DIGITAL TRACKING LOOP FOR HIGH-POWERED, EXPERIMENTAL RADAR

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DIGITAL TRACKING LOOP FOR HIGH-POWERED, EXPERIMENTAL RADAR

by

Edward Cleveland Fraser

ABSTRACT

The problem of utilizing a digital computer as one element of a radar system for tracking of space objects, such as satellites, is investigated. The basic principle of curve extrapolation for digital tracking is mentioned and the fitting of either a first or second order curve to three points is developed.

The facilities of the Millstone Radar site, where the experimental work was done, are described in detail, with emphasis on the role each component plays as an element of a closed-loop tracking system. For purposes of comparison, a brief analysis of the operation of the present analog tracking system is presented. A computer program for the digital computer is written and each phase of the program explained in detail. The emphasis is on logical methods and techniques for tracking rather than on arithmetic operations.

The system is tested with simulated and real time data. The arithmetic and logical tests indicated perfect operation of the program. Testing with taped data showed the presence of a noise problem. In actual operation, the linear extrapolation produced satisfactory tracking, whereas the second order extrapolation was unstable due to pulse-to-pulse noise.

A method is developed for measuring the mean square tracking error of either the analog or digital tracking system using tape-recorded error signals and an analog computer. The ratio of the value obtained for the digital tracker to that for the analog system is 1.12.

It is recommended that further work be done in several aspects of this field. Since tests showed that noise is an important consideration, the form of the system should be modified to include greater signal smoothing. It is also felt that quadratic extrapolation is still a valid approach but that considerably more than three points must be used. In addition, further real time experimentation should permit the optimization of the variable parameters of the system.

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CHAPTER I
INTRODUCTION

1.1 Millstone Facilities

There is located at the Millstone Hill Field Station of the Massachusetts Institute of Technology, Lincoln Laboratory, a high-powered experimental radar installation. This radar is capable of very long-range observation and is equipped with a fully steerable 84-foot parabolic antenna. One of the principal uses for which this installation was intended is the tracking of satellites, missiles, and other objects in order to help establish their exact location or trajectory in space. A second objective was to provide a radar installation not fully committed to service and hence available for experimental purposes. It is a combination of these two objectives that make the site of interest to this study.

The present tracking capability of the Millstone radar site is accomplished by means of a special-purpose analog computer. This computer interprets the returning radar echoes and from them develops error signals which are in turn used to drive the antenna. This method of tracking has the disadvantage that should the target fade or become invisible to the radar for any length of time, the computer loses its input information and ceases to function as it was intended. As a result, it may lose a track simply due to a fading or scintillating target. The need for a different tracking method which would overcome this difficulty is therefore apparent.

In addition to the facilities mentioned above, there is available at the Millstone site a high-speed digital computer which is capable of both ingesting data directly from the radar equipment, and generating command signals which
may be used to direct the antenna. This computer may therefore be used in place of the analog computer for the processing of the radar data and generation of tracking information. The capability of a digital computer to store information unchanged over periods of time allows utilization of a tracking technique which makes use of past points on the trajectory as well as the present data. In addition, the dynamics of such a tracking system may be easily adjusted to optimum values, since both time and gain factors are represented by numbers stored in the memory of the computer.

