A METHOD FOR FITTING LINEARLY SEGMENTED
MULTI-LAYER ATMOSPHERIC MODELS TO AN ARBITRARY
SET OF UPPER AND LOWER BOUNDARY CONDITIONS

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UNITED STATES AIR FORCE
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Atmospheric models are frequently generated for specific regions of the atmosphere in accordance with a particular set of observations. For example, for the region 20 to 90 km or for the region 120 to 100 km altitude. These models are not necessarily extended to sea level, neither are they designed to be continuous with some other existing model at either end of the included range. In some instances it becomes desirable to exactly connect two such models with a transition model atmosphere which for the examples cited would extend between 90 and 120 km. This report discusses a method suitable for generating such transition models.
ABSTRACT

Any set of temperature, pressure and density values which are realistic for one altitude can be exactly connected to another set of temperature, pressure and density values at a second altitude, with a model atmosphere defined by the appropriate linearly segmented two-layer temperature-altitude profile provided one of these layers is isothermal and the second is a nonzero constant gradient of temperature with respect to height.

The altitude comprising the intersection of the two segments of the required temperature-altitude profile is determined mathematically. The method for generating two-layer models is applied to the generation of three-layer and four-layer models.
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SECTION I

REVIEW AND EXTENSION OF THE METHOD FOR DEVELOPING TWO-LAYER TRANSITION MODELS

1. DEFINITION OF THE GENERAL TWO-LAYER TRANSITION MODELS AND THE SPECIFICATION OF THE PARTICULAR CLASS TO BE EXAMINED

Ninzner (1966, pp. 5 and 6) has shown that, under a particular though not highly restrictive set of conditions, a two-layer transition model can be made to fit exactly between two sets of fixed boundary conditions. The first consists of a set of realistic values \( H_0 \), \( p_0 \) and \( T_0 \), where \( H_0 \) is the geopotential corresponding to one arbitrary geometric altitude, \( p_0 \) is a corresponding realistic value of density, and \( T_0 \) is a realistic value of the temperature parameter at \( H_0 \). The second consists of a set of realistic values \( H_n \), \( p_n \) and \( T_n \), where \( H_n \) is the geopotential corresponding to another arbitrary geometric altitude, \( p_n \) is a corresponding realistic value of density, and \( T_n \) is a corresponding realistic value of the temperature parameter. The subscript immediately to the right of the symbols \( H \), \( p \), or \( T \) is associated with the particular geopotential level to which these three quantities are related. In this discussion of transition models, the words altitude or height will be intended to imply the equivalent geopotential without such a statement necessarily being made.

The restrictive conditions defining the particular class of two-layer models to be considered are as follows:

1. One of the two layers is characterized by a non-zero constant gradient of the temperature parameter relative to the geopotential such that this non-zero-gradient layer extends from \( H_0 \) toward \( H_n \) for an unspecified altitude interval to the level \( H_{XO} \).

2. The second layer extending from \( H_{XO} \) to \( H_n \), is characterized by a constant zero gradient of the temperature parameter relative to the geopotential. The symbol \( H_{XO} \) designates the altitude of the interface between the isothermal layer and the non-isothermal layer extending up to \( H_n \).

The equations and computational procedures for generating the class of transition models as discussed in this report are developed on the basis of two further assumptions: (1) Geopotential \( H_n \) represents a lower altitude level than that represented by \( H_0 \) such that the isothermal layer is the lower of the two layers of the transition model, and (2) the value of the temperature decreases with decreasing altitude from \( H_0 \) to the isothermal layer. This class of model applies to situations similar to that portion of the U. S. Standard Atmosphere 1962 between altitudes of 117 to 79 kilometers.

The temperature parameter employed in the transition-model development is the molecular scale temperature which combines two variables, the kinetic
temperature and the mean molecular weight of the atmosphere, into a single variable in accordance with the following definition:

The molecular scale temperature, at any level \( H \), is equal to the ratio of the kinetic temperature to the mean molecular weight, at that same level, times \( M_s \), the sea-level value of the molecular weight.

