AN ATTITUDE CONTROL SYSTEM TO CONSTRAIN THE SKIN TEMPERATURE
OF A MANNED LIFTING SPACECRAFT DURING REENTRY INTO THE EARTH'S ATMOSPHERE

by

Jerome H. Fine

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An attitude control system to regulate the temperature of a manned lifting spacecraft during reentry into the Earth's atmosphere is proposed. Its use prevents the peak skin temperature that is experienced during the reentry from rising moderately beyond that which would occur during an equilib-rium glide of the same vehicle.

The effects of Earth rotation and oblateness upon the performance of the attitude control system were found to be moderate and predictable. The maximum temperature increment associated with them was found to be only 100 °F for the worst set of initial conditions. The cross range shift of the foot-print due to rotation was found to be within 70 miles of the value that would occur for the corresponding orbit in vacuum. Oblateness could generally be accounted for by using the effective initial glide angle relative to the Earth's surface rather than the geocentric initial value relative to the central coordinate system.

The results of density variations in the Earth's atmosphere were not serious. Large increases in the maximum skin temperature occurred only when extremely large spatially random density disturbances were encountered by the vehicle.
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NOMENCLATURE

Roman Letters

$C_D$ drag coefficient $(D/\frac{1}{2}pV^2S)$

$C_{D_{\text{min}}}$ minimum drag coefficient (at zero angle of attack)

$C_{n,m}$ general spherical harmonic coefficient - see Eq. (4.1)

$C_{2,0}$ spherical harmonic coefficient associated with polar flattening

$C_L$ lift coefficient $(L/\frac{1}{2}pV^2S)$

$C_p$ pressure coefficient in Newtonian flow - see Eq. (3.1)

$D$ aerodynamic drag on the vehicle (lbf)

$F$ force (lbf)

$F_A$ total aerodynamic force acting on the vehicle (lbf)

$f_c$ defined by Eq. (4.17)

$f_r$ defined by Eq. (4.18)

$f_p$ the Earth's polar flattening $(\frac{R_o - R_{\text{pole}}}{R_o} = \frac{1}{298.28})$

$f_q$ the Earth's equatorial flattening $(\frac{R_{\text{major}} - R_{\text{minor}}}{R_o} \approx 10^{-5})$

$g$ total acceleration due to the Earth's gravitational field (ft/sec$^2$)

$g_r$ acceleration in the direction of the geocentric radius - see Eq. (4.4a) (ft/sec$^2$)

$g_{\lambda}$ acceleration in the direction of the geocentric latitude - see Eq. (4.4b) (ft/sec$^2$)

$g_{\theta}$ acceleration in the direction of the geocentric longitude - see Eq. (4.4c) (ft/sec$^2$)

$[\mathbf{r}]$, $[\mathbf{j}, \mathbf{k}]$ third intermediate reference system - see Fig. 21

$I_{\alpha}$ vehicle moment of inertia about the pitch axis (slug*ft$^2$)

$[\mathbf{I}, \mathbf{j}, \mathbf{k}]$ basic reference system - see Fig. 18

$[\mathbf{I}', \mathbf{j}', \mathbf{k}']$ first intermediate reference system - see Fig. 21

$[\mathbf{I}, \mathbf{j}, \mathbf{k}]$ flight axes reference system - see Fig. 20
K parameter defined by Eq. (A.8)

\( k_\alpha \) angular acceleration in pitch (sec\(^{-2}\))

\( k_\phi \) angular acceleration in roll (sec\(^{-2}\))

L aerodynamic lift on the vehicle (lbf)

L/D ratio of lift to drag

M moment exerted by the pitch reaction jets (lbf x ft)

m mass of the vehicle (slugs)

\( \frac{\theta}{S} \) heat capacity of the skin of the vehicle (BTU/ft\(^2\))

\( [\bar{e}, \bar{e}, \bar{v}] \) second intermediate reference system - see Fig. 21

\( P_{n}^m \) general spherical harmonic - see Eq. (4.2)

Q parameter defined by equation (3.9)

q convective heating rate at any point on the vehicle (BTU/ft\(^2\) sec)

q\(_n\) nose stagnation point convective heating rate (BTU/ft\(^2\) sec)

R\(_n\) radius of curvature of the nose stagnation point (feet)

R\(_o\) average equatorial radius (feet)

r geocentric distance to the vehicle (feet)

S reference wing area of the vehicle (ft\(^2\))

\( S_{n,m} \) general spherical harmonic coefficient - see Eq. (4.1)

s distance along the flight path starting at the initial position (feet)

T radiation-equilibrium nose stagnation point temperature (degrees Rankine or degrees Fahrenheit if specified)

\( \hat{T} \) actual skin temperature at any point on the vehicle (degrees Rankine)

T\(_{\text{last max}}\) the higher of the present temperature and the values measured since testing for a local maximum started - see Fig. 12

T\(_{\text{last min}}\) the lower of the present temperature and the values measured since testing for a local minimum started - see Fig. 12

T\(_{\text{deadband}}\) the allowable temperature variation before any action is taken - see Fig. 12

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time starting at the initial point (seconds) 
Earth's gravitational potential - see Eq. (4.1)
vehicle speed - see Fig. 19
perpendicular distance to the surface of the Earth from the vehicle - defined by Eq. (4.12) (feet)

Greek Letters

\[ \alpha \] angle of attack of the vehicle (radians or if specified degrees)

\[ \gamma \] geocentric (with respect to the local geocentric horizontal) glide angle of the velocity vector - see Fig. 19 and 20 (radians or if specified degrees)

\[ \gamma_e \] initial geocentric glide angle - see Fig. 37

\[ \gamma_{\text{eff}} \] effective initial glide angle, defined by Eq. (B2) - see Fig. 37

\[ \Delta t \] defined by Eq. (2.4)

\[ \Delta y \] the difference in the perpendicular distance to the Earth's surface from the vehicle for the two latitude positions at the beginning and end of Phase II as measured at the average geocentric radius - see Fig. 37 (feet)

\[ \Delta \alpha \] defined by Eq. (2.6)

\[ \epsilon \] emissivity of the material on the surface of the vehicle

\[ \theta \] geocentric longitude of the position of the vehicle relative to the Prime Meridian - see Fig. 19 (radians or if specified degrees)

\[ \theta_{2,2} \] longitude of the equatorial major axis (radians or if specified degrees)

\[ \lambda \] geocentric latitude of the position of the vehicle - see Fig. 19 (radians or if specified degrees)

\[ \rho \] density of the atmosphere (slugs/ft^3)

\[ \sigma \] Stefan-Boltzmann constant \((4.756 \times 10^{-13} \text{ Btu/ft}^2 \text{ sec} \cdot \text{R}^4)\)

\[ \varphi \] angle of roll of the vehicle (radians or if specified degrees)

\[ \psi \] geocentric azimuth of the position of the vehicle - see Fig. 19 (radians or if specified degrees)

\[ \Omega \] angular velocity of the \([I, J, K]\) flight axes system relative to the \([\hat{I}, \hat{J}, \hat{K}]\) basic reference system (radians/sec)

\[ \omega \] angular velocity of the Earth on its axis (radians/sec or if specified degrees/sec)
**Superscripts**

(·) \hspace{1cm} \text{first derivative of a variable with respect to time}

(··) \hspace{1cm} \text{second derivative of a variable with respect to time}

(Ⓡ) \hspace{1cm} \text{denotes three-dimensional vector}

**Subscripts**

(·)_{\text{max}} \hspace{1cm} \text{maximum value}

(·)_{\text{min}} \hspace{1cm} \text{minimum value}

(·)_{\text{s}} \hspace{1cm} \text{value when the last use of the reaction jets was started}
I. INTRODUCTION

1.1 Historical Background

When the problems of manned space flight were being solved in the late 1950's and the early 1960's, the state of the art with regard to protecting reentry nose cones was oriented toward an ablative type of material since the heat pulse for ballistic missiles is characterized by intense heating rates which normally last for quite short intervals of time. Thus, the occurrence of manned space flight so soon after the successful implementation of the ballistic missile program produced technological solutions based primarily on concepts developed for the ICBM system. As a consequence, the heat shield concept which is presently associated with the manned capsule configuration seemed both a natural and logical extension of the techniques which were and still are generally accepted practice.

However, the capsule shape is able to attain only a very low ratio of lift to drag (L/D). Therefore, only very limited manoeuvring capability is available to the pilot. He is unable to land the spacecraft outside a relatively small area once the retro rockets have been fired and the deboost phase has been completed. As a result, only a small portion of the Earth's surface can be reached during any given twenty-four hour period. This is because each successive orbital track is displaced by about 22.5 degrees as a result of the Earth's rotation. Therefore, during that time period there would be only about sixteen passes over the whole Earth each yielding only a thin strip of coverage.

As a result of the above restrictions, a large pickup and recovery force must be maintained in the principal landing zone during the entire course of an exercise in case of an emergency. This procedure is unsatisfactory for many reasons. First, and perhaps the most obvious, is the sheer cost. However, and this reason is probably of greater importance, weather conditions may dictate the choice of landing area and a sudden storm might require the movement of the pickup force to a more suitable position within an impossibly short time interval. Other difficulties are also apparent upon even a preliminary analysis. The efficient solution to the landing problem appears to require a different shape of reentry vehicle. The basic requirement is greater manoeuvrability, which requires a higher L/D. Thus, the capsule configuration must be abandoned in favour of a winged vehicle or lifting body.

Another obvious requirement for space flight for the foreseeable future will be the minimization of space vehicle weight. It is not necessary to point out in any detail the well-known correlation between the cost of the first stage booster and the mass of the final payload. Therefore, the first step is to design a vehicle with a moderate L/D which possesses no excess protection to enable it to reenter the Earth's atmosphere and land on the Earth's surface. Considering the minimal degree of protection available, the basic flight path upon which the preliminary design is based should be a simple trajectory for which the problems of protecting the spacecraft and its occupants are as moderate as possible. However, when the choice for the basic path and the method of protection is being made, consideration must be given to incorporating provision for departures from the basic path. The departures are necessary to allow the vehicle to manoeuvre to landing sites other than the basic terminal position.

Before considering the choice of the vehicle and the basic path, two general areas should be reviewed. The concepts involved in protecting the
vehicle against extreme heating rates will be presented first. Then, the manoeuvring capability or 'landing footprint' available to a vehicle will be discussed.

1.2 Types of Protection Against Heating

There are two basic methods of withstanding reentry heating. Ablation materials have successfully been used on both ICBM nose cones and the current series of manned capsules in addition to other classes of vehicles. Very broadly speaking, the basic principle is to use a material which has a large heat of sublimation in order to absorb as large a total heat load as possible for a given mass. The actual heating rates experienced during the reentry are usually unimportant for such a design. However, the maximum heat load is very strictly defined. When exceeded, the heat shield will fail and the spacecraft will disintegrate.

On the other hand, a radiative material can withstand much higher total heat loads as long as the maximum heating rate does not exceed the capability of the material. The permissible heating rate is primarily a function of the temperature versus time capability of the material (see Fig. 1), its emissivity, and (for higher than average heating rates of a short duration) its heat capacity. If the material is thin enough, its heat capacity may be neglected and the temperature then assumes the radiation-equilibrium value. In this situation, the rate at which heat is absorbed locally by the skin of the spacecraft equals the rate at which heat is being radiated from its surface. Changes in the heating rate are then quickly reflected by a change in the skin temperature. A higher rate requires a higher temperature to radiate more heat and vice versa. In general, the position at which the heating rate is most severe is the nose stagnation point of the vehicle, since the nose cap has the smallest effective radius of curvature. Therefore, the nose stagnation point will have the highest temperature on the spacecraft. Thus, if a suitable material can be found for the nose stagnation point, it can safely be assumed that the rest of the vehicle can also be adequately protected. Consequently, unless explicitly stated, the radiation-equilibrium nose stagnation point temperature will be the value to which reference is being made whenever temperature is referred to in this paper.

Two other points which should be mentioned in comparing ablative and radiative cooling. The materials which employ the latter method must always be replaced after use. On the other hand, while recoating may be necessary for a severe reentry, it should be possible to reuse in a conventional manner a vehicle which employs radiation cooling. It should be noted, however, that ablative materials are able to handle very large heating rates (until the maximum heat load is reached) that are usually far larger than the heating rates which the maximum temperature of a radiation cooled material can support.

1.3 Footprint Terminology

In order to initiate a return to the Earth's surface, an orbiting spacecraft will fire its retro rockets. Once this deboost phase is over, the vehicle becomes a hypersonic glider which is capable of influencing the position of its landing site only by means of aerodynamic force controlled by the attitude of the vehicle. A representative manoeuvre boundary or footprint from Ref. 1 is shown in Fig. 2. Note that the initial position is at the altitude at which aerodynamic forces begin to affect the path of the reentry vehicle. Throughout this investigation, the computations were started when the perpendicular distance to the Earth's surface was 350,000 feet. As a consequence, the reentry
values of velocity and glide angle at the initial position were treated independently of the orbital altitude at which retro-fire takes place. The actual flight path is effectively separated for computational purposes from the point and conditions of retro-fire. The only restriction is that the deboost cannot occur at an altitude that is greater than the maximum height which is compatible with conditions at the initial position.

In Fig. 3, additional terminology is given regarding the projection of the footprint onto an imaginary sphere which is used in place of the Earth's surface. Geocentric spherical trigonometry is used to define the geographical position over the imaginary sphere. Since the footprint boundary is given for an altitude of about 100,000 feet, the usual conversion problems of positioning the vehicle in terms of its geodetic latitude and longitude and of determining the distance travelled over an oblate Earth can be avoided.

The actual method of presenting the results requires some explanation. Assume for the moment that the initial position of the vehicle is in the Earth's equatorial plane and that the initial azimuth is either due East or West. Note that the initial glide angle is quite immaterial to the range definition (although it is of great importance to the pilot since too large a value would cause the spacecraft to disintegrate through overheating). In this situation, the equatorial plane, the great circle projection, the orbital track projection, and the footprint midline all coincide. The 'down range' is the absolute value of the longitude change from the initial position (by definition, down range is positive). The cross range is the final latitude (cross range may be positive or negative) if the initial azimuth was due East and the negative of the final latitude if the initial azimuth was due West. Note that the definition must be consistent with the reentry trajectory track over the Earth's surface, not with the Earth's co-ordinate system. A third range parameter, the azimuth or heading angle change, may also be simply defined in this situation as the final azimuth minus (or plus in the case of a due West reentry) ninety degrees.

At the initial position (which has been arbitrarily defined to be the point over the Earth's surface when the vehicle's altitude is 350,000 feet), the initial values of speed, altitude, glide angle, azimuth, latitude, and longitude define the path which the vehicle will travel over the Earth's surface. For a non-rotating, spherical Earth, the final pattern or shape of the projection of the path on the Earth's surface will not be affected by the initial values of the last three parameters. The resultant equivalent down range, cross range and azimuth change must be the same as for the case of the due East equatorial reentry with the same initial values of speed, altitude, and glide angle. For this situation, the great circle projection, the orbital track projection, and the footprint midline will still coincide with each other, but they will no longer coincide with the equatorial plane. However, for a rotating, oblate Earth, all three will tend to diverge. In this situation, the great circle projection remains in the same relative position. Consequently, all range parameters are measured in equivalent degrees over the imaginary sphere relative to the great circle projection through the initial position. Figure 3 illustrates a typical footprint over a rotating, oblate Earth. The subroutine which computes these range values is listed in Appendix C under the name of SUBROUTINE ICAME.
The type of vehicle chosen can be seen in Fig. 4. It satisfies the requirement of moderate lift to drag ratio since \((L/D)_{\text{max}}\) can range from 1.3 to 1.7 for this type of configuration.

With regard to the choice of the basic path, the manoeuvrability requirement is first used to determine the method of preventing the spacecraft from burning up during the reentry. When the vehicle must reach landing zones corresponding to large down range distances, prolonged flight times must result. Consequently the total heat input to an ablative material would be extremely large and might become prohibitive. However, in a design which radiates most of the heat away from the vehicle, the total heat load is not an important consideration. Therefore, a wide variation in the time interval over which the heating occurs will have a negligible effect on the vehicle design as long as the peak temperature experienced during the flight is independent of the point at which the spacecraft lands. Consequently, radiation cooling offers the only practical solution for protecting the vehicle against the extreme heating which is encountered during the reentry. This method then requires a trajectory that satisfies the requirement on the peak temperature.

An equilibrium glide was chosen as the basic path. This trajectory is characterized (see Appendix A for a more comprehensive discussion) by the absence of any long period or 'phugoid' oscillations along the trajectory. This situation results from the equality between the lift force and the difference between gravitational and centrifugal forces. There were a number of important reasons for the choice of the equilibrium glide. First, it satisfies the requirement that it be a simple trajectory since it is a fixed-attitude reentry. Second, it offers the lowest possible value for the peak temperature which is experienced by the vehicle during its reentry without the introduction of a sophisticated optimal programming procedure. Third, the equilibrium glide also satisfies the requirement that departures from this basic path may easily be made in order to provide the manoeuvring capability necessary to land at sites other than the basic terminal position.

The reader should also note the simple analytical result for the peak temperature which is experienced during an equilibrium glide (see Appendix A). The ease of obtaining this comparison standard and its simplicity of definition will greatly outweigh the benefits of obtaining any reduction in the standard peak temperature by complex optimal procedures.

The problem, then, is to conceive of as simple a procedure as possible which will allow the next generation of reentry vehicles the freedom to manoeuvre to their desired landing zones. Ideally, implementation of this procedure will not require any additional protection for the pilot or the vehicle.

As a solution to this problem, the author proposes a CONstant Temperature Attitude Control (CONTAC) system which will limit the peak temperature which is experienced by the vehicle during a reentry to a value which is only moderately (by 20°F or 30°F) greater than the peak temperature in a standard equilibrium glide. As a consequence, the CONTAC system will operate within the constraint that a negligible weight penalty is incurred relative to an
equilibrium glide, which it is assumed the vehicle can safely fly. A further constraint will be that neither predictive nor memory logic (with a minor exception) will be used by the CONTAC system. This requirement means that the CONTAC system must produce the solution on a real time basis when operating in the actual reentry environment.

It will be shown that the CONTAC system gives the pilot the extensive manoeuvrability which he requires in order to land the spacecraft at any point under or near the orbital envelope (Fig. 5). When coupled with the proper range controller, the complete system will provide automatic guidance to the final destination and prevent any dangerous temperature peaks which might destroy the vehicle. In addition, no undue waiting periods will be required. For most landing areas within the orbital envelope, the delay before retrofire will be less than twelve hours and no point will require more than twenty-four hours. For orbits of small inclination to the equatorial plane the waiting times would be usually less than one orbital period. Furthermore, the peak temperature will always be very near to the peak value encountered during an equilibrium glide except in those cases for which the atmospheric density variations about the mean are both large and random. Under these extreme conditions, it will be impossible to prevent a moderate rise in the peak temperature whenever the CONTAC system encounters a sequence of density variations for which no further correction by means of attitude control is available (i.e. when the optimum attitude - optimum angle of attack and zero roll angle - has already been assumed). Since adverse conditions which occur randomly are impossible to predict, a statistical result must be given and vehicle protection provided for the worst sequence of events which might occur with a certain design probability.

1.6 Comparison with Other Systems

As detailed in Ref. 1, a number of authors have studied the problem of guiding a spacecraft to its final destination. However, this investigation is not primarily concerned with the actual guidance, but rather with the means by which the heating problems may be minimized. In addition, the effects of Earth rotation and oblateness upon both the heating and the guidance problem will be ascertained. Finally, a test to determine the effect of random variations in atmospheric density will be presented.

With regard to the heating problem, a solution for minimizing the total heat load can be found in Refs. 2 and 3. However, since these solutions must operate within the inherent limitation imposed by the maximum allowable heat load, the manoeuvring capability is restricted to landing sites which are relatively close to the initial position.

The only other solution for minimizing peak temperatures that has been found in the open literature is a Temperature Rate Flight Control System which has been developed by Stalony-Dobrzanski (Ref. 4). The results show that the TRFCS is also able to prevent the peak temperature which occurs during a reentry from becoming greater than the value which is experienced during an equilibrium glide. (The use of the equilibrium glide as the basic path for the TRFCS is an additional reason for its choice as the standard of comparison in the present investigation.) However, in order to implement the TRFCS, the derivative of the temperature is required along with an elaborate analogue system to analyze the rate of temperature change. The attitude of the vehicle is adjusted using either the angle of attack or the bank angle as the primary control variable.
In contrast, the CONTAC system uses a simple digital program to analyze directly the actual temperature. In addition, the attitude is controlled via pitch and roll channels simultaneously.

With regard to the effects on the Earth's rotation and oblateness, there are no papers available which analyze the resulting changes in the footprint when constraints upon the peak temperature (or for that matter maximum heat load) are present. There have been a few studies which analyzed the effects of the Earth's rotation, but no constraints in connection with heating problems were incorporated.

Due to the lack of significant statistical data, the influence of density variations in the Earth's atmosphere is still extremely difficult to analyze. A number of average curves of density versus altitude are available for both summer and winter for latitudes from the equator to 75°N. Based on this type of information, a number of papers have appeared which deal with the effect of a constant percentage change in the density upon the guidance problem. However, even if heating effects were included in these investigations, the results cannot take account of the fact that the atmosphere is a dynamic system whose state can only be described in a statistical manner. With regard to the effects of random density variations about an average curve of density versus altitude, the author is not aware of any other studies which deal with the heating problem either from an optimal point of view (such as those represented by Refs. 2 and 3) or from a system approach (such as proposed by Stalony-Dobrzanski). Indeed, from a detailed examination of the way in which optimal procedures are applied, the inclusion of random density variations would seem to rule out this concept. The iteration procedure used depends on accurate external environmental specifications. If the density is a random quantity, it would not be possible under the presently used procedures to arrive at a final solution. Only a real time approach inherent in both the TRFCS and the CONTAC system can attempt to provide a solution. The present investigation will include the results of the attempts by the CONTAC system to deal with random density variations as well as constant percentage changes in the density.

1.7 Trajectory Phases

In order to simplify the description and analysis of the various aspects of the CONTAC system, the reentry trajectory has been divided into five rather distinct phases. Each phase is described rather briefly at this point in order to acquaint the reader with the overall concept. What the CONTAC system is capable of doing is also explained. In addition, problems which it cannot solve are mentioned and some suggestions are made regarding the solution of these difficulties.

It is assumed that the vehicle is under the primary control of a navigation and guidance system, that monitors and adjusts the two primary attitude angles, \( \alpha \) and \( \varphi \), in order to home on the desired landing site. At certain short but critical periods of the flight, the CONTAC system, as a result of monitoring vehicle temperature, automatically cuts in and overrides the navigation system. This situation occurs only during the portions of Phases III and IV of the reentry when the CONTAC system is in active operation. There are, of course, additional constraints on the attitude angles. These will be presented during the discussion of each phase.
The five phases of the trajectory are shown in Fig. 6. Although no such sharply defined divisions will occur during an actual flight, it is simpler to divide the reentry into the five parts shown for the purpose of explaining what takes place within each phase.

The CONTAC system has been designed for space vehicles entering the Earth's atmosphere from altitudes up to five hundred miles. It is conceivable that some application might be found for reentries from positions higher than this altitude. However, the primary requirement is that the maximum convective heating rate first experienced during the reentry (Fig. 6) must not produce a temperature greater than the peak value which would be experienced during an equilibrium glide. If this constraint is satisfied, the CONTAC system is capable of preventing all subsequent maximum temperatures from becoming very much greater than the peak experienced during an equilibrium glide. But, if the deboost altitude is above five hundred miles, the spacecraft must reenter the atmosphere at a speed which is substantially greater than the circular value. In order to prevent skipout, the reentry glide angle must also be increased. As a result, the first maximum experienced for the convective heating rate exceeds the peak for an equilibrium glide. Unless this single pulse can be tolerated, the vehicle fails.

