	1.05518	3	-4.75965	3	-1.64746	2	.491815		.118102		-.255489		-.435486		.53
.54	1.14708	4	7.49787	4	-3.36406	3	-1.19331	2	.4[illegible]3455		.090299		-.201376		-.39573		.54
.55	7.66775	5	5.12308	4	-2.35121	3	-8.54672	3	.337653		6.81922	2	-.156641		-.31797		.55
.56	5.07158	5	3.46276	4	-1.62613	3	-6.05163	3	.275[illegible]4		5.08596	2	-.120244		-.251873		.56
.57	3.31818	5	2.31477	4	-1.11167	3	-4.23533	3	.221453		3.74581	2	-.091037		-.196704		.57
.58	2.14894	5	1.52994	4	-7.51229	4	-2.92921	3	.17933[illegible]		2.72391	2	-.063085		-.151454		.58
.59	1.37327	5	9.99548	5	-5.01696	4	-2.00153	3	.134128		1.95544	2	-5.02111	2	-.114968		.59
.6	8.68044	6	6.45315	5	-3.31023	4	-1.35086	3	4.66951	3	1.38553	2	-3.65293	2	-8.60338	2	.6
.61	5.42205	6	4.11546	5	-2.15723	4	-9.00285	4	3.16795	3	9.68754	3	-2.62122	2	-6.34618	2	.61
.62	3.34413	6	2.59188	5	-1.33811	4	-5.92296	4	2.13205	3	6.68244	3	-1.85481	2	-4.61366	2	.62
.63	2.03537	6	1.61137	5	-8.31623	5	-3.84547	4	1.41637	3	4.54637	3	-1.29399	2	-3.30516	2	.63
.64	1.22268	6	9.88457	6	-5.52493	5	-2.46300	4	[illegible]23279	4	3.04981	3	-3.29779	3	-2.33271	2	.64
.65	7.24072	7	5.98091	6	-3.61505	5	-1.55565	4	5.[illegible]9621	4	2.01660	3	-6.02879	3	-.016216		.65
.66	4.22018	7	3.56605	6	-2.08130	5	-9.68555	5	3.[illegible]1732	4	1.31385	3	-4.02376	3	-1.10998	2	.66
.67	2.42331	7	2.09451	6	-1.24959	5	-5.94170	5	2.37407	4	8.43112	4	-2.64441	3	-7.47891	3	.67
.68	1.36808	7	1.21226	6	-7.39100	6	-3.58981	5	1.47452	4	5.32657	4	-1.71060	3	-4.95851	3	.68
.69			6.88932	7	-4.30406	6	-2.13490	5	4.[illegible]	5	3.31154	4	-1.08867	3	-3.23353	3	.69
.7			3.82426	7	-2.46511	6	-1.24914	5	[illegible]37314	5	2.02491	4	-6.81336	4	-2.07308	3	.7
.71			2.09189	7	-1.38769	6	-7.18567	6	3.15327	5	1.21711	4	-4.15090	4	-1.30602	3	.71
.72			1.12660	7	-7.72081	7	-4.06204	6	1.92317	5	7.18682	5	-2.53207	4	-8.08038	4	.72
.73					-4.13261	7	-2.25426	6	1.23381	5	4.16593	5	-1.50166	4	-4.90672	4	.73
.74					-2.24723	7	-1.22771	6	5.74459	6	2.36884	5	-8.73601	5	-2.92232	4	.74
.75					-1.17877	7	-6.55090	7	3.13214	6	1.32036	5	-4.98103	5	-1.70573	4	.75
.76							-3.43230	7	1.67[illegible]	6	7.20613	6	-2.73113	5	-9.74910	5	.76
.77							-1.77752	7	8.7[illegible]243	7	3.84743	6	-1.51912	5	-5.45136	5	.77
.78									4.421[illegible]	7	2.00670	6	-8.10770	6	-2.97875	5	.78
.79									2.17414	7	1.02265	6	-4.22023	6	-1.58880	5	.79
.8									1.[illegible]463	7	5.01655	7	-2.14482	6	-8.26163	6	.8
.81											2.46029	7	-1.05956	6	-4.18353	6	.81
.82											1.16016	7	-5.08218	7	-2.05988	6	.82
.83													-2.38538	7	-9.86385	7	.83
.84													-1.07945	7	-4.59770	7	.84
.85															-2.11226	7	.85
.86																	.86
.87																	.87
.88																	.88
.89																	.89
.9																	.9
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.93																	.93
.94																	.94
.95																	.95
.96																	.96
.97																	.97
.98																	.98
.99																	.99
1																	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	$F_8(x)$	p	$F_9(x)$	p	$F_{10}(x)$	p	$F_{11}(x)$	p	$F_{12}(x)$	p	$F_{13}(x)$	p	$F_{14}(x)$	p	$F_{15}(x)$	p	x
0	1.07507		0		1.01216		[illegible]		.961247		0		.918303		[illegible]		0
.01	1.02018		.346325		[illegible]		[illegible]		.890636		.365738		.84094		.371998		.01
.02	.861118		.653414		.767431		[illegible]		.689172		.673851		.621873		[illegible]		.02
.03	.614134		.886514		.490076		[illegible]		.386419		.875816		.297968		.831769		.03
.04	.30442		1.01931		.15149		.983271		2.67719	2	.939866		-7.43021	2	.891987		.04
.05	-3.65144	2	1.03691		-.206229		.969592		-.3371		.855966		-.438045		.766733		.05
.06	-.374103		.937474		-.53071		.785476		-.652004		.637352		-.726564		.497085		.06
.07	-.67428		.732354		-.804357		.516042		-.87202		.318371		-.893516		.14641		.07
.08	-.906948		.444728		-.97193		.17666		-.965184		-5.09854	2	-.911011		[illegible]		.08
.09	-1.04899		.106876		-1.01955		-.187038		-.91808		-.412985		-.776186		-.576996		.09
.1	-1.08655		-.243522		-.942103		-.526364		-.737681		-.711221		-.511584		-.814027		.1
.11	-1.01626		-.567666		-.749316		-.796127		-.450202		-.8994		-.161264		-.997535		.11
.12	-.845483		-.829966		-.464064		-.960638		-9.71627	2	-.948474		.21664		-.840496		.12
.13	-.591371		-1.00194		-.123431		-.998369		.270725		-.851013		.559642		[illegible]		.13
.14	-.279		-1.06523		.235032		-.904643		.600919		-.622184		.811259		-.599667		.14
.15	6.12394	2	-1.01348		.534174		-.692053		.846571		-.297214		.930266		7.94561	6	.15
.16	.396719		-.852747		.328400		-.38853		.973123		.074195		.897329		.449926		.16
.17	.695772		-.600667		.999607		-3.37318	2	.963048		.435596		.717969		.730079		.17
.18	.9307		-.284273		1.04722		.326757		.818088		.732438		.421423		.899774		.18
.19	1.08029		.063067		.97707		.646909		.558733		.920272		5.56632	2	.908909		.19
.2	1.13159		.405324		.794107		.886385		.221091		.971291		-.320542		.762941		.2
.21	1.08087		.707624		.519767		1.01548		-.148357		.878239		-.647168		.484303		.21
.22	.933644		.939811		.154461		1.01865		-.499304		.655176		-.872514		.123114		.22
.23	.703796		1.07937		-.171138		.896125		-.784533		.335065		-.961268		-.260573		.23
.24	.412036		1.11343		-.510661		.663496		-.962214		-3.53718	2	-.899819		-.602623		.24
.25	8.37735	2	1.03973		-.796409		.34944		-1.02076		-.402681		-.698041		-.843826		.25
.26	-.253306		.866392		-.998098		-8.11771	3	-.941636		-.714494		-.387364		-.990194		.26
.27	-.57172		.610739		-1.03516		-.366811		-.739758		-.926947		-1.55584	2	-.898815		.27
.28	-.846489		.297239		-1.07854		-.684976		-.441661		-1.01079		.36087		-.700352		.28
.29	-1.05707		-4.51328	2	-.951042		-.926501		-8.56343	2	-.954917		.685402		-.387191		.29
.3	-1.18875		-.385797		-.726719		-1.06479		.283483		-.767675		.909976		-9.77956	3	.3
.31	-1.23345		-.695433		-.428855		-1.08544		.620127		-.475062		1.00193		.371783		.31
.32	-1.18991		-.948529		-8.71935	2	-.987373		.883676		-.116742		.948428		.697475		.32
.33	-1.06327		-1.12539		.265315		-.782362		1.04323		.259678		.757808		.316764		.33
.34	-.86416		-1.21344		.595933		-.493201		1.08096		.605196		.457804		.996236		.34
.35	-.607457		-1.20781		.875204		-.150715		.993683		.875844		9.10437	2	.92451		.35
.36	-.310799		-1.11114		1.07554		.209892		.792605		1.03815		-.291401		.712414		.36
.37	6.95916	3	-.932775		1.19297		.55291		.50136		1.07298		-.637358		.392631		.37
.38	[illegible].27214		-.687428		1.20603		.845769		.152731		.977289		-.900712		1.23167	2	.38
.39	6.2899		-.393536		1.12597		1.06207		-.215465		.763822		-1.04738		-.573317		.39
.4	.909495		-7.14673	2	.995439		1.18375		-.564779		.458796		-1.05936		-.709442		.4
.41	1.14572		.258233		.711734		1.20217		-.860261		9.81276	2	-.936347		-.949402		.41
.42	1.3339		.576403		.414461		1.11821		-1.07389		-.27727		-.695015		-1.06094		.42
.43	1.46999		.866528		8.55182	2	.941364		-1.18694		-.626433		-.366056		-1.03005		.43
.44	1.55338		1.11561		-.25205		.688147		-1.19119		-.912924		1.03302	2	-.862087		.44
.45	1.58642		1.31469		-.579224		.379931		-1.08885		-1.10851		.389835		-.380132		.45
.46	1.57384		1.45888		-.876122		.040617		-.891475		-1.19557		.739545		-.221085		.46
.47	1.5221		1.54721		-1.12933		-.305679		-.61798		-1.16814		.992838		.169307		.47
.48	1.43872		1.58209		-1.32928		-.636669		-.292188		-1.03155		1.1531		.545125		.48
.49	1.33161		1.56863		-1.47103		-.933502		5.98604	2	-.800852		1.19593		.86124		.49
.5	1.20856		1.51389		-1.55391		-1.18185		.412474		-.498418		1.11975		1.08331		.5
.51	1.07676		1.4261		-1.58096		-1.37244		.742519		-.151056		.934782		1.18873		.51
.52	.942611		1.3139		-1.55815		-1.50107		1.03108		.213113		.660963		1.16879		.52
.53	.811282		1.18576		-1.49344		-1.56818		1.20452		.567232		.324879		1.02844		.53
.54	.686869		1.04943		-1.35596		-1.5781		1.43484		.888175		-.043594		.784246		.54
.55	.57229		.911647		-1.2751		-1.5381		1.53955		1.15804		-.414592		.492851		.55
.56	.469389		.77789		-1.1399		-1.45736		1.581		1.36503		-.761141		9.43491	2	.56
.57	.379072		.65232		-.993493		-1.34598		1.56541		1.50357		-1.06133		-.288518		.57
.58	.301476		.537814		-.857760		-1.21408		1.50176		1.57383		-1.29903		-.655323		.58
.59	.236138		.436076		-.723154		-1.07107		1.4006		1.58081		-1.46744		-.980417		.59
.6	.182171		.347813		-.590623		-.925207		1.27297		1.5351		-1.56272		-1.24478		.6
.61	.138417		.272923		-.460747		-.783186		1.12949		1.44157		-1.58922		-1.4369		.61
.62	.103578		.210706		-.338065		-.650084		.979673		1.31808		-1.55508		-1.55269		.62
.63	.076326		.16005		-.305292		-.529362		.831464		1.17435		-1.47145		-1.59947		.63
.64	5.53784	2	.119607		-.235555		-.423015		.691018		1.02111		-1.35096		-1.57076		.64
.65	3.95538	2	8.79291	2	-.178624		-.331795		.562665		.86749		-1.2064		-1.49236		.65
.66	2.78046	2	6.35795	2	-.133117		-.255473		.449044		.720667		-1.04968		-1.37292		.66
.67	1.92312	2	4.52083	2	-9.74822	2	-.193103		.351326		.585794		-.891026		-1.22631		.67
.68	1.30833	2	3.16027	2	-7.01355	2	-.143278		.269504		.466095		-.73857		-1.06556		.68
.69	8.79166	3	2.17119	2	-4.95642	2	-.104341		.202704		.363106		-.598198		-.901948		.69
.7	5.75367	3	1.46549	2	-3.43943	2	-7.45636	2	.149475		.276995		-.473628		-.744452		.7
.71	3.71601	3	9.71395	3	-.023428		-5.22726	2	.108046		.206915		-.366674		-.59957		.71
.72	2.35644	3	6.32011	3	-1.56576	2	-3.59371	2	7.85381	2	.151336		-.277598		-.471389		.72
.73	1.46632	3	4.03397	3	-1.02626	2	-2.42185	2	5.31158	2	.10835		-.205307		-.361872		.73
.74	8.94748	4	2.52435	3	-6.55272	3	-1.59906	2	3.60962	2	7.59124	2	-.148744		-.271265		.74
.75	5.35005	4	1.54765	3	-4.14833	3	-1.05381	2	2.40025	2	5.20247	2	-.105226		-.193537		.75
.76	3.13207	4	9.28882	4	-2.55422	3	-6.53995	3	1.56195	2	3.46569	2	-7.27281	2	-.141838		.76
.77	1.79361	4	5.45292	4	-1.53873	3	-4.04509	3	9.93240	3	2.28177	2	-.049084		-9.88723	2	.77
.78	1.00367	4	3.12787	4	-9.05465	4	-2.44405	3	6.16825	3	1.45823	2	-3.23256	2	-6.72121	2	.78
.79	5.48142	5	1.75115	4	-5.20016	4	-1.44104	3	3.73740	3	9.08999	3	-2.07572	2	-4.45254	2	.79
.8	2.91772	5	9.55660	5	-2.91117	4	-8.28160	4	2.20695	3	5.52113	3	-1.29834	2	-2.87198	2	.8
.81	1.51112	5	5.07634	5	-1.58644	4	-4.63286	4	1.28846	3	3.24357	3	-7.90157	3	-1.80183	2	.81
.82	7.60312	6	2.62021	5	-8.49244	5	-2.51692	4	7.02530	4	1.87476	3	-4.67265	3	-1.09817	2	.82
.83	3.70542	6	1.31152	5	-4.31727	5	-1.32872	4	3.84013	4	1.04490	3	-2.68077	3	-6.49242	3	.83
.84	1.74486	6	6.35361	6	-2.14788	5	-6.78669	5	2.01522	4	5.63953	4	-1.48939	3	-3.71680	3	.84
.85	7.89033	7	2.97014	6	-1.03228	5	-3.34814	5	1.02150	4	2.94090	4	-7.99606	4	-2.05613	3	.85
.86	3.38615	7	1.33301	6	-4.78027	6	-1.59154	5	4.58911	5	1.47786	4	-4.13765	4	-1.09653	3	.86
.87	1.34213	7	5.72397	7	-2.12874	6	-7.26743	6	2.33943	5	7.13388	5	-2.05745	4	-5.62050	4	.87
.88	—	—	2.30571	7	-9.08871	7	-3.17732	6	1.04873	5	3.29529	5	-9.79556	5	-2.79927	4	.88
.89			—	—	-3.73354	7	-1.32722	6	4.47185	6	1.44959	5	-4.44573	5	-1.29188	4	.89
.9					-1.48387	7	-5.26691	7	1.79784	6	6.03387	6	-1.91303	5	-5.73794	5	.9
.91							-2.01635	7	6.71462	7	2.35521	6	-7.75244	6	-2.40216	5	.91
.92									2.25488	7	8.50265	7	-2.93301	6	-9.39358	6	.92
.93											2.74935	7	-1.02695	6	-3.40099	6	.93
.94													-3.26609	7	-1.12142	6	.94
.95															-3.35595	7	.95
.96																	.96
.97																	.97
.98																	.98
.99																	.99
1																	1

