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TECHNICAL TRANSLATION

COMPARISON OF THE ACCURACY OF TRIANGULATION, TRILATERATION, AND TRIANGULATION-TRILATERATION

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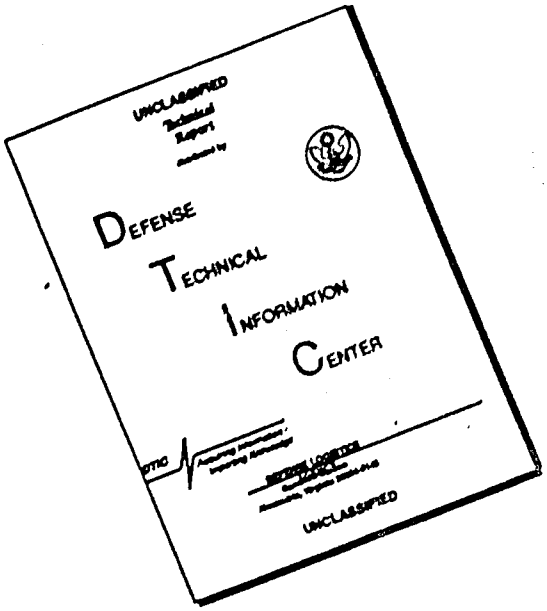

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TRANSLATION

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COMPARISON OF THE ACCURACY OF TRIANGULATION, TRILATERATION, AND TRIANGULATION-TRILATERATION
(Sravneniye tochnosti uglovoy, lineynoy, i lineyno-uglovoy triangulyatsii)

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COMPARISON OF THE ACCURACY OF TRIANGULATION, TRILATERATION, AND TRIANGULATION-TRILATERATION

1

The application of precise radar and light-beam distance-measuring equipment makes it possible to perfect methods of constructing geodetic networks. Along with triangulations, consisting of triangles with measured angles, geodetic networks may be established by the method of trilateration when one measures only the sides, and by the method of triangulation-trilateration when the triangles' angles and sides are measured. The method of polygonometry should obtain wider application.

With modern methods of geodetic measurements, the problems of creating a single system of coordinates over the whole territory of the Soviet Union may be solved most successfully by constructing polygons of an astrogeodetic net. The accuracy of constructing this net is characterized by the accuracy of lengths and directions of the polygons' sides which are the diagonals of the links. Hence, special attention in constructing the link must be paid to achieving a high accuracy of length and direction for the diagonals (that is, the longitudinal and transverse displacement of the link).

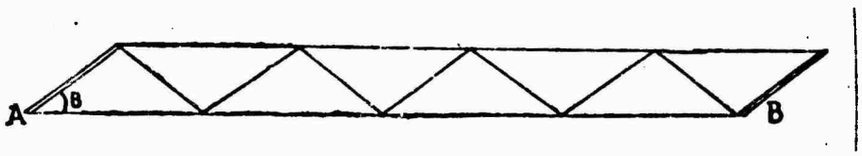

Fig. 1

Table 1

Survey Link	Values of connecting angles of triangles: Mean square errors (in meters)		
	60°	30°	15°
1. Triangulation			
m_t	± 0,65	± 1,95	± 4.20
m_u	0,65	0,65	0,65
2. Trilateration			
m_t	0,20	0,21	0,21
m_u	1,11	6,9	17,7

Table 1 shows the longitudinal (m_t) and transverse (m_u) mean square errors of link AB extending 200 km. and consisting of 16 isosceles triangles with various connecting angles B (figure 1). The errors are found by rigorous formulas deduced for the link by triangulation adjusted by angles for figure, azimuth, and base conditions; and for the link by trilateration adjusted for the azimuth condition. The mean square error m" of a measured angle was accepted equal to ±0".7, and the same for a measured side, m_s, equal to ±0.1 m.* This corresponds to a relative error of 1:250,000 for a 25-km. length of an intermediate side of a triangle in the investigated link.

From the table it is seen that in a link by triangulation the transverse error m_u is completely independent of the value of the connecting angles of isosceles triangles, while the longitudinal error m_t rises sharply as the connecting angles grow smaller. In contrast, for a trilaterated link the longitudinal error is practically independent of the connecting angles' values, while the transverse error rises sharply as these angles diminish. The overall displacement M for the end of diagonal AB, found by the formula:

$$M = \sqrt{m_t^2 + m_u^2} \qquad (1)$$

with connecting angles of 60° amounts to ±0.92 m. for the triangulated link and ±1.13 m. for the trilaterated link. At angles of 30° and less, the overall displacement in both cases will become unacceptably large. Thus, the methods of triangulation and trilateration guarantee a satisfactorily high accuracy of constructing a link only with connecting angles not less than 50 or 60°.

* The error of measuring sides was taken to be constant, independent of the side length, which approximately corresponds to the character of measurement errors for geodimeter-type equipment.