1.2 Tracking Techniques

Several techniques for digital radar tracking have been described in the literature, some in connection with gun fire control systems. Most of these follow the same basic principle of extrapolating a curve that has been fitted to previously obtained points. The method of obtaining points and curve fitting varies among the several proposals. For the purposes of this study, the method to be employed is dictated to a great extent by the existing equipment at the Millstone site. The antenna drive system is divided into two axes, azimuth and elevation. Likewise, the data processing equipment is so arranged that each returning radar echo may be identified as being in the azimuth or elevation plane. Since the position of an object in space may be specified insofar as tracking is concerned by its azimuth and elevation angles, the natural conclusion was to use two identical but separate channels, one for each axis. The function of these channels would be to establish a curve of either elevation or azimuth angle as a function of time, and provide the facility for extrapolating these curves so as to continually position the antenna so that it is aimed at the point in space where the target is expected to be. The specific manner in which the data points are obtained involves a twofold process. First, the radar echoes are processed to obtain an angular error, the difference between the antenna pointing angle and the actual position of the target. This error angle is then added to the antenna pointing angle to obtain a true point on the curve. When seven successive such points have been collected, a weighted average is taken about the median point. The resulting value, and its corresponding value of time, determine what is called a "smoothed point", which is considered to be an exact point on the trajectory of the object being tracked. The purpose of performing this smoothing operation is to reduce the effects of system noise that may perturb individual points. After three such smoothed points have been obtained, a parabola is fitted to them by the method of Lagrangian polynomials. The curvature of this parabola is examined by comparing the coefficient of the $t^2$ term with a reference. If it is larger, the parabola is used as the extrapolating curve. If it is smaller, then a straight line is fitted to the three points in the least mean square error sense. The equation of the parabola or straight line is then evaluated for future values of time and the resulting angles used to position the antenna. The extrapolating equation is recalculated every time a new smoothed point is obtained. It is inherent in this scheme that the tracking, once begun, will be relatively insensitive to a loss of data, but will correct itself as soon as it receives fresh information.

1.3 Summary of Investigation

The computer program required to enable the digital computer to perform this automatic tracking has been written and thoroughly tested. Checkout of the program was begun by supplying it with data generated from a curve, the equation of which was already known. In this test, the program succeeded in
matching the previously calculated coefficients for the test curve to within one bit in the least significant place. These derived equations were then evaluated at an arbitrary preselected value of time and the results compared with known points on the curve. For both the case of quadratic and linear prediction, the predicted values differed by not more than 3 minutes of arc from the expected values -- this being the granularity of the code used to represent angles in the arithmetic elements of the computer.

The second phase of system testing involved the recording in digital form on punched paper tape of the information that is supplied to the computer during a tracking operation. This tape was then read into the computer at a reduced speed and the open-loop tracking operation of the program observed. It was noted that the output appeared somewhat noisy. Further investigation showed that this was due to noise carried on the received signal, and that in spite of the smoothing operations, it propagated through to the system output. This, however, was not considered extremely serious since the antenna and associated drive equipment may be considered as essentially a long time constant multiple integration, or very low pass filter. Any noise contained in the output commands of the computer will therefore be filtered out by the relatively low bandwidth of the antenna system.

Final testing involved the operation of the entire site as a closed-loop tracking system with the digital computer being used in place of the present analog tracking system. The analog tracker was used for the initial acquisition stages of the track. Once it was determined that the site was tracking satisfactorily, the computer indicated that it had assimilated sufficient data to generate prediction equations. Control of the antenna was transferred to the digital servo outputs of the computer. In each of four such tests the system achieved a successful track, each lasting for a period of at least three minutes. During each of these trials, tape recordings were made of the error signals generated by the analog tracker for each axis. The mean square value of these error signals was determined and used as a figure of merit for comparison with the performance of the analog tracker. The two sets of figures compare quite favorably, even though the parameters of the digital tracking system may not be set to optimum values.

1.4 Results and Conclusions

The results of this study demonstrate that a digital computer may be used to considerable advantage as a data processing element in a real-time tracking operation. It has the capability not only to track objects equally as well as the present tracker, but in addition, it can maintain a track through a target fade or scintillation. Also, the time required by this program to execute one complete cycle is less than one-half of the radar interpulse period. It is therefore possible that other data processing programs may also be run in the computer at the same time that the digital auto track program is operating.
CHAPTER II
PRESENT SITE FACILITIES

Before discussing the more specific features of this tracking system, and particularly the computer program, it would be appropriate to more completely describe each of the components used in the overall closed-loop system.