For simplicity, the notation commonly used for molecular scale temperature will be avoided in favor of \( T \) in this discussion of transition models, and the word temperature will imply molecular scale temperature.

2. DENSITY AND TEMPERATURE RELATIONSHIPS FOR THE SPECIFIED TYPE OF TWO-LAYER MODEL

In the type of model hypothesized, where an isothermal layer at temperature \( T_n \) extends upward from \( H_n \) to \( H_{xo} \) and a layer with a constant non-zero temperature-altitude gradient extends upward from \( H_{xo} \) to \( H_o \), the density \( \rho_x \) at \( H_x \) may be expressed in terms of \( \rho_n \) and \( T_n \) by the following equations:

\[
\rho_{xo} = \rho_n \exp \left( -\frac{(H_{xo} - H_x)}{T_n} Q \right)
\]

where

\[
Q = \frac{GM_s}{R} = 0.03416319470 K/m^7
\]

and

\( G \) is the geopotential transformation coefficient equal to 9.80665 m^7 sec^-2 (m')^-1
\( M_s \) is the sea-level value of the molecular weight of air, 28.944 kilograms per kilomole (kg/kmol)
\( R \) is the universal gas constant 8.31432 x 10^3 joules (K^-1) (kg m^-3) (mol^-1)

In such a model the density \( \rho_x \) at \( H_x \) may also be expressed relative to \( T_n \) and the upper-level boundary condition values, \( \rho_o \), \( T_o \), and \( H_o \), which through \( H_{xo} \) imply the value of the non-zero gradient of \( T \) between \( H_{xo} \) and \( H_o \). Thus,

\[
\rho_{xo} = \rho_o \left[ \frac{T_o}{T_n} \right]^{\frac{(H_o - H_{xo}) Q}{T_o - T_n}} \left\{ 1 + \frac{(H_o - H_{xo}) Q}{T_o - T_n} \right\}
\]

or

\[
\rho_{xo} = \rho_o \left[ \frac{T_o}{T_n} \right]^{\frac{(H_o - H_{xo}) Q}{T_o - T_n}}
\]
The value of $H_{xo}$ is found by the simultaneous solution of Equations (1) and (3) as expressed by:

$$
\exp \left( \frac{-(H_{xo} - H_n) Q}{T_n} \right) = \frac{T_o}{T_n}^n 
$$

The temperature-altitude gradient between $H_{xo}$ and $H_o$ specified by $L_{xo}$ is given by

$$
L_{xo} = \frac{T_o - T_n}{H_o - H_{xo}} 
$$

Here, the symbol for the gradient has two subscripts, the first designating the unknown lower end of the associated layer and the second designating the known upper end of the associated altitude interval.

3. ANALYTICAL EXPRESSION FOR THE GENERATION OF THE TWO-LAYER MODELS

Because it was not at first suspected that an analytical solution for $H_{xo}$ in Equation (4) could be found, numerical methods involving digital computers were used to determine the value of $H_{xo}$ for a number of transition atmospheres (Nininger, 1966, pp. 7 to 24). It may now be shown, however, that an analytical solution for $H_{xo}$ in Equation (4) does exist.

Dividing both sides of Equation (4) by $\rho_o$ and taking the natural logarithm of both sides of the resulting expression leads to

$$
\ln \left[ 1 + \left( \frac{H_o - H_{xo}}{T_o - T_n} \right) Q \right] - \ln \left[ \frac{T_o}{T_n} \right] = \ln \left[ \frac{\rho_n}{\rho_o} \right] 
$$

