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EFFECT OF NOZZLE GEOMETRY
ON OFF-DESIGN PERFORMANCE OF
PARTIAL ADMISSION IMPULSE TURBINES

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EFFECT OF NOZZLE GEOMETRY ON OFF-DESIGN PERFORMANCE OF PARTIAL ADMISSION IMPULSE TURBINES

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SUMMARY

1.0 SUMMARY

In attaining optimum turbine performance, the turbine nozzle is the single most influential component. Small, high specific energy turbines, generally operating at high pressure ratios, require supersonic converging-diverging nozzles for high efficiency. Such nozzles, however, yield poor performance at off-design pressure ratios. The object of the present study was to obtain information to improve the off-design performance of supersonic turbine nozzles.

The scope of the study consisted of a literature search, a theoretical analysis, and an experimental program. The literature search consisted of an evaluation of available information on nozzle performance characteristics in the off-design regime and of nozzle concepts which might yield superior performance. The theoretical analysis consisted of an investigation of the physical phenomena involved with turbine nozzle operation in the off-design regime and the development of a theory to predict performance. The experimental portion consisted of the design, fabrication, testing and evaluation of conventional and non-conventional nozzle designs.

During the literature search, 60 reports were reviewed and brief comments concerning each one is included. Turbine and rocket nozzle data presented in the literature were correlated and compared; this established that both qualitative and quantitative similarity exists between turbine and rocket nozzle performance when operating at off-design pressure ratios.

Nozzle performance was derived from test measurements of a complete turbine-nozzle system. All nozzles were tested with nitrogen and with the identical rotor. A total of twelve nozzles were tested, being made up of the following four types: 7 conical nozzles with design Mach numbers ranging from 1.5 to 4.6; one shock cancellation (contoured) nozzle designed for Mach 4.0; one unique plug type nozzle designed for Mach 4.0; and three free-expansion nozzles designed for Mach 4.0.

Results show that contoured nozzles provide the best performance for turbines which operate at near design pressure ratios or at greater-than-design pressure ratios. For operation at pressure ratios below design, the conical or plug types are superior. The free expansion nozzles generally produced inferior performance as compared to the other types. Axial spacing between nozzle and rotor had little effect on off-design performance.

The theoretical analysis shows that the performance of nozzles operating at off-design pressure ratios can be predicted by a combination oblique and normal shock calculation procedure.

This analysis together with limited test data indicates that the gas ratio of specific heats has a strong effect on off-design nozzle performance. The indication is that performance may be greatly reduced when operating with gases with low ratios of specific heat. The design point performance, however, does not appear to be affected.

SECTION 2.0

INTRODUCTION

2.0 INTRODUCTION

The mission profiles of many turbine power systems require operation over a wide range of pressure ratios. For example, systems which are designed to operate in space with very large pressure ratios, are required to operate on the launch pad as well, at low or intermediate pressure ratios. Also, open-cycle torpedo propulsion systems operate at pressure ratios which may vary by an order of magnitude or more due to changes in back pressure caused by depth changes. This off-design regime often accounts for a significant portion of the mission profile. The research program conducted herein was concerned with improving performance in this regime.

Turbine performance is dependent on a large number of factors, including nozzle and rotor performance, leakage, Reynolds number effects, and parasitic losses. Basically, turbine performance depends on the momentum change across the rotor which is described by the turbine hydraulic efficiency equation. This equation reflects the basic turbine efficiency from which the losses and effects mentioned above detract. The effects of nozzle and rotor performance are included in the hydraulic efficiency equation which for an impulse turbine may be written:

$$\eta_H = 2\,U/C_o \;(1 + \Psi_R)\,(\Psi_N \cos\alpha - U/C_o) \tag{2.1}$$

where:

- η_H = hydraulic efficiency
- U = turbine tip speed, fps
- C_o = isentropic spouting velocity, fps
- Ψ_R = rotor velocity ceefficient
- Ψ_N = nozzle velocity coefficient
- α = nozzle angle, degrees

Equation (2.1) defines the maximum attainable turbine efficiency if there are no parasitic losses or leakage. As may be noted the turbine hydraulic efficiency is a strong function of U/C_o. Maximum efficiency is obtained when the turbine is operating at optimum U/C_o. The optimum U/C_o is obtained by differentiating equation (2.1) with respect to U/C_o and solving for U/C_o when the differential is equated to zero. This yields:

$$(U/C_o)_{opt} = \Psi_N \cos\alpha / 2 \tag{2.2}$$

Solutions of equation (2.2) yield optimum values of U/C_o in the range of 0.4 to 0.5 for typical values of Ψ_N, and α.

At a U/C_o of 0.5 a change in nozzle coefficient will result in a change in the turbine hydraulic efficiency which is 2.1 times as large as the percentage change in the nozzle coefficient as shown in Figure 1. For example, if the nozzle coefficient is 0.90 instead of 1.0 at a U/C_o of 0.5, the hydraulic efficiency will be reduced by 2.1 x 10% or 21 percent. The effect of a change in rotor coefficient, however, is much smaller; for example a 10 percent change in rotor coefficient can only result in a change in hydraulic efficiency of 4 or 5 percent for obtainable values of rotor coefficients.

This illustrates that the nozzle and rotor velocity coefficients have a large effect on the ultimate obtainable turbine efficiency, with the nozzle coefficient being much more important than the rotor coefficient. When operating a turbine at the design pressure ratio, nozzle coefficients of approximately 0.95 may be realized. However, for pressure ratios greater or less than design, this coefficient decreases rapidly, particularly for nozzles with high area ratios and, hence, high design Mach numbers. This fact is shown in Figure 2.

The nozzle velocity coefficient presented in Figure 2 is representative of the performance of conical and contoured convergent-divergent nozzles. However, the performance deficit indicated in this figure was not acceptable to rocket designers. Considerable work has been done to improve the off-design performance of rocket nozzles, resulting in concepts such as the plug nozzle, the annular internal-external-expansion nozzle, and aerodynamically controlled nozzles.

The objectives of this study were to investigate the literature to obtain insight of nozzle performance characteristics and to obtain nozzle concepts which may produce superior turbine performance; to design and test various turbine nozzle types to obtain a performance comparison; and to derive a theoretical calculation procedure which would provide a means of applying the information gained in this study to other nozzle configurations and nozzles operating on other gases or fluids. The area of research was basically at pressure ratios below and above design rather than the area of the design point.

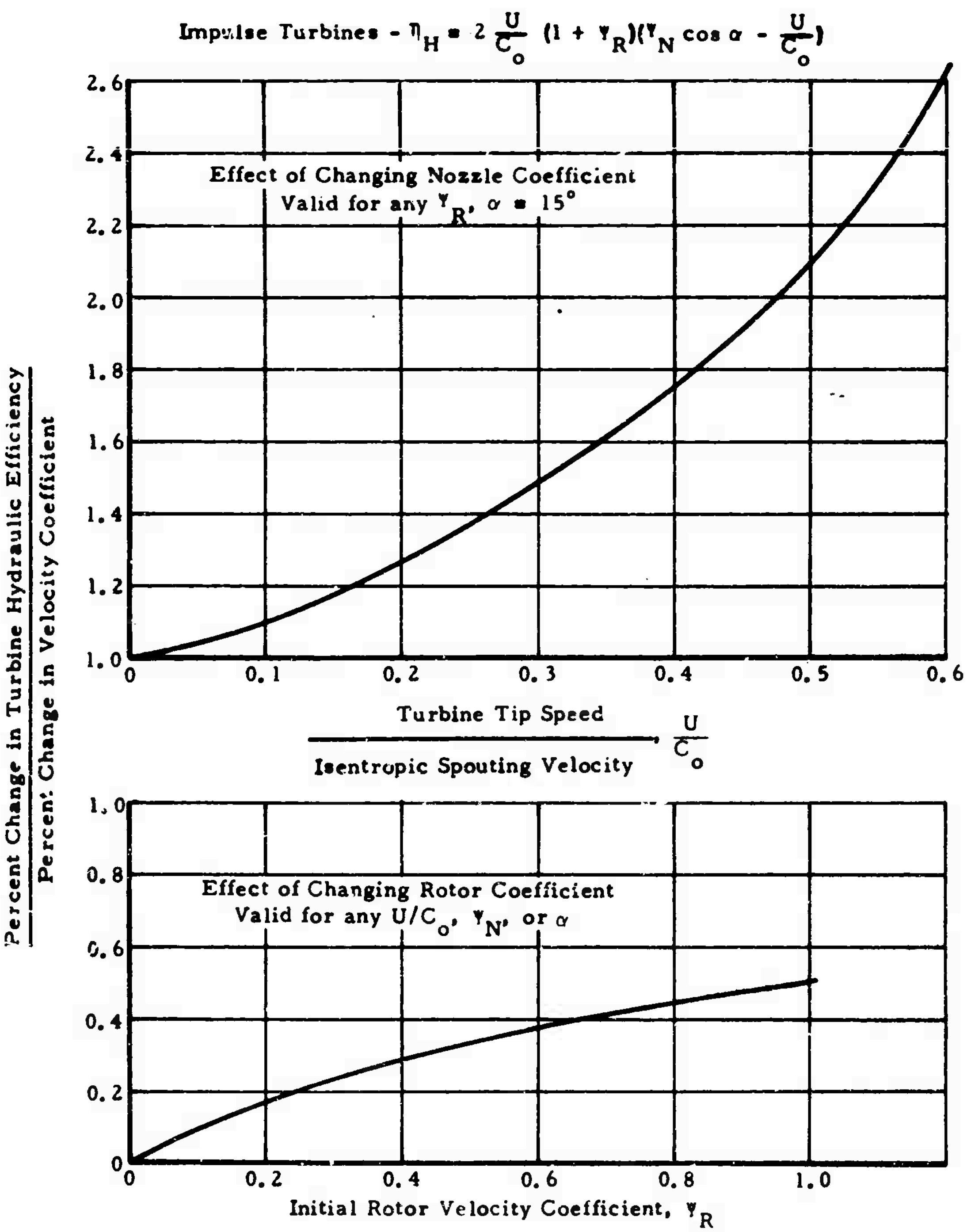

Figure 1. Comparison of the Effect of Nozzle and Rotor Velocity Coefficient on Turbine Hydraulic Efficiency

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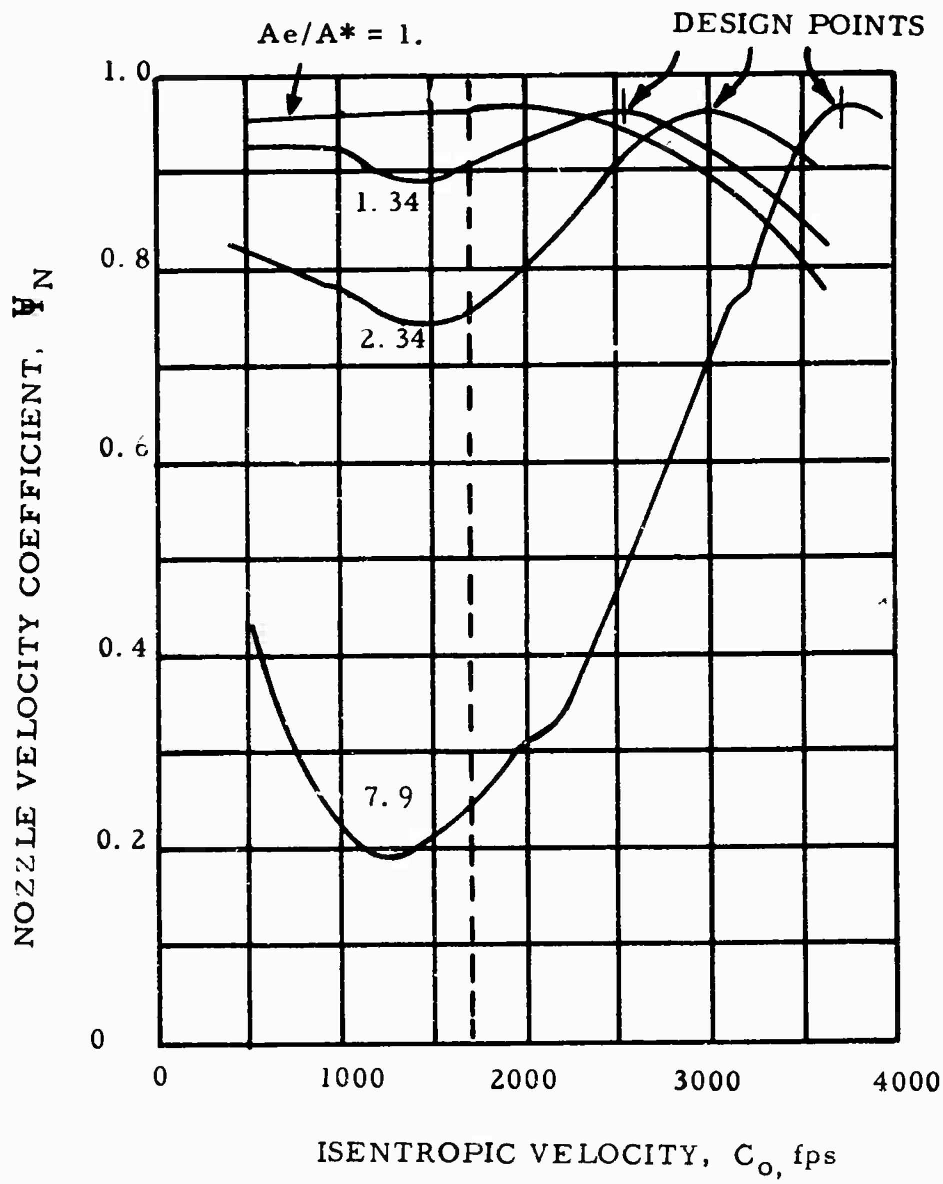
FIGURE 2
TYPICAL VARIATION OF NOZZLE VELOCITY
COEFFICIENT AT OFF-DESIGN PRESSURE RATIOS



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SECTION 3.0

LITERATURE SEARCH

3.0 LITERATURE SEARCH

A detailed literature search was conducted to make available for this study as much turbine nozzle and rocket nozzle information as possible. As was expected, little turbine nozzle data were uncovered; however, considerable rocket nozzle data are available. The rocket nozzle data are not directly applicable to turbine nozzle performance, but an understanding of the aerodynamics of rocket nozzles lends insight into turbine nozzle performance.

3.1 DISCUSSION OF THE LITERATURE

A large number of nozzle reports were obtained and reviewed. A listing of the reports and short comments relating to their applicability to small turbines is presented in Appendix I. These reports range from turbine nozzle studies with test data to theoretical rocket nozzle studies.

Appendix I is divided into three sections: Section 1 is devoted to papers which are theoretical in nature; Section 2 includes papers which are experimental in nature; and Section 3 summarizes papers which include both theoretical and experimental information. The order in which the papers are presented in each section has no significance.

Most of the nozzle data reviewed were obtained by reaction tests. In these tests the nozzles were mounted on a balancing mechanism and the thrust was measured. This thrust value was then corrected for various external torque and pressure forces unique to the particular test configuration. The resultant corrected thrust, F, is described in aerodynamic terms by the momentum equation:

$$F = \frac{\dot{w}}{g} (V_e - V_o) + A_c (P_c - P_a) \tag{3.1}$$

Where:

$\dot{w}$ = Nozzle weight flow, lb/sec.
g = Acceleration of gravity, ft/sec.^{2}
V_e = Nozzle exit velocity, ft/sec.
V_o = Nozzle entrance velocity, ft/sec.
A_c = Nozzle exit area, in.^{2}
P_c = Nozzle exit plane pressure, lb/in.^{2}
P_a = Ambient pressure, lb/in.^{2}

In order to compare nozzle performance, it is desirable to determine the nozzle velocity coefficient, ψ_N, and hence, the nozzle exit velocity must be evaluated from the thrust measurements. This is readily done at the design pressure ratio since the second term of equation (3.1) (the pressure-area term) is zero. At off-design operating points the pressure-area term may not be zero. Fortunately, the exit plane pressure is felt to contribute only a small percentage of thrust since the ambient pressure very often feeds into the nozzle through the boundary layer, thereby eliminating the pressure force on the nozzle walls during below-design pressure ratio operation.

Another test technique used in turbine nozzle testing is one of pressure surveys in the nozzle exit plane. The pressure surveys are used to calculate the mass or area-averaged velocity coefficient and the flow angle. This procedure generally has the disadvantage of neglecting the effects of the "dead" space between nozzles; however, it has the advantage of eliminating the unknown pressure-area term associated with thrust measurement techniques.

These methods of evaluating nozzle performance are the most common. However, in addition to the drawbacks mentioned, they do not include any effect that the rotor may have on the nozzle. Therefore, in the present study, the overall performance was measured and a "derived" nozzle velocity coefficient was calculated from efficiency data.

3.1.1 Turbine Nozzle Reports

Undoubtedly the two most significant turbine nozzle reports are those by Kraft (11.2.1)* and Keenan (11.2.3)*. These papers were presented in the 1940's and little information has been presented since. Since the success or failure of a turbine design is largely dependent on the nozzle design, each manufacturer considers his design technique (and the data upon which it is based) highly proprietary; hence the lack of information in the open literature.

The converging nozzle data presented by Kraft and the converging-diverging nozzle data by Keenan cover a large range of pressure ratios both above and below design. These data are the bases for many turbine designs; however, it is felt by the authors of this report that the nozzle velocity coefficient data of Kraft and Keenan are pessimistic at below-design pressure ratios. This is believed to result from the measurement and calculation technique used to obtain the velocity coefficients. Since the data of Kraft and Keenan were obtained on the same test apparatus, these comments pertain to both references. As Keenan stated, "the velocity deduced is a hypothetical one, and the application to turbines need not be answered here". No attempt was made to correct the data for the pressure-area term of equation (3.1). In following sections pertaining to the correlation of test information, these data will be discussed in more detail.

A very interesting and informative turbine nozzle report is presented by Stratford (11.2.2). The tests were conducted in a rather unusual manner in that the cascade pressure ratio was varied by adjusting the tunnel side walls downstream of the cascade. This adjustment had the same effect, in Stratford's opinion, as a rotor operating at off-design (or incidence) conditions. The results led Stratford to conclude that supersonic turbines may not actually operate at incidence to the rotor.

*Numbers correspond to the reference number in Appendix I.

This conclusion was based on measurements which showed that when the wheel speed is such that incidence should occur, what resulted was an adjustment of the nozzle pressure ratio, and some reaction in the rotor developed to result in zero incidence at the rotor inlet. The nozzle velocity ratios, measured by Stratford with a traversing probe, were between 0.98 & 0.99. These high coefficients, at design point, are typical of cascade tests (see 11.2.9) and rocket nozzle tests, however, the values presented by Keenan only approach 0.96. The reason for this discrepancy may be the measurement technique that Keenan used, or the rather unusual cross-sectional shape of his nozzles. Reaction tests of rocket nozzles yield velocity coefficients as high as 0.98 or even 1.0; therefore, the reason for the disagreement is not solely reaction technique as compared to survey technique.

Another significant report is that presented by Ohlsson in his MIT thesis (11.3.26). This report summarizes the theoretical and experimental effort which includes the study of two different families of converging-diverging, two-dimensional contoured turbine nozzles. In this report Ohlsson rejects one family as inferior. The results of the superior family is presented in an ASME paper (11.3.25). The nozzle performance data presented in this report is low, probably due to the measurement technique. Ohlsson measured stall torque on a rotor with axial exhaust to determine nozzle performance. This procedure has two major faults: (1) rotor entering losses affect the measured torque, and (2) the rotor choked before the design pressure ratio was reached with several of the nozzles. The rotor losses are quite likely even higher than the design rotor losses since, for the largest area ratio nozzle (AR = 2.6), the rotor relative Mach number is 2.8 at stall and only 1.7 at the design U/C_o. The test data bear out this point in that the design nozzle coefficient based on the stall torque varies from 0.92 at an area ratio of 1.0, to 0.85 at an area ratio of 3.6. Since the nozzle design coefficient was excessively low, these data were not used in the following correlation of nozzle data. However, Ohlsson's work is worthwhile and the nozzle data presented may be used for qualitative information rather than quantitative. Considerable off-design turbine performance data are presented in Ohlsson's reports which include variations in turbine speed as well as pressure ratio.

Some researchers have spent considerable effort examining turbine blading on a boundary layer basis. Included are the boundary layer experts Schlicting (11.2.19), Deich (11.3.27), and the NASA personnel Rohlik (11.2.9), and Whitney and Stewart (11.2.16). The boundary layer approach, while valid for large turbomachinery, is too detailed for turbines with blade heights in the range of 0.2 to 0.8 inch such as considered herein. (This is not the case when the operational regime of the nozzle falls into the Reynolds number area where boundary layer effects become very important. The work reported by Dollin (11.3.5) occurred in this area. He shows that the low Reynolds number accounts for a decrease in nozzle performance of approximately 2%.)

A particularly interesting turbine study was conducted in England. Initially, Johnson and Dransfield were conducting a high pressure ratio turbine study (11.3.29); they encountered losses that were greater than expected. Cascade studies were then conducted by Stratford and Sansome to evaluate the rotor blades (11.3.20), the nozzle blades (11.2.2) and the interaction between the nozzle and the rotor (11.2.15) and (11.2.20). These studies showed that rotor choking had occurred in the initial tests and that the rotor determined the nozzle exit angle. Hence, the nozzle pressure ratio varies with speed at a constant overall system pressure ratio. They also point out that the rotor leading edge thickness has more effect of rotor incidence angle than the nozzle angle. They stated that the rotor operated at incidence angles from 2 to 5 degrees at all pressure ratios studied, even though the calculated incidence varied over a much larger range.

Many converging nozzle tests have been conducted over the years, with the first organized research beginning in 1920 with the formation of the Steam Nozzle Research Committee of the Institute of Mechanical Engineers in England. They published six reports (11.3.7-11.3.10) over an eight year period. The information they obtained was analyzed, correlated, or amended by Guy (11.3.6), Ainley (11.3.22), Kraft (11.2.1), Bartocci (11.3.11), Proskuryokov (11.3.21) and Schlichting (11.2.20). These studies were mainly concerned with two-dimensional nozzles and the effects of geometric variables on nozzle performance. These variables included parameters such as: nozzle throat length, inlet radius, exit angle, blade height, trailing edge thickness, aspect ratio, and surface finish.

Converging nozzle performance is well defined, but, these data lend little insight into supersonic nozzle performance. However they are useful in determining a nozzle design, particularly in the area of nozzle entrance geometry up to the throat. Other parameters, such as aspect ratio, the ratio of rotor height to nozzle height, axial spacing etc., provide guidelines to supersonic designs. References which discuss this information include Kraft (11.2.1) for subsonic nozzles, and Keenan (11.2.3), Ohlsson (11.3.25 and 11.3.26), and Stratford (11.2.2) for supersonic nozzle.

3.1.2 Rocket Nozzle Reports

These reports were reviewed to provide insight into the understanding of nozzle phenomenon and trends of supersonic nozzle operation at off-design conditions. The objective in rocket nozzle design is to obtain good performance with the minimum weight and/or minimum length. Therefore, most parametric studies are based on optimizing these parameters.

Rocket nozzles operate over a large range of pressure ratios, both less-than and greater-than design. Considerable interest has been placed on improving rockets in recent years and much off-design nozzle data are available. The performance of conical nozzles was studied by Bloomer (11.2.10), Lovell (11.2.14), and Campbell (11.3.13). A large amount of data are presented by Bloomer for conical nozzles with area ratios from 1.0 to 75 and cone-half angles of 15 to 30 degrees. Campbell presented data for area ratios of 10 to 30 and cone-half angles of 15 to 30°. Axisymmetric contoured nozzles were evaluated by Bloomer (11.2.13) and Farley (11.3.15). More conical data are presented by Krull (11.2.4) for four nozzles of area ratios from 1.0 to 2.65. Krull showed that converging-diverging nozzles performed better than predicted by normal shock theory when overexpanded, indicating that oblique rather than normal shocks probably occur in the nozzle.

Farley presents data for contoured nozzles with area ratios from 10 to 25 for three families of nozzles. These families were of decreasing overall length and of increasing wall angles. The data of Bloomer compares the performance of contoured nozzles with conical nozzles. He concludes that the design point performance of contoured nozzles is slightly superior to conical nozzles.

The report by Barakauskas (11.2.6) presents some data with free expansion nozzles. These data did not include thrust or velocity measurements; however, the pressure measurements agreed with the same configuration using a conical nozzle. These interesting results led to the inclusion of free expansion nozzles in the present study.

A number of studies have examined the area of shock formation in overexpanded nozzles. These studies indicate that most nozzles do not shock down through a normal shock but rather through oblique shocks. Work in this area has been both theoretical, Darwell (11.1.1) and experimental, Kalt (11.2.7) and Arens (11.3.3). Of these studies Arens is the most complete and includes a good correlation of the large amount of experimental data presented.

Rocket nozzle design is generally based on minimum length or weight. Migdol (11.9.4) used the method of characteristics to predict the effects of geometric parameters on conical nozzle performance. Axisymmetric contoured nozzles are the object of the studies of Guentert (11.1.5), Beckwith (11.1.11), and Ahlbert (11.3.4). Guentert is concerned with the effects of the isentropic exponent of nozzle design and Beckwith presents a rapid method to design shock concellation nozzles for gases having K = 1.4.

Design techniques for plug nozzles are presented by Angelino (11.1.2) and Greer (11.1.8). Both of these papers present a calculation procedure for representative nozzle contours; one method contours only the plug (Angelino); the other method contours both the cowl and the plug (Greer). Since these nozzles are constructed on a bias and are reported to have superior off-design performance, they appear as attractive candidates for a new type of turbine nozzle. For such applications the design technique of Greer is not as applicable as that of Angelino due to a poor flow configuration at the nozzle throat entrance.

3.2 CORRELATION OF DATA IN THE LITERATURE

All nozzle test data found in the literature search were examined and considered for inclusion herein. The data is presented as a ratio of the nozzle velocity coefficient to the design nozzle velocity coefficient as a function of the dependent variables, the design Mach number and the isentropic operating Mach number. This isentropic Mach number ratio, M'/M'_D, is important since it represents the location of an operating point relative to the design point more effectively than either the nozzle isentropic velocity, the actual velocity, or the pressure ratio. The nozzle design Mach number is that Mach number at which optimum nozzle velocity coefficients are measured. This Mach number should be very nearly the same as the calculated Mach number based on the nozzle area ratio. Some of the data in the literature did not meet this requirement, and thus were not included in the correlations made in this study. Other causes for rejection of data were the lack of data at the design point and the use of unusual nozzle shapes or testing practices.

Most nozzle data in the literature were presented in terms of thrust parameters. As mentioned in the previous section and shown in equation (3.1) there is a pressure-area term in the thrust measurement which makes the exact calculation of the nozzle exit velocity from thrust data impossible without the pressure measurements to evaluate the pressure-area term. However, in high velocity nozzles the momentum term in equation (3.1) is by far the largest contributor to thrust, and the rocket nozzle velocity coefficients were evaluated on the basis that:

$$V_c = \frac{gF}{\dot{w}} \quad , \quad \text{ft/sec} \tag{3.2}$$

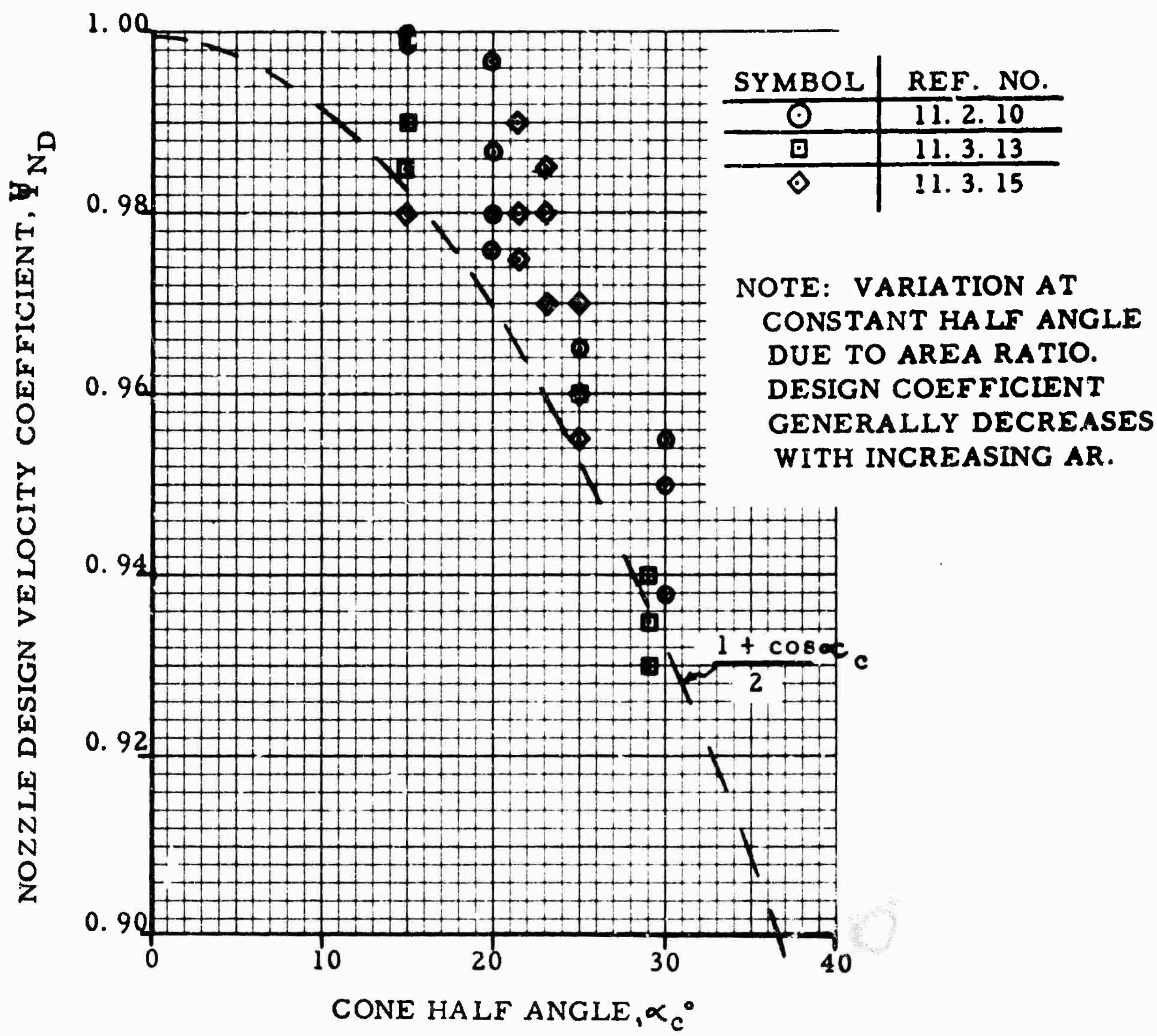
This calculation procedure was used by many investigators in their turbine nozzle studies; therefore, the rocket nozzle and turbine nozzle data will be compared on equal terms.

The primary concern of the present study is off-design nozzle performance. However, the design point performance is important in as much as a nozzle which has poor design point performance is expected to have reduced off-design performance also. Therefore, the nozzle velocity coefficient is correlated as the ratio of nozzle velocity coefficient to design nozzle velocity coefficient. In determining the design point coefficient, the geometric parameters of area ratio and cone half angle have a strong influence as seen in Figure 3. It may be noted that the data generally follow the variation of:

$$\psi_{N_D} \qquad (1 + \cos \alpha_c)/2 \tag{3.3}$$

Where α_c is the cone half angle. Equation (3.3) is proposed by Englert and Kochendorfen in Reference 3 to account for the deviation from axial flow at the nozzle exit in a conical nozzle.

FIGURE 3
ROCKET NOZZLE DESIGN VELOCITY
COEFFICIENT



It is interesting to note that with cone half-angles up to 25 degrees, the velocity coefficient is 0.96 or greater. With turbine nozzles (cone half-angles of 5 to 12 degrees) coefficients run from 0.92 to 0.97 with most near 0.96. The reason for this lack of agreement is not clear; however, it may be caused by the normal force on the canted exit area of the turbine nozzle, which would result in a negative thrust value when reaction test measurements are being made. In the case of derived nozzle coefficients from turbine test data, such as those obtained in this study, the values are based upon rotor empirical data which were in turn based upon an assumed design nozzle coefficient which itself was based upon limited nozzle test data. Hence, one goes full circle and ends up with a derived design coefficient of 0.96 if the turbine performance reaches expectation!

3.2.1 Correlations of Turbine Nozzle Data

There is only limited turbine nozzle test data available and some of the data are not included herein because of the reasons mentioned in the previous section. The data of Kraft (11.2.1), Keenan (11.2.3), and some of the data of Dollin (11.3.5) and Stodola, (Reference 1) are correlated in the form of a nozzle velocity coefficient ratio versus an isentropic mach number ratio (Figure 4).

The shape of the curves shown in Figure 4 are typical of all that are presented. Basically there is a decrease in velocity coefficient at both above ($M'/M'_o > 1.0$) and below ($M'/M'_o < 1.0$) design pressure ratio. The decrease is more rapid in the below-design point area; however, performance generally improves at Mach number ratios below 0.5, with the nozzle performance being a strong function of the nozzle area ratio or design Mach number. At above design Mach number the performance decrease is not so rapid and the effect of design Mach number is not so pronounced. The theoretical analysis in Section 4 will show that these trends are predictable.

Considering the data of Figure 4, the three investigators agree well in the limited areas where comparison can be made. The nozzle geometry is generally two dimensional of unique design. In the case of the large area ratios of Keenan, the nozzles had circular throats and basically rectangular exits. Therefore, the correlation with conical nozzle data is remarkable as will be seen later.