Where no values are listed the absolute value is less than 10^{-7}

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	$F_{16}(x)$	p	$F_{17}(x)$	p	$F_{18}(x)$	p	$F_{19}(x)$	p	$F_{20}(x)$	p	$F_{21}(x)$	p	$F_{22}(x)$	p	$F_{23}(x)$	p	x
0	[illegible]		[illegible]		.847578		[illegible]		.817173		0		.768914		0		0
.01	.79746		[illegible]		.758609		[illegible]		.723287		[illegible]		.690696		.381283		.01
.02	.56285		.637847		.510356		[illegible]		.463186		[illegible]		.420469		.664129		.02
.03	.221503		.343215		.154871		[illegible]		9.65532	2	[illegible]		.045438		.7755		.03
.04	-.162006		.8415[illegible]		-.233333		.749934		-.272485		.73[illegible]		-.341136		.686603		.04
.05	-.515171		.671284		-.572917		[illegible]		-.414682		[illegible]		-.643124		.420304		.05
.06	-.771277		[illegible]		-.792764		[illegible]		-.796136		[illegible]		-.785422		.045216		.06
.07	-.881996		[illegible]	2	-.846849		[illegible]		-.795213		[illegible]		-.732591		-.342003		.07
.08	-.826464		[illegible]		-.723827		[illegible]		-.61204		[illegible]		-.497626		-.661565		.08
.09	-.615134		[illegible]		-.449329		[illegible]		-.248424		[illegible]		-.138696		-.776243		.09
.1	-.287711		[illegible]		-8.05398	2	[illegible]		.101768		[illegible]		.255316		-.711223		.1
.11	9.44074	2	[illegible]		.305802		[illegible]		.46752		[illegible]		.586853		-.463023		.11
.12	.459722		[illegible]		.629414		[illegible]		.730996		[illegible]		.773825		-9.52006	2	.12
.13	.740047		[illegible]		.823144		[illegible]		.826001		[illegible]		.769868		.297992		.13
.14	.8832		[illegible]	3	.846833		[illegible]		.73447		[illegible]		.57573		.615795		.14
.15	.862622		[illegible]		.69551		[illegible]		.475326		[illegible]		.239011		.776736		.15
.16	.682143		[illegible]		.400263		[illegible]		.107663		[illegible]		-.157634		.739425		.16
.17	.375077		[illegible]		2.16814	2	[illegible]		-.285586		[illegible]		-.516857		.513058		.17
.18	-2.12118	3	[illegible]		-.362717		[illegible]		-.615828		[illegible]		-.750519		.154997		.18
.19	-.380386		[illegible]		-.674425		.5		-.808751		5.33359		-.801231		-.243902		.19
.2	-.690763		[illegible]		-.849952		[illegible]	3	-.820933		[illegible]		-.656289		-.582414		.2
.21	-.876948		[illegible]		-.853617		-.3[illegible]		-.649423		[illegible]		-.350654		-.774616		.21
.22	-.905363		[illegible]		-.684584		[illegible]		-.332212		-.817237		4.16862	2	-.771578		.22
.23	-.770978		[illegible]		-.37678		[illegible]		6.04127	2	[illegible]		.425772		-.573623		.23
.24	-.497903		[illegible]		8.18214	3	-.832966		.44161		[illegible]		.708705		-.230125		.24
.25	-.134759		[illegible]		.393539		-.640297		.247251		[illegible]		.822042		.173017		.25
.26	.254241		[illegible]		.702778		-.312396		.854407		[illegible]		.738156		.534988		.26
.27	.6008		[illegible]		.874774		8.[illegible]		.79501		[illegible]		.476692		.76528		.27
.28	.84455		[illegible]		.875672		.462754		.5617		.777[illegible]		9.96752	2	.806186		.28
.29	.943421		[illegible]		.70523		.749744		.204868		[illegible]		-.30345		.647013		.29
.3	.880615		[illegible]		.396494		[illegible]		-.19861		[illegible]		-.637164		.326552		.3
.31	.667065		[illegible]		9.00669	3	[illegible]		-.562091		[illegible]		-.822474		-7.66822	2	.31
.32	.339086		[illegible]		-.382968		[illegible]		-.807803		[illegible]		-.8154		-.463902		.32
.33	-4.82088	2	[illegible]		-.70482		[illegible]		-.88332		[illegible]		-.617079		-.740313		.33
.34	-.430483		[illegible]		-.895743		[illegible]		-.772419		[illegible]		-.27321		-.838196		.34
.35	-.745003		[illegible]		-.919966		[illegible]		-.498112		[illegible]		.136851		-.733204		.35
.36	-.940868		[illegible]		-.773029		[illegible]		-.117406		[illegible]		.518657		-.450002		.36
.37	-.986974		[illegible]		-.482082		[illegible]		.29094		[illegible]		.784482		-5.60512	2	.37
.38	-.876492		[illegible]		-.100333		[illegible]		.642933		[illegible]		.873313		.354807		.38
.39	-.627295		[illegible]		.303127		[illegible]		.866612		[illegible]	2	.764476		.684869		.39
.4	-.278462		[illegible]		.656079		[illegible]		.916505		[illegible]		.481931		.855735		.4
.41	.116407		[illegible]		.896117		[illegible]		.782397		[illegible]		8.84574	2	.826617		.41
.42	.497821		[illegible]		.981419		[illegible]		.490805		[illegible]		-.32874		.603577		.42
.43	.80953		[illegible]		.897533		[illegible]		9.91035	2	[illegible]		-.677602		.237705		.43
.44	1.00672		[illegible]		.659148		[illegible]		-.31629		[illegible]		-.881486		-.187111		.44
.45	1.06209		[illegible]		.306784		[illegible]		-.675129		[illegible]		-.895669		-.573762		.45
.46	.968995		[illegible]		-.100777		[illegible]		-.908836		-.165379		-.716574		-.834186		.46
.47	.741477		[illegible]		-.496796		[illegible]		-.973295		[illegible]		-.381932		-.909164		.47
.48	.411296		[illegible]		-.817727		[illegible]		-.85651		[illegible]		3.78784	2	-.781236		.48
.49	2.27248	2	[illegible]		-1.0133		[illegible]		-.580114		-.970069		.455171		-.478027		.49
.5	-.374031		[illegible]		-1.05393		[illegible]		-.1944		-.906669		.783514		-6.55062	2	.5
.51	-.729701		[illegible]		-.934441		[illegible]		.231546		-.667376		.955493		.367		.51
.52	-1.00215		[illegible]		-.673744		[illegible]		.622746		-.297945		.936035		.726497		.52
.53	-1.16108		[illegible]		-.310841		[illegible]		.911671		[illegible]		.728731		.936264		.53
.54	-1.19084		[illegible]		.101938		[illegible]		1.04962		[illegible]		.374184		.951755		.54
.55	-1.09103		[illegible]		.507199		[illegible]		1.01423		[illegible]		-5.89089	2	.769222		.55
.56	-.875193		[illegible]		.850828		[illegible]	3	.812124		[illegible]		-.487819		.425565		.56
.57	-.567926		[illegible]		1.08918		[illegible]		47645		[illegible]		-.831947		-1.01824	2	.57
.58	-.201015		[illegible]		1.19416		[illegible]		6.03067	2	[illegible]		-1.02784		-.451564		.58
.59	.190836		[illegible]		1.15563		[illegible]		-.372669		.519763		-1.04023		-.812294		.59
.6	.574038		[illegible]		.981155		1.19[illegible]		-.758662		[illegible]		-.867331		-1.02304		.6
.61	.919487		[illegible]		.693416		1.17[illegible]		-1.04323		-.343571		-.539703		-1.04409		.61
.62	1.20493		[illegible]		.325872		1.00[illegible]		-1.18861		-.743211		-.113438		-.871711		.62
.63	1.41621		[illegible]		-8.23931	2	.715[illegible]		-1.17778		-1.0397		.340679		-.537331		.63
.64	1.54746		[illegible]		-.491904		.345591		-1.0151		-1.1917		.74975		-.100292		.64
.65	1.60028		[illegible]		-.867354		-7.15652	2	-.723708		-1.12036		1.05073		.364033		.65
.66	1.5824		[illegible]		-1.18099		-.492699		-.340591		-1.00398		1.19971		.77813		.66
.67	1.50578		[illegible]		-1.41457		-.373613		8.97674	2	-.705612		1.17727		1.07554		.67
.68	1.3848		[illegible]		-1.55988		-1.19939		.521908		-.30717		.989352		1.21105		.68
.69	1.23443		[illegible]		-1.61795		-1.43555		.915171		.137438		.664011		1.1663		.69
.7	1.06879		[illegible]		-1.59735		-1.57[illegible]		1.23799		.579618		.245184		.950472		.7
.71	.900139		[illegible]		-1.51199		-1.6301		1.47031		.976469		-.215253		.59628		.71
.72	.738464		[illegible]		-1.37863		-1.[illegible]393		1.60415		1.29942		-.665564		.152724		.72
.73	.589818		[illegible]		-1.2147		-1.50349		1.64248		1.51674		-1.06109		-.323882		.73
.74	.459392		[illegible]		-1.03647		-1.[illegible]		1.59696		1.63339		-1.36912		-.778832		.74
.75	.348839		[illegible]		-.857722		-1.1[illegible]		1.48498		1.65206		-1.57131		-1.16645		.75
.76	.258246		[illegible]		-.689054		-1.[illegible]		1.32669		1.53545		-1.66369		-1.45504		.76
.77	.186347		[illegible]		-.537675		-.[illegible]		1.14219		1.45368		-1.65464		-1.62898		.77
.78	.131016		[illegible]		-.407614		-.[illegible]		.949469		1.27943		-1.56157		-1.6881		.78
.79	8.97015	2	[illegible]		-.300205		[illegible]		.763009		1.03347		-1.40712		-1.64497		.79
.8	5.97628	2	[illegible]		-.21473		-.369403		.593163		.384455		-1.2153		-1.5209		.8
.81	3.87095	2	[illegible]		-.140004		-.266511		.446177		[illegible]		-1.0085		-1.34149		.81
.82	2.43483	2	[illegible]		-.100389		-.486239		.324675		[illegible]		-.805222		-1.13256		.82
.83	1.48521	2	[illegible]		-6.54919	2	-.126051		.228435		.3387[illegible]		-.618988		-.917021		.83
.84	8.77140	3	[illegible]		-4.13375	2	-8.24678		.155257		.27[illegible]		-.458129		-.712774		.84
.85	5.00560	3	[illegible]		-2.52011	2	-5.20885		.101802		.18[illegible]		-.32629		-.53193		.85
.86	2.75381	3	[illegible]		-1.48084	3	-.031702		6.42893	2	.123335		-.223393		-.380946		.86
.87	1.45640	3	[illegible]	3	-8.36577	3	-1.85474		3.90162	2	.077753		-.146784		-.261499		.87
.88	7.37929	4	[illegible]	3	-4.52954	3	-1.04004		2.26923	2	4.70109	2	-9.23555	2	-.171739		.88
.89	3.56724	4	[illegible]	4	-2.34142	3	-5.56912		1.26045	2	[illegible]		-5.54793	2	-.107626		.89
.9	1.63680	4	[illegible]		-.00115		-2.83462		6.65710	3	.01439		-3.16957	2	-6.41334	2	.9
.91	7.08267	5	[illegible]		-5.33422	4	-1.36344		3.32443	3	7.73057	3	-1.71349	2	-3.61675	2	.91
.92	2.86627	5	[illegible]		-2.31849	4	-6.15071	4	1.55837	3	3.77069	3	-8.70740	3	-1.91822	2	.92
.93	1.07312	5	[illegible]		-9.36528	5	-2.57639		6.79116	4	1.71146	3	-4.12255	3	-9.48753	3	.93
.94	3.65699	6	[illegible]		-3.44391	5	-9.88402		2.71519	4	7.13[illegible]7	4	-1.79639	3	-4.32512	3	.94
.95	1.11408	6	[illegible]		-1.13752	5	-3.40570		7.77324	5	[illegible]	4	-7.07903	4	-1.78702	3	.95
.96	2.93436	7	[illegible]		-3.26212	6	-1.02248		3.07737	5	8.37665	5	-2.45650	4	-6.52280	4	.96
.97			[illegible]		-7.71262	7	-2.54622		8.03746	6	[illegible]	5	-7.18327	5	-2.01630	4	.97
.98					-1.32893	7	-4.72780		1.62036	6	[illegible]	6	-1.62903	5	-4.87204	5	.98
.99									1.99793	7	6.96563	7	-2.34096	6	-7.57757	6	.99
1															-2.86435	7	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	$F_{24}(x)$	p	$F_{25}(x)$	p	$F_{26}(x)$	p	$F_{27}(x)$	p	$F_{28}(x)$	p	$F_{29}(x)$	p	$F_{30}(x)$	p	$F_{31}(x)$	p	x
0	.762162		[illegible]		.736353		[illegible]		.710931		0		.685254		0		0
.01	.660196		.339236		.631251		[illegible]		.603351		.373053		.57594		.368299		.01
.02	.38153		.655963		.345913		[illegible]		.313135		.631693		.282838		.616254		.02
.03	6.95701	4	.749324		-3.82937	2	[illegible]		-7.19764	2	[illegible]		-.100633		.662808		.03
.04	-.380578		.636[illegible]73		-.411804		[illegible]		-.435538		.539[illegible]		-.452226		.492665		.04
.05	-.660361		.346434		-.668079		[illegible]		-.667584		.217167		-.659811		.161234		.05
.06	-.763839		-3.[illegible]34	2	-.733966		[illegible]		-.697857		-.17251		-.657095		-.22315		.06
.07	-.663257		-.613336		-.590541		[illegible]		-.517037		-.509034		-.444755		-.535106		.07
.08	-.385313		-.67[illegible]278		-.278493		[illegible]		-.179571		-.687822		-9.02528	2	-.672584		.08
.09	-4.01524	3	-.759672		.113458		-.707471		.212744		-.653169		.295608		-.590457		.09
.1	.379061		-.619327		.473805		-.41142		.541476		-.415546		.584566		-.31523		.1
.11	.66184		-.316373		.699965		-.176243		.707277		-4.83913	2	.689798		.063568		.11
.12	.768912		7.41659	2	.72743		.213332		.659851		.334614		.575478		.422422		.12
.13	.671555		.445251		.548084		[illegible]		.413131		.61474		.277577		.644249		.13
.14	.395342		.694127		.212502		.721256		4.11172	2	.704966		-.109498		.656374		.14
.15	1.32998	2	.754138		-.184393		.691[illegible]		-.344373		.576933		-.462846		.454361		.15
.16	-.37347		[illegible]		-.530223		.442931		-.627314		.269737		-.67007		.103524		.16
.17	-.662578		.232124		-.726956		7.30432	2	-.722349		-.12191		-.664961		-.29223		.17
.18	-.777401		-.103457		-.716608		-.313339		-.600439		-.477284		-.448559		-.577383		.18
.19	-.687272		-.472239		-.507048		-.61[illegible]073		-.297615		-.[illegible]36655		-.088913		-.685593		.19
.2	-.415497		-.713[illegible]		-.151453		-.740555		9.57837	2	-.684427		.300383		-.571077		.2
.21	-3.31571	2	-.761149		.248365		-.646[illegible]		.462119		-.471163		.59608		-.270274		.21
.22	.359617		-.6[illegible]181		.580034		-.364[illegible]17		.691629		-.111672		.704238		.1137		.22
.23	.659921		-.279535		.750178		2.44302	2	.715227		.263907		.589964		.472633		.23
.24	.768978		.122702		.710573		.409622		.525148		.544077		.288528		.673867		.24
.25	.712642		.442343		.47165[illegible]		.676519		.17731		.723251		-.105638		.657546		.25
.26	.450196		.737[illegible]7		9.94559	2	.75034		-.225277		.631091		-.468591		.428015		.26
.27	6.92321	2	.77342		-.502608		.608072		-.563087		.344881		-.68583		5.85799	2	.27
.28	-.331955		.603754		-.522747		.289812		-.73553		-.049627		-.68832		-.331959		.28
.29	-.649821		.279232		-.771726		-.113744		-.690765		-.429696		-.474261		-.61766		.29
.3	-.802239		-.127089		-.707579		-.43727		-.441067		-.682209		-.109793		-.705732		.3
.31	-.749519		.502275		-.447169		-.723781		-5.92555	2	-.728075		.291573		-.56677		.31
.32	-.504425		-.746384		-6.16803	2	-.755145		.342717		-.552835		.604356		-.244246		.32
.33	-.128757		-.794167		.343951		-.571464		.646651		-.208325		.730271		.159206		.33
.34	.282523		-.632183		.659008		-.223872		.762779		.201977		.628939		.514526		.34
.35	.62545		-.302201		.794373		.189959		.656066		.554339		.330595		.707484		.35
.36	.813297		.109902		.71501		.55342		.356476		.742053		-.073138		.675257		.36
.37	.79828		.496725		.440419		.763812		-4.97227	2	.707577		-.457529		.426656		.37
.38	.583317		.757341		4.39768	2	.761174		-.444997		.459384		-.703282		3.35555	2	.38
.39	.221048		.323643		-.367959		.545034		-.714605		.071879		-.733414		-.363659		.39
.4	-.199465		.677446		-.68471		.17449		-.779702		-.341295		-.537113		-.654823		.4
.41	-.574887		.355334		-.820917		-.243283		-.620353		-.656518		-.173224		-.740626		.41
.42	-.813042		-6.11374	2	-.73923		-.636394		-.280726		-.779286		.247634		-.592267		.42
.43	-.855288		-.46633		-.460078		-.800521		.143012		-.671[illegible]		.596827		-.254792		.43
.44	-.690498		-.757657		-5.60869	2	-.776191		.530419		-.3[illegible]		.76696		.166837		.44
.45	-.357374		-.360357		.367213		-.538615		.770976		5.46514	2	.704662		.540533		.45
.46	6.52316	2	-.749373		.699137		-.151138		.795392		.462079		.426935		.743434		.46
.47	.47741		-.449436		.852683		.282082		.595129		.733761		1.61822	2	.723954		.47
.48	.781977		-3.44613	2	.78695		.644321		.224756		.802816		-.404533		.472202		.48
.49	.907204		.394339		.517424		.837684		-.213388		.633784		-.708446		7.00383	2	.49
.5	.823203		.75253		.111693		.809216		-.597754		.278584		-.803132		-.35933		.5
.51	.548303		.897533		-.328054		.5648		-.8213		-.161986		-.658213		-.682945		.51
.52	.144242		.848767		-.691077		.167352		-.820977		-.562163		-.314545		-.79966		.52
.53	-.298416		.576604		-.885904		-.279968		-.594917		-.807143		.127776		-.671275		.53
.54	-.681011		.199385		-.862932		-.660385		-.202779		-.825715		.538885		-.334432		.54
.55	-.918792		-.250354		-.626296		-.97621		.250617		-.610364		.79732		.110374		.55
.56	-.959254		-.650207		-.232316		-.869231		.643688		-.219325		.829606		.52817		.56
.57	-.792885		-.[illegible]96945		.224729		-.639877		.870687		.239736		.612884		.77265		.57
.58	-.454294		-.962165		.635711		-.244265		.869719		.640021		.218123		.821628		.58
.59	-1.38858	2	-.3[illegible]2437		.90281		.219837		.638817		.870116		-.246857		.603402		.59
.6	.437704		-.461939		.962368		.637976		.235075		.865052		-.649435		.200058		.6
.61	.80876		-1.37696	2	.799158		.906913		-.235846		.62355		-.873988		-.270774		.61
.62	1.02505		.446397		.44903		.960039		-.656473		.208157		-.854612		-.670576		.62
.63	1.04397		.921568		-1.02214	2	.732905		-.918353		-.271841		-.593587		-.880617		.63
.64	.861743		1.0337		-.477168		.415437		-.953993		-.689732		-.140604		-.836675		.64
.65	.51258		1.03912		-.849688		-5.79332	2	-.752209		-.934853		.326711		-.54774		.65
.66	6.06004	2	.336256		-1.04716		-.528967		-.360119		-.941296		.73526		-9.34614	2	.66
.67	-.415366		.464236		-1.02704		-.890774		.129199		-.704475		.952763		.393632		.67
.68	-.827171		-6.16150	3	-.792569		-1.06187		.599849		-.281623		.917893		.789359		.68
.69	-1.11179		-.437943		-.390388		-1.00349		.940977		.223407		.636287		.967052		.69
.7	-1.22222		-.894456		.100579		-.726694		1.07249		.686633		.178503		.878606		.7
.71	-1.14315		-1.15531		.586181		-.288238		.962613		.994654		-.339008		.543301		.71
.72	-.888983		-1.2277		.975945		.222777		.6337		1.07177		-.784599		5.00051	2	.72
.73	-.498748		-1.10168		1.19985		.705073		.155495		.897206		-1.04408		-.472763		.73
.74	-2.75609	2	-.779124		1.21978		1.06545		-.371675		.508289		-1.05079		-.885913		.74
.75	.16355		-.367095		1.03395		1.23669		-.[illegible]3941		-9.61233	3	-.799177		-1.07924		.75
.76	.917401		.132306		.674108		1.18855		-1.15398		-.543989		-.34569		-.998939		.76
.77	1.28842		.633417		.196972		.930529		-1.25435		-.980865		.206527		-.660406		.77
.78	1.5474		1.07754		-.327869		.506771		-1.12198		-1.22919		.733025		-.142956		.78
.79	1.68302		1.42079		-.830761		-1.50843	2	-.781107		-1.23849		1.11703		.431153		.79
.8	1.70032		1.63833		-1.25309		-.558072		-.290567		-1.00642		1.27487		.927322		.8
.81	1.61702		1.72575		-1.55486		-1.05		.270405		-.575513		1.17213		1.23058		.81
.82	1.45878		1.69247		-1.71799		-1.43474		.818545		-2.10369	2	.867601		1.27095		.82
.83	1.25413		1.56437		-1.74514		-1.67897		1.28163		.566568		.305349		1.03657		.83
.84	1.03023		1.37089		-1.65606		-1.77389		1.60955		1.10014		-.30[illegible]244		.572448		.84
.85	.809767		1.1431		-1.48092		-1.73232		1.77849		1.51122		-.89[illegible]192		-3.36057	2	.85
.86	.60921		.908643		-1.25406		-1.58275		1.79235		1.75962		-1.38745		-.673121		.86