Therefore, the CONTAC system is intended to be used for reentries from low orbital altitudes, up to about five hundred miles, with vehicles that use radiative cooling as the primary protection against failure due to heating. Examples of this type of operation include shuttle service for space stations, low orbit reconnaissance, and low orbit satellite service and repair.

1.8 Phase I - Pre-Atmospheric Flight

It should be noted at this point that although the CONTAC system may go into operation as soon as the retro rockets have been fired (or some other method is used to push the spacecraft out of its vacuum orbit), the angle of attack of the vehicle is quite immaterial for a period of between ten and thirty minutes, depending on the height of orbital ejection. This delay is due to the fact that aerodynamic forces do not become appreciable until an altitude of about 350,000 feet is reached (Fig. 7). Thus the pilot will have ample time to jettison any service modules or extraneous externals and to perform the tasks associated with the alignment of the spacecraft prior to the buildup of atmospheric density.

One point of importance which is associated with this phase is the deboost attitude alignment and velocity increment. Both values must be carefully computed and equally carefully implemented since a large error in either could cause excessive heating rates leading to the failure of the spacecraft. Figure 8 shows the effect of errors in retro-fire attitude alignment and velocity increment. However, past experience in the Gemini program indicates that the required accuracies for attitude alignment and velocity increment will not be difficult to achieve. Interpretation of the results indicates that the Gemini retro-fire system experienced maximum errors of one degree in attitude alignment and three feet per second in velocity increment (Ref. 5). A simultaneous maximum error in both would yield a maximum resultant error at the end of Phase I at 350,000 feet of 0.1 degrees in flight path angle, 10 feet per second in speed and 2 degrees in down range. Cross range might also be affected, but to a negligible extent; the maximum error could amount to only 0.2 degrees. Note that all of the above nominal error values are maximum and would be very unlikely to occur simultaneously.
Since experience has proven that a highly accurate calculation of the alignment attitude and velocity increment values is possible, there should be no difficulty in this area. However, it may be necessary to change the method by which the calculation is made. Consequently, the allowable margin of error is described and discussed in Section VI where the effects of incorrect reentry conditions at the beginning of Phase II are considered.

1.9 Phase II - Overall Down Range Control

For the purposes of this investigation, the reentry begins at 350,000 feet. This is due to the fact that for all practical purposes, the atmosphere has no effect on the flight path until the spacecraft reaches this altitude. Since the pilot has no control over the trajectory between the time he fires the retro rockets and the time he descends to this height, it is best to separate the problems associated with achieving the correct reentry conditions at 350,000 feet (by the proper attitude alignment and retro velocity increment at orbital altitudes) from the problems associated with controlling the vehicle's down range (by the proper angle of attack once lift and drag become appreciable). In fact, in order to simplify the problem even further, the portion of the flight within the atmosphere has been sub-divided into four parts, Phases II, III, IV, and V of the reentry.

During Phase II, the CONTAC system must be in operation; however, it exercises no actual control except to signal the end of Phase II at the point when the first $q_{max}$ is reached (Fig. 9). At this position, the CONTAC system switches from passive to active operation and Phase III begins. Therefore, during Phase II, the pilot (or the automatic guidance system) has the function of devising the angle of attack program that will eliminate down range errors which may initially be present or which may occur during Phase II due to the Earth's departure from a circular, non-rotating sphere with a standard time invariant atmosphere. Since Phase II lasts for about four minutes, there is ample time for any down range corrections which are required in order to ensure that the vehicle lands on target.

To some extent, it will be possible to compensate, by adjustment of the initial retro-fire, for down range errors which it is anticipated will occur during Phase II due to some of the above non-ideal conditions. As a first approximation, a constant angle of attack can be used during the calculation for Phase II. In this situation, down range errors which will occur due to rotation and oblateness can be evaluated rather easily for this portion of the flight. The same calculation procedure can also be used to find the down range errors which will occur during Phase I. Thus, at the deboost position, it should be feasible to make the necessary corrections for down range errors which will occur during Phases I and II due to Earth rotation and oblateness. Note that the time of retro-fire may also be varied in addition to the attitude alignment and velocity increment. This additional parameter will probably provide most of the necessary correction.

Errors in down range which occur during Phase II due to variations in the atmospheric density are much more difficult to evaluate. Since it is unlikely that an accurate prediction of the density can be made, an estimate based on the available statistical data should be made. It may then be possible to compensate for the anticipated down range error by adjusting the retro-fire parameters.
The only restriction on the angle of attack during Phase II (angle of roll is zero) is that it must not be so low as to cause the first \( \theta_{\text{max}} \) to exceed the maximum allowable value nor so high as to endanger the rear portion of the vehicle from excessive heating rates. In general, the optimum value lies very near the angle of attack which yields the maximum lift coefficient. Larger values for the angle of attack will reduce the down range and smaller values will increase it. For the vehicle studied, the optimum angle of attack was 55 degrees. The largest angle of attack used during Phase II was 65 degrees and the smallest value was 35 degrees. These two limits yielded a down range interval of about 75 degrees or about 5000 nautical miles.

It should be noted that almost all of the incremental down range achieved by modulating the angle of attack during Phase II is not apparent until the end of Phase III. This is because the primary effect of varying the entry angle of attack is a variation in the vehicle's speed at the end of Phase II without any major difference in altitude, glide angle, or down range distance (Fig. 9). A comprehensive discussion and analysis of this point is given in Sec. 6.2.

1.10 Phase III - Equilibrium Glide Attainment

The most critical portion of the reentry is Phase III, the various sections of which are shown in Fig. 10. During this part of the flight, the CONTAC system must remove energy from the vehicle at a rate designed to place the vehicle on or very near to an equilibrium glide trajectory. However, the rate of energy dissipation must also be flexible enough to allow the pilot (or the automatic guidance system) to control the vehicle's range in order to counteract all departures from non-ideal conditions as well as any errors present at the start of Phase III. Failure of the CONTAC-guidance combination to cope with adverse conditions will result in the vehicle over-heating during the flight or landing at the wrong location, either of which could be disastrous.

The actual details of the CONTAC system are given in Section II. At this point, it is sufficient to state that the phugoid or long period oscillation can be completely eliminated before the peak temperature is experienced and as a result, that value is very nearly minimized.

For most reentries, the attitude which is adopted subsequent to the first \( \theta_{\text{max}} \) is held for between two and four minutes. During this period, the pilot again has an opportunity to continue correcting any down range errors as well as beginning to reduce any cross range discrepancy. The additional incremental down range available during Phase III varies from about 1500 miles when the entry angle of attack is 65 degrees up to about 5000 miles when the entry angle of attack is 35 degrees. It should be repeated that practically all of the down range increment available due to Phase II attitude control actually occurs during Phase III due to the variation in speed at the beginning of Phase III. The pilot or the guidance system must be aware of and be able to compensate for this speed difference.

During Phase III, a number of pairs of values are used for the upper and lower limits on the attitude angles of the vehicle. These pairs of values are inputs to the CONTAC system and may be regarded as constants within this system. During an actual or simulated reentry in which a pilot or range controller is available, the attitude limits will be changed to provide the
necessary range control in order to land the spacecraft at the proper point on the Earth's surface. It is a feature of the CONTAC system that it is unaffected by the values for these limits as long as the upper and lower values of certain key pairs do not coincide. As long as this requirement is satisfied, the CONTAC system can damp the long period oscillations from the trajectory and place the vehicle on the equilibrium glide before the peak temperature occurs.

In very simple terms, the pilot will select the upper and lower limits for the angles of attack and roll which best fit the choice of the landing area toward which he desires to proceed, although certain practical considerations will restrict the pilot's freedom of choice (the upper and lower limits must not coincide for both pitch and roll). The CONTAC system then automatically adjusts the attitude angles (or the pilot himself if the associated control hardware should fail, but not the temperature readout equipment) to some value between the upper and lower limits of the angles of pitch and roll.

It has definitely been determined that the CONTAC system will hold the temperature at very nearly a constant value as long as the vehicle's attitude has not reached either an upper or lower limit. A single skip usually occurs just after the first \( q_{\text{max}} \), but this is quite intentional, and is designed to provide down range control capability during the early part of the reentry. Indeed, in some situations, this short skip is completely eliminated in order to substantially reduce the down range.

Thus, by the time the vehicle descends to about 260,000 feet, it has been placed upon an equilibrium glide corresponding to the attitude at which the spacecraft is currently being held. The lowest value for the peak temperature is achieved by using the optimum attitude. It occurs at maximum vertical force (see Appendix A) which is at 55 degrees angle of attack and zero roll angle. This optimum attitude is then held by the CONTAC system quite automatically until the peak temperature is attained. At that point, Phase IV begins and the attitude is changed to provide down range and cross range maneuvering capability. In practically all the re-entries studied, Phase IV began at an altitude of about 245,000 feet. However, in cases where the density variations were quite large, Phase IV began at an altitude as much as 30,000 feet lower than the above value.

1.11 Phase IV - Attitude Control Release

There will, of course, be problems associated with unpredictable increases in air density which may drive the temperature during this phase beyond its previous maximum. In such a situation, it might be expected that the CONTAC system would have to revert to Phase III operation. However, the reader may recall that no such sharply defined division occurs during the actual flight. Therefore, since the CONTAC system has neither memory nor predictive logic, it is impossible for any such division to exist in the controlling concept. When in operation, the CONTAC system is attempting to keep the temperature constant, and if an increase occurs, whatever the cause, the CONTAC system will change the attitude so as to reduce the temperature. If the attitude is already at its optimum value and the CONTAC system is unable to prevent a further increase in the temperature, then the conclusion to be drawn is quite obvious, i.e. Phase III had not yet been completed. But the reversion is only with respect to the author's arbitrary division of the flight. The CONTAC system is not affected.
In fact, the pilot (or the automatic guidance system) need not in principle even know that the CONTAC system exists, let alone how it operates. Naturally, he would wonder why he is unable to assume a particular attitude for range control during certain portions of the reentry. However, the CONTAC system is completely automatic so that it can cope with unpredictable situations about which the pilot may be unaware.

The pilot would, of course, actually be informed about both the existence and the operation of the COMAC system and he may, therefore, depend on it to properly reduce the attitude from the optimum position at the beginning of Phase IV to the desired values of pitch and roll sometime before the end of Phase IV. In addition, with regard to where Phase III ends and Phase IV begins, the pilot can leave this quite arbitrary decision entirely to the built-in logic of the CONTAC system. It is quite immaterial, as far as he is concerned, just where the peak temperature occurs during the flight. The only problem is to prevent that value from becoming any larger than necessary, and this problem the CONTAC system will solve all by itself.

Therefore, during Phase IV, the CONTAC system will gradually reduce the angles of pitch and roll until at some particular point in the flight, both values are under the complete control of the pilot. The transfer of this control may be revoked at any time after it is first made; however, after the temperature has fallen several hundred degrees, the pilot will realize that the transfer is complete and that the CONTAC system is no longer in operation. This transfer always occurs by the time the speed and height have dropped to values in the vicinity of 12,000 feet per second and 150,000 feet, respectively. At that time, a procedure to increase the cross range will have come into effect. A paper by Wagner (Ref. 6) discusses the variation of the bank angle of the spacecraft in relation to cross range control. The results indicate that large angles of roll should be used at the beginning of the reentry when the speed is high and that small angles of roll are best near the end of the flight when the speed is low. Various modes of dependence between the angle of roll and the flight parameters speed, altitude, and energy, were investigated to determine the best relationship. The calculations showed that a simple linear variation of bank angle versus speed gives the best overall performance.

1.12 Phase V - Final Range Control and Touchdown

As this paper is being written, the procedures for the final landing of a spacecraft are being developed in principle and tested in practice. However, an analysis of this phase is outside the scope of this investigation. It is therefore, of little value to speculate upon the best method to reach the landing site and perform the touchdown. Several papers (among them Refs. 7, 8, and 9) have appeared which deal with this manoeuvre. No, that's some ideas are already available concerning the problems which may be encountered and the methods of solution.

To this date, all of the flight experience in the supersonic regime has been with the X-15 vehicle. Thus, it may be quite some time before the best solutions to the various problems are found. However a basic outline of the final phase has emerged from this test program. The additional local down range which can be attained is about 200 miles for the conditions given in Fig. 3 for the footprint boundary. But, it may be noted that the final azimuth is approximately perpendicular to the boundary line. Consequently, the footprints which are presented in Sections VI and VII are a conservative estimate of the
vehicle's capability. One point of considerable importance should be noted. At the end of Phase IV, the values of speed and altitude within and on the footprint boundary are independent of the final position. Consequently, the same procedure can always be used for the approach during Phase V.

II. ATTITUDE CONTROL LOGIC

2.1 Introduction

This section discusses the detailed concepts behind the CONTAC system. The methods by which the system may be implemented on the spacecraft itself will also be suggested although the actual details may require modifications in order to conform to hardware requirements.

In order to give the reader some insight as to why the CONTAC system is used, a typical reentry is shown in Fig. 1a in which no attitude control was attempted except at the first skip point. At this position, the angle of attack was reduced from the optimum value of 55 degrees and zero roll angle to 22.7 degrees with a roll angle of 50 degrees to the left in order to obtain greater down range and to produce some cross range. However, due to the reduction in the angle of attack, a phugoid oscillation is initiated which results in a large temperature oscillation as well. Because the latter is synchronized almost exactly with the phugoid, it was logical to attempt to control both the phugoid and the temperature oscillations by monitoring only the temperature. The reason for the use of the temperature as the controlling variable is that this value can be measured directly on board the spacecraft with a high degree of accuracy. On the other hand, the altitude is a very difficult quantity to measure when the vehicle is still 250,000 feet above the Earth's surface. In addition, the temperature oscillations are the ones which must be controlled. It is quite immaterial whether or not the phugoid oscillation is controlled and damped out as soon as possible or is amplified and made larger so long as the temperature peaks are reduced. Of course, the fact that the elimination of the temperature oscillation also results in the elimination of the phugoid (Fig. 1b) is an added bonus which results in greater range control over the vehicle by the pilot.

It may be of interest to note that attitude control based upon altitude measurements was also investigated at an early stage in the development of the CONTAC system. In view of the results, which were inferior to the CONTAC concept, and the difficulty associated with the measurement of the altitude during an actual reentry, all effort on this aspect was abandoned.

A discussion of the basic philosophy behind the CONTAC system will provide a greater understanding of how it works. The main purpose is to dissipate as much of the vehicle's kinetic energy at as high an altitude as possible. In order to achieve this result, the CONTAC system is designed to prevent any decrease in the temperature after the first maximum is reached during the initial pass into the atmosphere. In order to prevent the decrease, the angle of attack is reduced which increases the L/D (actually the ratio of vertical force to drag, but at constant bank angle, L/D is just as appropriate) and thereby extends the glide path over a longer distance at the higher altitudes. However, the angle of attack eventually reaches its minimum value and at that point the temperature will begin to fall. However, when at some later time the temperature again starts to rise, a decision must be made as to when is the best time to increase
the angle of attack in order to prevent a subsequent temperature peak from exceeding the highest temperature experienced prior to that point in the flight. With no prediction scheme available, the best choice of the time to increase the angle of attack is immediately, i.e. as soon as the temperature again begins to rise. Keeping the temperature constant from that point on implies a gradual reduction in L/D which in turn may result in lower altitudes sooner than is desirable. There may be an optimum period of time to wait before initiating any increase in the angle of attack. However the absence of both memory and predictive logic within the CONTAC system precludes any use of a time delay interval.

In summary, the CONTAC system operates on the principle that the best place to lose speed is at as high an altitude as possible. Therefore, the angle of attack is gradually reduced in such a manner as to prevent the temperature from falling after it achieves a local peak value. That action results in an extension of the glide path at higher altitudes due to the resulting higher values of L/D. On the other hand, it was found best (by increasing the angle of attack) to prevent for as long as possible any subsequent rise in temperature after a minimum value occurs. The result is a system which always attempts to keep the temperature constant as long as angle of attack changes are possible, i.e. as long as the angle of attack was at neither its lower nor upper limit following a maximum or minimum, respectively, in the equilibrium nose stagnation point temperature.

The same principles are also used to control the bank angle in conjunction with the angle of attack. The only difference is that the "wings level" position is substituted for the upper limit since the maximum value for the ratio of vertical force to drag occurs for zero bank angle. On the other hand, the minimum value for this ratio occurs at the largest angle of roll. Therefore, the "lower limit" is the largest bank angle to the right or left depending on which direction the landing zone is situated with respect to the orbital track.

2.2 Choice of Attitude Limits

There are certain practical limitations on the freedom of choice for the upper and lower limits for both the angle of attack and the bank angle. Most of the limitations are due to the problems associated with obtaining as small a value as possible for the peak temperature experienced during the reentry of the spacecraft. In addition, the allowable range of values for the upper and lower limits changes with each phase of the reentry. Consequently, the best procedure will be to take each phase in turn and describe the reasons for the restrictions during that part of the trajectory.

(a) During Phase I, there are no restrictions and no practical limitations since the atmosphere is not effective in controlling the trajectory. The only point at which the attitude must be controlled is at the beginning of Phase I during the retro-fire. At that time, the attitude control must be quite precise, but there is, of course, no restriction on the choice of the angles. Near the end of Phase I, however, the angle of attack should be somewhere near the value to be used during Phase II since, otherwise, stability and control problems might be encountered.

(b) During Phase II, the bank angle should always be zero in order to produce the maximum possible vertical force for a given angle of attack. It might be feasible to introduce some aerodynamic size force on the vehicle at this
time, but the results would hardly be worth the added complexity necessary in the guidance system. This is because only large angles of roll would really be effective in producing cross range at this high a speed (Ref. 6), but the associated reduction in vertical force could not be tolerated.

The angle of attack should always be near the optimum value of 55 degrees which gives the maximum lift coefficient. In the presence of a predetermined atmosphere in which there were no density variations about the mean curve of density versus altitude, it would be possible to determine the desired angle of attack which would then be held throughout Phase II. However, in general, the guidance system will probably be making small changes throughout Phase II in order to compensate for predicted errors in the terminal down range position. Since the CONTAC system does not go into active operation until the end of Phase II, the angle of attack is completely under control of the guidance system during this time.

The largest angle of attack which was used herein during Phase II was 65 degrees. However, values as high as 75 degrees would probably be feasible if the spacecraft could withstand the resulting higher heating rates at the rear of the undersurface of the vehicle. The smallest value used was 35 degrees. When still smaller values were tried, the first maximum temperature was higher than the peak temperature for an equilibrium glide.

The designation of the upper limit on the attitude (no lower limit was used) was the entry angle of attack (which varied between 35 degrees and 65 degrees) and the entry angle of roll (which was always zero).

(c) During Phase III, the CONTAC system is in active operation about half the time. The vehicle is initially rolled to one side or the other and the nose pitched down in order to prevent the temperature from falling.

The lower limits are designated as the "skip angle of attack" and the "skip bank angle". The reason for this terminology is that the spacecraft usually performs a nearly constant altitude glide at the beginning of Phase III, involving a very small skip, or increase in altitude, of between zero and 20,000 feet. For some situations, especially when the entry angle of attack is large, the skip does not occur; however, the designation is still retained.

Note that the skip angle of attack and the skip bank angle are controlled by the range guidance system. These attitude angles are always the actual values held by the vehicle whenever the temperature is falling. The skip angles may, of course, be changed at any time during Phase III, but an immediate change in predicted down range will not result if the skip angles are not in current use due to override by the CONTAC system.

The largest value used for the skip angle of attack was 55 degrees; the smallest value was 5 degrees. The smallest value might require adjustment if its use results in upper surface heating which cannot be tolerated.

The "largest" value used for the skip angle of roll was zero and the "smallest" value -90 degrees. Notice that the use of a bank angle to the left preserves the algebraic definition of largest and smallest. If roll angles to the right are employed, the largest would still be zero and the "smallest" would then become +90 degrees. The key point to keep in mind is that the largest roll angle must always occur for the maximum vertical force (which obviously forces
the largest roll angle to be zero) and the "smallest" bank angle for the minimum vertical force.

The upper limits during Phase III are simply designated the "upper limits". The values are always the ones which give the maximum vertical force which is, of course, obtained at the angle for the maximum lift coefficient. Therefore, the upper limit for the angle of attack is 55 degrees and the upper limit for the bank angle is zero.

(d) Phase IV always starts with the attitude at the upper limits, the same as the upper limits for Phase III.

The lower limits for Phase IV are designated "lower limits". They are used when the speed falls appreciably and the CONTAC system goes into passive operation. Within the limitations imposed by the stability and available aerodynamic control of the vehicle, the guidance system is free to choose any values which it finds necessary to control the range. In general, the lower limit for the angle of attack will lie in the vicinity of the angle for \((L/D)_{\text{max}}\) while the lower limit for the bank angle is varied in the way which will result in the maximum cross range (if necessary).

(e) During Phase V, the CONTAC system is no longer in operation. All values for the attitude are dictated by guidance and control problems.

2.3 CONTAC Program Flow Diagram (see Fig. 12)

The basic purpose of the CONTAC system is to prevent a change in the temperature. The digital implementation of the basic concept is quite simple. The program to simulate and test the concept is, of course, much more detailed only because of other requirements such as program error checks and printouts of intermediate calculations. In general, conflicts often occur and the extra effort to take account of the numerous possibilities is considerable.

In the absence of these other details, the CONTAC system logic is shown in Fig. 12. There must be some memory (but no predictive requirement) so that the system can remember whether it is testing for a local minimum or maximum, and for the storage of \(T_{\text{last min}}\) or \(T_{\text{last max}}\), respectively. If the latter is currently being experienced, then when the temperature has fallen below the current local maximum by a reasonable margin \((T_{\text{deadband}})\), corrective action is taken. The determination of a local minimum is similar.

However, the simulation of the CONTAC system by a digital computer is prone to difficulties in logic flow and numerical calculation which have no counterpart in the real situation. The basic cause is the method by which a digital computer must perform the calculations. As opposed to a continuous value for all variables, as is found in the output of an analogue computer, or analogue instruments, a digital computer calculates the flight path only at specific points along that path. As a consequence, the deadband test shown in Fig. 12 may not be satisfied at one point in the calculation, but will be satisfied by too great a margin at the next point. If the digital calculation is proceeding in steps of two or three seconds, the time lag introduced by the digital calculation is totally unacceptable (Fig. 13) and measures must be found
to correct the situation*. The simplest method would be to reduce the time interval (actually, the flight path distance) between the points in the calculation. However, what value should be used? A step size small enough to satisfy the deadband test requirement produces an unacceptably large increase in computing time on the machine. A step size which increases computer time by only a reasonable amount will not always be able to provide an acceptably short time delay.

The solution adopted was to provide the calculation section with backup capability. As shown in Fig. 14, new points are calculated at shorter and shorter intervals until both questions (Fig. 12) are satisfied. At that point, the distance travelled and the corresponding time delay between the actual position and the position at backup point No. 5 (Fig. 14) is acceptable and the commands to adjust the angle of attack and the bank angle are calculated and implemented.

The backup capability allows the calculation section to proceed with step sizes which are as large as calculation errors permit and thereby perform the calculation as fast as possible. The choice of the smallest step size for the test is based upon practical experience with the runs and usually results in a maximum time delay of between ten and twenty milliseconds. During test runs to determine the maximum acceptable value, it was found that delays which were an order of magnitude greater produced results which were only negligibly different. However, for still larger time delays, a discrepancy was noted sometimes.

