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EXTENSION TABLES FOR THE U.S. STANDARD ATMOSPHERE, 1962, WITH SPECIAL ATTENTION TO THE CALCULATION OF GEOPOTENTIAL

R. A. Minzner

GCA CORPORATION
Bedford, Massachusetts

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Contract Monitor
Frank A. Marcos
Upper Atmosphere Physics Laboratory

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Prepared for
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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Washington, D.C.

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PREFACE

This report is submitted in fulfillment of items 2 and 3 of Contract AF19(628)-6085 monitored for AFCRL by Frank Marcos. It presents a set of tables consisting of an extension of the U.S. Standard Atmosphere in the altitude region of 90 to 120 km for which region tables had originally been published only as a function of integral multiples of one geometric kilometer. The tables in this report are presented as a function of integral multiples of one standard geopotential kilometer and includes a discussion of the various equations and constants involved.

This report also includes the development of a simplified function for accurately relating geopotential and geometric altitude.

ABSTRACT

The "United States Standard Atmosphere, 1962", was published with two kinds of metric-unit tables for the altitude interval from -5000 to 90,000 meters. One kind of table presented the atmospheric properties as a function of integral multiples of particular numbers of geopotential meters while the second presented the atmospheric properties as a function of integral multiples of similar numbers of geometric meters. For the region above 90,000 meters, altitude only one type of metric table was published. This type presented atmospheric properties in integral multiples of particular numbers of geometric meters. A similar situation prevailed for the English-unit tables. The need for both metric-unit and English-unit tables as a function of integral multiples of specific numbers of geopotential meters for altitudes above 90 kilometers has prompted a new set of calculations, which required the use of equations not specifically presented in the United States Standard Atmosphere, 1962. The development of these equations is discussed and the value of all constants employed are given. The calculations involve a transformation between geopotential and geometric altitude, and the development of an empirical analytical expression relating these quantities is presented. This empirical function yields results which differ by less than 0.1 meter at 700 km altitude, from those computed in an unspecified manner for the United States Standard Atmosphere, 1962.

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SECTION I

PRESSURE AND DENSITY EQUATIONS FOR TEMPERATURE-ALTITUDE PROFILES LINEAR WITH RESPECT TO GEOMETRIC ALTITUDE

Above 90 km, the pressure-altitude profile of the U.S. Standard Atmosphere, 1962(Ref. 1) is defined in terms of a segmented temperature-altitude profile for which each segment is linear in terms of geometric altitude, i.e.,

$$T_M = T_{M_r} + L(Z-Z_r) \tag{1}$$

where

T_M is the molecular scale temperature at altitude Z

T_{M_r} is the reference molecular scale temperature at reference altitude Z_r

Z_r is the base of any layer characterized by a single constant value of L

L is the gradient of T_M with respect to Z, i.e., dt_M/dZ.

This condition is in contrast to that below 90 km where the segmented temperature-altitude profile has segments which are linear in terms of geopotential i.e.,

$$T_M = T_{M_r} + L' (H-H_r) \tag{2}$$

where

T_M is the molecular scale temperature at geopotential H

T_{M_r} is the molecular scale temperature at geopotential H_r

H_r is the base of any layer characterized by a single constant value of L

L' is the gradient of T_M with respect to H, i.e., dT_M/dH.

Equation I.2.10-(3) of the document of the U.S. Standard Atmosphere, 1962 is suitable for calculating pressures associated with a segmented temperature-altitude profile having segments which are linear with respect to geopotential, but is not suitable for calculating Standard-Atmosphere pressures above 90 km where the temperature-altitude profile is defined to have segments which are linear with respect to geometric altitude. For the region above 90 km the Standard-Atmosphere document does not contain a detailed equation for calculating

pressure or density but presents only a general integral form, not suitable for direct numerical evaluation. The equation actually used for calculating the standard-atmosphere pressures for integral geometric altitudes was probably a corrected form of one of a pair of equations published with some format errors by Champion and Minzner in 1963(Ref. 2). A redevelopment of those pressure-altitude equations and a related density-altitude equation show the correct forms to be as follows:

$$\frac{-RL}{M_o} \ln \frac{p}{p_r} = \left[\ln \frac{T_{M_r} + L(z - z_r)}{T_{M_r}} \right] \left[\sum_{i=0}^{6} (-1)^i g_i a^i \right]$$

$$+ \sum_{j=0}^{5} \left[\frac{z^{j+1} - z_r^{\,j+1}}{j+1} \right] \left[\sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right] \tag{3}$$

$$p = p_r \left[\frac{T_{M_r} + L(z - z_r)}{T_{M_r}} \right]^{-\left[\sum_{i=0}^{6} (-1)^i g_i a^i \right] \frac{M_o}{RL}}$$

$$\exp \left[\frac{-M_o}{RL} \sum_{j=0}^{5} \left(\frac{z^{j+1} - z_r^{\,j+1}}{j+1} \right) \left(\sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right) \right] \quad ($$

$$\rho = \rho_r \left[\frac{T_{M_r} + L(z - z_r)}{T_{M_r}} \right]^{-1-\left[\sum_{i=0}^{6} (-1)^i g_i a^i \right] \frac{M_o}{RL}}$$

$$\exp \left[\frac{-M_o}{RL} \sum_{j=0}^{5} \left(\frac{z^{j+1} - z_r^{\,j+1}}{j+1} \right) \left(\sum_{i=j+1}^{6} (-1)^{i+j+1} g_i a^{i-j-1} \right) \right] \tag{5}$$

where

R is the universal gas constant

M_o is the sea-level value of the molecular weight

p is the pressure at altitude Z

p_r is the pressure at reference altitude Z_r

ρ is the density at altitude Z

ρ_r is the density at reference altitude A_r

g_i is the coefficient of the ith term of Lambert's gravitational formula given below

$$g(Z) = \sum_{i=0}^{6} g_i Z^i$$

$$= 9.8066500 - 3.0854195 \times 10^{-6} Z + 7.2539455 \times 10^{-13} Z^2$$
$$- 1.5167771 \times 10^{-19} Z^3 + 2.9724620 \times 10^{-26} Z^4$$
$$- 5.5905936 \times 10^{-33} Z^5 + 1.0219762 \times 10^{-39} Z^6 \qquad (6)$$

At one point in the derivation of the above equations it was necessary to use the following interesting transformation in order to obtain a form suitable for integration with respect to Z:

$$\frac{K + AZ + BA^2 + CZ^3 + DZ^4 + EZ^5 + FZ^6}{a+Z} = \qquad (7)$$

$$\frac{1}{a+Z} [K - Aa + Ba^2 - Ca^3 + Da^4 - Ea^5 + Fa^6] +$$
$$Z [A - Ba + Ca^2 - Da^3 + Ea^4 - Fa^5] +$$
$$Z^2 [B - Ca + Da^2 - Ea^3 + Fa^4] +$$
$$Z^3 [C - Da + Ea^2 - Fa^3] +$$
$$Z^4 [D - Ea + Fa^2] +$$
$$Z^5 [E - Fa] +$$
$$Z^6 [F]$$

The pressure equation actually programmed in the digital machine calculation for tables presented in this report was Equation (4) of this document. The density was then computed from the gas law

$$\rho = \frac{P\,M_o}{T_M\,R} \tag{8}$$

using the values of millibar pressure computed from Equation (4) (multiplied by 100 to transform millibars to newtons per m^2) and the values of temperature computed from Equation (1).

Constants and Boundary Conditions

The constants used in the calculation of the thermodynamic properties are those defined in the standard atmosphere:

$$R = 8.31432 \times 10^3 \text{ Joules } (^\circ K)^{-1} \text{ (kilomole)}^{-1}, \text{ and}$$

$$M_o = 28.9646 \text{ kg (kilomole)}^{-1},$$

such that

$$\frac{M_o}{R} = 3.483676 \times 10^3\ ^\circ K \text{ sec}^2\ m^{-2}.$$

The values of T_{M_r} and p_r in the program are redefined for the base of each layer in accordance with the values tabulated in the Standard-Atmosphere publication.

Geometric Altitude Meters	Temperature Degrees K	Temperature Gradient deg K/m	Pressure Millibars
90,000	180.65	0.003	1.6438×10^{-3}
100,000	210.65	0.005	3.0075×10^{-4}
110,000	260.65	0.010	7.3544×10^{-5}
120,000	360.65	0.020	2.5217×10^{-5}

The coefficients of Lambert's equation for the acceleration of gravity are given in Equation (6).

The thermodynamic properties calculated in English units follow the defined conversions:

1 foot = 0.3048 meters,

1 pound = 0.45359237 kilogram,

1 Celcius degree = 1.8 Rankin degrees

a temperature of 0 degree Celsius = a temperature of 273.15°K.

From the above definitions the following derived relationships apply:

1 meter = 3.2808399 feet,

1 kg m^{-3} = 6.242797 x 10^{-2} lbs ft^{-3}.

SECTION II

GEOPOTENTIAL TO GEOMETRIC ALTITUDE CONVERSION

The above-defined constants are sufficient to compute the Standard-Atmosphere values of pressure temperature and density as a function of geometric altitude between 90 and 150 geometric kilometers or between 295469.47 and 492125.98 geometric feet. The computation of these thermodynamic properties as a function of geopotential altitudes in a manner compatible with the relationships of the U.S. Standard-Atmosphere is a far more complicated problem, since the expressions used in this transformation in the standard atmosphere were never published in the Standard-Atmosphere document. In the absence of these expressions, a simple empirical correction term has been developed for use with the previously used simple expression for converting geopotential to geometric altitude. Thus, while the expression used by Minzner and Ripley in 1956 (Ref. 3) is

$$Z = \frac{r\,H}{r - H} \cdot \frac{g_o}{G} \tag{9}$$

the revised expression is

$$Z = \frac{r\,[H + F(H)]}{r - [H + F(H)]} \cdot \frac{g_o}{G} \tag{10}$$

where $F(H) = AA + ABH + ACH^2 + ADH^2 + ADH^3 + AEH^4$ (11)

where $AA = 0.0000$

$AB = -0.2161710$

$AC = +0.1807561$

$AD = +0.9153012$

$AE = +0.2006785$

and where

g_o/G is numerically but not dimensionally unity.

It is estimated that the differences between the values of Z for a given value of H as computed by Equation (11) and the values of Z for a given value of H as computed by the methods used in the Standard-Atmosphere do not exceed 0.2 meter over the entire altitude region of 0 to 700 kilometers. The development of Equations (10) and (11) and expressions satisfying the inverse of the relationship of Equation (10) is given in Appendix A.