.87	.438467		.683932		-1.00814		-1.36183		1.67546		1.83611		-1.71866		-1.24129		.87
.88	.301506		.497983		-.77003		-1.10716		1.46554		1.75875		-1.86427		-1.65753		.88
.89	.197654		.342647		-.558527		-.851458		1.20479		1.56476		-1.83297		-1.87811		.89
.9	.123142		.223849		-.384017		-.619056		.932233		1.30044		-1.6596		-1.89916		.9
.91	7.25987	2	.138319		-.242494		-.424575		.678461		1.31148		-1.39385		-1.75078		.91
.92	4.02652	2	8.04094	2	-.152445		-.273606		.463128		.735564		-1.08842		-1.48519		.92
.93	.020842		.04365		-8.70038	2	-.166665		.295016		.493301		-.789081		-1.16265		.93
.94	5.95691	3	2.18953	2	-4.59272	2	-9.17207	2	.173964		.21233		-.528453		-.837592		.94
.95	4.32022	3	7.99486	3	-.022103		-4.66528	2	9.37923	2	.179131		-.323844		-.551597		.95
.96	1.66130	3	4.0[illegible]26	3	-9.48575	3	-2.12216	2	4.53442	2	9.23399	2	-.178698		-.327431		.96
.97	5.43692	4	.001403		-3.49921	3	-8.33848	3	1.90265	2	4.15042	2	-8.63498	2	-.170863		.97
.98	1.40257	4	3.38537	4	-1.03512	3	-2.65027	3	6.51445	3	1.53566	2	-3.46469	2	-7.46747	2	.98
.99	2.36671	5	7.14129	5	-2.07580	4	-5.81799	4	1.56975	3	4.07745	3	-1.01744	2	-.024377		.99
1	9.60890	7	3.43893	6	-1.20509	5	-4.33635	5	1.32145	4	4.16168	4	-1.26279	3	-3.70823	3	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	F32(x)	p	F33(x)	p	F34(x)	p	F35(x)	p	F36(x)	p	F37(x)	p	F38(x)	p	F39(x)	p	x
0	.658437		[illegible]		.628866		[illegible]		.599612		0		[illegible]		0		0
.01	.548283		[illegible]		.519074		[illegible]		[illegible]		.329306		.440565		.305112		.01
.02	.25464		[illegible]		.227994		[illegible]		[illegible]		.537259		.174124		.494495		.02
.03	-.124334		[illegible]		-.142829		[illegible]		-.1[illegible]		.547186		[illegible]		.196273		.03
.04	-.461928		[illegible]		-.463998		.402838		-.4[illegible]		.355331		-.430634		.300679		.04
.05	-.645208		[illegible]		-.623375		6.83461	2	-.[illegible]	2	3.22764	2	[illegible]	3	5.37284	3	.05
.06	-.612758		[illegible]		-.565196		-.290824		-.[illegible]		-.303007		[illegible]		-.301274		.06
.07	-.375224		[illegible]		-.309549		-.546852		-.[illegible]		-.526932		[illegible]		-.433507		.07
.08	-1.18018	2	[illegible]		4.45674	2	[illegible]		[illegible]		-.556803		.140917		-.449267		.08
.09	.35616		[illegible]		.400213		[illegible]		[illegible]		-.581385		[illegible]		-.315032		.09
.1	.605658		[illegible]		.606723		-.136716		[illegible]		-6.46553	4	[illegible]		-1.08302	2	.1
.11	.653101		[illegible]		.601787		[illegible]		[illegible]		.276183		[illegible]		[illegible]		.11
.12	.482264		[illegible]		.386717		.514651		[illegible]		.516093		[illegible]				.12
.13	.149782		[illegible]		3.61036	2	.617315		-5.[illegible]	2	.566189		-1.4[illegible]				.13
.14	-.233647		[illegible]		-.328047		.500472		[illegible]		.407532		-.41194		[illegible]		.14
.15	-.540115		[illegible]		-.57876		.205113		-.[illegible]		9.80681	2	[illegible]		1.63958	2	.15
.16	-.667124		[illegible]	2	-.628293		-.1[illegible]		-.[illegible]		-.248329		-.475711		-.259613		.16
.17	-.571804		[illegible]		-.458795		-.376249		[illegible]		-.504578		[illegible]		-.496321		.17
.18	-.285296		[illegible]		-.128673		-.619333		6.[illegible]	3	-.575374		.107496		-.509103		.18
.19	9.74737	2	[illegible]		.247554		-.543568		.352231		-.454168		[illegible]		-.323433		.19
.2	.449265		-.426433		.538967		-.273518		.5[illegible]		-.132234		.54[illegible]		-2.22098		.2
.21	.652794		-7.42[illegible]	2	.643755		.034353		.583921		.219746		.489162		.293731		.21
.22	.639758		[illegible]		.524704		.431287		[illegible]		.492222		.246131		.499387		.22
.23	.413655		[illegible]		.222335		.615649		[illegible]	2	.564409		-9.03132	2	.516116		.23
.24	4.87981	2	[illegible]		-.158933		.582411		-.311642		.461301		-.354386		.336669		.24
.25	-.334166		[illegible]		-.486848		.34184		-.[illegible]		.167763		-.552363		2.82703	2	.25
.26	-.608147		[illegible]		-.647153		-.021468		-.[illegible]		-.189415		-.504331		-.292536		.26
.27	-.681721		[illegible]		-.583311		-.379188		-.43[illegible]		-.478749		-.266439		-.503879		.27
.28	-.529314		[illegible]		-.316338		-.604392		-.101047		-.593257		7.20145	2	-.524601		.28
.29	-.200803		[illegible]		6.21743	2	-.616475		[illegible]		-.48975		.339905		-.346219		.29
.3	.19654		[illegible]		.421512		-.409926		[illegible]		-.205047		.557118		-3.47929		.3
.31	.531485		-.317913		.636945		-.09665		[illegible]		.157125		.520622		.291925		.31
.32	.693008		[illegible]		.632929		.319185		.47715		.463891		.288714		.509805		.32
.33	.626762		[illegible]		.409468		.584603		[illegible]		.601931		-5.31004	2	.535238		.33
.34	.353181		[illegible]		4.24735	2	.644953		-.218470		.519241		-.377601		.357157		.34
.35	-3.89584	2	[illegible]		-.342083		.477456		-.420213		.244649		-.563145		.041756		.35
.36	-.42144		[illegible]		-.611396		.139674		-.636417		-.122247		-.539188		-.292075		.36
.37	-.668526		[illegible]	2	-.671601		-.250323		-.723724		-.447183		-.313028		-.517507		.37
.38	-.698172		-.331158		-.500334		-.355226		-.22096		-.610292		3.23334	2	-.547875		.38
.39	-.499062		-.623333		-.154837		-.666748		.164274		-.550227		.368624		-.360929		.39
.4	-.134611		-.70363		.247268		-.543945		.49384		-.287126		.569874		-4.34898	2	.4
.41	.277397		-.551266		.568047		-.228074		.642521		8.41318	2	.539513		.292852		.41
.42	.602932		-.212333		.696483		.171345		.571021		.428148		.339404		.327047		.42
.43	.735239		.201176		.586728		.514521		.287035		.618266		-1.01459	2	.562894		.43
.44	.629625		.551434		.274237		.680245		-.133309		.583136		-.359655		.384697		.44
.45	.318242		.721941		-.135821		.608541		-.460761		.333524		-.573354		5.79823	2	.45
.46	-.099935		.653369		-.504019		.322161		-.656953		-.041792		-.583002		-.294546		.46
.47	-.490722		.365775		-.704027		-.080719		-.61864		-.405956		-.369449		-.330001		.47
.48	-.727772		-4.67174	2	-.665769		-.460206		-.358017		-.625523		-1.50235	2	-.581047		.48
.49	-.733219		-.449922		-.399698		-.68321		3.44243	2	-.61829		.349436		-.402201		.49
.5	-.502903		-.707535		6.14001	3	-.669946		.41967		-.384141		.537789		-6.77525	2	.5
.51	-.108088		-.735642		.415374		-.422218		.659877		-5.97054	3	.603313		.296999		.51
.52	.326871		-.521193		.389119		-2.35439	2	.666751		.379661		.405079		.55358		.52
.53	.663801		-.131877		.732929		.339104		.435305		.63176		4.30165	2	.602772		.53
.54	.794472		.306625		.529053		.672333		4.47534	2	.656279		-.338547		.422839		.54
.55	.67499		.651341		.143077		.725971		-.3678		.440984		-.599415		7.88719	2	.55
.56	.339984		.748505		-.207393		.526123		-.655635		6.09487	2	-.640341		-.300667		.56
.57	-.107841		.670536		-.644989		.143638		-.714416		-.347638		-.442591		-.571786		.57
.58	-.529491		.332402		-.781729		-.296937		-.519680		-.636186		-7.60841	2	-.629414		.58
.59	-.792836		-.119692		-.658692		-.642836		-.136567		-.697541		.325278		-.447932		.59
.6	-.813693		-.541415		-.312968		-.77314		.30215		-.50349		.612404		-9.22252	2	.6
.61	-.582115		-.796618		.143824		-.[illegible]		.641987		-.125468		.676175		.305396		.61
.62	-.165833		-.300934		.561778		-.283189		.760971		.307762		.486649		.594233		.62
.63	.310111		-.648305		.601819		.177603		.613028		.637812		.114824		.652074		.63
.64	.701219		-.116303		.781587		.583268		.245277		.742872		-.309826		.478531		.64
.65	.887264		.36274		.703037		.809475		-.217122		.580293		-.628288		.108192		.65
.66	.808641		.733123		5.26721	2	[illegible]		-.613597		.203331		-.71978		-.311937		.66
.67	.484457		.39022		-.425573		.449025		-.803699		-.256324		-.54519		-.622870		.67
.68	7.11736	3	.767361		-.777118		-2.25171	2	-.716381		-.634479		-.165208		-.703514		.68
.69	-.484767		.402342		-.68508		-.49[illegible]		-.375848		-.792786		.289308		-.517384		.69
.7	-.846923		-9.71226	2	-.709627		-.812702		.10523		-.668841		.646165		-.128542		.7
.71	-.971362		-.583812		-.300771		-.867011		.563131		-.300053		.772432		.32025		.71
.72	-.817536		-.941296		.216434		-.632996		.838608		.187636		.615136		.659591		.72
.73	-.423887		-.353299		.680602		-.181005		.831683		.622717		.224304		.756903		.73
.74	.103153		-.729323		.945256		.345497		.536969		.848061		-.262369		.567662		.74
.75	.619004		-.27423		.919882		.775699		4.73205	2	.776956		-.667989		.154818		.75
.76	.981185		.277434		.607488		.964[illegible]		-.475777		.425426		-.840164		-.331761		.76
.77	1.08788		.7637		9.53437	2	.84895		-.855395		-9.09033	2	-.707438		-.709181		.77
.78	.904954		1.[illegible]713		-.466128		.451505		-.956318		-.594886		-.307769		-.82952		.78
.79	.474071		1.35606		-.000095		-.107676		-.740443		-.908222		.221339		-.636877		.79
.8	-9.93155	2	.753111		-1.09800		-.656785		-.263272		-.913515		.692399		-.191798		.8
.81	-.674066		[illegible]		-.569351		-1.02314		.324[illegible]3		-.596642		.9[illegible]769		.347072		.81
.82	-1.10935		-.374722		-.250429		-1.08644		.832261		-5.45518	2	.334283		.778724		.82
.83	-1.2988		.913611		4.92767	2	-.819678		1.09136		.534726		.42783		.933473		.83
.84	-1.19414		-1.25476		.438717		-.280175		1.03665		.970224		-.15316		.737949		.84
.85	-.813235		-1.27453		1.11938		.372198		.58956		1.09515		-.720992		.247062		.85
.86	-.231695		-1.[illegible]791		1.33125		.955913		-4.39015	2	.850622		-1.03609		-.370088		.86
.87	.438406		-.446[illegible]		1.18[illegible]		1.30157		-.737279		.29311		-1.02918		-.887021		.87
.88	1.07615		.19[illegible]4		.725[illegible]		1.30253		-1.19916		-.397059		-.626196		-1.10044		.88
.89	1.57847		[illegible]		4.93533	2	.94253		-1.36261		-1.01585		3.69279	2	-.906398		.89
.9	1.87948		1.49347		-.695057		.297008		-1.13047		-1.35316		.745647		-.34448		.9
.91	1.95903		1.[illegible]43		-1.[illegible]32		-.190752		-.543069		-1.28126		1.23814		.406933		.91
.92	1.83974		2.1[illegible]		-1.85[illegible]		-1.25024		.263659		-.788975		1.37966		1.08461		.92
.93	1.57512		1.[illegible]377		-2.07022		-1.82891		1.10533		1.52607	2	1.02866		1.42654		.93
.94	1.23306		1.[illegible]		-2.0[illegible]		-2.12836		1.79726		.93658		.[illegible]479		1.26137		.94
.95	.879364		1.3[illegible]		-1.7[illegible]		-2.12301		2.19427		1.75174		-.7[illegible]379		.570725		.95
.96	.565112		.912371		-1.[illegible]		-1.65746		2.24297		2.27333		-1.69406		-.497451		.96
.97	.320226		[illegible]		-.93[illegible]		-1.4259		1.97548		2.4002		-[illegible]38834		-1.67006		.97
.98	.153234		[illegible]		-.547[illegible]		-.941426		1.49399		2.13964		-[illegible]64538		-2.63154		.98
.99	5.59163	2	12[illegible]		-.627641		-.50504		.935094		1.59629		-2.42343		-3.12116		.99
1	1.06468	2	[illegible]	2	-7.37[illegible]	2	-.153137		.428347		.933636		-1.91432		-3.01303		1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	$F_{40}(x)$	p	$F_{41}(x)$	p	$F_{42}(x)$	p	$F_{43}(x)$	p	$F_{44}(x)$	p	$F_{45}(x)$	p	x
0	.526571		0		.568037		0		.584876		0		0
.01	.422243		.329313		.456988		.344402		.46527		.362593		.01
.02	.149261		.530441		.167686		.555331		.155213		.574751		.02
.03	-.187691		.525022		-.185627		.555925		-.218872		.548391		.03
.04	-.459356		.314992		-.463384		.307328		-.504395		.294321		.04
.05	-.56144		-1.81236	2	-.555768		-4.70913	2	-.584762		-8.21866	2	.05
.06	-.454615		-.34485		-.426092		-.383347		-.427031		-.424958		.06
.07	-.179661		-.538064		-.125359		-.565881		-9.52556	2	-.591643		.07
.08	.158088		-.522435		.227777		-.521162		.275335		-.512776		.08
.09	.429049		-.303795		.493776		-.266771		.553474		-.220725		.09
.1	.529059		3.29995	2	.567297		9.54944	2	.573527		.163562		.1
.11	.419372		.35694		.418886		.420312		.378687		.480569		.11
.12	.141651		.541748		.10677		.57708		2.80379	2	.598362		.12
.13	-.197846		.515026		-.24595		.502394		-.335492		.467459		.13
.14	-.468883		.286568		-.499793		.22569		-.563318		.141621		.14
.15	-.567165		-5.52318	2	-.554		-.142314		-.56188		-.244162		.15
.16	-.454434		-.377568		-.386491		-.455847		-.331045		-.52952		.16
.17	-.173356		-.554796		-6.28615	2	-.583348		3.55043	2	-.595502		.17
.18	.168601		-.517386		.28927		-.473027		.38822		-.41331		.18
.19	.440201		-.279211		.530522		-.179374		.582624		-5.84203	2	.19
.2	.536742		6.75475	2	.56487		.193143		.538441		.322138		.2
.21	.420369		.387993		.377843		.489799		.272744		.369763		.21
.22	.134878		.556906		4.26155	2	.590738		-.106564		.580517		.22
.23	-.210743		.507595		-.30856		.454368		-.444481		.348753		.23
.24	-.483888		.258174		-.556455		.134523		-.602167		-2.99802	2	.24
.25	-.579161		-.095078		-.549999		-.240711		-.513948		-.398217		.25
.26	-.459014		-.414851		-.342649		-.520191		-.214687		-.601945		.26
.27	-.168526		-.576144		4.54003	3	-.590518		.173894		-.554972		.27
.28	.181455		-.515104		.354526		-.422011		.492613		-.27542		.28
.29	.456547		-.254317		.568303		-8.12301	2	.609947		.121113		.29
.3	.550345		.105471		.560056		.295227		.476279		.469211		.3
.31	.425512		.424166		.331589		.555391		.144906		.622535		.31
.32	.128582		.576771		-2.77569	2	.593206		-.249086		.515307		.32
.33	-.227359		.502314		-.375953		.391754		-.543761		.190581		.33
.34	-.505699		.228147		-.574313		3.08202	2	-.616803		-.216872		.34
.35	-.598674		-.140065		-.542664		-.344578		-.43628		-.536334		.35
.36	-.469055		-.45902		-.291719		-.582527		-7.46308	2	-.632564		.36
.37	-.165028		-.6035		8.03924	2	-.584766		.320235		-.463045		.37
.38	.197643		-.515503		.426396		-.348779		.585278		-9.68451	2	.38
.39	.479592		-.227534		.608065		.032011		.609558		.313308		.39
.4	.571298		.149271		.551376		.404252		.380574		.594524		.4
.41	.435522		.467902		.276664		.616496		-9.01935	3	.626525		.41
.42	.122334		.602678		-.108499		.580752		-.399476		.393118		.42
.43	-.249309		.478704		-.451285		.30898		-.628516		-9.67444	3	.43
.44	-.536687		.194138		-.614112		-9.06023	2	-.59911		-.412841		.44
.45	-.628048		-.19374		-.529772		-.456411		-.320514		-.645091		.45
.46	-.48601		-.51361		-.22914		-.63866		9.41772	2	-.60555		.46
.47	-.162808		-.639068		.169951		-.560602		.473684		-.307646		.47
.48	.218796		-.518538		.50874		-.250837		.659086		.124911		.48
.49	.511989		-.196471		.650551		.166897		.570294		.509163		.49
.5	.602223		.202855		.535866		.522928		.240695		.680053		.5
.51	.451307		.523078		.207283		.670404		-.195016		.561272		.51
.52	.111476		.636767		-.206057		.545798		-.555572		.199445		.52
.53	-.273454		.49596		-.554992		.196305		-.688325		-.25568		.53
.54	-.581184		.152233		-.556133		-.23768		-.533827		-.604469		.54
.55	-.671662		-.262054		-.5068		-.578642		-.152512		-.69991		.55
.56	-.512684		-.584586		-.146862		-.684315		.298479		-.49437		.56
.57	-.161904		-.686557		.281655		-.506941		.629659		-7.15034	2	.57
.58	.247861		-.524043		.607316		-.11479		.698558		.38918		.58
.59	.558676		-.136852		.695869		.333706		.470989		.688023		.59
.6	.648079		.273156		.507263		.6536		3.79058	2	.691515		.6
.61	.477063		.596528		.112695		.709462		-.420361		.39202		.61
.62	.106723		.68265		-.331616		.472662		-.708704		-8.57476	2	.62
.63	-.523415		.492218		-.645881		3.57629	2	-.700051		-.53588		.63
.64	-.647787		9.47611	2	-.699157		-.423068		-.391394		-.759441		.64
.65	-.738314		-.356469		-.463721		-.712172		9.16045	2	-.65221		.65
.66	-.554789		-.683316		-2.89049	2	-.706105		.544975		-.253465		.66
.67	-.162434		-.752813		.432284		-.399951		.771942		.267015		.67
.68	.290883		-.531125		.731513		8.51703	2	.668051		.68019		.68
.69	.629912		-9.95998	2	.741665		.549256		.269292		.797875		.69
.7	.719239		.374459		.450673		.795222		-.258869		.558373		.7
.71	.516828		.701982		-3.00182	2	.712245		-.689055		5.88246	2	.71
.72	9.28912	2	.746875		-.507294		.325633		-.829074		-.482479		.72
.73	-.394116		.481987		-.763135		-.209043		-.607544		-.82012		.73
.74	-.756337		3.49788	3	-.736141		-.667317		-.11016		-.792107		.74
.75	-.848505		-.503372		-.374541		-.849025		.45156		-.397749		.75
.76	-.625745		-.835405		.162202		-.665131		.828595		.194397		.76
.77	-.16446		-.852258		.655989		-.180921		.843761		.716062		.77
.78	.362969		-.535349		.897586		.405045		.47474		.918908		.78
.79	.751525		-4.05858	4	.774454		.840546		-.127685		.692094		.79
.8	.84193		.54328		.322587		.926833		-.698325		.122871		.8
.81	.284476		.873023		-.281044		.608662		-.971643		-.532254		.81
.82	6.32064	2	.842178		-.785531		7.11843	3	-.804762		-.959325		.82
.83	-.530488		.445576		-.968086		-.622849		-.25032		-.934613		.83
.84	-.966945		-.179775		-.730618		-.996991		.451899		-.441777		.84
.85	-1.06549		-.783945		-.150017		-.927245		.974144		.302424		.85
.86	-.767432		-1.11717		.542283		-.414481		1.04913		.935186		.86
.87	-.166245		-1.02236		1.04829		.335529		.604671		1.1176		.87
.88	.518743		-.50863		1.12738		.987561		-.180766		.712834		.88
.89	1.01838		.238734		.707114		1.21961		-.93777		-.11652		.89
.9	1.11371		.918811		-6.74407	2	.876301		-1.26779		-.954303		.9
.91	.724068		1.22771		-.873041		6.49872	2	-.943813		-1.32052		.91
.92	-4.39529	2	.98367		-1.33021		-.857528		-5.79512	2	-.932312		.92
.93	-.947223		.216991		-1.16649		-1.41891		.967402		.088879		.93
.94	-1.62657		-.813171		-.359375		-1.24672		1.54047		1.19156		.94
.95	-1.78165		-1.68822		.802808		-.284887		1.19629		1.61969		.95
.96	-1.25132		-1.96771		1.77771		1.08268		-.077998		.853882		.96
.97	-8.65468	2	-1.35031		1.93759		2.07231		-1.63192		-.881492		.97
.98	1.45122		.198502		.799922		1.7206		-2.18005		-2.27079		.98
.99	2.96251		2.40063		-1.75892		-.769126		-.122665		-.939016		.99
1	4.031		4.72416		-5.3888		-5.6654		5.89781		6.01841		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-7. Eigenvalues for $c = 25\pi$ or aperture length = 25λ