The separation of the numerators of the two fractions containing $H_k$, so as to permit the extraction of $H_{xo}$, followed by a rearrangement of the terms leads to the expression

$$
\frac{H_{xo} Q}{T_n} - \frac{H_{xo} Q}{T_o - T_n} \ln \left[ \frac{T_o}{T_n} \right] = \ln \left[ \frac{\rho_n}{\rho_o} \right] + \frac{H_n Q}{T_n} - \left[ 1 + \frac{H_o Q}{T_o - T_n} \right] \ln \left[ \frac{T_o}{T_n} \right] 
$$
Factoring $H_{x_0}$ out of the left-hand side of the equation and dividing both sides of that equation by the remainder of the left-hand side yields

$$H_{x_0} = \frac{T_0}{T_n} \left( \frac{T_0}{T_n} \right)^{-1} \left( \frac{Q_0}{T_0 - T_n} \right)^{-1} \left( \frac{n}{T_n} \right)^{-1} \left( \frac{H_n Q_n}{T_n} \right)^{-1} \left( 1 + \frac{H_n Q_n}{T_0 - T_n} \right)$$

An evaluation of this expression using the boundary conditions of model Ar (Minzner 1966, Table 2, p. 13) yielded $H_{x_0} = 101.9646$ km. This value agrees, to within one part in the sixth significant figure, with the value obtained by the numerical methods (Minzner, 1966, Table 3, p. 14), and thereby simultaneously substantiates the validity of the numerical method as well as of the analytical solution of $H_{x_0}$ given by Equation (8).

The subscript $o$ to the right of $x$ in $H_{x_0}$ is used to indicate that this particular value of $H_x$ is calculated with the upper boundary conditions at level $H_o$. If, as will be seen in the generation of a three-layer model, $H_x$ is calculated from some other upper-level boundary condition, as at $H_m$, the notation associated with $x$ will be $m$ as in $H_{x_m}$.

The value $H_{x_0}$ from Equation (8), and the value $H_{x_0}$ from Equation (5), along with the upper and lower set of boundary conditions completely define the only two-layer transition model, of the type under consideration which satisfies the boundary conditions. There are multilayer models (three or more layers), however, which satisfy the same upper and lower set of boundary conditions and the unique two-layer model serves as a basis for the generation of these multilayer models. Computer Programs for the generation of two-layer models are given in Appendix A.
SECTION II
GENERATION OF THREE-LAYER MODELS

The method for the generation of a two-layer model between upper and lower boundary conditions can be extended to the generation of an infinite number of three-layer models. The method consists essentially of arbitrarily selecting an altitude interval for the uppermost layer, computing the conditions at the base of that layer from the initial upper-boundary conditions, and using these new values as the basis for the computation of the related two-layer model. The detailed steps to be used are as follows:

1. Compute the value of \( H_{x0} \) and \( T_{x0} \) for that two-layer model which fits the initial boundary conditions.

2. Select a suitable value \( H_1 \) as the base of the uppermost or first layer of the desired three-layer model. A suitable value meets the condition \( H_{x0} < H_1 < H_0 \). An infinite number of possible values of \( H_1 \) exist between \( H_{x0} \) and \( H_0 \).

3. Select a suitable value of \( L_1 \) for the layer \( H_0 \) to \( H_1 \). It may be demonstrated that if the gradients of the several layers are to decrease monotonically from \( H_0 \) to \( H_n \), the value of \( L_1 \) must meet the condition \( L_{x0} < L_1 \).

Furthermore, it will be seen that \( L_1 \) must meet the condition \( L_1 < L_{max} \) where \( L_{max} \) is an upper limit associated with \( T_{min} \) at \( H_1 \) as in Figure 1. The determination as to whether \( L_1 < L_{max} \) is based upon the result of a test following a later step in the procedure. At this point, however, a trial value of \( L_1 \) is chosen. An infinite number of possible values of \( L_1 \) meet the necessary condition \( L_{x0} < L_1 < L_{max} \).

4. Compute \( T_1 \) at the selected value \( H_1 \) from the selected value \( L_1 \) using the relationship

\[
T_1 = T_0 - L_1 (H_0 - H_1)
\]

(9)

5. Compute the value of \( \rho_1 \), the density at \( H_1 \), using the relationship

\[
\rho_1 = \rho_0 \left\{ \frac{T_0}{T_1} \right\}^{\frac{1}{3}} + \frac{(H_0 - H_1)}{T_0 - T_1} Q
\]

(10)
The set of values $h_1$, $T_1$, and $L_1$ serve as a new upper-boundary condition which with $h_n$, $T_m$, and $P_n$ provide the means for generating a two-layer model between $h_1$ and $h_n$, where these two layers are designated layers 2 and 3 of the three-layer model. The interface between layers 2 and 3 is designated $h_{x1}$.