2.1 Antenna and Feed

The antenna is a parabolic dish, 84 feet in diameter, whose axis may be aimed in any direction by means of azimuth and elevation drive systems. The antenna, together with its drive and support structure, may be seen in Fig. 2.1 which is an aerial view of the entire site. The parabolic reflector is illuminated by a waveguide-supported mutating feed horn whose aperture is located at the focus of the parabola. Figure 2.2 shows the details of this feed structure. The mutation of the feed horn about its axis causes the center line of the antenna beam to describe a circular cone in space whose vertex is located at the center of the antenna dish. The included angle between diametrical elements of this cone is determined by the angular offset of the center of the feed horn aperture from the central axis of the reflector and is commonly referred to as the "squint angle". At the Millstone site, this angle has been measured as 1.7°. This conical scanning, together with the relatively narrow 2.25° beamwidth of the Millstone antenna, produces an amplitude modulation of the returning radar echoes. The amplitude of this modulation is a measure of the error between the antenna pointing angle and the true target angle, and its phase, relative to a reference signal, is proportional to the target position relative to the antenna axis; that is, the angle between a plane containing the target and the antenna axis and a vertical plane passed through the axis of the antenna.
2.2 Scanner Amplitude Modulation

The exact nature of this modulation is not known as it is affected by irregularities in the antenna pattern, propagation characteristics, and the position of the target relative to the antenna axis. The fundamental principle, however, may be illustrated by Fig. 2.3. Assume for purposes of illustration that the attenuation of power illuminating the target, due to its being off the axis of the antenna beam, is proportional to the square of the angular displacement between the beam axis and the target direction, i.e.,

\[ \alpha = \sigma d^2 \]

where \( d \) is the displacement in radians and \( \sigma \) has units of decibels. The experimentally measured antenna pattern shown in Fig. 2.4 indicates that this assumption is valid over a reasonable arc of the pattern. Since this off-axis attenuation is effective for both transmission and reception, the total attenuation of any signal will be \( 2\sigma d^2 \). This figure, however, applies to the diminution of power, whereas in this study, the quantity of primary interest is amplitude. Since power is proportional to the square of the amplitude, then the attenuation of amplitude, when expressed in decibels, will be half that for the corresponding value of power. The total amplitude attenuation for any off-axis condition is then given by

\[ \sigma_a = \sigma d^2 \]

The amplitude of any given pulse may then be expressed as

\[ A = A_{\text{max}} \cdot 10^{-\sigma_d^2} \]

\[ = A_{\text{max}} \cdot e^{-\frac{\sigma}{m} d^2} \]

where \( A_{\text{max}} \) represents the amplitude for the case of zero attenuation. If the power series expansion for the exponential is used, the above expression
ATTENUATION (db)

\[ A = A_{\text{max}} \left[ 1 - \frac{a}{m} d^2 + \frac{a^2}{m^2} d^4 - \ldots \right] \]

If only the first two terms are used, the error involved does not exceed 10% when \( d \) is within the confines of the antenna beamwidth. The expression then reduces to

\[ A = A_{\text{max}} (1 - ka^2) \]

It is convenient at this point to assume that the target is located on an element of the cone described by the axis of the antenna beam. The displacement of the target from the beam axis may then be described by

\[ d = 2r \cos \left( \frac{\theta}{2} + \varphi \right) \]

where \( \theta \) represents the rotation of the beam axis around the scanning cone, and \( \varphi \) is an arbitrary phase angle to allow for different positions of the target on the cone. The amplitude of the received signal will therefore be given by

\[ A = A_{\text{max}} \left[ 1 - k\pi^2 \cos^2 \left( \frac{\theta}{2} + \varphi \right) \right] \]

\[ = A_{\text{max}} \left[ 1 - 2k \pi^2 (1 + \cos \theta + 2 \varphi) \right] \]

\[ = A_{\text{max}} \left[ (1 - 2k \pi^2) - 2k \pi^2 \cos (\theta + 2 \varphi) \right] \]

If \( \theta \) is now taken as a linear function of time, as it would be for the case of the scanner rotating at a constant velocity, then the received signal is seen to be amplitude-modulated by a sinusoidal signal (see Fig. 2.5) whose magnitude is proportional to \( 2k \pi^2 \), and whose phase is determined by the \( 2 \varphi \) term which specifies the location of the target on the scanning cone.