n	λ_n	n	λ_n
0	.28284	28	.28284
1	j.28284	29	j.28284
2	-.28284	30	-.28284
3	-j.28284	31	-j.28284
4	.28284	32	.28284
5	j.28284	33	j.28284
6	-.28284	34	-.28284
7	-j.28284	35	-j.28284
8	.28284	36	.28284
9	j.28284	37	j.28284
10	-.28284	38	-.28284
11	-j.28284	39	-j.28284
12	.28284	40	.28284
13	j.28284	41	j.28283
14	-.28284	42	-.28281
15	-j.28284	43	-j.28277
16	28284	44	.28267
17	j.28284	45	j.28239
18	-.28284	46	-.28144
19	-j.28284	47	-j.27768
20	.28284	48	.26430
21	j.28284	49	j.22848
22	-.28284	50	-.16622
23	-j.28284	51	-j.10076
24	.28284	52	.05421
25	j.28284	53	j.02320
26	-.28284	54	-.00908
27	-j.28284	55	-j.00368

Table 3-8. Eigenfunctions for $c = 25\pi$ or aperture length = 25λ

$$\psi_n(x) = F_n(x) \times 10^{p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	2.23337		0		1.5715		0		1.35403		0		1.22952		0		0
.01	2.2247		.277483		1.54104		.335315		1.3072		.369814		1.16863		.393947		.01
.02	2.19889		.548569		1.45094		.652594		1.16997		.708591		.99203		.743178		.02
.03	2.15653		.807102		1.30488		.934886		.991976		.988		.71728		1.00815		.03
.04	2.09857		1.0474		1.10881		1.16729		.66842		1.13486		.371613		1.15901		.04
.05	2.02631		1.26445		.870602		1.33794		.338932		1.28309		-1.08737	2	1.17901		.05
.06	1.94132		1.4541		.59962		1.4381		-1.39196	2	1.27504		-.392714		1.0663		.06
.07	1.84541		1.61317		.306271		1.46565		-.366239		1.16199		-.736855		.833992		.07
.08	1.74054		1.73951		1.45579	3	1.41406		-.694835		.953879		-1.01035		.508334		.08
.09	1.62879		1.83209		-.303949		1.293		-.97826		.668293		-1.18745		.125882		.09
.1	1.51226		1.89092		-.599599		1.10765		-1.19883		.328616		-1.25212		-.271151		.1
.11	1.39302		1.91703		-.876124		.868217		-1.34341		-3.77219	2	-1.19923		-.639527		.11
.12	1.27307		1.91233		-1.1255		.587157		-1.40421		-.402013		-1.03477		-.939817		.12
.13	1.15423		1.87949		-1.34133		.273366		-1.37901		-.736472		-.774956		-1.14062		.13
.14	1.03816		1.82179		-1.519		-4.37119	2	-1.27105		-1.01646		-.444313		-1.22174		.14
.15	.926321		1.74293		-1.65574		-.364922		-1.08846		-1.22231		-7.30932	2	-1.17597		.15
.16	.819903		1.64484		-1.75062		-.672157		-.84341		-1.34058		.305763		-1.0094		.16
.17	.719867		1.53758		-1.80436		-.953986		-.551049		-1.36474		.659735		-.740309		.17
.18	.626921		1.4191		-1.81918		-1.20113		-.22827		-1.29526		.959607		-.396695		.18
.19	.541533		1.29515		-1.79852		-1.40674		.107502		-1.15907		1.18186		-1.30336	2	.19
.2	.463948		1.16917		-1.74675		-1.56655		.43946		-.908575		1.31034		.373521		.2
.21	.394206		1.0442		-1.65887		-1.67878		.75241		-.620305		1.33722		.726939		.21
.22	.332174		.92281		-1.57019		-1.74399		1.03355		-.293381		1.26296		1.01574		.22
.23	.277568		.807108		-1.45609		-1.76475		1.27302		5.20949	2	1.09574		1.21573		.23
.24	.229991		.598699		-1.33176		-1.74524		1.46412		.396549		.850099		1.31171		.24
.25	.188957		.398728		-1.20197		-1.69085		1.60339		.722237		.545215		1.29829		.25
.26	.15392		.307901		-1.071		-1.60773		1.69041		1.01426		.20298		1.17933		.26
.27	.124302		.426543		-.942475		-1.50235		1.72738		1.26125		-.153983		.967838		.27
.28	9.95129	2	.354645		-.819331		-1.38117		1.71867		1.45569		-.504124		.682063		.28
.29	7.89708	2	.291933		-.703824		-1.25027		1.67027		1.59389		-.828462		.345311		.29
.3	.062116		.23792		-.597541		-1.11516		1.58919		1.67579		-1.11169		-1.74659	2	.3
.31	4.84232	2	.191971		-.501462		-.98058		1.48289		1.70433		-1.3428		-.381729		.31
.32	3.74091	2	.153352		-.416035		-.850425		1.35884		1.68494		-1.51537		-.725294		.32
.33	2.86373	2	.121277		-.341257		-.727697		1.22406		1.62476		-1.62729		-1.02983		.33
.34	2.17209	2	9.49481	2	-.276775		-.614547		1.08487		1.53191		-1.68037		-1.28176		.34
.35	1.63218	2	7.35852	2	-.221964		-.512331		.946607		1.41487		-1.67955		-1.47271		.35
.36	1.21493	2	5.64302	2	-.17602		-.421717		.813614		1.28139		-1.63211		-1.59937		.36
.37	8.95746	3	4.28629	2	-.138028		-.342791		.689141		1.14055		-1.54679		-1.66296		.37
.38	6.54051	3	3.22109	2	-.107028		-.275185		.575428		.997434		-1.43293		-1.66847		.38
.39	4.72909	3	2.39549	2	-8.20613	2	-.218194		.473791		.857943		-1.29981		-1.62366		.39
.4	3.38551	3	1.76284	2	-6.22122	2	-.170886		.384763		.726238		-1.15604		-1.53801		.4
.41	2.39934	3	1.28354	2	-.046632		-.1322		.308238		.605256		-1.00913		-1.42178		.41
.42	1.68313	3	9.24568	3	-3.45569	2	-.101022		.245624		.496815		-.86528		-1.28513		.42
.43	1.16851	3	6.58785	3	-2.53159	2	-7.62534	2	.189993		.40176		-.72927		-1.13748		.43
.44	8.02730	4	4.64265	3	-1.83323	2	-5.68516	2	.146206		.32013		-.604456		-.98699		.44
.45	5.45571	4	3.23550	3	-1.31209	2	-4.18643	2	.111024		.251436		-.492906		-.840318		.45
.46	3.66777	4	2.22948	3	-9.28059	3	-3.04462	2	8.31939	2	.194644		-.395574		-.702493		.46
.47	2.43860	4	1.51872	3	-6.48633	3	-2.18662	2	6.15152	2	.148535		-.312512		-.576965		.47
.48	1.60318	4	1.02253	3	-4.47888	3	-1.55066	2	4.48818	2	.111742		-.243087		-.465754		.48
.49	1.04191	4	6.80360	4	-3.05305	3	-.010857		.032309		8.28706	2	-.186197		-.369669		.49
.5	6.49250	5	4.47242	4	-2.05812	3	-7.50409	3	.022946		6.05869	2	-.140455		-.288558		.5
.51	4.24777	5	2.90399	4	-1.36913	3	-5.11929	3	1.60757	2	4.36646	2	-.104347		-.221565		.51
.52	2.66349	5	1.86196	4	-8.99187	4	-3.44644	3	1.11085	2	3.10185	2	-7.63483	2	-.167368		.52
.53	1.64933	5	1.17852	4	-5.82903	4	-2.28928	3	7.57007	3	2.17175	2	-5.50157	2	-.12439		.53
.54	1.00837	5	7.36060	5	-3.72889	4	-1.50005	3	5.07620	3	1.49844	2	-3.90405	2	-.090961		.54
.55	6.08475	6	4.53391	5	-2.35338	4	-9.65380	4	3.31940	3	.010187		-.02728		-6.54448	2	.55
.56	3.62310	6	2.75247	5	-1.46494	4	-6.17667	4	2.19180	3	6.82247	3	-1.87681	2	-.046326		.56
.57	2.12716	6	1.64329	5	-8.99141	5	-3.87947	4	1.41570	3	4.50066	3	-1.27109	2	-3.22601	2	.57
.58	1.23166	6	9.66658	6	-5.43972	5	-2.40116	4	8.96896	4	2.92356	3	-8.47296	3	-2.20976	2	.58
.59	7.01543	7	5.55809	6	-3.24312	5	-1.46411	4	5.59792	4	1.86961	3	-5.55779	3	-1.48864	2	.59
.6	3.93055	7	3.12948	6	-1.90457	5	-8.79183	5	3.43989	4	1.17673	3	-3.58651	3	-9.86099	3	.6
.61	2.16487	7	1.69988	6	-1.10136	5	-5.19726	5	2.08038	4	7.28699	4	-2.27628	3	-6.42144	3	.61
.62	1.16873	7	8.75939	7	-6.26897	6	-3.02341	5	1.23784	4	4.43836	4	-1.42046	3	-4.10972	3	.62
.63			4.08999	7	-3.51046	6	-1.72997	5	7.24330	5	2.65789	4	-8.71234	4	-2.58420	3	.63
.64			1.48484	7	-1.93330	6	-9.73438	6	4.16649	5	1.56432	4	-5.25023	4	-1.59597	3	.64
.65			—	—	-1.04859	6	-5.38237	6	2.35459	5	9.04425	5	-3.10725	4	-9.67712	4	.65
.66			—	—	-5.57949	7	-2.92348	6	1.30671	5	5.13436	5	-1.80524	4	-5.75840	4	.66
.67			-1.05683	7	-2.90487	7	-1.55770	6	7.11750	6	2.86048	5	-1.02904	4	-3.36114	4	.67
.68			-1.24437	7	-1.48673	7	-8.15785	7	3.80149	6	1.56302	5	-5.75211	5	-1.92344	4	.68
.69			-1.33850	7	—	—	-4.19044	7	1.99014	6	8.37141	6	-3.15116	5	-1.07852	4	.69
.7			-1.37110	7			-2.10768	7	1.02077	6	4.39159	6	-1.69059	5	-5.92181	5	.7
.71			-1.37527	7			-1.02918	7	5.11298	7	2.25594	6	-8.87618	6	-3.18159	5	.71
.72			-1.35418	7			—		2.51873	7	1.13346	6	-4.55597	6	-1.67129	5	.72
.73			-1.33582	7					1.19988	7	5.56064	7	-2.28391	6	-8.57423	6	.73
.74			-1.31826	7					—	—	2.65390	7	-1.11921	6	-4.29225	6	.74
.75			-1.28723	7							1.24211	7	-5.34587	7	-2.09633	6	.75
.76			-1.27814	7							—		-2.48508	7	-9.96110	7	.76
.77			-1.26075	7									-1.12688	7	-4.59269	7	.77
.78			-1.24888	7									—		-2.02969	7	.78
.79			-1.23074	7											—		.79
.8			-1.20890	7													.8
.81			-1.21045	7													.81
.82			-1.18339	7													.82
.83			-1.18475	7													.83
.84			-1.16417	7													.84
.85			-1.14231	7													.85
.86			-1.13001	7													.86
.87			-1.11993	7													.87
.88			-1.10067	7													.88
.89			-1.09252	7													.89
.9			-1.08489	7													.9
.91			-1.07709	7													.91
.92			-1.06179	7													.92
.93			-1.04955	7													.93
.94			-1.03187	7													.94
.95			-1.02729	7													.95
.96			-1.00255	7													.96
.97			-1.01017	7													.97
.98																	.98
.99																	.99
1																	1

(Where no values are listed the absolute value is less than 10^{-7})