SECTION III

CALCULATION OF THE TABLES

Equations (10) and (11) plus the previously discussed expressions, Equations (1), (4), (6), and (8) permit the computation of the desired extensions to the Standard-Atmosphere Tables, presented as Table 1 in metric units and as Table 2 in English units. Successive integral multiples of 1000 geopotential meters ranging from 90,000 to 120,000 are arbitrarily selected as values of H for the metric tables. These values are converted to the related values of Z through Equations (10) and (11). The resulting values of Z when introduced into Equation (4), into which the coefficients of Equation (6) have already been introduced, yield the values of pressure at altitude Z. The temperatures which are implicitly calculated in Equation (4) are independently calculated as a function of Z using Equation (1). The pressures and temperatures at a given value of Z then yield densities for that altitude through Equation (8).

In the case of the table presented in English units successive integral multiples of 5000 geopotential feet are selected, in the range 295,000 to 390,000, and are converted to geopotential meters. The procedure then follows that used for the metric tables except that the temperature, temperature gradient, and density are converted to the appropriate value in English units. The pressure is retained in millibars which is a metric-related unit. In addition, temperature is also calculated in degrees Celsius.

The above procedures have been specified in a fortran program presented in Appendix B.

TABLE 1

GEOPT ALTITUDE GEOPOTEN METERS	GEOMETRIC ALTITUDE GEOMETRIC METERS	TEMP GRAD DEG K PER M	MOLEC SCALE TEMP DEG K	PRESSURE MB	DENSITY KG/CU M
88743.3	90000.00	.003	180.65	.16438E-02	.3170E-05
90000.00	91292.75	.003	184.53	.12993E-02	.2453E-05
91000.00	92321.85	.003	187.62	.10814E-02	.2007E-05
92000.00	93351.28	.003	190.70	.90272E-03	.1649E-05
93000.00	94381.03	.003	193.79	.75574E-03	.1358E-05
94000.00	95411.12	.003	196.88	.63448E-03	.1122E-05
95000.00	96441.53	.003	199.97	.53413E-03	.9304E-06
97000.00	98503.35	.003	206.16	.38151E-03	.6446E-06
96000.00	97472.28	.003	203.07	.45083E-03	.7734E-06
98000.00	99534.75	.003	209.25	.32364E-03	.5388E-06
99000.00	100566.49	.005	213.48	.27529E-03	.4492E-06
100000.00	101598.56	.005	218.64	.23502E-03	.3744E-06
101000.00	102630.95	.005	223.80	.20139E-03	.3134E-06
102000.00	103663.68	.005	228.97	.17318E-03	.2634E-06
103000.00	104696.72	.005	234.13	.14942E-03	.2223E-06
104000.00	105730.11	.005	239.30	.12934E-03	.1882E-06
105000.00	106763.83	.005	244.47	.11230E-03	.1600E-06
106000.00	107797.87	.005	249.64	.97802E-04	.1364E-06
107000.00	108832.25	.005	254.81	.85413E-04	.1167E-06
108000.00	109866.96	.005	259.98	.74796E-04	.1002E-06
109000.00	110901.99	.010	269.67	.65732E-04	.8491E-07
110000.00	111937.36	.010	280.02	.58048E-04	.7221E-07
111000.00	112973.06	.010	290.38	.51494E-04	.6177E-07
112000.00	114009.10	.010	300.74	.45873E-04	.5313E-07
113000.00	115045.46	.010	311.10	.41025E-04	.4593E-07
114000.00	116082.16	.010	321.47	.36824E-04	.3990E-07
115000.00	117119.19	.010	331.84	.33167E-04	.3481E-07
116000.00	118156.56	.010	342.22	.29970E-04	.3050E-07
117000.00	119194.25	.010	352.59	.27163E-04	.2683E-07
118000.00	120232.27	.020	365.30	.24691E-04	.2394E-07
119000.00	121270.63	.020	386.06	.22544E-04	.2034E-07
120000.00	122309.33	.020	406.84	.20682E-04	.1771E-07

TABLE 2

GEOPT ALTITUDE	GEOMETRIC ALTITUDE	TEMP GRAD	MOLEC SCALE TEMP	MOLEC SCALE TEMP	PRESSURE	DENSITY
GEOPOTEN FEET	GEOMETRIC FEET	DEG R PER FT	DEG R	DEG C	MB	LB/CU FT
291152.56	295275.59	.0016459	325.17	-92.50	.16438E-02	.1979E-06
295000.00	299233.35	.0016459	331.68	-88.87	.13197E-02	.1597E-06
300000.00	304379.14	.0016459	340.15	-84.17	.99843E-03	.1209E-06
305000.00	309527.43	.0016459	348.63	-79.46	.76056E-03	.9240E-07
310000.00	314678.23	.0016459	357.11	-74.75	.58315E-03	.7092E-07
315000.00	319831.54	.0016459	365.59	-70.04	.44992E-03	.5817E-07
320000.00	324987.36	.0016459	374.07	-65.32	.34920E-03	.4634E-07
325000.00	330145.68	.0027432	384.83	-59.35	.27266E-03	.2773E-07
330000.00	335306.54	.0027432	398.98	-51.48	.21466E-03	.2186E-07
335000.00	340469.89	.0027432	413.15	-43.61	.17041E-03	.1614E-07
340000.00	345635.77	.0027432	427.32	-35.74	.13635E-03	.1249E-07
345000.00	350804.18	.0027432	441.50	-27.86	.10988E-03	.9743E-08
350000.00	355975.08	.0027432	455.68	-19.98	.89169E-04	.7660E-08
355000.00	361148.53	.0154864	470.58	-11.71	.72821E-04	.6057E-08
360000.00	366324.49	.0154864	498.97	4.06	.60017E-04	.4788E-08
365000.00	371502.96	.0154864	527.38	19.84	.49997E-04	.3711E-08
370000.00	376684.00	.0154864	555.81	35.63	.42051E-04	.2961E-08
375000.00	381867.53	.0154864	584.25	51.43	.35674E-04	.2398E-08
380000.00	387053.65	.0154864	612.70	67.24	.30502E-04	.1948E-08
385000.00	392242.27	.0154864	641.17	83.05	.26267E-04	.1603E-08
390000.00	397433.41	.0109728	690.13	110.25	.22802E-04	.1293E-08

295275.59 GEOMETRIC FEET = 90000.00 GEOMETRIC METERS

APPENDIX A

DEVELOPMENT OF AN EMPIRICAL FUNCTION RELATING THE NUMERICAL VALUES OF GEOPOTENTIAL AND GEOMETRIC ALTITUDE AS PUBLISHED IN THE UNITED STATES STANDARD ATMOSPHERE

A-1 COMPARISON OF DIFFERENT SETS OF CALCULATIONS OF GEOPOTENTIAL

Geopotential in the 1956 ARDC Model Atmosphere (Ref. A-1) was calculated by the expression

$$H_{56} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} \tag{A-1}$$

where r = 6,356,766 meters, and where H_{56} is used to differentiate the results of this particular calculation of geopotential from the values tabulated in U.S. Standard Atmosphere, for which values the designation is H_{62}.

In the 1962 U.S. Standard Atmosphere (Ref. A-2), geopotential H_{62} at any altitude Z was computed by the integration of a complicated gravity-acceleration expression along a path identical to the curved line of gravitational force passing through the point in question (where the point in question was defined relative to geometric latitude and relative to the distance from the center of an ellipsoid rather than relative to sea level) such that for each successive altitude, the integration must be performed along a different line of force. No specific equation suitable for direct numerical evaluation of H_{62} as a function of Z was given in the standard-atmosphere document, nor has one been developed by the writer from the fundamental considerations which were given. Instead, a simple approximation formula in the nature of Equation (A-1) with a correction term was developed; i.e.,

$$H_{62/8} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} - f(Z) \tag{A-2}$$

where $H_{62/8}$ represents the eight-significant-figure values which when rounded lead to $H_{62/R}$, the values of geopotential published in the Standard Atmosphere.

A comparison of the tabulated six-significant-figure values (above 100,000 m) of H_{62} with $H_{56/8}$ the eight-significant-figure values of H_{56} as obtained from Equation (A-1) suggest the following observations:

(1) The tabulated values of H_{62} are always less than the corresponding value of $H_{56/8}$ for altitudes greater than sea level.

(2) The tabulated values of H_{62} are rounded from original seven- or eight-significant-figure values. (These rounded values will henceforth be designated as $H_{62/R}$).

(3) The values of $H_{62/8}$ which have been rounded to $H_{62/R}$ would most likely differ from $H_{56/8}$ in accordance with f(Z), some smooth monotonically increasing function of Z, which function has a value of zero at sea

level and a value of about 33 geopotential meters (m′) at the geometric altitude Z of 700 km; i.e.,

$$f(Z) = H_{56/8} - H_{62/8} \qquad \text{(A-3)}$$

Thus, a curve fit to the difference between $H_{56/8}$ and $H_{62/8}$ (if these were available) would provide the desired function f(Z), and the desired approximation expression for calculating $H_{62/8}$ as in Equation (A-2) would have been determined. Unfortunately, values of $H_{62/8}$ are not available, and an indirect approach must be pursued.

A-2 AN APPROACH FOR GENERATING f(Z)

An imaginary set of eight-significant-figure values of H_{62} is hypothesized. If the hypothetical numerical values of $H_{62/8}$ were compared with Z in the region of sea level and immediately above, one would find that at Z = 0, $H_{62/8}$ = 0, and as Z increases above zero, $H_{62/8}$ would also increase but at a lesser rate than Z, so that at $Z = Z_{0.4999}$, which is the symbol for a specific altitude located somewhere between Z = 1750 and Z = 1800 meters, the value of $H_{62/8}$ would lag behind that of Z by exactly 0.4999 of a meter. At $Z = Z_{0.5000}$ which is the symbol for an altitude immediately above $Z_{0.4999}$, the value $(Z-H_{62/8})$ would become 0.5000. Between Z = 0 and Z = 4000 m, the value of $(Z-H_{62/8})$ would increase smoothly as Z increases in accordance with the values presented in Table A-1.

The exact numerical values of the symbolic altitudes $Z_{0.4999}$, $Z_{0.5000}$, $Z_{1.4999}$, etc. are not known, but from the U.S. Standard Atmosphere 1962 we may infer that these values are bounded within particular limits indicated in Table A-1. Thus $Z_{0.4999}$ and $Z_{0.5000}$ have values between 1750 and 1800 meters, while $Z_{1.4999}$ and $Z_{1.5000}$ would be found between the altitudes 3050 and 3100 meters, etc.