3.2.2 Correlation of Rocket Nozzle Data

Considerable rocket nozzle data are available in NASA reports which are presented in Figures 5, 6, and 7. While these data follow the basic trends of those shown in the turbine nozzle data of Figure 4, some additional insight into performance is required. The most important point illustrated here is that there appears to be an improvement in nozzle performance at below-design Mach numbers when large cone half-angles are used. This effect is evident in Figures 5 and 7 where the larger angles had much better performance at Mach number ratios near 0.5 than did the lower cone angles for the same design Mach number. This effect is believed to be the result of flow

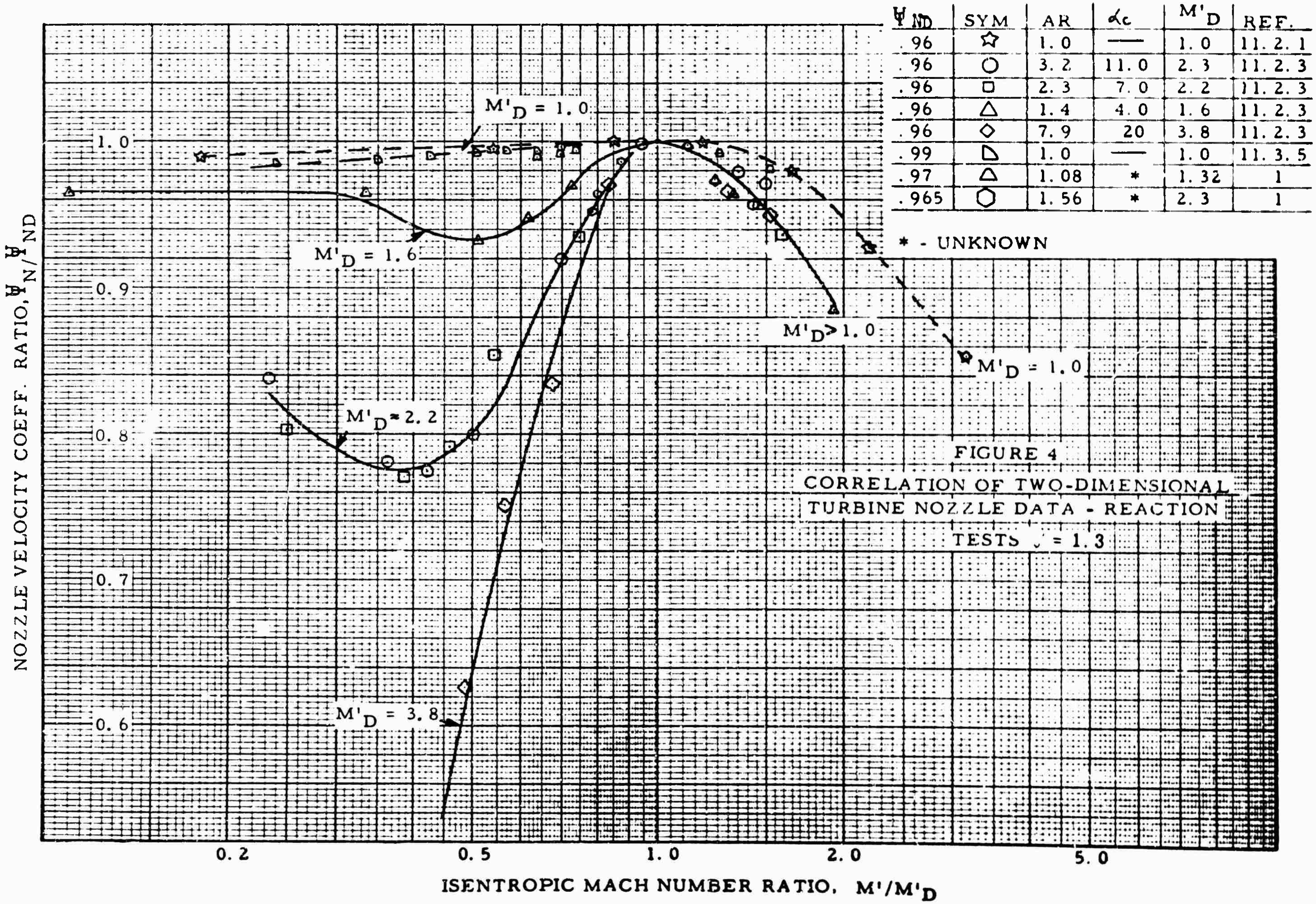
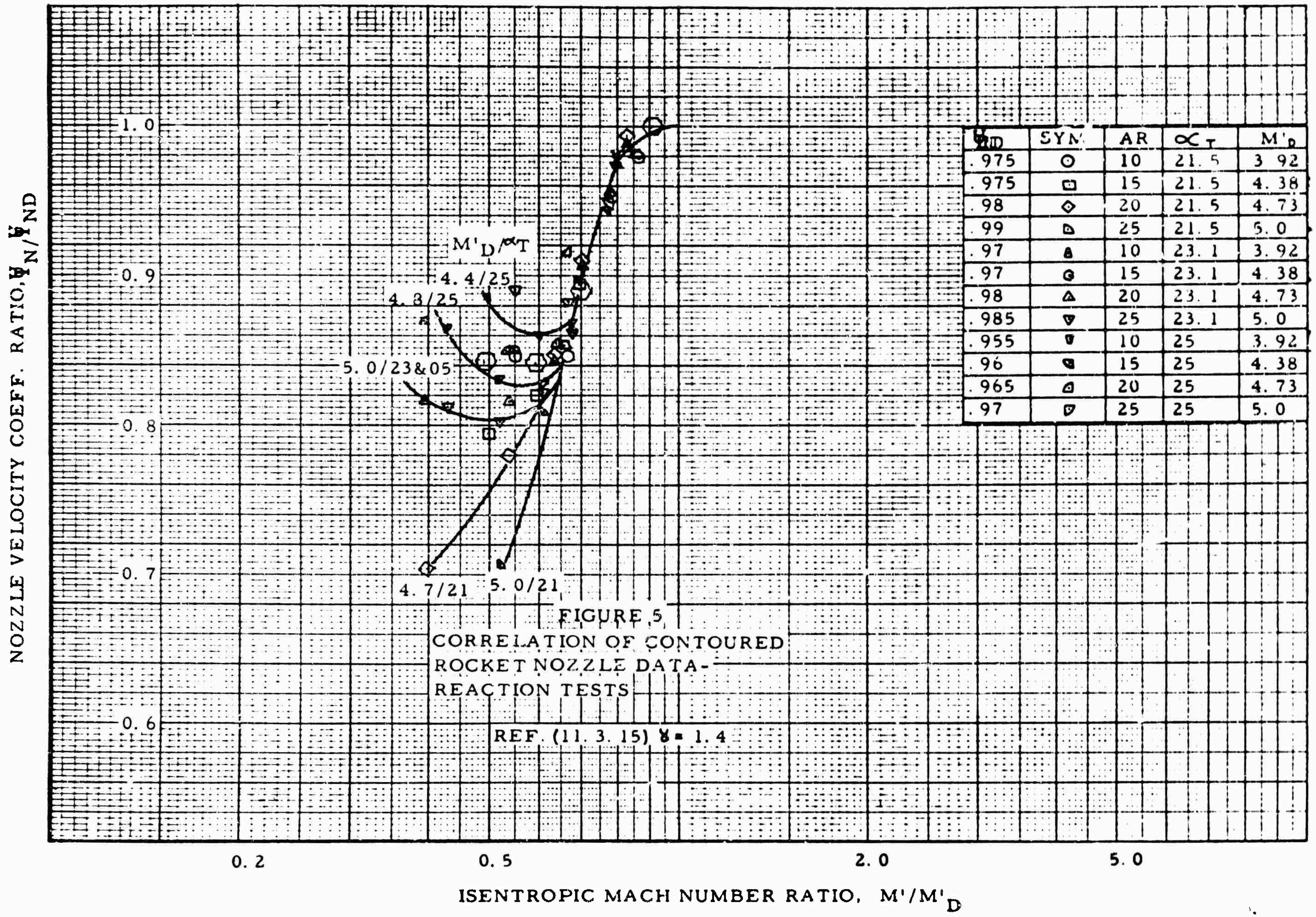


FIGURE 4

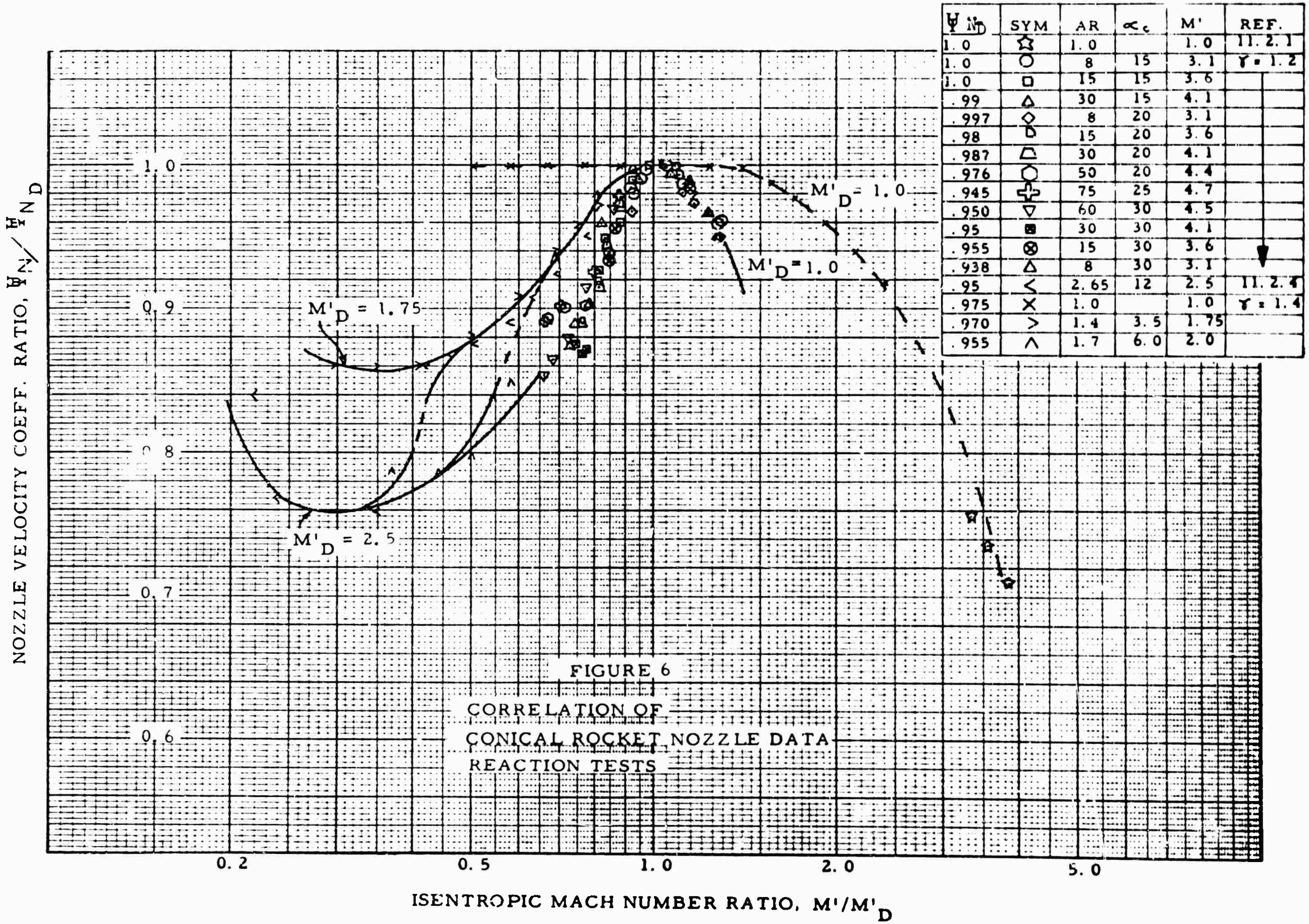
CORRELATION OF TWO-DIMENSIONAL TURBINE NOZZLE DATA - REACTION TESTS ῤ = 1.3



ψ_{ND}	SYM	AR	α_T	M'_D
.975	○	10	21.5	3.92
.975	□	15	21.5	4.38
.98	◇	20	21.5	4.73
.99	[illegible]	25	21.5	5.0
.97	[illegible]	10	23.1	3.92
.97	[illegible]	15	23.1	4.38
.98	△	20	23.1	4.73
.985	▽	25	23.1	5.0
.955	[illegible]	10	25	3.92
.96	[illegible]	15	25	4.38
.965	[illegible]	20	25	4.73
.97	[illegible]	25	25	5.0

FIGURE 5
CORRELATION OF CONTOURED ROCKET NOZZLE DATA-REACTION TESTS

REF. (11.3.15) $\gamma = 1.4$



Ψ_N/Ψ_{N_D}	SYM	AR	α_c	M'	REF.
1.0	☆	1.0		1.0	11.2.1
1.0	○	8	15	3.1	$\gamma = 1.2$
1.0	□	15	15	3.6	
.99	△	30	15	4.1	
.997	◇	8	20	3.1	
.98	⊳	15	20	3.6	
.987	⏢	30	20	4.1	
.976	⬡	50	20	4.4	
.945	✚	75	25	4.7	
.950	▽	60	30	4.5	
.95	■	30	30	4.1	
.955	⊗	15	30	3.6	
.938	△	8	30	3.1	
.95	<	2.65	12	2.5	11.2.4
.975	×	1.0		1.0	$\gamma = 1.4$
.970	>	1.4	3.5	1.75	
.955	∧	1.7	6.0	2.0	

FIGURE 6

CORRELATION OF CONICAL ROCKET NOZZLE DATA REACTION TESTS

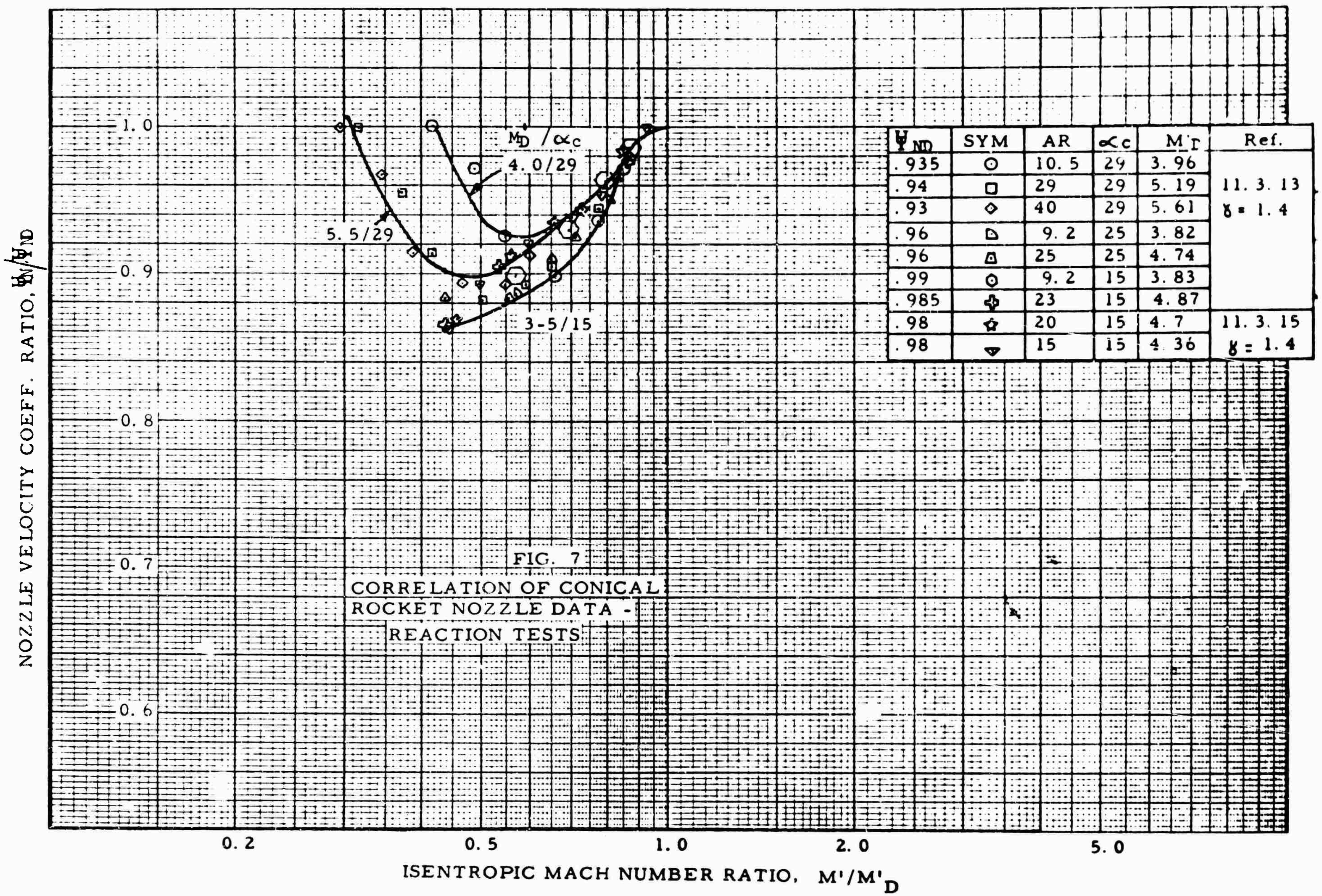


FIG. 7
CORRELATION OF CONICAL ROCKET NOZZLE DATA - REACTION TESTS

Ψ_{ND}	SYM	AR	α_c	M'_T	Ref.
.935	⊙	10.5	29	3.96	
.94	□	29	29	5.19	11.3.13
.93	◇	40	29	5.61	$\gamma = 1.4$
.96	◺	9.2	25	3.82	
.96	△	25	25	4.74	
.99	⬡	9.2	15	3.83	
.985	✤	23	15	4.87	
.98	☆	20	15	4.7	11.3.15
.98	▽	15	15	4.36	$\gamma = 1.4$

separation in the nozzle, without reattachment to the walls, at a lower area ratio in the nozzles with severe pressure gradients (large cone angle), than in the nozzles with lesser pressure gradients. When the nozzles separate at lower area ratios the shock Mach number is lower and hence the shock losses are lower. This effect was noted by Rao (11.3.12) and Farley (11.3.15).

This effect is quite important when one notes the difference between the Mach 4.7 nozzles with throat cone half-angles of 21 and 25 degrees shown in Figure 5. These data show an improvement in nozzle coefficient of 18 percentage points at a Mach number ratio of 0.4. This improvement would result in at least an equal improvement in turbine efficiency if this effect occurred in turbine nozzles. The price paid for this off-design performance improvement is reduced design point performance. In the case of the illustration above, the design point performance is reduced by 2 percentage points for the nozzles compared. This is small if the nozzle operates off-design much of the time.

Figures 5, 6, and 7 generally follow the trends of the turbine nozzle data in Figure 4. The values at which the performance improves at below design Mach numbers is quite variable; however, it may be noted that the data generally agree qualitatively at Mach number ratios from 0.5 to 1.0. An exception is the data of Krull (11.2.4) in Figure 6. At Mach number ratios greater than 1.0 the trends of the turbine nozzle data are repeated by the data presented in Figure 6.

Similarities between the rocket and the turbine nozzle are definitely established by these figures. Hence a valid theoretical analysis may consider the case of the nozzle without a canted exit, with assurance that the theory is also applicable to the turbine nozzle.

SECTION 4.0

THEORETICAL ANALYSIS

4.0 THEORETICAL ANALYSIS

This section contains a theoretical analysis which was derived to extend the nozzle information obtained in this study to nozzles of other geometries and operating on other gases. This procedure allows prediction of the nozzle velocity coefficient as a function of nozzle geometries and gas properties.

When considering off-design nozzle performance, basically four regimes occur in supersonic nozzles as shown below in the order of increasing pressure ratio.

1.0 Subsonic flow throughout the nozzle.

2.0 Overexpanded flow.

- 2.1 Choked throat with normal shock and subsonic diffusion to nozzle exit.
- 2.2 Oblique shock at nozzle wall coalescing into a normal shock at center of nozzle.
- 2.3 Same as 2.2 except oblique shock originates at the nozzle exit. As the pressure ratio approaches design, the normal shock and oblique shock weakens to near isentropic flow at design pressure ratio.

3.0 Near isentropic flow at design pressure ratio.

4.0 Underexpanded flow--expansion wave originates at nozzle exit, no pressure effect in nozzle.

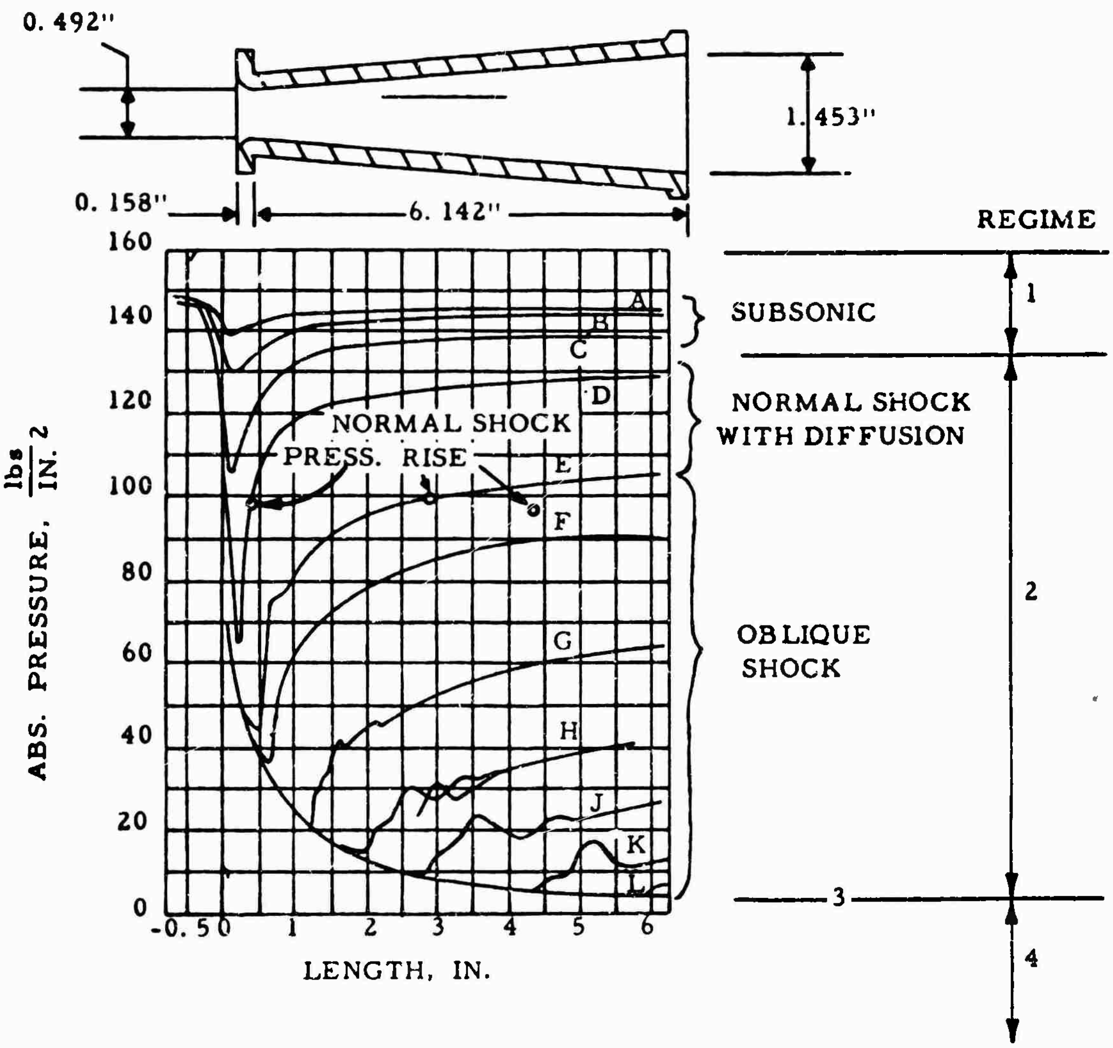
The subsonic flow regime is not of practical interest in this discussion since this regime generally occurs at nozzle pressure ratios of less than 1.1. The other regimes will be examined in detail.

4.1 OVEREXPANDED FLOW, REGIME 2

Regime 2 is the most difficult to describe with an analytical model. The ideal conditions of a normal shock with subsequent subsonic diffusion to the nozzle exit as presented by Shapiro in his classic text, "The Dynamics and Thermodynamics of Compressible Fluid Flow", does not adequately define the flow field as illustrated by the test data of Krull and Steffen (11.2.4). In this study of rocket nozzles Krull compared the measured thrust to the theoretical thrust, based on normal-shock diffusion calculation, and found variations as great as a factor of 2. This lack of correlation with the normal shock calculation is typical of all experimental investigations with nozzles designed for Mach numbers greater than 1.5.

The reason for this disagreement is seen when one examines the nozzle static pressures. A typical example is presented by Stodola in Reference 1 and reproduced here as Figure 8. The regimes are indicated on this figure. At the two test pressure ratios shown in Figure 8,

FIGURE 8
TYPICAL STATIC PRESSURE
VARIATION IN CONICAL NOZZLE
AT VARIOUS NOZZLE PRESSURE RATIOS



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STODOLA AND LOEWENSTEIN VOL. I PAGE 83

a normal shock pressure rise was possible, curves D and E. It appears that curve D did experience a normal shock; however, it is not clear if curve E had a normal shock or a strong oblique shock and subsequent diffusion. All other curves do not have a pressure rise equal to the normal shock and hence an oblique shock must occur in these cases.

Considerable data are available in the literature concerning the pressure rise associated with the oblique shock and boundary layer separation in the nozzle. Arens and Spiegles (11.3.3) show that the data of most investigators correlates as some function of the Mach number before the shock and the ratio of the wall static pressure before the shock to the nozzle exit ambient pressure. These data can then be transformed, by one-dimensional theory, into a flow deflection angle or wedge angle versus shock Mach number as shown in Arens paper and reproduced here as Figure 9. Also shown in Figure 9 is the correlation of Farley and Campbell (11.3.15) which suggests a shock Mach number ratio of 0.77. As may be noted both correlations yield very similar values.

The maximum deflection without shock detachment is also shown on Figure 9. When the flow deflection angle and hence the pressure ratio reaches this value the result is a normal shock. As noted in this figure, a normal shock should occur over the range of shock Mach numbers from 1.0 to 1.35 or 1.5. Therefore, in the following discussion it was assumed that a normal shock occurs at shock Mach numbers up to 1.5 and oblique shocks with the strength shown in Figure 9 occur at Mach numbers greater than 1.5. Hence, Regine 2.1 is assumed to exist over the shock Mach number range of 1.0 to 1.5, and Regime 2.2 exists at shock Mach numbers from 1.5 to the design Mach number.

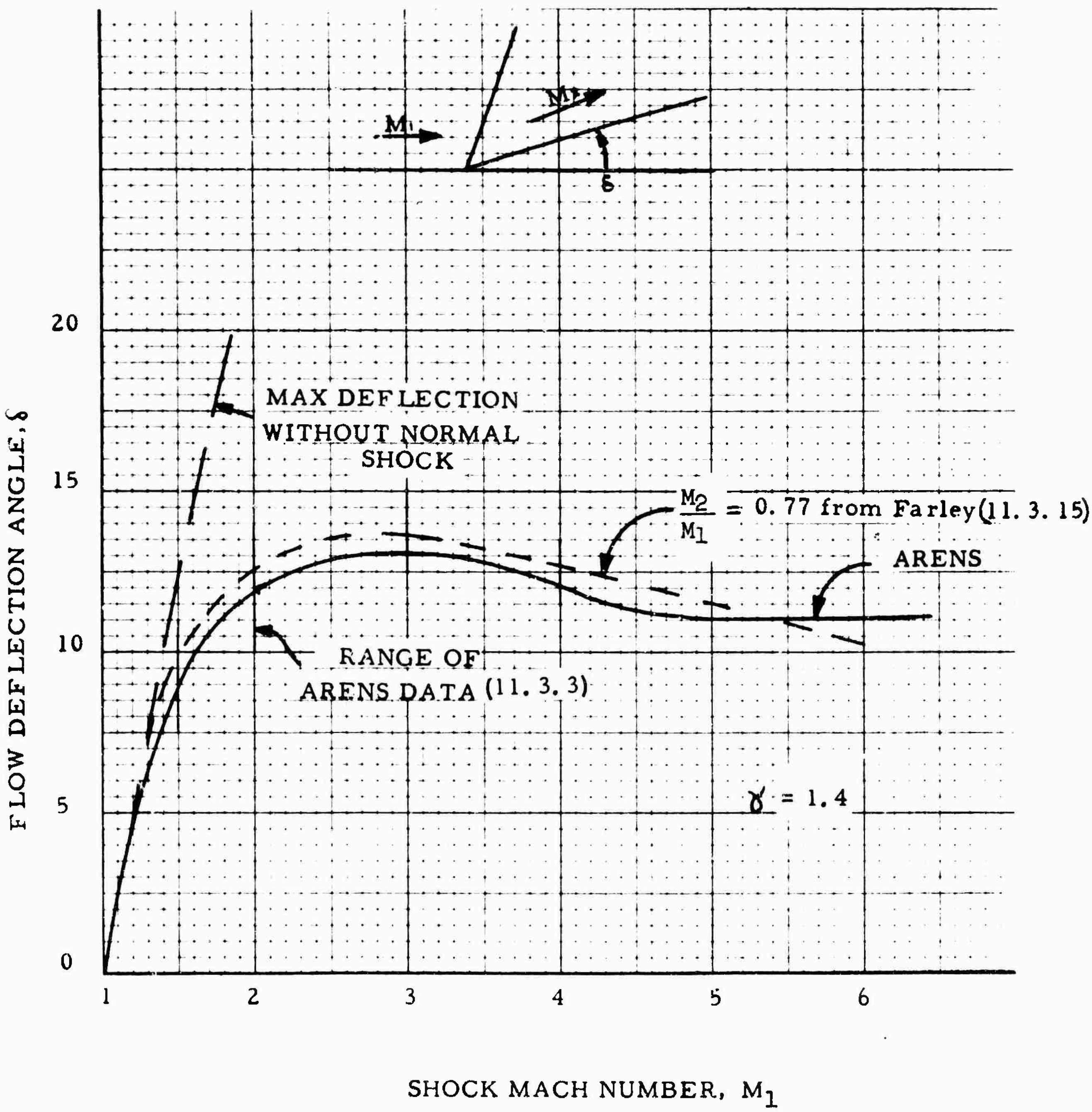
It is believed that Regime 2.2 contains an oblique as well as a normal shock for three reasons: (1) in internal flow, normal shocks almost always are made up of what is termed a (λ) lambda shock at the wall, as noted and shown in Schlieren photographs in Reference 2; (2) In axi-symmetric nozzles the oblique shock is strengthened as it approaches the center due to three-dimensional effects and must reach normal shock strength toward the center; and (3) Oblique shock losses alone are very small and do not approach the measured thrust and velocity losses.

Assuming the oblique-normal shock conditions do occur, it still remains to define the portion of the channel which is experiencing each type of shock. The oblique shock is generally becoming stronger near the nozzle center, and a discontinuity cannot occur at the nozzle center. Therefore, it was assumed that the shock strength followed the shape of an ellipse across the flow path, ranging from normal shock strength at the flow center to oblique shock strength at the wall. Therefore, the mean shock strength is:

$$S_{mean} = \pi/4 \, (S_{os} + S_{ns}) \tag{4.1}$$

Where: S_{os} is oblique shock strength
S_{ns} is normal shock strength

FIGURE 9
VARIATION OF FLOW DEFLECTION
ANGLE WITH SHOCK MACH NUMBER
DERIVED FROM SEPARATION
PRESSURE RATIO DATA



Since a linear variation would result in $(\frac{1}{2})$ in place of $(\frac{\pi}{4})$ in equation (4.1), it is apparent that the proposed equation weights the mean shock strength in the direction of the normal shock.

When the oblique shock reaches the nozzle exit, Regime 2.3 begins. As the pressure ratio is increased it is assumed that the normal and oblique shock strengths decrease until the design point is reached. Figure 10 presents the results of a theoretical calculation for a nozzle designed for Mach 2; the calculation procedure is derived in Appendix II.

All Regimes are pointed out on this curve, however, it will be noted that Regime 2 covers the majority of the operational range of a nozzle. It was found that the calculated curve in Regime 2.3 is optimistic considering the data presented in Section 3 and the data obtained in this study. Therefore, because of the shape and location of the calculated points of the extremes of each sub-regime of Regime 2, it is recommended that these extremes be calculated and a curve be faired in between them. This greatly reduces the calculations required and allows the person performing the analysis to utilize the data of Section 3 and that obtained in this study to result in a more realistic curve (Regime 2.3).

As shown in Section 3, little variation in nozzle performance occurs in Regime 2.3 regardless of nozzle design Mach number. Therefore, the point at which Regime 2.2 occurs is the most important to determine. It will be shown later that this point is accurately described by the calculation procedure described.

4.2 DESIGN POINT AND UNDEREXPANDED FLOW, REGIMES 3 AND 4

The design point performance of a rocket nozzle is dependent upon the nozzle geometric parameters described in Section 3. It appears that turbine nozzles have slightly lower design point performance than rocket nozzles, velocity coefficients being in the area of 0.96 versus 0.98 to 0.99 for geometrically similar rocket nozzles. Nozzle area ratio has an effect on design performance because of the frictional effects. Since the correlations presented herein use the ratio of the off-design nozzle velocity coefficient to the design velocity coefficient, it is felt that the effect of nozzle design coefficient is eliminated from the correlation.