6) Determine $h_{x1}$ using Equation (8) with suitable subscript modifications, i.e., subscript zero is changed to one. If $L_1$ has been properly selected in step 3, $h_{x1}$ will be found to have a value such that $h_n < h_{x1} < h_{xo}$, and this condition serves as the test for suitability of $L_1$.

7) Compute the gradient for the layer between $h_{x1}$ and $h_1$ by using Equation (5) with the subscript zero increased to subscript 1.

The conditions for the test of suitability of $L_1$ based on the value of $h_{x1}$ have been developed through the following argument:

It is apparent that if $L_1$ is selected to be equal to $L_{xo}$, $h_1$ is not really the base of a new layer, but is a level within the layer $h_{xo}$ to $h_0$. Computations based on $h_1$, $T_1$, and $L_1$ would then yield $h_{x1} = h_{xo}$, and one would have determined only a redundant case of the original two-layer model. As the value of $L_1$ is allowed to increase from $L_{xo}$, for a fixed value of $h_1$, the value of $h_{x1}$ progressively decreases from $h_{xo}$ until, in the limit for $L_1 = L_{max}$, the value of $h_{x1}$ becomes identically $h_1$ in which case the isothermal layer has vanished as indicated in Figure 1. For values of $L_1 > L_{max}$, the computed value of $h_{x1}$ become less than $h_1$, indicating an impossible situation.

It is also possible to select values of $L_1$ such that $l_{max} < L_1 < L_{xo}$. As $L_1$ is decreased from $L_{xo}$, the value of $h_{x1}$ increases from $h_{xo}$ until for $L_{min}$, $h_{x1} = h_1$. For this case, layer 2 has the impossible conditions of zero thickness and an infinite temperature gradient. All values of $L_1 < L_{xo}$ are impossible however, if the model is constrained to one having a monotonically decreasing set of values of $L$ for successive layers. It follows therefore, that the only suitable values of $L_1$ are those which lead to values of $h_{x1}$ such that $h_n < h_{x1} < h_{xo}$. An infinite number of such values exist.

If values of $L_1$ are constrained to be integral multiples of some fixed values $L_1$, and if they are constrained to be within the range $L_{xo}$ to $L_{max}$, the smallest suitable values of $L_1$ may be expressed as

$$L_{1a} = J_1 \cdot N_1$$  \hspace{1cm} (11)

where $J_1$ satisfies the inequality

$$\frac{L_{xo}}{J_1} < J_1 \leq 1 + \frac{L_{xo}}{N_1}$$  \hspace{1cm} (12)

6
Figure 1. Schematic temperature-altitude diagram for a two-layer model atmosphere, for an acceptable related three-layer model, and for a limiting case of the three-layer models.
The series of acceptable values of \( l_1 \) and the related series of values of \( h_{\chi 1} \) are then

\[
\begin{align*}
L_{1a} &= (j_1 + 1 - 1) \Delta l_1, & H_{x1a} \\
L_{1b} &= (j_1 + 2 - 1) \Delta l_1, & H_{x1b} \\
L_{1c} &= (j_1 + 3 - 1) \Delta l_1, & H_{x1c}
\end{align*}
\]

etc., until a value \( L_{1\delta} \) is reached for which \( H_{x1\delta} \) is less than \( H_n \). This situation is shown schematically in Figure 2.
Figure 2. A schematic temperature-altitude diagram showing four possible three-layer models and one impossible three-layer model.
SECTION III
GENERATION OF MULTILAYER MODELS

The above procedure may be extended to the generation of four-layer models and on to the generation of n-layer models. The procedure consists of generating a three-layer model as indicated above, and then taking the lower two layers of this model, follow the procedure of steps 2 through 7 of Section II, but with subscript 1 replaced by subscript 2, and subscript 0 replaced by subscript 1. In a similar manner, the procedure may be extended to a model of any number of layers.