Returning momentarily to the reality of the U.S. Standard Atmosphere 1962, we can examine the difference between the tabulated integral values of geometric altitude Z_i and the rounded values of geopotential altitude $H_{62/R}$ where Z_i is increased discontinuously in integral steps of one meter as Z increases, with successive discontinuities occurring between $Z_{0.4999}$ and $Z_{0.5000}$, and again between $Z_{1.4999}$ and $Z_{1.5000}$, etc. as indicated in Table A-1. The differences $(Z-H_{62/8})$ are also tabulated.

Two differences given in Table A-1 are shown in Figure A-1 where the hypothetical quantity $(Z - H_{62/8})$ is shown as the solid-line, smooth-curve function, and the realistic quantity $(Z_i - H_{62/R})$ is shown as the discontinuous function represented as a series of alternate horizontal and vertical line segments, where these line segments connect the series of discrete points derived from the finite number of tabulated values. A graph of the difference $(Z - H_{56/8})$ is also shown as a smooth, dashed-line curve in Figure A-1 where $H_{56/8}$ represents the eight-significant-figure values obtained from Equation A-1.

TABLE A-1

DIFFERENCES $(A - H_{62/8})$ and $(Z_i - H_{62/R})$ AS A FUNCTION OF ALTITUDE

Numerical Altitude (meters)	Symbolic Altitude	Difference $(Z - H_{62/8})$ (meters)	Difference $(Z_i - H_{62/R})$ (meters)
0	Z_0	0.0000	0
1750			0
	$Z_{0.4999}$	0.4999	0
	$Z_{0.5000}$	0.5000	1
1800			1
3050			1
	$Z_{1.4999}$	1.4999	1
	$Z_{1.5000}$	1.5000	2
3100			2
3950			2
	$Z_{2.4999}$	2.4999	2
	$Z_{2.5000}$	2.5000	3
4000			3

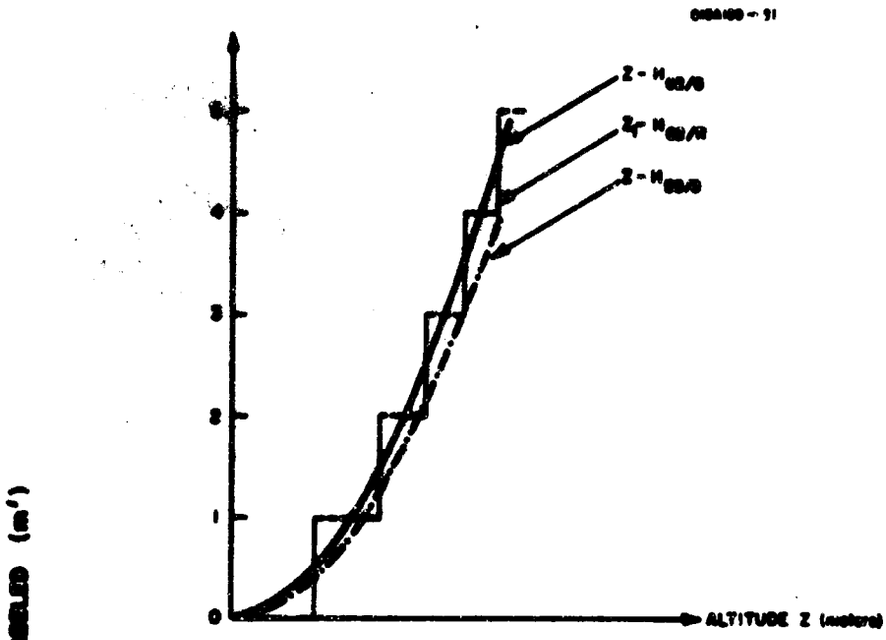

Figure A-1 Differences $Z - H_{62/8}$, $Z_1 - H_{62/R}$ and $Z - H_{56/8}$ vs altitude.

Figure A-2 Difference $H_{62/8} - H_{62/R}$ vs altitude.

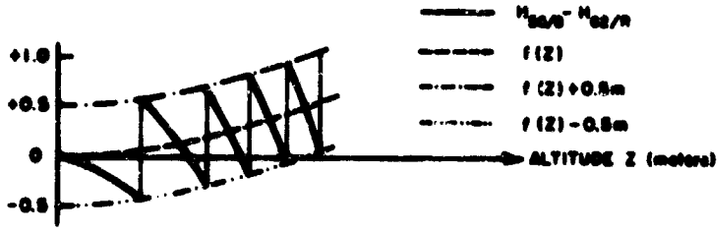

Figure A-3 Difference $H_{56/8} - H_{62/R}$, as well as f(Z), f(Z) + 0.5 m, and f(Z) - 0.5 m all as a function of altitude.

Since $Z - H_{56/8}$ is always less than $Z - H_{62/8}$, it is apparent that $H_{56/8}$ departs from Z less rapidly than does $H_{62/8}$, and the difference $(H_{56/8} - H_{62/8}) = f(Z)$ increases from zero as Z increases from zero.

If the graphs of Figure A-1 were extended to include the region from 0 to 90 kilometers altitude, the graph of $(Z_i - H_{62/R})$ would show 1257 vertical line segments representing the same number of abrupt discontinuities while, if extended to include the region from 90 to 700 kilometers altitude, the graph would include an additional 68, 216 such discontinuities. In these same two altitude intervals a graph (not presented in Figure A-1) showing the difference between Z and rounded values of H_{56}, i.e., $(Z_i - H_{56/R})$ would show one less and 33 less discontinuities, respectively, than would be seen in the extended graph of $Z_i - H_{62/R}$.

If the hypothetical graph $(Z - H_{62/8})$ and the realistic graph $(Z_i - H_{62/R})$ are compared, we find that at the particular points for which Z is an integral multiple of one meter, $Z - Z_i = 0$, and the difference $H_{62/8} - H_{62/R})$ varies between +0.5 and about -0.5 meter in accordance with the graph of Figure A-2. A careful examination shows that the limiting differences at the points of discontinuity are separated by values of 0.999 m', 0.999 m', or 0.99 m', consistent with the fixed eight significant figures in $H_{62/8}$ and the varying number in $H_{62/8}$ depending upon the altitude region. It is also apparent that these discontinuities are symmetrical about the horizontal axis.

A comparison of the discontinuous graph $(Z_i - H_{62/R})$ and the continuous graph $(Z - H_{56/8})$ at those points for which Z has an integral value yields a difference $(H_{56/8} - H_{62/R})$ which follows the pattern of Figure A-3. In this figure, the discontinuities occur at the same altitudes as for Figure A-2. The limiting differences at the points of discontinuity are the same as for the corresponding discontinuity of Figure A-2. Contrary to the situation in Figure A-2, these limiting points are not symmetrical about the horizontal axis but are symmetrical about the curved-line function f(Z) which is the function being sought for use in Equation (A-2).

Unfortunately, the number of values of $H_{62/R}$ in the U.S. Standard Atmosphere is not sufficient so that a graph of these points would show the detailed type of pattern of Figure A-3: there are only 420 values of $H_{62/R}$ for altitudes between 90 to 700 km compared with 68216 regions of discontinuity for that same range of altitudes. Consequently, any single value $(H_{56/8} - H_{62/R})$ from available data represents but a single point on a section of graph which, on the average, might have 170 regions of discontinuity. Obviously the detailed step-function graph cannot be produced from the available data. On a graph in which the altitude scale has been sufficiently compressed, however, a plot of the available 420 data points (above 90 km) appears as a band of randomly scattered points in which the extreme ordinate values for any single value of abscissa may never exceed a difference corresponding to one geopotential meter, as in Figure A-3. Hopefully, a sufficient number of data points lie near enough to the two extremes to permit the establishment of a good approximation to the locus of the upper and lower boundary of the 1-m' band.

The smoothed lower bound of the envelope of the set of scattered data ($H_{56/8} - H_{62/R}$) represents a subset of values which are never more than 0.4999 m′ below the desired function ($H_{56/8} - H_{62/8}$) = f(Z). Similarly, the smoothed upper bound of the envelope of the scattered data represents a set of values which are never more than 0.5 m′ greater than the desired f (Z). Thus, if 0.4999 m′, 0.499 m′, or 0.49 m′ is added to the individual values of the difference set ($H_{56/8} - H_{62/R}$), each in the appropriate altitude region, a set of data called Source Set 1 is formed. The smoothed lower bound of the values of Source Set 1 consist of a smaller set of data called Subset 1. The points of Subset 1 will have values equal to or very slightly greater than the values of the desired function f(Z) at the corresponding altitudes.

Similarly if 0.5000 m′ is subtracted from the individual values of the difference set ($H_{56/8} - H_{62/R}$), to form Source Set 2, the smoothed upper bound of these downwardly adjusted data form a set of points called Subset 2. These data points of Subset 2 will have values equal to or slightly less than the values of the desired function f(Z).

The points of Subset 1 and Subset 2 are then subjected to a further graphical selection to form Smoothed Subsets 1 and 2. The points of these Smoothed Subsets 1 and 2 are then combined, and when processed in a curve-fitting program determine a close approximation to the desired function f(Z). The numerical and graphical processes employed depend upon the assumption that the function f(Z) as well as its first derivative are monotonically increasing with Z, a situation which is apparently true for the upper and lower bounds of the band of data points.

The detailed steps of the process are as follows:

(1) Prepare Source Set 1 by performing the following operations as appropriate:

add 0.4999 to ($H_{56/8} - H_{62/R}$) for 1.0000 km $\leq Z \leq$ 9.9999 km,

add 0.499 to ($H_{56/8} - H_{62/R}$) for 10.000 km $\leq Z \leq$ 99.999 km,

add 0.49 to ($H_{56/8} - H_{62/R}$) for 100.00 km $\leq Z \leq$ 999.99 km,

(2) Prepare Subset 1 as follows:

(a) Scan the entire Source Set 1 for the smallest positive number (all of the numbers of this set will be positive), and store this value with its corresponding value of Z as the first entry of Subset 1.

(b) Remove this value and those for lower altitudes from Source Set 1 and discard.

(c) Scan the remaining numbers of Source Set 1 for the lowest value and store this value with its corresponding value of Z as the 2nd entry of Subset 1.

(d) Remove this member along with those associated with lower altitudes from Source Set 1. Repeat steps (c) and (d) until all the values of Source Set 1 have been removed, and Subset 1 has been developed. If there are two or more lowest values at any of the above steps store only that one corresponding to the greatest altitude, and reject the others.

(3) Punch and print the stored values of Subset 1. This is presented as column 5 DIF+INC, and column 1 in Table A-2 which consists of an ordered listing of the appropriate cards extracted from Source Set 1.