A simple two-dimensional flow model adequately describes the nozzle performance in the Underexpanded Regime 4. This model considers the theoretical control surface to enclose the nozzle and the expansion shocks originating from the nozzle exit edge. In this case the control surface exit pressure is equal to ambient pressure and no pressure correction need be applied. Therefore, the nozzle velocity coefficient is equal to the cosine of the Prandtl-Meyer expansion angle. The calculated velocity coefficient in this Regime is shown in Figure 10. A derivation of this theory is presented in Appendix II.

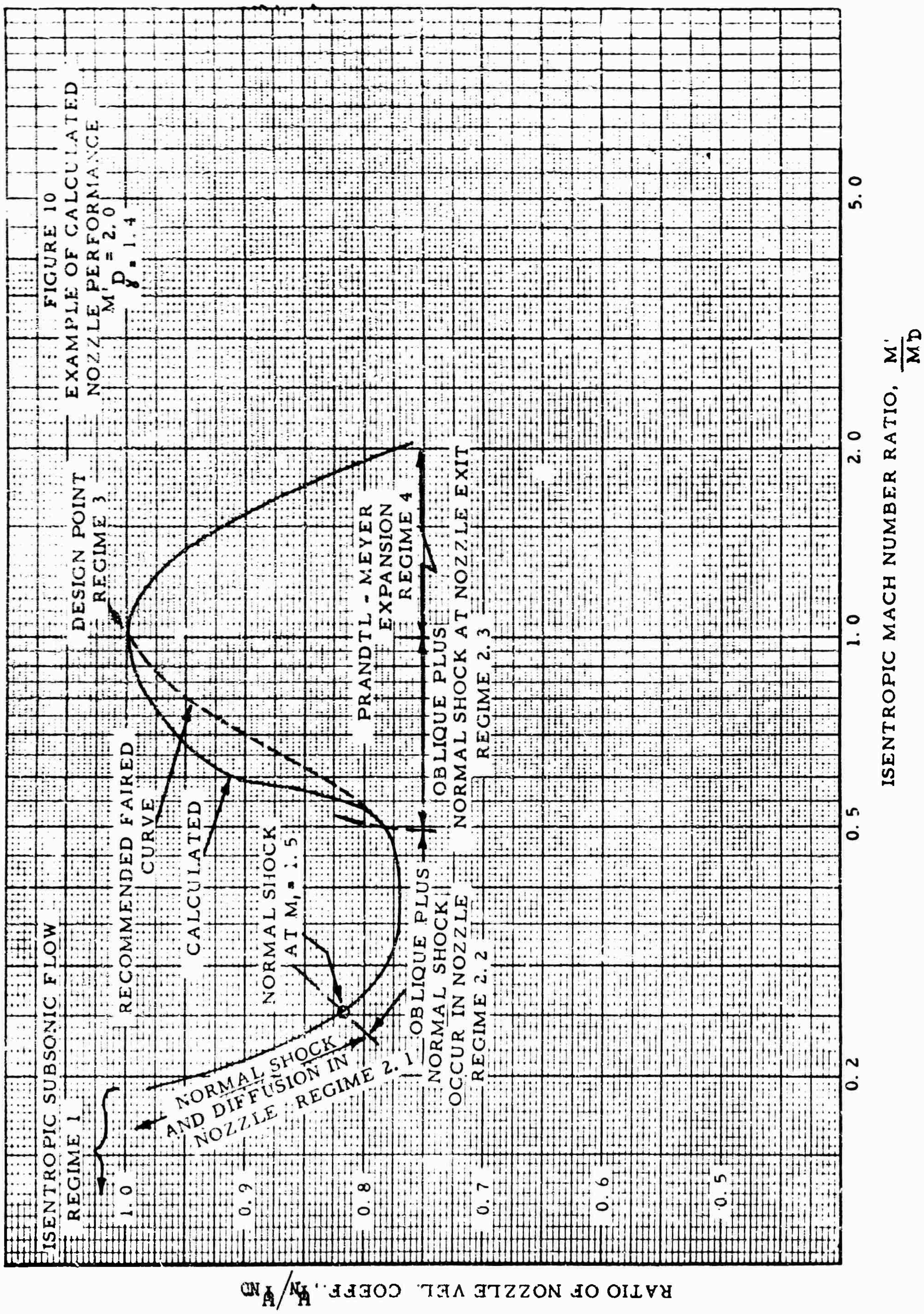


FIGURE 10
EXAMPLE OF CALCULATED NOZZLE PERFORMANCE

4.4 COMPARISON OF THEORY WITH DATA

The effect of design Mach number on the calculated nozzle performance is shown in Figure 11. The qualitative and quantitative agreement between Figure 11 and the nozzle data of Section 3 is quite good.

The limit line shown in the lower left hand corner of Figure 11 indicates the limit at which a frictionless model applies. In the area noted, the pressure ratio is low and the area ratio high, so that a large percentage of the available energy may be absorbed in friction. For example, (with a nozzle designed for Mach 2.5) when the Mach 1.5 normal shock with subsequent expansion occurs, Regime 2.1, the nozzle pressure ratio is only 1.1. This pressure ratio is unreasonably low, hence the theory is unapplicable in this area.

It is interesting to note that a straight line between the calculated interface point between Regime 2.2 and 2.3 (at a design Mach number of 4.0) describes one limit with the interface point for a design Mach number of 1.5 describing the other limit to aid in locating the faired lines of Regime 2.3. This range correlates well with the data of Section 3 and also with the data obtained in this study as will be discussed later.

Another significant point is the relatively small effect of Design Mach number in Regime 4. This same effect was shown in the nozzle data of Section 3. The theoretical values shown in Figure 11 indicate a larger difference in performance between sonic and Mach 1.5 nozzles than between Mach 1.5 and Mach 4.0 nozzles. This interesting fact was also noted in the data of Section 3.

With turbine nozzles designed for Mach numbers of 1.5 or greater, rarely will the nozzle operate at isentropic Mach number ratios less than 0.5 or greater than 2.0, since the pressure ratio will be excessively far from the design value shown in Figure 12. Therefore, the procedure of Regimes 2.2, 2.3 and 4.0 are the most important in the practical sense.

4.4 EFFECT OF GAS RATIO OF SPECIFIC HEATS

Turbines do operate on working fluids with specific heat ratios from 1.667 for monatomic gases to 1.02 for fluorochemical and organic fluids. The calculated effect of specific heat ratios is shown in Figure 13. Theoretically this effect is seen to be quite large; however, little data are available for comparison.

A comparison of one turbine operating on fluids with different ratios of specific heats was made in tests at Sundstrand. This turbine was operated on nitrogen ($\gamma = 1.4$) and Freon 12 ($\gamma = 1.13$). The results of these tests generally agree with the effect shown in Figure 13. These data will be discussed in more detail in Section 6.

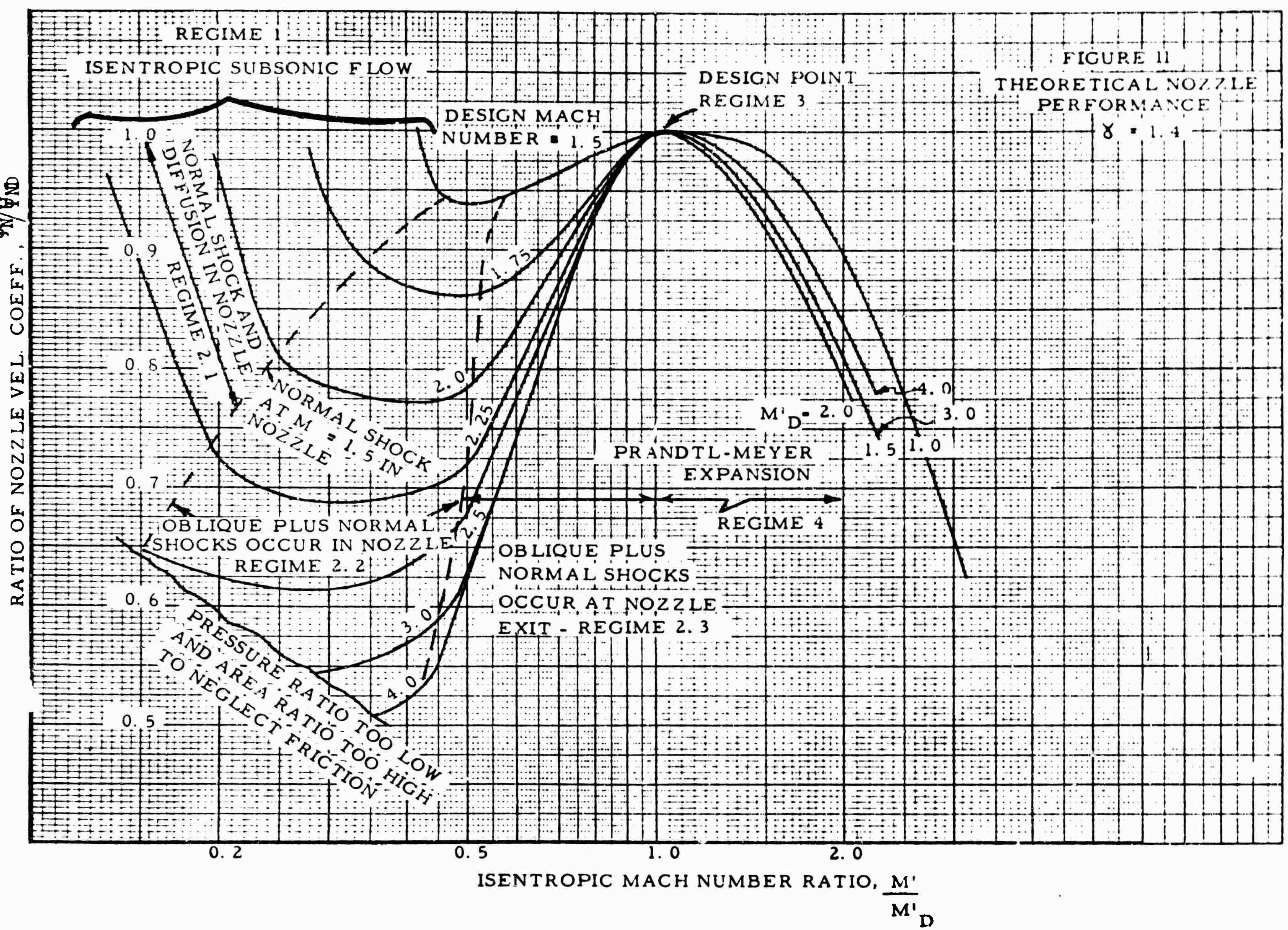


FIGURE 11
THEORETICAL NOZZLE PERFORMANCE
γ = 1.4
REGIME 1
ISENTROPIC SUBSONIC FLOW
DESIGN MACH NUMBER = 1.5
DESIGN POINT REGIME 3
NORMAL SHOCK AND DIFFUSION IN NOZZLE
REGIME 2.1
NORMAL SHOCK AT M = 1.5 IN NOZZLE
OBLIQUE PLUS NORMAL SHOCKS OCCUR IN NOZZLE
REGIME 2.2
OBLIQUE PLUS NORMAL SHOCKS OCCUR AT NOZZLE EXIT - REGIME 2.3
PRESSURE RATIO TOO LOW AND AREA RATIO TOO HIGH TO NEGLECT FRICTION
PRANDTL-MEYER EXPANSION
REGIME 4
M'D = 2.0
1.75
2.0
2.25
2.5
3.0
4.0
1.5
1.0
1.0
0.9
0.8
0.7
0.6
0.5
0.2
0.5
1.0
2.0
ISENTROPIC MACH NUMBER RATIO, M'/M'D
RATIO OF NOZZLE VEL. COEFF., ψN/ψND

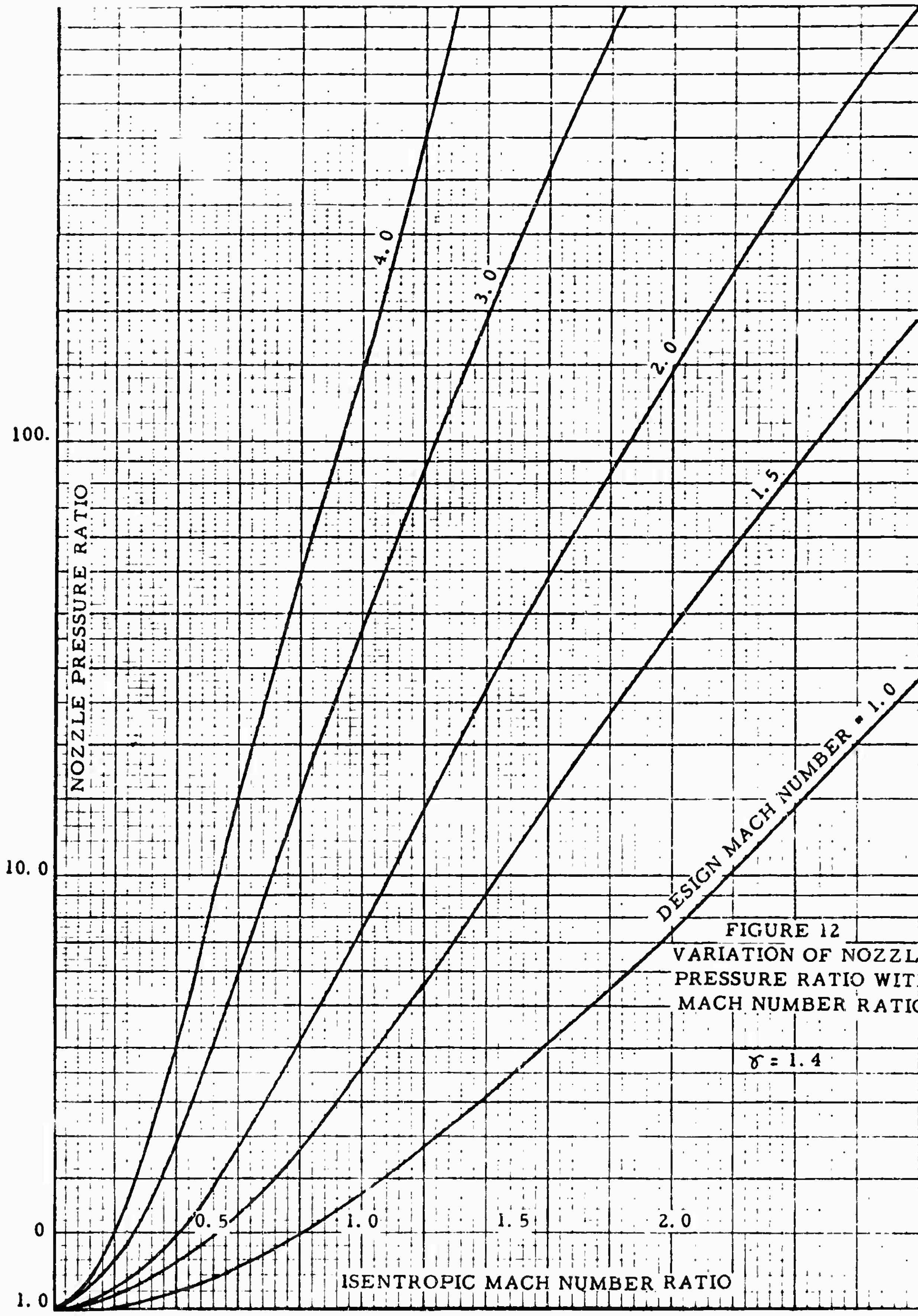


FIGURE 12
VARIATION OF NOZZLE PRESSURE RATIO WITH MACH NUMBER RATIO

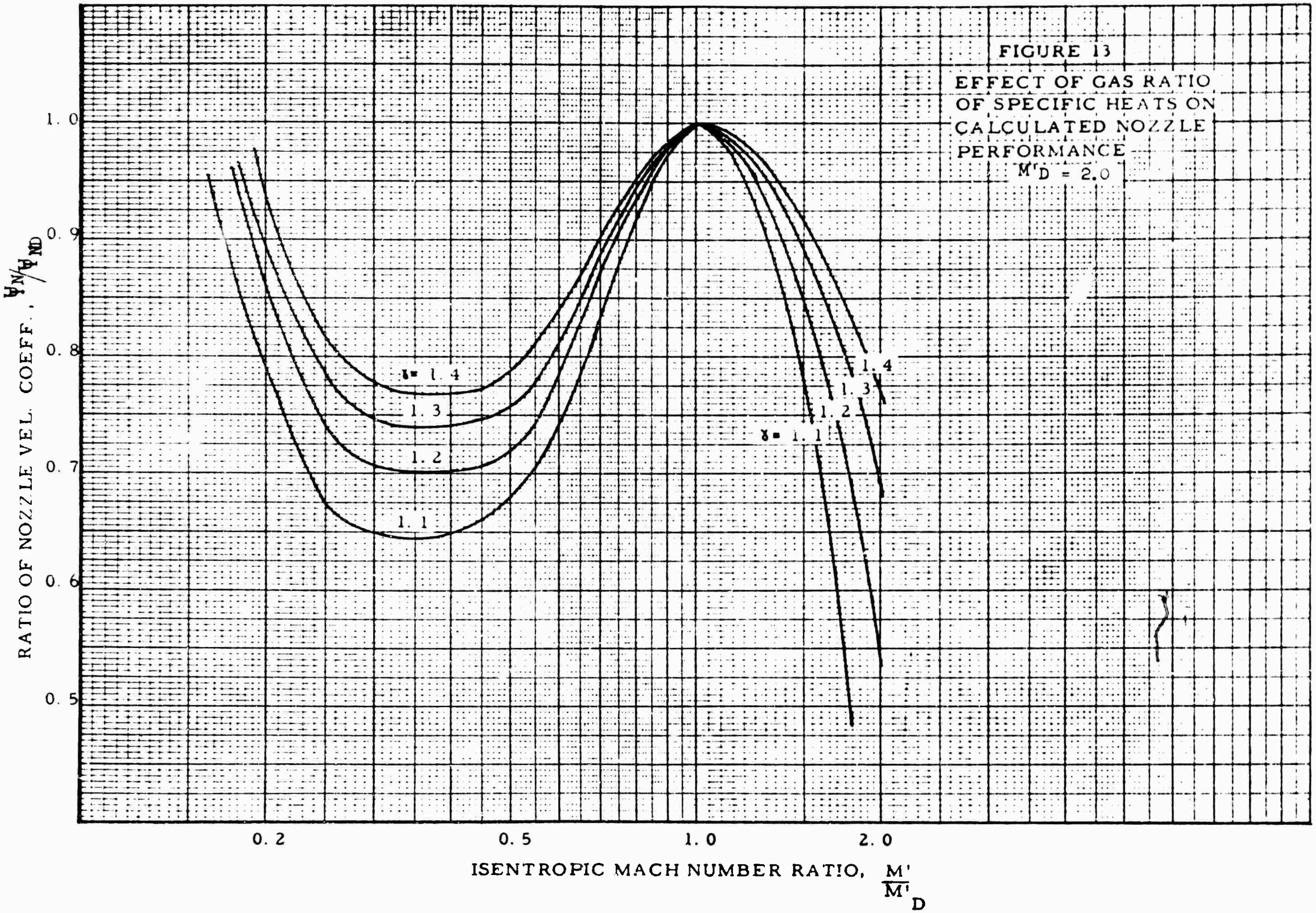


FIGURE 13

EFFECT OF GAS RATIO OF SPECIFIC HEATS ON CALCULATED NOZZLE PERFORMANCE

$M'_D = 2.0$

SECTION 5.0

EXPERIMENTAL APPARATUS AND TECHNIQUES

5.0 EXPERIMENTAL APPARATUS AND TEST TECHNIQUES

The experimental study was made using a test dynamometer designed and built by Sundstrand. The test apparatus and techniques were developed over the past two years while performing various turbine research and development studies.

5.1 TURBINE TEST DYNAMOMETER

The turbine test facility is specifically designed to evaluate the effects of turbine geometry on performance. Rapid changes in rotors, nozzles, and rotor tip and side clearances are easily made. Shaft power is determined by a torque arm force measurement, the load being applied by an electrical homopolar dynamometer. Turbine bearing losses are included in the torque arm measurement since the entire rotating group is supported by hydrostatic gas bearings. The dynamometer is shown in Figures 14 and 15; typical test hardware is shown in Figure 16.

The facility is capable of testing turbines on cold (400 to 700°R) gases (usually nitrogen) at pressure ratios up to 200, without Reynolds number effects, while pressure ratios to 2500 are possible. The entire unit is located in a vacuum chamber during tests such that exhaust pressures as low as 0.05 psia are possible. The maximum test turbine speed is 40,000 rpm. Turbine diameters up to approximately 7 inches can be accommodated. Output shaft power levels of up to 20 hp can be absorbed by the electrical load bank.

Tests to obtain nozzle flow coefficients and turbine efficiency as a function of turbine geometry, pressure ratio, speed, and Reynolds number can be conducted. It is planned that provisions to obtain turbine disc friction and pumping losses will also be available in the future.

Testing shows that turbine efficiency data repeats with less than 2% scatter. The nozzle flow is measured by a venturi meter which shows scatter in the flow rate of 0.5%. Instrumentation to obtain pressure and temperature data throughout the turbine is utilized.

All tests were made with the shrouded turbine wheel shown in Figure 17, and the turbine exhaust housing shown in Figure 18. During all tests the radial tip clearance was 0.010 inch; the axial clearance between the nozzle and the wheel was 0.035 inch except during the tests made to determine the effects of axial clearance. Table I presents geometric parameters that were common during all tests.

5.2 TURBINE NOZZLE TEST CONFIGURATIONS

Four types of turbine nozzles were studied, including conical, axisymmetric shock cancellation (contoured), plug, and free expansion nozzles. The design Mach number varied from 1.6 to 4.6 for the conical nozzles and was 4.0 for the contoured, plug, and free expansion nozzles.

TABLE I

GEOMETRIC PARAMETERS OF THE TEST NOZZLES AND TEST TURBINE COMMON TO ALL TEST CONFIGURATIONS*

Nozzle:

Nozzle Angle	16°
Exit Diameter	0.35 inch
Pitch Diameter	6.775 inches
Number of Nozzles	1.0

Major axis of nozzle exit ellipse is tangent to wheel pitch diameter at center of ellipse.

Rotor:

Pitch Diameter	6.775 inches
Blade Angles (inlet and exit)	25°
Blade Height	0.42 inch
Blade Chord	0.3 inch

Clearances:

Rotor Tip Radial	0.010 inch
Nozzle to Rotor	0.035 inch
Rotor to Exhaust Housing	0.065 inch

Gas Conditions (pure nitrogen):

Inlet Temperature	Approx. 80°F
Inlet Pressure	Up to 325 psia

*Unless exception noted on figure.

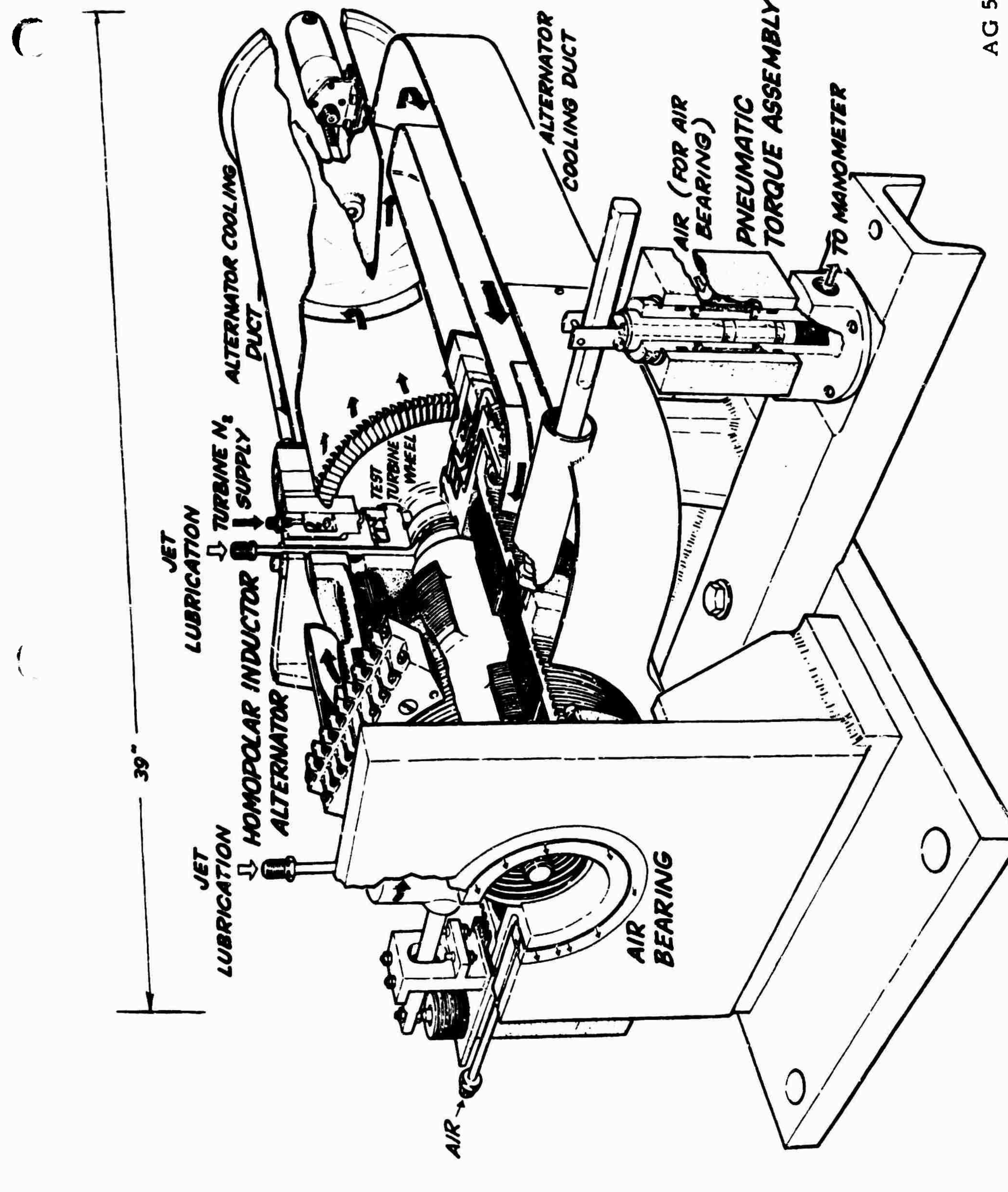


Figure 14. Sectional View of Dynamometer

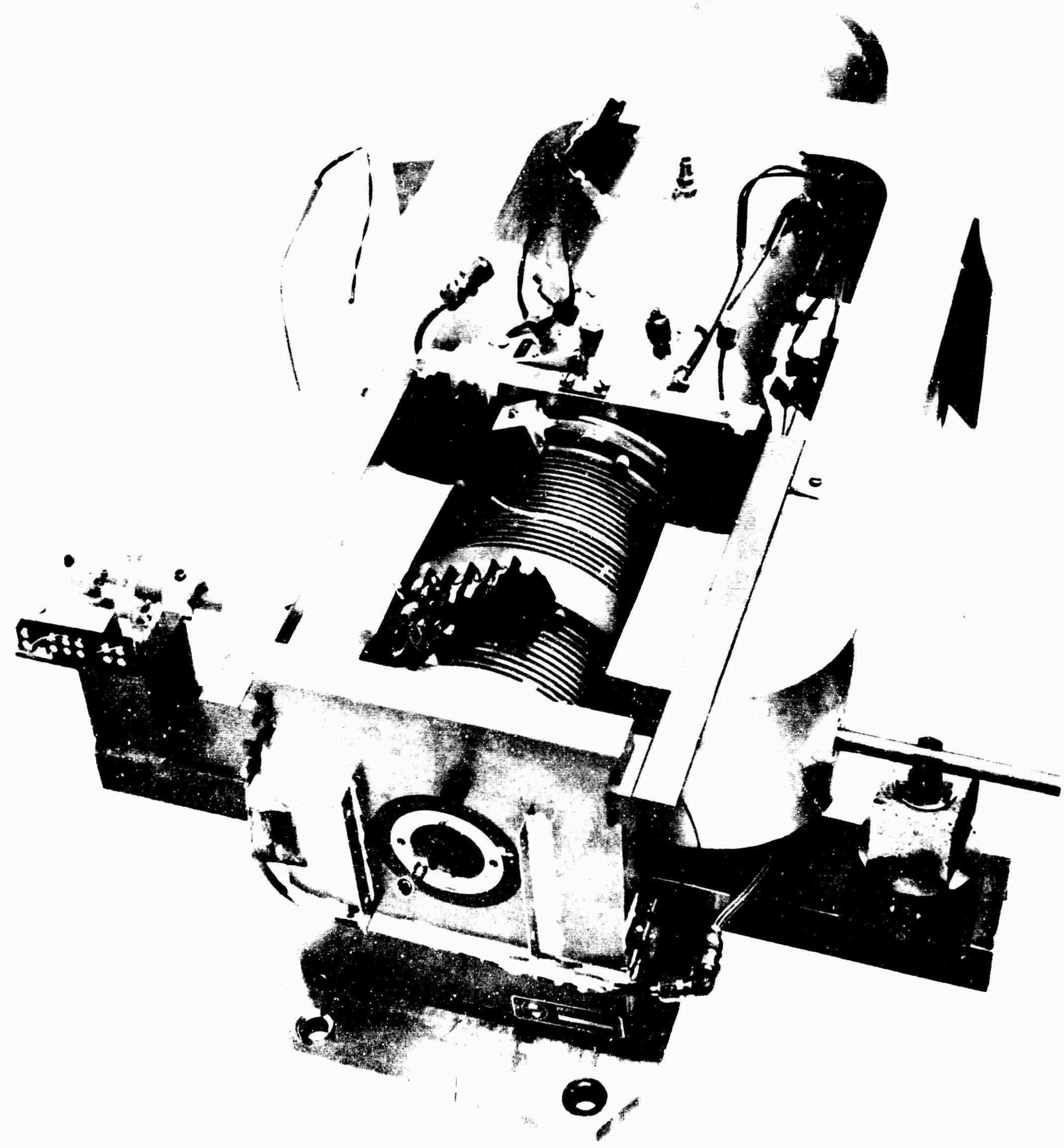


Figure 15. Turbine Dynamometer

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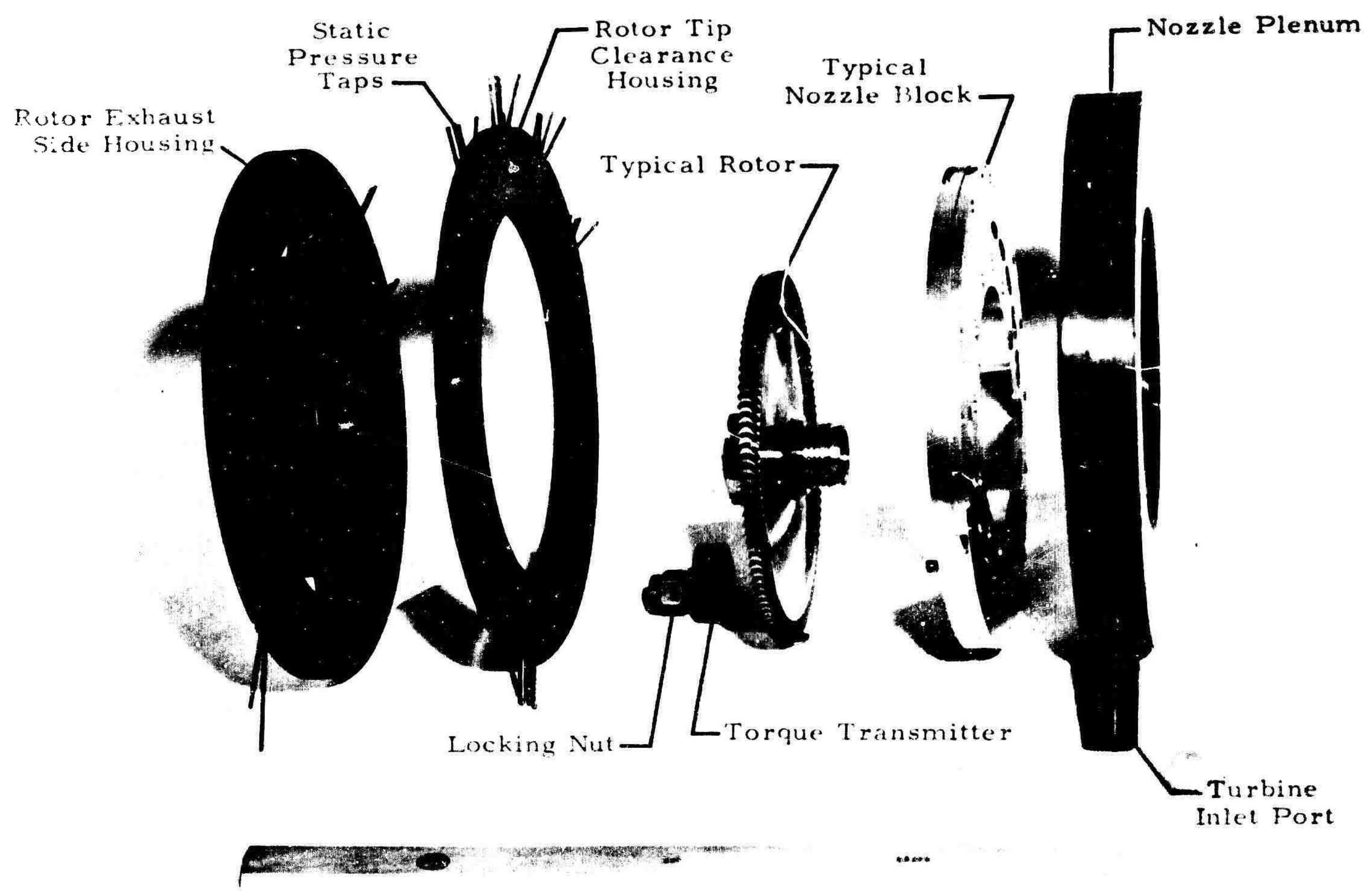
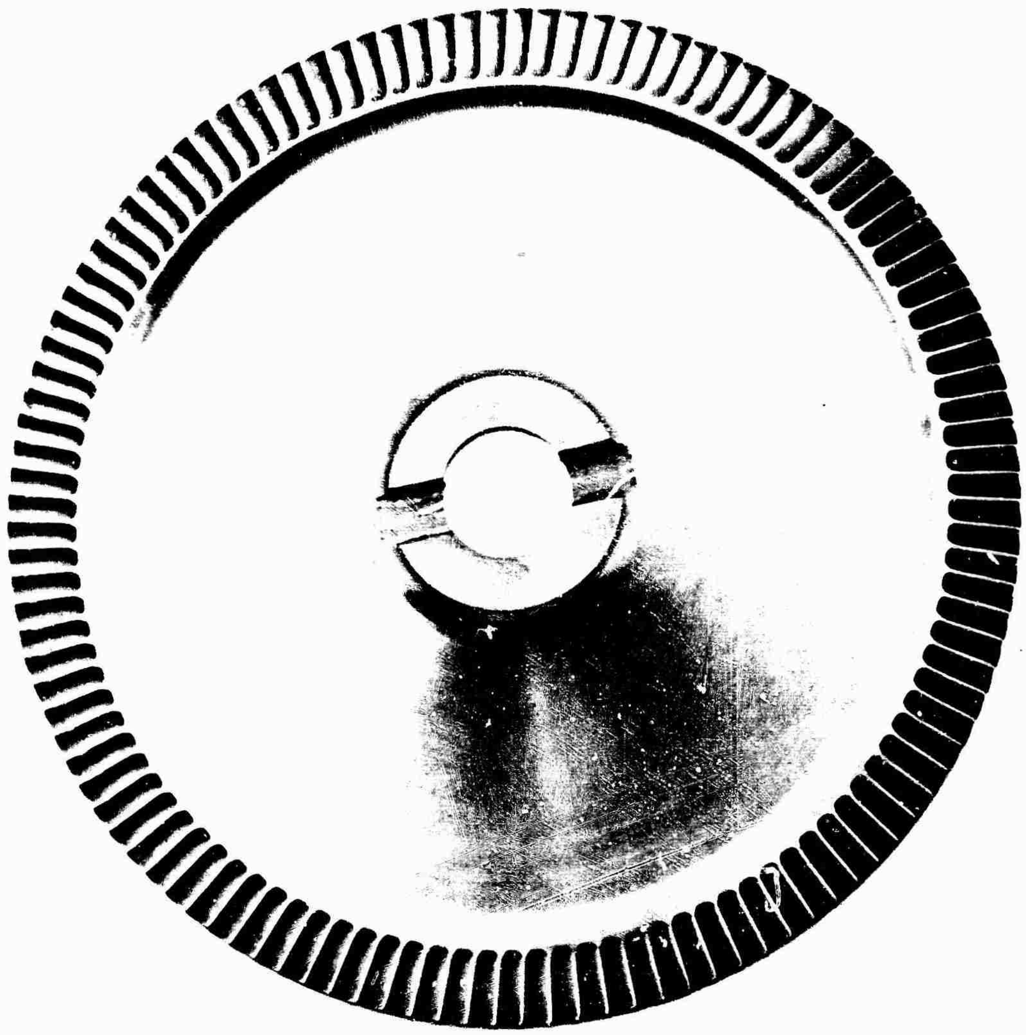


Figure 16. Typical Axial Flow Turbine Components

AG 4706



Tip Diameter = 6.2 in.
Blade Height = 0.42 in.
Chord = 0.3 in.
25° Symmetrical Blades
115 Blades

Figure 17. Shrouded Turbine Wheel

AG 7711

Figure 18. Turbine Assembly 831091 Exhaust Housing Adapted for Use on the Turbine Test Lab Dynamometer

AG 7715

One nozzle of each type was fabricated. All nozzles had an exit diameter of 0.35 inch; the rotor had a blade height of 0.42 inch. Hence, the lap (blade height to nozzle height ratio) for all tests was 1.2; the admission area was 7 percent. The relationship between the nozzle exit height and the blade height is shown in Figure 19.