The altitudes specifying the boundaries of the four layers of each of a set of several possible four-layer models for a specified set of boundary conditions are given in Table 1.
**TABLE 1**

Definitions of possible four-layer models consistent with the boundary conditions:

P_0 = 0.246100E-07 kg/m^3, T_0 = 982.244 deg K at H_0 = 117776.0 m
P_0 = 0.198200E-04 kg/m^3, T_0 = 190.650 deg K at H_0 = 79000.0 m

And with the assigned values of H and DL:
H_1 = 110000.0 m, DL = 0.100000E-02
H_2 = 105000.0 m, DL = 0.100000E-02

Minimum height for base of layer 1 is H_0 = 100503.0 m.
Minimum gradient for layer 1 is GL_X = 110921F-01 deg/m.

Set 01 models defined by layer 1 gradient of 12 deg/km.

Minimum height for base of layer 2 is H_1 = 99731.2 m.
Minimum gradient for layer 2 is GL_X = 0.95709E-02 deg/m.

Model 1 of set 01 defined by layer-2 and -1 gradients 10 and 12 deg/km.

<table>
<thead>
<tr>
<th>Level</th>
<th>H (meters)</th>
<th>RHO (kg/m^3)</th>
<th>T (deg K)</th>
<th>L (deg/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>117776.0</td>
<td>0.246100E-07</td>
<td>382.244</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>110000.0</td>
<td>0.722249E-07</td>
<td>288.932</td>
<td>0.120000E-01</td>
</tr>
<tr>
<td>2</td>
<td>105000.0</td>
<td>0.167158E-06</td>
<td>233.932</td>
<td>0.110000E-01</td>
</tr>
<tr>
<td>3</td>
<td>99731.2</td>
<td>0.193486E-06</td>
<td>190.650</td>
<td>0.86019E-02</td>
</tr>
<tr>
<td>4</td>
<td>90000.0</td>
<td>0.198200E-04</td>
<td>190.650</td>
<td>0.00000E+00</td>
</tr>
</tbody>
</table>

Model 2 of set 01 defined by layer-2 and -1 gradients 11 and 12 deg/km.

<table>
<thead>
<tr>
<th>Level</th>
<th>H (meters)</th>
<th>RHO (kg/m^3)</th>
<th>T (deg K)</th>
<th>L (deg/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>117776.0</td>
<td>0.246100E-07</td>
<td>382.244</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>110000.0</td>
<td>0.722249E-07</td>
<td>288.932</td>
<td>0.120000E-01</td>
</tr>
<tr>
<td>2</td>
<td>105000.0</td>
<td>0.171473E-06</td>
<td>233.932</td>
<td>0.110000E-01</td>
</tr>
<tr>
<td>3</td>
<td>98452.7</td>
<td>0.607072E-06</td>
<td>190.650</td>
<td>0.66106E-02</td>
</tr>
<tr>
<td>4</td>
<td>79000.0</td>
<td>0.198200E-04</td>
<td>190.650</td>
<td>0.00000E+00</td>
</tr>
</tbody>
</table>

Model 3 of set 01 defined by layer-2 and -1 gradients 12 and 12 deg/km.

<table>
<thead>
<tr>
<th>Level</th>
<th>H (meters)</th>
<th>RHO (kg/m^3)</th>
<th>T (deg K)</th>
<th>L (deg/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>117776.0</td>
<td>0.246100E-07</td>
<td>382.244</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>110000.0</td>
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<td>288.932</td>
<td>0.120000E-01</td>
</tr>
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<td>0.120000E-01</td>
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<tr>
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<td>190.650</td>
<td>0.49551E-02</td>
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<td>79000.0</td>
<td>0.198200E-04</td>
<td>190.650</td>
<td>0.00000E+00</td>
</tr>
</tbody>
</table>
REFERENCES