(4) Prepare Source Set 2 by performing the following operation for all altitudes of interest:

subtract 0.5000 from $(H_{56/8} - H_{62/R})$

(5) Prepare Subset 2 as follows:

(a) Scan the entire Source Set 2 for negative values, which will be found at the low-altitude end of the set, removing and discarding these members of the set.

(b) Scan the remainder of Source Set 2 for the largest positive value and store this value along with its associated altitude value as the first member of Subset 2.

(c) Remove this member from Source Set 2 along with all members associated with greater altitudes.

(d) Repeat steps (b) and (c) until all members of Source Set 2 have been removed and Subset 2 has been developed. If there are two or more greatest values at any point in the scanning operation, store only that one associated with the lowest altitude and discard the others.

(6) Punch and print the stored values of Subset 2. This is presented as column 6 DIF-0.5, and column 1 in Table A-3 which consists of an ordered listing of the appropriate cards extracted from Source Set 2.

(7) Plot the data points of Subset 1 on large-scale graphs (not shown in this report) for further data selection. The suggested scales for these graphs are indicated in Table A-4.

(8) Select certain points of Subset 1 which appear to form a smooth monotonically increasing lower bound to the total of Subset 1 data. These selected points comprise Smooth Subset 1 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies below all those points in Subset 1 having altitude values within the altitude interval encompassed by the particular line segment.

(b) The successive line segments have slopes which are monotonically increasing for increasing altitudes.

TABLE A-2

VALUES OF SUBSET 1 AS A FUNCTION OF GEOMETRIC ALTITUDE
WITH THE RELATED VALUES OF H62/R, H56/8;
AND THE DIFFERENCES H56/8 - H62/R

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
31[illegible].	.30850000E 05	.30849557E 05	-.4430	.0560	-.9430
58[illegible].	.57476000E 05	.57475585E 05	-.4150	.0840	-.9150
82[illegible].	.80956000E 05	.80955700E 05	-.3000	.1990	-.8000
96[illegible].	.94572000E 05	.94571776E 05	-.2240	.2750	-.7240
105[illegible].	.10329400E 06	.10329381E 06	-.1900	.3000	-.6900
111[illegible].	.10909500E 06	.10909501E 06	.0100	.5000	-.4900
140[illegible].	.13698300E 06	.13698311E 06	.1100	.6000	-.3900
144[illegible].	.14081000E 06	.14081022E 06	.2200	.7100	-.2800
145[illegible].	.14176600E 06	.14176626E 06	.2600	.7500	-.2400
152[illegible].	.14845000E 06	.14845032E 06	.3200	.8100	-.1800
156[illegible].	.15226300E 06	.15226333E 06	.3300	.8200	-.1700
166[illegible].	.16177500E 06	.16177542E 06	.4200	.9100	-.0800
173[illegible].	.16841600E 06	.16841652E 06	.5200	1.0100	.0200
179[illegible].	.17409700E 06	.17409759E 06	.5900	1.08[illegible]	.0900
190[illegible].	.18448500E 06	.18448582E 06	.8200	1.3100	.3200
192[illegible].	.18637000E 06	.18637085E 06	.8500	1.3400	.3500
193[illegible].	.18731200E 06	.18731292E 06	.9200	1.4100	.4200
209[illegible].	.20234600E 06	.20234716E 06	1.1600	1.6500	.6600
213[illegible].	.20609300E 06	.20609428E 06	1.2800	1.7700	.7800
224[illegible].	.21637400E 06	.21637536E 06	1.3600	1.8500	.8600
228[illegible].	.22010400E 06	.22010541E 06	1.4100	1.9000	.9100
233[illegible].	.22476000E 06	.22476162E 06	1.6200	2.1100	1.1200
239[illegible].	.23033800E 06	.23033975E 06	1.7500	2.2400	1.2500
243[illegible].	.23405100E 06	.23405286E 06	1.8600	2.3500	1.3600
252[illegible].	.24238900E 06	.24239094E 06	1.9400	2.4300	1.4400
253[illegible].	.24331400E 06	.24331600E 06	2.0000	2.4900	1.5000
255[illegible].	.24516300E 06	.24516525E 06	2.2500	2.7400	1.7500
261[illegible].	.25070400E 06	.25070634E 06	2.3400	2.8300	1.8400
267[illegible].	.25623500E 06	.25623739E 06	2.3900	2.8800	1.8900
275[illegible].	.26359400E 06	.26359656E 06	2.5600	3.0500	2.0600
278[illegible].	.26634900E 06	.26635165E 06	2.6500	3.1400	2.1500
283[illegible].	.27093500E 06	.27093798E 06	2.9800	3.4700	2.4800
287[illegible].	.27459900E 06	.27460216E 06	3.0600	3.5500	2.5600
291[illegible].	.27734400E 06	.27734722E 06	3.2200	3.7100	2.7200
298[illegible].	.28465200E 06	.28465558E 06	3.5800	4.0700	3.0800
299[illegible].	.28556400E 06	.28556788E 06	3.8800	4.3700	3.3800
30[illegible].	.28647600E 06	.28647992E 06	3.9200	4.4100	3.4200
312[illegible].	.29739900E 06	.29740300E 06	4.0000	4.4900	3.5000
320[illegible].	.30465900E 06	.30466323E 06	4.2300	4.7200	3.7300
322[illegible].	.30647100E 06	.30647554E 06	4.5800	5.0700	4.0800
336[illegible].	.31912700E 06	.31913164E 06	4.6400	5.1300	4.1400
344[illegible].	.32633500E 06	.32633993E 06	4.9300	5.4200	4.4300

TABLE A-2 (Continued)

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/S GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
346[illegible]	.328134[illegible]E 06	.32813931E 06	5.3100	5.8000	4.8100
348[illegible]	.329932[illegible]E 06	.32993763E 06	5.6300	6.1200	5.1300
362[illegible]	.342490[illegible]E 06	.34249582E 06	5.8200	6.3100	5.3200
368[illegible]	.347856[illegible]E 06	.34786190E 06	5.9000	6.3900	5.4000
380[illegible]	.358559[illegible]E 06	.35856539E 06	6.3900	6.8800	5.8900
382[illegible]	.360339[illegible]E 06	.36034559E 06	6.5900	7.0800	6.0900
384[illegible]	.362118[illegible]E 06	.36212473E 06	6.7300	7.2200	6.2300
386[illegible]	.363896[illegible]E 06	.36390284E 06	6.8400	7.3300	6.3400
3[illegible]	.367449[illegible]E 06	.36745586E 06	6.8600	7.3500	6.3600
398[illegible]	.374542[illegible]E 06	.37454930E 06	7.3000	7.7900	6.8000
4[illegible]	.378[illegible]82[illegible]E 06	.37808971E 06	7.7100	8.2000	7.2100
406[illegible]	.381618[illegible]E 06	.38162595E 06	7.9500	8.4400	7.4500
410[illegible]	.385150[illegible]E 06	.38515801E 06	8.0100	8.5000	7.5100
416[illegible]	.390443[illegible]E 06	.39044826E 06	8.2600	8.7500	7.7600
418[illegible]	.392201[illegible]E 06	.39220959E 06	8.590[illegible]	9.0800	8.0900
420[illegible]	.393961[illegible]E 06	.39396988E 06	8.8800	9.3700	8.3800
422[illegible]	.395720[illegible]E 06	.39572915E 06	9.1500	9.6400	8.6500
424[illegible]	.397478[illegible]E 06	.39748736E 06	9.3600	9.8500	8.8600
426[illegible]	.399235[illegible]E 06	.39924454E 06	9.5400	10.0300	9.0400
428[illegible]	.4[illegible]0991[illegible]E 06	.40100068E 06	9.6800	10.1700	9.1800
442[illegible]	.413255[illegible]E 06	.41326479E 06	9.7900	10.2800	9.2900
450[illegible]	.420240[illegible]E 06	.42025019E 06	10.1900	10.6800	9.6900
454[illegible]	.423726[illegible]E 06	.42373674E 06	10.7400	11.23[illegible]0	10.2400
460[illegible]	.428948[illegible]E 06	.42895889E 06	10.8900	11.3800	10.3900
464[illegible]	.432424[illegible]E 06	.43243521E 06	11.2100	11.7000	10.7100
468[illegible]	.435896[illegible]E 06	.43590747E 06	11.4700	11.9600	10.9700
472[illegible]	.439364[illegible]E 06	.43937566E 06	11.6600	12.1500	11.1600
476[illegible]	.442828[illegible]E 06	.44283976E 06	11.7800	12.2700	11.2800
479[illegible]	.444558[illegible]E 06	.44457032E 06	12.3200	12.8100	11.8200
482[illegible]	.448[illegible]16[illegible]E 06	.44802837E 06	12.3700	12.8600	11.8700
488[illegible]	.453195[illegible]E 06	.45320787E 06	12.8700	13.3600	12.3700
494[illegible]	.458365[illegible]E 06	.45837829E 06	13.2900	13.7800	12.7900
5[illegible]	.463526[illegible]E 06	.46353966E 06	13.6600	14.1500	13.1600
506[illegible]	.468678[illegible]E 06	.46869201E 06	14.0100	14.5000	13.5100
512[illegible]	.473821[illegible]E 06	.47383536E 06	14.3600	14.8500	13.8600
514[illegible]	.475533[illegible]E 06	.47554781E 06	14.8100	15.3000	14.3100
52[illegible]	.480664[illegible]E 06	.48067919E 06	15.1900	15.6800	14.6900
526[illegible]	.485756[illegible]E 06	.48580162E 06	15.6200	16.1100	15.1200
528[illegible]	.487491[illegible]E 06	.48750711E 06	16.1100	16.6000	15.6100
532[illegible]	.490899[illegible]E 06	.49091514E 06	16.1400	16.6300	15.6400
536[illegible]	.494303[illegible]E 06	.49431920E 06	16.2000	16.6900	15.7000
538[illegible]	.496[illegible]03[illegible]E 06	.49601975E 06	16.7500	17.2400	16.2500
542[illegible]	.499[illegible]1[illegible]E 06	.49941790E 06	16.[illegible]000	17.3900	16.4000
546[illegible]	.5[illegible]27950[illegible]E 06	.50281[illegible]9E 06	17.0900	17.58[illegible]0	16.5900
55[illegible]	.5[illegible]61850[illegible]E 06	.50620237E 06	17.3700	17.86[illegible]0	16.8700
554[illegible]	.5[illegible]9571[illegible]E 06	.50958872E 06	17.7200	18.21[illegible]0	17.2200

TABLE A-2 (Continued)