The important geometric parameters and the design techniques used with each nozzle type are give in the following sections.

5.2.1 Conical Nozzles

The conical nozzles are quite simple and employ a conical supersonic expansion area. Figure 20 is a sketch of a typical nozzle. The table below the sketch gives the significant geometric parameters for the conical nozzles tested. In this design, the nozzle cone does not extend beyond the area designated as "L" in Figure 20. Figure 16 is a photograph of a typical conical nozzle.

The conical nozzles were selected for testing in this study as the basis of comparison, since they are the most common type used in partial admission turbines.

5.2.2 Contoured Nozzle

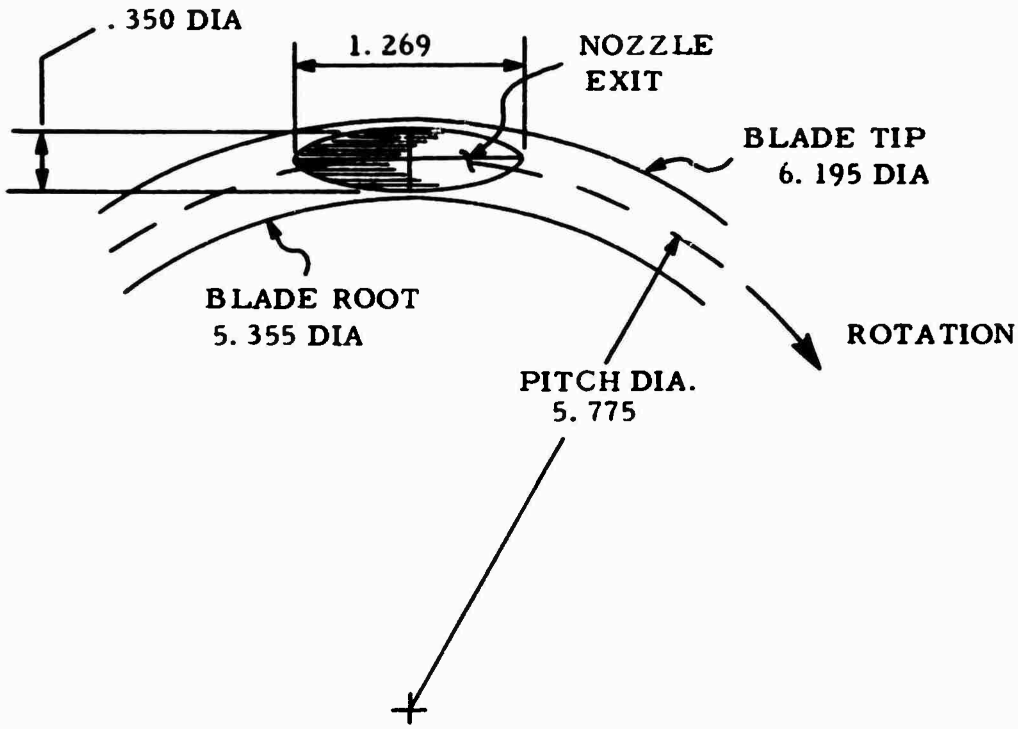
Figure 21 shows a sketch and the contour of the nozzle. Up to the nozzle throat the design is identical to that of the Mach 4 conical nozzle. Downstream of the throat the surface is designed to cancel the expansion waves which originated at the throat. This nozzle was designed by the method presented by Rao (11.1.13) for a Mach number of 4. The contoured nozzle was included in this study because improved performance is expected as compared to the conical nozzle. This improvement occurs because the exit flow vector is parallel to the nozzle axis, and the compression shocks do not occur in the nozzle during design point operation. The data of Farley and Campbell (11.3.15) shows improved performance for the contoured rocket nozzles as compared to the conical nozzles.

5.2.3 Plug Nozzles

The contoured plug nozzle is the result of rocket nozzle research to obtain improved off-design performance. A sketch of the plug nozzle as adapted for turbine nozzle use is shown in Figure 22. This nozzle was designed by the method of characteristics to cancel a family of Prandtl-Meyer expansion waves which originate at the sharp corner at the nozzle throat. The nozzle contour is described in the table of Figure 22.

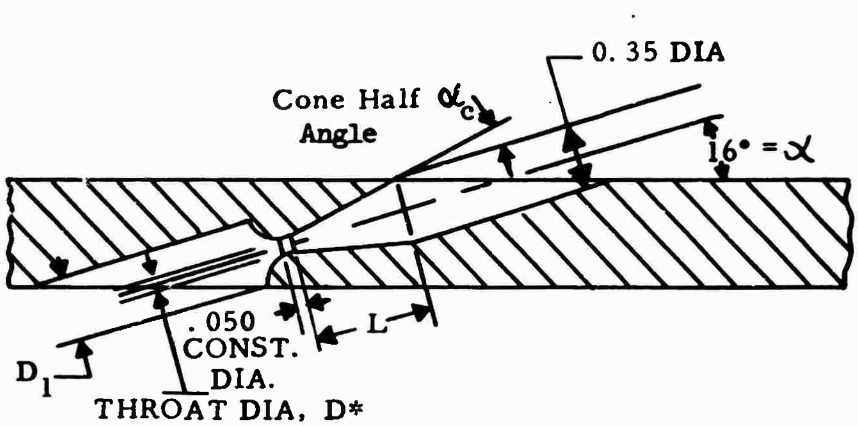
Techniques for designing plug nozzles are presented by Angelino (11.1.2) and Greer (11.1.8). The procedure of Greer is basically that used in the design for this study since the procedure of Angelino

FIGURE 19
SKETCH OF CONICAL NOZZLE EXIT
AND WHEEL OUTLINE
VIEW LOOKING THROUGH ROTOR BLADES
AT NOZZLE EXIT



NOTE: NOZZLE DESIGNED WITH 20% LAP
EQUALLY SPACED

FIGURE 20
SKETCH OF CONICAL NOZZLE
CONFIGURATION



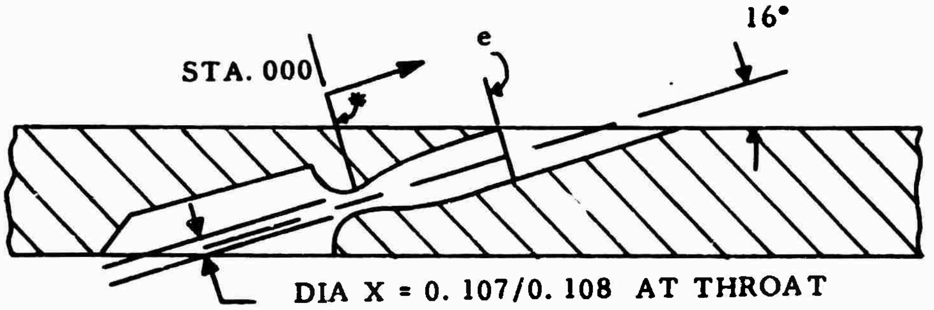
M'_D	AR	D_1, IN	D*, IN	L, IN	$\alpha_c°$	Y_{nd}
+2. 11	1. 7	0. 5	0. 268	0. 50	4. 7	. 975
+4. 0	10. 7	0. 5	0. 106	0. 572	12	. 996
4. 6	18	0. 5	0. 082	0. 625	12	. 933
1. 6	1. 25	0. 6	0. 315	0. 444	2. 0	. 949
±3. 7	8. 38	0. 5	0. 105	0. 470	12	. 967
2. 4	2. 5	0. 4	0. 221	0. 613	6	. 945
Δ1. 5	1. 4	0. 54	0. 268	0. 410	4	. 950

\+ NOZZLES DESIGNED SPECIFICALLY FOR THIS STUDY

± NOZZLE EXIT DIAMETER = 0. 305

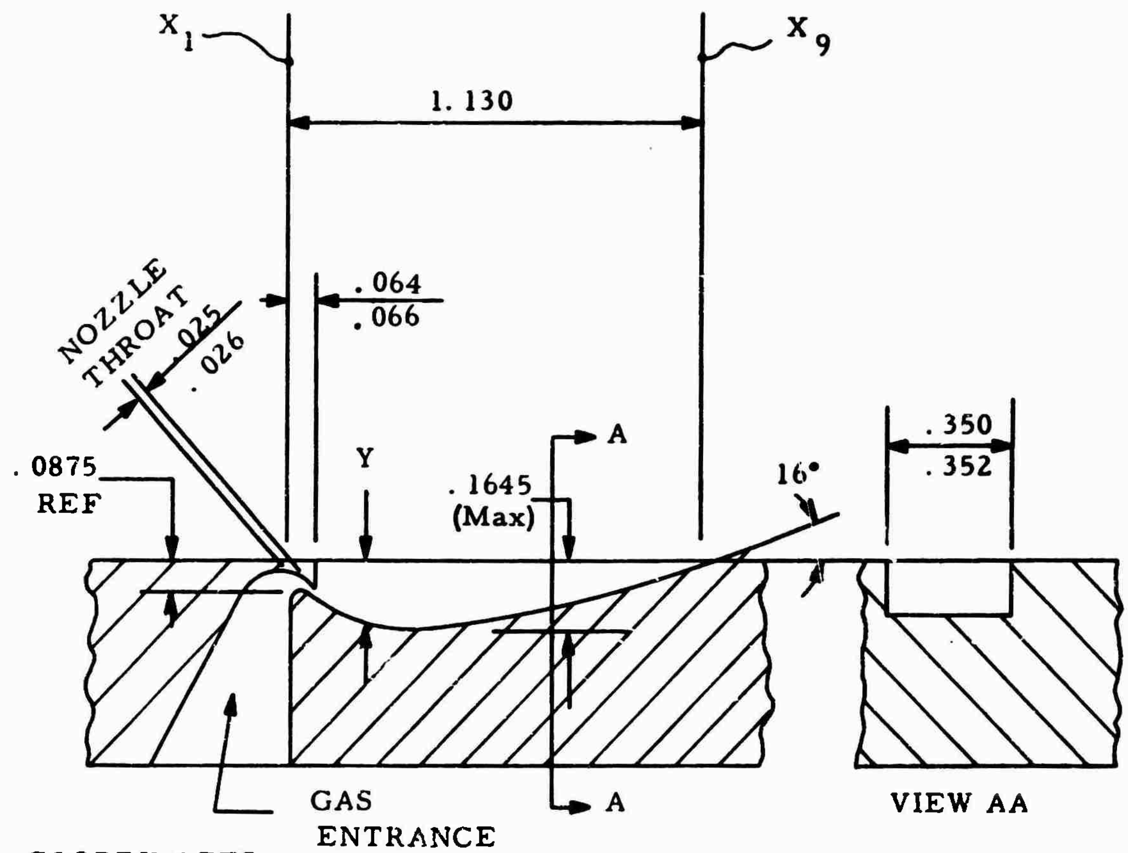
Δ NOZZLE EXIT DIAMETER = 0. 323

FIGURE 21
SKETCH OF AXI-SYMETRIC
SHOCK CANCELLATION NOZZLE



COORDINATES FOR M'_D = 4. 0			
STA. IN	DIA X, IN	STA. IN	DIA X, IN
0. 00*	0. 107	0. 35	0. 280
0. 05	0. 136	0. 40	0. 296
0. 10	0. 166	0. 45	0. 310
0. 15	0. 193	0. 50	0. 322
0. 20	0. 219	0. 60	0. 339
0. 25	0. 241	0. 70	0. 348
0. 30	0. 261	e = 0. 75	0. 350

FIGURE 22
SKETCH OF PLUG NOZZLE



COORDINATES

STA	X±. 001	Y±. 001
1	0	. 100
2	. 140	. 143
3	. 300	. 164
4	. 460	. 158
5	. 620	. 137
6	. 780	. 104
7	. 940	. 064
8	1. 100	. 010
9	1. 130	0

yields a poor flow configuration entering the nozzle throat. This type was selected for the study since it had improved off-design performance over other rocket nozzle types. Another advantage of this design is that the nozzle flow is not affected by slanting the nozzle exit for turbine application.

5.2.4 Free-Expansion Nozzles

The free expansion nozzle design was based on the information of Barakauskas (11.2.6). This report indicated that this type of nozzle could have performance comparable to a conical nozzle and is less costly to design and fabricate. As noted in Figure 23, four nozzles were fabricated with various free-expansion lengths. The paper by Barakauskas indicates the optimum length to be approximately two exit diameters ($L^* = 0.70$). The shortest length nozzle is essentially a conical nozzle with a 51 degree half-angle cone.

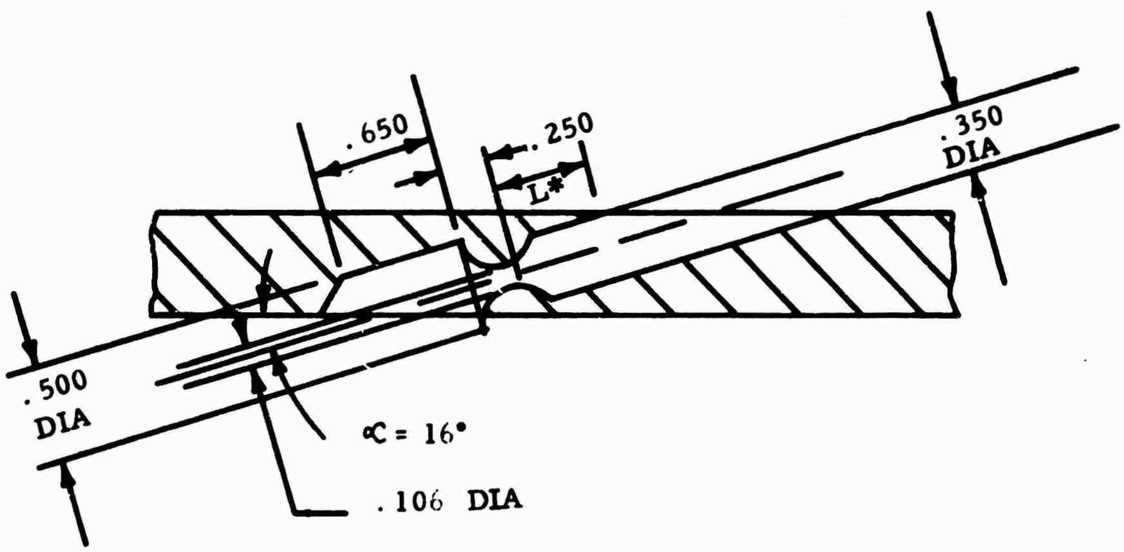
5.3 TEST TECHNIQUES

A test run was made by pressurizing the turbine inlet with bottled nitrogen to a prescribed value (not greater than 325 psia) and adjusting the turbine back pressure to the prescribed level to obtain the desired pressure ratio. Data was then taken at 3 to 5 turbine speeds from 2,000 to 30,000 rpm. The speed was controlled by adjustment of the homopolar alternator load resistance (coarse adjustment) and the field voltage (fine adjustment). After the speed range was covered the back pressure was adjusted to the next prescribed pressure ratio and the speed points repeated. The test pressure ratios were varied from the minimum pressure ratio (approximately 5) at which the unloaded generator would reach speeds of approximately 10,000 rpm to the maximum pressure ratio (approximately 1200) at which Reynolds number effects become significant. Data were taken at eight to ten pressure ratios and 3 to 7 speeds at each pressure ratio.

The turbine pressure ratio was evaluated by measuring the pressure in the turbine nozzle plenum chamber, which the nozzle plate seats against, and three pressures in the exhaust duct just upstream of the exhaust control valve, Figure 14. The turbine flow rate was measured by a venturi meter in the nitrogen line leading to the turbine plenum. At each data point conditions were allowed to stabilize and then two or three identical readings of the torque load cell pressure were made before recording the data and then proceeding to the next data point. Approximately two minutes of run time were required at each data point.

All pressures greater than 25 psia were measured with large Bourdon tube pressure gauges and the lower pressures were read on mercury manometers. The temperatures through-out the turbine were determined by the use of copper-constantan thermocouples and a Bristol read-out device. The turbine inlet temperature was maintained at approximately 80°F by using hot water in nitrogen heat exchangers. The nitrogen was heated before it entered the venturi flow meter.

FIGURE 23
SKETCH OF FREE EXPANSION
NOZZLE



NOTE: NOZZLES WITH THE FOLLOWING
VALUES OF L* WERE FABRICATED

1) 0.10
2) 0.50
3) 0.75
4) 1.00

The torque read-out device was calibrated before and after each run by the application of dead weight to the torque arm. If significant variations in the pre-and post-calibration points occured, the data was discarded and a rerun made.

The turbine speed was measured with an electronic counter which monitored the alternator frequency. This resulted in a count which was thirty times the rpm of the alternator and turbine.

The data were recorded manually and later reduced by a computer program to obtain the desired performance parameters. The flow rate was determined from the venturi measurements as well as the perfect gas choked flow equation. In this manner the flow discharge coefficient was obtained; if an unusual variation in this coefficient was found the data was discarded as unreliable and the test was rerun.

5.4 DATA REDUCTION PROCEDURE

The data are reduced utilizing an IBM 1620 computer which includes curve fits of compressibility factor and ratio of specific heats for nitrogen, obtained from Reference 4. These properties are calculated as functions of local static temperature and pressure, and can be input if a gas other than nitrogen is used as the test fluid. Two calculation procedures are used, one to obtain turbine performance data from the test measurements, and the other to calculate the derived nozzle velocity coefficient that would yield the measured turbine performance.

The first program calculates the turbine gas flow from both the venturi measurements and from the perfect gas relationship for choked nozzles. Comparison of these two values yields the nozzle discharge coefficient. The turbine efficiency is then calculated based upon (1) the measured torque, speed, and venturi flow rate values and (2) the measured temperature difference. The second calculation is for a comparison only, since no attempt to obtain an adiabatic process was made. Other parameters such as the specific speed, specific diameter, torque coefficient, shaft horsepower, adiabatic head, velocity ratio, and wheel tip speed are also calculated.

The second program determines the apparent nozzle velocity coefficient based on the measured turbine performance. The nozzle coefficient is obtained by solving equation 2.1 after the hydraulic efficiency and the rotor coefficient are found. This procedure is complex and requires an iterative procedure since the rotor coefficient and some losses are dependent upon the nozzle coefficient. The hydraulic efficiency is obtained by adding the calculated parasitic losses to the measured turbine efficiency.

Obviously, the derived velocity coefficient is most accurate when the calculated losses and, hence, corrections are smallest. Therefore, the data were taken, when possible, at conditions which resulted in minimum parasitic losses and no Reynolds number correction. The reduced turbine efficiency is estimated as having an accuracy within 1%, those of gas properties and speed within 0.5%.

SECTION 6.0

TEST RESULTS

6.0 TEST RESULTS

A large amount of test data was obtained and reduced to evaluate the apparent nozzle velocity coefficient. Typical turbine efficiency and torque coefficient data for a design Mach number 2 conical nozzle is shown in Figures 24 and 25. The derived nozzle velocity coefficient is shown in Figure 26.

As may be noted in Figure 26, the velocity coefficient generally varies with U/C_o. Intuitively, one would expect the velocity coefficient of a nozzle to be dependent upon pressure ratio only. However, Stratford (11.3.20) reaches the conclusion that the nozzle pressure ratio adjusts itself to enter the rotor incidence-free. This adjustment of pressure ratio with speed would result in a variation in rotor reaction with speed (even with impulse turbines) causing the apparent nozzle velocity coefficient to change. Another cause of this variation is a possible slight error in parasitic losses or incidence losses. It was thus concluded that the most significant derived nozzle velocity coefficient is that obtained at the zero incidence operating point. This would correspond to the design speed and pressure ratio where the rotor would be least likely to affect the nozzle.

The velocity ratio at which zero incidence occurs is derived from a velocity diagram. From such a diagram, the gas angle relative to the blade is obtained as:

$$\tan \beta_g = \Psi_N C_o \sin \alpha / (\Psi_N C_o \cos \alpha - U) \tag{6.1}$$

If equation (6.1) is solved for U and divided by C_o, the result is:

$$U/C_o = \Psi_N (\cos \alpha - (\sin \alpha / \tan \beta_g)) \tag{6.2}$$

For incidence free operation the relative gas angle β_g must be equal to the entering blade angle β_1. Then the U/C_o at zero incidence is:

$$U/C_o \Big|_{i=0} = \Psi_N (\cos \alpha - \sin \alpha / \tan \beta_1)) \tag{6.3}$$

FIGURE 24
VARIATION OF TURBINE EFFICIENCY
WITH VELOCITY RATIO
CONICAL NOZZLE
DES. MACH. NO. = 2.0
AR = 1.7
C_D = 0.916

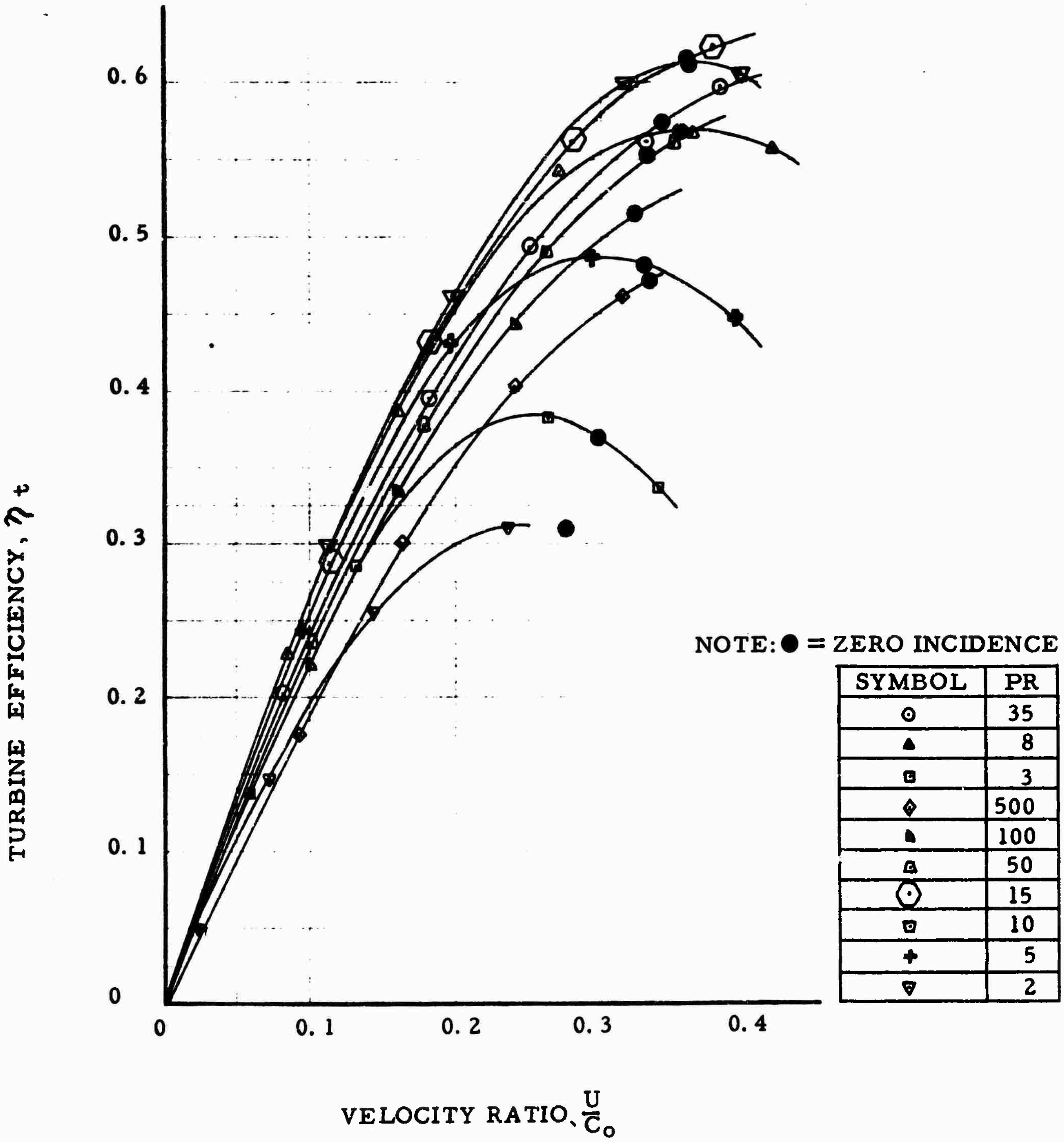


FIGURE 25
VARIATION OF TORQUE COEFFICIENT
WITH VELOCITY RATIO
CONICAL NOZZLE
DES. MACH NO. = 2.0
AR = 1.7
C_D = 0.916

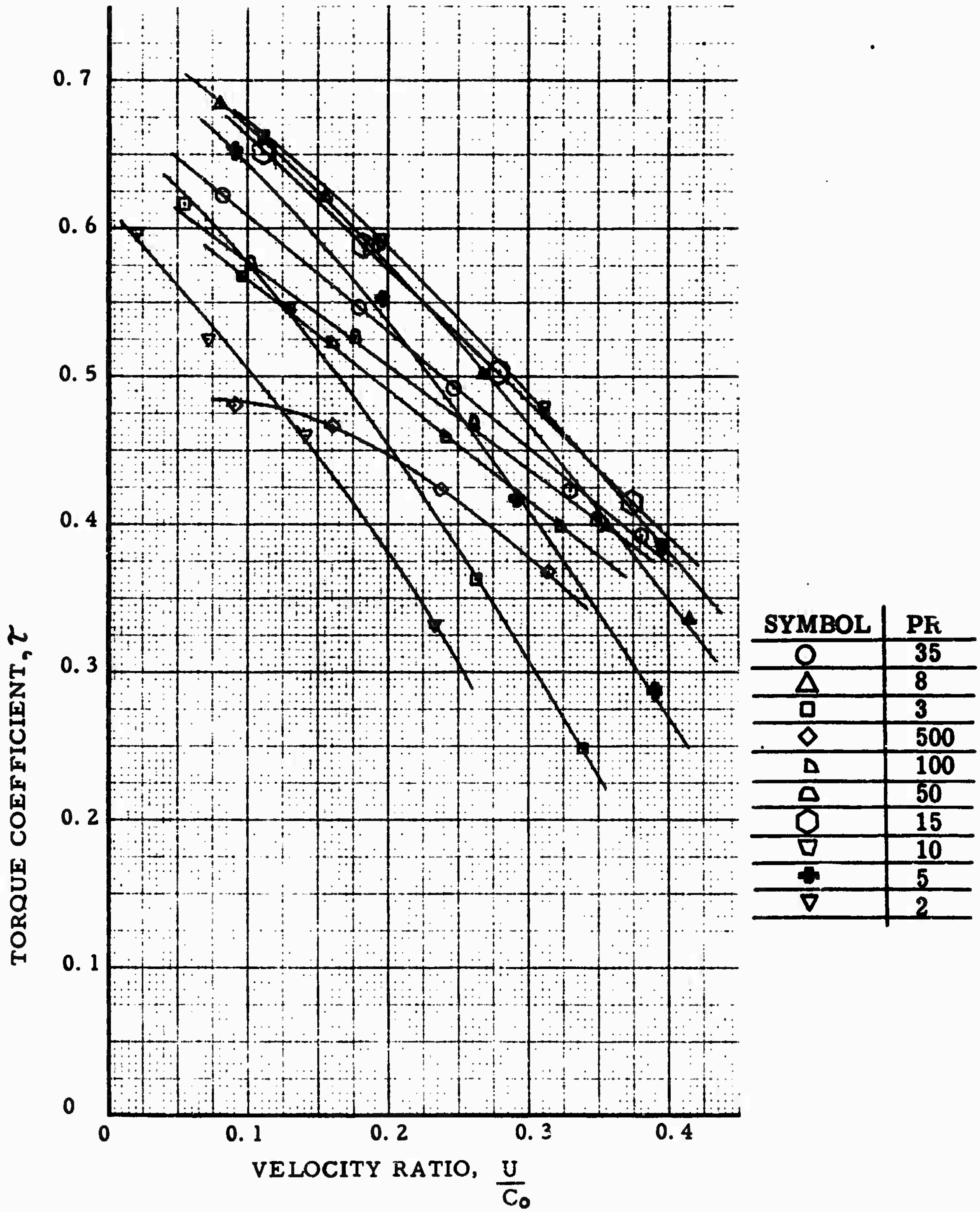
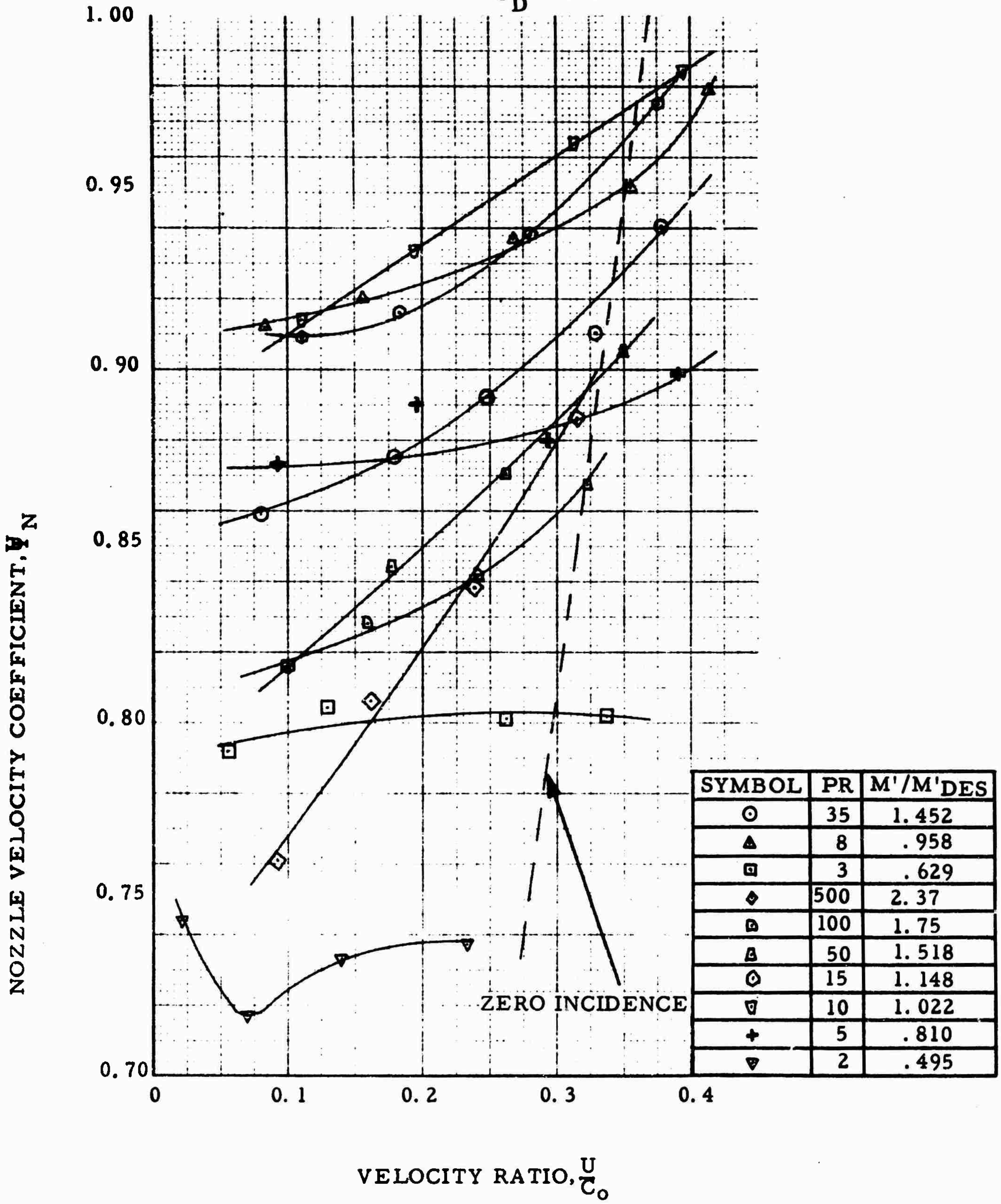


FIGURE 26
VARIATION OF NOZZLE VELOCITY COEFF.
WITH VELOCITY RATIO
CONICAL NOZZLE
DES. MACH NO. = 2.0
AR = 1.7
C_D = 0.916



SYMBOL	PR	M'/M'_{DES}
⊙	35	1.452
▲	8	.958
⊡	3	.629
◆	500	2.37
⧈	100	1.75
⧍	50	1.518
⬡	15	1.148
⛛	10	1.022
✚	5	.810
▼	2	.495

For all the tests in this study the entering blade angle was 25° and the nozzle angle was 16°, hence equation (6.3) reduces to:

$$U/C_o\Big|_{i=0} = .370\ \Psi_N \tag{6.4}$$

The line of zero incidence is shown on Figure 26, which results in a design nozzle velocity coefficient of 0.975 for this Mach 2 nozzle. It is interesting to note that zero incidence performance occurs near the peak efficiency at low pressures ratios as shown in Figure 24. This lends credence to the selection of the zero incidence point as the nominal value at each pressure ratio.