2.4 Vehicle Implementation

The problems of isolating the control logic from the details of the calculation will, of course, disappear once the system is placed on board the spacecraft and actually put into operation. As shown in Fig. 15, the calculation section of Fig. 12 will be replaced by the vehicle dynamics. The vehicle sensors will provide the forward interface to the control hardware and the reaction jets will be the rearward interface. In the case of a digital control system, the sampling rates for the analogue to digital convertor will probably be on the order of ten milliseconds or less for the temperature. Other variables need to be sensed less often and perhaps only when needed.

The absence of inner return loops in the vehicle control logic merely indicates that no signal is necessary until the appropriate test is satisfied. However, since the calculation will be in real time, a great deal of attention must be given to the way in which the available computational time is divided. Because the CONTAC system is often in the passive mode, i.e. no active attitude control is being performed, time sharing procedures will probably make the best use of the available computational capacity. The range controller will undoubtedly use the major share of the computer's time for predictive calculations since in view of the complex corrections required by the disturbing effects of the Earth's rotation and oblateness it is doubtful whether any simple predictive scheme would be satisfactory for the size of footprint within which the CONTAC system is able to operate. However, a simple interrupt system

* It is not so much the length of the time lag that complicates the situation, but the fact that it is unpredictable and varies from zero to perhaps five or even ten seconds. Since no real system will possess such a random variation in the size of the deadband, results computed on this basis would be highly suspect.
will allow for the monitoring of the temperature on a scheduled basis and the subsequent calculation of the attitude control moments when required.

The control scheme proposed here requires the measurement or computation of angles of attack and roll and their derivatives. No specific means are recommended for obtaining this information. However, one possibility is to compute them from the outputs of an inertial platform, which can provide both velocity and attitude in inertial space. Other techniques are also possible within the state of the art. When the CONTAC system commands a change of $\alpha$ or $\phi$, it is assumed that pitch or roll jets are turned on, producing a constant moment with zero time lag. Both $|\alpha|$ and $|\phi|$ are constrained to be less than certain selected maxima, and when these are reached, the control jets are turned off.

An extensive effort to produce a thermocouple which is capable of measuring the temperature at the nose stagnation point of the vehicle is described in Ref. 4. Actual tests were conducted which showed that such a device was able to measure values in excess of 3500°F for several minutes. Since the peak temperature which occurred for the vehicle used in this investigation was usually below 3000°F, there will not be any difficulty in procuring a satisfactory measuring instrument.

Another basic problem is to determine the actual stagnation point temperature. The position of the thermocouple is, of necessity, fixed in the nose of the vehicle. When changes are made in the angle of attack, the thermocouple will no longer be at the stagnation point and some method must be found to determine the actual stagnation point temperature.

The solution to this problem is also given in Ref. 4. The relationship between the measured and the stagnation point temperatures is given by

$$T_{\text{measured}} = T_{\text{stagnation}} \cos^{3/8} (\alpha - \alpha_{\text{stagnation}})$$

At high Mach numbers and for positions up to 40 degrees off the stagnation point, this relationship is in excellent agreement with both theoretical and experimental data for both hemispheres and swept cylinders. When the CONTAC system is active, the maximum variation in the angle of attack was from 5 degrees to 55 degrees. The thermocouple position could be at the stagnation point when the angle of attack of the vehicle is at 30 degrees. Consequently, the maximum offset would be only 25 degrees. For a stagnation point temperature of 3000°F, the temperature measured when the angle of attack of the vehicle is 55 degrees is only 4.2 percent less or 2875°F. The derivative of the temperature with respect to the angle of attack is also very small. The corresponding value would be 9.7°F/degree change in angle of attack.

In general, the vehicle will experience a small, but variable degree of side slip, owing to the presence of inadvertent yawing movements from asymmetries and control action. Consequently, two extra thermocouples could be provided to measure the sideslip angle and to control reaction jets which would achieve a null angle. The suggested positions are on the wing leading edges; offset nose positions would also be satisfactory.

Note that regardless of the position used for the thermocouples, several extra sensors would be used in parallel at each location in order to guarantee reliability.
2.5 Vehicle Attitude Control

As part of any simulation of a complex physical situation, a decision must be made as to just how accurately the mathematical model should be able to duplicate the real situation. In the case of the attitude control system, information is available (Refs. 10 and 11) which makes possible a considerable simplification in the complexity of the mathematical model. These references indicate that the short period mode possesses time constants orders of magnitude greater than the characteristic times of the CONAVAC system. As a result, most of the terms in the general equations describing the attitude of the vehicle may be eliminated - in particular, both the damping and spring constants are deemed negligible. Consequently, only the inertial term remains in the equations for rotation about any axis. It is also assumed that both the Earth's rotation and the movement of the vehicle within the Earth's reference system will have a negligible effect on the attitude equations.

In keeping with the above assumptions, the equations of motion for the attitude become very simple. Only the equation for the angle of attack will be developed since the bank angle equation is similar. The basic equation is

\[ I_\alpha \ddot{\alpha} = M \]  

(2.1)

where \( M \) is the constant pitching moment supplied by the pitch control jets when they are turned on. Thus

\[ \ddot{\alpha} = k_\alpha \]  

(2.2)

where \( k_\alpha \) is zero when the control jets are off, positive when set for nose-up change, and negative for nose-down change.

Assuming that the angle of attack was decreasing when the pitch jets were first turned on and that the angle of attack should now begin to increase (i.e. \( k_\alpha \) is positive), integration gives

\[ \alpha = \left\{ \begin{array}{ll}
\dot{\alpha}_s + k_\alpha(t-s) & \text{for } (t-s) < \Delta t \\
\dot{\alpha}_\text{max} & \text{for } (t-s) \geq \Delta t
\end{array} \right. \]  

(2.3a)

where

\[ \Delta t = (\dot{\alpha}_\text{max} - \dot{\alpha}_s)/k_\alpha \]  

(2.3b)

A final integration yields

\[ \alpha = \left\{ \begin{array}{ll}
\dot{\alpha}_s + \dot{\alpha}_s(t-s) + \frac{1}{2}k_\alpha(t-s)^2 & \text{for } (t-s) < \Delta t \\
\dot{\alpha}_s + \Delta \alpha + \dot{\alpha}_\text{max} \left( (t-s) - \Delta t \right) & \text{for } (t-s) \geq \Delta t
\end{array} \right. \]  

(2.5a)

where

\[ \Delta \alpha = (\dot{\alpha}_\text{max} + \dot{\alpha}_s)(\dot{\alpha}_\text{max} - \dot{\alpha}_s)/2k_\alpha \]  

(2.6)

Figure 16 shows the various relationships. Since equations (2.5a) and (2.5b) are simple analytical expressions which are a function only of time, they are easily introduced into the equations of motion presented in Section V. One point which may require explanation is the reason for the limit placed on \( \dot{\alpha} \). On the one hand, as large a value as possible is desirable in order to be able to respond as quickly as possible to rapid changes in atmos-
pheric density that require a rapid adjustment of the angle of attack. On the other hand, if the inertia of the vehicle (with its negative angular velocity—recall that \( \alpha_0 \) was negative) is to be quickly overcome, Eq. (2.4) shows that a large \( \dot{\alpha}_{\text{max}} \) requires a large angular acceleration. In addition, high angular rates lead to frequent cycling of the control which requires a high fuel expenditure for the reaction jets. Consequently, a reasonable compromise must be found between \( \dot{\alpha}_{\text{max}} \), the available angular acceleration, and the available mass of fuel. The values used in this investigation were quite conservative. Appendix D gives a sample set of values prior to the actual output.

Slight variations in sign in Eqs. (2.3) to (2.6) are required when \( k_\alpha \) is negative. Also, two sets of bank angle equations are required, one for positive \( k_\phi \) and one for negative \( k_\phi \). The bank angle equations are a little more complicated to use since the optional right or left hand turn requires additional logic within the digital computer program. For a left hand turn, all the \( \alpha \) symbols need only be replaced by \( \varphi \) to convert equations (2.3) to (2.6) into bank angle equations for a positive \( k_\phi \). Of course, the basic equation for the bank angle

\[
\varphi = k_\phi
\]  

which is exactly the parallel of Eq. (2.2) and applies equally to all situations.

III. VEHICLE CHARACTERISTICS

3.1 Introduction

In recent years, the general trend in lifting vehicle programs has been a shift away from configurations with relatively high lift to drag ratio \((L/D > 2.5)\), such as the X-20 Dynasoar, to a lifting body style as typified by the M2-F2, the HL-10, and the SV-5 (Ref.12) wingless reentry vehicles. The reasons for the choice seem to have been based primarily upon the advantages associated with much lower temperatures resulting from the elimination of small nose cap and leading edge radii. The disadvantages caused by a much smaller footprint or landing area capability are not severe enough to cause serious concern. By reason of the Earth's rotation under what is essentially a fixed inertial orbit, these wingless vehicles are still able to land at any point on the Earth which falls underneath or at least near the orbital envelope (Fig. 5) without having to wait more than 24 hours. Naturally, the coverage is severely restricted for equatorial or near-equatorial orbits; but even the Dynasoar could not claim complete coverage for equatorial orbits without incurring extremely high temperatures.

3.2 Vehicle Geometry

The vehicle chosen for this study is quite similar to current design concepts. Its mass and wing area never individually enter the equations of motion; consequently, they have been left in the form of the wing loading ratio. This parameter was then chosen from about the middle of the range which is presently under review. The value of 40 lb/ft\(^2\) is, therefore, simply representative.

The actual vehicle configuration is a delta wing planform with a leading edge sweep of about 70 degrees. Although the mass and wing area are not needed \( \alpha_0 \) the equations of motion, nevertheless some estimate of the nose cap and leading edge radii is desirable. A mass of about 8000 lb. gives a wing area of about 200 sq. ft. which would then lead to a base length of about 17 ft.
However, the body would be only about 20 ft. long since around 5 ft. of the nose would probably be cut off to provide a nose cap radius in the vicinity of 2.5 ft. The leading edge radius would also lie in this region.

3.3 Vehicle Aerodynamics

One of the most vital requirements for any investigation which seeks to predict the reentry flight path of a spacecraft with a reasonable degree of accuracy is a proper representation of aerodynamic forces throughout all phases of the trajectory. However, since the primary object of this study is to determine the effectiveness of the CONTOC system under varying operating conditions, only that portion of the flight where the heating rates are extreme will require highly accurate models for the aerodynamic forces which are exerted on the vehicle. Portions of the trajectory that are relatively unimportant to the primary purpose of the analysis may employ a model which is able to give only a reasonable approximation for the lift and drag coefficients.

Such is the case with the aerodynamic model chosen to represent the lift and drag forces on the vehicle under consideration. Several experimental studies (Refs. 13 to 15) show that expressions based on Newtonian theory adequately represent the sum total of the aerodynamic forces which are developed over a spacecraft during its reentry into the Earth's atmosphere as long as the Mach number is greater than about six or eight. As a result, a number of authors (Refs. 3, 16, and 17) have used either an exact or modified form of the flat plate, hypersonic, Newtonian approximation. Based on Ref. 14, the best estimate for the pressure coefficient on a flat plate is

\[ C_p = 1.75 \sin^2 \alpha \] (3.1)

The value assigned to the minimum drag coefficient, \( C_{D_{\text{min}}} \), is determined by the vehicle configuration, primarily by the degree of nose cap and leading edge bluntness. The nose cap radius should be as large as possible in order to reduce heating (See Sec. 3.4); however, a large value will produce a maximum lift-drag ratio which is too small to permit the required maneuverability. The compromise value of nose radius given in Sec. 3.2 yields a value for the minimum drag coefficient of

\[ C_{D_{\text{min}}} = 0.0625 \] (3.2)

If the value of the nose cap radius which has been suggested does not produce the above value for \( C_{D_{\text{min}}} \), an adjustment to the former may easily be made without unduly affecting the peak temperature, since the latter varies only as the eighth root of the nose radius (see Eq. 3.7).

The use of the preceding values for \( C_p \) and \( C_{D_{\text{min}}} \) gives the modified Newtonian expressions for the lift and drag coefficients, which are

\[ C_L = 1.75 \sin^2 \alpha \cos \alpha \] (3.3)
\[ C_D = 0.0625 + 1.69 \sin^3 \alpha \] (3.4)

These expressions are valid for \( 0^\circ \leq \alpha \leq 90^\circ \) degrees and are quite accurate down to Mach numbers of six or seven. For Mach numbers smaller than five, there is every indication that Eqs. (3.3) and (3.4) continue to be quite representative even down through the transonic region. References 18 to 20 indicate that for
angles of attack below about twenty or thirty degrees, the lift and drag coefficients continue to exhibit the same general pattern while the L/D slowly increases with decreasing Mach number. Consequently, Eqs. (3.3) and (3.4) may continue in use down through the high subsonic range if conservative estimates of manoeuvring capability are acceptable.

3.4 Equilibrium Nose Stagnation Point Temperature

Since the primary goal of this investigation is to determine the applicability of the CONTAC system, the exact numerical magnitude of the nose stagnation point convective heating rate is relatively unimportant. Indeed, it is even questionable whether extremals in the stagnation point temperature (see Fig. 14 as to how a maximum or minimum T is handled) which occur as a result of reversals in the gradient of the heating rate must be accurately predicted by the model which has been adopted. On the other hand, the heating rate must be simulated in a way which is sensitive to changes in atmospheric and flight conditions, especially at the times when the model predicts that the maximum rate is being experienced. In other words, it is not at all important that the predicted heating rate reach a maximum or minimum at precisely the same time that the actual heating rate reaches its extremal values (in fact, it is very dubious whether any model which simulates the heating rate, no matter how complex or sophisticated it might be, could predict the exact time at which the heating rate derivative changes sign); it is only necessary that if the heating rate is to remain relatively constant as a result of attitude control for the simulated model that this condition also occur in the real situation and at about the same time in the flight. This rather lax requirement is due to the adaptive nature of the CONTAC system which must only prevent any large changes in the heating rate as long as possible, i.e. as long as the attitude angle has not reached its upper or lower limit.

The particular form of the nose stagnation point convective heating rate which is used in this investigation has also been adopted by several other authors (Refs. 3, 17, 21, and 22) as being sufficiently representative of experimental results. The basic relationship is

$$q_n = \frac{17600}{\vartheta_n} \left( \frac{V}{26000} \right)^{3.15} \left( \frac{\rho}{0.002376} \right)^{0.5}$$  \(3.5\)

Since the value which is to be measured on board the vehicle is the temperature, this heating rate is then inserted into the temperature rate equation which is

$$\frac{\vartheta}{S} \frac{dT}{dt} = q - \varepsilon \sigma \vartheta^n$$  \(3.6\)

The radiation-equilibrium temperature is defined by setting the left hand side of Eq. (3.6) equal to zero. When an actual measurement is being made, the equilibrium value is produced by using such a small value of \(\frac{\vartheta}{S}\) at the temperature sensor that the left hand side becomes negligible and an excellent approximation to the equilibrium value is achieved very quickly. The data in Ref. 4 indicates that there will be a relatively negligible time lag in reaching the equilibrium temperature since the time constants associated with the cyclic commands produced by the CONTAC system are of the order of five to ten seconds.

Then, setting to zero the left hand side of Eq. (3.6) and substituting Eq. (3.5) for the heating rate, \(q_n\), gives the radiation-equilibrium nose
The stagnation point temperature is given by

\[ T^4 = \frac{17,600}{\epsilon\sqrt{R_n}} \left( V \right)^{3.15} \left( \frac{\rho}{0.002378} \right)^{0.5} \]  

(3.7)

The only terms which must have values assigned are \( \epsilon \) and \( R_n \). Normally, \( \epsilon \) is between 0.5 and 1.8 while \( R_n \) is about 2.5 feet for the vehicle under consideration. Thus, the products of \( \epsilon\sqrt{R_n} \) would range from 0.8 to 1.25 due to the uncertainty in \( \epsilon \). As a consequence, a "standard" value for the temperature is used for which a value of unity is taken for \( \epsilon\sqrt{R_n} \). The advantage is a standard from which the proper result for the temperature may be computed when the correct values of \( \epsilon \) and \( R_n \) are known without the disadvantage of being greatly in error. This standard stagnation point temperature is used in all the calculations as a measure of the actual temperature which would result for a device (e.g. a sophisticated thermocouple) placed at the nose stagnation point of the spacecraft. This variable will henceforth be referred to simply as the "temperature" (as already noted in Section 1.2). Then, in view of the above assumptions, the temperature is given by

\[ T^4 = Q \left( V \right)^{3.15} \left( \frac{\rho}{0.002378} \right)^{0.5} \]  

(3.8)

where

\[ Q = \frac{17,600}{\epsilon\sqrt{R_n}} \left( \frac{V}{26,000} \right)^{3.15} \left( \frac{\rho}{0.002378} \right)^{0.5} \]  

(3.9)

or

\[ Q = 9,397 \]  

(3.10)

IV. GEOPHYSICAL ENVIRONMENT

4.1 Introduction

One of the most important goals of this investigation is to study the effect of non-ideal conditions upon the peak temperature, primarily with a view to determining the extent to which these conditions impair the performance of the COVTAC system.

There are two distinct types of non-ideal conditions which are of concern. The first includes all the effects which may be predicted well in advance of the reentry. These effects include the Earth's rotation, the Earth's oblateness (i.e. the distortions of both shape and gravitational potential), and the geographical variations of the time average density (which are a function of both latitude and longitude as well as altitude). The second type includes those effects which are the result of departures from the time averaged mean values of the density.

A third source of disturbance is atmospheric wind. No data could be found showing the spatial gradient of the wind at the altitudes of concern of this investigation (the same problem was encountered with density variations and will be discussed in Sec. 4.6). However, local wind velocities of up to 500 feet per second do exist. Actually, almost all values are lower than 250 feet per second, but on isolated occasions they approach the higher figure. Assuming a step increase of 500 feet per second (from -500 to +500 feet per second seems far too large a difference to support even if both conditions did occur within a reasonable proximity), the percentage increase in the velocity at a speed of 20,000 feet per second would amount to only 2.5 percent. The
resulting increase in temperature would be slightly less (see Eq. 3.8). The percentage increase would be greater at lower velocities, but by the time the speed falls below about 16,000 feet per second, the temperature is no longer a problem. Since it would be the changes in wind speed that cause the difficulties (see Sec. 8.4), it is clear that winds should not present a problem for the CONTAC system.

4.2 Rotation of the Earth

There is a relative pseudowind present due to the Earth's rotation. The West to East speed at the equator amounts to about 1500 feet per second relative to inertial space. For the calculation of aerodynamic heating and forces, this relative velocity may be subtracted from the initial entry speed for equatorial West to East reentries and provides a small but not insignificant reduction in the peak temperature experienced during the flight.

The actual rotational motion of the Earth is quite complex. Parvin (Reference 23) has done an excellent summary of the various motions and lists five separate parts to the rotation of the Earth.

The primary motion is, of course, the familiar rotation of the Earth about its axis which produces the normal sunrise and sunset. This movement takes by definition, exactly one solar day to complete. However, the rotational speed of interest is with respect to inertial space and for this figure the sidereal day may be used. The length of time taken for one revolution in inertial space is 23 hours, 56 minutes, and 4.09 seconds. This works out to a rotational speed of 0.00417807 degrees of longitude per second.

The other four parts of the rotation will be mentioned for interest, but are all negligible with regard to the effect they have on the reentry of a spacecraft. First, there is the precessional rotation about the 23.5 degree inclination to the ecliptic pole which takes about 25,800 years. Second, other bodies in the solar system tend to cause a slight irregularity in the precessional rate called nutation; one component with less than ten seconds of arc has a period of nineteen years while the other component with less than one-half second of arc has a period of only two weeks. Next, the pole itself wanders, which would tend to affect the alignment of the Earth's axis; however, although the two periods are relatively short - twelve and fourteen months - the angles are less than four seconds of arc. Finally, there is an additional revolution due to the spiral motion of the galaxy which occurs once approximately every 200 million years. A better and much more complete description of these four additional motions may be found in the reference. It is quite clear, however, that they are too small to have any effect on the reentry.

4.3 Earth's Oblate Gravitational Field

During the past ten years, the use of artificial satellites to measure the potential field of the Earth has shown the inadequacy of representing the gravitational attraction as a simple central force. Using the notation of this paper, the general representation of the Earth's potential field as given by Kozai (Ref. 24) is
\[
U = \frac{GM}{r} \left[ 1 + \sum_{m=2}^{\infty} \sum_{m=0}^{\infty} \left( \frac{R_0}{r} \right)^n P_n^m (\sin \lambda) \left[ c_{n,m} \cos m \theta + s_{n,m} \sin m \theta \right] \right] \quad (4.1)
\]

where
\[
P_n^m (\sin \lambda) = \frac{1}{2^n n!} \cos^m \lambda \frac{d^{n+m}(\sin^2 \lambda - 1)^n}{d(\sin \lambda)^{n+m}} \quad (4.2)
\]

are spherical harmonics. Several other formulations exist, but all are essentially the same in that they use an infinite series of spherical harmonics to represent the potential field.