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	$F_8(x)$	p	$F_9(x)$	p	$F_{10}(x)$	p	$F_{11}(x)$	p	$F_{12}(x)$	p	$F_{13}(x)$	p	$F_{14}(x)$	p	$F_{15}(x)$	p	x
0	1.14377		0		1.07882		0		1.02667		0		.983074		0		0
.01	1.07038		.411991		.99411		.425933		.931608		.436867		.8785		.445465		.01
.02	.85964		.765267		.753281		.779051		.66402		.7d688		.587019		.790219		.02
.03	.538632		1.00956		.394153		.99906		.273431		.980515		.17059		.936566		.03
.04	.148501		1.11026		-2.69696	2	1.04851		-.16796		.979401		-.282352		.906437		.04
.05	-.260921		1.05329		-.444238		.919157		-.578725		.78389		-.67576		.651783		.05
.06	-.637597		.847026		-.792649		.635217		-.883303		.432F47		-.9264		.249882		.06
.07	-.933985		.520904		-1.01822		.239421		-1.02592		-4.23306	3	-.981439		-.208786		.07
.08	-1.11307		.121047		-1.08634		-.195481		-.980716		-.441137		-.829471		-.62124		.08
.09	-1.15291		-.296443		-.986948		-.598186		-.756288		-.792053		-.502685		-.895177		.09
.1	-1.04923		-.673506		-.755801		-.901278		-.393798		-.988442		-6.08022	2	-.969582		.1
.11	-.815582		-.958278		-.371742		-1.0545		4.08292	2	-.992307		.37861		-.82813		.11
.12	-.481347		-1.11219		.049612		-1.03291		.469135		-.803311		.749238		-.50248		.12
.13	-8.77335	2	-1.11502		.464596		-.840721		.814422		-.458432		.965458		-6.48529	2	.13
.14	.317564		-.967503		.811243		-.510088		1.01548		-2.43602	2	.983018		.388293		.14
.15	.686334		-.68976		1.03864		-9.53795	2	1.03727		.415702		.798778		.757704		.15
.16	.975657		-.319998		1.11406		.336109		.876661		.778178		.450828		.963065		.16
.17	1.15283		9.29566	2	1.02757		.715297		.562446		.994952		1.01559	2	.960239		.17
.18	1.19886		.495462		.792725		.982424		.149665		1.02608		-.434218		.750332		.18
.19	1.11011		.836391		.444061		1.09631		-.290452		.866473		-.79343		.378808		.19
.2	.898103		1.07361		3.14601	2	1.04034		-.683071		.546403		-.99651		-.074904		.2
.21	.587528		1.179		-.387368		.82435		-.96254		.125052		-1.0041		-.514814		.21
.22	.212864		1.14135		-.755206		.482316		-1.08318		-.32092		-.815555		-.84887		.22
.23	-.185889		.96703		-1.02318		6.64041	2	-1.02632		-.711642		-.46821		-1.0081		.23
.24	-.567806		.678435		-1.15705		-.361487		-.802521		-.978387		-2.93204	2	-.960443		.24
.25	-.895404		.310317		-1.14112		-.739241		-.448945		-1.07539		.417552		-.716481		.25
.26	-1.13828		-9.47745	2	-.979459		-1.01359		-2.24533	2	-.987167		.788661		-.326349		.26
.27	-1.27569		-.491941		-.694364		-1.14737		.4101		-.730161		1.01592		.131359		.27
.28	-1.29786		-.839122		-.322556		-1.12403		.782669		-.348887		1.05879		.565967		.28
.29	-1.20605		-1.10154		9.02222	2	-.9489		1.04011		9.25933	2	.910937		.892906		.29
.3	-1.01148		-1.25495		.495344		-.647336		1.14598		.522029		.600104		1.04996		.3
.31	-.733275		-1.28731		.847297		-.260158		1.08705		.871078		.182001		1.00852		.31
.32	-.395994		-1.19906		1.10878		.16261		.874001		1.08607		-.270536		.777804		.32
.33	-2.67822	2	-1.00188		1.25433		.56876		.53858		1.13574		-.680782		.401835		.33
.34	.347304		-.716458		1.27219		.910604		.127898		1.0149		-.981268		-.030047		.34
.35	.701634		-.369507		1.16436		1.15047		-.302991		.743825		-1.12455		-.496753		.35
.36	1.0156		9.43583	3	.945091		1.26442		-.699		.363691		-1.08993		-.860392		.36
.37	1.27473		.391115		.638106		1.24383		-1.01221		-7.06631	2	-.885317		-1.07973		.37
.38	1.46922		.749206		.273156		1.09494		-1.20742		-.499467		-.544321		-1.11987		.38
.39	1.59574		1.06229		-.117669		.836704		-1.26542		-.86665		-.119511		-.976682		.39
.4	1.65587		1.31506		-.503383		.497285		-1.18388		-1.12713		.326725		-.675731		.4
.41	1.65335		1.49874		-.856932		.110031		-.97594		-1.25183		.732006		-.266325		.41
.42	1.60288		1.61082		-1.1572		-.290645		-.667025		-1.22998		1.04785		.187816		.42
.43	1.50898		1.65426		-1.39008		-.67293		-.290599		-1.06862		1.22136		.619376		.43
.44	1.38487		1.63627		-1.54869		-1.01645		.116529		-.789928		1.24897		.967793		.44
.45	1.24149		1.56703		-1.63279		-1.28397		.51861		-.42682		1.12698		1.18754		.45
.46	1.08878		1.45832		-1.64767		-1.43229		.884589		-1.78257	2	.874328		1.25323		.46
.47	.935223		1.32225		-1.60274		-1.60188		1.19063		.39802		.523243		1.16119		.47
.48	.787517		1.17033		-1.50996		-1.64591		1.42141		.785583		.113665		.927692		.48
.49	.650561		1.01269		-1.38234		-1.62284		1.57027		1.11688		-.312495		.584537		.49
.5	.527525		.857659		-1.23271		-1.54458		1.63835		1.37296		-.716517		.173141		.5
.51	.420063		.711551		-1.07272		-1.42487		1.63314		1.54436		-1.06681		-.261894		.51
.52	.328587		.578695		-.912193		-1.27769		1.5665		1.63051		-1.34132		-.678644		.52
.53	.252557		.461613		-.758782		-1.11605		1.45282		1.63809		-1.52834		-1.04244		.53
.54	.190773		.361299		-.617917		-.931122		1.30712		1.57909		-1.626		-1.32878		.54
.55	.141635		.277556		-.492944		-.791798		1.14365		1.46855		-1.64062		-1.52436		.55
.56	.10336		.209326		-.385413		-.644494		.974823		1.32254		-1.58453		-1.62664		.56
.57	7.41403	2	.155004		-.295441		-.513282		.81059		1.15648		-1.47361		-1.6421		.57
.58	5.22714	2	.112706		-.222095		-.400183		.658208		.983798		-1.32499		-1.58378		.58
.59	3.62194	2	8.04706	2	-.163757		-.305561		.522335		.815404		-1.15516		-1.46863		.59
.6	2.46621	2	5.64152	2	-.118438		-.228555		.40533		.659186		-.978522		-1.31492		.6
.61	1.64988	2	3.88313	2	-.084027		-.1675		.307699		.520199		-.806631		-1.14012		.61
.62	1.08422	2	2.62381	2	-5.84739	2	-.120284		.228573		.400977		-.647814		-.959409		.62
.63	6.99708	3	1.74005	2	-3.99091	2	-8.46388	2	.166181		.302028		-.507301		-.784776		.63
.64	4.43318	3	1.13231	2	-2.67101	2	-5.83542	2	.118257		.222369		-.387603		-.624715		.64
.65	2.75657	3	7.22803	3	-1.75257	2	-3.94145	2	8.23673	2	.160055		-.289067		-.484372		.65
.66	1.68154	3	4.52454	3	-1.12707	2	-2.60756	2	5.61468	2	.112631		-.210482		-.366013		.66
.67	1.00587	3	2.77626	3	-7.10152	3	-1.68924	2	3.74507	2	7.74846	2	-.149656		-.269653		.67
.68	5.89743	4	1.66910	3	-4.38239	3	-1.07122	2	2.44372	2	5.21053	2	-.103905		-.193734		.68
.69	3.38719	4	9.82708	4	-2.64745	3	-6.64707	3	.015594		3.42419	2	-7.04375	2	-.135746		.69
.7	1.90451	4	5.66293	4	-1.56487	3	-4.03402	3	9.72760	3	2.19838	2	-4.66124	2	-9.27583	2	.7
.71	1.04781	4	3.19197	4	-9.04497	4	-2.39316	3	5.92907	3	1.37831	2	-3.01023	2	-6.18022	2	.71
.72	5.63562	5	1.75861	4	-5.10886	4	-1.38695	3	3.52904	3	8.43474	3	-1.89636	2	-4.01377	2	.72
.73	2.96093	5	9.46312	5	-2.81781	4	-7.84707	4	2.04992	3	5.03532	3	-1.16479	2	-2.55994	2	.73
.74	1.51833	5	4.96893	5	-1.51637	4	-4.33079	4	1.16120	3	2.93033	3	-6.97130	3	-1.56528	2	.74
.75	7.59222	6	2.54324	5	-7.95406	5	-2.32944	4	6.40908	4	1.66112	3	-4.06263	3	-9.38809	3	.75
.76	3.69855	6	1.26745	5	-4.06266	5	-1.21992	4	3.44343	4	9.16403	4	-2.30337	3	-5.47582	3	.76
.77	1.75604	6	6.14300	6	-2.01814	5	-6.21330	5	1.79896	4	4.91507	4	-1.26930	3	-3.10325	3	.77
.78	8.11098	7	2.88830	6	-9.73697	6	-3.07396	5	9.12758	5	2.55994	4	-6.79091	4	-1.70697	3	.78
.79	3.62681	7	1.31452	6	-4.55980	6	-1.47533	5	4.49133	5	1.29301	4	-3.32288	4	-9.10224	4	.79
.8	1.59418	7	5.74216	7	-2.06161	6	-6.85825	6	2.13981	5	6.32396	5	-1.76944	4	-4.69869	4	.8
.81	—	—	2.40069	7	-8.98361	7	-3.08209	6	9.85103	6	2.99026	5	-8.59156	5	-2.34437	4	.81
.82			—	—	-3.71123	7	-1.33664	6	4.37107	6	1.36436	5	-4.02477	5	-1.12645	4	.82
.83					-1.50985	7	-5.57684	7	1.85928	6	5.93288	6	-1.81538	5	-5.22974	5	.83
.84					—	—	-2.26410	7	7.58048	7	2.53143	6	-7.86454	6	-2.32775	5	.84
.85							—	—	2.91084	7	1.02400	6	-3.26576	6	-9.92484	6	.85
.86									1.04074	7	3.98995	7	-1.29833	6	-4.04279	6	.86
.87									—	—	1.50647	7	-4.96881	7	-1.57333	6	.87
.88											—	—	-1.84823	7	-5.86413	7	.88
.89													—	—	-2.14832	7	.89
.9			Where no values are stated the absolute value is less than 10^{-7}														.9
.91																	.91
.92																	.92
.93																	.93
.94																	.94
.95																	.95
.96																	.96
.97																	.97
.98																	.98
.99																	.99
1																	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	$F_{16}(x)$	p	$F_{17}(x)$	p	$F_{18}(x)$	p	$F_{19}(x)$	p	$F_{20}(x)$	p	$F_{21}(x)$	p	$F_{22}(x)$	p	$F_{23}(x)$	p	x
0	.945531		0		.913348		0		.882746		0		.855653		0		0
.01	.833139		.452169		.791047		.457282		.753319		.461018		.720011		.46553		.01
.02	.513357		.790046		.459116		.78705		.40501		.781733		.356058		.774466		.02
.03	8.19281	2	.928262		4.90508	3	.897372		-6.22359	2	.864553		-.12092		.83045		.03
.04	-.375339		.831942		-.450874		.757514		-.511623		.684276		-.55938		.613028		.04
.05	-.743302		.52564		-.787263		.406435		-.812243		.295685		-.821872		.193664		.05
.06	-.933984		8.60639	2	-.915068		-5.81759	2	-.876593		-.183197		-.823997		-.28984		.06
.07	-.902077		-.375554		-.800464		-.5972		-.68936		-.607052		-.765533		-.678695		.07
.08	-.655302		.743415		-.473825		-.816033		-.295699		-.847388		-.178021		-.845274		.08
.09	-.[illegible]239		-.929517		-2.14423	2	-.899132		.181031		-.831427		.350627		-.734896		.09
.1	.211125		-.875849		.437464		-.733552		.606232		-.563951		.719706		-.383[illegible]		.1
.11	.695735		-.607499		.780277		-.365032		.857056		-.125624		.86315		9.39495	2	.11
.12	.893891		-.187202		.922653		.104932		.861194		.35159		.735966		.541913		.12
.13	.952853		.280464		.827002		.547271		.61746		.724302		.3[illegible]7813		.814369		.13
.14	.789104		.680401		.506688		.840885		.195781		.880768		-9.9[illegible]235	2	.822635		.14
.15	.441051		.913657		5.87258	2	.905684		-.283216		.774163		-.547386		.563871		.15
.16	-1.04078	2	.923393		-.405902		.724131		-.682942		.436223		-.826145		.121648		.16
.17	-.461047		.707445		-.767471		.345532		-.889805		-3.26494	2	-.849481		-.3614		.17
.18	-.807555		.318671		-.933321		-.12 7824		-.845255		-.493574		-.610063		-.729791		.18
.19	-.971157		-.148751		-.861238		-.56866		-.561904		-.813485		-.181552		-.865193		.19
.2	-.915185		-.582343		-.569955		-.859055		-.119395		-.890164		.304541		-.724102		.2
.21	-.652758		-.878563		-.133166		-.921803		.338483		-.709248		.699509		-.351297		.21
.22	-.[illegible]4305		-.967347		.339153		-.740467		.738721		-.320475		.882938		.134946		.22
.23	.222658		-.828191		.729027		-.363024		.916066		.163314		.799062		.580805		.23
.24	.641731		-.494192		.939883		.11151		.841769		.602366		.473111		.845688		.24
.25	.922909		-4.34252	2	.9201		.55957		.536319		.870518		3.00312	3	.846301		.25
.26	1.00591		.419927		.674961		.865397		8.30047	2	.891155		-.470647		.58224		.26
.27	.873734		.78999		.264417		.950714		-.395464		.65846		-.808878		.135408		.27
.28	.55531		.98331		-.212374		.794139		-.770547		.238206		-.906141		-.356099		.28
.29	.118234		.957186		-.641543		.435654		-.942323		-.25165		-.739179		-.741076		.29
.3	-.346347		.718256		-.921833		-3.45386	2	-.865568		-.674608		-.354764		-.901665		.3
.31	-.743212		.319824		-.988142		-.499446		-.560761		-.913736		.135478		-.788878		.31
.32	-.992605		-.130995		-.825851		-.844731		-.107581		-.903543		.590165		-.436653		.32
.33	-1.0458		-.59302		-.472966		-.98663		.376826		-.647022		.87914		4.94719	2	.33
.34	-.893736		-.912994		-1.01038	2	-.891553		.768639		-.21365		.920367		.524622		.34
.35	-.567355		-1.04496		.458829		-.582929		.968967		.260472		.702328		.848257		.35
.36	-.130122		-.96305		.830418		-.134192		.928153		.704401		.286051		.925355		.36
.37	.335355		-.685338		1.02441		.349769		.656965		.947124		-.213141		.733444		.37
.38	.743554		-.268538		1.00034		.757776		.222412		.946051		-.65839		.327894		.38
.39	1.02185		.2048		.764694		.997926		-.270048		.702005		-.929004		-.175335		.39
.4	1.12791		.643496		.367857		1.01763		-.702913		.277426		-.95263		-.633773		.4
.41	1.0317		.965584		-.108114		.813904		-.974826		-.220976		-.723421		-.919026		.41
.42	.766228		1.11324		-.567484		.432151		-1.0237		-.669999		-.301932		-.952204		.42
.43	.372478		1.06214		-.920551		-4.55003	2	-.839651		-.960642		.202376		-.724497		.43
.44	-8.49991	2	.823939		-1.10088		-.519095		-.465255		-1.02399		.660627		-.297873		.44
.45	-.534781		.442031		-1.07681		-.892358		1.59677	2	-.846258		.957761		.213405		.45
.46	-.910286		-1.80038	2	-.855407		-1.09206		.499574		-.469878		1.02083		.674512		.46
.47	-1.15953		-.480954		-.478814		-1.08137		.883559		1.82948	2	.835473		9.65926	2	.47
.48	-1.25123		-.874998		-1.44249	2	-.864951		1.0897		.508699		.417282		1.01373		.48
.49	-1.17875		-1.14284		.458657		-.485564		1.07839		.894318		-.051436		.806806		.49
.5	-.955717		-1.24927		.863944		-1.36185	2	.854493		1.09385		-.545175		.397339		.5
.51	-.614507		-1.18429		1.13997		.467509		.463587		1.06766		-.922743		-.114469		.51
.52	-.198352		-.961951		1.24882		.877145		-1.94047	2	.823667		-1.10201		-.606621		.52
.53	.245438		-.615584		1.1797		1.15089		-.506348		.412807		-1.04639		-.964958		.53
.54	.672037		-.190821		.947728		1.24984		-.912944		-8.39145	2	-.770123		-1.10924		.54
.55	1.01427		.262185		.58869		1.16434		-1.1733		-.573221		-.332105		-1.00959		.55
.56	1.35589		.695994		.151331		.912319		-1.24954		-.908019		.179107		-.690007		.56
.57	1.53281		1.07155		-.31119		.533356		-1.13528		-1.20273		.665056		-.219926		.57
.58	1.63257		1.[illegible]617		-.749166		8.00401	2	-.853215		-1.24284		1.03717		.303461		.58
.59	1.64243		1.55249		-1.1224		-.391387		-.448045		-1.08765		1.23248		.777102		.59
.6	1.57663		1.64242		-1.40356		-.829468		2.31777	2	-.766572		1.22258		1.11297		.6
.61	1.45341		1.64022		-1.58074		-1.19382		.501277		.330678		1.01502		1.2536		.61
.62	1.29223		1.56182		-1.65345		-1.45865		.933827		.198194		.847816		1.17983		.62
.63	1.11147		1.427		-1.63295		-1.61352		1.28145		.638443		.179333		.91003		.63
.64	.926843		1.25643		-1.5373		-1.66175		1.52099		1.05783		-.323931		.492454		.64
.65	.750492		1.06921		-1.388		-1.61722		1.6456		1.3795		-.798974		-6.86678	3	.65
.66	.590765		.881296		-1.20685		-1.50049		1.66249		1.58443		-1.19535		-.517559		.66
.67	.452445		.704654		-1.0134		-1.33483		1.5891		1.67069		-1.48052		-.976817		.67
.68	.337319		.547138		-.823474		-1.14291		1.44865		1.65027		-1.64127		-1.33825		.68
.69	.2449		.412868		-.648461		-.944388		1.26605		1.54452		-1.68179		-1.57552		.69
.7	.173173		.302923		-.495351		-.75448		1.06446		1.37934		-1.61952		-1.6829		.7
.71	.119266		.216158		-.367303		-.583534		.863072		1.18083		-1.47983		-1.67174		.71
.72	7.99905	2	.150024		-.264475		-.437303		.675969		.972051		-1.29087		-1.56519		.72
.73	5.22307	2	.101266		-.184953		-.317706		.511979		.771076		-1.07929		-1.39234		.73
.74	3.31896	2	6.64599	2	-.125612		-.223822		.375257		.590239		-.867235		-1.18283		.74
.75	2.05151	2	4.23914	2	-8.28513	2	-.152904		.266263		.436388		-.670897		-.962868		.75
.76	1.23232	2	2.62647	2	-5.30117	2	-.101271		.182907		.311775		-.500249		-.752751		.76
.77	7.18989	3	1.57955	2	-3.29086	2	-6.50003	2	.121621		.215278		-.359753		-.565919		.77
.78	4.07010	3	9.21256	3	-1.98005	2	-.040406		7.82456	2	.143641		-.24958		-.409506		.78
.79	2.25294	3	5.20538	3	-1.15361	2	-2.43064	2	4.86748	2	.092574		-.167012		-.285295		.79
.8	1.18565	3	2.84583	3	-6.50062	3	-1.41347	2	2.92518	2	5.72877	2	-.107751		-.191343		.8
.81	6.08387	4	1.50318	3	-3.53809	3	-7.93591	3	1.69653	2	3.45443	2	-6.69731	2	-.123484		.81
.82	3.01132	4	7.65754	4	-1.85691	3	-4.29520	3	9.47912	3	1.99563	2	-4.00607	2	-7.66168	2	.82
.83	1.43483	4	3.75528	4	-9.37968	4	-2.23695	3	5.09543	3	1.10856	2	-2.30283	2	-4.56488	2	.83
.84	6.56499	5	1.76839	4	-4.54963	4	-1.11859	3	2.62939	3	5.90976	3	-1.26986	2	-2.60756	2	.84
.85	2.87614	5	7.97422	5	-2.11344	4	-5.35687	4	1.29932	3	3.01646	3	-6.70268	3	-1.42513	2	.85
.86	1.20217	5	3.43204	5	-9.37218	5	-2.44928	4	6.13033	4	1.46998	3	-3.37727	3	-7.43346	3	.86
.87	4.77642	6	1.40404	5	-3.95266	5	-1.06523	4	2.75173	4	6.81587	4	-1.61912	3	-3.68878	3	.87
.88	1.79316	6	5.43168	6	-1.57822	5	-4.38732	5	1.17007	4	2.99445	4	-7.35598	4	-1.73481	3	.88
.89	6.32617	7	1.97563	6	-5.93380	6	-1.70181	5	4.68795	5	1.24013	4	-3.15131	4	-7.65518	4	.89
.9	2.06340	7	6.67092	7	-2.08521	6	-6.17345	6	1.75818	5	4.81069	5	-1.26519	4	-3.20026	4	.9
.91			2.04423	7	-6.80024	7	-2.07712	6	6.12193	6	1.73428	5	-4.72356	5	-1.23844	4	.91
.92					-2.04204	7	-6.40254	7	1.95761	6	5.75394	6	-1.62391	5	-4.41664	5	.92
.93							-1.75258	7	5.63593	7	1.73113	6	-5.07292	6	-1.43317	5	.93
.94									1.40080	7	4.62795	7	-1.41416	6	-4.16204	6	.94
.95											1.10905	7	-3.46340	7	-1.06483	6	.95
.96															-2.37407	7	.96
.97																	.97
.98																	.98
.99																	.99
1																	1