The longitudinal and transverse errors of diagonal AB in a triangulated-trilaterated link (figure 1), adjusted for figure, side, and azimuth conditions with L = 220 km., N = 16 (L is length of diagonal, N is number of triangles in link), m" = ±0".7 and m_s = ±0.1 m., are shown in table 2.

Table 2

Survey Link	Mean Square Errors (in meters) Values of Triangles' Connecting Angles			
	60°	30°	15°	0°
1. Triangulation-trilateration				
m_l	±0,16	±0,18	±0,18	±0.18
m_u	0.57	0,64	0.64	0,64
2. Triangulation-trilateration without measurement of connecting angles.				
m_l	0,16	0,18	0,18	0.18
m_u	0,73	0.78	0,78	0.78
3. Triangulation-trilateration without measurement of intermediate sides.				
m_l	0,23	0,35	0,39	0,40
m_u	0,73	0,67	0,64	0,64
4. Triangulation-trilateration without measurement of connecting angles or intermediate sides (polygonometry).				
m_l	0,25	0.35	0.39	0.40
m_u	0.86	0,81	0,79	0,78

It is seen from table 2 that in a triangulated-trilaterated link constructed of isosceles triangles (example 1), both the longitudinal and transverse errors are practically independent of the values of connecting angles. This conclusion remains true even for the case where the connecting angles are reduced to 0° (last column of table 2) and the survey link essentially degenerates into a polygonometric link. The overall displacement M of the diagonal end, found by formula (1), varies within limits of 10% (from 0.65 to 0.59 m.) for connecting angles from 0 to 60°.

In the discussed link, if we do not measure the connecting angles and limit ourselves to measurement of all sides and only the intermediate angles of triangles (example 2), the longitudinal error will remain the same as in example 1, while the transverse error will increase by about 20%.

If in the survey link we measure all angles and only the connecting sides (example 3), then in comparison with example 1 the transverse error will vary little while the longitudinal error will rise sharply.

Finally, if in the triangulation-trilateration we measure neither the connecting angles nor the intermediate sides (example 4), we obtain the usual polygonometric link: in this case, the longitudinal and transverse errors will be quite great in comparison to the first three examples.

The case where we measure, in a link, all angles and intermediate sides of odd-numbered or all triangles is not examined, since the accuracy of point positions will drop sharply with decreasing connecting angles.

It must be noted that the sharp rise of longitudinal errors with decreasing connecting angles in examples 3 and 4 is a consequence of the constancy of the error m_s of the sides' measurement, which leads to the growth of the relative error m_s/s of the connecting sides. In the first two examples of table 2, the longitudinal error is determined basically by errors of the intermediate sides, for which the m_s/s values will also be the same with constant m_s, since the lengths of these sides do not depend on the value of the connecting angles. Therefore the longitudinal error in the first two examples remains constant. If in examples 3 and 4 the relative error of connecting sides is made a uniform value equal to 1:250,000 (that is, the same as is obtained for intermediate sides in examples 1 and 2), the transverse error will scarcely change, while the longitudinal error will prove to be about the same as in examples 1 and 2 (see table 3).

Table 3

Survey Link	Mean Square Errors (in meters)			
	Value of Triangles' Connecting Angles			
	60°	30°	15°	0°
1. Triangulation-trilateration without measurement of intermediate sides.				
m_l	± 0,23	± 0,20	± 0,20	± 0,20
m_u	0,73	0,65	0,64	0,64
2. Triangulation-trilateration without measurement of connecting angles or intermediate sides (polygonometry).				
m_l	0,25	0,21	0,20	0,20
m_u	0,86	0,79	0,78	0,78

The precise formulas for errors of elements of an adjusted triangulation link are quite cumbersome. Besides, the mean square errors themselves are obtained from the adjustment with errors up to 10-15%. Hence, in estimating the accuracy of a link it is worthwhile to use approximate formulas, which may be considered to have the same value with an error down to a few centimeters for trilaterated, triangulated, and trilaterated-triangulated layouts, as well as for polygonometry (if the link lies in a sufficiently straight line). These formulas are the following:

Longitudinal Error

a) when measuring three sides in each triangle (example 2, table 1; examples 1 and 2, table 2):

$$m_l = m_s \sqrt{\frac{N}{5}}, \qquad (2)$$

b) when measuring only the connecting sides -- when the error of side measurement is constant (that is, m_t = const) (examples 3 and 4, table 2):

$$m_l = m_s \cos B \sqrt{N}, \qquad (3)$$

When the error of side measurement is proportional to the distance, m_s/s = const (examples 1 and 2, table 3):

$$m_l = \frac{m_s}{s} \frac{L}{\sqrt{N}}, \qquad (4)$$

c) In the case where the sides are not measured (example 1, table 1):

$$m_l = \frac{m''}{\rho''} L \operatorname{ctg} B \sqrt{\frac{N}{6}}, \qquad (5)$$