Its value must be such that \( A \) has its maximum value when the beam axis and the target direction are coincident. Note that the frequency of the modulating signal is exactly equal to the rate of rotation of the scanner.
The fact that the form of the modulating waveshape reduced to a pure sinusoid is due to the assumptions made regarding the beam pattern and the target location. In general the modulation will not be sinusoidal, but it will be periodic with a fundamental frequency which is exactly equal to the rate of rotation of the scanner. A derivation of an exact representation of the modulation resulting from the mutation of the scanner has not been attempted since the grossness of some of the assumptions required would render the result of questionable validity.

2.3 Effect of Pulses

In the above discussion, no mention has been made of the fact that the Millstone radar is a pulsed radar rather than continuous wave, nor was the effect of transit time considered. Each of these considerations modifies the foregoing discussion to some extent. The pulse repetition rate of the Millstone radar is 30 pulses per second. The mutating scanner makes 7.5 revolutions per second, resulting in 1/4 radar pulses per scanner revolution. The scanner drive system is so arranged that alternate pulses occur either in a vertical or horizontal plane. Referring to Fig. 2.5, the arrows indicate the received amplitudes for a target located as shown in Fig. 2.6.

The amplitude of two alternate pulses may be expressed as

\[ A_1 = A + B \cos (\theta + 2 \phi) \]
\[ A_2 = A + B \cos (\theta + \pi + 2 \phi) \]
\[ A_3 = A + B \cos (\theta - 2 \phi) \]
\[ A_4 = A - B \cos (\theta + 2 \phi) \]
\[ A_5 = A + B \cos \phi \]
\[ A_6 = A - B \cos \phi \]
where $\beta = \theta + 2 \phi$ and must equal the angular scanner rotation between the target and the point at which the pulse is fired, as shown in Fig. 2.6. It is easily shown from the figure that

$$\cos \beta = \frac{b-a}{b+a}.$$  

The signals then become

$$A_1 = A + B \frac{b-a}{b+a},$$

$$A_2 = A - B \frac{b-a}{b+a},$$

$$A_1 - A_2 = 2B \frac{b-a}{b+a},$$

$$b-a = \frac{b+a}{2B} (A_1 - A_2).$$

It is evident from the diagram that the quantity $b-a$ is twice the component of the error in the plane of the two pulses being considered. The error is then expressible as

$$e = \frac{b-a}{2} = \frac{b+a}{AB} (A_1 - A_2).$$

For this special case, then, the angular pointing error for the axis considered is directly proportional to the difference in magnitude of the two pulses associated with that axis. Again, the fact that this relationship resolves to a simple linear form is due to the simplifying assumptions made. In general the angular error is not a linear function of the difference in amplitude between two consecutive pulses in either axis, but experience indicates that a linear approximation is sufficiently valid for operational purposes. One further point is worthy of note; the amplitude of a given pulse is not only affected by the component of error in the axis associated with that pulse, but also by the error in the orthogonal axis. This would appear to cause 

\[21\]
talk, but, fortunately, the attenuation caused by an error in the orthogonal axis is equally effective on each of the two pulses concerned. Hence, although the effective constant of proportionality may now be different, the difference of the amplitudes is still a function only of the error in the axis under consideration.

2.4 Transit Time Effect

In the previous discussion the radar pulse has been presumed to be transmitted and received at the same instant of time, when the rotating antenna is located at each of the cardinal points of the scanning circle. Since this installation is designed for long-range observation, this condition is not true, nor can it be assumed true as an approximation. To compensate for this transit time, the scanner is caused to lag slightly behind the performance described earlier. Its phase is so adjusted that the feed horn passes through a cardinal point of the scanning circle exactly one-half of the round-trip transit time after the radar pulse has been transmitted. This delay time is, of course, a direct function of the range of the target. Provision has been made to automatically adjust this delay time proportional to range. The effect of this arrangement is to equally space the transmission and reception points on either side of a cardinal point of the scanning circle. In this way, the phase of the apparent amplitude modulation of the received pulses is held fixed, independent of range.