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF. H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
558000.	.51295300E 06	.51297114E 06	18.1400	18.6300	17.6400
562000.	.51633100E 06	.51634966E 06	18.6600	19.1500	18.1600
568000.	.52139100E 06	.52141012E 06	19.1200	19.6100	18.6200
578000.	.52980500E 06	.52982475E 06	19.7500	20.2400	19.2500
580000.	.53148400E 06	.53150478E 06	20.7800	21.2700	20.2800
582000.	.53316300E 06	.53318382E 06	20.8200	21.3100	20.3200
584000.	.53484100E 06	.53486190E 06	20.9000	21.3900	20.4000
586000.	.53651800E 06	.53653902E 06	21.0200	21.5100	20.5200
588000.	.53819400E 06	.53821517E 06	21.1700	21.6600	20.6700
590000.	.53986900E 06	.53989035E 06	21.3500	21.8400	20.8500
592000.	.54154300E 06	.54156457E 06	21.5700	22.0600	21.0700
600000.	.54823000E 06	.54825182E 06	21.8200	22.3100	21.3200
602000.	.54989900E 06	.54992122E 06	22.2200	22.7100	21.7200
604000.	.55156700E 06	.55158968E 06	22.6800	23.1700	22.1800
608000.	.55490100E 06	.55492370E 06	22.7000	23.1900	22.2000
610000.	.55656600E 06	.55658928E 06	23.2800	23.7700	22.7800
614000.	.55989400E 06	.55991756E 06	23.5600	24.0500	23.0600
620000.	.56487900E 06	.56490284E 06	23.8400	24.3300	23.3400
626000.	.56985500E 06	.56987954E 06	24.5400	25.0300	24.0400
628000.	.57151100E 06	.57153654E 06	25.5400	26.0300	25.0400
630000.	.57316700E 06	.57319260E 06	25.6000	26.0900	25.1000
632000.	.57482200E 06	.57484770E 06	25.7000	26.1900	25.2000
634000.	.57647600E 06	.57650186E 06	25.8600	26.3500	25.3600
636000.	.57812900E 06	.57815508E 06	26.0800	26.5700	25.5800
638000.	.57978100E 06	.57980734E 06	26.3400	26.8300	25.8400
644000.	.58473200E 06	.58475848E 06	26.4800	26.9700	25.9800
646000.	.58638000E 06	.58640697E 06	26.9700	27.4600	26.4700
650000.	.58967400E 06	.58970114E 06	27.1400	27.6300	26.6400
654000.	.59296400E 06	.59299155E 06	27.5500	28.0400	27.0500
660000.	.59789200E 06	.59792012E 06	28.1200	28.6100	27.6200
662000.	.59953200E 06	.59956110E 06	29.1000	29.5900	28.6000
664000.	.60117200E 06	.60120115E 06	29.1500	29.6400	28.6500
666000.	.60281100E 06	.60284028E 06	29.2800	29.7700	28.7800
668000.	.60444900E 06	.60447845E 06	29.4500	29.9400	28.9500
670000.	.60608600E 06	.60611570E 06	29.7000	30.1900	29.2000
672000.	.60772200E 06	.60775203E 06	30.0300	30.5200	29.5300
678000.	.61262500E 06	.61265539E 06	30.3900	30.8800	29.8900
682000.	.61588900E 06	.61591966E 06	30.6600	31.1500	30.1600
686000.	.61914900E 06	.61918023E 06	31.2300	31.7200	30.7300
690000.	.62240500E 06	.62243709E 06	32.0900	32.5800	31.5900
692000.	.62403200E 06	.62406414E 06	32.1400	32.6300	31.6400
694000.	.62565800E 06	.62569026E 06	32.2600	32.7500	31.7600
696000.	.62728300E 06	.62731545E 06	32.4500	32.9400	31.9500
698000.	.62890700E 06	.62893974E 06	32.7400	33.2300	32.2400
700000.	.63053000E 06	.63056309E 06	33.0900	33.5800	32.5900

TABLE A-3

VALUES OF SUBSET 2 AS A FUNCTION OF GEOMETRIC ALTITUDE
WITH THE RELATED VALUES OF H62/R, H56/8,
AND THE DIFFERENCES H56/8 - H62/R

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
700000.	.63053000E 06	.63056309E 06	33.0900	33.5800	32.5900
698000.	.62890700E 06	.62893974E 06	32.7400	33.2300	32.2400
696000.	.62728300E 06	.62731545E 06	32.4500	32.9400	31.9500
694000.	.62565800E 06	.62569026E 06	32.2600	32.7500	31.7600
692000.	.62403200E 06	.62406414E 06	32.1400	32.6300	31.6400
688000.	.62077700E 06	.62080912E 06	32.1200	32.6100	31.6200
684000.	.61751900E 06	.61755040E 06	31.4000	31.8900	30.9000
680000.	.61425700E 06	.61428800E 06	31.0000	31.4900	30.5000
676000.	.61099100E 06	.61102186E 06	30.8600	31.3500	30.3600
674000.	.60935700E 06	.60938741E 06	30.4100	30.9000	29.9100
672000.	.60772200E 06	.60775203E 06	30.0300	30.5200	29.5300
670000.	.60608600E 06	.60611570E 06	29.7000	30.1900	29.2000
668000.	.60444900E 06	.60447845E 06	29.4500	29.9400	28.9500
666000.	.60281100E 06	.60284028E 06	29.2800	29.7700	28.7800
664000.	.60117200E 06	.60120115E 06	29.1500	29.6400	28.6500
662000.	.59953200E 06	.59956110E 06	29.1000	29.5900	28.6000
656000.	.59460700E 06	.59463534E 06	28.3400	28.8300	27.8400
652000.	.59131900E 06	.59134681E 06	27.8100	28.3000	27.3100
648000.	.58802700E 06	.58805453E 06	27.5300	28.0200	27.0300
642000.	.58308200E 06	.58310905E 06	27.0500	27.5400	26.5500
640000.	.58143200E 06	.58145866E 06	26.6600	27.1500	26.1600
638000.	.57978100E 06	.57980734E 06	26.3400	26.8300	25.8400
636000.	.57812900E 06	.57815508E 06	26.0800	26.5700	25.5800
634000.	.57647600E 06	.57650186E 06	25.8600	26.3500	25.3600
632000.	.57482200E 06	.57484770E 06	25.7000	26.1900	25.2000
630000.	.57316700E 06	.57319260E 06	25.6000	26.0900	25.1000
628000.	.57151100E 06	.57153654E 06	25.5400	26.0300	25.0400
622000.	.56653800E 06	.56656270E 06	24.7000	25.1900	24.2000
616000.	.56155600E 06	.56158028E 06	24.2800	24.7700	23.7800
612000.	.55823000E 06	.55825390E 06	23.9000	24.3900	23.4000
610000.	.55656600E 06	.55658928E 06	23.2800	23.7700	22.7800
606000.	.55323400E 06	.55325717E 06	23.1700	23.6600	22.6700
604000.	.55156700E 06	.55158968E 06	22.6800	23.1700	22.1800
598000.	.54655900E 06	.54658145E 06	22.4500	22.9400	21.9500
596000.	.54488800E 06	.54491011E 06	22.1100	22.6000	21.6100
594000.	.54321600E 06	.54323782E 06	21.8200	22.3100	21.3200
592000.	.54154300E 06	.54156457E 06	21.5700	22.0600	21.0700
590000.	.53986900E 06	.53989035E 06	21.3500	21.8400	20.8500
588000.	.53819400E 06	.53821517E 06	21.1700	21.6600	20.6700
586000.	.53651800E 06	.53653902E 06	21.0200	21.5100	20.5200
584000.	.53484100E 06	.53486190E 06	20.9000	21.3900	20.4000
582000.	.53316300E 06	.53318382E 06	20.8200	21.3100	20.3200

TABLE A-3 (Continued)

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
580000.	.53148400E 06	.53150473E 06	20.7800	21.2700	20.2800
572000.	.52307500E 06	.52309499E 06	19.9900	20.4800	19.4900
564000.	.51801800E 06	.51803745E 06	19.4500	19.9400	18.9500
560000.	.51464200E 06	.51466090E 06	18.9000	19.3900	18.4000
556000.	.51126200E 06	.51128042E 06	18.4200	18.9100	17.9200
552000.	.50787800E 06	.50789603E 06	18.0300	18.5200	17.5300
548000.	.50449000E 06	.50450773E 06	17.7300	18.2200	17.2300
544000.	.50109800E 06	.50111548E 06	17.4800	17.9700	16.9800
540000.	.49770200E 06	.49771931E 06	17.3100	17.8000	16.8100
538000.	.49600300E 06	.49601975E 06	16.7500	17.2400	16.2500
534000.	.49260100E 06	.49261766E 06	16.6600	17.1500	16.1600
530000.	.48919500E 06	.48921163E 06	16.6300	17.1200	16.1300
524000.	.48407900E 06	.48409514E 06	16.1400	16.6300	15.6400
518000.	.47895400E 06	.47896973E 06	15.7300	16.2200	15.2300
516000.	.47724400E 06	.47725927E 06	15.2700	15.7600	14.7700
510000.	.47210700E 06	.47212191E 06	14.9100	15.4000	14.4100
504000.	.46696100E 06	.46697557E 06	14.5700	15.0600	14.0700
498000.	.46180600E 06	.46182021E 06	14.2100	14.7000	13.7100
492000.	.45664200E 06	.45665583E 06	13.8300	14.3200	13.3300
486000.	.45146900E 06	.45148238E 06	13.3800	13.8700	12.8800
484000.	.44974300E 06	.44975587E 06	12.8700	13.3600	12.3700
480000.	.44628700E 06	.44629986E 06	12.8600	13.3500	12.3600
478000.	.44455800E 06	.44457032E 06	12.3200	12.8100	11.8200
474000.	.44109600E 06	.44110823E 06	12.2300	12.7200	11.7300
470000.	.43763000E 06	.43764207E 06	12.0700	12.5600	11.5700
466000.	.43416000E 06	.43417186E 06	11.8600	12.3500	11.3600
462000.	.43068600E 06	.43069756E 06	11.5600	12.0500	11.0600
456000.	.42546700E 06	.42547848E 06	11.4800	11.9700	10.9800
452000.	.42198300E 06	.42199397E 06	10.9700	11.4600	10.4700
444000.	.41500200E 06	.41501268E 06	10.6800	11.1700	10.1800
436000.	.40800500E 06	.40801494E 06	9.9400	10.4300	9.4400
434000.	.40625300E 06	.40626292E 06	9.9200	10.4100	9.4200
432000.	.40450000E 06	.40450988E 06	9.8800	10.3700	9.3800
430000.	.40274600E 06	.40275580E 06	9.8000	10.2900	9.3000
428000.	.40099100E 06	.40100068E 06	9.6800	10.1700	9.1800
426000.	.39923500E 06	.39924454E 06	9.5400	10.0300	9.0400
424000.	.39747800E 06	.39748736E 06	9.3600	9.8500	8.8600
422000.	.39572000E 06	.39572915E 06	9.1500	9.6400	8.6500
420000.	.39396100E 06	.39396988E 06	8.8800	9.3700	8.3800
414000.	.38867700E 06	.38868587E 06	8.8700	9.3600	8.3700
408000.	.38338400E 06	.38339249E 06	8.4900	8.9800	7.9900
404000.	.37985000E 06	.37985836E 06	8.3600	8.8500	7.8600
400000.	.37631200E 06	.37632003E 06	8.0300	8.5200	7.5300
392000.	.36922300E 06	.36923080E 06	7.8000	8.2900	7.3000
378000.	.35677700E 06	.35678411E 06	7.1100	7.6000	6.6100
372000.	.35142700E 06	.35143398E 06	6.9800	7.4700	6.4800