When [illegible] the [illegible] value is less than 10^{-7}

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	$F_{24}(x)$	p	$F_{25}(x)$	p	$F_{26}(x)$	p	$F_{27}(x)$	p	$F_{28}(x)$	p	$F_{29}(x)$	p	$F_{30}(x)$	p	$F_{31}(x)$	p	x
0	.830993		0		.80712		0		.784869		0		.763527		0		0
.01	.688729		.464926		.659614		.465278		.632299		.464626		.606478		.46298		.01
.02	.311559		.765529		.270969		.755129		.23386		.743411		.199891		.730472		.02
.03	-.172187		.795556		-.216877		.760249		-.255665		.724821		-.289098		.689494		.03
.04	-.597439		.544346		-.625775		.478641		-.646112		.416207		-.659471		.357249		.04
.05	-.819117		.100553		-.806417		.01633		-.785793		-5.91578	2	-.758935		-.126147		.05
.06	-.741586		-.379223		-.692792		-.452618		-.620356		-.511359		-.546469		-.556781		.06
.07	-.44435		-.725736		-.326204		-.751653		-.21383		-.759718		-.109144		-.752943		.07
.08	2.47066	2	-.816772		.159879		-.768162		.276271		-.704915		.373653		-.631718		.08
.09	.486146		-.620128		.588522		-.495737		.660051		-.368562		.703842		-.243895		.09
.1	.783231		-.204807		.803742		-3.64767	2	.788781		.115477		.745854		.247445		.1
.11	.815091		.283402		.727136		.437445		.612398		.554492		.482143		.63559		.11
.12	.570764		.673235		.386304		.748398		.198752		.773662		2.03026	2	.757128		.12
.13	.13268		.828014		-9.55214	2	.779794		-.292724		.685542		-.451011		.560582		.13
.14	-.351432		.693384		-.544063		.519566		-.672602		.324705		-.739202		.127977		.14
.15	-.718524		.316076		-.797266		6.44918	2	-.794429		-.166099		-.726329		-.359619		.15
.16	-.845096		-.172669		-.763518		-.416088		-.610971		-.592617		-.417116		-.697985		.16
.17	-.688483		-.603005		-.45462		-.743413		-.192178		-.785988		6.30856	2	-.745207		.17
.18	-.30081		-.82555		.018831		-.1[illegible]		.301841		-.669395		.519363		-.480928		.18
.19	.188809		-.763026		.487367		-.55313		.682228		-.288213		.766424		-1.47673	2	.19
.2	.617595		-.436652		.783443		-.104894		.803537		.208054		.703663		.459452		.2
.21	.843273		4.15692	2	.801188		.334011		.619035		.624674		.355749		.744401		.21
.22	.791019		.507713		.533836		.733746		.19822		.798115		-.137512		.721196		.22
.23	.477731		.802283		7.57216	2	.815674		-.29984		.659938		-.577677		.398609		.23
.24	5.92779	3	.824586		-.411513		.599266		-.686926		.263301		-.787558		-9.06964	2	.24
.25	-.47034		.566629		-.756208		.162866		-.816753		-.238027		-.682274		-.545092		.25
.26	-.796028		.115289		-.837034		-.335255		-.639853		-.649631		-.303054		-.777157		.26
.27	-.86539		-.377591		-.625198		-.714634		-.221789		-.811845		.199481		-.690713		.27
.28	-.655736		-.746623		-.19396		-.837927		.281788		-.661298		.625537		-.320145		.28
.29	-.234034		-.868279		.307416		-.660142		.68349		-.25501		.805617		.183879		.29
.3	.265294		-.701683		.705763		-.244288		.833899		.251894		.667557		.616098		.3
.31	.683744		-.301489		.863788		.26174		.676664		.665865		.264756		.800291		.31
.32	.889078		.200945		.726857		.678262		.268804		.828833		-.245204		.660723		.32
.33	.816604		.641291		.340991		.857634		-.240799		.678074		-.662688		.252561		.33
.34	.488621		.876117		-.163567		.736048		-.66631		.269508		-.824168		-.260688		.34
.35	6.45567	3	.829117		-.617014		.3553		-.852743		-.24392		-.665756		-.673164		.35
.36	-.481762		.514943		-.867295		-.152601		-.731872		-.670084		-.247597		-.819212		.36
.37	-.827015		3.34605	2	-.830619		-.611962		-.346209		-.849604		.26988		-.659348		.37
.38	-.924646		-.462993		-.518496		-.864369		.167018		-.714869		.683108		-.203543		.38
.39	-.745169		-.818294		-3.28879	2	-.822817		.625487		-.314453		.846569		.316928		.39
.4	-.341848		-.92139		.468167		-.500626		.86673		.205515		.683754		.71737		.4
.41	.166466		-.740066		.822503		-5.86853	3	.805146		.655471		.259918		.840099		.41
.42	.631449		-.329592		.916045		.49599		.461222		.871659		-.266366		.635788		.42
.43	.918703		.18558		.718515		.83794		-4.66495	2	.775072		-.698304		.182094		.43
.44	.946215		.650604		.29172		.906668		-.54406		.399073		-.874502		-.347024		.44
.45	.706532		.927061		-.251323		.678976		-.860926		-.123562		-.72849		-.748696		.45
.46	.266175		.533509		-.588907		.227985		-.889194		-.608638		-.312424		-.868826		.46
.47	-.252869		.668025		-.940847		-.301931		-.618412		-.885803		.223015		-.660257		.47
.48	-.709787		.206888		-.910634		-.742403		-.137829		-.857746		.684239		-.199781		.48
.49	-.982505		-.319124		-.607059		-.95464		.394632		-.532909		.904912		.341616		.49
.5	-.999687		-.762974		-.119631		-.872183		.805212		-2.11606	2	.805092		.763161		.5
.51	-.757663		-1.0026		.409643		-.519898		.960898		.504954		.418533		.908688		.51
.52	-.318911		-.973451		.828699		-4.33251	3	.811279		.869164		-.120628		.723565		.52
.53	.2063		-.683[illegible]		1.01903		.520531		.402828		.99013		-.625917		.272484		.53
.54	.689245		-.210504		.927889		.899286		-.13778		.720323		-.923531		-.283471		.54
.55	1.01484		.324777		.580673		1.0224		-.644688		.253106		-.911357		-.747273		.55
.56	1.10832		.787288		7.11958	2	.854965		-.964376		-.302972		-.592112		-.955068		.56
.57	.950447		1.06397		-.465967		.443148		-1.00137		-.772961		-7.04217	2	-.833057		.57
.58	.576655		1.08286		-.892365		-9.81593	2	-.74467		-1.01078		.483704		-.421445		.58
.59	7.31537	2	.860957		-1.10127		-.622987		-.268166		-.943333		.891502		.141381		.59
.6	-.452862		.431572		-1.04278		-.992871		.293498		-.590429		1.02282		.66743		.6
.61	-.902239		-.101766		-.73289		-1.11318		.78497		-5.58397	2	.835777		.982634		.61
.62	-1.1854		-.624566		-.245732		-.955094		1.07366		.506018		.387087		.983469		.62
.63	-1.25499		-1.03028		.307466		-.559184		1.08424		.936275		-.188406		.668412		.63
.64	-1.10439		-1.24171		.806127		-2.12294	2	.813363		1.11634		-.720813		.135419		.64
.65	-.765261		-1.22373		1.14743		.534185		.335863		.998473		-1.0563		-.451795		.65
.66	-.297127		-.985683		1.2665		.984475		-.236823		.614823		-1.10042		-.916081		.66
.67	.026726		-.574411		1.14294		1.23621		-.768393		6.44002	2	-.841707		-1.12034		.67
.68	.733674		-6.08654	2	.814047		1.24254		-1.14045		-.516356		-.350266		-1.00581		.68
.69	1.16358		.475889		.333816		1.00821		-1.27599		-.989834		.245677		-.604555		.69
.7	1.47629		.963575		-.213511		.582557		-1.15226		-1.25014		.796775		-2.74171	2	.7
.71	1.65416		1.34771		-.745838		4.42109	2	-.739708		-1.24464		1.1715		.574421		.71
.72	1.70033		1.59753		-1.19483		-.518041		-.289215		-.98021		1.28646		1.04837		.72
.73	1.63387		1.70544		-1.51524		-1.02321		.287467		-.51493		1.12129		1.28105		.73
.74	1.48344		1.68504		-1.68813		-1.41137		.838605		.06067		.716422		1.22226		.74
.75	1.28099		1.56293		-1.71867		-1.65025		1.28974		.6466		.15677		.89042		.75
.76	1.05451		1.3724		-1.63012		-1.75537		1.59427		1.15461		-.452391		.360392		.76
.77	.834546		1.1469		-1.4561		-1.68488		1.73641		1.52307		-1.01022		-.260566		.77
.78	.632445		.915222		-1.23297		-1.55151		1.72759		1.72351		-1.43981		-.860661		.78
.79	.460282		.698879		-.993753		-1.31395		1.59846		1.7591		-1.69881		-1.34775		.79
.8	.321835		.511299		-.764309		-1.06928		1.38931		1.65715		-1.78065		-1.66467		.8
.81	.216182		.358565		-.561759		-.827882		1.1411		1.43883		-1.70813		-1.79381		.81
.82	.13943		.241023		-.394813		-.610872		.888824		1.20868		-1.52249		-1.75221		.82
.83	8.62664	2	.155202		-.265322		-.429885		.657868		.94644		-1.27154		-1.58054		.83
.84	5.11306	2	.095637		-.170377		-.2885		.462074		.702		-1.00012		-1.32976		.84
.85	2.89788	2	5.63068	2	-.104417		-.184502		.309961		.49367		-.742546		-1.0493		.85
.86	1.56683	2	3.16072	2	-6.09627	2	-.112277		.197109		.32309		-.520958		-.7788		.86
.87	8.05797	3	1.68699	2	-3.38232	2	-6.48754	2	.118878		.207718		-.345267		-.544221		.87
.88	3.92725	3	8.53161	3	-1.77753	2	-3.54888	2	6.78216	2	.12388		-.215848		-.357867		.88
.89	1.80555	3	4.07035	3	-8.81179	3	-.018308		3.64729	2	6.95879	2	-.126955		-.221027		.89
.9	7.78524	4	1.82177	3	-4.09871	3	-8.86209	3	1.84015	2	3.66593	2	-6.99804	2	-.127798		.9
.91	3.12511	4	7.53501	4	-1.77658	3	-3.99878	3	8.65387	3	1.80056	2	-3.59559	2	-6.88385	2	.91
.92	1.15672	4	2.92225	4	-7.11209	4	-1.66752	3	3.76488	3	8.18057	3	-1.70926	2	-3.43056	2	.92
.93	3.89712	5	1.02497	4	-2.59834	4	-6.35270	4	1.49742	3	3.40155	3	-7.44148	3	-1.56651	2	.93
.94	1.17375	5	3.22325	5	-8.52207	5	-2.17609	4	5.36292	4	1.27579	3	-2.92508	3	-6.46582	3	.94
.95	3.07410	6	8.86043	6	-2.45094	5	-6.55097	5	1.69163	4	4.22065	4	-1.01689	3	-2.36493	3	.95
.96	6.61746	7	[illegible].35211	6	-5.96263	6	-1.67411	5	4.54319	5	1.19331	4	-3.02934	4	-7.43553	4	.96
.97	1.02792	7	3.[illegible]372	7	-1.15803	6	-3.45295	6	9.82742	6	2.73074	5	-7.33752	5	-1.91053	4	.97
.98					-1.60116	7	-5.07120	7	1.53373	6	4.57225	6	-1.30724	5	-3.64533	5	.98
.99									1.31062	7	4.41950	7	-1.33256	6	-4.09670	6	.99
1															-1.05493	7	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	F32(x)	p	F33(x)	p	F34(x)	p	F35(x)	p	F36(x)	p	F37(x)	p	F38(x)	p	F39(x)	p	x
0	.74281		0		.722438		0		.707111		0		.68147		0		0
.01	.581886		.46032		.558274		.456584		.535393		.451656		.512965		.445335		.01
.02	.16879		.716352		.140338		.701032		.114361		.684413		.090724		.666279		.02
.03	-.317615		.654427		-.341555		.619718		-.36116		.585405		-.376561		.551439		.03
.04	-.666704		.301912		-.66851		.250299		-.66544		.202486		-.657882		.198542		.04
.05	-.727256		-.184918		-.691934		-.235757		-.653339		-.278924		-.614052		-.314597		.05
.06	-.472881		-.590176		-.403986		-.612752		-.331893		-.625599		-.266492		-.625635		.06
.07	-1.34368	2	-.73406		7.24781	2	-.705309		.14813		-.669439		.213275		-.627695		.07
.08	.452489		-.552536		.51371		-.470679		.558526		-.388881		.588279		-.309402		.08
.09	.72342		-.125711		.722451		-1.68887	2	.704537		8.05688	2	.673083		.165324		.09
.1	.682033		.357626		.603727		.445585		.516565		.511862		.425379		.557631		.1
.11	.345806		.683547		.210907		.702252		8.32187	2	.696153		-3.30204	2	.669856		.11
.12	-.140541		.707597		-.278427		.63379		-.390566		.543838		-.476149		.445		.12
.13	-.567456		.41865		-.642831		.271349		-.680459		.127963		-.685253		-4.44907	3	.13
.14	-.750841		-5.61672	2	-.717151		-.217793		-.648859		-.350934		-.556668		-.452868		.14
.15	-.611287		-.507969		-.467195		-.607525		-.310026		-.661555		-.153041		-.674745		.15
.16	-.208191		-.736377		-5.22267	3	-.717385		.176534		-.655464		.327326		-.557972		.16
.17	.28578		-.641263		.460668		-.495935		.581272		-.329753		.647949		-.160313		.17
.18	.658806		-.263341		.720324		-4.45837	2	.712859		.194271		.65049		.319317		.18
.19	.750687		.231898		.696194		.429069		.508444		.565727		.332748		.640396		.19
.2	.521381		.627234		.296254		.70478		6.33931	2	.706173		-.150009		.641315		.2
.21	6.80664	2	.748963		-.198509		.66009		-.41543		.507128		-.561052		.320515		.21
.22	-.416376		.542954		-.605309		.309435		-.697761		6.31587	2	-.698245		-.162411		.22
.23	-.725578		9.83149	2	-.743955		-.185115		-.655272		-.413275		-.493241		-.566241		.23
.24	-.727453		-.391517		-.54888		-.597147		-.30472		-.693572		-4.52335	2	-.688572		.24
.25	-.420217		-.713094		-.107153		-.737546		.19066		-.642529		.427276		-.467053		.25
.26	6.69377	2	-.725835		.384877		-.541013		.599385		-.283224		.693112		-1.08527	2	.26
.27	.528568		-.423066		.708038		-9.59981	2	.72987		.213737		.621357		.453363		.27
.28	.769725		6.51156	2	.717621		.395506		.519935		.611447		.245492		.694094		.28
.29	.688016		.528159		.408568		.709816		.065979		.719988		-.25256		.590271		.29
.3	.316506		.765978		-8.39823	2	.703198		-.421682		.485189		-.630675		.191782		.3
.31	-.190277		.675128		-.542692		.377142		-.716401		.018093		-.704162		-.304809		.31
.32	-.621069		.293203		-.764844		-.122193		-.68035		-.460892		-.43563		-.653387		.32
.33	-.79597		-.217358		-.651412		-.569906		-.328489		-.724496		4.64861	2	-.680149		.33
.34	-.641142		-.638797		-.250574		-.763708		.178122		-.646584		.509706		-.369872		.34
.35	-.219218		-.790923		.263397		-.614964		.606435		-.262002		.729674		.125995		.35
.36	.296809		-.607675		.666319		-.185564		.75862		.249524		.598541		.563596		.36
.37	.694992		-.164939		.781838		.32603		.563026		.647713		.177291		.726577		.37
.38	.811418		351401		.558077		.699339		.107211		.744533		-.333119		.53271		.38
.39	.597058		.724406		9.00725	2	.764076		-.40188		.492509		-.687893		7.48086	2	.39
.4	.13741		.796651		-.421351		.389501		-.732165		.007256		-.715649		-.424174		.4
.41	-.382539		.536115		-.756127		-4.91187	3	-.732011		-.486138		-.400662		-.71988		.41
.42	-.753276		4.93918	2	-.769396		-.502188		-.399149		-.757732		.109152		-.665959		.42
.43	-.825002		-.462996		-.453373		-.783328		.118117		-.679545		.572209		-.285888		.43
.44	-.567301		-.78954		5.94886	2	-.723396		.587638		-.285094		.767806		.237786		.44
.45	-8.07358	2	-.794753		.552879		-.346102		.79774		.245755		.600857		.651535		.45
.46	.443537		-.474386		.818139		.186551		.652123		.669346		.147258		.753511		.46
.47	.799443		4.32847	2	.742008		.644592		.212974		.790035		-.381377		.491437		.47
.48	.846695		.5499		.353835		.829471		-.326595		.549851		-.736857		-1.15416	2	.48
.49	.564957		.841283		-.186991		.659417		-.728083		5.52167	2	-.750615		-.515395		.49