2.5 Receivers

The received signal is conducted by means of waveguide and rigid coaxial cable from the feed horn of the antenna to the input of the receiver. A block diagram of the receiving equipment is shown in Fig. 2.7. The input of the
receiver is protected from possible damage due to coupling between the transmitting and receiving lines by means of a TR tube. This tube is fired whenever the transmitter is delivering energy to the antenna, thus making the input of the receiver appear as a short circuit. A directional coupler is installed at the input of the receiver to allow the insertion of a test signal. The circulator is a passive device which provides isolation between the parametric amplifiers and the antenna to prevent spurious self-oscillation to develop within the parametric amplifier due to changing reflected impedances from the antenna. The parametric amplifiers which provide the "front end" of the receiver equipment are pumped at such a frequency that the intermediate frequency output is 30 megacycles. This is fed through a gated 30 Mcps I.F. amplifier. This amplifier incorporates a variable attenuator allowing the output level to be set wherever desired. A gating signal is supplied to the amplifier which biases it off during the period that the transmitter is delivering power. This operation contributes in eliminating the large amount of noise that is detected by the receiver when the transmitter is radiating. The output of the 30 Mcps I.F. amplifier is supplied to the Automatic Gain Control amplifier which is also supplied with a control signal from the analog tracker. Its purpose is to maintain as constant a level as possible at the output of the receiving equipment. It accomplishes this by changing its internal gain in accord with the signal supplied to it from the average value sensing equipment of the analog tracker. The time constants of this circuitry are long enough, being in the order of one second, that it would not affect the amplitudes of the alternate pulses and thus interfere with subsequent data processing and tracking.

2.6 Doppler Filter Bank

The output of the Automatic Gain Control amplifier is then fed through two heterodyne converters to obtain a 200 kc signal. This signal is amplified and limited to an envelope of ± 3 volts. The resulting signal is impressed on a bank of matched crystal filters, each having a bandwidth of 160 cps and covering the frequency range from 175 kc to 225 kc. Following the filter bank there is a diode network which selects that filter whose response is the greatest and delivers its output waveshape at the output of the filter circuitry. This bank of filters and its associated diode network serve a twofold purpose. First, resonant frequency of the particular crystal filter that produces maximum response for any given pulse is a measure of the doppler shift in the carrier frequency and indicative of the radial velocity of a target. The second—and more important from a tracking point of view—function of the filter system is the generation of a relatively noise-free pulse from a received signal which may be only slightly greater than the accompanying background noise. The output waveshape for any of the individual filters, when excited by a 2-millisecond pulse of energy at its resonant frequency, is shown in Fig. 2.9. This characteristic shape, together with the extremely narrow effective bandwidth of 160 cps, serves to accomplish a considerable noise rejection. A typical signal-to-noise ratio at the input of the filter bank may be 1:8:1, whereas this same signal will produce an output signal-to-noise ratio of 10:1. The peak amplitude of the filter bank output pulse is 2 volts, always positive. These pulses are passed through a variable attenuator to the input of a Transicon Datrac analog-to-digital converter. The purpose of the attenuator is twofold: first, it allows reduction of signal amplitude so as not to exceed the maximum signal input to the converter of one volt.
Secondly, it allows the adjustment of the noise level in the system thereby permitting system adjustment so that the noise never exceeds a predetermined number of binary bits.