The design velocity coefficients for the nozzles tested in this study are shown in Figure 27. As expected, the contoured nozzle had the best design point performance with the conical and plug nozzles as close seconds. The free expansion nozzles have quite low coefficients. The variation in coefficient with area ratio is not significant since the absolute value of the derived coefficient is dependent upon a number of empirical coefficients which could cause such variation. However, a comparison of absolute values for nozzles of the same area ratio may be made. For comparison of different area ratio a comparison of velocity coefficient ratio removes any basic variation due to the empirical coefficients.

6.1 CONICAL NOZZLE PERFORMANCE

Seven conical nozzles were tested with the geometric parameters shown in Figure 20. The derived velocity coefficients for the Mach 2 and Mach 4 nozzles are given in Figures 28 and 29. The range of the derived coefficient as well as the coefficient at zero incidence is shown.

The discharge coefficient of the Mach 2 nozzle was unusually low (0.916 rather than 0.98), hence the optimum velocity coefficient occurred at an isentropic Mach number of approximately 2.1 rather than 2.0. This Mach number corresponds to an apparent area ratic of 1.83. The values were calculated based upon a design Mach number of 2.11 (noted in Figure 28), consistent with the calculation procedure in the literature (Section 3.0). This low discharge coefficient was probably due to the area ratio between the nozzle entrance and throat being approximately 2 (rather than a more normal area ratio of approximately 4), and by the throat approach having a sharp angular discontinuity. These "defects" were not present on the other test nozzles; the design Mach number was based upon the nozzle area ratio. Typical discharge coefficients for properly fabricated nozzles are 0.97 to 0.99. In some cases the coefficient may exceed 1.0 since it may be impossible to determine the nozzle throat area within 2 or 3 percent.

FIGURE 27
NOZZLE DESIGN VELOCITY
COEFFICIENTS FOR
NOZZLES TESTED IN PRESENT STUDY

SYM	TYPE
⊙	CONICAL
⊡	FREE EXPANSION
◇	PLUG
▲	CONTOURED

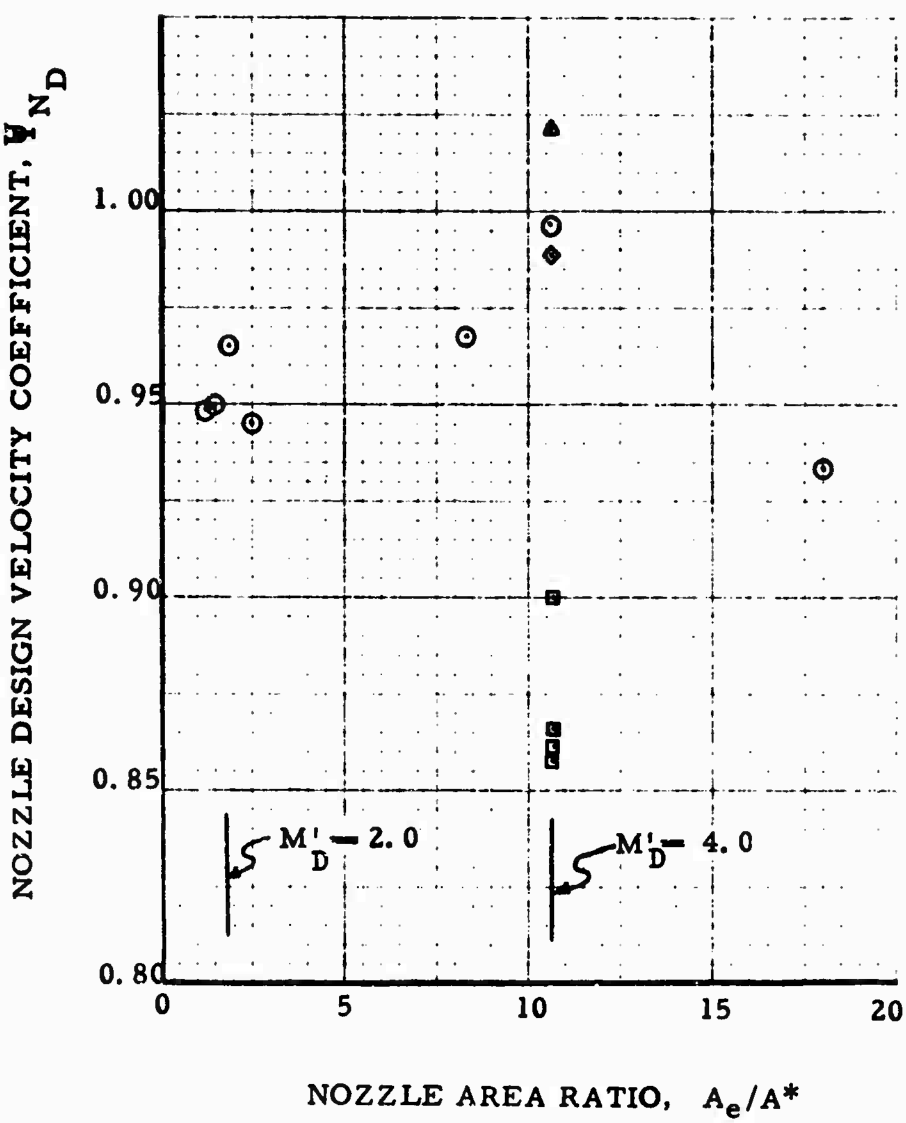


FIGURE 28
CONICAL NOZZLE DERIVED NOZZLE
VELOCITY COEFFICIENT

$M'_D = 2.01$ $\Psi_{N_D} = 0.975$ $C_D = 0.916$
$AR = 1.7$

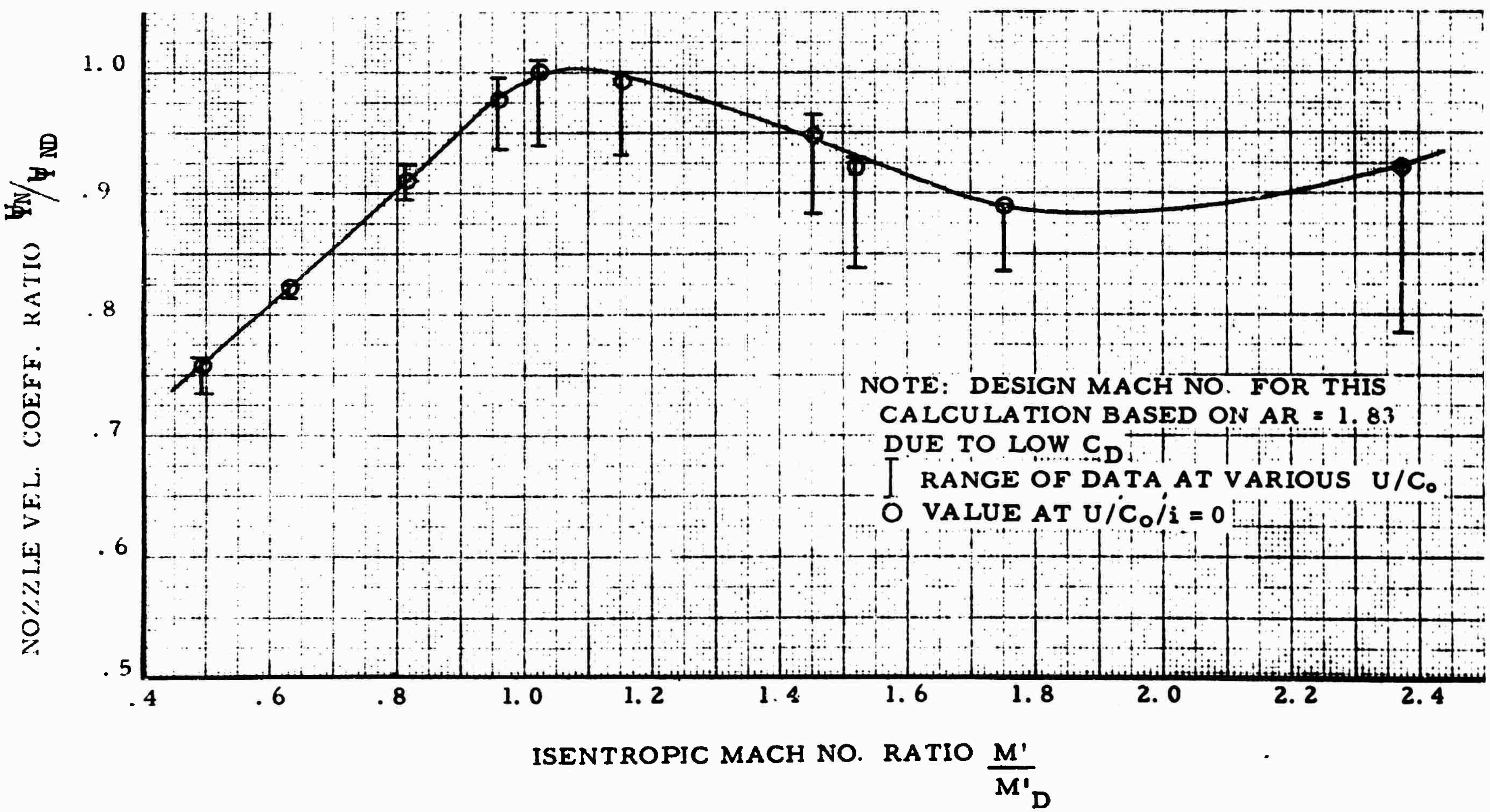
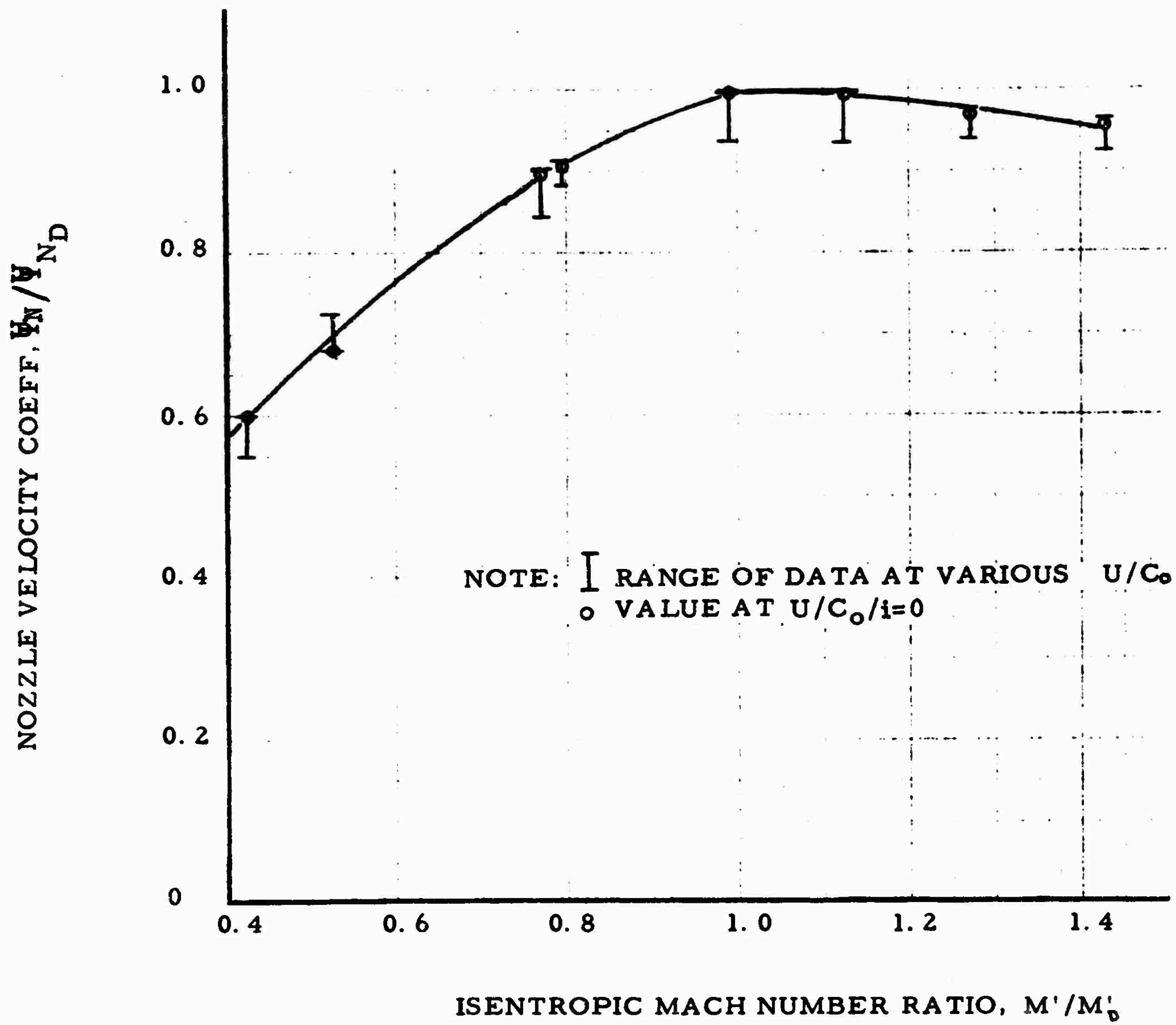


FIGURE 29
CONICAL NOZZLE DERIVED
NOZZLE VELOCITY COEFFICIENT

$M'_o = 4.0$ $\quad C_D = 0.99$
$AR = 10.7$
$\Psi_{ND} = 0.996$



A summary of all the conical nozzle data is presented in Figure 30. The basic agreement between these data and those of Keenan (11.2.3) shown in Figure 4 is remarkable considering the different test techniques. A comparison of the conical nozzle data with the theoretical analysis is shown in Figure 31. The agreement is good at design Mach numbers up to approximately 2.5; however, the theory appears to be somewhat pessimistic at design Mach numbers above 2.5.

Another interesting result is shown in Figure 32 where the test results of two conical nozzles are compared. Both nozzles were designed for a Mach number of 4; however, the cone half angles are 12 and 51 degrees. Here again, at very low Mach number ratios, the nozzle with the large cone angle outperforms the nozzle with the lower cone angle. This same result was found in the rocket nozzle test results shown in Figures 5 and 7.

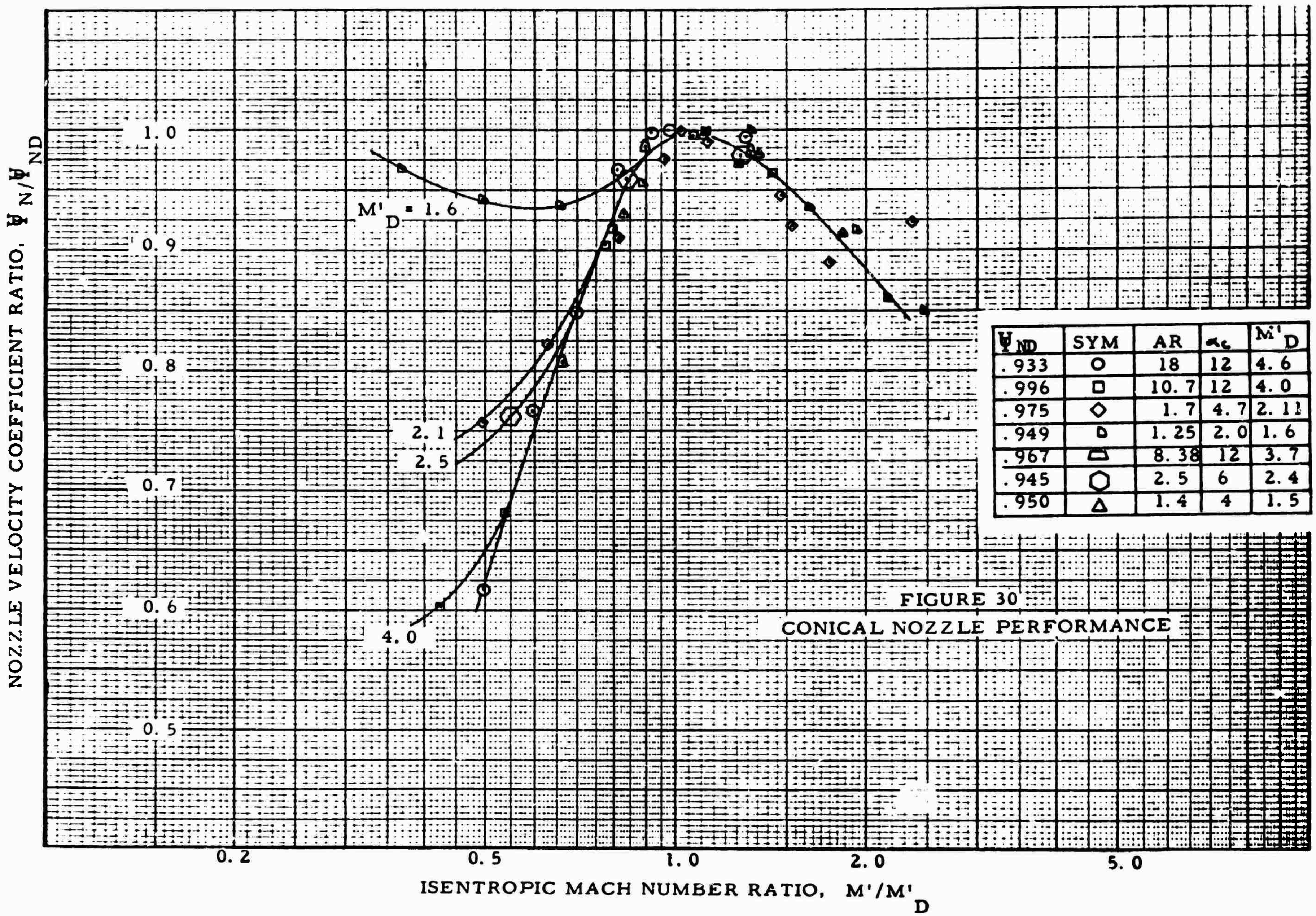
These data indicate the need for further investigation into the effects of cone half angle on off-design nozzle performance. The rocket nozzle data of Figure 7 indicates that off-design improvement may be attained with contoured nozzles with a smaller decrease in design point performance than occurs in conical nozzles. For example, nozzles which perform well at isentropic Mach numbers ratios around 0.4 have a design nozzle coefficient of approximately 0.94 for contoured nozzles as compared to 0.86 for the conical nozzle (Figure 32).

6.2 CONTOURED NOZZLE PERFORMANCE

The contoured nozzle is designed for expansion shock cancellation and axial flow at the exit plane. This nozzle resulted in the highest velocity coefficient of 1.021 (Figure 33). This high value resulted from a design point turbine efficiency of 0.635 as compared to 0.60 for the Mach 4 conical nozzles. Because of the small area ratio at Mach 2.0 (AR = 1.7), it was felt that an insignificant difference would be measured between the contoured and conical nozzles; therefore, only the Mach 4 design was fabricated. The nozzle contour is described in Figure 21.

Comparing the performance of the low divergence angle contoured and conical nozzles, it is noted that the velocity coefficient at low Mach number ratios is lower for the contoured nozzle than for the conical. For example, at an isentropic Mach number ratio of 0.42 the contoured nozzle velocity coefficient ratio is only 69 percent of the corresponding conical nozzle velocity coefficient ratio. This is significantly lower than the 2.5 percent increase in the nozzle design velocity coefficient. This effect was also noted by Farley and Campbell (11.3.15) during contoured rocket nozzle tests.

The performance of contoured nozzles at Mach number ratios greater than 1.0 is essentially identical to conical nozzles. In fact, all nozzle types appear to have equal performance at greater than design pressure ratios. Hence, it is recommended that contoured nozzles be used for



ψ_{ND}	SYM	AR	α_c	M'_D
.933	○	18	12	4.6
.996	□	10.7	12	4.0
.975	◇	1.7	4.7	2.11
.949	⊃	1.25	2.0	1.6
.967	⏢	8.38	12	3.7
.945	⬡	2.5	6	2.4
.950	△	1.4	4	1.5

FIGURE 30

CONICAL NOZZLE PERFORMANCE

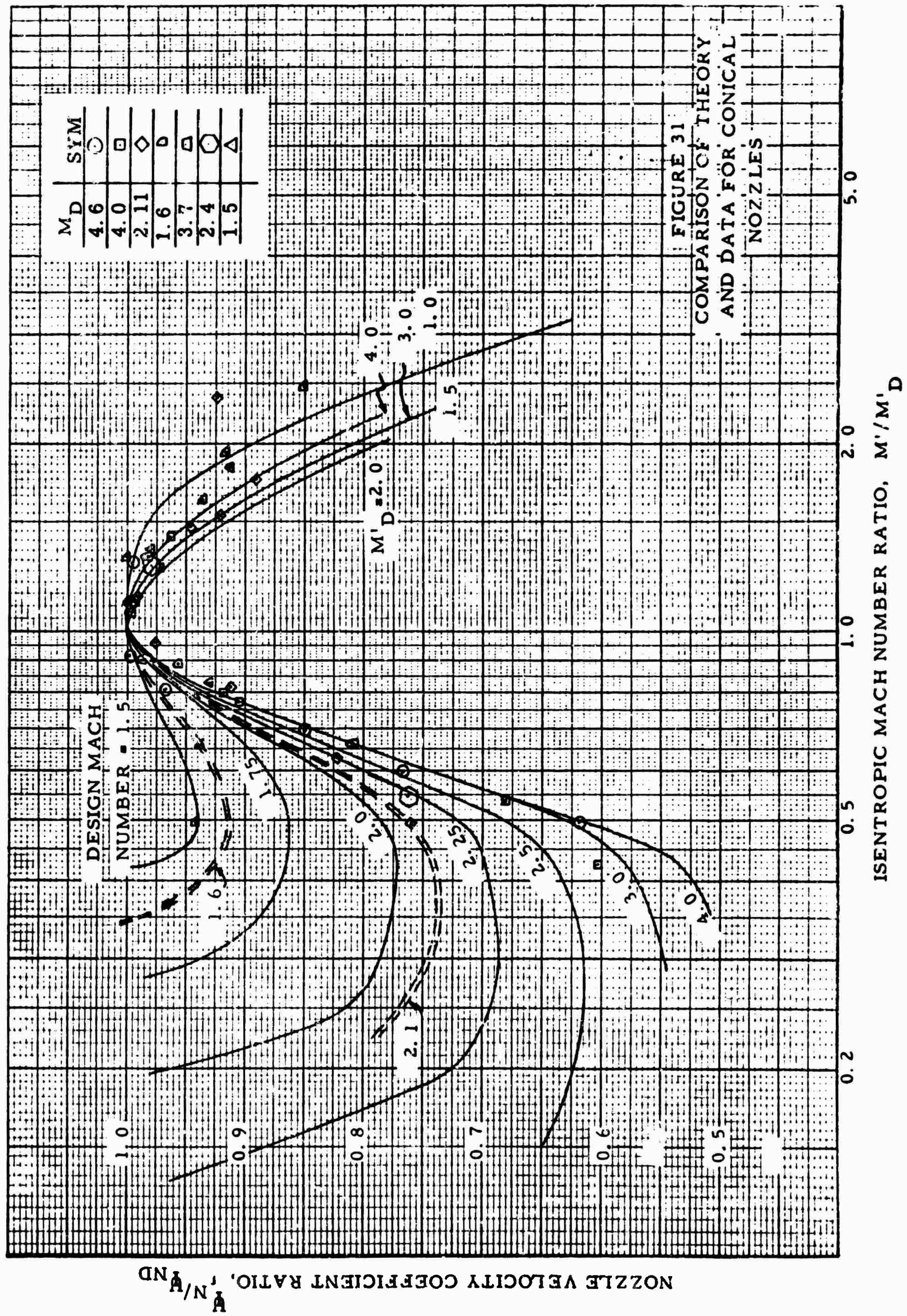


FIGURE 31
COMPARISON OF THEORY AND DATA FOR CONICAL NOZZLES

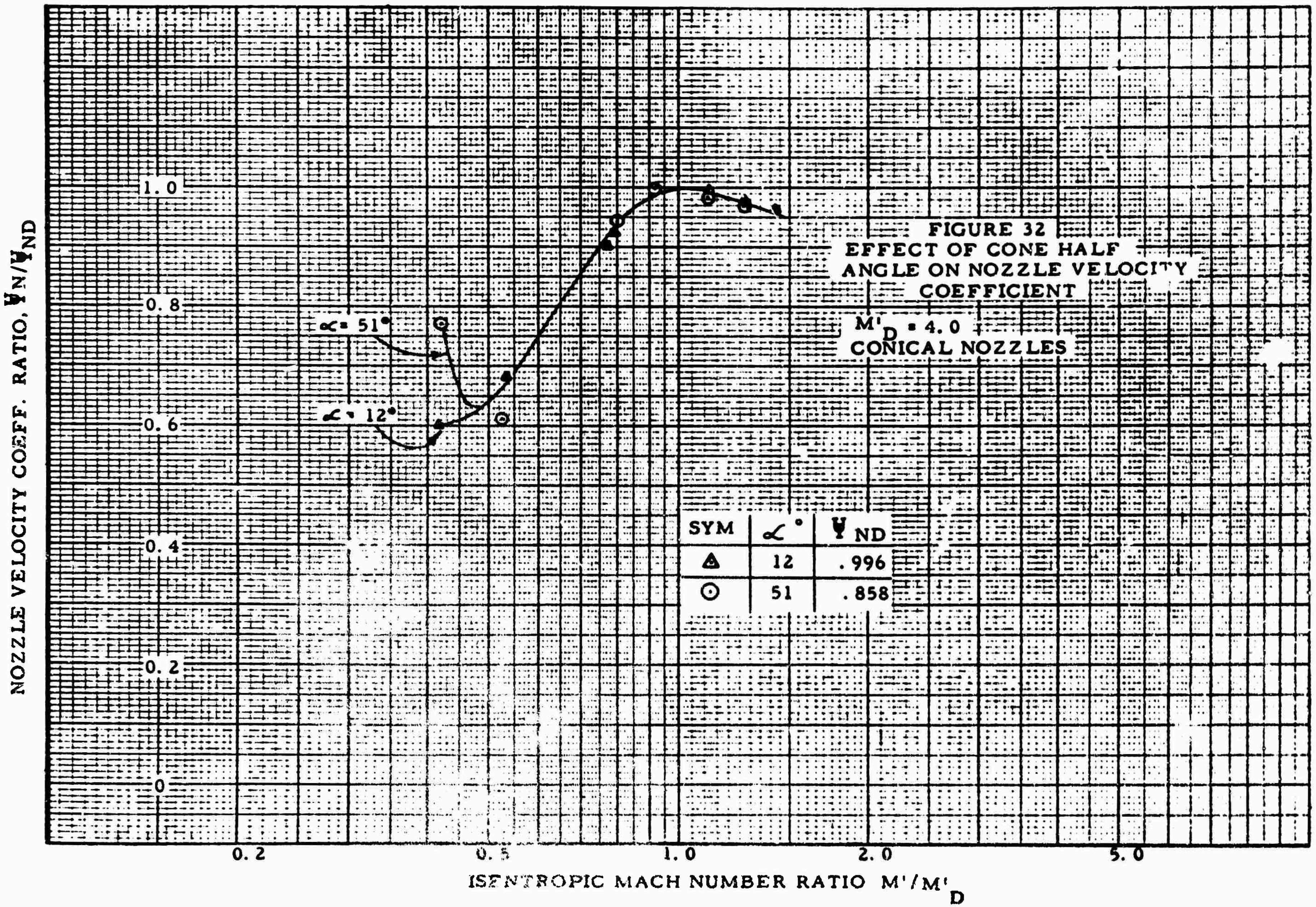
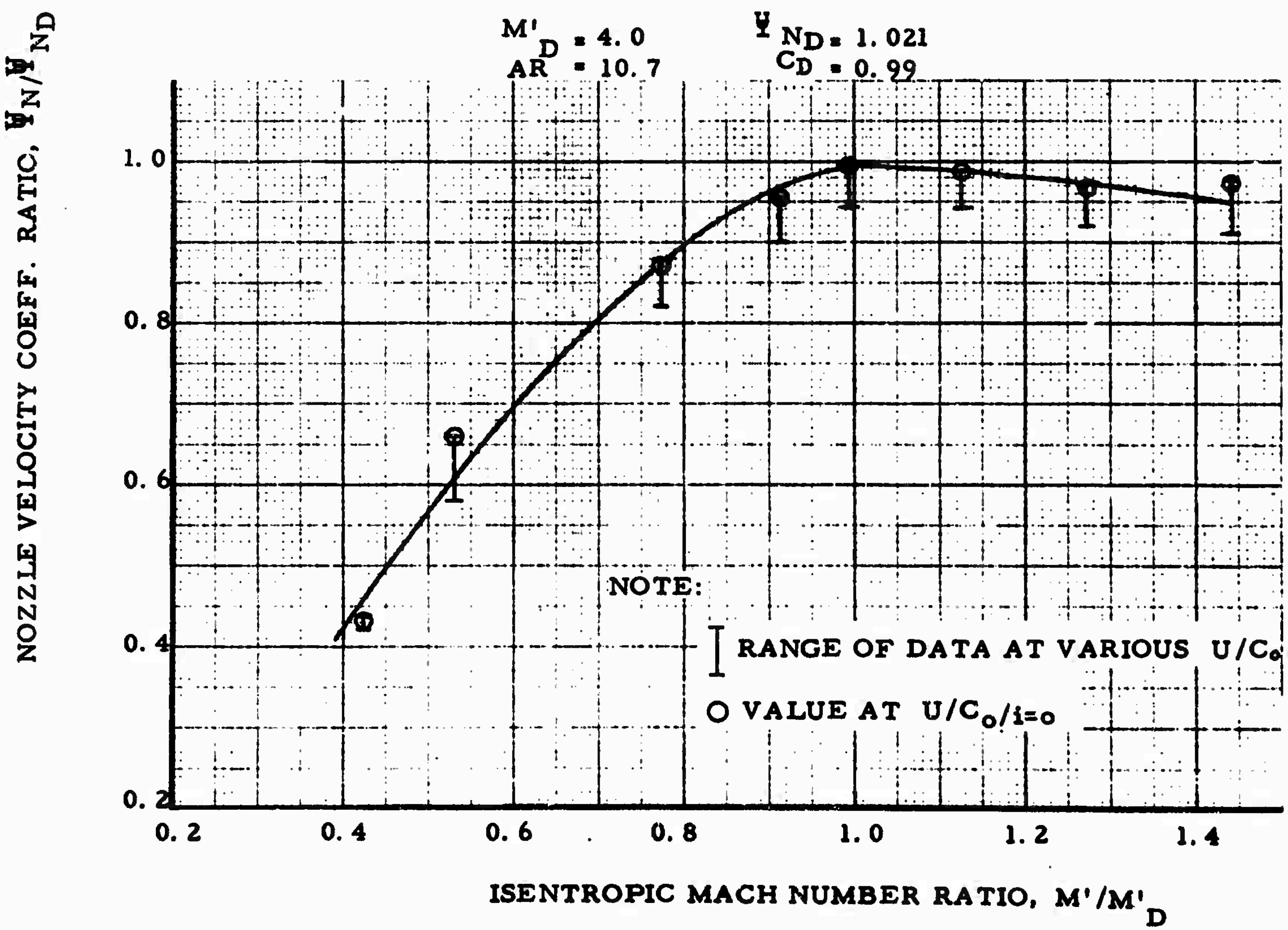


FIGURE 32
EFFECT OF CONE HALF
ANGLE ON NOZZLE VELOCITY
COEFFICIENT

FIGURE 33
CONTOURED NOZZLE DERIVED NOZZLE
VELOCITY COEFFICIENT



turbines that operate over ranges from slightly below design to greater than design pressure ratios, as other types have better performance at far below design pressure ratios.