TABLE A-3 (Continued)

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
364[illegible]	.34427900E 06	.34428558E 06	6.5800	7.0700	6.0800
356[illegible]	.33711400E 06	.33712015E 06	6.1500	6.6400	5.6500
354[illegible]	.33532000E 06	.33532613E 06	6.1300	6.6200	5.6300
352[illegible]	.33352500E 06	.33353102E 06	6.0200	6.5100	5.5200
350[illegible]	.33172900E 06	.33173496E 06	5.8600	6.3500	5.3600
348[illegible]	.32993200E 06	.32993763E 06	5.6300	6.1200	5.1300
342[illegible]	.32453400E 06	.32453947E 06	5.4700	5.9600	4.9700
338[illegible]	.32093000E 06	.32093532E 06	5.3200	5.8100	4.8200
33[illegible]	.31370900E 06	.31371410E 06	5.1000	5.5900	4.6000
328[illegible]	.31190100E 06	.31190609E 06	5.0900	5.5800	4.5900
324[illegible]	.31009200E 06	.31009700E 06	5.0000	5.4900	4.5000
324[illegible]	.30828200E 06	.30828683E 06	4.8300	5.3200	4.3300
318[illegible]	.30284500E 06	.30284981E 06	4.8100	5.3000	4.3100
314[illegible]	.29921500E 06	.29921968E 06	4.6800	5.1700	4.1800
3[illegible]	.29194200E 06	.29194638E 06	4.3800	4.8700	3.8800
3[illegible]	.29012100E 06	.29012532E 06	4.3200	4.8100	3.8200
332[illegible]	.28829900E 06	.28830316E 06	4.1600	4.6500	3.6600
296[illegible]	.28282600E 06	.28283013E 06	4.1300	4.6200	3.6300
292[illegible]	.27917200E 06	.27917597E 06	3.9700	4.4600	3.4700
291[illegible]	.27825800E 06	.27826173E 06	3.7300	4.2200	3.2300
277[illegible]	.26543000E 06	.26543357E 06	3.5700	4.0600	3.0700
273[illegible]	.26175500E 06	.26175842E 06	3.4200	3.9100	2.9200
268[illegible]	.25715500E 06	.25715826E 06	3.2600	3.7500	2.7600
266[illegible]	.25531300E 06	.25531625E 06	3.2500	3.7400	2.7500
263[illegible]	.25254800E 06	.25255115E 06	3.1500	3.6400	2.6500
258[illegible]	.24793400E 06	.24793705E 06	3.0500	3.5400	2.5500
254[illegible]	.24423800E 06	.24424077E 06	2.7700	3.2600	2.2700
2450[illegible]	.23590500E 06	.23590774E 06	2.7400	3.2300	2.2400
237[illegible]	.22847900E 06	.22848149E 06	2.4900	2.9800	1.9900
227[illegible]	.21917100E 06	.21917333E 06	2.3300	2.8200	1.8300
225[illegible]	.21730600E 06	.21730830E 06	2.3000	2.7900	1.8000
223[illegible]	.21544000E 06	.21544213E 06	2.1300	2.6200	1.6300
215[illegible]	.20796400E 06	.20796612E 06	2.1200	2.6100	1.6200
2[illegible]	.19671600E 06	.19671792E 06	1.9200	2.4100	1.4200
194[illegible]	.18825300E 06	.18825472E 06	1.7200	2.2100	1.2200
188[illegible]	.18259800E 06	.18259965E 06	1.6500	2.1400	1.1500
179[illegible]	.17315000E 06	.17315146E 06	1.4600	1.9500	.9600
175[illegible]	.17031000E 06	.17031138E 06	1.3800	1.8700	.8800
162[illegible]	.16462200E 06	.16462336E 06	1.3600	1.8500	.8600
164[illegible]	.15987400E 06	.15987533E 06	1.3300	1.8200	.8300
162[illegible]	.15797300E 06	.15797409E 06	1.0900	1.5800	.5900
158[illegible]	.15416700E 06	.15416808E 06	1.0800	1.5700	.5800
150[illegible]	.14654100E 06	.14654206E 06	1.0600	1.5500	.5600
146[illegible]	.14272100E 06	.14272201E 06	1.0100	1.5000	.5100
142[illegible]	.13985300E 06	.13985388E 06	.8800	1.3700	.3800
131[illegible]	.12835400E 06	.12835496E 06	.8600	1.3500	.3600

TABLE A-3 (Continued)

GEOMETRIC ALTITUDE METERS	H62/R GEOPOTENTIAL METERS	H56/8 GEOPOTENTIAL METERS	DIF H56-H62 GEOPT. METERS	DIF+INC GEOPT. METERS	DIF-0.5 GEOPT. METERS
112000.	.11006000E 06	.11006083E 06	.8300	1.3200	.3300
108000.	.10619500E 06	.10619575E 06	.7500	1.2400	.2500
107000.	.10522800E 06	.10522874E 06	.7400	1.2300	.2400
91000.	.89715000E 05	.89715680E 05	.6800	1.1790	.1800
72000.	.71193000E 05	.71193624E 05	.6240	1.1230	.1240
70000.	.69237000E 05	.69237564E 05	.5640	1.0630	.0640
44000.	.43697000E 05	.43697536E 05	.5360	1.0350	.0360

TABLE A-4

SCALE VALUES OF $f_1(Z)$ AND Z EMPLOYED IN DIFFERENT PORTIONS OF THE GRAPHS OF SUBSETS 1 AND 2

Altitude Interval km	Meters of f (Z) Per 1 cm of Graph	km of Altitude Z Per 1 cm of Graph
0 - 150	0.02	4
100 - 250	0.04	4
220 - 370	0.10	4
360 - 550	0.10	4
550 - 700	0.10	1

The series of straight-line segments meeting these conditions is designated as f(SS-1). The ordinate values of the desired function f(Z) may be equal to or less than the ordinate values of the end points of the segments of f(SS-1), but will always be less than the ordinate values of all other parts of f(SS-1).

(9) Plot the data from Subset 2 on the same graphs with Subset 1.

(10) Select certain points of Subset 2 which appear to form a smooth monotonically increasing upper bound to the total of Subset 2 data. These selected points comprise Smooth Subset 2 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies above all those points of Subset 2 having altitude values within the altitude interval encompassed by the particular line segment.

(b) Same as condition b under step 8.

This series of straight-line segments associated with Smooth Subset 2 data is designated as f(SS-2) and will be seen to be close to but below the segments f(SS-1) prepared in Step 8. The ordinate values of the desired function f(Z) may be equal to or greater than the corresponding ordinate values of f(SS-2) for all values of Z, but there is a small probability that the ordinate values of f(Z) may sometimes be slightly less than the corresponding ordinate values of f(SS-2) for some values of Z in between the end points of some of the line segments, i.e., the points of smooth Subset 2. For any particular set of abscissa values Z_2 corresponding to the abscissa of the data points in Smooth Subset 2, the desired smooth function f(Z) has ordinate values which are equal to or greater than the ordinate values of the related points of Subset 2, but which are less than the corresponding ordinate values of f(SS-1). Thus, the value of f(Z) corresponding to each member of the set of ordinates Z_2 is bounded within small limits, and it is possible to estimate the value of f(Z) for each value of Z_2 to be midway between the appropriate limits. It is also possible to estimate the value of $\delta f(Z)$, the range of uncertainty of f(Z), to be equal to the separation between the above specified boundaries.

Values of f(Z) and $\delta f(Z)$ may similarly be made for the set of abscissa values Z corresponding to the abscissas of the data points in Smooth Subset 1. In these instances, however, the ordinate values of f(Z) will be equal to or less than the ordinate values of the related data points of Smooth Subset 1, and may be as low as or even slightly lower than the ordinate value of the corresponding points of f(SS-2).

(11) From the graphical representations f(SS-1) and f(SS-2) connecting the data points of Smooth Subsets 1 and 2 respectively, estimate values of f(Z) and $\delta f(Z)$ for the abscissas in the set Z_1 and Z_2 in accordance with the discussion under step 10.

The set of graphically estimated values of f(Z) and uncertainty of f(Z) in the form of 100 $\delta f(Z)/f(Z)$ are presented as a function of Z in Table A-5.

The f(Z) Polynomial

The data of Table A-5 were fed into a digital-machine curve-fitting program designed to yield the coefficients of best-fit, first-, second-, third-, and fourth-degree polynomials as well as the difference between each point of input data and the polynomial values for the same abscissas. From considerations of mean differences, the fourth-degree curve was found to give the best fit of the four curves considered. For the case when Z is expressed as kilometers, we find

$$f(Z) = A + BZ + CZ^2 + DZ^3 + EZ^4 \qquad \text{(A-4)}$$

where f(Z) is expressed as m′

$$A = 0.4858124 \times 10^{-2}\ m'$$

$$B = 0.1338918 \times 10^{-3}\ m'/km$$

$$C = 0.1903029 \times 10^{-4}\ m'/km^2$$

$$D = 0.8288881 \times 10^{-9}\ m'/km^3$$

$$E = 0.1822113 \times 10^{-10}\ m'/km^4$$

When Z is expressed in meters, the values of the several coefficients are multiplied by $(10^{-3})^x\ (km/m)^x$, where x is the power of Z in the term with which the particular coefficient is associated. In both instances the dimensions of f(Z) is geopotential meters. A list of the functional values of f(Z) as defined above and a list of the difference between the graphical and functional value of f(Z) are also given in Table A-5.