2.7 Analog-to-Digital Conversion

The Transico Datrac analog-to-digital converter generates an eight-bit binary number from an input signal whose amplitude may be between 0 and 1 volt. The smallest distinguishable division is therefore 1/256 or .0041 volts. In addition, an extra bit is provided to indicate the polarity of the input voltage. Since the output pulses of the filter bank are always positive, this bit is not used for tracking operations. The time at which the converter samples its input to produce an output number is determined by a trigger pulse derived from the range gate. This pulse is automatically positioned in time so that the sampling occurs at the time indicated by the vertical bar in Fig. 2.9. Small variations of this time from its correct value, due to system jitter, will tend to introduce noise into the signal since the waveshape being sampled is not a constant but has a definite slope at the sampling time. This situation could perhaps be remedied, but it is unavoidable with present equipment. The input to the converter is adjusted for each operation in such a manner that the background noise will produce a number no larger than the 3 least significant bits, whereas the signals usually encountered produce numbers of at least 6 and occasionally 7 bits.

2.8 Buffers

The numerical data from the converters are transferred to the computer by means of two time-sharing buffers, one associated with the data processing equipment and the second with the computer. The data-processing buffer supplies to the computer buffer either the digital amplitude information, as described...
above, or digital range and range rate information as obtained from the data processing equipment. One word of each of these types of information is supplied during each radar cycle, the range and range rate information being supplied first. The data processing buffer has provision for storing the digital amplitude data until they are called for by the computer buffer. It also generates the required synchronizing pulses to properly load the information into the computer buffer.

The computer buffer is essentially a matching element between external circuitry and the circuitry of the CO 24 digital computer. It contains three 25-bit flip-flop registers corresponding to the three live registers of the computer. The first of these registers is used to transfer the digital range and range rate or the digital amplitude data into the \( \nu^1 \) live register of the computer. When handling amplitude data, the 8 bits appear in bits 5 through 12 inclusive of the computer word. Bits 1 through 4 and 13 through 24 of the \( \nu^2 \) register are not used. The second register is divided into two sections. Bits 1 through 11 are used to indicate the antenna elevation angle, and bits 12 through 24 indicate the corresponding azimuth angle. This information is transferred into the \( \nu^2 \) live register. The code used to represent angles in binary form is to consider any angle as its equivalent binary fraction of a revolution, with the radix point being at the left end of each of the numbers. A full revolution is represented by a 13-bit binary number, yielding a granularity of slightly less than 3' per least significant bit. The elevation angle is represented by only 11 bits since its range is limited to 1/4 of a revolution. There are therefore two zeroes understood to be at the left end of the number representing the elevation angle. The digital representation of the antenna pointing angles are obtained from two Norden-Ketay digital shaft position encoders which are driven by a servo repeater system direct from the antenna mounts. The encoders are equipped with a V-brush logic to prevent ambiguities in the angular representation, and also are so arranged that they may not be read while any of the bits are changing state.

The third buffer register is a binary clock which presents real time as a binary number. It is operated by counting a 512 cps tuning fork controlled oscillator. The least significant bit of the clock number that is transferred to the computer represents 1/128 of a second. The smallest increment of time that is therefore detectable using this clock is just less than 6 milliseconds. The digital time is stored in the \( \psi^2 \) live register. Two further pieces of information are transferred into the live registers of the computer from the buffer. The zeroth bit of the \( \psi^2 \) register contains the so-called U-pulse. If this bit is a one, the \( W^1 \) register contains range and range rate data; if it is a zero, the \( W^2 \) register contains amplitude data. The zeroth bits of registers \( \nu^1 \) and \( \nu^2 \) indicate the position of the beam axis relative to that of the antenna. Bit \( W^1 \) is one if the beam is in the azimuth plane, and zero if it is in the elevation plane. Similarly, bit \( W^2 \) is one if the beam is down or to the left, and zero if it is up or to the right. These directions are referenced to a point at the base of the dish facing out into space. The scanner, of course, will be diametrically out of phase with this. The two bits, \( W^1 \) and \( W^2 \), form a two-bit counter where the \( W^1 \) bit is the least significant. This counter counts radar transmitted pulses and changes state by one shortly after the trailing edge of the transmitted pulse.