APPENDIX A

Program 1

The program for defining a two-layer transition model atmosphere by the method of successive approximation and from the definition computing the various properties as indicated in Tables 3 through 10 of the final report on Contract No. NASA-1225 (Minzner, 1966).
PROGRAM 1

C SOURCE DECK FOR THE COMPUTATION OF TWO-LAYER TRANSITION MODEL
C ATMOSPHERES BETWEEN UPPER LAYER AND LOWER LAYER BOUNDARY CONDITIONS
C WHEN THE FOLLOWING CONDITIONS REGARDING TEMPERATURE-ALTITUDE
C GRADIENT PREDICT
C 1. IT HAS A DIFFERENT CONSTANT VALUE FOR EACH LAYER,
C 2. IT HAS A POSITIVE VALUE FOR THE UPPERMOST LAYER,
C 3. IT HAS A ZERO VALUE FOR THE LOWEST LAYER,
C THE METHOD INVOLVES THE SIMULTANEOUS NUMERICAL SOLUTION OF
C TWO EQUATIONS USING THE METHOD OF SUCCESSIVE APPROXIMATIONS
C AS DISCUSSED BY MINZNER IN GCA TECH RPT. NO. 66-14-N
C JUNE 1965, REVISED MARCH 1966
C
C READ 101, DEG, HR, HDP, HPA, PHA, FMON, T1, T2
C READ IN INITIAL CONDITIONS FOR MODEL
C T1 AND T2 ARE THE TITLF FOR THE MODEL
C
C READ 115, FL, HBA
C
C FORMAT(F4.1,1XF5.0)
C F=2.7R2123
C DPS=1.1E-08
C X=HR+1
C PUNCH 11?
C
C FORMAT(35HPROPRIETIES OF TRANSITION ATMOSPHERE)
C PUNCH 106, 11, 17, DEG, FMON
C
C FORMAT(2AX5.2X1H(*,A3.1H),2XA4,4X17HTO JACCHIA MODELS/)
C SOLVE THE TWO EQUATIONS SIMULTANEOUSLY USING
C THE METHOD OF SUCCESSIVE APPROXIMATIONS
C
C X(BETA)= (HA-X)/(HPA-HBA)
C Y= (1+BETA)*LOG(HPA/HBA)+LOG(PHA)
C X= Y-LOG(HPA)*HBA/LOG(E)+HBA
C IF(SENSWITCH 1) 4, 3
C SENSE SWITCH 1 ON WILL LET THE OPERATOR
C WATCH THE DIRECTION THE X VALUE IS
C GOING IN.
C
C PRINT 103, X
C
C FORMAT(F14.8)
C 4 IF(FABS(XOLD-X)-EPS) .LT. 1,2
C CONTINUE
C Y=EXP(Y)
C X=BETA/X
C X=P(X+0)
C PUNCH 107, DEG, FMON, X, Y
C
C FORMAT(A3,1X,A4,2X3HX =F10.5,7X2HKM4X7HFEN X =F14.8,2X4HG/M3)
C PUNCH 113, HPA, HPA, HPR
C
C FORMAT(19HGEOPOT SCALE HT AT F9.4.1X4HKM =F9.5,11H, AT X KM =,1
C 1F10.7)
C PUNCH 109, BETA
C
C FORMAT(35HGRADIENT OF GEOP SCALE HT ABOVE X =F14.8)
C PUNCH 114, FL, HBA, HR
C
C FORMAT(23HGRADIENT OF MOL* TEMP. =F4.1,10H FOR BTWN 5.0,4H TO ,F4.0)
C PUNCH 113, HR
C
C FORMAT(23HGRADIENT OF MOL* TEMP. ZERO FOR X BTWN ,F4.0,6H AND X)
C PUNCH 107
C
C FORMAT(3X11HALITUDE KM10X15HTEMP DEG KELVIN4X6HMCOL WT2X,