The gravitational vector is divided into its three principal components which results in (see Sec. 5.2 for a definition of the \([\vec{\Pi}, e, \vec{\nabla}]\) reference system)
\[
\vec{g} = \vec{g}_r + \vec{g}_\lambda + \vec{g}_\theta \quad (4.3)
\]

and requires the expressions for each of the gravitational components which are defined by the spherical gradient of the potential as
\[
\vec{g}_r = \frac{\partial U}{\partial r} \quad (4.4a)
\]
\[
\vec{g}_\lambda = \frac{1}{r} \frac{\partial U}{\partial \lambda} \quad (4.4b)
\]
\[
\vec{g}_\theta = \frac{1}{r \cos \lambda} \frac{\partial U}{\partial \theta} \quad (4.4c)
\]

Substitution of \(U\) into equations (4.4a), (4.4b), and (4.4c) and expansion of the spherical harmonics by means of Eq. (4.2) is tedious, but can easily be carried out. The result for the first few terms in the series is shown in order to provide the reader with some idea of the actual results. When all the substitutions are made, Eq. (4.3) for \(\vec{g}\) becomes
\[
\vec{g} = \frac{GM}{r^2} \left[ 1 + \left( \frac{R_0}{r} \right)^2 \left\{ \frac{3}{2} (3 \sin^2 \lambda - 1)c_{2,0} + 9 \cos^2 \lambda (c_{2,2} \cos \theta + s_{2,2} \sin \theta) \right\} 
+ \left( \frac{R_0}{r} \right)^3 \left\{ 2(5 \sin^3 \lambda - 3 \sin \lambda)c_{3,0} 
+ 2(15 \sin^2 \lambda - 3) \cos \lambda (c_{3,1} \cos \theta + s_{3,1} \sin \theta) 
+ 60 \sin \lambda \cos^2 \lambda (c_{3,2} \cos \theta + s_{3,2} \sin \theta) 
+ 60 \cos^3 \lambda (c_{3,3} \cos \theta + s_{3,3} \sin \theta) \right\} + ... \right] 
\]
\[
\begin{align*}
&+ \pi \frac{GM}{r^2} \left[ \left( \frac{R_0}{r} \right)^2 \left\{ 3 \sin \lambda \cos \lambda C_{2,0} - 6 \sin \lambda \cos \lambda (C_{2,2} \cos 2\theta + S_{2,2} \sin 2\theta) \right\} \\
&\quad + \left( \frac{R_0}{r} \right)^3 \left\{ \frac{1}{2} (15 \sin^2 \lambda - 3) \cos \lambda C_{2,0} \right\} \\
&\quad + \frac{3}{2} (11 - 15 \sin^2 \lambda) \sin \lambda (C_{3,1} \cos \theta + S_{3,1} \sin \theta) \\
&\quad + 15 (1 - 3 \sin^2 \lambda) \cos \lambda (C_{3,2} \cos 2\theta + S_{3,2} \sin 2\theta) \\
&\quad + 45 (\sin^2 \lambda - 1) \sin \lambda (C_{3,3} \cos 3\theta + S_{3,3} \sin 3\theta) \right\} + \ldots \ldots \\
&+ \bar{\omega} \frac{GM}{r^2} \left[ \left( \frac{R_0}{r} \right)^2 \left\{ 6 \cos \lambda (S_{2,2} \cos 2\theta - C_{2,2} \sin 2\theta) \right\} \\
&\quad + \left( \frac{R_0}{r} \right)^3 \left\{ \frac{1}{2} (15 \sin^2 \lambda - 3) (S_{3,1} \cos \theta - C_{3,1} \sin \theta) \\
&\quad + 30 \sin \lambda \cos \lambda (S_{2,2} \cos 2\theta - C_{2,2} \sin 2\theta) \\
&\quad + 45 (1 - \sin^2 \lambda) (S_{3,3} \cos 3\theta - C_{3,3} \sin 3\theta) \right\} + \ldots \ldots \right]
\end{align*}
\]

Although the forces due to even higher order harmonics will influence the orbit of an artificial satellite to some extent, the presence of terms other than those containing \( C_{2,0} \) has a negligible effect upon the reentry of a spacecraft. By neglecting all such terms, the complexity of the gravitational vector is considerably reduced, with the \( \{ \bar{\omega} \} \) component disappearing altogether. The result is

\[
\bar{g} = \bar{\omega} \frac{GM}{r^2} \left[ 1 + \left( \frac{R_0}{r} \right)^2 \frac{3}{2} (3 \sin^2 \lambda - 1) C_{2,0} \right] \\
+ \pi \frac{GM}{r^2} \left( \frac{R_0}{r} \right)^2 3 \sin \lambda \cos \lambda C_{2,0}
\]

The three remaining constants are given by

\[
GM = 3.986032 \times 10^{20} \text{ cm}^3/\text{sec}^2
\]

or

\[
GM = 1.407654 \times 10^{16} \text{ ft}^3/\text{sec}^2
\]

or

\[
R_0 = 6.378196 \times 10^8 \text{ cm}
\]

or

\[
R_0 = 20,925,840 \text{ feet}
\]

\[
C_{2,0} = -1.08248 \times 10^{-3}
\]

They were taken as the best average of all the values suggested by Refs. 24 to 26.
4.4 Oblate Shape of the Earth

The primary departure of the Earth from a perfect sphere is the flattening of the poles. Several papers by Kaula (Refs. 25 and 26 in particular) have also indicated the additional distances by which the Earth departs from its ellipsoidal shape, which in turn gives rise to the additional coefficients in Eq. (4.1). The variations are systematic to the extent that they indicate a triaxial tendency. Wagner (Refs. 27 to 29) has done extensive work on this subject with, however, conflicting results. Consequently, only an approximate (one figure accuracy) average was adopted with the major axis at about twenty degrees longitude west and a minor axis about 210 feet shorter ($f_q = 10^{-5}$) situated at seventy degrees longitude east.

In order to calculate the perpendicular height above a triaxial ellipsoid, it is sufficient to know the geocentric geographical position. The three variables will be the geocentric radius, the geocentric latitude, and the geocentric longitude. Several different methods were found in the literature which provided suggestions on how to calculate the perpendicular height above an oblate spheroid. While most were unnecessarily complex, a very simple solution is given by Baker (Ref. 30) which is deemed to be the best. In the notation of this paper, the suggested equation is

$$y = r - R_o \left[ 1 - f_p \sin^2 \lambda - \frac{1}{2} (f_p)^2 \left( \frac{R_o}{r} - \frac{1}{2} \right) \sin^2 2\lambda \right]$$ (4.10)

where $y$ is the perpendicular height above an ellipsoidal Earth.

In order to compensate for the triaxial tendency of the Earth's equator, an additional term of the form $R_o f_q \sin^2 \theta$ (which is derived in the same way as the $R_o f_p \sin^2 \lambda$ term for the polar flattening) is required. The higher order term in $f_p^2$ is, of course, negligible in comparison. This additional term must then be multiplied by $\cos^2 \lambda$ in order to reduce the triaxial effect to zero at the poles. A biasing constant is also required in order to raise the major axis and lower the minor axis above and below, respectively, the Earth's average equatorial radius, $R_o$. This biasing constant (of $\frac{1}{2}$) in equation (4.11) is required due to the definition of $f_q$ as given in the nomenclature.

In addition, $\sin^2 2\lambda$ is expanded using the usual trigonometric relationship in order to adapt the formula for digital computation (both $\sin \lambda$ and $\cos \lambda$ are already available, but $\sin 2\lambda$ would have to be calculated). The result is

$$y = r - R_o \left[ 1 - f_p \sin^2 \lambda - \frac{1}{2} (f_p)^2 \left( \frac{R_o}{r} - \frac{1}{2} \right) \sin^2 2\lambda \right] + R_o f_q \left( \sin^2 (\theta - \theta_{1,2}) - \frac{1}{2} \right) \cos^2 \lambda$$ (4.11)

An exact procedure which required several iterations to yield the result and which was not, therefore, useful during the actual computation of trajectories was then compared with the answer: from Eq. (4.11). The exact procedure indicated that a correction factor should be added to take account of higher order terms which were omitted from the power series approximation given in Eq. (4.10). The actual formula used during calculation of trajectories was

$$y = r - R_o \left[ 1 - f_p \sin^2 \lambda + \left( \frac{f_r}{r} + f_c \right) \sin^2 \lambda \cos^2 \lambda - \frac{1}{2} \right] \sin^2 2\lambda$$

$$y = r - R_o \left[ 1 - f_p \sin^2 \lambda + \left( \frac{f_r}{r} + f_c + f_f (\sin^2 \lambda - \frac{1}{2}) \right) \sin^2 \lambda \cos^2 \lambda \right] + R_o f_q \left[ \sin^2 (\theta - \theta_{1,2}) - \frac{1}{2} \right] \cos^2 \lambda$$ (4.12)
where  
\[ R_0 = 6.378196 \times 10^8 \text{ cm} \]  
or  
\[ R_0 = 20,925,840 \text{ feet} \]  
\[ f_p = 1/298.28 \]  
\[ f_q = 10^{-5} \]  \[ \theta_{2,2} = -20 \text{ degrees} \]  
were taken as the best average of all the values suggested by Refs. 24 to 29.

Also  
\[ f_r = \frac{1}{2} (R_0 f_p)^2 \]  
\[ f_c = R_0 (f_p)^2/8 \]  
And  
\[ f_r = -3.328 \]  
was optimized for the adopted values of \( R_0 \) and \( f_p \).

In practice, Eq. (4.12) evaluated the last three terms in single precision arithmetic and then used double precision addition when adding all five values together to achieve the final result. Comparison with the exact iterative procedure showed that the accuracy achieved with Eq. (4.12) was always better than 0.05 feet over the tested altitude range of zero to one million feet.

4.5 Mean Standard Atmosphere

In this investigation, the basic model used to define the variation of density with altitude was the U.S. Standard Atmosphere, 1962 (Ref. 31). As far as corrections for oblateness were concerned, the true perpendicular altitude above the surface was used to define the height as per Eq. (4.12). In essence, the atmosphere was assumed to be a uniformly thick blanket enveloping the Earth. Figure 17 from a report by Cole and Kantor (Ref. 32) gives ample evidence to support his assumption.

Additional information regarding standard atmospheres for different latitudes has recently appeared in the literature (Ref. 33). However, in view of the lack of detail concerning the space between each mean set of values, the additional accuracy afforded by the extra reference curves would not have been worth the additional programming effort necessary to make them available to the program calculating the flight trajectories. In addition, there must certainly be longitudinal variations, as well as those associated with latitude, and the current absence of information about these is further justification for ignoring both effects. Perhaps the most important point in favour of using a single standard atmospheric model over the whole Earth is the necessity of determining the effects of departures from a non-rotating, circular Earth in a manner which distinguishes them from those due to atmospheric density variations.
Atmospheric Density Variations

The presence of spatial density variations about some mean values in the atmosphere poses a problem with which any control system must cope. Whether the purpose of the control system is to achieve lower temperatures, a terminal position, or any other of several possibilities, the most difficult part of the problem is the fact that the density variations about the mean are random and can only be specified by way of statistical data. Since this paper deals with the feasibility of the CONTAC system rather than with the exact details of the hardware, a single mean curve, the U.S. Atmosphere, 1962, was deemed sufficiently representative for all but a few reentries. Then, the problem was to find a model that would permit a reasonably accurate picture of the spatial density variations to be found in the atmosphere about the mean curve of density versus altitude.

One question that was deemed important, however, was whether the shape of the mean curve of density versus altitude was itself of any great importance. In order to answer it, a number of quite arbitrary exponential curves were used with widely varying values of the scale height. Actually, each new reference curve was composed of two exponentials. The higher altitude portion was defined by the scale height and the point at which it intersected the U.S. Standard Atmosphere, 1962. The lower curve was defined by the cut-off point for the upper curve and a second point of intersection with the U.S. Standard Atmosphere, 1962 which was below the cut-off point. The results are given in Sec. 8.2.

Two types of spatial density variations about the mean were used. One was based on altitude and the other was generated as a set of Gaussian random numbers which were spaced at equal intervals along the flight path length. Considerable attention was paid to the choice of these two types due to the almost complete lack of data regarding spatial density variations in the atmosphere. A great many papers could be found which gave day to day variations at a given point. Other investigators gave the results of calculations which defined new mean atmospheres at various latitudes. However, Cole and Cantor (Ref. 32) did provide some information regarding the maximum spatial density variations about the mean which might be expected as well as the distances over which the departures might occur.

For the density variations based on altitude, an actual experimental result shown in Ref. 32 confirmed a suggestion by Etkin (Ref. 34) that a simple sinusoidal variation about the mean of density versus altitude would be realistic. The necessary parameters were the amplitude of the variation about the mean, the wave length, and the point of zero variation or the nodal position. A range of values were used and the results can be found in Sec. 8.3.

In the case of random departures from the basic curve, a statistical approach was required. The method chosen to simulate the variations about the mean was a series of random numbers representing fractional deviations from the mean, with a Gaussian distribution. After specifying the variance of the series, the numbers were then assigned to equidistance positions along the flight path. In the absence of any other data, linear interpolation was used between the points to find the fractional departure from the mean value for the density. The fraction could be positive or negative, but was limited to an absolute magnitude of three times the variance (which it would be less than 99%
of the time in any case) in order to prevent bizarre results such as a negative density. In order to provide a statistical result, a number of trajectories was computed, one for each of several different series of random numbers having the same variance and correlation interval. (The effect of the random density model is presented in Sec. 8.4.)

While the above method of simulating the random variation to be expected in the atmosphere is certainly not the best possible model, it should at the very least be adequate. In any case, the lack of any concrete data upon which to base a more realistic model does not warrant a more sophisticated approach. It is, of course, possible that the information exists in classified reports. If so, the results presented in Sec. 8.4 will provide an immediate answer as to whether the CONTAC system will perform in a satisfactory manner under all expected atmospheric variations. On the other hand, if such data actually does not exist, then an evaluation of the CONTAC system must wait until it has been determined.

V. EQUATIONS OF MOTION

5.1 Basic Reference System

The choice of a basic reference system from among the multitude of possibilities is usually dictated by the problem which is to be solved as well as by the methods of solution which are available. In many situations, as in the present case, the choice is easily made due to the many restrictions imposed by unfavourable reference systems which might require an unnatural set of equations or perhaps extra terms in the equations. Initially, this investigation was begun using an analoge computer of limited capacity. When the problem outgrew the available equipment, a large scale digital computer was substituted. The original reference system was retained, however, due to the background of experience and confidence.

The basic reference system chosen is a set of axes $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ fixed at the mass centre of a rotating Earth (Fig. 18). Thus, the position of the vehicle is naturally expressed in terms of the geocentric latitude, $\lambda$, the geocentric longitude, $\theta$ (positive to the East), and the geocentric radius, $r$ (Fig. 19); the velocity of the vehicle is then given by the geocentric azimuth, $\psi$ (heading with respect to the North-positive to the East), the glide angle, $\gamma$ (angle between the local geocentric horizontal and the velocity vector-positive downward), and the magnitude of the velocity vector, $\mathbf{V}$ (Fig. 19). However, since all aerodynamic data is always referenced to a set of axes which are aligned in some way with the vehicle, a second reference system which will be termed flight axes $(\mathbf{i}_f, \mathbf{j}_f, \mathbf{k}_f)$ must be introduced (Fig. 20). It should be noted that these flight axes are not fixed to the body; on the contrary, the $(\mathbf{j}_f)$ axis always remains in the local geocentric horizontal plane, thereby forcing the $(\mathbf{i}_f, \mathbf{k}_f)$ axes to remain in the local geocentric vertical plane. In order to complete the orientation requirements, the $(\mathbf{i})$ axis is directed along the velocity vector, $\mathbf{V}$, and the $(\mathbf{k})$ axis is directed downward (in this study $|\gamma| < 90$ degrees). As a consequence of this definition for the flight axes, the side force equation will include the component of the lift in that direction, namely: $L \sin \varphi$. For those readers who have not encountered this vehicle reference system of flight axes (for example, Refs. 35 and 36), it should again be mentioned that this investigation is concerned with a three degree of freedom system of equations.
for a point mass vehicle. Since the pitch, roll, and yaw angles which are dependent variables in a six degree of freedom system are now control variables, it is rather useful to simplify the three force equations by using the required components of the lift and drag. The point mass vehicle is then rotated in pitch and roll (sideslip is assumed negligible and the yaw angle is, therefore, set identically equal to zero) so as to develop these required lift and drag components. The validity of the assumption regarding the representation of aero-dynamic forces has been examined in Section III.

5.2 Conversion Between Reference Systems

Since the equations of motion are written in the flight axes reference system, it is necessary to find the components of all forces in this final reference system. (Fig. 21). Thus the conversion matrices from any of the basic or intermediate reference systems to the flight axes reference system must be found. A rotation about the \((\hat{K})\) axis (Fig. 22) gives the transformation from the \((\hat{I},\hat{J},\hat{K})\) set of axes to the \((\hat{I}',\hat{J}',\hat{K})\) set of axes. The second rotation about the \((\hat{J}')\) axis (Fig. 23) gives the transformation to the \((\hat{n},\hat{e},\hat{v})\) set of axes. The third rotation about the \((\hat{v})\) axis (Fig. 24) gives the transformation to the \((\hat{O},\hat{\eta},\hat{V})\) set of axes. Finally, the fourth rotation about the \((\hat{j})\) axis gives the transformation to the \((\hat{3},\hat{j},\hat{g})\) reference system of flight axes (Fig. 25). These four rotations naturally require the four simple conversion matrices given with each figure. The overall conversion matrices from any of the intermediate reference systems are then easily found to be

\[
\begin{bmatrix}
\hat{I} \\
\hat{J} \\
\hat{K}
\end{bmatrix}
= \begin{bmatrix}
\cos\gamma & 0 & -\sin\gamma \\
0 & 1 & 0 \\
\sin\gamma & 0 & \cos\gamma
\end{bmatrix}
\begin{bmatrix}
\hat{I} \\
\hat{J} \\
\hat{K}
\end{bmatrix}
\]  
(5.1)

\[
\begin{bmatrix}
\hat{n} \\
\hat{e} \\
\hat{v}
\end{bmatrix}
= \begin{bmatrix}
\cos\gamma\cos\psi & -\sin\gamma & -\sin\gamma\cos\psi \\
\cos\gamma\sin\psi & \cos\psi & -\sin\gamma\sin\psi \\
\sin\gamma & 0 & \cos\gamma
\end{bmatrix}
\begin{bmatrix}
\hat{I} \\
\hat{J} \\
\hat{K}
\end{bmatrix}
\]  
(5.2)

\[
\begin{bmatrix}
\hat{I} \\
\hat{J} \\
\hat{K}
\end{bmatrix}
= \begin{bmatrix}
\cos\gamma\cos\psi\sin\lambda + \sin\gamma\cos\lambda & -\sin\gamma\sin\psi\sin\lambda - \sin\gamma\cos\psi\sin\lambda & -\sin\gamma\cos\psi\sin\lambda - \sin\gamma\cos\psi\sin\lambda \\
-\cos\gamma\sin\lambda & \cos\gamma\cos\lambda & \cos\gamma\sin\lambda
\end{bmatrix}
\begin{bmatrix}
\hat{I} \\
\hat{J} \\
\hat{K}
\end{bmatrix}
\]  
(5.3)

The conversion matrix between the basic reference system \((\hat{I},\hat{J},\hat{K})\) and the flight axes reference system \((\hat{i},\hat{j},\hat{k})\) could be found and expressed in the same manner; however, it is unnecessary since it is not used.

5.3 Vector Equation of Motion

An evaluation was made of the relative magnitudes of all forces which may act upon a spacecraft during a reentry into the Earth's atmosphere. Forces due to other planetary bodies including the moon, the solar wind, and magnetic fields often have an appreciable effect on the orbits of particular satellites when these forces are experienced over long periods of time. However,
for a reentry trajectory, it can easily be shown (Ref. 38) that only aerodynamic forces and the Earth’s gravitational field have a non-negligible effect upon the flight path. In addition, based on the information in Ref. 23, it may be assumed that the basic reference system, \((I,J,K)\) (see Fig. 18), is rotating in inertial space at a constant angular velocity.

The single vector equation of motion which is required for a point mass moving in such a rotating reference system is given by (Ref. 39)

\[
\frac{\mathbf{F}}{m} = \mathbf{a} + 2(\mathbf{\omega} \times \mathbf{V}) + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r})
\]  
(5.4)

where \(\mathbf{a}\) is the acceleration relative to the \((I,J,K)\) reference system, i.e. relative to the Earth.

The only external forces are either aerodynamic or gravitational. It is appropriate, therefore, to separate the left hand side of Eq. (5.4) and write

\[
\mathbf{F} = \mathbf{F}_A + mg
\]  
(5.5)

The relative acceleration, \(\mathbf{a}\), in the \((I,J,K)\) system has components along the \((I,J,K)\) axes that are obtained by the rule for transforming a derivative between rotating frames of reference (Ref. 40), i.e.

\[
\mathbf{\bar{a}} = \dot{\mathbf{V}} + (\mathbf{\Omega} \times \mathbf{V})
\]  
(5.6)

where \(\mathbf{\Omega}\) is the angular velocity of the \((I,J,K)\) system relative to the \((I,J,K)\) and the components of \(\mathbf{V}\) are the rates of change of the components of \(\mathbf{V}\) along the \((I,J,K)\) axes.

Equations (5.4) to (5.6) can be combined to give

\[
\frac{\mathbf{F}_A}{m} + \mathbf{g} = \dot{\mathbf{V}} + (\mathbf{\Omega} \times \mathbf{V}) + 2(\mathbf{\omega} \times \mathbf{V}) + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{V})
\]  
(5.7)

5.4 Expansion of the Single Vector Equation of Motion

The expansion of each of the terms in Eq. (5.7) in the \((I,J,K)\) flight axes system is not difficult although it is somewhat involved and lengthy. The three components of each term will first be found in whichever axes system is most convenient then, if necessary, transferred to the \((I,J,K)\) flight axes system, and finally collected to give the three complete force equations. Starting with the first term, the results are as follows.

The aerodynamic forces are given by

\[
\mathbf{F}_A = - I C_D(\alpha) \frac{\rho(y)}{2} V^2 S
\]

\[
+ J C_L(\alpha) \frac{\rho(y)}{2} V^2 S \sin \phi
\]

\[
- K C_L(\alpha) \frac{\rho(y)}{2} V^2 S \cos \phi
\]  
(5.8)

\(C_L(\alpha)\) and \(C_D(\alpha)\) are given by Eqs. (3.3) and (3.4). The expression for the density, \(\rho(y)\), has been discussed in Sec. 4.5 and the evaluation of the height \(y\), above a triaxial ellipse is given by Eq. (4.12).
The gravitational force vector is given by Eq. (4.3) in the $\mathbf{e}, \mathbf{e}, \mathbf{v}$ reference system. Transforming the components from this intermediate reference system to the $\{i,j,k\}$ flight axes system via equation (5.2) gives the gravity vector as

$$
\mathbf{g} = \mathbf{T} (-g_r \sin \gamma + g_\lambda \cos \gamma \cos \psi + g_\theta \cos \gamma \sin \psi) \\
+ \mathbf{J} (-g_\lambda \sin \psi + g_\theta \cos \psi) \\
+ \mathbf{K} (-g_r \cos \gamma - g_\lambda \sin \gamma \cos \psi - g_\theta \sin \gamma \sin \psi)
$$

The expressions for $g_r$, $g_\lambda$, and $g_\theta$ are developed and stated in Sec. 4.3. Although, as noted in Sec. 4.3, $g_\theta$ is too small to affect the reentry trajectory of a spacecraft, the expressions have purposely been left in a generalized form so that the precise effect of any term in the series may be ascertained should it be desirable at some future date. In addition, the values of $g_r$ and $g_\lambda$ will normally include only the major effect of the Earth's oblate gravity potential, namely $C_2^0$ term. However, here again, the expressions are left in their generalized form. The truncated expressions which were actually used in the calculations may be found by a comparison of Eqs. (4.3) and (4.6) to be

$$
ge_r = \frac{-GM}{r^2} \left[ 1 + \left( \frac{R_0}{r} \right)^2 \frac{3}{2} (3 \sin^2 \lambda - 1) C_2^0 \right] 
$$

$$
ge_\lambda = \frac{-GM}{r^2} \left[ \left( \frac{R_0}{r} \right)^2 3 \sin \lambda \cos \lambda C_2^0 \right] 
$$

$$
ge_\theta = 0
$$

The terms on the right hand side of Eq. (5.7) are either very simple or quite complex. The first term is simply

$$\mathbf{V} = \mathbf{T} \mathbf{V}$$

Before proceeding with the next term, the expression for $\mathbf{V}$ should be found. With reference to Figs. 22 to 25, it can easily be seen that the sum total of all the rotations which are required to rotate from the $\{i,j,k\}$ basic reference system to the $\{i,j,k\}$ flight axes reference system yields the relative rotational velocity which is

$$\mathbf{\Omega} = \mathbf{K} \dot{\theta} + \mathbf{J} \dot{\lambda} + \mathbf{V} \dot{\psi} - \mathbf{J} \dot{\gamma}
$$

The negative sign in the last term is required due to the rotation of $\mathbf{\Omega}$ into $\mathbf{V}$ which is in the opposite sense from the normal right hand rule.