It is noted that the function determined does not pass exactly through zero at Z = 0, but has a value of 0.004858124 meter at this altitude. This value, in effect, increases the value of the entire function by less than five thousandths of a meter, an amount which is trivial in the determination of geopotential to the nearest hundreth of a meter. This discrepancy undoubtedly results because the graphically derived values of f(Z) have only limited accuracy. In addition, only three graphical values of f(Z) resulted from the study for the region 0 to 90 km in which region the basic data consisted of only those 90 tabulated values of Z for integral multiples of one km′. There are, however, approximately 600 tabular entries of Z and H_{62}/R between 0 and 90 km′, and more than three graphical values of f(Z) could possibly have been obtained for that region. A better fit at Z = 0 might have been determined if all the available data had been used. No great error is made, however, if zero is assigned to the coefficient of the Z^0 term.

TABLE A-5

GRAPHICALLY DETERMINED ESTIMATES OF $f_1(Z)$ COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF $f(Z)$

Z (km)	Graphical Estimates Of $f_1(Z)$, m	Percentage Degree of Certainty	Functional Value of f(Z), m	Graphical Value Minus Functional Value, m
.000E-50	.00000E-50	100.000	.4858124E-02	-.4858124E-02
.440E+02	.42000E-01	90.576	.4280204E-01	-.8020470E-03
.720E+02	.13000E+00	96.923	.1243193E+00	.5680610E-02
.112E+03	.34200E+00	98.830	.3421639E+00	-.1639800E-03
.144E+03	.86000E+00	97.680	.8471755E+00	.1282441E-01
.203E+03	.14500E+01	98.620	.1424355E+01	.2564490E-01
.215E+03	.16450E+01	98.480	.1640593E+01	.4407000E-02
.225E+03	.18210E+01	98.050	.1835597E+01	-.1459790E-01
.245E+03	.22550E+01	99.334	.2267670E+01	-.1267070E-01
.258E+03	.25700E+01	99.222	.2579805E+01	-.9805100E-02
.266E+03	.27750E+01	99.097	.2784586E+01	-.9586400E-02
.273E+03	.29550E+01	98.815	.2971894E+01	-.1689450E-01
.373E+03	.65100E+01	99.538	.6506622E+01	.3377400E-02
.455E+03	.11010E+02	99.728	.1097247E+02	.3752700E-01
.486E+03	.12915E+02	99.652	.1293304E+02	-.8047000E-02
.492E+03	.13360E+02	99.776	.1334955E+02	.1045000E-01
.518E+03	.15265E+02	99.771	.1525077E+02	.1422500E-01
.530E+03	.16175E+02	99.725	.1618200E+02	-.7009000E-02
.570E+03	.19510E+02	99.897	.1953848E+02	-.2848800E-01
.612E+03	.23450E+02	99.787	.2349433E+02	-.4433200E-01
.642E+03	.26650E+2	99.625	.2660026E+02	.4973800E-01
.676E+03	.30405E+02	99.852	.3041133E+02	-.6333000E-02

A-3 APPLICATION OF f(Z) TO THE DETERMINATION OF GEOPOTENTIAL FOR Z = 90 km

The function f(Z) when used in Equation (A-2) provides the means for computing the value of $H_{62/8}$ for specified values of Z. An evaluation of Equation (A-4) for Z = 90 km shows f(Z) to be 0.201 m′ when the coefficient A is taken to be zero. The corresponding value of H from Equation (A-2) is found to be 88743.335 m′. This value should be a close approximation to that value of geopotential at which the defining temperature-altitude profiles of the U.S. Standard Atmosphere 1962 are switched from being linear in terms of geopotential, in the lower regime, to being linear in terms of geometric altitude in the upper regime. The calculation of the pressure for this transition altitude is considered under the discussion related to pressures at critical altitudes at 90 and below 90 km in the next section.

Application of f(Z) Data to the Determination* of an Expression for Z versus H

The calculation of geometric altitude in terms of integral multiples of one geopotential kilometer for the required tables for the upper regime of the Standard Atmosphere may not be made by means of Equation (A-2), involving the functional expression for f(Z), without an undesirable iteration process. If Equation (A-2) is solved for Z without considering f(Z) explicitly in terms of Z, one obtains

$$A = \frac{r[H + f(Z)]}{r - [H + f(Z)]} \quad . \tag{A-5}$$

Since f(Z) represents a set of values associated with a specific set of geometric altitudes, it is apparent that this same set of values may become f(H) by being associated with a corresponding specific set of geopotentials, related to the geometric altitude through Equation (A-2). With this transformation of f(Z) to f(H), Equation (A-5) becomes

$$Z_{62/8} = \frac{r[H + f(H)]}{r - [H + f(H)]} \tag{A-6}$$

The function f(H) is found from the same basic graphical data employed in finding f(Z). In this case the value -0.342 m′, for example, previously associated with 112,000 geometric meters in Table A-5 is now associated with 110,060.488 geopotential meters, i.e.,

$$\frac{rZ}{r+Z} - 0.342 = 110060.83 - 0.342 = 110,060.488 \text{ m}' \quad .$$

Similar relationships apply for each of the other data points. These revised data points presented in Table A-6 yield a new polynomial fit,

$$f(H) = AA + ABH + ACH^2 + ADH^3 + AFH^4 \quad , \tag{A-7}$$

*The development of Equations (A-5) through (A-6) and the Table A-6 form the basis for the geopotential tables included in the United States Standard Atmosphere Supplements, 1966 (Ref. A-3).

TABLE A-6

GRAPHICALLY DETERMINED ESTIMATES OF $f_1(H)$
COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF $f_1(H)$

Geopotential km	Graphical Estimates of $f_1(H)$, m	Percentage Degree of Certainty	Functional Value of $f_1(H)$, m	Graphical Value Minus Functional Value, m
.0000000E-50	.00000E-50	100.000	.2579651E-02	-.2579651E-02
.4369749E+02	.42000E-01	90.576	.4386026E-01	-.1860263E-02
.7119349E+02	.13000E+00	96.923	.1262009E+00	.3799100E-02
.1100604E+03	.34200E+00	98.830	.3441282E+00	-.2128220E-02
.1598744E+03	.86200E+00	97.680	.8482695E+00	.1373044E-01
.1967164E+03	.14500E+01	98.620	.1424622E+01	.2537790E-01
.2079644E+03	.16450E+01	98.480	.1640626E+01	.4373300E-02
.2173064E+03	.18210E+01	98.050	.1835452E+01	-.1445220E-01
.2359054E+03	.22550E+01	99.334	.2267215E+01	-.1221530E-01
.2479344E+01	.25700E+01	99.222	.2579188E+01	-.9188600E-02
.2553134E+03	.27750E+01	99.097	.2783888E+01	-.8888100E-02
.2617554E+03	.29550E+01	98.815	.2971135E+01	-.1613520E-01
.3514274E+03	.65100E+01	99.538	.6505996E+01	.4003100E-02
.4254674E+03	.11010E+02	99.728	.1097266E+02	.3733800E-01
.4514694E+03	.12925E+02	99.652	.1293344E+02	-.8443000E-02
.4566424E+03	.13360E+02	99.776	.1334997E+02	.1002700E-01
.4789544E+03	.15265E+02	99.771	.1525125E+02	.1374500E-01
.4891954E+03	.16175E+02	99.725	.1618248E+02	-.7483000E-02
.5230754E+03	.19510E+02	99.897	.1953878E+02	-.2878400E-01
.5582304E+03	.23650E+02	99.787	.2349426E+02	-.4426000E-01
.5830824E+03	.26650E+02	99.625	.2659992E+02	.5007600E-01
.6109914E+03	.30405E+02	99.852	.304.092E+02	-.5925000E-02

where f(H) is expressed as m′

$$AA = 0.2579651 \times 10^{-2} \quad m'$$
$$AB = 0.2161710 \times 10^{-7} \quad m'/m'$$
$$AC = 0.1807561 \times 10^{-10} \quad m'/(m')^2$$
$$AD = 0.9153012 \times 10^{-16} \quad m'/(m')^3$$
$$AE = 0.2006785 \times 10^{-22} \quad m'/(m')^4$$

and where H is the altitude in geopotential meters. If H is to be expressed in kilometers, the values of the various coefficients must be multiplied by $(10^3)^x$ $(m'/km')^x$ where x is the power to which H is raised in that term to which the coefficient applies.

Equation (A-7) introduced into Equation (A-6) now yields values of Z for integral values of H in substantial agreement with the related values tabulated in the U.S. Standard Atmosphere 1962. The application of this equation to the expansion of the Standard Atmosphere above 90 km is considered in the main body of this report.