(ctg=cotangent)

Transverse Error

a) When measuring three angles in triangles (example 1, table 1; examples 1 and 3, table 3; example 1, table 3):

$$m_u = \frac{m''}{\rho''} L \sqrt{\frac{N}{18}}, \qquad (6)$$

b) When measuring only the intermediate angles (examples 2 and 4, table 2; example 2, table 3):

$$m_u = \frac{m''}{\rho''} L \sqrt{\frac{N}{12}}, \qquad (7)$$

c) In the case where the angles are not measured (example 2, table 1):

$$m_u = m_s N \operatorname{ctg} B \sqrt{N(9 + 10\cos 2B)}. \qquad (8)$$

3

Analysis makes possible several general conclusions concerning the construction of links in the astrogeodetic net.

Constructing the link by the method of triangulation guarantees high accuracy of the diagonal's elements. A disadvantage of this method is the sharp drop of accuracy of linear elements with decreasing connecting angles. This calls forth the necessity of paying attention to the forms of triangles, to the detriment of the best observance of other conditions in laying out the triangulation and in its employment (best access to points, minimum height of signals, best conditions for observation, most suitable emplacement of points for geodetic work, etc.).

In the trilateration link, with modern accuracy of line measurement by geodimeter, the length of the diagonal is determined with high accuracy, while the accuracy of its direction (transverse displacement) falls rapidly with decrease of the connecting angles. A disadvantage of this method is the poor verification of measurements with only one condition equation available.

The essential disadvantage of triangulation-trilateration methods is the great interdependence of longitudinal and transverse displacements obtained by the identical measured elements, which to a certain extent disturbs the rigor of polygons' adjustment in the astrogeodetic net by the method of F. N. Krasovskiy.

In the triangulated-trilaterated link, both the length and the direction of the diagonal are determined in the best way, and their accuracy is practically independent of the triangles' form. The latter circumstance greatly simplifies the reconnaissance of points, doubtlessly leads to a lower height of signals, and probably produces a somewhat higher accuracy of field measurements, due both to the lower signal height and to the choice of point locations corresponding to the most favorable measurement conditions. At the same time, the organization of field work is greatly eased, since all points may be placed along a narrow strip of land confined to a certain natural or artificial alignment.

In triangulation-trilateration there is an extremely rigid check on linear and angular measurements, since the number of condition equations in this case exceeds the number of triangles by more than three times. It should also be noted that with all variations of constructing the triangulated-trilaterated link shown in table 2 (particularly with sufficiently small connection angles), an extremely small dependence of length errors on the diagonal's direction is provided.

A disadvantage of the method of triangulation-trilateration is the great volume of field measurements (angles and lines). From the viewpoint of the least volume of field work, the most advantageous method of constructing a link is the method of polygonometry, which at the same time fully provides a satisfactory accuracy of adjusted elements. However, the poor check on angular measurements (only azimuth condition available) and lack of a check on linear measurements compel us to apply polygonometry with great caution in constructing the links of an astrogeodetic net, especially in regions of difficult access where the measurements must be made with great certainty in view of the huge waste of funds in case of repeated measurements.

An acceptable variation of link construction may be the method of constructing a triangulation-trilateration without measurement of connecting angles, or with their measurement (for a check on intermediate angles) in three or four observation sets as a maximum, for example. If Laplace azimuths are determined in such a link every tenth triangle, then with $m_s = \pm 0.1$ m. the longitudinal error will amount to ± 0.15 m., while the transverse error, with $m'' = \pm 0''.7$ and L = 125 km., will amount to ± 0.46 m., which corresponds to an error of $\pm 0''.64$ in the diagonal's direction. In accuracy, this method is not far behind the method of complete triangulation-trilateration and at the same time has a sufficient check on linear and angular measurements. This method also preserves all the other technical-economic advantages of triangulation-trilateration.

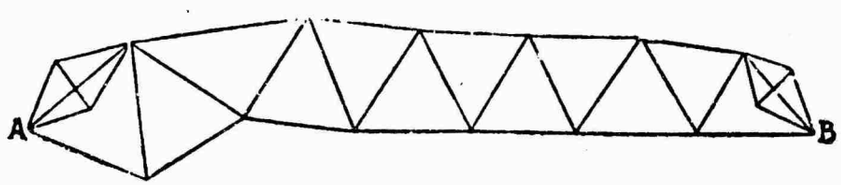

Fig. 2

Figure 2 shows a proposed first-order triangulation link in a region of Western Siberia, set up according to cartographic data. With $m'' = \pm 0''.7$, the longitudinal and transverse errors of diagonal AB, having length L = 189 km., are found from adjustment to be:

$$m_t = \pm 0.51 \text{ m.}$$
$$m_u = \pm 0.48 \text{ m.}$$

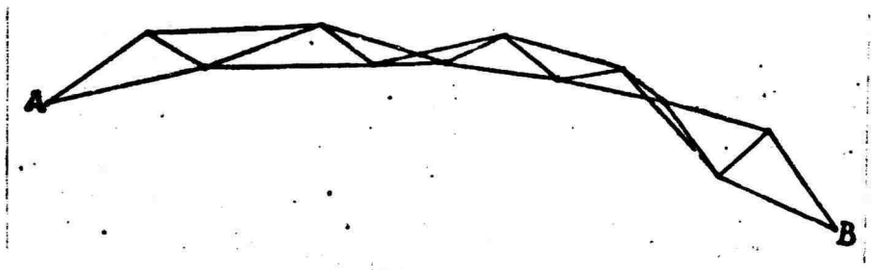

Fig. 3

Figure 3 shows a triangulated-trilaterated link without measurement of connecting angles proposed for the same end points A and B as in the triangulated link. The accuracy indices of this link, with $m'' = \pm 0''.7$ and $m_s = \pm 0.1$ m. are found from adjustment to be the following:

$$m_t = \pm 0.16 \text{ m.}$$
$$m_u = \pm 0.51 \text{ m.}$$

4

The accuracy of the continuous triangulation net has been investigated in quite some detail. It is known that modern second-order continuous triangulation nets represent extremely rigid geodetic structures characterized by high accuracy of adjusted elements. The accuracy of homogeneous elements in various parts of the net is approximately homogeneous and the mutual longitudinal and transverse displacements of any two (adjacent or non-adjacent) points are mutually equal. In continuous trilateration nets, the mutual position of points is also determined with high accuracy, though the transverse error in this case is about 1.5 times greater than the longitudinal.

Table 4 shows the errors of mutual position for adjacent points of a continuous net of right triangles, found by the formulas:

a) for a continuous triangulated net:

$$m_t = m_u = \frac{sm''}{10\rho''}\sqrt{N+15}, \qquad (9)$$

b) for a continuous trilaterated net:

$$m_t = \frac{5}{6} m_s; \quad m_u = \frac{6}{5} m_s. \qquad (10)$$

Table 4

Mean Square Errors				
Relative Error of Measured Side	Of Measured Angle	Of Mutual Position of Adjacent Points (in cm.)		
$m_s : s$	m°	m_l	m_β	M
1. Continuous triangulation net.				
—	1",0	±2,1	±2,1	±3,0
2. Continuous trilateration net.				
1 : 200000	—	3,2	4,5	5,5
1 : 300000	—	2,1	3,0	3,7
3. Continuous triangulation-trilateration net.				
1 : 200000	1",0	1,8	1,9	2,6
1 : 300000	1",0	1,5	1,7	2,3

Here N is the number of triangles between sides of departure, and s is the length of the triangle's side. In computing errors, we took N = 16, s = 7.5 km.

Thus, the profit of applying the method of triangulation or trilateration in constructing continuous nets, in view of modern abilities to measure angles and distances, must be decided on the basis of the organizational-economic advantages, since the accuracy of point positioning is sufficiently high in both cases. The advantage of triangulation is the more rigid check on field measurements. Thus, this method gives rise to a number of conditions equal to the number of triangles plus twice the number of central systems (not counting the base, azimuth, and coordinate conditions), while in the continuous trilaterated net only one condition arises for each central system.

In the general case, obtaining the formulas for a continuous trilaterated net is extremely cumbersome. Therefore, in computing the errors shown in table 4; this formula was applied:

$$\frac{1}{m^2_{\lambda-\gamma}} = \frac{1}{m^2_\gamma} + \frac{1}{m^2_\lambda}, \qquad (11)$$

It was obtained by us for the angular elements of a triangulation link and was extended (with an error of not more than 20%) to errors of linear elements. In this formula, $m_{\lambda-\gamma}$ is the mean square error of an adjusted element of the triangulated-trilaterated net, and m_γ and m_λ are the corresponding errors of a triangulated and a trilaterated net. As is seen from table 4, the construction of a continuous net by the method of triangulation-trilateration provides a higher accuracy

of adjusted net elements. However, the application of this method in extensive work is considered unprofitable due to the great volume of field measurements. Apparently, the need to set up a triangulated-trilaterated net arises only in particular cases, for special nets of the highest accuracy.

We are not considering here the question of applying polygonometry to the construction of continuous nets, which is a separate great problem.

The author confronts the problem of examining the possible (from his viewpoint) variations of constructing a geodetic net on the basis of known methods of measuring geodetic quantities. The final solution of this responsible and difficult question may be achieved only as a result of judging the various viewpoints, and by taking account of all organizational-technical details of conducting and utilizing the state geodetic work.