6.3 PLUG NOZZLE PERFORMANCE

The plug nozzle was included in this study since it is naturally adapted to a slanted configuration, and since it showed good below-design pressure ratio performance as a rocket nozzle. The plug contour is designed to cancel the Prandlt-Meyer expansion fan originating from the throat. The configuration is described in Figure 22.

The plug nozzle had generally good performance (Figure 34). The design coefficient is one percent less than that of the conical nozzle, and the velocity coefficient ratio is essentially equal to that of the conical at off-design pressure ratios. As noted in Figure 34 this data may be conservative since the discharge coefficient is high, i.e., 1.06. This may be caused in part by inaccuracy in measurement of the nozzle throat area. However, this inaccuracy should be only 2 or 3 percent; the other 4 or 5 percent could be caused by leakage. Unfortunately, the turbine test laboratory was dismantled for reassembly in a different location which prevented re-testing of this configuration.

The effect of leakage is to decrease the value of turbine efficiency and hence the derived nozzle velocity coefficient, since the flow through the turbine is less than that shown by the venturi. The effect of leakage on the values of Figure 34 would be primarily a change in the design nozzle velocity coefficient.

This nozzle type indicates promise as a turbine nozzle since these first tests resulted in performance which was nearly as good as any other type. The geometry of this nozzle could be quite significant, particularly that in the throat area. Also, the relationship between the nozzle exit rectangle and the rotor blades is probably quite important. Optimization of this nozzle type may result in a significant improvement in off-design turbine performance. The advantage gained in rocket performance was not realized in these first turbine tests.

6.4 FREE EXPANSION NOZZLE PERFORMANCE

Four free expansion nozzles were fabricated for a design Mach number of 4. These nozzles had expansion lengths of 0.1, 0.5, 0.75, and 1.0 inch as shown in Figure 23. The data of Barakauskas (11.2.6) indicates that the free expansion length of 0.75 inch should be optimum. The test data for that particular nozzle are presented in Figure 35. These data are typical of all the free expansion nozzles. The nozzle design velocity coefficient and the velocity coefficient ratio were lower than those of the other nozzle types. The shortest free expansion length, 0.1 inch, resulted in the best off-design performance as shown in Figure 36; however, the design nozzle coefficient was only 0.86.

FIGURE 34
PLUG NOZZLE DERIVED NOZZLE
VELOCITY COEFFICIENT

$M'_D = 4.0$ $\quad$ $\Psi_{N_D} = 0.988$
$AR = 10.7$ $\quad$ $C_D = 1.06$*

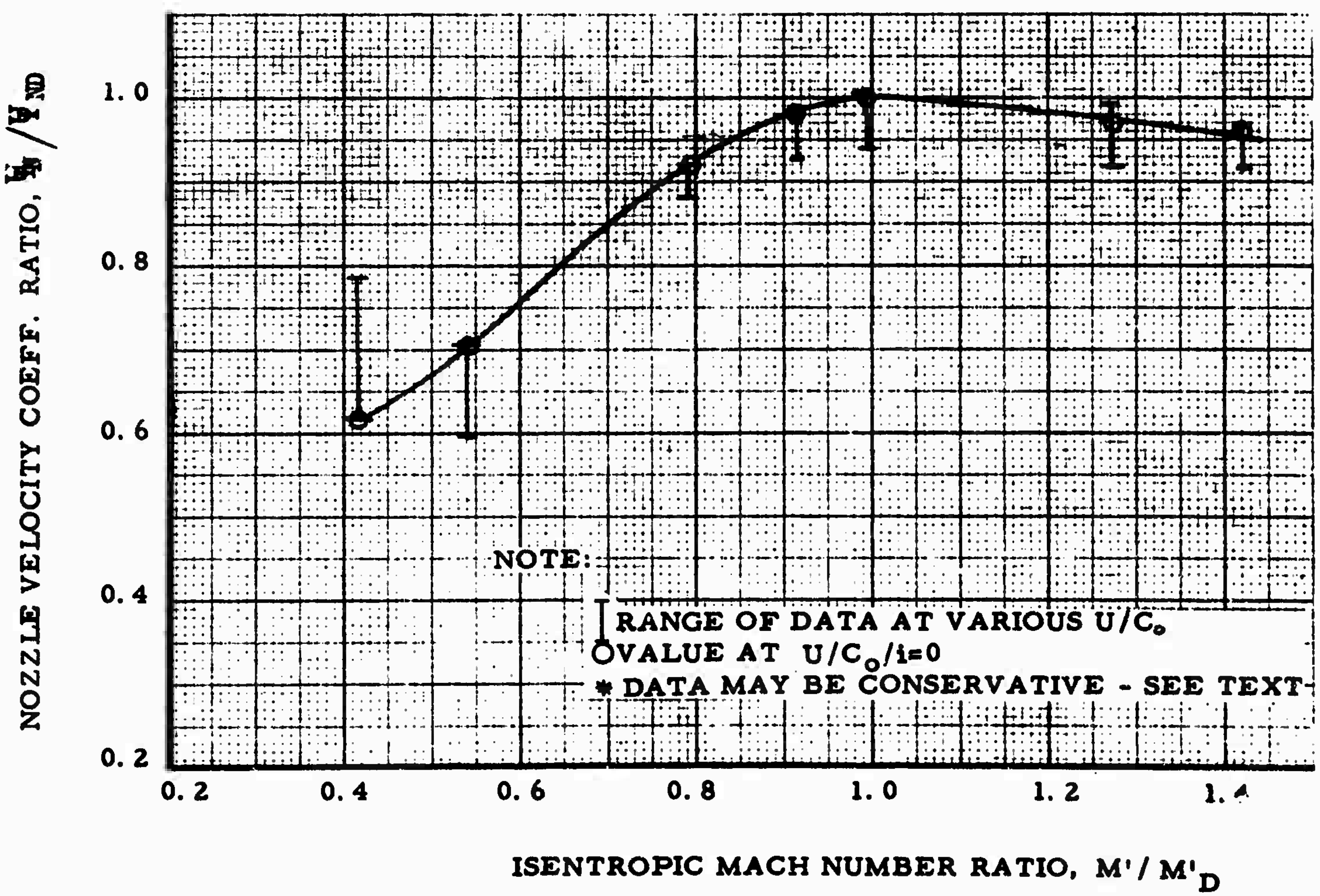
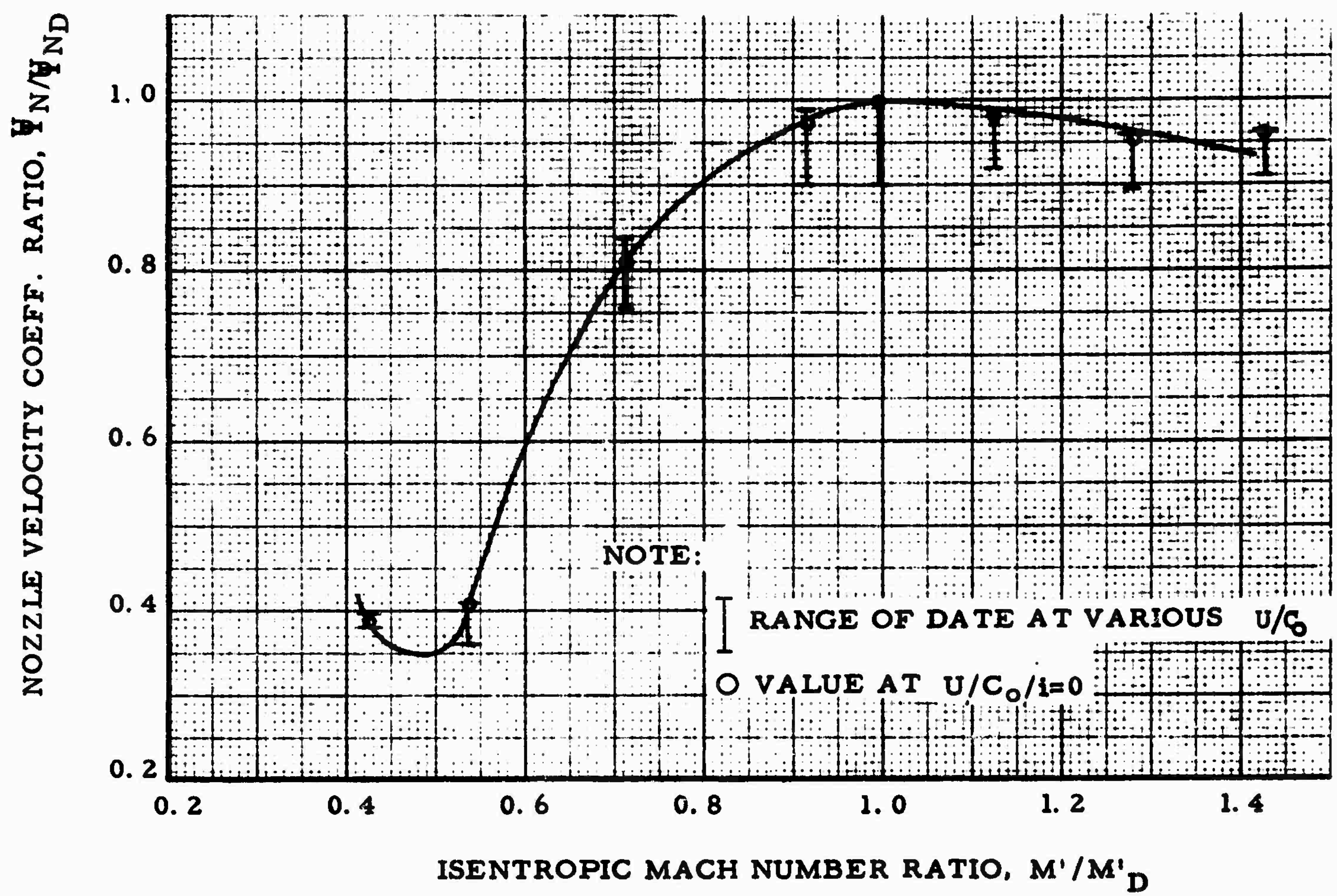


FIGURE 35
FREE EXPANSION NOZZLE DERIVED
NOZZLE VELOCITY COEFFICIENT
EXPANSION LENGTH $L^* = 0.75$ IN

$M'_D = 4.0$ $\quad \Psi_{ND} = 0.900$
$AR = 10.7$ $\quad C_D = 0.98$



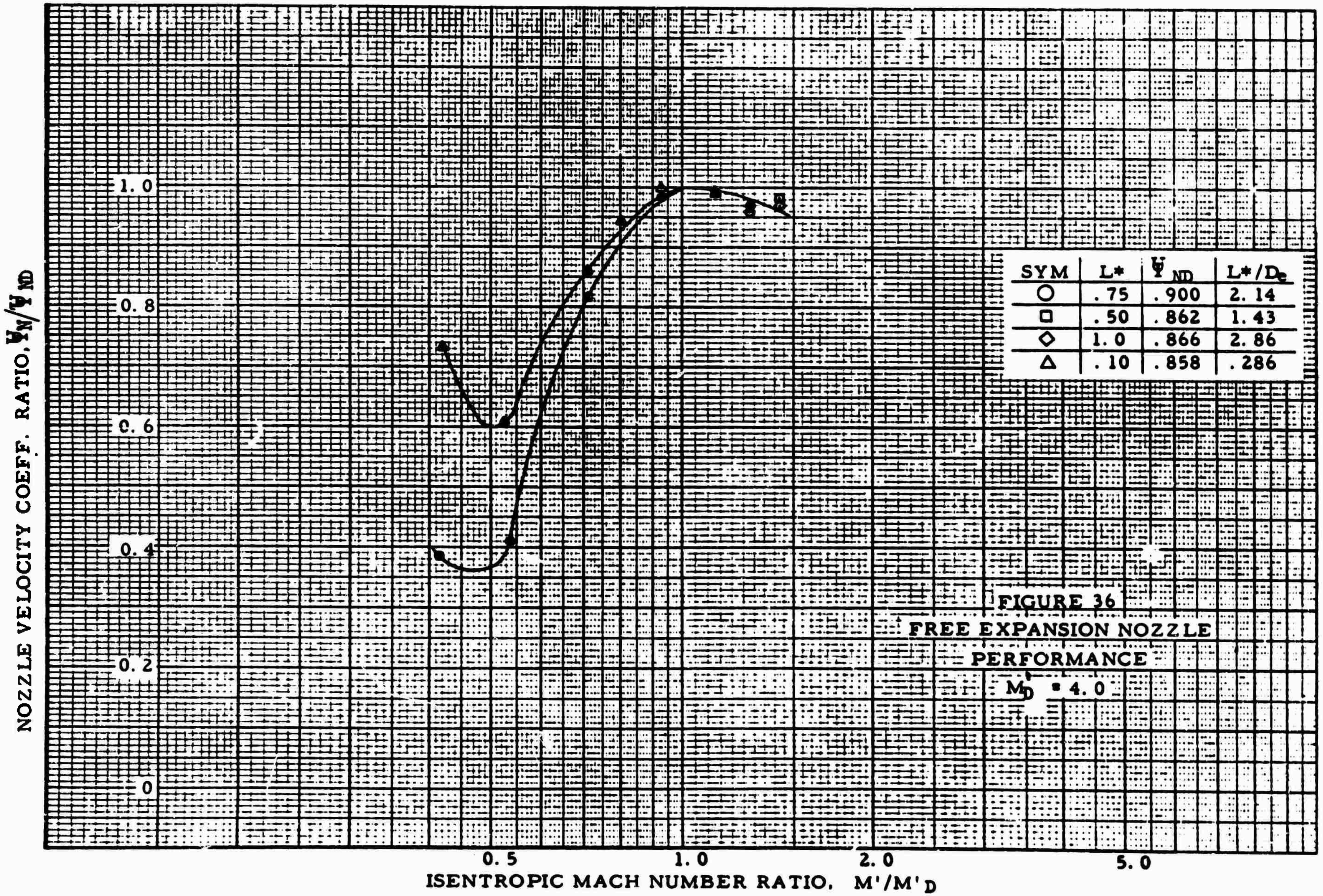


FIGURE 36
FREE EXPANSION NOZZLE
PERFORMANCE
$M'_D = 4.0$

The nozzle design velocity coefficient is very dependent on free expansion length as shown in Figure 37. The data of this study agree with those of Barakauskas as he also found optimum design point performance with a free expansion length of approximately 2 exit diameters.

The general performance of this nozzle type is inferior to the other nozzle types, and since the fabrication advantage is small it is recommended that in general some other nozzle type be utilized.

6.5 COMPARISON OF NOZZLE TYPES

A comparison of the data for all the Mach 4 nozzles tested is presented in Figure 38. In this figure it is apparent that at below-design pressure ratios the plug and conical nozzle have essentially the same performance while the contoured and the free expansion nozzles show reduced performance. At above-design pressure ratios, the conical, plug, and contoured nozzles have equivalent performance; the free expansion nozzle is somewhat lower. Note that the theoretical curve of Section 4.0 agrees well with the experimental data.

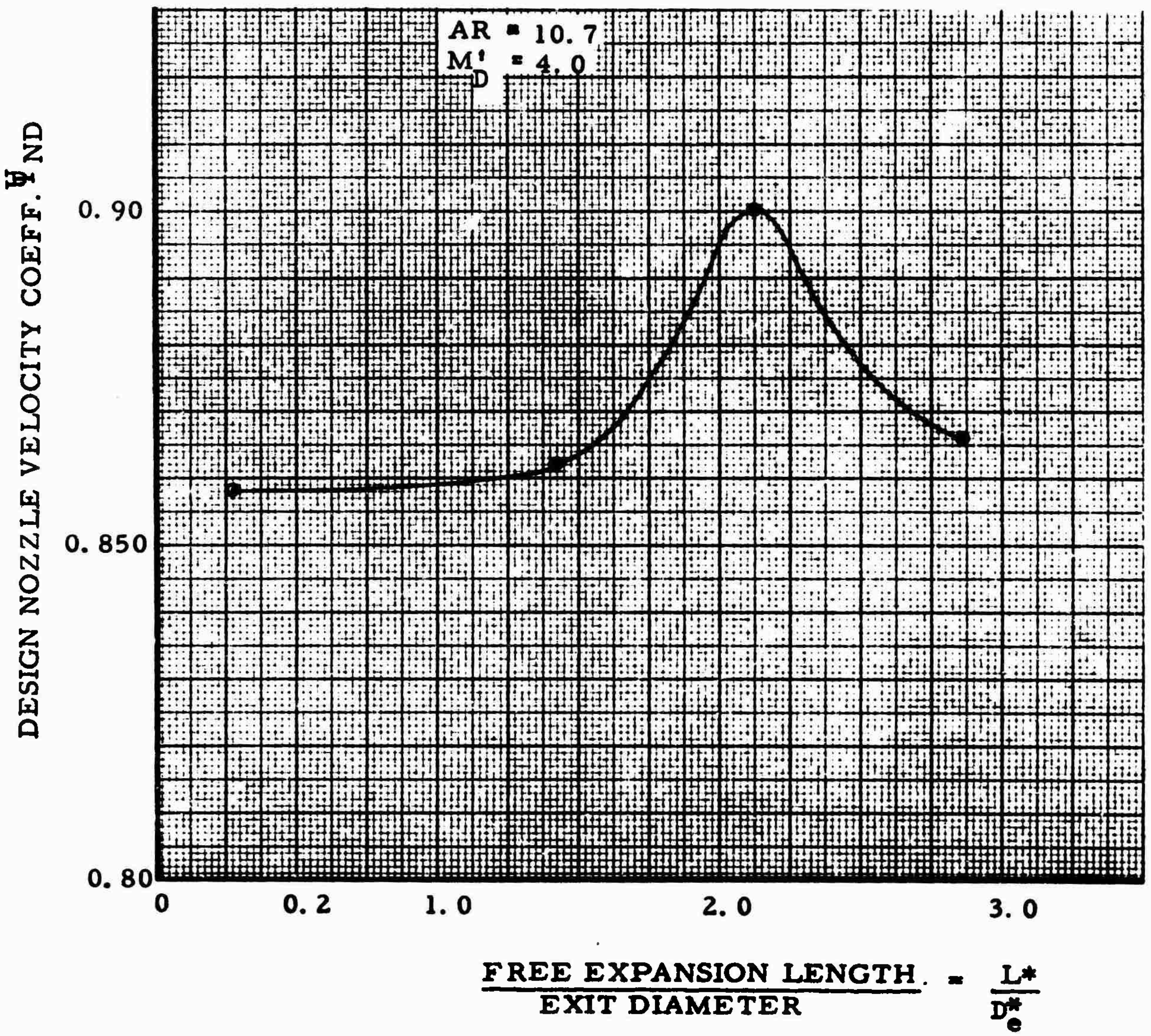
There is a significant variation in the design velocity coefficient for the different nozzle types. A curve of the nozzle coefficient has a considerably different character than does the coefficient ratio as shown in Figure 39. This shows that the contoured nozzle is superior at Mach number ratios above 0.6 rather than 1.0 as indicated in Figure 38. It is interesting to note that the free expansion nozzle has the highest nozzle coefficient at the Mach number ratio of 0.425 even with a very low design point coefficient.

When comparing performance of nozzles with different area ratios, a correlation of velocity coefficient versus isentropic Mach number is useful. Figure 40 gives the test data for three conical nozzles in this manner. Curves of this type are helpful in determining the optimum nozzle area ratio for a particular operational range of isentropic Mach numbers. The nozzle performance decreases more rapidly at below-design pressure ratios than at above-design pressure ratios. Hence, in general, the design Mach number should be selected near the lower end of the operating Mach number range. To accurately determine the optimum nozzle, the turbine operational time at each pressure ratio must be used to evaluate a time-weighted-average for each nozzle area ratio being considered. The theoretical procedure provides the information necessary to generate a family of curves which allows the calculation of the time-weighted nozzle coefficient as a function of nozzle area ratio in order to determine the optimum nozzle area ratio for a particular application.

6.6 EFFECT OF AXIAL SPACING

Spacing between the nozzle exit and the rotor provides a free expansion area where the nozzle exit flow can adjust to the exit pressure before entering the rotor. It has been shown in the literature that axial clearance has little effect at the design point. However, it was felt that the free expansion area may improve nozzle performance at greater than design pressure ratios.

FIGURE 37
VARIATION OF DESIGN NOZZLE
VELOCITY COEFFICIENT WITH
FREE EXPANSION LENGTH



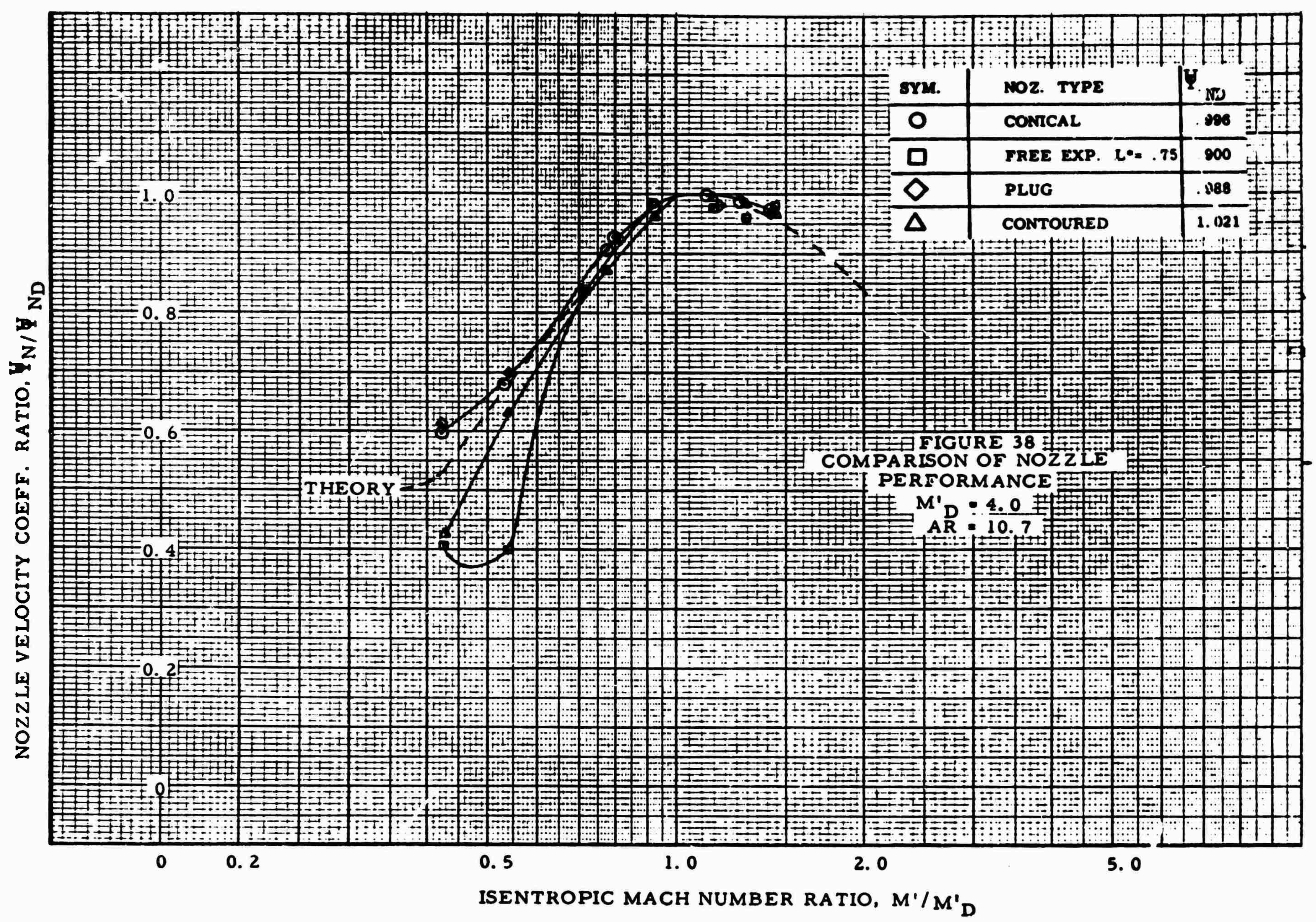


FIGURE 38
COMPARISON OF NOZZLE PERFORMANCE

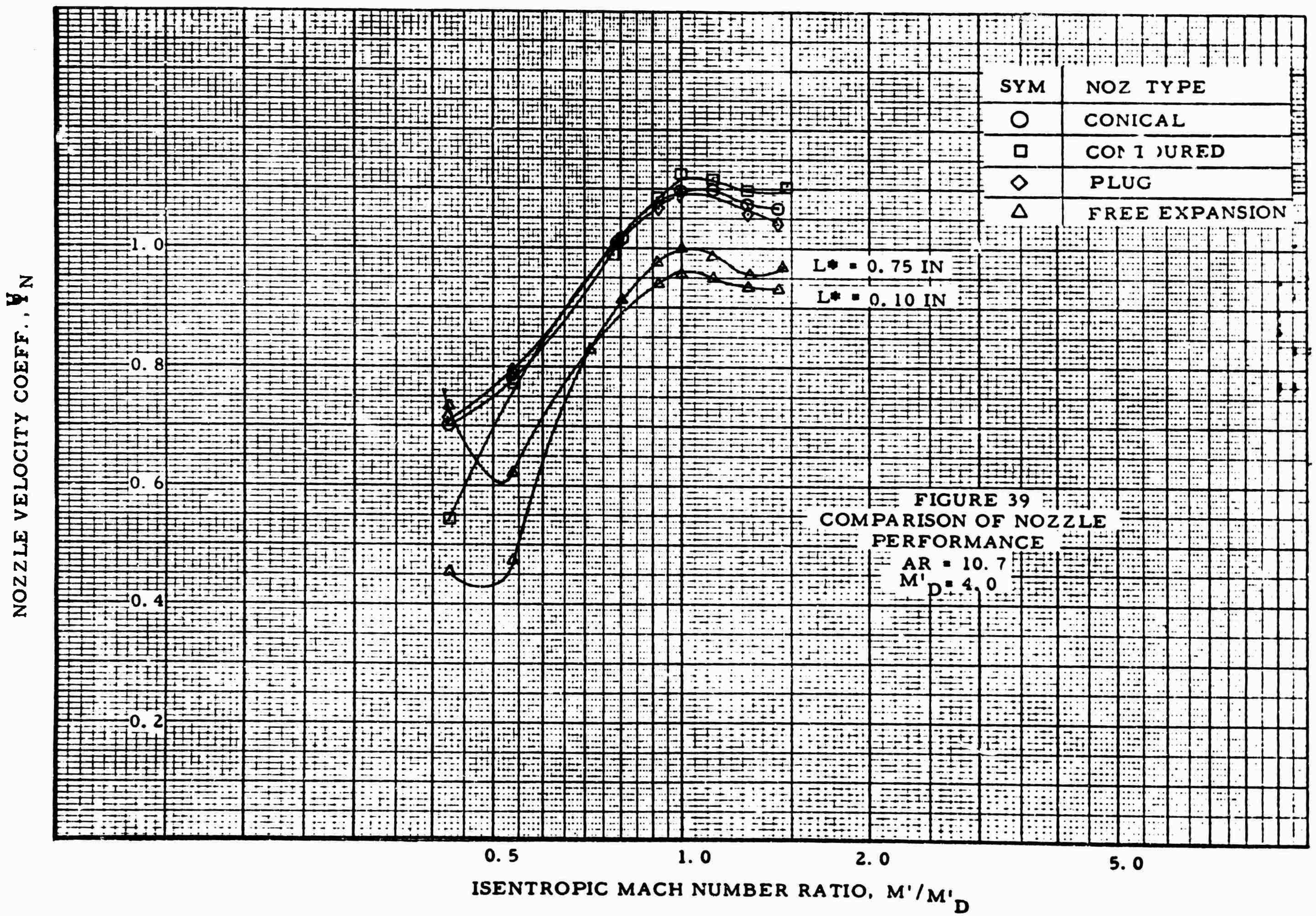


FIGURE 39
COMPARISON OF NOZZLE PERFORMANCE

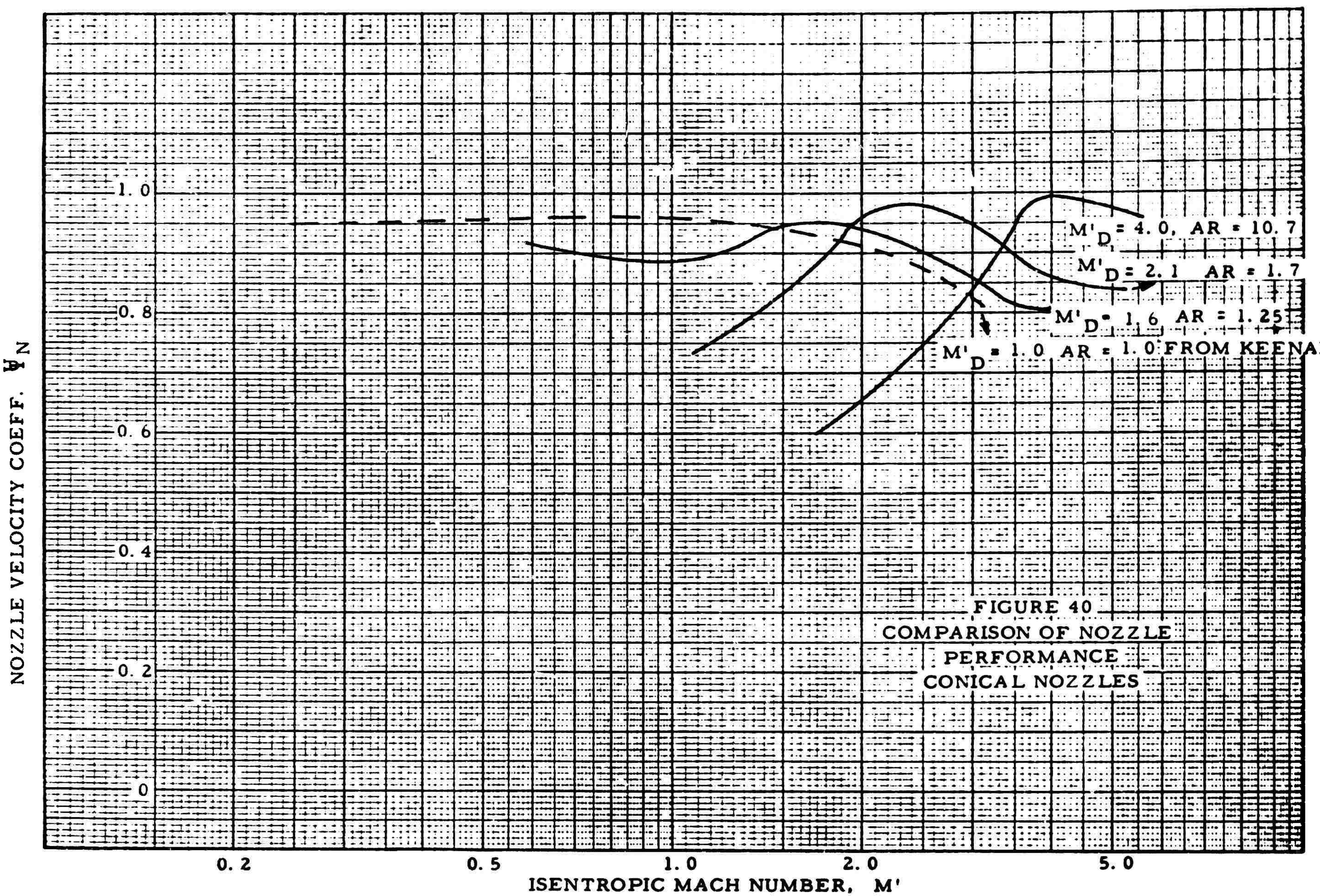


FIGURE 40
COMPARISON OF NOZZLE
PERFORMANCE
CONICAL NOZZLES

The conical and plug nozzles designed for Mach 4.0 were tested at three axial spacings, 0.017, 0.035, and 0.055 inch. The results of these tests are shown in Figures 41 and 42. The conical nozzle data shows no significant effect of axial spacing. The plug nozzle data indicates an improvement in turbine efficiency for larger axial clearance from slightly below design pressure ratio to the highest pressure ratio measured. However, this small increase is not adequate to define a significant trend.