16
117DENSITY G/M3x11HSAtF HT KM)
21108 FORMAT(IX6,HE0MFT4X6,FGFOPOT7X4HMOL.3X7H,HN3TIC28X6,HE0MFT1X.
3RH0X=Y
4READ 104,DFG+R,GPHI
5104 FORMAT(A9,F9.0,F8.5)
6H=XPT
7Z=R*H/(GPHI>R/9.80665-H)
8C Z IS THE GEOMETRIC ALTITUDE
9C RH0H=PHA*(HPA/(HPA+BETA*(H-HA)))**((1.+RFTA)/BETA)
10C RH0H IS THE DENSITY
11C HH=(HPA+BFTA*(H-H1))**9.80665/GPHI*((R+Z)/R)**2
12C HH IS THE SCALE HEIGHT
13C TM=34.1632*(HPA+BFTA*(H-X))
14C T IS THE TEMPERATURE
15C HPH=HPA+BFTA*(H-X)
16C SM=28.9644-.06881*(Z-90.)
17C SM IS THE MOLECULAR WEIGHT
18C TM=TM+.005
19T=T+.005
20SM=SM+.005
21HH=HH+.0005
22HPH=HPH+.0005
23C ROUND THE VALUES
24105,Z,T,H,TM,T,SM,RH0H,HPH
25PUNCH DATA CARD
26106 FORMAT(F8.3,7XF8.3,3XF7.2,3XF6.2sXF12*5
2716HGFOPOT/)
28C METHOD FOR FINDING NEXT EVEN KM.
29XNFW=DRH(X)
30XNFW=XNFW/2.
31XNEW=DRH(XNEW)
32XNEW=(XNEW*2.)+2.
33XP=X*10.
34XP=DRH(XP)/10.
35IF(XNEW-XP=7.)8,9,8
368 H=XP
37 GO TO 7
388 H=XNEW
397 IF(H-XPT)595921
4021 Z=R*H/(GPHI>R/9.80665-H)
41C RH0H=PHA*(HPA/(HPA+BETA*(H-HA)))**((1.+BETA)/BETA)
42C HH=(HPA+BFTA*(H-X))**9.80665/GPHI*((R+Z)/R)**2
43C TM=34.1632*(HPA+BFTA*(H-X))
44C T IS THE TEMPERATURE
45C HPH=HPA+BFTA*(H-X)
46C SM=28.9644-.06881*(Z-90.)
47C FLM IS THE GR,DIFNT
48C TM=TM+.005
49T=T+.005
50SM=SM+.005
51HH=HH+.0005
5217
HPH=HPH+.005
IF (Z-120=140).6.6
40 PINCH 105,Z+H,T,M,T+5M+RHOM+HH+HPH
5 H=H+.05
GO TO 7
6 Z=R*X/(GPHI*R/9.80665-X)
7=DRH(7)/2*
Z=DRH(Z)+.0+.70
12 H=R*(R+Z)*GPHI/9.80665
RHOM=PHA*(HPA/(HPA+8ETA*(H-HA)))*((1+BETA)/BETA)
SM=78.9644-.068133*(Z-90t)
HH=(HPR+BFTA*(H-X))*(R+Z)/(R+Z)-90.
TM=34.1637*(HPR+BFTA*(H-X))
T=TM*(1.0-2.37581E-03*(Z-90t))
FLM=34.1637*BFTA
HPH=HPR+BFTA*(H-X)
TM=TM+.005
T=TM+.005
SM=SM+.005
HH=HH+.005
HPH=HPH+.005
PINCH 105,Z+H,T,M,T+5M+RHOM+HH+HPH
IF (Z-120=140).10,11,11
10 Z=Z+.70
GO TO 12
11 H=HR
X=HR
BFTA=0
16 Z=R*X/(GPHI*R/9.80665-H)
HH=(HPR+BFTA*(H-X))*(R+Z)/(R+Z)-90.
TM=34.1637*(HPR+BFTA*(H-X))
IF (Z-90t=126).26,27
26 T=TM
SM=28.96
GO TO 28
27 T=TM*(1.0-2.37581E-03*(Z-90t))
SM=28.9644-.068133*(Z-90t)
28 RHOM=PHA*EXP(-(H-HA)/HPR)
HPH=HPR+BFTA*(H-X)
XPRM=DRH(XPT)
TM=TM+.005
T=TM+.005
SM=SM+.005
HH=HH+.005
HPH=HPH+.005
IF (H-XPRM)=14,14,15
14 PINCH 105,Z+H,T,M,T+5M+RHOM+HH+HPH
IF (H-HPR)=16,41,16
41 IF (SENSE SWITCH)=16,17
SENSE SWITCH 2 ON, INCREASE THE HEIGHT BY 2.
SENSE SWITCH 2 OFF, INCREASE THE HEIGHT BY 1.
17 H=H+.05
GO TO 18
16 H=H+.25
GO TO 18