.5	6.09162	2	.799192		-.657454		.208316		-.812885		-.471155		-.412939		-.777965		.5
.51	-.47355		.43796		-.863038		-.344276		-.54083		-.790944		.122199		-.670892		.51
.52	-.835136		-.101917		-.717334		-.75454		-2.77481	2	-.757729		.608074		-.241374		.52
.53	-.886067		-.609742		-.276812		-.852211		.505024		-.382889		.819128		.309552		.53
.54	-.605166		-.887276		.282692		-.593489		.826635		.169296		.654999		.721697		.54
.55	-9.45498	2	-.875377		.737272		-8.29112	2	.796252		.654657		.186915		.798445		.55
.56	.459391		-.445164		.904602		.470445		.42332		.45726		-.374436		.499584		.56
.57	.855041		.111215		.713897		.839365		-.136717		.684591		-.774464		-.039203		.57
.58	.948627		.635445		.24212		.871376		-.648569		.208117		-.829885		-.569022		.58
.59	.704763		.931631		-.334797		.55015		-.896299		-.367916		-.51143		-.843064		.59
.6	.207203		.888599		-.793213		1.24634	3	-.773469		-.795049		4.28031	2	-.730947		.6
.61	-.372508		.519416		-.956932		-.558919		-.326842		-.887573		.588689		-.278993		.61
.62	-.835857		-4.52359	2	-.761386		-.909292		.263578		-.601765		.884122		.310266		.62
.63	-1.0254		-.695721		-.276385		-.910707		.758693		-5.40826	2	.795555		.770013		.63
.64	-.8765		-.96512		.32162		-.558953		.957336		.52839		.356349		.889821		.64
.65	-.43615		-.997611		.815716		1.51778	2	.776503		.90295		-.247836		.610487		.65
.66	.155476		-.689889		1.02745		.597152		.283189		.91205		-.759186		5.07543	2	.66
.67	.712624		-.142733		.879459		.970109		-.333384		.346947		-.957942		-.545202		.67
.68	1.06348		.465624		.420284		.994835		-.836453		-4.96592	2	-.75524		-.915255		.68
.69	1.10202		.93986		.19554		.658843		-1.03283		-.642156		-.229773		-.893818		.69
.7	.816627		1.13029		-.762708		7.80805	2	-.845011		-.996249		.405751		-.483403		.7
.71	.288698		.97757		-1.09479		-.54698		-.33786		-.970485		.89273		.146833		.71
.72	-.335692		.52652		-1.084		-1.00286		.30805		-.569405		1.03177		.753425		.72
.73	-.890076		-.092524		-.732097		-1.13561		.863995		6.06383	2	.761709		1.02956		.73
.74	-1.2334		-.706258		-.14466		-.899517		1.13492		.688883		.181967		.907202		.74
.75	-1.28422		-1.14963		.506245		-.36728		1.02554		1.08663		-.488512		.4084		.75
.76	-1.03523		-1.30972		1.0364		.298612		.568723		1.10925		-.996874		-.273517		.76
.77	-.54691		-1.15055		1.30237		.900526		-9.08532	2	.744309		-1.15164		-.872616		.77
.78	7.49851	2	-.714189		1.2373		1.26646		-.748535		.110795		-.891137		-1.1551		.78
.79	.709665		-.101285		.86116		1.29731		-1.20662		-.586424		-.301422		-1.00783		.79
.8	1.25014		.560544		.265982		.987697		-1.3331		-1.12753		.419436		-.47741		.8
.81	1.62366		1.15006		-.416115		.417611		-1.09446		-1.34769		1.03367		.251949		.81
.82	1.80036		1.57220		-1.03042		-.270681		-.55938		-1.16274		1.34416		.929145		.82
.83	1.79049		1.8023		-1.53146		-.953841		.150044		-.679105		1.25427		1.32585		.83
.84	1.63338		1.82421		-1.80169		-1.48544		.862697		3.15429	2	.789114		1.31111		.84
.85	1.38289		1.68156		-1.85475		-1.80068		1.44282		.779084		7.58945	2	.886139		.85
.86	1.09349		1.43038		-1.72572		-1.88357		1.8014		1.40595		-.704862		.171685		.86
.87	.81006		1.13218		-1.474		-1.76652		1.91217		1.60602		-1.37714		-.641694		.87
.88	.562729		.835604		-1.16085		-1.51189		1.80462		1.94199		-1.81664		-1.35862		.88
.89	.366262		.575734		-.854663		-1.19992		1.54445		1.84055		-1.97446		-1.83535		.89
.9	.222803		.369831		-.582457		-.866389		1.20963		1.57139		-1.8747		-2.01085		.9
.91	.12614		.220753		-.367957		-.582077		.869816		1.22007		-1.5921		-1.90721		.91
.92	6.60476	2	.121768		-.214504		-.360038		.573718		.863812		-1.22099		-1.60561		.92
.93	3.16963	2	.061557		-.114543		-.203754		.345504		.556438		-.847033		-1.2108		.93
.94	1.37623	2	2.81747	2	-5.53984	2	-.10443		.188301		.325828		-.529228		-.81786		.94
.95	5.30471	3	1.14665	2	-2.38573	2	-4.77068	2	9.15317	2	.168091		-.294615		-.491017		.95
.96	1.76288	3	4.03503	3	-8.90773	3	-1.39444	2	.038757		7.61355	2	-.143283		-.257585		.96
.97	4.81071	4	1.17162	3	-2.75740	3	-6.26563	3	1.37301	2	2.83733	2	-5.87711	2	-.114339		.97
.98	9.82344	5	2.56773	4	-6.49750	4	-1.59105	3	3.76654	3	8.61067	3	-1.89838	2	-4.02956	2	.98
.99	1.19605	5	3.41286	5	-9.45285	5	-2.54119	4	6.62266	4	1.67184	3	-4.08465	3	-9.64689	3	.99
1	2.56661	7	7.98019	7	-2.62815	6	-8.58399	5	2.73314	5	5.42696	5	-2.52961	4	-7.38164	4	1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	$F_{40}(x)$	p	$F_{41}(x)$	p	$F_{42}(x)$	p	$F_{43}(x)$	p	$F_{44}(x)$	p	$F_{45}(x)$	p	$F_{46}(x)$	p	$F_{47}(x)$	p	x
0	.660332		0		.637039		0		.611028		0		.578217		0		0
.01	.490633		.437269		.467865		.42678		.443648		.412266		.415368		.38939		.01
.02	6.93335	2	.646203		5.01397	2	.623317		3.31526	2	.599475		1.84968	2	.356752		.02
.03	-.387731		.517632		-.394389		.483509		-.395675		.447755		-.388955		.406581		.03
.04	-.646015		.11859		-.629674		8.26368	2	-.607934		5.10387	2	-.577505		.024366		.04
.05	-.572847		-.342793		-.530634		-.363166		-.487202		-.374372		-.4408		-.372059		.05
.06	-.205514		-.625513		-.14959		-.615334		-9.93391	2	-.592074		-5.55651	2	-.356589		.06
.07	.267769		-.581792		.311387		-.5328		.343438		-.480854		.56145		-.423736		.07
.08	.604279		-.234082		.607602		-.164499		.598617		-.102119		.575354		-4.88829	2	.08
.09	.63119		.236469		.581538		.293195		.52613		.334009		.465341		.354474		.09
.1	.334236		.584378		.246535		.593474		.165163		.585218		9.28581	2	.556255		.1
.11	-.134821		.62782		-.270091		.574053		-.287055		.511545		-.332718		.440955		.11
.12	-.535777		.343545		-.570889		.244785		-.582979		.15328		-.571717		7.37048	2	.12
.13	-.662939		-.12078		-.613423		-.217587		-.560169		-.291087		-.48906		-.336516		.13
.14	-.450529		-.523309		-.339241		-.563499		-.230359		-.574799		-.130529		-.555731		.14
.15	-6.56681	3	-.453932		.121817		-.606607		.226613		-.539764		.302602		-.458337		.15
.16	.442144		-.443816		.519661		-.322519		.560853		-.204552		.566516		-9.89936	2	.16
.17	.665868		-1.53751	3	.643116		.136276		.588983		.245477		.512005		.318048		.17
.18	.549436		.443114		.425881		.523334		.294627		.360647		.168736		.554989		.18
.19	.151347		.658499		-1.80097	2	.629717		-.162177		.565396		-.270913		.475978		.19
.2	-.325765		.531675		-.454126		.397122		-.532003		.255955		-.559634		.124922		.2
.21	-.63832		.127429		-.651418		-5.04406	2	-.612152		-.197015		-.534202		-.298921		.21
.22	-.626041		-.344953		-.504489		-.47282		-.357613		-.542529		-.207645		-.553987		.22
.23	-.293929		-.64007		-8.97099	2	-.642542		9.38008	2	-.528267		.237429		-.493978		.23
.24	.189871		-.603771		.374599		-.467172		.496114		-.307485		.550913		-.15168		.24
.25	.579313		-.253457		.643095		-3.94697	2	.629152		.45499		.555654		.272959		.25
.26	.675524		.230425		.572917		.411776		.418889		.320134		.247429		.552668		.26
.27	.428154		.597414		.19948		.644119		-2.15412	2	.608132		-.201886		.51244		.27
.28	-5.86727	2	.656608		-.28156		.531569		-.452793		.359123		-.540145		.179478		.28
.29	-.488696		.375715		-.616882		.132689		-.639333		-9.06899	2	-.576331		-.25796		.29
.3	-.69325		-.101661		-.628788		-.340017		-.477921		-.493067		-.288275		-.550959		.3
.31	-.547158		-.529601		-.30912		-.633345		-.054528		-.624657		.16397		-.53147		.31
.32	-.122467		-.696537		.175714		-.388877		.401559		-.410803		.527057		-.208556		.32
.33	.367377		-.489725		.5715		-.228283		.641893		3.23053	2	.596168		.235683		.33
.34	.674923		-3.84933	2	.669437		.257373		.534037		.460827		.330381		.548761		.34
.35	.643549		.436123		.416024		.608952		.134291		.637395		-.123308		.551184		.35
.36	.286829		.689775		-.058093		.637387		-.341837		.462409		-.511303		.239193		.36
.37	-.217622		.590336		-.50569		.320077		-.635835		2.99792	2	-.615053		-.211836		.37
.38	-.616824		.186334		-.691862		-.163738		-.586378		-.42279		-.373965		-.545943		.38
.39	-.709231		-.317076		-.51704		-.569488		-.217559		-.645759		7.94469	2	-.571704		.39
.4	-.446084		-.662459		-6.98079	2	-.675053		.272953		-.513751		.492435		-.271717		.4
.41	4.34748	2	-.671587		.418279		-.421553		.619927		-9.65335	2	.632809		.186063		.41
.42	.516182		-.336962		.692661		5.91207	2	.633829		.378173		.419256		.542331		.42
.43	.735599		.173433		.608293		.513297		.304022		.648973		-3.18372	2	.593164		.43
.44	.590021		.600773		.205833		.699292		-.194129		.564524		-.469876		.306521		.44
.45	.148679		.726586		-.308298		.51572		-.592636		.167772		-.649175		-.13792		.45
.46	-.371742		.463998		-.667987		5.62304	2	-.674926		-.326		-.466505		-.537694		.46
.47	-.713762		-7.44807	3	-.68498		-.438505		-.393173		-.646013		-2.01967	2	-.615711		.47
.48	-.706254		-.501175		-.346885		-.70705		.104487		-.614266		.442876		-.344087		.48
.49	-.349371		-.747398		.175175		-.604913		.552045		-.244149		.663767		.126822		.49
.5	.184623		-.619097		.613519		-.181781		.707728		.265087		.515976		.531717		.5
.51	.635035		-.176748		.741087		.343031		.484201		.635512		7.74546	2	.639502		.51
.52	.77999		.360817		.488508		.694437		-3.06663	3	.662274		-.410447		.385011		.52
.53	.545068		.725		-1.90605	2	.683512		-.495753		.326142		-.674027		-9.20989	2	.53
.54	4.03827	2	.73138		-.524491		.316265		-.729621		-.193769		-.56794		-.523965		.54
.55	-.491874		.372256		-.769039		-.224637		-.575815		-.615632		-.141114		-.664709		.55
.56	-.793974		-.17828		-.624292		-.656606		-.111128		-.707481		.371271		-.450055		.56
.57	-.71695		-.649389		-.158617		-.752533		.420726		-.414223		.685137		5.27231	2	.57
.58	-.292857		-.8067		.395846		-.457474		.737034		.110201		.622655		.513827		.58
.59	.275582		-.568094		.75927		8.10713	2	.665954		.583864		.212407		.691508		.59
.6	.728889		-4.51496	2	.744969		.587465		.239018		.748261		-.323549		.480195		.6
.61	.839451		.510052		.354223		.799011		-.323123		.50878		-.689881		-7.40362	3	.61
.62	.553831		.826828		-.222651		.501512		-.725011		-1.18256	2	-.680331		-.50042		.62
.63	4.33644	4	.747226		-.699728		8.96142	2	-.751317		-.536712		-.293127		-.720069		.63
.64	-.564813		.303248		-.837075		-.479361		-.381221		-.782068		.264782		-.536738		.64
.65	-.878828		-.297204		-.560308		-.815064		.198066		-.609971		.688411		-4.56897	2	.65
.66	-.791505		-.768671		1.05508	3	-.741639		.686538		-.10461		.741039		.482434		.66
.67	-.336213		-.883674		.575432		-.288345		.826538		.469206		.385595		.750922		.67
.68	.284979		-.580948		.880941		.322777		.537558		.804958		-.191385		.601461		.68
.69	.792529		4.69807	3	.762786		.786451		-3.93922	2	.717404		-.67784		.109199		.69
.7	.955219		.604115		.269539		.866248		-.611473		.243332		-.804532		-.457844		.7
.71	.693614		.938357		-.368417		.513492		-.882718		-.374075		-.492938		-.782877		.71
.72	.115428		.847274		-.847715		-.106247		-.705984		-.810325		9.80433	2	-.676861		.72
.73	-.53172		.363395		-.935846		-.692338		-.160522		-.829444		.653479		-.187101		.73
.74	-.967988		-.30209		-.581672		-.955114		.484794		-.409815		.869837		.42338		.74
.75	-1.00152		-.853735		5.91782	2	-.757403		.904635		.240323		.619451		.816848		.75
.76	-.609438		-1.04309		.695223		-.182828		.880198		.787024		2.34729	2	.766557		.76
.77	4.36936	2	-.777986		1.03256		.501894		.410492		.941626		-.607308		.285704		.77
.78	.721052		-.962296		.908866		.972184		-.283663		.61096		-.934305		-.373432		.78
.79	1.12298		.549183		.367029		.998415		-.865156		-5.06352	2	-.771129		-.351387		.79
.8	1.09424		1.05908		-.363473		.554227		-1.04392		-.714935		-.186628		-.876276		.8
.81	.955316		1.15107		-.967273		-.169642		-.718467		-1.04274		.524652		-.415715		.81
.82	-8.74164	2	.776709		-1.17955		-.85105		-2.74309	2	-.854547		.991144		.297582		.82
.83	-.817491		7.16316	2	-.89794		-1.18086		.713124		-.223624		.956309		.383436		.83
.84	-1.29578		-.70159		-.223561		-.999856		1.15585		.554914		.415123		1.01503		.84
.85	-1.35547		-1.23671		.583541		-.367574		1.08242		1.10398		-.375976		.597095		.85
.86	-.971219		-1.38957		1.21094		.464525		.503651		1.14507		-1.02221		-.174181		.86
.87	-.255525		-1.04561		1.41555		1.16005		-.344559		.632457		-1.18562		-.903959		.87
.88	.591093		-.327333		1.11071		1.43526		-1.10459		-.222023		-.75511		-1.19891		.88
.89	1.35238		.554476		.387234		1.16804		-1.45005		-1.04338		9.28483	2	-.872553		.89
.9	1.86424		1.36124		-.533179		.435577		-1.21923		-1.46029		.972122		-5.00499	2	.9
.91	2.05233		1.90539		-1.3869		-.528444		-.473149		-1.26605		1.46452		.882205		.91
.92	1.93785		2.09987		-1.96093		-1.43194		.541202		-.301712		1.31084		1.45847		.92
.93	1.61044		1.96595		-2.15421		-2.03309		1.49881		.57145		.525305		1.357		.93
.94	1.18735		1.6044		-1.99014		-2.21589		2.12435		1.58975		-.61677		.552365		.94
.95	.774327		1.14772		-1.58438		-2.00837		2.28536		2.23775		-1.70635		-.670792		.95
.96	.440695		.714035		-1.08836		-1.54599		2.01771		2.36374		-2.37827		-1.85121		.96
.97	.217737		.377149		-.634005		-1.00391		1.48519		2.01498		-2.45572		-2.55946		.97
.98	8.21603	2	.160499		-.299326		-.530415		.887423		1.38837		-2.00018		-2.58096		.98
.99	.021931		4.82974	2	-.101961		-.206265		.398216		.729323		-1.25463		-1.93971		.99
1	2.00077	3	5.74117	3	-1.52673	2	-3.92153	2	.097009		.229978		-.517769		-1.08666		1