6.7 EFFECT OF GAS RATIO OF SPECIFIC HEATS

Tests were made with identical turbine geometry while operating on nitrogen and Freon 12 to evaluate the effects of gas ratio of specific heats. Two conical turbine nozzles with area ratios of 18 were used. Two adjacent nozzles were used because the maximum test speed of the turbine when using Freon 12 was too low with a single nozzle. The turbine was tested at the same isentropic Mach number ratios with each gas.

A summary of the test results is presented in Table II. It is interesting to note that the design point efficiency was identical with both gases. However, the off-design performance on Freon was lower than on nitrogen. This result was predicted during the theoretical analysis and the effect is shown in Figure 13 for a nozzle with design Mach number of 2.0. As may be seen in Table II the theory correlates well with the measured data.

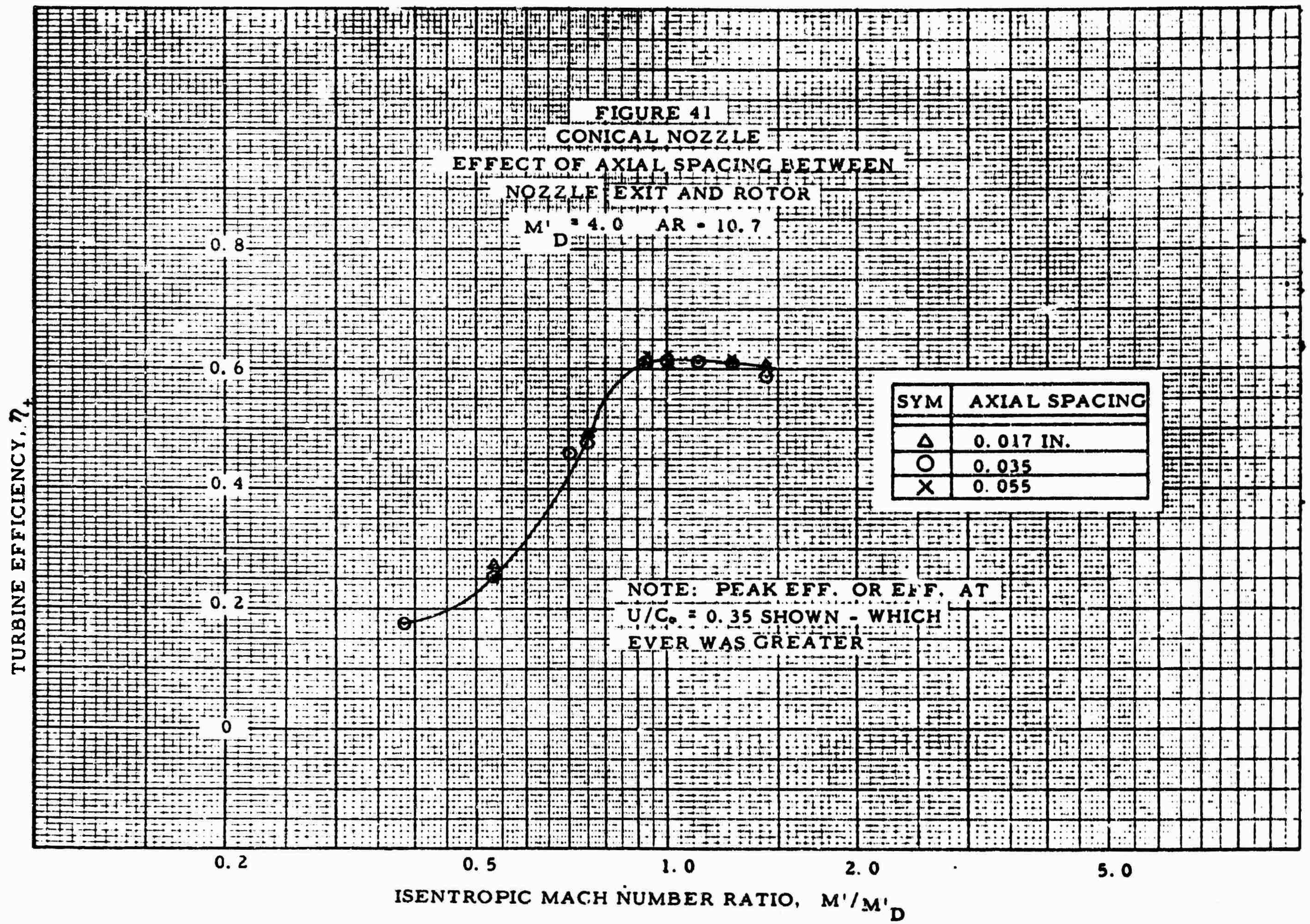


FIGURE 41
CONICAL NOZZLE
EFFECT OF AXIAL SPACING BETWEEN
NOZZLE EXIT AND ROTOR
$M'_D = 4.0$ AR = 10.7
SYM
AXIAL SPACING
△
0.017 IN.
○
0.035
×
0.055
NOTE: PEAK EFF. OR EFF. AT
U/C_o = 0.35 SHOWN - WHICH
EVER WAS GREATER
0.8
0.6
0.4
0.2
0
TURBINE EFFICIENCY, η_t
0.2
0.5
1.0
2.0
5.0
ISENTROPIC MACH NUMBER RATIO, M'/M'_D

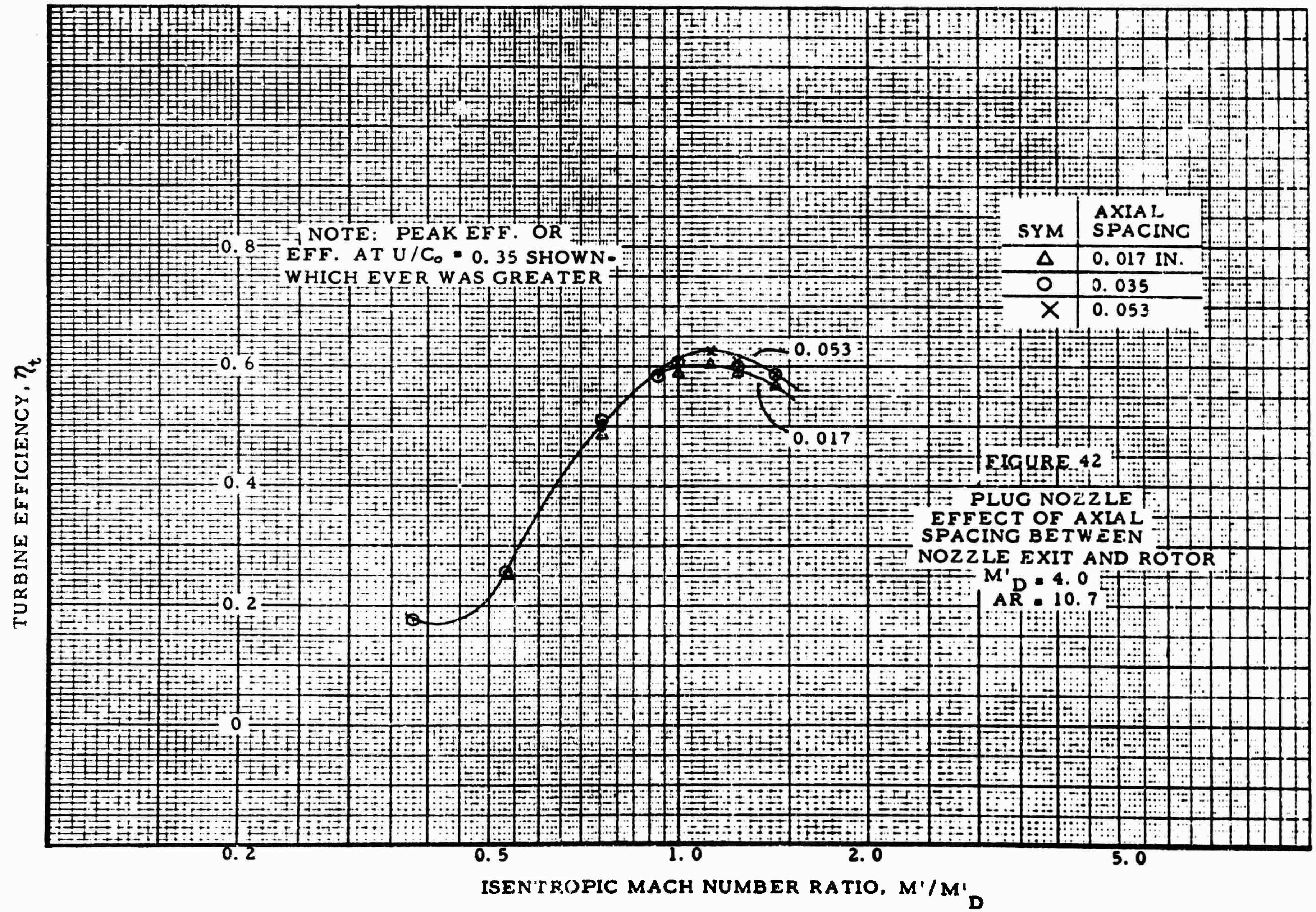


FIGURE 42
PLUG NOZZLE
EFFECT OF AXIAL SPACING BETWEEN NOZZLE EXIT AND ROTOR
$M'_D = 4.0$
AR = 10.7

TABLE II

EFFECT OF GAS RATIO OF SPECIFIC HEATS
ON TURBINE NOZZLE PERFORMANCE

AR = 18
C_D = 1.01
2 Nozzles

	DATA						THEORY	
	PR	η_T*	M'	M'/M'_D	Ψ_N	Ψ_N/Ψ_{ND}	K=1.4 Ψ_N/Ψ_{ND}	K=1.13 Ψ_N / Ψ_{ND}
FREON-12	50	0.520	2.98	0.86	0.897	0.935		0.94
	139	0.625	3.46	1.0	0.96	1.0		1.0
	300	0.600	3.80	1.099	0.94	0.978		0.985
NITROGEN	140	0.605	3.94	0.856	0.938	0.991	0.965	
	323	0.625	4.60	1.0	0.946	1.0	1.0	
	500	0.625	4.95	1.075	0.946	1.0	0.995	

*Turbine Efficiency at U/C_o = 0.35

SECTION 7.0

CONCLUSIONS

7.0 CONCLUSIONS

1. Turbines operating over a range from slightly below to far above design pressure ratio should employ contoured, shock cancellation nozzles to obtain best performance.

2. Turbines operating over a range of pressure ratios from far below to slightly above design pressure ratio should use conical or plug type nozzles to obtain best performance.

3. Turbines operating near design pressure ratio at all times should use contoured nozzles.

4. Axial spacing between nozzle and rotor has little effect on performance.

5. The performance of nozzles operating at off-design pressure ratios can be theoretically predicted by a combination oblique and normal shock calculation procedure.

6. Qualitative and quantitative similarity exists between turbine and rocket nozzle performance when operating at off-design pressure ratios.

7. Divergent nozzles with large half-cone angles may have improved performance over nozzles with low half-cone angles at below-design pressure ratios.

8. Limited data and the theoretical analysis indicates that the performance of nozzles (at equal design Mach number and isentropic Mach number ratio), when operating off-design, decreases as the gas specific heat ratio is decreased.

SECTION 8.0

RECOMMENDATIONS

8.0 **RECOMMENDATIONS**

Results of the present effort indicate that further study in the following areas could result in a significant contribution to the advancement of turbine nozzle design.

1. Effect of nozzle divergent half-cone angle. The type studied should be the contoured type with various nozzle throat angles. At least two area ratios should be examined.

2. Plug nozzle geometric parameters. Results of this study indicate that improvement in performance may be obtained if the geometric parameters of the plug nozzle were optimized.

3. Effect of gas ratio of specific heat. The theoretical analysis and the three data points measured indicate a large decrease in nozzle performance at off-design pressure ratio with gases with a low ratio of specific heat. Since many current turbines are operating on gases with specific heat ratios as low as 1.02, such a study would be timely. At least two nozzle area ratios should be tested on gases with ratio of specific heats from 1.667 to 1.02. These gases could be:

Argon	$\gamma = 1.667$
Nitrogen	$\gamma = 1.40$
CO_2	$\gamma = 1.30$
Freon 12	$\gamma = 1.12$
FC-75	$\gamma = 1.02$

SECTION 9.0

LIST OF SYMBOLS

9.0 LIST OF SYMBOLS

A	nozzle area	in.^{2}
AR	nozzle area ratio	
a_t	speed of sound based on total temperature	$\frac{ft}{sec}$
C_D	nozzle discharge coefficient	$\frac{\dot{w}\text{ - theoretical}}{\dot{w}\text{ - measured}}$
C_o	isentropic spouting velocity	ft/sec
D_e	diameter of nozzle exit	in.
D_1	diameter of nozzle entrance	in.
D^*	diameter of nozzle throat	in.
F	resultant corrected thrust	lbs.
i	rotor entering incidence angle	degrees
L	nozzle cone axial length	in.
L^*	free expansion length	in.
M	mach number	
P	static pressure	psia
P_T	total pressure	psia
PR	total to static pressure ratio	
S_{mean}	mean shock strength	
U	turbine tip speed	ft/sec
V	absolute velocity at nozzle exit	ft/sec
$\dot{w}$	nozzle weight flow	lb/sec
α	nozzle angle	degrees
α_c	nozzle cone half angle	degrees
α_t	throat divergence angle	degrees

β_g	relative gas angle	degrees
β_1	entering blade angle	degrees
γ	gas ratio of specific heats	
δ	flow deflection angle	degrees
η_t	turbine efficiency	
η_h	hydraulic turbine efficiency	
$\Delta\nu$	Prandtl-Meyer flow deflection angle	degrees
ψ_N	nozzle velocity coefficient	
ψ_R	rotor velocity coefficient	

SUBSCRIPTS

o	nozzle entrance
*	nozzle throat
e	nozzle exit plane
a	conditions downstream of nozzle exit (ambient)
D	design
1	conditions before shock
2	conditions after shock
os	oblique shock
ns	normal shock
opt.	optimum
i = o	conditions at zero incidence

SUPERSCRIPT

'	isentropic condition

SECTION 10.0

LIST OF REFERENCES NOT INCLUDED IN APPENDIX I

10.0 REFERENCES

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SECTION 11.0

APPENDICES

11.1 APPENDIX I-SUMMARY OF AVAILABLE TURBINE NOZZLE REFERENCE MATERIAL

SECTION:

11.1 Theoretical Only

11.2 Experimental Only

11.3 Comparison of Theoretical and Experimental Information

APPLICABILITY RATING:

1. Of particular interest and applicable to small turbines.
2. Of general interest and applicable to small turbines.
3. Applicable to small turbines but information incomplete or non-conclusive.
4. Not applicable to small turbines.

11.1 THEORETICAL ONLY

11.1.1 "SHOCK FORMATION IN CONICAL NOZZLES"

H. M. Darwell and H. Badham, AIAA Journal, Vol. 1, No. 8, August 1963.

APPLICABILITY 2

A theoretical evaluation of the gas flow in conical nozzles was made using the method of characteristics. This study showed that it is possible that a strong shock forms in the nozzle. Computer calculations show that the shock formation can be removed by changes in the wall contour near the throat.

SIGNIFICANT CONCLUSION: Method to improve conical nozzle performance.

11.1.2 "APPROXIMATE METHOD FOR PLUG NOZZLE DESIGN"

Gianfranco Angelino, AIAA Journal, Vol. 2, No. 10, June 3, 1964.

APPLICABILITY 2

This report describes a method of designing plug nozzles. This type of nozzle has good performance at off-design pressure ratios and is constructed so that positioning the nozzle on a bias, as in a turbine, would not (theoretically) affect the flow.

SIGNIFICANT CONCLUSION: Method of designing nozzles.

11.1.3 "THEORETICAL STUDIES OF SUPERSONIC TWO-DIMENSIONAL AND AXISYMMETRIC NONEQUILIBRIUM FLOW, INCLUDING CALCULATIONS OF FLOW THROUGH A NOZZLE"

James J. Der, Ames Research Center, NASA Technical Report TR R-164, December 1963.

APPLICABILITY 2

Methods of calculating the characteristics of a supersonic nozzle with a dissociated gas flow are presented. The report is directed toward rocket type nozzles. Few turbine nozzles operate with a dissociated gas because of material limitations to temperature; however, this theory appears to be applicable for a dissociated gas.

SIGNIFICANT CONCLUSION: Method to calculate nozzle flow characteristics in a dissociated gas.

11.1.4 "CHARACTERISTICS OF CONICAL SUPERSONIC NOZZLES"

David Migdal and Fred Landis, ARS Journal, December 1962.

APPLICABILITY 2

Conical supersonic nozzles were analyzed by the method of characteristics to show the effects of wall angle, throat-to-cone fairing, area ratio, and the thermodynamic properties as characterized by the isentropic expansion coefficient. The study is primarily concerned with rocket nozzles; however, some of the information may be useful in turbine nozzle design.

SIGNIFICANT CONCLUSION: Parametric Study.

11.1.5 "DESIGN OF AXISYMMETRIC EXHAUST NOZZLES BY METHOD OF CHARACTERISTICS INCORPORATING VARIABLE ISENTROPIC EXPONENT"

E. C. Guentert and H. E. Neumann NASA TR R33, September 1959.

APPLICABILITY 2

Analytical method for including thermodynamic data with variable isentropic exponent in method to design exhaust nozzles. Several nozzle contours are presented.

11.1.6 "THRUST OF CONICAL NOZZLES"

E. M. Landsbaum, ARS Journal, Vol. 29, N3 P.212, 1959.

APPLICABILITY 4

This paper points out an area of slight error in the calculation of jet thrust. However, this error is generally negligible.

QUESTIONABLE AREA: Only applicable to one particular method of calculating thrust.

11.1.7 "CONTOUR NOZZLES"

E. M. Landsbaum, ARS Journal, Vol. 30, P. 244, 1960.

APPLICABILITY 4

Theoretical paper on optimization of rocket nozzles. Most information is concerned with minimum weight and a comparison between conical and contoured nozzles.

11.1.8 "RAPID METHOD FOR PLUG NOZZLE DESIGN"

H. Greer, ARS Journal, Vol. 31, Pg. 560, 1961.

APPLICABILITY 2

An approximate method for determining the contour of a plug nozzle for a given area ratio and ratio of specific heats. The procedure is based on a centered Prandtl-Meyers wave expansion.

SIGNIFICANT RESULT: Quick calculation procedure.

11.1.9 "THE ANALYTICAL DESIGN OF AN AXIALLY SYMMETRIC DE LAVAL NOZZLE FOR A PARALLEL AND UNIFORM JET"

K. Foelsch, Journal of Aero Sciences, Vol. 16, No. 3, Pg. 161, March 1949.

APPLICABILITY 3

The equations for the nozzle contours are derived by integration of the characteristic equations. Conical source flow at the throat is converted into a parallel stream of uniform velocity.

QUESTIONABLE AREA: This detailed and laborious procedure is not substantiated by experiment. To determine validity of the procedure would require extensive study.

11.1.10 "THE AERODYNAMIC DESIGN OF HIGH MACH NUMBER NOZZLES UTILIZING AXISYMMETRIC FLOW WITH APPLICATION TO A NOZZLE OF SQUARE TEST SECTION"

I. E. Beckwith, H. W. Ridyard, and N. Cromer, NACA TN 2711, June 1952.

APPLICABILITY 4

This paper presents a method to design high Mach number wind tunnels using the method of characteristics. The shape of a Mach 10 nozzle is presented.

11.1.11 "AN ACCURATE AND RAPID METHOD FOR THE DESIGN OF SUPERSONIC NOZZLES"

I. E. Beckwith and J. A. Moore, NACA TN 3322.

APPLICABILITY 2

A method is presented for determining the contour of two-dimensional shock cancellation nozzles. The parameters of length-to-height ratio, Mach number, and wall angle at the inflection point are considered. In general, a nozzle is determined by specifying any two of these three parameters. Tables for K = 1.4.

SIGNIFICANT RESULT: Rapid calculation procedure.

11.1.12 "ANALYSIS OF TURBOMACHINE VISCOUS LOSSES AFFECTED BY CHANGES IN BLADE GEOMETRY"

James W. Miser, Warner L. Stewart, and Warren J. Whitney, Levis Flight Propulsion Laboratory, Cleveland, Ohio, NACA RM E56F21, October 1956.

APPLICABILITY 2

The effect of blade geometry on viscous losses was analyzed. The variables studied were blade number, solidity, aspect ratio, Reynolds number, and trailing edge blockage. The results of the analysis indicated that the viscous losses can be expressed as a function of: blade height-to-spacing ratio; solidity, and height Reynolds number. However, the trailing edge blockage also affects performance.

SIGNIFICANT CONCLUSION: Optimum solidity for turbine design.

11.1.13 "APPROXIMATION OF OPTIMUM THRUST NOZZLE CONTOURS"

G. V. R. Rao, Jet Propulsion Vol. 30, June 1960

APPLICABILITY 2

This report provides a rapid method to evaluate the contour of a shock cancellation nozzle. This contour is estimated to be within 3% of the contour obtained by the lengthy process of the method of characteristics.

SIGNIFICANT RESULT: Method to design shock cancellation nozzles.

QUESTIONABLE AREA: Variation in specific heat ratio ignored.

11.2 EXPERIMENTAL ONLY

11.2.1 "REACTION TESTS OF TURBINE NOZZLES FOR SUBSONIC VELOCITIES"

Hans Kraft, Transactions of the ASME, October 1949.

APPLICABILITY 1

This report presents considerable data of converging nozzles operating at various pressure ratios. The primary objective of the study was to find the effect of nozzle geometry on performance. Geometry that varied was aspect ratio, stator blade inlet angles, and stator blade shape. Kraft points out that Keenan's supersonic nozzle data are pessimistic for all supersonic data because of the method of testing (the same method was used in Kraft's tests) and "the velocity coefficients must be corrected upward by an uncertain amount."

SIGNIFICANT CONCLUSIONS: (1) Converging nozzle performance, and (2) Keenan's supersonic nozzle data are pessimistic.

QUESTIONABLE AREA: Effect of test method.

11.2.2 "THE PERFORMANCE OF SUPERSONIC TURBINE NOZZLES"

B. S. Stratford, G. E. Sansome - Aeronautical Research Council Report and Memoranda, R. & M. No. 3273, 1962.

APPLICABILITY 1

Experimental investigation of a Mach 2.5 supersonic nozzle. Tests were made at pressure ratios from 9 to 19, the design pressure ratio being 16.6. The objective was to find the effect of nozzle exit angle on nozzle pressure ratio.

SIGNIFICANT RESULT: The conditions immediately downstream of the nozzles exert an over-riding influence on the nozzle outlet flow angle.

11.2.3 "REACTION TESTS OF TURBINE NOZZLES FOR SUPERSONIC VELOCITIES"

J. H. Keenan, Transactions of the ASME, October 1949.

APPLICABILITY 1

Most complete testing of off-design supersonic nozzles available. Nozzles with area ratios of 1, 1.38, 2.34, 3.19 and 7.9 were tested. The nozzle shape was quasi-two-dimensional. All nozzles were designed for an inlet angle of 20°. All tests were reaction tests.

SIGNIFICANT RESULT: Off-design nozzle performance data.

QUESTIONABLE AREAS: (1) Unusual nozzle cross section. (2) No account taken of the pressure force on the nozzle.

11.2.4 "PERFORMANCE CHARACTERISTICS OF ONE CONVERGENT, AND THREE CONVERGENT-DIVERGENT NOZZLES"

H. George Krull and Fred W. Steffen, Lewis Flight Propulsion Laboratory, NACA Research Memo RM E52H12, September 29, 1952.

APPLICABILITY 2

An experimental study of 4 nozzles, applicable to rockets, was made. The nozzles area ratios were 1.0, 1.39, 1.69, and 2.65. An air flow parameter and thrust parameter are presented for all nozzles at pressure ratios from 1.2 to 25. The converging-diverging nozzles performed better than predicted when over-expanded, indicating oblique rather than normal shocks in the nozzle. Although these data are for rocket type nozzles some insight into off-design nozzle performance can be obtained.

SIGNIFICANT CONCLUSION: Off-design nozzle performance data.

QUESTIONABLE AREA: Rocket type nozzles.

11.2.5 "COMPARISON OF EXPERIMENTAL WITH PREDICTED WALL STATIC-PRESSURE DISTRIBUTIONS IN CONICAL SUPERSONIC NOZZLES"

L. H. Back, P. F. Massier, and H. L. Gier, Jet Propulsion Laboratory, Technical Report No. 32-654, October 15, 1964.

APPLICABILITY 2

Tests were made with room temperature and heated air in several conical convergent-divergent nozzles with an area ratio of 6.6. The nozzle geometry varied during the tests was the divergent angle of the convergent area. Test data are presented at off-design pressure ratios. This report should give some insight into the nozzle flow conditions at off-design pressure ratios.

SIGNIFICANT CONCLUSION: Conical nozzle data at off-design pressure ratio.

11.2.6 "SUDDEN EXPANSION OF A BOUNDED JET AT A HIGH PRESSURE RATIO"

Edward J. Barakauskas, AIAA Journal, Vol. 2, No. 9, April 10, 1964.

APPLICABILITY 2

Tests of the flow field of converging nozzles in a circular duct. The data show that the flow field is similar to that obtained with a convergent-divergent nozzle and if a duct is placed around the jet it has no influence unless the duct diameter is less than the jet diameter. This concept has interesting possibilities as a turbine nozzle.

SIGNIFICANT CONCLUSION: A sudden expansion nozzle has a similar flow field to convergent-divergent nozzle.

11.2.7 "CONICAL ROCKET NOZZLE PERFORMANCE UNDER FLOW-SEPARATED CONDITIONS:

Sherwin Kalt and David L. Badal, Engineering Notes, AIAA Journal of Spacecraft, May-June, 1965.

APPLICABILITY 2

More information on flow separation in conical nozzles. Should provide insight into off-design turbine nozzle performance.

SIGNIFICANT CONCLUSION: Off-design pressure ratio performance data for conical nozzles.

11.2.9 "SECONDARY FLOWS AND BOUNDARY-LAYER ACCUMULATIONS IN TURBINE NOZZLES"

Harold E. Rohlik, Milton G. Kofskey, Hubert W. Allen, and Howard Z. Herzig, NACA Report 1168, 1954.

APPLICABILITY 2

This report summarizes an investigation of secondary-flow loss patterns in three sets of converging turbine nozzle blade passages. The test technique used was flow-visualization (by means of paint on the blade surfaces and smoke jets) and detailed flow measurements with pressure and hot-wire probes. Overall mass-averaged blade velocity coefficients were 0.98 to 0.99.

SIGNIFICANT RESULT: Microscopic examination of nozzle flow.

QUESTIONABLE AREA: Neglects effect of rotor on nozzle performance.

11.2.10 "EXPERIMENTAL STUDY OF EFFECTS OF GEOMETRIC VARIABLES ON PERFORMANCE OF CONICAL ROCKET ENGINE EXHAUST NOZZLES"

H. E. Bloomer, R. J. Ontl, and P. E. Renos, NASA TN D-846 June 1961.

APPLICABILITY 1

A study of performance and separation characteristics of a family of conical exhaust nozzles was made. Tests were made with JP-4 and oxygen. Nozzles with divergent angles of 15 to 30° and area ratios of 8 to 75 were tested at pressure ratios from 35 to 840. The separation data may provide insight into off-design nozzle performance.

SIGNIFICANT CONCLUSIONS: Considerable conical nozzle data presented. Some separation data presented.

11.2.11 "EXPERIMENTAL INVESTIGATION OF FLOW IN AN ANNULAR CASCADE OF TURBINE NOZZLES BLADES OF CONSTANT DISCHARGE ANGLE"

K. E. Kofskey, H. E. Rohlik, and D. E. Monroe, NASA RM E52A09, March 1952.

APPLICABILITY 3

An investigation of the three-dimensional flow downstream of an annular cascade of nozzle blades was made at Mach numbers of 1.18, 1.31, and 1.41. Blade efficiencies from 0.978 to 0.983 were measured. This study was designed for insight into large gas turbine type nozzles.

QUESTIONABLE AREA: Cascade tests only.

11.2.12 "EFFECT OF NOZZLE SECONDARY FLOWS ON TURBINE PERFORMANCE AS INDICATED BY EXIT SURVEYS OF A ROTOR"

Warren J. Whitney, Howard A. Buckner, Jr., and Daniel E. Monroe, Lewis Flight Propulsion Laboratory, NACA Research Memo RM E54B03, April 5, 1954.

APPLICABILITY 2

Detailed circumferential and radial surveys of total temperature and pressure were made downstream of a turbine rotor at design conditions.

SIGNIFICANT CONCLUSION: It was observed that nozzle secondary-flow vortices, while small at nozzle exit, can induce much greater losses in the turbine rotor.

11.2.13 "EXPERIMENTAL STUDY OF EFFECTS OF GEOMETRIC VARIABLES ON PERFORMANCE OF CONTOURED ROCKET-ENGINE EXHAUST NOZZLES"

H. E. Bloomer, et al, NASA TN D-1181, January 1961.

APPLICABILITY 2

This study was conducted to determine the effects of nozzle contouring on separation and performance characteristics. The range of variables included area ratios of 16, 25 and 30 and pressure ratios from 35 to 450.

SIGNIFICANT RESULT: When compared to conical nozzles the contoured nozzles had performance 1 or 2 percent higher. The separation data for both contoured and conical nozzles agrees.

11.2.14 "EXPERIMENTAL PERFORMANCE OF NOZZLES HAVING AREA RATIOS UP TO 300 ON JP-4 FUEL-LIQUID OXYGEN ROCKET ENGINE"

J. C. Lovell and H. E. Somonich, NASA TN D-847 June 1961.

APPLICABILITY 2

The performance of 15° conical nozzles having area ratios of 50, 100, 200 and 300 and a bell-shaped nozzle with area ratio of 200 was made.

SIGNIFICANT RESULTS: The conical nozzles outperform the bell-shaped nozzles.

11.2.15 "TUNNEL TESTS ON A DOUBLE CASCADE TO DETERMINE THE INTERACTION BETWEEN THE ROTOR AND THE NOZZLES OF A SUPERSONIC TURBINE"

B. S. Stratford and G. E. Sansome, Aeronautical Research Council, C. P. No. 693, 1963.

APPLICABILITY 1

Good report of cascade study of a simulated turbine nozzle and rotor. Rotor relative inlet Mach number was 1.9. Tests were made at various pressure ratios and the rotor incidence was determined by Schlieren photographs and deduced from nozzle pressures. The rotor operated at +2 to +5° incidence for all pressure ratios studied.

11.2.16 "EXPERIMENTAL INVESTIGATION OF TURBINE STATOR-BLADE-OUTLET BOUNDARY LAYER CHARACTERISTICS AND A COMPARISON WITH THEORETICAL RESULTS"

Warren J. Whitney, Warner L. Stewart, and James W. Miser, Lewis Flight Propulsion Laboratory, NACA Research Memo RM E55K24, March 16, 1956.

APPLICABILITY 3

The boundary layer characteristics immediately downstream of a typical turbine stator blade were investigated experimentally. A comparison of the calculated and measured momentum thickness was made. Fair agreement was obtained.

11.2.17 "INVESTIGATION OF DISTRIBUTION OF LOSSES IN A CONSERVATIVELY DESIGNED TURBINE"

Rose L. Whitney, Jack A. Heller, and Cavour H. Hauser, Lewis Flight Propulsion Laboratory, NACA Research Memo RM E53A16, March 16, 1953.

APPLICABILITY 2

Surveys of the flow conditions and of the losses at the exit of both the stator and rotor of a turbo-jet type turbine were made.

SIGNIFICANT CONCLUSIONS: The major losses occur at the blade hub and tip with the tip losses being greater than the hub.

11.2.18 "SURVEY OF AVAILABLE INFORMATION ON INTERNAL FLOW LOSSES THROUGH AXIAL TURBOMACHINES"

Chung-Hua Wu, Lewis Flight Propulsion Laboratory, Cleveland, Ohio, NACA RM E50J13.

APPLICABILITY 3

This report summarizes an early study of turbine flow losses. Some data from other references is presented. Data showing optimum stator to rotor axial clearances of 0.15 to 0.40 inch is presented.

11.2.19 "APPLICATION OF BOUNDARY-LAYER THEORY IN TURBOMACHINERY"

H. Schlichting, Journal of Basic Engineering, December 1959.

APPLICABILITY 2

This report presents some data of subsonic NACA type airfoil shapes. The object of the report is to include boundary layer theory to predict the performance of cascade tests. Good agreement with data is shown.

SIGNIFICANT CONCLUSION: Theoretical analysis of subsonic cascades.

QUESTIONABLE AREAS: (1) Subsonic cascade data on low turning angle blades. (2) Little consideration of the application to rotating blades is given.

11.2.20 "RECENT RESEARCH ON CASCADE FLOW PROBLEMS"

H. Schlichting, Deutsche Forschungsanstalt Fur Luft - Und Raumfahrt E. V. Institut for Aerodynamik, DFL-Bericht Nr. 202, 1963. (Report in English)

APPLICABILITY 2

A summary is given on extensive research work on cascade flow problems carried out in recent years in Germany. The primary results presented are concerned with secondary flow losses in subsonic type blading (NACA blade profiles) at subsonic velocities. The results reported were obtained for compressible flow. However, tests at high subsonic velocities showed little influence of Mach number on secondary losses provided no shock waves occur.

SIGNIFICANT CONCLUSION: Presentation of recent German cascade data.

QUESTIONABLE AREA: Subsonic cascade data only.

11.3 THEORETICAL AND EXPERIMENTAL

11.3.1 "STUDY OF THE TWO-DIMENSIONAL FLOW THROUGH A CONVERGING-DIVERGING NOZZLE"

Arthur Kantrowitz, Robert E. Street, and John R. Erwin, NASA CB 3D24, 1942.