At this point, the three kinematic relationships for the position of the vehicle $\mathbf{i}$, $\mathbf{\lambda}$ and $\mathbf{\theta}$ should be defined since the last two are required immediately. They are

$$\dot{i} = -V \sin \gamma$$

$$\dot{\lambda} = \frac{V}{r} \cos \gamma \cos \psi$$

$$\dot{\theta} = \frac{V}{r} \cos \gamma \sin \psi$$

Then, in order to transform $\mathbf{\Omega}$ into the $\{i,j,k\}$ flight axes system, the trans-
formation given in Eqs. (5.2) and (5.3) must be used, along with the substitution of Eqs. (5.16) and (5.17). The result is

$$\vec{U} = \vec{I} (\dot{\psi} \sin \gamma - \frac{V}{r \cos \lambda} \sin \gamma \cos \lambda \sin \alpha \sin \psi)$$

$$+ \vec{J} (-\dot{\gamma} - \frac{V}{r} \cos \gamma)$$

$$+ \vec{K} (\dot{\psi} \cos \gamma - \frac{V}{r \cos \lambda} \cos^2 \gamma \sin \lambda \sin \psi)$$

(5.18)

Finally, combining the expression for $\vec{U}$ with $\vec{V} = \vec{I} V$ yields the desired cross product for the second term on the right hand side of Eq. (5.7). The result is

$$(\vec{U} \times \vec{V}) = \vec{I} (0)$$

$$+ \vec{J} (\dot{\psi} V \cos \gamma - \frac{V^2}{r \cos \lambda} \cos \gamma \sin \lambda \sin \psi)$$

$$+ \vec{K} (\dot{\gamma} V + \frac{V^2}{r} \cos \gamma$$

(5.19)

The next term is somewhat easier to express. It is easily seen (Fig. 23) that

$$\omega = \vec{K}$$

(5.20)

which upon substitution of Eq. (5.3) and combination with $\vec{V} = \vec{I} V$ results in

$$2(\vec{U} \times \vec{V}) = \vec{I} (0)$$

$$- \vec{J} (2 \omega V [\sin \gamma \cos \lambda \cos \psi + \cos \gamma \sin \lambda])$$

$$+ \vec{K} (2 \omega V \cos \lambda \sin \psi)$$

(5.21)

Finally, the last term is easiest to expand in the reverse direction. It is also easy to see (Fig. 23) that

$$\vec{r} = -\vec{V} r$$

(5.22)

or

$$\vec{r} = \vec{I} r \cos \lambda - \vec{K} r \sin \lambda$$

(5.23)

which upon combination with Eq. (5.20) gives

$$\vec{w} \times (\vec{w} \times \vec{r}) = \vec{I} \omega^2 r \cos \lambda$$

(5.24)

and after substitution of Eq. (5.3) results in

$$\vec{w} \times (\vec{w} \times \vec{r}) = \vec{I} (\omega^2 r \cos \lambda [\sin \gamma \cos \lambda + \cos \gamma \sin \lambda \cos \psi])$$

$$- \vec{J} (\omega^2 r \cos \lambda \sin \psi)$$

$$+ \vec{K} (\omega^2 r \cos \lambda [\cos \gamma \sin \lambda - \sin \gamma \cos \lambda \cos \psi])$$

(5.25)

All of the terms in Eq. (5.7) have now been expanded into the $\{\vec{I}, \vec{J}, \vec{K}\}$ flight axes reference system. Since each of the three components are
independent of the other, three first order, non-linear, differential equations may be written by collection of all the terms in Eqs. (5.8), (5.9), (5.13), (5.19), (5.21), and (5.25). The result is

\[ \dot{V} = -C_D(\alpha) \frac{\rho(y)V^2 S}{m} - g_r \sin \gamma + g_\lambda \cos \gamma \cos \psi + g_0 \cos \gamma \sin \psi - \omega^2 \cos \lambda (\sin \gamma \cos \lambda + \cos \gamma \sin \lambda \cos \psi) \]  
(5.26)

\[ \dot{V} = -C_L(\alpha) \frac{\rho(y)V^2 S}{m} \cos \phi - \frac{V^2}{r} \cos \gamma - g_\lambda \cos \gamma - g_\lambda \sin \gamma \sin \psi - \omega^2 \cos \lambda (\cos \gamma \cos \lambda - \sin \gamma \sin \psi) - 2\omega \psi \cos \lambda \sin \psi \]  
(5.27)

\[ \dot{W} = C_L(\alpha) \frac{\rho(y)V^2 S}{m} \sin \phi + \frac{V^2 \cos \gamma \sin \lambda \sin \psi}{r \cos \lambda} - g_\lambda \sin \psi + g_0 \cos \psi + \omega^2 \cos \lambda \sin \psi + 2\omega (\sin \gamma \cos \lambda \cos \psi + \cos \gamma \sin \psi) \]  
(5.28)

In addition to the above three differential equations, and the positional differential equations (5.15), (5.16), and (5.17), several other relationships are required which have already been stated or developed previously. For convenience, they will be mentioned again at this point. Equations (3.3) and (3.4) give the relationship between the lift and drag coefficients, \( C_L(\alpha) \) and \( C_D(\alpha) \), and the angle of attack, \( \alpha \). The relationship between the density, \( \rho(y) \), and the perpendicular height above the Earth's surface, \( y \), has been discussed in Sections 4.5 and 4.6. The perpendicular height above the surface of a triaxial Earth, \( y \), is given by Eqs. (4.12) to (4.18). Finally, the expressions for \( g_r \), \( g_\lambda \), and \( g_0 \) are given by Eqs. (5.10), (5.11), and (5.12). Should it be desirable to include additional terms in the oblate gravity field of the Earth, equations (4.1) to (4.5) will serve to illustrate the manner by which it may be done.

At this point in the development of the equations of motion, it was found to be convenient to make a change in the independent variable from time, \( t \), to path length along the trajectory, \( s \). The original reasons for making this change are no longer a justification for it. However, it was convenient to maintain the new independent variable and the following first order, non-linear, differential equations were the ones which were actually integrated.

The three force equations become

\[ \frac{d(y^2)}{ds} = -\frac{C_d(\alpha)S}{m} \rho(y)V^2 - 2(g_r \sin \gamma - g_\lambda \cos \gamma \cos \psi - g_0 \cos \gamma \sin \psi) - 2\omega^2 \cos \lambda (\sin \gamma \cos \lambda + \cos \gamma \sin \lambda \cos \psi) \]  
(5.29)

\[ \frac{dy}{ds} = -\frac{C_L(\alpha)S \cos \phi}{2m} \rho(y) - \left( \frac{1}{r} + \frac{g_r}{V^2} \right) \cos \gamma - \frac{1}{V^2} (g_\lambda \sin \gamma \cos \psi - g_0 \sin \gamma \sin \psi) - \frac{\omega^2 \cos \lambda}{V^2} (\cos \gamma \cos \lambda - \sin \gamma \sin \lambda \cos \psi) - \frac{2\omega}{V} \cos \lambda \sin \psi \]  
(5.30)
\[ \frac{d\psi}{ds} = \frac{C_L(c)\sin\psi}{2m} \rho(y) \frac{1}{\cos\gamma} + \frac{\cos\gamma\sin\psi}{r \cos\lambda} \]

\[ - \frac{1}{V^2 \cos\gamma} (g_\lambda \sin\psi - \rho \cos\psi) + \frac{\omega^2 \cos\lambda}{V^2} \sin\lambda \sin\psi \]

\[ + \frac{2\omega}{V} \left( \frac{\sin\gamma \cos\psi \cos\lambda}{\cos\gamma} + \sin\lambda \right) \]  

(5.30)

The three positional equations become

\[ \frac{dr}{ds} = - \sin \gamma \]  

(5.32)

\[ \frac{d\lambda}{ds} = \frac{\cos\gamma \cos\psi}{r} \]  

(5.33)

\[ \frac{d\theta}{ds} = \frac{\cos\gamma \sin\psi}{\cos\lambda} \]  

(5.34)

Also, the differential equation for the time is

\[ \frac{dt}{ds} = \frac{1}{V} \]  

(5.35)

5.5 Numerical Procedures

A major problem in the development of digital computer programs is the problem of retaining accuracy during the course of the computation while keeping the number of points (and therefore the computation time) as low as possible. In particular, the calculation of flight trajectories of reentry vehicles often requires an accuracy of six or eight figures in the altitude or geocentric radius, respectively. The latter requirement certainly exceeds the precision of numbers in most scientific computers when round-off error* inherent in all integration schemes is taken into account. Truncation-error is easily reduced by using one of the standard, high-accuracy integration techniques such as the fourth order Runge-Kutta method and the proper size of interval. However, round-off error proves much more difficult to eliminate, especially when a small enough interval size (and therefore a large number of points) is used to reduce the truncation-error. Consequently, a practical method must be found for reducing round-off error and also of reducing the number of integration points while simultaneously keeping the truncation-error to a minimum.

* Error is introduced from two sources:

(a) Truncation-error may be defined as the difference between the true value and the value obtained from the numerical integration formula which makes use of a finite number of terms from an infinite Maclaurin's or Taylor's series.

(b) Round-off error is the result of the limited accuracy (usually 8 figures) available within the computer.
The solution to the problem of round-off error took advantage of the basic characteristic of digital calculations. All numerical integration schemes involve adding small increments to the variables in order to find the next value after each step. Accordingly, these increments possess far more absolute accuracy than their respective variables. Thus, if the variables could take advantage of this increased absolute accuracy when incrementation took place, the round-off error would be greatly reduced. Notice that the incrementation stage would be the only one at which the increased accuracy (and therefore greater computation time) would be used since the calculation of the increments would use the variables only to their original accuracy.

In practical computational terms (using popular languages such as FORTRAN), the scheme merely involves the use of double-precision addition whenever the increments are added to the variables. Normal single-precision computation of the increments makes use only of the most significant part of the variables. In addition, an analysis of the whole program should be made as other sections may benefit from this technique. An example is the calculation of the altitude over an oblate Earth when only the geocentric latitude and the geocentric radius are available. Having calculated the "corrections" in single precision, they should be added in double precision.

A major reduction in the number of steps is also possible, by using a continuously varying interval step size dictated by truncation-error. It has been found that the truncation-error depends on how quickly the variables are changing as well as the step size. In the calculation of flight trajectories of entry vehicles using geographical coordinates, the key variables are the glide angle, the azimuth, and the latitude. Experience has shown that whenever all of these variables are reasonably stationary, the interval step size can be rather large. The criterion which was used to determine the interval size (within a certain preset range) was that the change in all these angles per step should be kept below a certain minimum value. By varying the interval size parameter, the value could be found for which the truncation-error was acceptable, since the double-precision incrementation always reduced the total round-off error to a negligible value.

Two basic methods were used to check the results of the computations. First, the atmosphere was removed (by setting the density to zero) and orbits were calculated for all the situations for which simple analytical solutions were known. Circular and elliptical orbits were compared over non-rotating and rotating Earth reference systems. In every case, the additional terms involving the Earth's rotation produced the proper result when changes were expected. On the other hand, parameters such as the orbital period and the major axis (for elliptical orbits) remained the same. Typical accuracies were six or seven figures over several orbits. These latter results also gave confidence in the numerical integration procedure referred to above. The check on the accuracy of Eq. (4.12) was made via the test program developed to check the original validity of its results.

In order to check the accuracy of the digital program when the atmosphere was present, the terms due to rotation and oblateness were removed. The equations were then further simplified (the atmosphere was made exponential, only zero roll angles were considered, and gravity was held constant at its average value) and the results compared with the output of the identical equations which had been wired into an analogue computer. Over the range for which the density approximation on the analogue computer was reasonably accurate (from 150,000 feet up to 300,000 feet) there was no discernable difference between the two results.
VI. RESULTS AND DISCUSSION FOR A CIRCULAR, NON-ROTATING EARTH

6.1 Introduction

The next three sections of this paper present an analysis of the performance of the CONTAC system with results obtained from the digital computer program listed in Appendix C. Section VI is devoted to presenting the overall capabilities of the CONTAC system as well as its limitations.

The method of presenting the results has been discussed in Sec. 1.3. Apart from the peak temperature, the two main quantities of interest are the equivalent down range and cross range. These values are illustrated and defined in Fig. 3.

In order to give the reader a better understanding of the way in which the CONTAC system works, a detailed history of all important variables is given in Figs. 11b and 11c. Figure 11a is a similar history of an uncontrolled reentry and may be used as a comparison. It is quite clear that the CONTAC system is easily able to control the temperature and prevent the additional rise of 650°F which occurred for the uncontrolled reentry.

In order to provide the details of the way in which the altitude angles are controlled, an expanded scale has been used for a portion of the reentry in Fig. 11d. Notice first that the line of time vs. down range is straight over this small portion of the reentry. Consequently, the abscissa of the graph has been labelled as time in addition to down range in order to provide a better feeling for the time scale of the cycling of the angle of attack by the CONTAC system. The most important information to be gained from Fig. 11d is the rates involved in the process. Notice that the fuel economy will be quite high due to infrequent use of the reaction jets.

6.2 Footprint Optimization

The standard reentry conditions at 350,000 feet altitude were chosen to be 26,000 feet per second for the speed and 1.0 degree for the glide angle. It might be noted that the speed is between 250 and 300 feet per second greater than the orbital velocity at this height, which therefore allows the spacecraft the possibility of having reached this position from an altitude of between 250 and 300 miles. A slight increase in the initial speed or glide angle would allow higher initial orbital altitudes up to 500 miles if that condition were desirable.

Two criteria were used in determining the best flight path which could be obtained from the initial conditions. The most important factor was to prevent throughout the reentry, the occurrence of a temperature greater than the peak value experienced during an equilibrium glide. In general, this requirement was slightly relaxed to allow temperature peaks which were greater by 20°F or 30°F. The second principal requirement was to obtain as large a cross range as possible throughout the whole extent of the down range capability.

In addition, some effort was expended to increase the available down range increment. It is here that a further reason for the choice of 26,000 feet per second as the initial velocity was found. On Fig. 9, it is shown that the speed lost up to the point of the first temperature maximum can easily be controlled to a value between 150 and 150 feet per second by the use
of the proper entry angle of attack. As noted in Sec. 1.9, this reduction in the vehicle's speed at the end of Phase II can be produced without causing a significant difference in altitude, glide angle, or down range distance at that position. This condition results from the variation of the drag coefficient with the angle of attack while the lift coefficient remains essentially constant. Consequently, due to the choice of speed at the initial position, the speed at the end of Phase II is either very slightly above (when low angles of attack with relatively low drag coefficients were used) or just below (when high angles of attack with much larger drag coefficients were used) circular orbital velocity. In a vacuum, the former speed results in a perigee condition which would produce an additional revolution around the Earth yielding 360 degrees of down range increment before the spacecraft descends to the same altitude. On the other hand, even in a vacuum, the latter speed would produce an apogee condition which brings about an immediate decrease in altitude. However, it is quite obvious that the presence of atmospheric drag will tend to alter the situation. Except when complicated by Earth oblateness, even the use of a very small angle of attack (at $5^\circ$, $D_\beta = D_{\beta_{\min}}$) for the former situation results only in an extended glide at practically constant altitude (the rise is limited to 30,000 feet) until the speed falls below the circular value by about 1000 feet per second. On the other hand, the continued use of a high angle of attack for the latter situation reduces the speed very quickly resulting in very little incremental down range. In both extreme situations as well as any between, the incremental down range which is available by means of subsequent attitude control, is very nearly the same once the speed falls below the circular value by about 1500 feet per second. Therefore, any small speed variations at the end of Phase II on the order of one hundred or even fifty feet per second will result in large variations in incremental down range during Phase III of the reentry. As a consequence, the situation provides the guidance system with the capability of large down range increments, without interference from the CONTAC system, due to the possibility of an extended glide at very low angles of attack during which the speed is lost very slowly.

It should be noted that the above discussion is not altered by the inclusion of Earth rotation. The immunity is due to the fact that although the relative speed can change by up to $\pm$ 1500 feet per second at the equator for initially due West or due East reentries, respectively, the relative circular orbital speed also changes by exactly the same value. For other initial azimuths, the same relative change also occurs. Consequently, the same perigee and apogee conditions result upon the use of low and high entry (i.e., during Phase II) angles of attack, respectively.

With regard to satisfying the first criterion, the CONTAC system behaved very well. The operational limits were difficult to obtain since impractical situations had to be imagined before any degradation was found in the peak temperature performance. The discussion of the parameters which were varied during the reentry is confined to the attitude at this particular point; the result of the differences or errors in initial conditions is presented in Sec. 6.3.

The entry angle of attack was limited to a maximum of 65 degrees in order to prevent excessive heating rates at the rear of the spacecraft. It may be questionable whether larger pitch angles could be maintained due to problems of stability and control. However, assuming that these problems can be solved, there is no reason why angles as high as 75 or even 80 degrees cannot be used.
The lower limit for the entry angle of attack was approximately 35 degrees. When smaller values were used, the first maximum temperature was too large, since too little lift was developed to prevent the vehicle from descending too low on the initial penetration into the atmosphere.

The entry roll angle was always held at zero during this investigation. It is probable that a moderate roll angle during Phase II could increase the cross range without affecting the peak temperature control. Further investigation into this question might prove useful.

The skip angle of attack (which is controlled by the guidance system) is employed as the lower limit during Phase III. The largest value was the lesser of 55 degrees and the entry angle of attack. The lowest value used was 5 degrees. The guidance system might employ even smaller values down to zero if the upper surface of the spacecraft is not subject to or can cope with large heating rates. It should be noted that when the large entry angles of attack of 55 or 65 degrees were used, the CONTAC system often prevented the vehicle from assuming the skip angle of attack. As soon as it commences active operation at the beginning of Phase III, the CONTAC system disallows lower limits if they would permit too high a peak temperature. Again, it is noted that this operation is completely automatic.

The skip angle of roll (which is also controlled by the guidance system) is employed as the lower limit during Phase III. The "smallest" value was a full minus ninety degrees to the left (or plus ninety to the right). Smaller values were not used since they resulted in less cross range which was not in accordance with the second criterion. The "largest" value used was zero degrees. The CONTAC system often prevented the vehicle from assuming the skip bank angle when it was near ninety degrees; the conditions which led to this situation were the same as those which prevented very small skip angles of attack from being used and again the CONTAC system automatically disallowed the value and prevented the temperature from achieving too high a value.

The upper limit on the angle of attack during both Phases III and IV was 55 degrees. This value was fixed at the attitude which yielded the maximum vertical force. The upper limit for the bank angle was also the value which yielded maximum vertical force, namely zero degrees. The reasons for this choice have already been given in Sec. 1.10.

The lower limit for both the angle of attack and the bank angle did not affect the operation of the CONTAC system. Therefore, the choice of the values used will be discussed under the second criterion of cross range optimization.

While operating within the above values for the attitude, the CONTAC system performed the task of restraining the peak temperature experienced during the reentry without any degradation in performance for any combination of these values. As shown in Fig. 26, the peak temperature experienced during the reentry varied only ±20 Fahrenheit degrees about the peak value experienced during an equilibrium glide. At the same time, cross range was practically constant (between 9 degrees and 11 degrees) throughout most of the available down range. Therefore, it is possible to obtain an "isothermal footprint" (with regard to the peak temperature experienced during the reentry) of considerable area when the CONTAC system is employed to constrain the temperature during reentry.
the flight. The dimensions are over one hundred degrees equivalent longitude in down range and about twenty degrees equivalent latitude in cross range. Considering the fact that the continental United States has a "length" of only forty degrees in equivalent longitude and is less than eighteen degrees of equivalent latitude wide, this "isothermal footprint" is about three times the area of that country. Consequently, during any given 12 (or at most 24) hour period, enough range control is available when using the COMAC system to land at any point within or near the United States, as long as the terminal position is underneath or at least near the orbital envelope.

The attainment of the second criterion, the maximum possible cross range, was attempted by programming the skip bank angle (Phase III) and the lower limit for the angle of roll (Phase IV) within the restraints imposed by the first criterion. During each Phase of the reentry, several possibilities were investigated until the best overall performance was attained throughout the flight.

Note that the final results for cross range presented in this paper should not be considered the maximum possible values since it was not feasible to consider every parameter combination. In addition, the calculation was terminated at approximately 100,000 feet where the speed was about 3000 feet per second. As a consequence, about 3 degrees of equivalent latitude or about 200 miles of additional cross range is available before the final touchdown. However, as pointed out in Section 1.12, this final portion of the flight has not been included in the investigation. Therefore, no details for Phase V can be given.

On the other hand, density variations in the atmosphere and other sources of error may reduce the anticipated cross range. It was invariably found that when the relationships which yielded maximum cross range were incorrectly adjusted, the result was not only a decrease in cross range, but also either increased or decreased down range depending on the direction of the mismatch. Therefore, it may be necessary to sacrifice a portion of the cross range capability in order to retain a degree of down range mobility in the latter phases of the flight.

At this point, a detailed description of the methods used to maximize the cross range will be given for Phases I to IV.

During Phase I, no cross range control is available by means of attitude adjustments since atmospheric forces are negligible. In conjunction with the rotation of the Earth, it might seem possible to provide additional cross range by appropriately delaying or advancing the retro-fire time. For instance, in the case of a polar orbit with the landing zone near the equator, a retro-fire delay of 20 minutes would appear to provide 5 additional degrees of cross range. However, the results in Sec. 7.2 are contrary to such a possibility since the midline of the footprint closely follows the vacuum orbital track over the Earth's surface.

During Phase II, no cross range control was attempted. For entry angles of attack in the vicinity of 35 degrees, a very slight increase in cross range may be possible by trading off some vertical force for small amounts of side force. However, the speed is so great that only large forces will produce a turning radius with any appreciable effect. Thus, since large bank angles cannot be used without degrading the vehicle's peak temperature performance, there is too little to be gained from complicating the reentry with an additional parameter.
During Phase III, about ten to twenty percent of the cross range may be realized by using the optimum bank angle. Figure 27 shows that the best value for the skip angle of roll is about 75 degrees and this value was adopted for all subsequent trajectories. Another possibility for producing the most cross range was that of using a bank angle based on speed or perhaps altitude as suggested by Wagner (Ref. 6). However, the result was usually so close to 75 degrees that the additional complexity was not considered worth while.

It is during Phase IV that most of the cross range capability is realized. The lower limits for the angle of attack and the bank angle are not, of course, used until well after the beginning of Phase IV. This is because the CONTAC system retains control of the attitude until so much energy has been dissipated that the convective heating rate will continue to decline regardless of the attitude assumed. Once this condition has been reached, the guidance system will assume full control.

The basic concept behind the attempt to maximize the cross range is the relationship proposed by Wagner (Ref. 6). The problem in this investigation was how to implement the procedure on a practical basis. The method which was adopted is to base the maximum allowable bank angle (either left or right) on a variable connected with the shape of the trajectory. The three variables considered were the speed, the altitude, and the glide angle. The energy was also considered in order to combine both speed and altitude in some manner. However, the glide angle was rejected on the grounds that its variation during this part of the flight was an increase while a decrease in the bank angle was obviously required. This left the speed, altitude, and energy of the vehicle as a basis upon which to proceed.

The next step was to find the best angle of attack for the lower limit during Phase IV. The value found to yield the largest cross range was the angle for \((L/D)_{\text{max}}\) which occurs at 22.7 degrees for the vehicle under study. This value could be anticipated due to the fact that the maximum down range also results from the use of this angle.

At this point, a number of simultaneous tradeoffs were investigated. No attempt is made to justify the procedure nor even the final result. However, very briefly, it involved the determination of which power relationship (square root, linear, or quadratic) between the bank angle and one of the speed, the altitude, or the energy would yield the most cross range. The result that was consistently best for all the reentries studied is the relationship shown in Fig. 28 in which the bank angle was a linear function of the speed over the interval shown. A few results were found for which a slightly greater cross range was achieved by using the square root of the speed over the specified interval; however, there was no way of predicting when the result would be larger. Therefore, the linear variation was adopted as the best solution.

In general, the above combination of the angle of attack for \((L/D)_{\text{max}}\) and the relationship shown in Fig. 28 for the bank angle as the lower limits during Phase IV proved a proper choice in one other way. In most situations, the azimuth change at 100,000 foot altitude was about 70 degrees. A slight additional increase until a speed of 1000 feet per second is reached during Phase V would yield a final azimuth change of about 80 to 90 degrees. It is certainly convenient to be heading in the proper direction to achieve the maximum possible additional cross range if it is required during the last stages of the reentry.
However, no calculations were made for Phase V of the reentry. But, it is worth mentioning that any procedures which are used to optimize the cross range must take into account the final approach and landing. Certainly, some cross range capability must be sacrificed in order to obtain a sensible landing pattern and a touchdown with a margin of safety and an allowance for adverse conditions at the landing site.