15 \( Z = \frac{R^2 * X}{(GPH^I*R/9.80665 - X)} \)

\( Z = DRH(Z)/2 \)

\( Z = DRH(Z)/2 + Z \)

21 \( H = \frac{R * Z}{(R+Z)} * GPH^I/9.80665 \)

IF (H-HB) 25, 25, 22

27 RHON=PH^R*FXP(-H-HB)/HPB)

HH=HPR+RFTA*(H-X)***(R-Z)/R)**

TM=34.1632*(HPB+BETA*(H-X))

IF (Z-90) 29, 29, 30

29 T=TM

SM=28.96

GO TO 31

30 T=TM*(1-2.375811E-03*(Z-90))

SM=28.9644-068133*(Z-90)

31 HDH=HPR+BFTA*(H-X)

TM=TM+.005

T=T+.005

SM=SM+.005

HH=HH+.0005

HPH=HPH+.0005

IF (H-XPT119 20, 20

19 PUNCH 105+Z*H,TM,T,SM,HH,HH,HPH

25 Z=7+7

GO TO 21

20 PUNCH 111,FLM,HA

\( 111 \) FORMAT (23HR,9GRADIENT OF MOL TEMP F10.5 2X19HDEG/KM FOR H BTWN X,

14H AND,F9.4/)!

GO TO 13

FND
PROGRAM 2

The program for defining a two-layer model by the evaluation of an algebraic function, which replaces the successive approximation method.
PROGRAM 2

SOURCE DECK FOR TWO-LAYER TRANSITION MODEL ATMOSPHERE
BETWEEN UPPER-LEVEL AND LOWER-LEVEL BOUNDARY CONDITIONS
WHEN THE FOLLOWING CONDITIONS REGARDING TEMPERATURE-ALTITUDE
GRADIENT PREVAILS
1. IT HAS A DIFFERENT CONSTANT VALUE FOR EACH LAYER,
2. IT HAS A POSITIVE VALUE FOR THE UPPERMOST LAYER,
3. IT HAS A ZERO VALUE FOR THE LOWER LAYER.

READ 100,H0,RH0,TO
BEGIN TRACE
O=3.4163195E-02
H0=(LOGF(RH0/RHO0)+H0*O/TN-(1+H0*O/(TO-TN))*LOGF(TO/TN))
H0=H0/(I/Q-1/(TO-TN))*LOGF(TO/TN)
GLX=(TO-TN)/(H0-HX0)
RHOX=RH0*EXP(-Q*(HX0-H0)/TN)

FORMAT(21X33HDEFINITION OF THE TWO-LAYER MODEL)
FORMAT(10X5LEVEL,5X1HH,11X3HRHO,11X1HT,11X1HL)
FORMAT(18X3HMETERS KG/M3 DEG K DEG/M)
FORMAT(10X13,2XF10.3,2XE12.6,1XF10.3,2XF12.6)
FORMAT(10X13,2XF10.3,2XE12.6,1XF10.3,3X11H,00000000E+00)
PUNCH 111
PUNCH 111
PUNCH 120
PUNCH 111
PUNCH 112
PUNCH 113
PUNCH 111
LEVEL=0
PUNCH 114,LEVEL,H0,RH0,TO
LEVEL=LEVEL+1
PUNCH 114,LEVEL,HX0,RHOX,TO,GLX
LEVEL=LEVEL+1
PUNCH 115,LEVEL,H0,RH0,TO
END TRACE
END