APPLICABILITY 2

A study of flow through a converging-diverging shock cancellation nozzle was made. The design Mach number was 1.44. A comparison of the measured and theoretical pressure distribution was in good agreement. This report includes initial work in the nozzle field which is generally understood at the present time.

11.3.2 "INVESTIGATION OF NEW M.E.I. NOZZLE CASCADES FOR SUPERSONIC VELOCITIES"

M. Ye. Deych, A. V. Gubarev, et al, Teploenergetika, No. 10, 1962, Foreign Technology Division FTD-TT-63-99.

APPLICABILITY 2

A "new" Russian nozzle design is proposed that will improve supersonic turbine nozzle off-design performance. This design is very similar to Ohlsson's design at MIT. Test data are presented for 22 nozzle designs and the performance does appear good; however, considerable time will be required to understand the report.

SIGNIFICANT CONCLUSION: Converging nozzles are best for design Mach numbers up to 1.5.

QUESTIONABLE AREA: Cascade data only.

11.3.3 "SHOCK-INDUCED BOUNDARY LAYER SEPARATION IN OVEREXPANDED CONICAL EXHAUST NOZZLES"

M. Arens, E. Spiegler, AIAA Journal Vol. 1, No. 3, March 1963.

APPLICABILITY 1

The flow in overexpanded supersonic conical nozzles was reviewed. Experimental data showed that oblique shocks occurred in the nozzle when operating overexpanded. When oblique shocks occur the nozzle velocity coefficient is larger than if a normal shock occurs. The prediction of the pressure ratio at which the oblique shocks and hence boundary layer separation occurs is of value for turbine analysis at below design pressure ratios. Good correlation was obtained with experimental data for the assumptions of this paper. A large number of data were available for comparison.

11.3.4 "TRUNCATED PERFECT NOZZLES IN OPTIMUM NOZZLE DESIGN"

J. H. Ahlbert, S. Hamilton, D. Migdal, and E. N. Nilson, ARS Journal, May 1961.

APPLICABILITY 2

This report presents a method to design shock cancellation supersonic nozzles. A discussion of wall friction and separation effects is included. The work is directed toward large rocket nozzles; however, the data could provide insight into turbine nozzle performance.

SIGNIFICANT CONCLUSION: Method of design of shock cancellation nozzles.

QUESTIONABLE AREA: Information directed toward large nozzles.

11.3.5 "INVESTIGATION OF STEAM TURBINE NOZZLE AND BLADING EFFICIENCY"

F. Dollin, Proc. I. Mech. E., Vol. 114, No. 4, 1940.

APPLICABILITY 2

This report discusses the design of converging nozzle and rotor blade shapes. A discussion of the problems of nozzle testing is presented. Some nozzle test data are presented for a sonic nozzle. The design of an improved nozzle test rig is described. The author feels that the Reynolds number is an important parameter that has not been considered previously.

QUESTIONABLE AREA: Discussion on nozzle test techniques and the description of analytical techniques is incomplete, making it difficult to use the procedure with other data.

11.3.6 "SOME RESEARCHES ON STEAM-TURBINE NOZZLE EFFICIENCY"

Henry Lewis Guy, M. Inst. C. E., Journal of Inst. of Civil Engineering Vol. 13, 1939.

APPLICABILITY 2

This paper summarizes nozzle work done by the Steam-Nozzles Research Committee. Unfortunately, the information and data presented only concerns converging nozzles. Data are presented which shows the effect of nozzle throat length, exit velocity, blade inlet radius, exit angle, and blade height on nozzle velocity coefficient.

SIGNIFICANT CONCLUSION: Variation of Nozzle Velocity coefficient with nozzle parameters.

QUESTIONABLE AREA: Converging nozzles only.

11.3.7 "FIRST REPORT OF THE STEAM NOZZLES RESEARCH COMMITTEE"

Proceedings of the Institute of Mechanical Engineers, January 1923.

APPLICABILITY 2

This report describes the test equipment used by the steam nozzles committee in later tests.

11.3.8 "SECOND REPORT OF THE STEAM NOZZLES RESEARCH COMMITTEE"

Proceedings of the Institute of Mechanical Engineers, March 1923.

APPLICABILITY 2

This report presents the initial studies of the committee. The majority of this paper is concerned with comments made during the discussion after presentation of the report. Data of 20° converging nozzles with thin partitions (cascades) are presented.

11.3.9 "THIRD REPORT OF THE STEAM NOZZLES RESEARCH COMMITTEE"

Proceedings of the Institute of Mechanical Engineers, May 1924.

APPLICABILITY 2

This report presents the performance of convergent nozzles with 20° angles with thick partitions. In addition, a series of tests were run to determine the effect of chamfer on the nozzle exit edges.

SIGNIFICANT RESULT: Chamfer the nozzle edge to improve the velocity coefficient over an un-chamfered edge.

11.3.10 "FOURTH REPORT OF THE STEAM NOZZLES RESEARCH COMMITTEE"

Proceedings of the Institute of Mechanical Engineers, May 1925.

APPLICABILITY 2

Test results of 12° convergent nozzles with thick and thin partitions are presented. Tests to determine the effect of exit edge chamfer were conducted. Comments are made as to the effect on nozzle efficiency of: throat length, thickness of the partitions, chamfering, and the nozzle angle.

11.3.11 "CASCADE TESTS OF THE BLADING OF A HIGH-DEFLECTION, SINGLE-STAGE, AXIAL-FLOW IMPULSE TURBINE AND COMPARISON OF RESULTS WITH ACTUAL PERFORMANCE DATA"

J. E. Fartocci, U. S. Naval Postgraduate School Thesis, May 1966.

APPLICABILITY 2

This report summarizes cascade tests of the nozzle and rotor for a subsonic design, M = 0.3. Comparisons with turbine data are made.

SIGNIFICANT RESULT: The nozzle loss coefficient determined from turbine tests was approximately twice that determined from cascade tests at M = 0.3.

QUESTIONABLE AREA: The nozzle performance was derived from turbine data; however, the raw data was not presented.

11.3.12 "RECENT DEVELOPMENTS IN ROCKET NOZZLE CONFIGURATIONS"

G.V.R. Rao, ARS Journal, November 1961, Vol. 31.

APPLICABILITY 2

Comparisons are made between ideal, conical, optimum (shock cancellation) and plug nozzles. It is shown that plug nozzles yield best performance with shortest length.

SIGNIFICANT RESULTS: It is shown that the performance (thrust coefficient) of a conventional nozzle with separation is superior to the performance without separation. Large list of references presented.

11.3.13 "PERFORMANCE OF SEVERAL CONICAL CONVERGENT-DIVERGENT ROCKET-TYPE EXHAUST NOZZLES"

C. E. Campbell and J. M. Farley, NASA TN D-467, September 1960.

APPLICABILITY 2

Thrust tests were made with conical nozzles having area ratios of 10, 25 and 40, and with divergent angles of 15, 25 and 29°. The tests were made on 1200° F. air.

SIGNIFICANT RESULTS: Decreasing the divergent angle resulted in sizable increases in thrust ratio, particularly at low pressure ratio. Data are presented for pressure ratios from 5 to 120. It is interesting to note that the slope of the thrust ratio vs. pressure ratio becomesless when the nozzle separates.

QUESTIONABLE AREA: The design point performance or above was not obtained since the maximum pressure ratio was 120 and the design pressure ratio at the minimum area ratio is 145.

11.3.14 "THRUST CHARACTERISTICS OF UNDEREXPANDED NOZZLES"

F. P. Durham, Jet Propulsion, December 1955, Pg. 696.

APPLICABILITY 2

The thrust characteristics of underexpanded nozzles are investigated both analytically and experimentally. Two conical nozzles were made, each having a 15° half angle cone. One nozzle was smooth (20 micro inches), the other was rough (300 micro inches). The nozzles were tested at area ratios from 1 to 6 by machining the nozzle back. Tests were run on air.

SIGNIFICANT RESULT: The maximum nozzle thrust is obtained from an area ratio of little more than half that required for complete expansion.

QUESTIONABLE AREA: No off-design pressure ratio data.

11.3.15 "PERFORMANCE OF SEVERAL METHOD-OF-CHARACTERISTICS EXHAUST NOZZLES"

J. M. Farley and C. E. Campbell, NASA TN D-293, October 1960.

APPLICABILITY 2

Performance data were obtained with three contoured and one 15° conical nozzle. The area ratios were 10, 15, 20 and 25. Tests were made on air.

SIGNIFICANT CONCLUSIONS: The contoured nozzles had a 1% increase in thrust at design over the conical nozzle. However, at pressure ratios considerably below design the conical nozzle outperformed the contoured.

11.3.16 "THE TEST PERFORMANCE OF HIGHLY LOADED TURBINE STAGES DESIGNED FOR HIGH PRESSURE RATIO"

I. H. Johnston and D. C. Dransfield, Aeronautical Research Council Reports and Memoranda, R & M No. 3242, June 1959.

APPLICABILITY 1

Very good report discussing the design and testing of a supersonic two-stage turbine. Test data are presented for the two stage turbine and each stage of the two stage turbine operating with full admission. The two stage and the first stage of the two stage turbine were also tested at 50% admission. These partial admission tests showed a 2 point decrease in efficiency for both turbines tested. The test data indicated that the second stage choked causing a reduction in efficiency.

SIGNIFICANT CONCLUSIONS: (1) Lap improves the chance of starting a supersonic rotor. (2) Both nozzle and rotor blade losses were greater in the turbine than in cascade tests.

11.3.18 "COMPARISON BETWEEN PREDICTED AND OBSERVED PERFORMANCE OF GAS-TURBINE STATOR BLADE DESIGNED FOR FREE-VORTEX FLOW"

M. C. Huppert and Charles MacGregor, Lewis Flight Propulsion Laboratory, NACA Technical Note No. 1810, April 1949.

APPLICABILITY 2

Initial experimental and theoretical work in supersonic vortex rotor blading. A comparison was made between the calculated and measured performance of an annular cascade of converging nozzles. The comparison concerned surface pressures and exit flow angles radially across the nozzle exit.

11.3.19 "PRELIMINARY ANALYSIS OF AXIAL-FLOW COMPRESSORS HAVING SUPERSONIC VELOCITY AT THE ENTRANCE OF THE STATOR"

Antonio Ferri, Langley Aeronautical Laboratory, NACA Research Memo RM L9G06, September 12, 1949.

APPLICABILITY 2

A supersonic compressor was analyzed on the assumption of two-dimensional flow. Preliminary cascade tests were made to determine empirical coefficients for the analysis. The starting conditions, stability of the flow and interactions between rotor and stator is discussed.

SIGNIFICANT CONCLUSION: Discussions on starting, stability, and interaction between rotor and stator.

QUESTIONABLE AREA: Two-dimensional approach.

11.3.20 "TUNNEL TESTS ON A DOUBLE CASCADE TO DETERMINE THE INTERACTION BETWEEN THE ROTOR AND THE NOZZLES OF A SUPERSONIC TURBINE"

B. S. Stratford and G. E. Sansome, National Gas Turbine Establishment Memorandum 359, 1962.

APPLICABILITY 1

Cascade tests were made at an entering Mach number of 1.9 using rotor blades with 140° of turning. The blades were sharp edged with 0 to 7% blockage. Tests were made with three rotor blade configurations, the main difference between blades being the leading edge thickness.

SIGNIFICANT RESULT: The thickness of the leading edge rather than the nozzle exit angle governs the blade incidence and, hence, the gas exit angle from the nozzles.

QUESTIONABLE AREA: Accuracy of determining flow angles.

11.3.21 "ON DETERMINING FLOW COEFFICIENTS AND LEAVING ANGLES FROM BLADE ROWS OF AXIAL GAS TURBINES"

G. V. Proskuryakov, Ural Turbomotor Works, Thermal Engineering (Teploenergetika) Vol. II, No. 9, September 1964, Pergamon Press.

APPLICABILITY 2

The author describes the theoretical methods used in the Russian Ural Turbomotor Works for determining flow characteristics in a gas turbine. The nozzle flow coefficients for subsonic nozzle blades is presented.

SIGNIFICANT RESULT: Static tests of blade rows can lead to large errors in determining the leaving angles.

QUESTIONABLE AREAS: (1) Subsonic turbine data. (2) Turbine geometry unknown.

11.3.22 "AN EXAMINATION OF THE FLOW AND PRESSURE LOSSES IN BLADE ROWS OF AXIAL-FLOW TURBINES"

D. G. Ainley and G. C. R. Mathieson, Aeronautical Research Council Reports and Memoranda, R & M No. 2891, 1955.

APPLICABILITY 1

Very good report concerned with the effects on Mach number; incidence; Reynold's number; blade shapes; spacing, and aspect ratio; and partial admission on secondary losses in turbo-machinery. This report studies and analyzes the data available up to 1951. Considerable turbine data are presented.

SIGNIFICANT CONCLUSIONS: (1) Variation of blade profile loss with turbine geometric parameters. (2) Incidence losses.

QUESTIONABLE AREA: Some data is perhaps obsolete.

11.3.23 "LOSSES AND EFFICIENCIES IN AXIAL-FLOW TURBINES"

J. H. Horlock, Int. J. Mech. Sci., Vol. 2, pp. 48-75, Pergamon Press Ltd., 1960.

APPLICABILITY 1

This paper attempts to bring together turbine data and theories of a number of investigators. A large amount of data is presented, sometimes conflicting, and an attempt to choose the most applicable correlation is made. Careful study should yield some useful information.

SIGNIFICANT CONCLUSION: Comparison of turbine loss data.

11.3.24 "FUELS AND PRIME MOVERS FOR ROTATING AUXILIARY POWER UNITS"

Robert W. Mann, M. I. T. Dynamic Analysis and Control Laboratory, Report No. 121.

APPLICABILITY 1

Good report on early work connected with small turbine engines. The report is directed toward missile auxiliary power supplies with high energy fuels. Considerable attention is given to partial admission turbines and a discussion of the associated losses is made.

SIGNIFICANT CONCLUSIONS: (1) Variation of nozzle efficiency with pressure ratio. (2) Disc friction and blade pumping data. (3) Some partial-admission turbine data.

11.3.25 "SUPERSONIC TURBINES"

Gunnar O. Ohlsson, Research Dept., De Laval Ljungstrom Turbine Co., ASME Paper No. 62-WA-37, 1962.

APPLICABILITY 1

Very good report concerning experimental results of three supersonic nozzles and one converging nozzle. Turbine performance data are presented at design and off-design pressure ratio. The turbine nozzle design Mach numbers were .98, 1.21, 1.60, and 1.69. The measured turbine efficiency was approximately 0.6 at design pressure ratio for all turbines.

SIGNIFICANT RESULTS: (1) Turbine and nozzle off-design performance data. (2) Experimental results indicate rotor choking occurs when operating at greater than design pressure ratios with the Mach 1 nozzle.

QUESTIONABLE AREA: Nozzle data is questionable at high pressure ratios since the rotor used to measure the nozzle efflux momentum is choked.

11.3.26 "PARTIAL ADMISSION, LOW ASPECT RATIOS AND SUPERSONIC SPEEDS IN SMALL TURBINES"

G. Ohlsson, MIT Thesis, January 1956.

<u>APPLICABILITY 1</u>

Comprehensive study of single stage impulse turbines. Three problems wer studied; partial admission, low aspect ratio blades, and supersonic relative inlet Mach numbers. The performance data of 10 turbines are presented. Tests were made at admission ratios of 0.1, 0.2 and 1.0, aspect ratios of 0.07 to 0.70, and nozzle Mach numbers of 1.0, 1.5, 1.71, 2.08, 2.21, and 2.86.

QUESTIONABLE AREA: Nozzle data is questionable at high pressure ratios since the rotor used to measure the nozzle efflux momentum is choked.

11.3.27 "FLOW OF GAS THROUGH TURBINE LATTICES" (Translation)

M. E. Deich, Technical Gasdynamics, NACA Technical Memo 1393, May 1956.

<u>APPLICABILITY 2</u>

Very complete analytical study with a fair amount of experiment information as well. An aerodynamic approach to rotor flow is presented for both subsonic and supersonic flow.

QUESTIONABLE AREA: In order to convert the information presented here into useful data, a significant amount of time would be required.

11.3.28 "THEORY AND TUNNEL TESTS OF ROTOR BLADES FOR SUPERSONIC TURBINES"

B. S. Stratford and G. E. Sansome, Aeronautical Research Council Reports and Memoranda, R. & M. No. 3275, 1962.

<u>APPLICABILITY 1</u>

Experimental study of supersonic rotor blades with 140° of turning. Test Mach number was 1.9 and four basically vortex type rotor designs were tested. Measured rotor velocity coefficients were very high, in the order of 0.93.

SIGNIFICANT RESULTS: (1) Effects of geometry on supersonic flow in rotor blades. (2) The effect of lap on rotor choking and losses.

QUESTIONABLE AREAS: (1) Humidity affected rotor performance data (2) Cascade tests only.

11.3.29 "THE TEST PERFORMANCE OF HIGHLY LOADED TURBINE STAGES DESIGNED FOR HIGH PRESSURE RATIO"

I. H. Johnston and D. C. Dransfield, Aeronautical Research Council Reports and Memoranda, R & M No. 3242, June 1959.

APPLICABILITY 1

Very good report discussing the design and testing of a supersonic two-stage turbine. Test data are presented for the two stage turbine and each stage of the two stage turbine operating with full admission. The two stage and the first stage of the two stage turbine were also tested at 50% admission. These partial admission tests showed a 2 point decrease in efficiency for both turbines tested. The test data indicated that the second stage choked causing a reduction in efficiency.

SIGNIFICANT CONCLUSIONS: (1) Lap improves the chance of starting a supersonic rotor. (2) Both nozzle and rotor blade losses were greater in the turbine than in cascade tests.

11.2 APPENDIX II - THEORETICAL PROCEDURE, SAMPLE CALCULATION

The general description of the theoretical procedure is given in Section 4. A sample calculation is presented here as an illustration and derivation of the theoretical techniques. Frictionless flow, a nozzle design Mach number of 2.0, and a gas ratio of specific heats of 1.4 is assumed. The effect of design Mach number and ratio of specific heats is given in Section 4.

Equations for the calculation of Mach number functions are included here. Many of these functions are available in tabular form, such as in Reference 5, 6, and 7. Unfortunately these references only included tables for specific heat ratios of 1.4 and 1.286. Therefore, calculation may be necessary for specific heat ratios other than these.

In the following example each flow regime will be handled separat Regime 1, all subsonic low, is neglected since supersonic turbine nozzles do not operate in this regime.

11.2.1 Regime 2.0 - Overexpanded Flow

Two sub-regimes are defined here as described in Section 4. Each sub-regime is described separately.

11.2.1.1 Regime 2.1 - Choked Throat With Normal Shock and Subsonic Diffusion - The sketch below defines the station location and nomenclature used in the following discussion.

Consider the condition when the Mach number, M_1, at which the shock occurs is 1.35. Then the conditions after the normal shock are:

Mach Number

$$M_2 = \left(\frac{2 + (\gamma + 1)M_1^2}{2\gamma M_1^2 - \gamma + 1} \right)^{\frac{1}{2}} = .762 \qquad \text{(II-1)}$$

Static pressure ratio,

$$P_2/P_1 = 2\gamma/(\gamma+1)\, M_1^2 - (\gamma-1)/(\gamma+1) = 1.96 \qquad \text{(II-2)}$$

and Total pressure ratio,

$$\frac{P_{T2}}{P_{T1}} = \left[\frac{M_1^2\,(\gamma+1)/2}{M_1^2\,(\gamma-1)/2 + 1} \right]^{\gamma/(\gamma-1)} \left(\frac{P_2}{P_1} \right)^{1/(1-\gamma)} = .97 \qquad \text{(II-3)}$$

Since:

$$A/A^* = 1/M \left[\frac{2(1+M^2(\gamma -1)/2)}{\gamma + 1} \right]^{(\gamma+1)/2(\gamma-1)} \qquad \text{(II-4)}$$

$A_1/A^* = 1.089$ @ $M_1 = 1.35$

$A_e/A^* = 1.688$ @ $M_D = 2.0$

$A_2/A^* = 1.058$ @ $M_2 = 0.762$

The subsonic diffusion is described by:

$$A_e/A^* = (A_e/A^*)\ (A^*/A_1)\ (A_2/A^*) \qquad \text{(II-5)}$$

Since $A_1 = A_2$,

$$A_e/A^* = (1.6880)\ (1.058)/1.089 = 1.64$$

The actual exit Mach number is obtained by solving eqn. (II-4) for the subsonic Mach number solution when $Ae/A^* = 1.64$, hence,

$$M_e = 0.385 \quad @ \quad Ae/A^* = 1.64$$

The isentropic conditions are based on the nozzle static pressure ratio, which is:

$$P_e/P_{To} = (Pe/P_{Te})\ (P_{T2}/P_{T1}) \qquad \text{(II-6)}$$

since for frictionless flow $P_{Te} = P_{T2}$. The pressure ratio is obtained as a function of Mach number from:

$$P/P_T = (1 + M^2 (\gamma -1)/2)^{\gamma/(\gamma -1)} \qquad \text{(II-7)}$$

so that

$$P_e/P_{Te} = 0.902 \quad @ \quad M_e = 0.385$$

since, for frictionless flow, $P_{T0} = P_{T1}$ and $P_{T2} = P_{Te}$, equation (II-6) yields the nozzle overall static to total pressure ratio as:

$$P_e/P_{to} = (.902)\,(.97) = .875$$

The isentropic Mach number at this pressure ratio is obtained from equation (II-7) as:

$$M' = .440 \qquad @ \quad P/P_t = .875$$

Therefore, the isentropic Mach number ratio is:

$$M'/M'_D = .440/2. = .220$$

To obtain the calculated nozzle velocity coefficient, Ψ, the following velocity ratio is determined as a function of Mach number:

$$V/a_t = M(1+M^2(\gamma-1)/2)^{-\frac{1}{2}} \qquad \text{(II-8)}$$

where a_t is the speed of sound based on the total temperature. This is a convenient parameter to use with internal flow since the total temperature can only be changed by the addition or subtraction of heat or work from the gas. The velocity ratios calculated using equation (II-8) are:

$$V_e/V_{te} = .379 \qquad @ \quad M = .385$$

$$V'/V'_t = .432 \qquad @ \quad M' = .440$$

Then the nozzle velocity coefficient is obtained:

$$\Psi_N = V_e/V' = (V_e/a_{te})\,(a'_t/V') = .379/.432 = .876$$

since

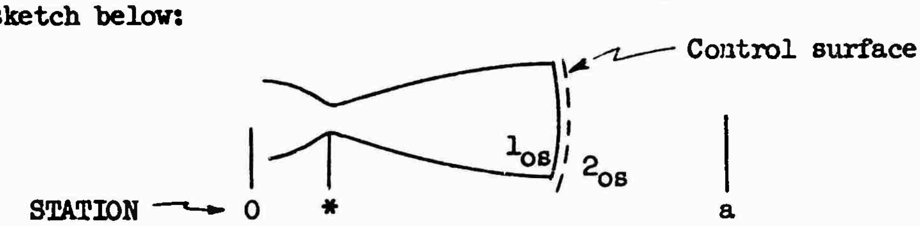
$$a_{te} = a'_t \qquad \text{(No heat or work addition)}$$

This procedure is continued at $1.0 < M_1 < 1.5$ to obtain the family of curves shown in Figure 11 in Regime 2.1. It may be noted that if the procedure illustrated here is continued to shock Mach numbers greater than 1.5, the calculated nozzle velocity becomes unrealistically low.

11.2.1.2 Oblique Shock at Nozzle Wall Coalescing Into a Normal Shock at Center of Nozzle

The analytical model for this regime is more complex than the previous regime. Therefore, as mentioned in Section 4, it is recommended that the theoretical performance be calculated for the case of an oblique shock occurring at design Mach number only. In fact, the actual isentropic Mach number range covered in this regime is small, and to define the operational range where the transition between the occurrence of Regime 2.1 and the fully developed oblique plus normal shock regime occurs is quite complex. Because of the complexity and the resultant assumptions necessary, a theory devised for this transition area is of little value.

The control surface considered for this analysis is shown in the sketch below:



The control surface is taken downstream of the shocks so that the pressure at the control surface exit is equal to ambient pressure and; therefore, no pressure-area correction is necessary. The oblique shock strength is obtained from Figure 9 for $\gamma = 1.4$. When calculations are made for other specific heat ratios the shock strength is evaluated as that necessary to stagnate flow at 60 percent of free stream velocity. This strength corresponds exactly to that shown in Figure 9 for $\gamma = 1.4$.

The actual calculation of oblique shock parameters is beyond the scope of this discussion and it will be assumed that the reader would use a procedure such as that outlined in Reference 5. This procedure shows that for a M_1[26] = 2.0 and the flow deflection angle equal to 11.6 degrees (see Figure 9), a plane (two dimensional) shock yields:

$$M_{2os} = 1.58$$

$$P_{T2os}/P_{T1os} = .975$$

Then from equation (II-8),

$$V/a_t \Big|_{2os} = 1.2904 \qquad @ \quad M_{2os} = 1.58$$

The normal shock portion yields:

from equation (II-1) $\quad M_2 = .577 \quad @ \quad M_1 = 2.$

and from equation (II-3) $\quad P_{T2}/P_{T1} = .721 \quad @ \quad M_1 = 2.$

From equation (II-8)

$$V/a_t \Big|_{2os} = .559 \qquad @ \quad M_2 = .577$$

It is necessary now to assume some shock strength variation from the oblique shock at the wall to the normal shock at the flow center along the control surface in order to integrate and obtain mean values across the shock. A linear variation is not reasonable since no discontinuity actually occurs at the stream center. In fact the distribution must have an inflection point at the nozzle center line in order to be realistic. Therefore, an elliptical variation across the nozzle was assumed. Integration to obtain mean values of total pressure ratio across the shock, and velocity ratio after the shock yields:

$$\overline{P_{T2}/P_{T1}} = P_{T2}/P_{T1}\Big|_{os} - \pi/4 \left(P_{T2}/P_{T1}\Big|_{os} - P_{T2}/P_{T1}\right) \qquad \text{(II-9)}$$

and $V_2/a_t = V_2/a_t\Big|_{os} - \Psi/4\,(V_2/a_t\Big|_{os} - V_2/a_t)$ (II-10)

For the example, equations (II-9) and (II-10) yield,

$$P_{T2}/P_{T1} = .775$$

$$V_2/a_t = .715$$

The mean Mach number is obtained from equation (II-8) as a function of $\dfrac{V_2}{a_t}$,

$$M_2 = .755 \quad @ \quad V_2/a_t = .715$$

and the pressure ratio from equation (II-7) as :

$$P_2/P_{T2} = .685 \quad @ \quad M_2 = .755$$

The nozzle pressure ratio based on inlet total pressure is:

$$P_a/P_{To} = (P_2/P_{T2})\ (P_{T2}/P_{T1})$$

since $P_{T1} = P_{To}$ and $P_2 = P_a$

$$P_a/P_{To} = P_2/P_{T1} = (.685)(.775) = .531$$

Now the isentropic values can be calculated by equations (II-7) and (II-8) as:

$$M' = .996 \quad @ \quad P/P_T = .531$$

$$V'/a_t = .910 \quad @ \quad M' = .996$$

so that: $\Psi_N = .715/.910 = .786$

and

$$M'/M'_D = .996/2 = .486$$

11.2.1.3 Regime 2.3 - Oblique Shock at Nozzle Exit Coalescing into Normal Shock As Pressure Ratio Nears Design the Shock Strength Weakens.

The calculation of this regime is identical to that of Regime 2.2 with the exception that equations (II-9) and (II-10) are calculated as the value $\pi/4$ approaches zero, and the oblique shock strength approaches zero. It is noteworthy that the calculated parameters ψ_N and M'/M_D follow the same curve when the oblique shock strength and the $\pi/4$ parameter are changed singly, or are both changed at once. The result of this calculation is shown in Figure 10. The calculated values in this regime, as the shock strength is weakened, are optimistic as compared to most of the measured nozzle performance. Therefore, it is recommended that the variation between the limit of Regime 2.2 and the design point be faired in as shown in Figure 10. This fairing can be accomplished by calculating the limit value of Regime 2.2 for a relatively high (approximately 4) design Mach number. Since the curve for the high Mach number point is steep in Regime 2.3, a straight line is adequate as shown in Figure 11. If the same thing is done for a nozzle with design Mach number of 1.5, the limits of the faired curves are established and the curves can be easily drawn.

It would seem that the model described here is reasonable for this application particularly when considering that the calculation basically agrees with the test data at the interface between Regime 2.2 and 2.3. Therefore, one would expect the theory to describe the actual shape of the curve in Regime 2.3. The data of Krull and Steffen (11.2.4), presented in Figure 5, follow the trend of the calculated curve of Figure 10. Although these data follow closely the trends of the calculation in Regime 2.3, these are the only data which do. Therefore, the faired curve in Regime 2.3 is still recommended.

11.2.2 Regime 3.0 - Design Point Performance

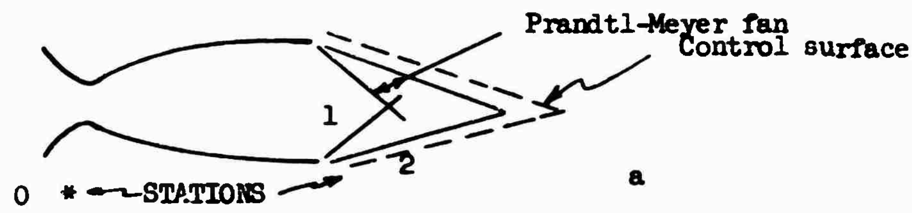
The design point performance is based on the particular nozzle geometry and is discussed in Section 3. Since the data correlation is based on the ratio of nozzle velocity coefficient to design nozzle velocity coefficient, the off-design performance is hypothesized to have an equal percentage loss in performance at all operational points. In other words, a nozzle having a 0.90 design point velocity coefficient is hypothesized to have the same ratio of velocity coefficients (for equal Mach number ratios and design Mach number) as a nozzle with design velocity coefficient of 0.98.

In the case of the theoretical analysis the calculated velocity coefficient is equivalent to the ratio of velocity coefficients since the design velocity coefficient is 1.0.

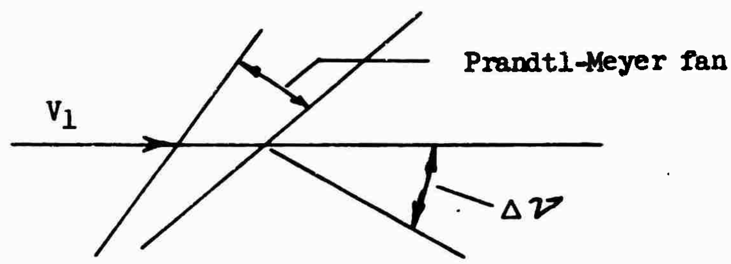
11.2.3 Regime 4.0 - Underexpanded Conditions

In the underexpanded regime, the calculation of the flow parameters after a two dimensional Prandtl-Meyer expansion adequately describe the flow.

This regime is shown in the sketch below:



Since the Prandtl-Meyer expansion is an isentropic process, the only theoretical evaluation necessary is that to obtain the nozzle axial velocity vector at station 2. This is done by evaluating the Prandtl-Meyer flow deflection angle $\Delta\nu$:



since $\psi_N = V_{e_{axial}}/V_e'$

and $V_e = V_e'$ for frictionless flow

therefore;

$$\psi_N = V_e \cos(\Delta\nu)/V_e = \cos(\Delta\nu)$$

The Prandtl-Meyer turning angle, ν, as a function of Mach number is:

$$\nu = \left(\frac{\gamma+1}{\gamma-1}\right)^{\frac{1}{2}} \tan^{-1}\left(\frac{\gamma-1}{\gamma+1}(M^2-1)\right)^{\frac{1}{2}} \tan^{-1}(M^2-1) \qquad \text{(II-11)}$$

For example at $M_1 = M_D^1 = 2$, evaluation of eqn. (II-11) yields:

$$\nu_1 = 26.38^\circ \quad @ \quad M_1 = 2$$

assuming $\Delta\nu = 10^\circ$

$$\nu_2 = 36.38^\circ$$

then solving eqn. (II-11) for M_2 yields

$$M_2 = 2.385 \quad @ \quad \nu = 36.38^\circ$$

since $M_2 = M'$

$$M'/M'_D = 2.385/2. = 1.1925$$

and $\psi_N = \cos 10° = .985$

This procedure is repeated at various $\Delta\nu$ to obtain the calculated values of Regime 4 shown in Figures 10, 11, and 13.

An interesting result from this analysis is that in Regime 4, nozzles designed for Mach numbers between 1.5 and 4.0 all yield basically the same performance with the Mach 1.0 nozzle having performance which is superior to the supersonic nozzles. These observations are also borne out by literature data, both rocket and turbine types.

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13. ABSTRACT

This report is presented as the final document to be submitted in accordance with ONR Contract N00014-66-C0204, with Sundstrand Aviation, Rockford, Ill., to satisfy the specific requirement of Section F(a) of the Contract Schedule wherein a final report shall be submitted at the end of the twelve months after initial date of contract. (U)
The object of this study was to obtain information to improve the off-design performance of supersonic turbine nozzles. The scope of study entailed a literature search, a theoretical analysis, and an experimental program. (U)

A total of 60 reports were reviewed during the literature search; comments on each are presented. The theoretical study shows that off-design performance can be successfully predicted. The experimental information obtained supports this theory. (U)

The ratio of specific heats of the working fluid was found to be influential on the off-design performance. Further study is required, however, before definite conclusions may be reached. (U)

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