6.3 Effect of Non-Standard Initial Conditions

The results presented in this section deal with the effects produced by a difference between the desired and the actual initial values of the speed and the glide angle. The cause of the difference has been dealt with in some detail in Section 1.8. However, the investigation should not be confined only to that limited range of errors in attitude misalignment and incorrect retro-fire velocity increment that have occurred in recent flights. Therefore, the results of large deviations in the speed and the glide angle will be examined.

Before any results are presented, it should be noted that there are actually two problems to consider. The more important is whether the CONTAC system can cope with the deviations from the expected initial values. This question is easily answered. If the peak temperature during the reentry does not exceed by more than a small value the peak temperature which is experienced during an equilibrium glide, then this test is successful. If unsuccessful, the first temperature maximum is always higher than the standard peak temperature. On the other hand, when the test is successful, the peak which occurs during the flight (whether it be the first or a subsequent maximum) will be equal to the standard peak value.

The second question is whether or not the vehicle may still land at its original destination. Due to changes in the initial conditions, the shape, size, and position of the footprint is altered. The alterations are analyzed and the basic differences in the footprint are given. It can then be determined whether or not the footprint still contains the desired landing site.

Figure 29 shows the results of deviations in the initial speed which range from 25,700 to 26,300 feet per second. With respect to the first problem, it can be concluded that positive errors in the initial speed will not cause any degradation in the performance of the CONTAC system. However, peculiarly enough, it is the negative errors (due to a lower than normal initial speed) which cause some difficulty. For small errors, between zero and -100 feet per second, the increase in the peak temperature is also small, amounting to only 50°F for most points on the footprint and only 90°F for the extreme down range positions. However, for errors in the vicinity of -300 feet per second, the performance definitely begins to decline; peak temperatures are 200°F greater for most points on the footprint and at the extreme down range position, the peak is another sixty degrees higher still. Calculations could be made for even greater errors in the initial speed, but a simple check on the initial altitude from which such a reentry could be initiated reveals that even for an error of -300 feet per second (i.e. an initial speed at 350,000 feet of 25,700 feet per second), the maximum height from which the vehicle could have descended is only 130 miles. In addition, the errors in retro-fire alignment attitude would have to be about 30 degrees in alignment and about 250 feet per second in velocity increment. Such large errors would lie outside the normal
operating conditions. Finally, still larger errors (on the negative side at least) have no chance at all of occurring unless some deliberate action to decrease the speed is taken at an altitude below 100 miles. Since this height is below normal orbiting altitudes, the action would have to take place subsequent to retro-fire and is not considered a feasible possibility in the production of errors.

With regard to the position of the altered footprint, no problems are apparent. The maximum down range possible with the -300 feet per second error is still 60 degrees of equivalent longitude greater than the minimum down range with the +100 feet per second error. And with the latter error, global down range is already available. Therefore, with the +300 feet per second error, the guidance system will only have to take the vehicle around the Earth one additional orbit if the minimum down range position is past the landing site. With regard to cross range, it is quite clear that no problems exist here either. The minimum footprint width is still almost equal to the equivalent latitude span of the United States. Also, note that the minimum down range increment or footprint length is still greater than the equivalent longitude span of that country.

Figure 30 shows the results of deviations in the initial glide angle which range from 0.65 to 1.5 degrees. Reference 5 indicates that such an interval is far larger than would be expected due to normal errors in attitude alignment and velocity increment. However, Fig. 8 shows that this interval is to be expected if large retro-fire errors are present. Negative errors in the initial glide angle are easily handled by the CONTAC system and without any degradation in performance. But for positive errors even as small as 0.25 degrees, the CONTAC system fails to prevent the peak temperature from rising beyond the standard peak value. An analysis of the actual trajectories indicates that in every case the first temperature maximum is higher than the standard peak value due to the inability of the spacecraft to produce enough vertical force. As a result, the vehicle descends to a lower than standard height in the atmosphere at that position. The decrease in height results in a higher density without a compensating decrease in speed. Therefore, although the CONTAC system is not directly responsible for the temperature increase (recall that it does not go into active operation until the beginning of Phase III and these temperature peaks occur at the end of Phase II), nevertheless, the vehicle must be designed to withstand the additional rise in the peak temperature of 20°F if the initial glide angle may be as high as 1.25 degrees. However, it should be noted that the additional rise for an initial glide angle of 1.5 degrees is only about 30°F over the standard peak temperature. Consequently, the relationship is highly non-linear.

With regard to the position of the altered footprint there may be a problem for initial glide angles greater than about 1.4 degrees. Beyond this value, the maximum down range may be less than the minimum down range for initial glide angles less than 0.75 degrees. Whether the additional down range (obtainable from permitting the vehicle to travel once around the Earth in order to overlap the original down range position) can be usefully employed by the down range guidance scheme is a question which must be answered by simulated real time runs. The cross range is not a problem; there is more than the usual value.
A few additional points which apply to both Figs. 29 and 30 should be mentioned. First, there has been no attempt to optimize the upper and lower limits for the attitude when incorrect initial conditions in the speed or glide angle were present. Second, the results of an incorrect initial value for the down range position has not been taken into account. A one degree error in the initial down range would require about a twelve second error in retro-fire time assuming that the initial conditions of speed and glide angle were correct. However, the initial down range is also affected by the conditions which produce errors in the initial reentry values. For instance, an increased initial glide angle of 1.35 degrees (which results in a decreased down range over the whole footprint) is accompanied by a decreased down range at the initial position of eleven degrees of equivalent longitude when the retro-fire takes place at 250 miles. Consequently, it can be seen that the range control problem may be quite difficult to solve, but a solution should be available which is also compatible with the concept and operation of the CONTAC system.

VII. RESULTS AND DISCUSSION - OBLATE, ROTATING EARTH

7.1 Introduction

Section VI presented the problems associated with variations in the initial reentry conditions of speed and glide angle. However, it will certainly be possible to design for the spacecraft the equipment necessary to compute the proper attitude alignment and retro-fire velocity increment at the given orbital position. Therefore, these problems should not be considered to be very serious. On the other hand, the effects of Earth rotation and oblateness must be compensated for this task; may not be simple in all situations.

In order to understand the individual effects of the Earth's rotation and oblateness, calculations were performed in which only one of these was present at a time. All the effects were then combined and the overall effect noted.

7.2 Effect of the Earth's Rotation

Figure 31 gives the results of calculations for a number of different reentry positions and directions when it was assumed that the Earth was circular, but rotating.

An analysis of the peak temperatures shows that the CONTAC system encounters a little difficulty in controlling the temperature when rotation and an initial Westerly heading along the equator combine to increase the initial relative speed between the vehicle and the atmosphere. On the other hand, this effect is reversed for an initial Easterly heading and lower than normal peak temperatures result. However, since the maximum difference in each case amounts to only ± 125°F for due West and due East equatorial reentries, respectively, the effect of rotation on temperature peaks cannot be considered a major problem. In most situations, the effect will probably be beneficial since the loss in inertial velocity during launch does not favour the establishment of orbits which travel in a Westerly direction against the rotation of the Earth.
With respect to the shape of the footprint, the most obvious part of the result is the shift towards negative values of cross range without any substantial change in down range. A comparison should be made (Fig. 29). In those cases, for a non-rotating Earth, errors in the initial speed of approximately 300 feet per second produced large shifts in final down range. For these cases in which there is a 21500 feet per second change in speed relative to the Earth due to an initial Westerly or Easterly heading, respectively, along the equator, the down range shift averages only about 8 degrees of equivalent longitude which is only about four percent of the total down range distance. It may be concluded, therefore, that for a given set of inertia initial conditions, the Earth's rotation is not a significant factor with respect to the final down range position of the vehicle. The physical explanation has already been presented in Sec. 6.2.

However, with regard to the cross range, an important shift occurs in the footprint. Fortunately, the extent of the shift is quite predictable. A comparison has been made with orbital tracks for vacuum trajectories. In almost every case studied, the extent of the shift is either the same as or algebraically less (i.e. whether the actual shift is positive or negative) by up to 1 degree than the value which would be found due to the rotation of the Earth under the equivalent circular orbit (Fig. 3). In most cases, the agreement was better than 1 degree. The example easiest to visualize is the case for which the vehicle is initially heading due north from the equator. For a circular vacuum orbit through the initial position, the Earth will have rotated 10.8 degrees by the time the spacecraft reaches the equator again at 180 degrees down range. The actual shift in this case also amounts to -10.8 degrees. A check upon all other cases yields similar results.

The observed result may be explained as follows. In symmetric flight, with zero bank and sideslip, neither the lift nor the drag have a horizontal component perpendicular to V. Thus the only force available to produce horizontal curvature of the path (in an Earth-fixed reference frame) is the Coriolis force. This is the same whether the atmosphere is present or not, (except for the gradual reduction of V by the drag). Hence the horizontal curvature of the path is largely unaffected by the atmosphere.

7.3 Effect of the Earth's Oblate Shape

Figure 32 gives the results of calculations for a number of different reentry positions and directions when it was assumed that the Earth's poles were depressed to give the correct oblate shape, but that the gravitational field was purely a central force and the Earth was not rotating.

An analysis of the peak temperatures shows that the CONTAC system also encountered a little difficulty when the oblate shape of the Earth caused the effective initial glide angle (see Appendix B) to be larger than normal. For situations in which the effective initial glide angle is smaller than normal, the CONTAC system was able to maintain the peak temperature at the standard value.

The situation may be compared directly with the results of Fig. 30 in which both smaller and larger than normal values were used for the geocentric initial glide angle. The extreme negative difference between the effective and the geocentric initial glide angle is -0.18 degrees, and occurred for an initial latitude of 30 degrees North and an initial heading of due
North. The peak temperature for both this footprint and the one where the geocentric initial glide angle was 0.75 degrees is the normal peak value. In addition, the cross range is the same for each. However, down range achieved in the oblate Earth case is a relatively small but significant amount less at both ends of the footprint than for the case of the geocentric initial glide angle of 0.75 degrees. This difference is in good agreement with the expected result.

The extreme positive difference between the effective and the actual initial glide angle is +0.18 degrees, and occurred for an initial latitude of 60 degrees South and an initial heading of due North. For this reentry, the peak temperature for both this footprint and the one where the initial glide angle was 1.25 degrees is somewhat greater than the standard peak value, but the difference is, as expected, less for the case of the oblate Earth reentry. The peaks were about 160°F - 190°F greater. On the other hand, both the down range and the cross range achieved under both situations is substantially the same, which indicates that the continued decrease in the height due to the oblate shape has a continued effect on range while for peak temperatures, the effect ceases at the first temperature maximum (which occurs at about 18.5 degrees down range from the initial position).

The use of other geographical locations for the initial position produced similar results. A slight rise in the peak temperature was noted for the South-to-North trajectory, which produces an increase in the effective initial glide angle of only 0.07 degrees. The equator-to-pole reentry, which produces a decrease of 0.07 degrees, experiences a slight increase in down range.

In general, the results due to the shape oblateness of the Earth often showed that much of the effect can be accounted for by the difference between the effective and the geocentric initial glide angle. In addition, although there is a substantial difference in many situations between results for the same effective (over an oblate Earth) and geocentric (over a spherical Earth) initial glide angles, there is at least qualitative agreement in all cases. However, it should not be considered serious if the correction factors for non-normal values of the effective and the geocentric initial glide angles do not agree. The guidance system must reduce to zero either type of error. Since they are equal in magnitude, errors due to non-normal values in the actual initial glide angle will provide the necessary information on the extent of the errors which may be expected on account of the shape oblateness of the Earth.

There is another possible approach to the problem of shape oblateness. Rather than relying on the CONTAC - guidance system to correct for the errors introduced by incorrect values in the effective initial glide angle, it should be possible to produce the correct down range by an appropriate adjustment of the actual geocentric initial glide angle. The adjustment would have to take account of the geographical location of the initial position. The change in the actual initial glide angle could be produced by correcting the retro-fire attitude alignment and velocity increment.

7.4 Effect of the Earth's Oblate Gravity Field

The results of including the oblateness term in the gravitational field for the Earth showed a negligible difference between the various foot-
prints which result from different initial positions and initial heading directions. Consequently, it may be concluded that even the $C_{2,0}$ coefficient has a negligible effect on the reentry of a spacecraft and that all other terms may be completely ignored.

One minor effect was found which tends to cancel the shape oblateness contraction in down range which resulted when proceeding from polar to equatorial regions. For these cases, the small reduction in gravitational force on the vehicle due to the additional distance between the vehicle and the equatorial bulge resulted in a small increase in down range of the order of 5 degrees at the minimum down range position and 10 degrees at the maximum down range position. Since this increase amounted to less than 5% of the total, and no other results were affected, it was not considered significant. Note that in the case of shape oblateness, the decrease in down range was double this value when the initial position was at the pole.

7.5 Effect of the Earth's Rotation and Oblateness

Figure 33 gives the results of calculations for a number of different reentry positions and directions for a rotating oblate Earth.

An analysis of the peak temperatures shows that the CONTAC system again encountered the same expected difficulty in controlling the peak temperature when rotation and the initial heading combine to increase the relative speed between the vehicle and the atmosphere. In these situations, an initial heading which is due West together with an initial equatorial position results in an increase in the peak temperature of about $100^\circ F$ above the standard peak temperature. Almost exactly the same result occurred in the case of a circular, but rotating Earth. Also, the decrease of $100^\circ F$ due to an initial Easterly heading is again present.

In addition, the expected increase in the peak temperature also occurs when the effective initial glide angle is larger than normal although in the rotating Earth environment the increase is relatively small. The worst situation studied occurred for an initial latitude of 50 degrees South with an initial heading of due North; for this case, the rise was only a modest $100^\circ F$ over all but the extreme down range portion of the footprint for which a rise of $200^\circ F$ was incurred.

It is conceivable that a higher temperature increase would result if the initial heading and initial latitude combined to produce both a higher than normal effective initial glide angle together with a higher than normal effective initial speed. However, in order to produce the former, almost due North initial headings are required while the latter requires almost due West values. The latitude must also be large if the former is to be significant, which in turn reduces the latter. Therefore, it is deemed quite unlikely that peak temperature will be more than $100^\circ F$ above the standard peak during reentries for which the inertial initial conditions are a speed of 26,000 feet per second and a flight path angle of 1.0 degrees. Finally, even if the effective initial glide angle is greater than 1.0 degrees and increases in the peak temperature higher than $100^\circ F$ occur, it would be possible to compensate for this in advance of the reentry by the appropriate adjustment in the retro-fire attitude alignment and velocity increment.
With regard to the shape and position of the footprint, the combined effects of rotation and oblateness upon both the down range and the cross range do produce a situation which is difficult to analyze. However, the results may be considerably simplified with the aid of Figs. 31 and 32 and the analysis which applies to each. Very broadly speaking, the two effects seem to combine in an almost linear fashion and produce the range distortions associated with shape oblateness together with the cross range shift due to rotation. For example, the rotational case used in Sec. 7.2 in which the vehicle is initially heading due North from the equator results in the identical cross range shift of 10.8 degrees of equivalent latitude at 180 degrees of equivalent longitude down range; in addition, this same case now manages to have an extended footprint with almost the same maximum down range as the oblate shape case.

Continuing with this same footprint, the cross range shift at 360 degrees of equivalent longitude down range is 21.4 degrees as compared to 21.7 degrees for the circular orbit in vacuum which goes through the initial position. The above results are illustrated in Fig. 33a. Many more examples (there were 40 pairs of point to choose from as well as the general outline of each footprint) far too numerous to mention bear out the conclusion that the effects of rotation and oblateness combine with very little interaction.

In conclusion, it should be noted that the effects of both rotation and shape oblateness upon the shape and size of the footprint are large. The effect of gravitational oblateness is usually negligible and is so completely masked by shape oblateness that it is never significant. With regard to shape oblateness, the primary effect is to change the down range position and span of the footprint without producing any major change in the cross range. On the other hand, rotation produces a dramatic cross range shift without any major effect upon the down range. However, the cross range shift due to rotation is accurately predicted by the vacuum orbital track over the Earth's surface of an object which passes through the initial position of the vehicle. It must have the same azimuth and also possess the correct speed (the glide angle should be zero) relative to the Earth's surface to place the object in a circular orbit. Consequently, although the cross range shift is quite large and cannot be ignored if accurate navigation is to be achieved, the guidance system will be able to predict the extent of the shift as a function of the down range.

VIII. RESULTS AND DISCUSSION - DENSITY PERTURBATIONS

8.1 Introduction

As noted in Sec. 4.6, there is very little data available on spatial density variations in the Earth's atmosphere. As a result, several different types of variations were proposed and calculations carried out to determine if they caused any degradation in the CONTAC system performance. Each sub-section gives a detailed account of the extent of the variation for which the CONTAC system could correct, so that its limitations are established.

8.2 Effect of Different Basic Models

An attempt was made to determine just how small the atmospheric scale height had to be before the CONTAC system would be unable to correct for larger than normal density gradients. The details of the density versus altitude relationship are shown in Fig. 34. Scale heights between 14,000 feet and 26,000 feet were used during the initial portion of the reentry. It can be
seen that the increase in density at 230,000 feet due to the use of the small-scale height produced a density increase of about 250% which is an extreme value in view of the available data (Ref. 33).

The results were very satisfactory. The use of the CONTAC system resulted in no increase in the peak temperature larger than 10°F above the standard peak value and the range results were within 2% of the values found when using the U.S. Standard Atmosphere, 1962. The 1959 ARDC Standard Atmosphere was also used. Again, as expected, there was no effect noted.

8.3 Effect of Altitude-Dependent Perturbations in the Density

In order to simulate variations dependent upon the altitude, information found in Ref. 32 was adopted. The resemblance to a sine wave suggested the curve shown in Fig. 35. The variations are all about the U.S. Standard Atmosphere, 1962. After the reentry was calculated for this shape, a number of others were investigated. The wave length was varied from 25,000 feet to 100,000 feet. The amplitude was increased to give a fractional variation of 0.2 and the nodal point was shifted to test the sensitivity to all phase relationships.

The most severe case used a wave length of 25,000 feet and an amplitude of 0.2 for the fractional variation. The above values together with a scale height of 18,000 feet for parts of the U.S. Standard Atmosphere, 1962 yield a density increase of 45% in an altitude range of 4,250 feet in the region one-half of a wave length from the nodal point. Since Ref. 32 indicates that the "density may vary by as much as 50 percent when descending through an altitude increment of 4,250 feet" and that this is the maximum increase to be expected, it was felt that further calculations were not necessary.

Again, the results were completely satisfactory in that the CONTAC system was able to maintain control of the temperature. The down range, however, did vary by as much as 5% from the values obtained by using the U.S. Standard Atmosphere, 1962.

8.4 Effect of Random Density Perturbations

The simulation of random spatial density variations presented an unavoidable difficulty. Data is not available from which to set up a realistic model of the variations which might be expected. A little information combined with intuitive judgements was used to postulate a random variability model based upon the distance travelled by the spacecraft during the reentry. The detailed description of the model is given in Sec. 2.6.

The results may best be described by reference to Fig. 36. The combination of variance and correlation interval above the hatched zone usually resulted in situations with which the CONTAC system could not cope. Increases in the peak temperature of about 30°F usually resulted for variances which were 0.05 to 0.1 greater than the variance shown acceptable for a given correlation interval. For combinations in the hatched zone, the peak temperature was usually less than 25°F greater than the standard peak value; however, in isolated examples, an increase of between 50°F and 100°F was experienced. For variances which were 0.05 to 0.1 less than the variance immediately under the hatched region, the peak temperature was always equal to the standard peak value.
An exhaustive study could not be made due to the statistical nature of the problem. A complete set of results would have required over a thousand individual runs, each requiring up to three or four minutes of computer time.

A large number of runs is needed because a probability analysis is required. For each combination of variance and correlation interval, a number of different random density distributions would have to be used, and a graph deduced of the probability of exceeding a given temperature for that combination. Repeating for a number of variance-correlation pairs would provide the data for a Figure like 36, in which the hatched area would be bounded by curves of constant probability of exceeding a set temperature, e.g., 1% and 10% probability of exceeding a 300°F rise. In order to have a reasonable degree of confidence in the results, a great many individual runs must be made for each pair of values for the variance and correlation interval. Since certain results would occur infrequently, the suggested figure for the number of runs in each set is one hundred. It seems likely that at least ten pairs or sets would be required. For the graph presented, about 2½ hours of time was used to make about 50 individual runs for 7 pairs of values for the variance and correlation interval.

One additional factor of importance must be noted. In addition to the radiation-equilibrium temperature, the actual temperature based on Eq. (3.6) was calculated for the nose stagnation point. In general, the equilibrium temperature is held at a constant value by the CONTAC system for a lengthy period. For this situation, the difference between the real peak temperature (in which the skin possesses some heat capacity) and the equilibrium peak temperature is usually less than 5°F. For this reason, the real peak temperature has heretofore been ignored. However, when random spatial density variations were encountered which had a correlation interval less than 200 miles (and especially when this distance was less than 100 miles - at 20,000 feet/sec, 100 miles are traversed in only 26 seconds), there was often a large difference between the two peak values. In one instance when the equilibrium peak temperature was about 300°F greater than the standard value, a heat capacity of 2.0 BTU/ft² (which might be provided by a 0.03 inch thick skin of Molybdenum weighing 1.6 lb/ft²) reduced the excess temperature by 20% to about 240°F. Only a few of the 50 runs provided a situation for which this result could occur. Consequently, no definitive conclusions could be drawn. But it will certainly be very fruitful to determine the extent to which even a limited heat capacity for the vehicle's skin can reduce excess peak temperatures caused by random spatial density variations in the Earth's atmosphere.

IX. CONCLUSIONS

9.1 Validity of the CONTAC System Concept

The results of this investigation indicate that the CONTAC system which has been proposed is a highly effective method for reducing the peak temperature experienced during the reentry of a manned, lifting spacecraft. The maximum temperature which a vehicle experiences during a fixed attitude reentry is lowest for an equilibrium glide. This maximum is taken as the standard peak value. The CONTAC system prevents the actual peak temperature experienced during the reentry from exceeding the value as long as the initial conditions are not such as to produce a value for the first temperature maximum.
that is higher than the peak value for the equilibrium glide. In addition, if the first peak temperature is higher, then the CONTAC system is able to prevent all subsequent peaks from exceeding the first.

With regard to range control, the CONTAC system is intended for use with any type of guidance scheme which uses closed loop control. The available down range increment for the vehicle in this study (which is similar to the M2-F2 now currently under investigation) is in excess of 100 degrees of equivalent longitude. The combined cross range for turns both to the left and to the right is greater than 20 degrees of equivalent latitude throughout all of the above value for down range. Within these limits, the footprint is essentially isothermal, i.e. the temperature experienced during the reentry is the same regardless of the terminal position of the vehicle within the footprint.

9.2 Effects of Adverse Conditions on the Reentry

The performance of the CONTAC system has been investigated under several types of adverse conditions. The first type is errors in the initial values of speed and glide angle. The second consists of those effects which can be predicted in advance, such as the Earth's rotation and oblateness. The third type consists of variations from standard values of the atmospheric density which are both altitude dependent and random.