APPENDIX B

```
C                                PROGRAM FOR COMPUTING 1962 STANDARD-ATMOSPHERE
C                                VALUES OF PRESSURE, TEMPERATURE,TEMPERATURE
C                                GRADIENT, AND DENSITY AS A FUNCTION OF INTEGRAL
C                                MULTIPLES OF ONE THOUSAND GEOPOTENIAL METERS,
C                                FROM 90,000 TO 120,000 GEOPOTENTIAL METERS, AND
C                                AS A FUNCTION OF INTEGRAL MULTIPLES OF 5000
C                                GEOPOTENTIAL FEET, FROM 295000 TO 390,000
C                                GEOPOTENTIAL FEET. PRESSURE IS COMPUTED IN
C                                MILLIBARS IN BOTH THE METRIC AND ENGLISH TABLES,
C                                WHILE TEMPERATURE, TEMPERATURE GRADIENT, AND
C                                DENISITY ARE COMPUTED IN TERMS OF DEGREES KELVIN,
C                                DEGREES KELVIN PER METER, AND KILOGRAMS PER CUBIC
C                                METER RESPECTIVELY IN THE METRIC TABLES, BUT IN
C                                TERMS OF DEGREES RANKIN, DEGREES RANKIN PER FOOT,
C                                AND POUNDS PER CUBIC FOOT RESPECTIVELY IN THE
C                                ENGLISH TABLES.   PROGRAM ORIGINALLY WRITEN BY
C                                MRS MELLO. REVIEWED AND COMMENTED BY R.A.  MINZNER
C                                NOVEMBER 1966.
      DIMENSION G(7),A(7),TMR(4),FL(4),D(4),P(4)
C                                THE QUOTIENT OF THE SEA-LEVEL VALUE OF THE
C                                MOLECULAR WEIGHT DIVIDIED BY THE UNIVERSAL GAS
C                                CONSTANT IS DESIGNATED FMOR.
   18 FMOR=3.483676E-03
C                                THE QUANTITIES AA,AB,AC, AD AND AE ARE THE
C                                COEFFICIENTS OF THE POLYNOMIAL DEFINING FX, THE
C                                CORRECTION TERM IN THE EXPRESSION FOR CONVERTING
C                                GEOPOTENTIAL TO GEOMETRIC ALTITUDE.
      AA=0.
      AB=-.2161710E-07
      AC=.1807561E-10
      AD=.9153012E-16
      AE=.2006785E-22
C                                TMR(1) THROUGH TMR(4) ARE THE DEFINED VALUES OF THE
C                                MOLECULAR SCALE TEMPERATURE (DEGREES KELVIN) AT
C                                90,100,110 AND 120 GEOMETRIC KILOMETERS ALTITUDE.
      TMR(1)=180.65
      TMR(2)=210.65
      TMR(3)=260.65
      TMR(4)=360.65
C                                D(1) THROUGH D(4) ARE THE PUBLISHED STANDARD-
C                                ATMOSPHERE VALUES OF DENSITY (KG PER CUBIC METER)AT
C                                90, 100,110 AND 120 KILOMETERS (THESE VALUES
C                                APPEAR NOT TO BE USED IN THIS PROGRAM).
      D(1)=3.170E-06
      D(2)=4.974E-07
      D(3)=9.829E-08
      D(4)=2.436E-08
C                                FL(1) THROUGH FL(4) ARE THE DEFINED VALUES OF THE
C                                DERIVATIVE OF TM WITH RESPECT TO Z (DEGREES
C                                KELVIN PER METER) FOR THE FOUR LAYERS WHOSE BASES
```

```
C                                   ARE 90, 100, 110 AND 120 KM.
      PL(1)=5.E-03
      PL(2)=5.E-03
      PL(3)=10.E-03
      PL(4)=20.E-03
C                                   P(1) THROUGH P(4) ARE THE MILLIBAR PRESSURES AT
C                                   ALTITUDES 90, 100, 110 AND 120 KILOMETERS.
      P(1)=1.6498E-03
      P(2)=3.8075E-04
      P(3)=7.3544E-05
      P(4)=2.5217E-05
C                                   G(1) THROUGH G(7) ARE THE COEFFICIENTS OF THE
C                                   LAMBERT EQUATION FOR EXPRESSING THE VARITION OF THE
C                                   ACCELERATION OF GRAVITY WITH GEOMETRIC HEIGHT Z
C                                   AT ABOUT 45 DEGREES LATITUDE.
      G(1)=9.80665
      G(2)=-3.0854195E-06
      G(3)=7.2539455E-13
      G(4)=-1.9167771E-19
      G(5)=2.9724620E-26
      G(6)=-5.5905936E-33
      G(7)=1.0219762E-39
      IF(SENSE SWITCH 1)19,20
C                                   HF IS THE HEIGHT IN GEOPOTENTIAL FEET OF THE
C                                   INITIAL ENTRY IN THE ENGLISH TABLES.
      HF=295000.
C                                   H=89916 IS THE HEIGHT IN GEOPOTENTIAL METERS
C                                   EQUAL TO 295,000 GEOPOTENTIAL FEET (EXACT).
   20 H=89916.
      GO TO 21
C                                   H=90.E+03 IS THE HEIGHT IN GEOPENTIAL METERS OF
C                                   THE INITIAL ENTRY IN THE METRIC TABLES.
   19 H=90.E+03
C                                   R IS THE EFFECTIVE EARTH RADIUS (FOR  PURPOSES OF
C                                   RELATING GEOMETRIC HEIGHT AND GEOPOTENTIAL) AT 45
C                                   DEGREES 32 MINUTES AND 33 SECONDS LATITUDE.
   21 R=6356.766E+03
      BEGIN TRACE
C                                   FX IS THE CORRECTION TERM IN THE H TO Z CONVERSION.
   11 FX=AA+AB*H+AC*H**2+AD*H**3+AE*H**4
C                                   THE NEXT EXPRESSION CONVERTS GEOPOTENTIAL H TO
C                                   GOEMETRIC HEIGHT Z.
      Z=R*(H+FX)/(R-(H+FX))
C                                   THE NEXT 14 STATEMENTS INVOLVE THE SETTING OF
C                                   INDICES FOR THE FOUR LAYERS AND FOUR REFERENCE
C                                   LEVELS OF THE CALCULATIONS.
      IF(Z-100.E+03)2,3,4
    2 K=1
      ZR=90.E+03
      GO TO 5
```

```
    3 K=2
      ZR=100.E+03
      GO TO 5
    4 IF(Z-110.E+03)3,6,7
    6 K=3
      ZR=110.E+03
      GO TO 5
    7 IF(Z-120.E+03) 6,28,28
   28 K=4
      ZR=120.E+03
C                                THE NEXT 23 STATEMENTS DEAL WITH THE CALCULATION OF
C                                PRESS,THE PRESSURE IN MILLIBARS, IN ACCORDANCE
C                                WITH THE CORRECTED VERSION OF EQUATION 27 OF
C                                CHAMPION AND MINZNER, REV. OF GEOPHYSICS 1,P57,1963
    5 SUMA0=0.
      SUMA1=0.
      SUM=0.
      DO 8 I=1,7
      A(I)=(TMR(K)/FL(K)-ZR)**(I-1)
    8 SUMA0=SUMA0+(-1.)**(I-1)*G(I)*A(I)
      DO 9 L=1,6
      GL=L
      DO 10 I=L,6
      M=I+L
      N=I-L+1
   10 SUM=SUM+(-1.)**(M)*G(I+1)*A(N)
      SHEL=Z**L-ZR**L
      SHEL=SHEL/GL*SUM
      SUMA1=SUMA1+SHEL
    9 SUM=0.
      SHE=-SUMA0*(FMOR/FL(K))
      SHEIL=SUMA1*FMOR/FL(K)
      PRESS=TMR(K)+FL(K)*(Z-ZR)
      PRESS=PRESS/TMR(K)
      PRESS=PRESS**SHE
      PRESS=PRESS*P(K)
      PRESS=PRESS*EXP(-SHEIL)
C                                THE NEXT STATEMENT EXPRESSES THE MOLECULAR SCALE
C                                TEMPERATURE TM AT ALTITUDE Z WITHIN ANY LAYER K.
      TM=TMR(K)+FL(K)*(Z-ZR)
C                                THE NEXT STATEMENT EXPRESSES THE DENSITY IN KG. PER
C                                CUBIC METER IN TERMS OF TM AND PRESS USING THE
C                                GAS LAW. THE FACTOR 1.E+02 IS REQUIRED TO CONVERT
C                                MILLIBAR PRESSURES TO NEWTONS PER SQUARE METER, THE
C                                PROPER UNITS OF PRESSURE IN THE MKS SYSTEM OF UNITS
      DENS=PRESS/TM*FMOR*1.E+02
      IF(SENSE SWITCH 1 )22,23
C                                THE NEXT STATEMENT CONVERTS TM IN DEGREES K TO TMD
C                                IN DEGREES RANKIN.
   23 TMD=1.8*TM
```

APPENDIX B CONCLUDED

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C                                THE NEXT STATEMENT CONVERTS DENSITY IN KILOGRAMS
C                                PER CUBIC METER TO POUNDS PER CUBIC FOOT.
      DENS =DENS*6.242797E-02
C                                THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                DECIMAL PLACES.
      TMD=TMD+.005
C                                THE NEXT STATEMENT CONVERTS Z IN METERS TO ZF IN-
C                                FEET.
      ZF=Z*3.2808399
C                                THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                DECIMAL PLACES.
      ZF=ZF+.005
C                                THE NEXT STATEMENT CONVERTS TM IN DEGREES KELVIN TO
C                                TMC IN DEGREES CELCIUS.
      TMC=TM-273.15
C                                THE NEXT STATEMENT ROUNDS THE QUANTITY TO TWO
C                                DECIMAL PLACES.
      TMC=TMC+.005
C                                THE NEXT STATEMENT CONVERTS FL IN DEGREES KELVIN
C                                PER METER TO RL IN DEGREES RANKIN PER FOOT.
      RL=FL(K)*.54864
C                                THE NEXT STATEMENT IS THE PUNCH STATEMENT FOR THE
C                                ENGLISH TABLES.
      PUNCH 101,HF,ZF,RL,TMD,TMC,PRESS,DENS
      GO TO 24
C                                THE NEXT TWO STATEMENTS ROUND THE RESPECTIVE
C                                QUANTITIES TO TWO DECIMAL PLACES, WHERE FZ IS THE
C                                ROUNDED VALUE OF Z IN METRIC UNITS.
   22 TM=TM+.005
      FZ=Z+.005
C                                THE NEXT STATEMENT IS THE PUNCH STATEMENT FOR THE
C                                METRIC TABLES.
      PUNCH 100,H,FZ,FL(K),TM,PRESS,DENS
C                                THE NEXT 8 STATEMENTS SET THE INCREMENTS OF BOTH
C                                THE METRIC AND ENGLISH TABLES.
   24 IF(SENSE SWITCH 1)12,13
   12 IF(H-120.E+03)14,15,15
   15 PAUSE
      GO TO 18
   14 H=H+1.E+03
      GO TO 11
   13 IF(Z-120.E+03)16,17,17
   16 HF=HF+5.E+03
C                                THE NEXT STATEMENT CONVERTS HF GEOPOTENTIAL HEIGHT
C                                IN FEET TO GEOPOTENTIAL HEIGHT IN METERS PRIOR TO
C                                CALCULATING THE THERMODYNAMIC PROPERTIES FOR HF.
      H=.3048*HF
      GO TO 11
  100 FORMAT(2F11.2,F6.3,F8.2,E12.5,E11.4)
  101 FORMAT(2F11.2,F10.7,2F8.2,E12.5,E11.4)
      END TRACE
      END
```

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The "United States Standard Atmosphere, 1962," was published with two kinds of metric-unit tables for the altitude interval from -5000 to 90,000 meters. One kind of table presented the atmospheric properties as a function of integral multiples of particular numbers of geopotential meters while the second presented the atmospheric properties as a function of integral multiples of similar numbers of geometric meters. For the region above 90,000 meters, altitude only one type of metric table was published. This type presented atmospheric properties in integral multiples of particular numbers of geometric meters. A similar situation prevailed for the English-unit tables. The need for both metric-unit and English-unit tables as a function of integral multiples of specific numbers of geopotential meters for altitudes above 90 kilometers has prompted a new set of calculations, which required the use of equations not specifically presented in the United States Standard Atmosphere, 1962. The development of these equations is discussed and the value of all constants employed are given. The calculations involve a transformation between geopotential and geometric altitude, and the development of an empirical analytical expression relating these quantities is presented. This empirical function yields results which differ by less than 0.1 meter at 700 km altitude, from those computed in an unspecified manner for the United States Standard Atmosphere, 1962.

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