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-8. (continued)

x	$F_{48}(x)$	p	$F_{49}(x)$	p	$F_{50}(x)$	p	$F_{51}(x)$	p	$F_{52}(x)$	p	$F_{53}(x)$	p	$F_{54}(x)$	p	$F_{55}(x)$	p	x
0	.532277		0		.510897		0		.543807		0		.570458		0		0
.01	.378751		.361887		.360526		.374244		.378754		.403051		.395182		.417723		.01
.02	6.68361	3	.512892		-2.12206	3	.525986		-1.72417	2	.560958		-2.27554	2	.574906		.02
.03	-.369389		.364946		-.363666		.36493		-.406233		.377575		-.426053		.373428		.03
.04	-.532539		4.13130	3	-.511287		-1.32989	2	-.554313		-3.57528	2	-.566311		-6.12008	2	.04
.05	-.3885		-.359382		-.357932		-.383923		-.372452		-.427764		-.356939		-.45795		.05
.06	-2.01004	2	-.513683		6.38525	3	-.526487		3.04699	2	-.539949		7.34032	2	-.569181		.06
.07	.360346		-.368583		.367378		-.355928		.412331		-.3516		.459809		-.325127		.07
.08	.533326		-8.29524	3	.512464		.026696		.543434		7.09072	2	.564089		.122348		.08
.09	.398711		.357412		.355367		.394046		.344527		.450691		.321525		.494172		.09
.1	3.36671	2	.515278		-.010696		.527492		-6.53108	2	.556427		-.118964		.558001		.1
.11	-.351554		.372832		-.371697		.347157		-.439903		.323159		-.486164		.273347		.11
.12	-.534648		1.23253	2	-.514444		-4.02921	2	-.553754		-.107819		-.553445		-.182711		.12
.13	-.409462		-.355969		-.354318		-.404689		-.337723		-.474484		-.278571		-.525569		.13
.14	-4.74882	2	-.517698		1.50966	2	-.529011		.078992		-.553297		.169768		-.540588		.14
.15	.342947		-.377736		.376667		-.338555		.446047		-.295442		.515377		-.217419		.15
.16	.536517		-.016857		.517256		5.41935	2	.542241		.142915		.544591		.242671		.16
.17	.420841		.35505		.353281		.415939		.308743		.495089		.238323		.552763		.17
.18	6.16754	2	.52098		-1.96236	2	.531059		-.114938		.545988		-.215451		.517776		.18
.19	-.334458		.383348		-.382343		.330054		-.474228		.26341		-.536933		.158504		.19
.2	-.538952		2.13288	2	-.52094		-6.85131	2	-.552351		-.181597		-.526912		-.301352		.2
.21	-.432948		-.354659		-.352759		-.427891		-.301057		-.518173		-.190173		-.574111		.21
.22	-.076351		-.525174		2.43198	2	-.533656		.12957		-.540411		.266512		-.487967		.22
.23	.326014		-.389737		.388793		-.321582		.480643		-.233295		.561202		-9.52119	2	.23
.24	.341979		-2.59834	2	.525555		8.33843	2	.539983		.217178		.510732		.359342		.24
.25	.445896		.354808		.352762		.440657		.27028		.536487		.144551		.590782		.25
.26	9.16521	2	.530345		-.029233		.536827		-.167472		.528756		-.312288		.452582		.26
.27	-.317536		.396986		-.396101		.313059		-.509967		.196779		-.577266		2.94172	2	.27
.28	-.54563		5.08696	2	-.531172		-9.89451	2	-.54985		-.258319		-.485001		-.411505		.28
.29	-.459816		-.35552		-.353306		-.454361		-.2612		-.558923		-9.04627	2	-.600199		.29
.3	-.107736		-.536579		.034419		-.540605		.183654		-.520097		.36357		-.409143		.3
.31	.308938		-.405198		.40437		-.304595		.516866		-.163367		.595716		4.10054	2	.31
.32	.549948		-.036044		.537888		.115364		.53619		.294952		.460243		.468613		.32
.33	.474864		.356826		.35442		.469155		.227655		.574723		3.86684	2	.604066		.33
.34	.124786		.543987		-3.99429	2	.545029		-.224561		.505052		-.409201		.359705		.34
.35	-.300117		.4145		-.41373		.295488		-.547926		.121095		-.605009		-.113331		.35
.36	-.554982		4.15742	2	-.545822		-.132839		-.545663		-.33936		-.424826		-.518281		.36
.37	-.491226		-.358767		-.356141		-.485216		-.216446		-.596357		2.23857	2	-.598693		.37
.38	-.143022		-.552706		4.58837	2	-.550148		.243072		-.490311		.460385		-.300803		.38
.39	.290953		-.425092		.42434		-.286213		.555515		-8.31272	2	.616044		.190432		.39
.4	.560793		-4.75416	2	.555128		.121605		.530026		.377646		.389573		.565817		.4
.41	.509127		.361401		.358517		.50276		.17877		.609036		-8.14211	2	.5863		.41
.42	.162711		.562915		-5.23378	2	.556021		-.288375		.46625		-.305208		.235225		.42
.43	-.281297		.437055		-.436403		.276418		-.589007		3.32627	2	-.616346		-.2685		.43
.44	-.567463		5.40475	2	-.565999		-.171954		-.538758		-.426233		-.341862		-.60647		.44
.45	-.528846		-.364904		-.361616		-.52205		-.164316		-.629345		.150767		-.561601		.45
.46	-.184184		-.574841		5.94268	2	-.562716		.310302		-.447687		.555404		-.158369		.46
.47	.270965		-.450765		.450175		-.265913		.597475		.011082		.617198		.351127		.47
.48	.575078		-6.12191	2	.578687		.19425		.520034		.466844		.293051		.642972		.48
.49	.550721		.369075		.365523		.543417		.120467		.637636		-.218367		.527888		.49
.5	.207862		.588777		-6.73056	2	.570318		-.36198		.413897		-.597889		7.39348	2	.5
.51	-.259714		.466513		-.465989		.254451		-.634255		-7.12686	2	-.6047		-.432992		.51
.52	-.583753		6.92211	2	-.593517		-.218957		-.527294		-.520848		-.229116		-.668646		.52
.53	-.575226		-.374343		-.370349		-.567278		-.100958		-.655391		.297586		-.477622		.53
.54	-.234289		-.605109		7.61777	2	-.578926		.388948		-.386538		.645004		2.39711	2	.54
.55	.247228		-.484735		.484282		-.241704		.64372		.124732		.590527		.318239		.55
.56	.593627		-7.82701	2	.610917		.246683		.503552		.564216		.162014		.636356		.56
.57	.602908		.380781		.376243		.594173		4.76433	2	.656742		-.37517		.414084		.57
.58	.264187		.624352		-8.63158	2	.588657		-.450016		.338431		-.681769		-.130348		.58
.59	-.233069		.506014		-.50564		.227224		-.684848		-.19947		-.558505		-.599153		.59
.6	-.604871		.08866		-.631464		-.27824		-.507809		-.624827		-7.56707	2	-.686442		.6
.61	-.634547		-.388623		-.383399		-.624815		-1.99157	2	-.669204		.465792		-.32774		.61
.62	-.298538		-.647209		9.80971	2	-.599645		.484632		-.297151		.721017		.252424		.62
.63	.216626		-.531156		.530872		-.210382		.695173		.266373		.52049		.679814		.63
.64	.617695		-.1008		.655956		.31475		.475395		.671059		-1.72111	2	.671144		.64
.65	.671195		.398194		.392085		.660173		-4.86676	2	.658288		-.554158		.221934		.65
.66	.338727		.674669		-.11206		.612041		-.560065		.226165		-.744386		-.382869		.66
.67	-.196999		.561308		-.561126		.190255		-.741884		-.362558		-.436398		-.748071		.67
.68	-.632357		.115283		-.685523		-.357809		-.473207		-.739142		.135847		-.625642		.68
.69	-.714538		-.409953		-.402667		-.701597		9.07064	2	-.659086		.656307		-8.34234	2	.69
.7	-.386773		-.70817		.129003		-.626002		.606869		-.16078		.764358		.329559		.7
.71	.172821		-.598158		.598099		-.165442		.752023		.450067		.377385		.805828		.71
.72	.649172		-.132998		.721834		.409784		.424324		.786329		-.266521		.54958		.72
.73	.766161		.424584		.413673		.751046		-.186047		.623682		-.752743		-8.40519	2	.73
.74	.445765		.749899		-.150172		.641651		-.705719		4.99278	2	-.755486		-.679525		.74
.75	-.141891		.644326		-.644412		.133693		-.805175		-.579607		-.255767		-.830933		.75
.76	-.668502		.155363		-.767476		-.474364		-.406937		-.86007		.433236		-.418052		.76
.77	-.830046		-.443147		-.431882		-.811487		.255863		-.596273		.859316		.298308		.77
.78	-.520712		-.803376		.177635		-.658973		.773453		6.17079	2	.726053		.836748		.78
.79	.10043		-.704125		.704591		-9.11583		.810447		.7		.098918		.815352		.79
.8	.690708		-.184782		.826716		.557681		.321666		.897944		-.619827		.222894		.8
.81	.911582		.467413		.452504		.887691		-.404935		.553365		-.944483		-.551959		.81
.82	.620395		.874726		-.215115		.677502		-.914423		-.252261		-.631504		-.968683		.82
.83	-4.13458	2	.785293		-.785774		3.06485	2	-.866608		-.88407		.138216		-.708289		.83
.84	-.715924		.225742		-.907228		-.670835		-.261376		-.960258		.855112		8.22236	2	.84
.85	-1.0209		-.500634		-.479528		-.987937		.539494		-.394894		1.01116		.859468		.85
.86	-.761994		-.975829		.27014		-.695287		1.0219		.468325		.456608		1.04695		.86
.87	-5.04618	2	-.90347		.904176		6.30072	2	.843083		1.0548		-.462523		.458558		.87
.88	.743089		-.287785		1.02463		.836572		7.03422	2	.932436		-1.10516		-.521828		.88
.89	1.17902		.549742		.516428		1.12822		-.826898		.124727		-.969735		-1.16827		.89
.9	.985171		1.13388		-.360848		.704824		-1.24978		-.836444		-8.03875	2	-.932941		.9
.91	.214093		1.09679		-1.0975		-.228461		-.857711		-1.31473		.96159		9.41961	2	.91
.92	-.755126		.395987		-1.21763		-1.1114		.168902		-.845565		1.35121		1.15485		.92
.93	-1.44089		-.633843		-.569257		-1.34462		1.17886		.321111		.675574		1.3314		.93
.94	-1.41258		-1.43237		.546605		-.670369		1.42529		1.37588		-.672646		.30816		.94
.95	-.594818		-1.49645		1.49476		.604057		.572924		1.42376		-1.92547		-1.17989		.95
.96	.731304		-.648958		1.6256		1.69684		-.965755		.17086		-1.16871		-1.70166		.96
.97	2.04587		.848536		.633489		1.73482		-2.10744		-1.61999		.68914		-.317016		.97
.98	2.8316		2.40281		-1.23943		.249136		-1.61397		-2.26623		2.3357		1.99889		.98
.99	2.80982		3.36551		-3.2744		-2.48923		1.18642		.044333		.902923		1.62662		.99
1	2.05278		3.34065		-4.59996		-5.49696		5.86122		6.14529		-6.37334		-6.48126		1

NOTE: (The exponent p is 0 unless otherwise stated.)

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RELATED CONTRACTS AND PUBLICATIONS

The following contracts relate to the study of antenna radiation problems in aircraft and space vehicles.

AF 19(604)-8386

AF 19(604)-3508

The following related reports have been published on the present contract:

J. H. Harris and J. Pachares, The Computation of Cylindrical Plasma Radiation Problems. Scientific Report No. 1 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-79, AFCRL-65-610, June 1965.

J. H. Harris, Radiation Pattern Characteristics of Line Sources in the Presence of Dielectric Layers. Scientific Report No. 2 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-91, AFCRL-65-666, July 1965.

J. H. Harris, The Nature of Leaky Waves in Multiply Layered Media. Scientific Report No. 3 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-96, AFCRL-65-667, August 1965.

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APPENDIX

A. PROGRAM FOR COMPUTATION OF THE EIGENFUNCTIONS AND THE EIGENVALUES

```
100 READ K,G
110 LET A(151)=-G
120 LET P=3.14159265
130 LET C=10*P
140 PRINT "C="C,"K="INT(K);K-INT(K)
150 LET W=C↑2
160 DIM A (500),Z(300),P(150)
170 LET A(149)=1E-40
180 DEF FNC(N)=-N*(N-1)*W/(2*N-3)/(2*N-1)
190 DEF FNB(N)=N*(N+1)-K+(2*N*(N+1)-1)*W/(2*N-1)/(2*N+3)
200 DEF FNA(N)=(N+2)*(N+1)*W/(2*N+3)/(2*N+5)
210 FOR N=149 TO 2 STEP -2
220 LET A(N-2)=(FNB(N)*A(N)+FNA(N)*A(N+2))/FNC(N)
230 NEXT N
240 LET S=0
250 LET D=0
260 LET R=0
270 LET L=0
280 LET J=-1
290 FOR X=0 TO 1 STEP .005
300 LET F=0
310 LET P(0)=1
320 LET P(1)=X
330 LET P(2)=1.5*X↑2-.5
340 FOR N=2 TO 61
350 LET P(N+1)=(X*(2*N+1)*P(N)-N*P(N-1))/(N+1)
360 NEXT N
370 FOR N=1 TO 61 STEP 2
380 LET F=F+A(N)*P(N)
390 NEXT N
400 LET S=S+F↑2
410 LET D=D+F*SIN(C*.005*X)
420 LET R=R+F*SIN(C*.015*X)
430 LET L=L+F*SIN(C*.025*X)
440 LET J=J+1
450 LET Z(J)=F
460 NEXT X
470 LET Q=.1*SQR(S-.5*(Z(0)↑2+Z(200)↑2))
480 PRINT "NORMALIZING FACTOR="Q
490 PRINT "EIGENVALUE"
500PRINT"U(.005)="(D-.5*Z(200)*SIN(C*.005))/Z(1)/100
510PRINT"U(.015)="(R-.5*Z(200)*SIN(C*.015))/Z(3)/100
520PRINT"U(.025)="(L-.5*Z(200)*SIN(C*.025))/Z(5)/100
530 PRINT "NORMALIZED EIGENFCN"
540 PRINT "X","V(X)"
550 FOR I=0 TO 200 STEP 2
560 PRINT I/200;Z(I)/Q
570 NEXT I
580 GOTO 100
590 DATA 1114.05,.0111241
600 END
```

Remarks: This program is for the odd-index eigenelements. For the even-index cases, statement 370 becomes FOR N=0 to 60 STEP 2, 210 becomes FOR N=150 TO 1 STEP-2, and all SIN become COS.

B. PROGRAM FOR DETERMINATION OF k IN EQUATION (9) TO A 9-PLACE ACCURACY

```
10 DIM G(100),F(100)
30 LET P 3.14159265
40 LET B (10*P)↑2
55 READ L
56 LET M 1
57 FOR A 0 TO 5
60 FOR K L-1/10↑A TO L+1/10↑A STEP 1/10↑(A+1)
70 FOR N 3 TO 71 STEP 2
75 DEF FNR(N) (2*N+3)*(2*N+5)/((N+2)*(N+1)*B)
76 DEF FNS(N) N*(N+1)-K+(2*N*(N+1)-1)*B/(2*N-1)/(2*N+3)
77 DEF FNT(N) N*(N-1)*B/(2*N-3)/(2*N-1)
80 LET F(N) FNR(N)*FNS(N)
90 LET G(N)=FNR(N)*FNT(N)
100 NEXT N
170 LET I=G(65)/(F(65)-G(67)/(F(67)-G(69)/F(69)))
180 LET H=G(59)/(F(59)-G(61)/(F(61)-G(63)/(F(63)-I)))
190 LET J=G(53)/(F(53)-G(55)/(F(55)-G(57)/(F(57)-H)))
200 LET R=G(47)/(F(47)-G(49)/(F(49)-G(51)/(F(51)-J)))
210 LET S=G(41)/(F(41)-G(43)/(F(43)-G(45)/(F(45)-R)))
220 LET T G(35)/(F(35)-G(37)/(F(37)-G(39)/(F(39)-S)))
230 LET U=G(29)/(F(29)-G(31)/(F(31)-G(33)/(F(33)-T)))
240 LET V=G(23)/(F(23)-G(25)/(F(25)-G(27)/(F(27)-U)))
250 LET W=G(17)/(F(17)-G(19)/(F(19)-G(21)/(F(21)-V)))
260 LET X=G(11)/(F(11)-G(13)/(F(13)-G(15)/(F(15)-W)))
270 LET Y=G(5)/(F(5)-G(7)/(F(7)-G(9)/(F(9)-X)))
280 LET Z=G(3)/(F(3)-Y)-1-11.6666667/B+5.83333333*K/B
290 LET Q=ABS(2.5-Z)
300 IF Q>=M THEN 340
310 IF Q<M THEN 320
320 LET M=Q
330 LET D=K
340 NEXT K
350 PRINT M;INT(L);D-INT(L)
355 LET L=D
360 NEXT A
370 PRINT
371 PRINT
380 GO TO 55
3000 DATA 139,322,502,676,846
9999 END
```

END

Remark: This program is for the odd-index cases. For the even-index cases, the arguments of F and G are even numbers and statement 280 becomes LET Z=G(2)/(F(2)-Y)+7.5*K/B.