Large errors in the initial speed and glide angle are considered unlikely since they can be produced only by extremely large retro-fire errors. (Past experience shows that retro-fire errors can be kept quite small.) At the deboost position, attitude alignment must be in error by about 5 degrees or the velocity increment must be in error by about 50 fps before any significant changes in the initial speed and glide angle are noted at the reentry altitude of 350,000 feet. However, if large retro-fire errors are present, the following results are noted. Negative errors in the initial speed up to 100 feet per second (larger errors are improbable) could cause an increase in the peak temperature up to 100 F. However, an increase in the initial glide angle of 0.5 degrees causes an increase of 300 F in the peak temperature.

The Earth's rotation causes a maximum increase of about 100 F in the peak temperature when the Earth's rotation and the initial heading combine to produce an effective initial speed 1500 feet per second above the standard value of 26,000 feet per second. When the Earth's oblateness and the initial heading and latitude combine to produce an effective initial glide angle of 0.18 degrees greater than the standard value of 1.0 degree, the maximum increase in the peak temperature is again about 100 F. However, the rotational condition is unlikely to occur in practice since satellite orbits are usually in the same sense as the Earth's rotation, and the oblateness effect may be compensated for in advance of the reentry by an appropriate adjustment of the retro-fire attitude alignment and velocity increment.

Atmospheric density perturbations based on altitude did not, within reasonable limits, produce any situations for which the CONTAC system could not correct. The maximum likely vertical density gradients had no effect on the CONTAC system performance. On the other hand, random spatial (roughly horizontal) density variations produced a very different result. The investigation described a basis for judging the adequacy of the CONTAC system when sufficient information about random density variations in the Earth's atmosphere becomes available. Insufficient data could be found for the altitudes of concern, namely between 220,000 feet to 290,000 feet, in order to arrive at a firm conclusion at this time.
REFERENCES


<table>
<thead>
<tr>
<th>Reference</th>
<th>Author(s)</th>
<th>Title and Details</th>
</tr>
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</table>
FIGURE 1 ENDURANCE TIME vs TEMPERATURE FOR COATED REFRACTORY METALS
(From 'Refractory Metals and their Protection', Thomas D. Cooper and Oscar O. Sharp, Aerospace Engineering, Vol. 22, No. 1, January 1963.)
FIGURE 2 TYPICAL MANOEUVRE BOUNDARY OR FOOTPRINT

NOTES:
1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.
2. Down range is always positive. It is in the direction parallel to the great circle.
3. Cross range is measured in the direction perpendicular to the great circle.
4. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 250,000 ft lb / 1lb
      (Nominal Speed = 3,000 ft/sec)
      (Nominal Altitude = 100,000 feet)
   (b) Heading is approximately perpendicular to the footprint boundary (110 degrees)

FIGURE 3 FOOTPRINT TERMINOLOGY
FIGURE 4  TYPICAL VEHICLE CONFIGURATION
MAXIMUM LATITUDE OF ORBITAL ENVELOPE EQUAL TO INERTIAL ORBITAL INCLINATION

SINGLE ORBITAL TRACK OVER EARTH'S SURFACE

EQUATOR

ORBITAL INCLINATION ANGLE RELATIVE TO THE EARTH

ORBITAL ENVELOPE (By reason of the Earth's rotation, complete surface under the envelope is covered at least once every 24 hours)

FIGURE 5 ORBITAL ENVELOPE
FIGURE 6 THE FIVE REENTRY PHASES
(Not To Scale)
INITIAL ORBIT

RETRO-FIRE

COMPLETION OF PHASE I OCCURS AT 350,000 FEET

FIGURE 7 PHASE I - PRE-ATMOSPHERIC FLIGHT
FIGURE 8a INITIAL CONDITIONS AT 350,000 FEET vs RETRO-FIRE ATTITUDE ALIGNMENT ERROR FOR AN INITIALLY CIRCULAR ORBIT
FIGURE 8b INITIAL CONDITIONS AT 350,000 FEET vs RETRO-FIRED
INCREMENTAL VELOCITY ERROR FOR AN INITIALLY CIRCULAR ORBIT
INITIAL POSITION FOR START OF CALCULATION
Phase II starts at 350,000 feet with an inertial speed of 26,000 ft/sec and an inertial glide angle of 1.00 degrees.

TYPICAL RESULTS AT FIRST \( \gamma_{\text{max}} \)

<table>
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<tr>
<th>ANGLE OF ATTACK (deg)</th>
<th>SPEED (fps)</th>
<th>ALTITUDE (feet)</th>
<th>RANGE (degrees)</th>
<th>GLIDE ANGLE (degrees)</th>
</tr>
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<tr>
<td>30°</td>
<td>25870</td>
<td>271900</td>
<td>18.52</td>
<td>0.022</td>
</tr>
<tr>
<td>40°</td>
<td>25820</td>
<td>273900</td>
<td>18.02</td>
<td>0.025</td>
</tr>
<tr>
<td>45°</td>
<td>25770</td>
<td>275100</td>
<td>17.69</td>
<td>0.031</td>
</tr>
<tr>
<td>55°</td>
<td>25630</td>
<td>276400</td>
<td>17.25</td>
<td>0.047</td>
</tr>
<tr>
<td>65°</td>
<td>25530</td>
<td>272400</td>
<td>16.76</td>
<td>0.074</td>
</tr>
</tbody>
</table>

NOTE: Orbital velocity at 275,000 feet is 25,770 ft/sec.

FIGURE 9 PHASE II - OVERALL DOWN RANGE CONTROL

--- CONTAC SYSTEM IN PASSIVE OPERATION (Attitude Controlled by Guidance System)

--- --- --- --- CONTAC SYSTEM IN ACTIVE OPERATION

SKIP PORTION OF THE REENTRY
(May be extended or reduced to provide medium down range control by the appropriate choice of the attitude. Some cross range control is also available during this portion by using large bank angles.)

SECOND \( \gamma_{\text{max}} \) AGAIN BRINGS CONTAC SYSTEM INTO ACTIVE OPERATION

PEAK TEMPERATURE USUALLY OCCURS AT ABOUT 245,000 FEET AND SIGNALS THE END OF PHASE III

SLOWLY BRINGS THE ATTITUDE TO THE OPTIMUM POSITION (35° for the angle of attack and 0° for the bank angle) WHILE KEEPING THE TEMPERATURE CONSTANT

NOTE:
Length of skip portion is dictated primarily by speed inherited from phase II of the reentry. When combined with a low angle of attack, a speed slightly higher than orbital velocity produces extended skips up to 120 degrees of equivalent longitude. When large angles of attack and large bank angles are combined with a speed slightly less than orbital velocity, the skip portion of phase III can be eliminated completely and the total down range increment during phase III can be reduced to about 10 degrees of equivalent longitude.

FIGURE 10 PHASE III - EQUILIBRIUM GLIDE ATTAINMENT
FIGURE 11c HISTORY OF ATTITUDE ANGLES FOR A CONTAC CONTROLLED REENTRY
Figure 11-4 Detailed History of Angle of Attack for a Portion of a Contac Controlled Reentry
TO TRY AGAIN WITH A SMALLER SIZE PATH STEP AND SET UP PATH STEP TO BE USED

ALERT CALCULATION SECTION TO TRY AGAIN WITH A SMALLER SIZE PATH STEP AND SET UP PATH STEP TO BE USED

CALCULATION OF TRAJECTORY OF SPACE VEHICLE

EXTERNAL SETTING TEST FOR T

IS \( (T_{lastmax} - T) > \tau_{deadband} \)

Maximum Test

IS \( (T_{lastmin} - T) > \tau_{deadband} \)

Minimum Test

HAS SMALLEST SIZE PATH STEP FOR THIS TEST BEEN REACHED

ALERT CALCULATION SECTION TO COMPUTE NRM VALUES OF \((N, t_0) AND (L, t_2)\)

FIGURE 12 CONTACT SYSTEM PROGRAM FLOW DIAGRAM

ORIGINAL POINTS IN DIGITAL CALCULATION

FIGURE 13 TEMPERATURE vs DISTANCE ALONG FLIGHT PATH

DIGITAL POINTS AND \( \tau_{deadband} \) DIFFICULITIES
Figure 14: Temperature vs Distance Along Flight Path

Digital Points and $T_{\text{deadband}}$ Difficulty

Figure 15: Onboard Digital Contac System Flow Diagram
FIGURE 16 CONTAC SYSTEM CONTROL OF THE ANGLE OF ATTACK
FIGURE 18 BASIC REFERENCE SYSTEM

PRIME MERIDIAN (GREENWICH)

FIGURE 17 DEPARTURES IN THE HEIGHT OF SPECIFIC DENSITY SURFACES FROM U.S. STANDARD ATMOSPHERE, 1962.

(From 'Horizontal and Vertical Distribution of Atmospheric Density up to 50 km', Allen E. Cole and Arthur J. Kantor, Air Force Surveys in Geophysics, No. 157, AFRCL-64-483, June 1964.)
Figure 19: Geocentric Coordinates for Position and Velocity

Figure 20: Flight Axes Coordinate Reference System
NOTE:
1. \( \vec{R} \) axis is pointing out of page from indicated position.

\[
\begin{pmatrix}
\vec{1}
\vec{2}
\vec{3}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\vec{1}'
\vec{2}'
\vec{3}'
\end{pmatrix}
\]

\[
\begin{pmatrix}
\vec{1}
\vec{2}
\vec{3}
\end{pmatrix} =
\begin{pmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi
\end{pmatrix}
\begin{pmatrix}
\vec{1}'
\vec{2}'
\vec{3}'
\end{pmatrix}
\]

**Figure 21**: Reference Systems

- **Basic**: \((\vec{1}, \vec{2}, \vec{3})\)
- **Intermediate**: \((\vec{1}', \vec{2}', \vec{3})\), \((\vec{5}, \vec{6}, \vec{7})\), \((\vec{8}, \vec{9}, \vec{10})\)
- **Flight**: \((\vec{1}, \vec{2}, \vec{3})\)

**Notes**:
1. \( \vec{1}, \vec{2}, \vec{3}, \vec{1}', \vec{2}', \vec{3}' \) are all in the equatorial plane.
2. \( \vec{5}, \vec{6}, \vec{7}, \vec{8}, \vec{9}, \vec{10} \) are all in the local geocentric horizontal plane.
3. \( \vec{1}, \vec{2}, \vec{3}, \vec{1}, \vec{2}, \vec{3} \) are all in the same local geocentric vertical plane.
NOTE:
1. \((\mathbf{g})\) axis is pointing into page at indicated position.

LOCAL GEOCENTRIC HORIZONTALS

PARALLEL OF LATITUDE THROUGH VEHICLE POSITION

\[
\begin{pmatrix}
\mathbf{i} \\
\mathbf{j} \\
\mathbf{k}
\end{pmatrix} =
\begin{pmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\mathbf{h} \\
\mathbf{j} \\
\mathbf{v}
\end{pmatrix}
\]

FIGURE 24 CONVERSION BETWEEN SECOND INTERMEDIATE REFERENCE SYSTEM \((\mathbf{h},\mathbf{e},\mathbf{v})\) AND THIRD INTERMEDIATE REFERENCE SYSTEM \((\mathbf{e},\mathbf{j},\mathbf{v})\)

NOTES:
1. \((\mathbf{f'})\) axis is pointing out of page from indicated position.
2. \((\mathbf{e})\) axis is pointing into page at indicated position.

EQUATOR

MERIDIAN OF LONGITUDE THROUGH VEHICLE POSITION

\[
\begin{pmatrix}
\mathbf{i} \\
\mathbf{j} \\
\mathbf{k}
\end{pmatrix} =
\begin{pmatrix}
\sin \alpha & 0 & \cos \alpha \\
0 & -1 & 0 \\
\cos \alpha & 0 & -\sin \alpha
\end{pmatrix}
\begin{pmatrix}
\mathbf{r} \\
\mathbf{f'} \\
\mathbf{e}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\mathbf{i} \\
\mathbf{j} \\
\mathbf{k}
\end{pmatrix} =
\begin{pmatrix}
\sin \alpha & 0 & \cos \alpha \\
0 & -1 & 0 \\
\cos \alpha & 0 & -\sin \alpha
\end{pmatrix}
\begin{pmatrix}
\mathbf{e} \\
\mathbf{f'} \\
\mathbf{r}
\end{pmatrix}
\]

FIGURE 23 CONVERSION BETWEEN FIRST INTERMEDIATE REFERENCE SYSTEM \((\mathbf{r},\mathbf{f'},\mathbf{e})\) AND SECOND INTERMEDIATE REFERENCE SYSTEM \((\mathbf{e},\mathbf{n},\mathbf{v})\)
NOTE:
1. ($\vec{j}$) axis is pointing out of page from indicated position

\[
\begin{align*}
\begin{pmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{pmatrix}
&= 
\begin{pmatrix}
cos\gamma & 0 & -sin\gamma \\
0 & 1 & 0 \\
sin\gamma & 0 & cos\gamma
\end{pmatrix}
\begin{pmatrix}
\hat{h} \\
\hat{j} \\
\hat{v}
\end{pmatrix}

\begin{pmatrix}
\hat{h} \\
\hat{j} \\
\hat{v}
\end{pmatrix}
&= 
\begin{pmatrix}
cos\gamma & 0 & sin\gamma \\
0 & 1 & 0 \\
-sin\gamma & 0 & cos\gamma
\end{pmatrix}
\begin{pmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{pmatrix}
\end{align*}
\]

FIGURE 25 CONVERSION BETWEEN THIRD INTERMEDIATE REFERENCE SYSTEM ($\hat{h}, \hat{j}, \hat{v}$) AND FLIGHT AXES REFERENCE SYSTEM ($\hat{i}, \hat{j}, \hat{k}$)
Figure 26: Vehicle performance for various entry and skip angles of attack.

Initial speed = 28,000 ft/sec
Initial glide angle = 1.00 degree
Initial altitude = 350,000 feet

Each curve represents a different entry angle of attack, \( \alpha_e \). For each entry angle of attack, the skip angle of attack was varied between 5° and the smaller of 35° and \( \alpha_g \).

Equilibrium glide point

Equilibrium glide point

Down range achieved at 100,000 feet (degrees)

Required initial conditions:
- Initial speed = 25,703 ft/sec
- Initial glide angle = 0.09857 degrees
- Initial altitude = 350,000 feet
- Angle of attack = 35.0 degrees
- Bank angle = 0.0 degrees

(Note: Cross range is zero.)
1. All positions of down range and cross range are given in degrees of equivalent latitude and longitude, respectively, relative to the great circle through the projection of the initial velocity vector.

2. Conditions on the footprint boundary are as follows:
   - Energy of Vehicle = 250,000 ft lb / lbm
     - Nominal Speed = 3,600 ft/sec
     - Nominal Altitude = 100,000 feet
   - Heading is approximately perpendicular to the footprint boundary (220 degrees)

3. Conditions at the initial position are as follows:
   - Glide Angle = 1.00 degree
   - Altitude = 350,000 feet

**Table**

<table>
<thead>
<tr>
<th>CODE</th>
<th>INITIAL SPEED</th>
<th>INITIAL ERROR</th>
<th>PEAK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26000 fpe</td>
<td>0 ft/sec</td>
<td>2828°F</td>
</tr>
<tr>
<td>2</td>
<td>26100 fpe</td>
<td>100 ft/sec</td>
<td>2828°F</td>
</tr>
<tr>
<td>3</td>
<td>26300 fpe</td>
<td>300 ft/sec</td>
<td>2828°F</td>
</tr>
<tr>
<td>4</td>
<td>25500 fpe</td>
<td>-100 ft/sec</td>
<td>3020°F</td>
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<tr>
<td>5</td>
<td>25700 fpe</td>
<td>-300 ft/sec</td>
<td>3107°F</td>
</tr>
</tbody>
</table>

Footprints 2 and 3 covered more than one complete revolution of the Earth. Consequently, the forward portion of 2 is actually at about 300 degrees down range and 3 is actually at about 135 degrees down range.

**Figure 29** Cross Range vs Down Range for Various Initial Speeds Over a Circular, Non-Rotating Earth
NOTES:
1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.
2. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 250,000 ft lb / l
      (Nominal Speed = 3,000 ft/sec)
      (Nominal Altitude = 100,000 feet)
   (b) Heading is approximately perpendicular to the footprint boundary (±20 degrees)
3. Conditions at the initial position are as follows:
   (a) Speed = 26,000 ft/sec
   (b) Altitude = 320,000 feet

<table>
<thead>
<tr>
<th>NO.</th>
<th>GLIDE ANGLE</th>
<th>INITIAL</th>
<th>INITIAL ERROR</th>
<th>PEAK</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00 degrees</td>
<td>0.00</td>
<td>0.00</td>
<td>2828°F</td>
</tr>
<tr>
<td>2</td>
<td>0.75 degrees</td>
<td>-0.25</td>
<td>-0.25</td>
<td>2828°F</td>
</tr>
<tr>
<td>3</td>
<td>0.65 degrees</td>
<td>-0.35</td>
<td>-0.35</td>
<td>2828°F</td>
</tr>
<tr>
<td>4</td>
<td>1.25 degrees</td>
<td>+0.25</td>
<td>+0.25</td>
<td>2853°F</td>
</tr>
<tr>
<td>5</td>
<td>1.50 degrees</td>
<td>+0.50</td>
<td>+0.50</td>
<td>3012°F</td>
</tr>
</tbody>
</table>

Footprints 2 and 3 covered more than one complete revolution of the earth. Consequently, the forward portion of 2 is actually at about 550 degrees down range and 3 is actually at about 800 degrees down range.

FIGURE 30 CROSS RANGE vs DOWN RANGE FOR VARIOUS INITIAL GLIDE ANGLES OVER A CIRCULAR, NON-ROTATING EARTH
NOTES:

1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.
2. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 250,000 ft lb / lbm (Nominal Speed = 3,000 ft/sec)
   (b) Nominal Altitude = 100,000 feet
   (c) Inertial Azimuth is value given in the table; no correction is necessary for these two reentries.

3. Inertial conditions (relative to inertial space) at the initial position are as follows:
   (a) Inertial Speed = 25,000 ft/sec
   (b) Inertial Glide Angle = 1.00 degree
   (c) Altitude = 350,000 feet
   (d) Inertial Azimuth is value given in the table; no correction is necessary for these two reentries.

4. Solid lines are the boundaries of the footprints.
5. Broken lines are the midlines of the footprints. They show the extent of the shift in cross range due to the Earth's rotation.

FIGURE 31b CROSS RANGE vs DOWN RANGE FOR TWO INITIAL POSITIONS
OVER A CIRCULAR, ROTATING EARTH
NOTES:

1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.

2. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 250,000 ft lb / lbm
      (Nominal Speed = 3,000 ft/sec)
      (Nominal Altitude = 100,000 feet)
   (b) Heading is approximately perpendicular to the footprint boundary (±20 degrees)

3. Inertial conditions (relative to inertial space) at the initial position are as follows:
   (a) Inertial Speed = 26,000 ft/sec
   (b) Inertial Glide Angle = 1.00 degree
   (c) Altitude = 350,000 feet
   (d) Inertial Azimuth is value given in table; a correction must be applied (±2.0 degrees) to obtain the azimuth relative to a rotating Earth.

4. Solid lines are the boundaries of the footprints.

5. Broken lines are the midlines of the footprints. They show the extent of the shift in cross range due to the Earth's rotation.

FIGURE 31b CROSS RANGE vs DOWN RANGE FOR TWO INITIAL POSITIONS OVER A CIRCULAR, ROTATING EARTH
1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.

2. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 250,000 ft lb / lbm
   (Nominal Speed = 3,000 ft/sec)
   (Nominal Altitude = 100,000 feet)
   (b) Heading is approximately perpendicular to the footprint boundary (220 degrees)

3. Inertial conditions (relative to inertial space) at the initial position are as follows:
   (a) Inertial Speed = 26,000 ft/sec
   (b) Inertial Glide Angle = 1.00 degree
   (c) Altitude = 350,000 feet
   (d) Inertial Azimuth is value given in table; the azimuth for footprint 4 is -3.4 degrees relative to a rotating Earth; no correction is necessary for footprint 7 since the relative rotational speed at the pole is zero.

4. Solid lines are the boundaries of the footprints.

5. Broken lines are the midlines of the footprints. They show the extent of the shift in cross range due to the Earth's rotation.

Figure 31c: Cross Range vs. Down Range for Two Initial Positions Over a Circular, Rotating Earth
### NOTES:

1. All positions of down range and cross range are given in degrees of equivalent longitude and latitude, respectively, relative to the great circle through the projection of the initial velocity vector.

2. Conditions on the footprint boundary are as follows:
   - **(a) Energy of Vehicle = 250,000 ft lb / 1 lb**
     - (Nominal Speed = 3,000 ft/sec)
   - (Nominal Altitude = 100,000 feet)

3. Conditions at the initial position are as follows:
   - (a) Glide Angle = 1.00 degree
   - (b) Altitude = 350,000 feet (perpendicular height above the Earth's oblate surface)

### CODE

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<th>LATITUDE</th>
<th>AZIMUTH</th>
<th>PEAK</th>
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<tbody>
<tr>
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<td>Circular Earth Ref.</td>
<td>2828°F</td>
<td>2828°F</td>
</tr>
<tr>
<td>2</td>
<td>0°</td>
<td>90°</td>
<td>2828°F</td>
</tr>
<tr>
<td>3</td>
<td>0°</td>
<td>90°</td>
<td>2828°F</td>
</tr>
<tr>
<td>4</td>
<td>0°</td>
<td>0°</td>
<td>2828°F</td>
</tr>
<tr>
<td>5</td>
<td>0°</td>
<td>0°</td>
<td>2828°F</td>
</tr>
<tr>
<td>6</td>
<td>-60°</td>
<td>0°</td>
<td>2984°F</td>
</tr>
<tr>
<td>7</td>
<td>-90°</td>
<td>0°</td>
<td>2937°F</td>
</tr>
</tbody>
</table>

*All of footprint 2 and the rear 30 degrees of footprint 3 coincided with footprint 1.*

Footprints 4 and 5 covered a little more than one complete revolution of the Earth. Consequently, the forward portions of both footprints are actually at about 300 degrees of down range.

**FIGURE 32** CROSS RANGE vs DOWN RANGE FOR VARIOUS INITIAL POSITIONS AND HEADINGS OVER AN OBLATE, NON-ROTATING EARTH
Figure 39a: Comparison between the Cross Range Shift of a Footprint Boundary and the Cross Range Shift of the Orbital Track of a Satellite — Oblate, Rotating Earth

Notes:
1. All positions of down range and cross range are given in degrees of equatorial longitude and latitude, respectively, relative to the given circle through the projection of the initial velocity vector.
2. Conditions on the footprint boundary are as follows:
   (a) Energy of Vehicle = 300,000 ft lb / lbf
   (b) Normal Range = 2,000 ft
   (c) Downrange Distances = 100,000 feet
3. Inertial conditions (relative to inertial space) at the initial positions are as follows:
   (a) Inertial Range = 20,000 ft/s
   (b) Inertial Downrange = 700 feet
4. Inertial Altitude = 300,000 feet (geocentric height above Earth's equator plane)
5. Inertial Attitude to be given in table; the errors for parameters 1 to 3.5 degrees relative to a rotating Earth.
CROSS RANGE (degrees)

INITIAL POSITION

DOWN RANGE (degrees)

<table>
<thead>
<tr>
<th>CODE</th>
<th>INITIAL LATITUDE</th>
<th>INITIAL AZIMUTH</th>
<th>PEAK TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>90°</td>
<td>2704°F</td>
</tr>
<tr>
<td>2</td>
<td>0°</td>
<td>135°</td>
<td>2730°F</td>
</tr>
<tr>
<td>4</td>
<td>0°</td>
<td>-135°</td>
<td>2831°F</td>
</tr>
<tr>
<td>5</td>
<td>0°</td>
<td>-90°</td>
<td>2915°F</td>
</tr>
</tbody>
</table>

NOTES:
1. All positions of cross range and cross range are given in degrees of equivalent longitude and latitude, respectively. The initial position vector of the initial velocity vector.
2. Conditions on the footprint boundary are as follows:
   a. Range of vehicle = 250,000 ft lb / Ibm
   b. Initial Slide Angle = 1.00 degree
   c. Altitude = 350,000 feet (perpendicular height above Earth's oblate surface)
   d. Inertial Azimuth in value given in table; no correction is necessary for footprints 1 and 3; a correction must be applied (-2.0 degrees) for footprints 2 and 4 to obtain the azimuth relative to a rotating Earth.

3. Inertial conditions (relative to inertial space) at the initial position are as follows:
   a. Inertial Speed = 24,000 ft/sec
   b. Inertial Slide Angle = 1.00 degree
   c. Altitude = 350,000 feet (perpendicular height above Earth's oblate surface)

4. Solid lines are the boundaries of the footprints.
5. Broken lines are the midlines of the footprints. They show the extent of the shift in cross range due to the Earth's rotation.

FIGURE 33 CROSS RANGE vS DOWN RANGE FOR FOUR INITIAL HEADINGS OVER AN OBLATE, ROTATING EARTH
Density relationships \( \rho = \rho_0 e^{-\gamma h} \)

<table>
<thead>
<tr>
<th>CURVE NO.</th>
<th>( \rho_0 ) (slugs/ft(^3))</th>
<th>( h^2 ) (ft)</th>
<th>( \rho_0 ) (slugs/ft(^3))</th>
<th>( h^2 ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>41.39</td>
<td>8.193</td>
<td>14,000</td>
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<td>2</td>
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<td>30.713</td>
<td>0.2118</td>
<td>17,000</td>
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<td>3</td>
<td>0.0011494</td>
<td>26.014</td>
<td>0.01639</td>
<td>20,000</td>
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<tr>
<td>4</td>
<td>0.0021107</td>
<td>22.370</td>
<td>0.002473</td>
<td>23,000</td>
</tr>
<tr>
<td>5</td>
<td>0.0033711</td>
<td>21.676</td>
<td>0.0015773</td>
<td>26,000</td>
</tr>
</tbody>
</table>

Figure 34: Density Altitude Relationships Used to Test the Performance of the Contac System.
Figure 36 Operational Region of Contac System

Figure 35 Fractional Variation of Density with Altitude
FIGURE 37 DEFINITION OF THE EFFECTIVE INITIAL GLIDE ANGLE

\[ \gamma_e = \sin^{-1}(-\Delta y/\Deltairst) \]
\[ \Delta y = \gamma_1 - \gamma_2 \]
\[ \gamma_{eff} = \gamma_e + \Delta y/\Deltairst \]  
(B.2)
**APPENDIX A**

**The Equilibrium Glide Trajectory**

An equilibrium glide trajectory is a coplanar reentry during which the lift is always equal to the net difference between the gravitational and the centrifugal forces on the vehicle. Since drag is not compensated by thrust, the speed is gradually reduced thereby requiring an increase in lift to offset the loss in centrifugal force. Since the angle of attack is fixed, this requirement is met by continually descending to a lower altitude, but a controlled rate so that the lift never becomes too large. As a consequence, the reentry is characterized by the absence of any phugoid or long period oscillations involving the exchange of speed for altitude and vice versa.

For a simple coplanar reentry over a circular non-rotating Earth, the equations of motion are given by

\[ m \ddot{V} + D - mg \sin \gamma = 0 \quad (A.1) \]
\[ m V \dot{\gamma} + L + m \left( \frac{V^2}{r} - g \right) \cos \gamma = 0 \quad (A.2) \]

For an equilibrium glide, all the important effects of interest occur between 100,000 feet and 350,000 feet. Consequently, the values of \( r \) and \( g \) will vary by less than \( \pm 1\% \) from the average values over the above altitude range. Therefore, it is reasonable to make \( r \) and \( g \) constant.

During an equilibrium glide, it is assumed that the \( m V \dot{\gamma} \) term in Eq. (A.2) is negligible compared to the lift. Note that this term cannot always be equal to zero since this would require a constant glide angle.

Finally, the analysis will be restricted to the region for which \( \gamma \ll 1 \). (For the vehicle used in this investigation, \( \gamma < 5^\circ \) until the speed drops to 4300 feet/sec at an altitude of 140,000 feet.) Also, the lift and drag are expressed in the usual manner, in terms of \( C_L \) and \( C_D \). Equations (A.1) and (A.2) become

\[ \dot{V} + C_D \frac{1}{2} \rho V^2 \frac{S}{m} - g \gamma = 0 \quad (A.3) \]
\[ C_L \frac{1}{2} \rho V^2 \frac{S}{m} = \left( g - \frac{V^2}{r} \right) \quad (A.4) \]

Equation (A.4) is then rewritten to give

\[ \rho = \frac{2 m}{C_L \frac{1}{2} \rho V^2} \frac{1 - s^2}{s^2} \quad (A.5) \]

where the speed ratio is

\[ s = V/\sqrt{rg} \quad (A.6) \]

Equation (3.7) can be used to find the temperature as a function of speed. The result is

\[ T^4 = K_s \left( 1 - s^2 \right)^{\frac{1}{2}} \quad (A.7) \]
where
\[ K = \frac{17,600 (\sqrt{gR})^{3.15}}{\epsilon \sigma \sqrt{R_n}(26,000)^{3.15} \sqrt{0.00237} \frac{C_L S}{2m} r} \] (A.8)

It is straightforward to show that the peak temperature occurs at
\[ s^* = \frac{\sqrt{2}}{3} = 0.8165 \] (A.9)

The use of GM and \( R_n \) from equations (4.7) and (4.8), respectively, and using \( g \) and \( r \) for the average height of 250,000 feet gives the values at the position of the peak temperature as
\[ T^* = 3035 \left( \frac{\rho^*}{\epsilon^2 R_n C_L S} \right)^{1/8} \] (A.10)
\[ V^* = 21,050 \text{ feet/sec} \] (A.11)
\[ \rho^* = 4.722 \times 10^{-8} \left( \frac{C_L S}{m} \right) \text{ slugs/ft}^3 \] (A.12)

Equations (A.10) to (A.12) show that the peak temperature always occurs at the same speed. Also, the height, \( y^* \) (actually the density) is dependent only on the lift coefficient and the wing loading. From Eq. (A.10), the smallest value for \( T^* \) is obtained for the maximum value of vertical force, i.e. of \( C_L S/m \). Consequently, the smallest \( T^* \) is obtained by holding the vehicle at the attitude for \( C_L \text{max} \). The expression for \( C_L \) in Eq. (3.3) is a maximum for an angle of attack of 55 degrees, which is, therefore, the optimum angle.

At the optimum angle of 55 degrees, \( C_L S/m \) is 0.5417 sq.ft/slug for the subject vehicle. Substitution of this value together with \( \epsilon R_n = 1.0 \) into Eqs. (A.10) and (A.12) yields the values in Table A.1.

An equilibrium glide can be realized very closely in practice if the proper initial conditions are achieved at the initial position. Shown on Figure 26 are the values of speed and glide angle which are required at 350,000 feet altitude above a circular, non-rotating Earth. Initially, the speed and glide angle are adjusted to produce a negligible value for \( mV^2/2 \) in Eq. (A.2). It is found that the required condition of a negligible \( mV^2 \) (relative to \( L \)) then prevails throughout the flight without any additional control. The values of \( T^*, V^*, \rho^* \) are also shown in Table A.1 for the trajectory computed from the exact equations. The value of \( y^* \) is also included for this case and may be compared to the value of \( y^*(\rho^*) \) which results from Eq. (A.12) when the U.S. Standard atmosphere, 1962 is the curve of density versus altitude. The same curve was used for the numerically computed trajectory.

The agreement between the two sets of data shows that equations (A.10) to (A.12) are quite accurate. Consequently, the standard peak temperature, \( T^* \), as given by Eq. (A.10) is shown to be independent of \( (L/D) \). Also, the smallest value of \( T^* \) is achieved for the smallest value of \( (m/\epsilon^2 R_n C_L S) \). In addition, because of the \( 1/8 \) power, the gains to be realized from even a 50% reduction in this parameter are not large; when \( T^* \) equals 3000°F, the reduction in \( T^* \) is only 320°F or only a little more than 10%.

A2
TABLE A1

Flight Conditions at the Position of Peak Temperature During an Equilibrium Glide

<table>
<thead>
<tr>
<th>Value</th>
<th>Approximate Analytical Results*</th>
<th>Numerical Integration of Exact Equations‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>T*</td>
<td>2817°F</td>
<td>2810°F</td>
</tr>
<tr>
<td>v*</td>
<td>21,050 feet/sec</td>
<td>20,800 feet/sec</td>
</tr>
<tr>
<td>ρ*</td>
<td>$8.72 \times 10^{-8}$ slugs/ft³</td>
<td>$9.27 \times 10^{-8}$ slugs/ft³</td>
</tr>
<tr>
<td>y*</td>
<td>245,500 feet***</td>
<td>243,800 feet***</td>
</tr>
</tbody>
</table>

+ equations (A.10) to (A.12)

‡ initial conditions of Fig. 26

*** corresponding to ρ* for the U.S. Standard Atmosphere, 1962
The definition of the effective initial glide angle over an oblate Earth is quite straightforward. It simply takes account of the effective change in height of the vehicle which occurs during Phase II of the reentry due to the change in the Earth's radius. This portion of the flight has been chosen since it is during this period that both the peak temperature and the overall down range are determined. As shown in Fig. 37, the change in the Earth's radius may be used to give

\[ \sin \Delta \gamma = \frac{\Delta y}{s} \]  

(8.1)

where \( \Delta y \) is defined in Fig. 37.

For this investigation, both \( \gamma \) and \( \Delta \gamma \) are always very small during Phase II. Consequently, the small angle approximation may be used to give:

\[ \gamma_{\text{eff}} = \gamma_e + \frac{\Delta y}{s} \]  

(8.2)

In practice, \( \Delta y \) is always very easy to find. It is the difference in the height over the Earth's surface when the same change that takes place during Phase II in geocentric latitude also occurs at the average geocentric radius which is used during Phase II. The value of \( s \) is the flight distance travelled during this portion of the reentry.
SUBROUTINE FEP'S. PAGE 5

110 IF (FEP'S. EQ. 1) THEN
   120 IF (FEP'S. EQ. 2) THEN
      130 IF (FEP'S. EQ. 3) THEN
         140 IF (FEP'S. EQ. 4) THEN
            150 IF (FEP'S. EQ. 5) THEN
               160 IF (FEP'S. EQ. 6) THEN
                  170 IF (FEP'S. EQ. 7) THEN
                     180 IF (FEP'S. EQ. 8) THEN
                        190 IF (FEP'S. EQ. 9) THEN
                           200 IF (FEP'S. EQ. 10) THEN
                              210 IF (FEP'S. EQ. 11) THEN
                                 220 IF (FEP'S. EQ. 12) THEN
                                    230 RETURN
   240 END IF
51 RETURN

SUBROUTINE FEP'S. PAGE 6

140 IF (FEP'S. EQ. 13) THEN
   150 IF (FEP'S. EQ. 14) THEN
      160 IF (FEP'S. EQ. 15) THEN
         170 IF (FEP'S. EQ. 16) THEN
            180 IF (FEP'S. EQ. 17) THEN
               190 IF (FEP'S. EQ. 18) THEN
                  200 IF (FEP'S. EQ. 19) THEN
                     210 IF (FEP'S. EQ. 20) THEN
                        220 IF (FEP'S. EQ. 21) THEN
                           230 RETURN
   240 END IF
52 RETURN

RETURN TO INITIALIZATION POINT SUBROUTINE FEP'S.

RETURN
SUBROUTINE CECST 6.8 PAGE 1

C

C COMMON STATEMENTS FOR GLOBAL VARIABLES
COMMON 00001,0002,0003,0004,0005,0006

C COMMON STATEMENTS FOR LOCAL VARIABLES
COMMON 0007,0008,0009,0010,0011,0012

C LOCAL DATA VALUES
DATA 0013,0014,0015,0016,0017,0018

SUBROUTINE CECST 6.8 PAGE 2

C 

C COMMON STATEMENTS FOR LOCAL VARIABLES
COMMON 0019,0020,0021,0022,0023,0024

C COMMON STATEMENTS FOR GLOBAL VARIABLES
COMMON 0025,0026,0027,0028,0029,0030

C LOCAL DATA VALUES
DATA 0031,0032,0033,0034,0035,0036

SUBROUTINE CECST 6.8 PAGE 3

C

C COMMON STATEMENTS FOR LOCAL VARIABLES
COMMON 0037,0038,0039,0040,0041,0042

C COMMON STATEMENTS FOR GLOBAL VARIABLES
COMMON 0043,0044,0045,0046,0047,0048

C LOCAL DATA VALUES
DATA 0049,0050,0051,0052,0053,0054

SUBROUTINE CECST 6.8 PAGE 4

C

C COMMON STATEMENTS FOR LOCAL VARIABLES
COMMON 0055,0056,0057,0058,0059,0060

C COMMON STATEMENTS FOR GLOBAL VARIABLES
COMMON 0061,0062,0063,0064,0065,0066

C LOCAL DATA VALUES
DATA 0067,0068,0069,0070,0071,0072

SUBROUTINE CECST 6.8 PAGE 5

C

C COMMON STATEMENTS FOR LOCAL VARIABLES
COMMON 0073,0074,0075,0076,0077,0078

C COMMON STATEMENTS FOR GLOBAL VARIABLES
COMMON 0079,0080,0081,0082,0083,0084

C LOCAL DATA VALUES
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SUBROUTINE CSTRM (real,parm,log)

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C EXTENDS THE SPACE DESIGN FOR LOCAL VARIABLES

C COMMON STATEMENTS FOR GLOBAL VARIABLES

C DATA STATEMENTS FOR GLOBAL VARIABLES

C COMMON STATEMENTS FOR LOCAL VARIABLES

C TYPE STATEMENTS FOR LOCAL VARIABLES

SUBROUTINE CSTRM (real, parm, log)

C COMM STATEMENTS FOR LOCAL VARIABLES

C EXTENDS THE SPACE DESIGN FOR LOCAL VARIABLES

C COMMON STATEMENTS FOR GLOBAL VARIABLES

C DATA STATEMENTS FOR GLOBAL VARIABLES

C COMMON STATEMENTS FOR LOCAL VARIABLES

C TYPE STATEMENTS FOR LOCAL VARIABLES

END

SUBROUTINE CSTRM (real, parm, log)

C COMM STATEMENTS FOR LOCAL VARIABLES

C EXTENDS THE SPACE DESIGN FOR LOCAL VARIABLES

C COMMON STATEMENTS FOR GLOBAL VARIABLES

C DATA STATEMENTS FOR GLOBAL VARIABLES

C COMMON STATEMENTS FOR LOCAL VARIABLES

C TYPE STATEMENTS FOR LOCAL VARIABLES

END
**SUBROUTINE MODULE 6.0 PAGE 1**

**COMPUTATION OF THE HARMONIC VARIABILITY FACTOR**

1. \( F_{12} \) = \( \frac{F_{12}}{F_{11}} \) \( \times \) \( \frac{F_{11}}{F_{12}} \) \( \times \) \( \frac{F_{12}}{F_{12}} \)

2. \( F_{11} \) = \( \frac{F_{11}}{F_{11}} \) \( \times \) \( \frac{F_{11}}{F_{11}} \) \( \times \) \( \frac{F_{11}}{F_{11}} \)

3. \( F_{12} \) = \( \frac{F_{12}}{F_{12}} \) \( \times \) \( \frac{F_{12}}{F_{12}} \) \( \times \) \( \frac{F_{12}}{F_{12}} \)

**SUBROUTINE MODULE 6.0 PAGE 2**

**COMPUTATION OF THE VARIABILITY FACTOR**

1. \( F_{i} \) = \( \frac{F_{i}}{F_{i}} \) \( \times \) \( \frac{F_{i}}{F_{i}} \) \( \times \) \( \frac{F_{i}}{F_{i}} \)

2. \( F_{j} \) = \( \frac{F_{j}}{F_{j}} \) \( \times \) \( \frac{F_{j}}{F_{j}} \) \( \times \) \( \frac{F_{j}}{F_{j}} \)

3. \( F_{k} \) = \( \frac{F_{k}}{F_{k}} \) \( \times \) \( \frac{F_{k}}{F_{k}} \) \( \times \) \( \frac{F_{k}}{F_{k}} \)

**SUBROUTINE MODULE 6.0 PAGE 3**

**COMPUTATION OF THE ALTITUDE DEPENDENT VARIABILITY FACTOR**

1. \( F_{i} \) = \( \frac{F_{i}}{F_{i}} \) \( \times \) \( \frac{F_{i}}{F_{i}} \) \( \times \) \( \frac{F_{i}}{F_{i}} \)

2. \( F_{j} \) = \( \frac{F_{j}}{F_{j}} \) \( \times \) \( \frac{F_{j}}{F_{j}} \) \( \times \) \( \frac{F_{j}}{F_{j}} \)

3. \( F_{k} \) = \( \frac{F_{k}}{F_{k}} \) \( \times \) \( \frac{F_{k}}{F_{k}} \) \( \times \) \( \frac{F_{k}}{F_{k}} \)

**SUBROUTINE MODULE 6.0 PAGE 4**

**CALCULATION OF THE BALANCE FACTOR**

1. \( \text{Balance} = \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \)

2. \( \text{Balance} = \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \)

3. \( \text{Balance} = \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \times \frac{\text{Balance}}{\text{Balance}} \)
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<td>Data 6</td>
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<tr>
<td>Data 7</td>
<td>Data 8</td>
<td>Data 9</td>
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Additional notes or comments:
<table>
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<tr>
<th><strong>PROGRAM D</strong></th>
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<tbody>
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<td><strong>-0.00</strong></td>
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<tr>
<td><strong>-0.00</strong></td>
</tr>
</tbody>
</table>

**NOTE**
- In order to minimize the temperature and acceleration peaks, the attitude of the vehicle will be slowly changed to the nominal values whenever the temperature is decreasing. The attitude will slowly change back to the nominal values when the temperature is increasing.
- The following rates of change will be used:
  - **Maximum Pitch Rate** = 4.000 DEGREES/SEC.
  - **Maximum Roll Rate** = 8.000 DEGREES/SEC.
- **Upper Pitch Accel** = 10.000 DEGREES/SEC#2
- **Upper Roll Accel** = 30.000 DEGREES/SEC#2
- **Lower Pitch Accel** = 1.000 DEGREES/SEC#2
- **Lower Roll Accel** = 2.000 DEGREES/SEC#2

**INITIAL VELOCITY** = 26000.00000000000 00 FT/SEC.
**INITIAL GEOCENTRIC RADIUS** = 21275839.8900132000 00 FEET

---

**PILE**

**GLIDE**

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<tr>
<th><strong>N</strong></th>
<th><strong>S</strong></th>
<th><strong>E</strong></th>
<th><strong>F</strong></th>
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**INITIAL ATTITUDE**

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<th><strong>NO.</strong></th>
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<th><strong>TEMPELS</strong></th>
<th><strong>CEN</strong></th>
<th><strong>DONN</strong></th>
<th><strong>GROSS</strong></th>
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<td><strong>2,257645</strong></td>
<td><strong>2,257645</strong></td>
<td><strong>2,257645</strong></td>
</tr>
</tbody>
</table>

---

**NOTE**
- In order to minimize the temperature and acceleration peaks, the attitude of the vehicle will be slowly changed to the nominal values whenever the temperature is decreasing. The attitude will slowly change back to the nominal values when the temperature is increasing.
- The following rates of change will be used:
  - **Maximum Pitch Rate** = 4.000 DEGREES/SEC.
  - **Maximum Roll Rate** = 8.000 DEGREES/SEC.
- **Upper Pitch Accel** = 10.000 DEGREES/SEC#2
- **Upper Roll Accel** = 30.000 DEGREES/SEC#2
- **Lower Pitch Accel** = 1.000 DEGREES/SEC#2
- **Lower Roll Accel** = 2.000 DEGREES/SEC#2

---

**NOTE**
- In order to maximize the velocity range, the nominal angle of roll minimum if temperatures are being minimized will vary according to the 1.0 power of the velocity between -50.000 DEGREES and -0.00000 DEGREES OVER A SPECIFIED VELOCITY RANGE.
- In order to extend (or shorten) the down range, the following pairs of limits will be used for the angles of attack and roll until the equilibrium stagnation point temperature reaches its second rise for some other appropriate condition is reached in the flight.

<table>
<thead>
<tr>
<th><strong>PHI</strong></th>
<th><strong>-75.000 DEGREES</strong></th>
<th><strong>COEFFICIENTS</strong></th>
<th><strong>L</strong></th>
<th>0.105704</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

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**INITIAL VELOCITY** = 26000.00000000000 00 FT/SEC.
**INITIAL GEOCENTRIC RADIUS** = 21275839.8900132000 00 FEET

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**ANGULAR MOVEMENT**

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<tr>
<th><strong>GLIDE</strong></th>
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<tbody>
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**PROGRAM D**

<table>
<thead>
<tr>
<th><strong>CASE NUMBER</strong></th>
<th>3312</th>
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<tbody>
<tr>
<td><strong>LETTER CODE</strong></td>
<td>EFQH</td>
</tr>
<tr>
<td><strong>OUTPUT</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

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**NOTE**
- In order to minimize the temperature and acceleration peaks, the attitude of the vehicle will be slowly changed to the nominal values whenever the temperature is decreasing. The attitude will slowly change back to the nominal values when the temperature is increasing.
- The following rates of change will be used:
  - **Maximum Pitch Rate** = 4.000 DEGREES/SEC.
  - **Maximum Roll Rate** = 8.000 DEGREES/SEC.
- **Upper Pitch Accel** = 10.000 DEGREES/SEC#2
- **Upper Roll Accel** = 30.000 DEGREES/SEC#2
- **Lower Pitch Accel** = 1.000 DEGREES/SEC#2
- **Lower Roll Accel** = 2.000 DEGREES/SEC#2

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**NOTE**
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**INITIAL VELOCITY** = 26000.00000000000 00 FT/SEC.
**INITIAL GEOCENTRIC RADIUS** = 21275839.8900132000 00 FEET
THE CALCULATION FOR CASE NUMBER 3312 HAS BEEN TERMINATED. TIME USED = 1 MINUTES, 34.267 SECONDS.
An attitude control system to regulate the temperature of a manned lifting spacecraft during reentry into the Earth's atmosphere is proposed. Its use prevents the peak skin temperature that is experienced during the reentry from rising moderately beyond that which would occur during an equilibrium glide of the same vehicle. The effects of Earth rotation and oblateness upon the performance of the attitude control system were found to be moderate and predictable. The maximum temperature increment associated with them was found to be only 100°F for the worst set of initial conditions. The cross range shift of the footprint due to rotation was found to be within 70 miles of the value that would occur for the corresponding orbit in vacuum. Oblateness could generally be accounted for by using the effective initial glide angle relative to the Earth's surface rather than the geocentric initial value relative to the central coordinate system. The results of density variation in the Earth's atmosphere were not serious. Large increases in the maximum skin temperature occurred only when extremely large spatially random density disturbances were encountered by the vehicle.
Reentry - heating
Reentry - lifting body
Reentry - temperature control
Reentry - effect of earth oblateness
Reentry - effect of density perturbations

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8b. Sc & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subcontract number, system numbers, task number, etc.
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