The counter is initially set to the CO state by a magnetic pip generator that is excited as the mutating feed horn passes through bottom dead center.
The 00 state therefore corresponds to the case of elevation plane, beam up. Since the counter is indexed before the feed horn passes through the cardinal points, the pip is not needed after the initial reset. It continues to operate, however, and will correct the counter in the event an error occurs.

2.9 CG 24 Digital Computer

The CG 24 computer is a high-speed, internally programmed, general purpose digital computer. It has a word length of 24 bits plus sign, and a magnetic core memory storage capacity of 8192 words. One memory cycle requires 12 microseconds for completion, and the range of instructions require from 1 to 25 memory cycles for their execution. At present the computer is equipped to recognize and execute 39 different instructions. Input channels include a punched paper tape reader, toggle switches, Flexowriter and the live registers via the computer buffer. Output equipment includes a high-speed paper tape punch, oscilloscope display, neon indicator lights, Flexowriter, and the digital servo. It is the latter element in each group that is of primary interest to this study. The three live registers are each 25 bit words where each bit is represented by the state of a flip flop. These flip flops are reset each time fresh information is loaded into them by the buffer. The buffer, however, is constrained not to supply new information until the contents of the live registers has been sampled by means of the programmed instruction "store data". This insures that new data will not be loaded into the live registers on top of old data, resulting in mere gibberish. The "store data" instruction stores the contents of the three live registers in three consecutive registers at any specified location in memory and clears the way for fresh information to be loaded into the live registers from the buffer.
2.10 Digital Servo

The computer may be used to generate position command signals for the antenna by means of the digital servo mentioned above. It consists of two identical channels, one for elevation and the other for azimuth. The block diagram of one of these is shown in Fig. 2.11. The input to each of these units is a flip-flop register which serves as a buffer between external circuitry and the computer. The contents of the B register of the computer may be transferred into these flip flops by means of the "load hoff" instruction. The digital servo circuitry interprets bits 2 and 3 of this register as indicators; if bit 2 is a one, the desired angle is in the elevation plane; if bit 3 is a one, the desired angle is in the azimuth plane. Bits 4 through 15 are interpreted as the sine of the desired angle where bit 4 is the algebraic sign, and, similarly, bits 16 through 24 are interpreted as the cosine of the desired angle where bit 14 is the algebraic sign. The digital sine and cosine numbers are used to actuate the mercury relays of a ladder-type digital-to-analog converter, thus producing voltages proportional to the sine and cosine of the desired angle. These voltages are amplified and impressed on the two orthogonal stator windings of a resolver and a conventional positional instrument servo system used to rotate the rotor winding to a null. The characteristic of the resolver is that the angular position of the null is in quadrature to the angle whose sine and cosine are impressed on its stator windings. Geared to the shaft of the resolver, but 90° out of phase with it, are the 1 and 36 speed control transformers used as input devices for the main antenna drive system. The response time for this system is not a constant since the mercury relays behave in a statistical manner. However, measurements have shown that the positional system will reach a condition of zero error in not more than 70 milliseconds after being excited by a 1° step input.

2.11 Antenna Drive System

The main drive system for either axis of the antenna is a fairly straightforward, two-stage, Ward Leonard positional control system employing both position and velocity feedback. The major components, as shown in Fig. 2.12, include a main servo amplifier, amplitidyne, DC generator, DC motors, gearing, and the mass of the antenna. The functional block diagram for this portion of the system, as well as the analog auto tracker, is given in Fig. 2.13. The transfer functions shown are valid only when the system is operated within its linear region, and for the values of moment of inertia and viscous friction assumed. The moment of inertia of either axis is not a constant but varies as a function of the position of the antenna due to its geometric irregularity. Similarly, the friction may not be constant as it is strongly related to temperature, the ambient range of which includes values from below zero up to 100°F. The gain constants are presently set in such a way that an angular error of ± 6 minutes is sufficient to cause saturation of the generator field. Except for very small displacements, the system may be considered to be a constant speed drive with a maximum slew rate of 4 degrees per second. The positional error information for the main drive system may be derived from one of several sources. When operating in the position mode, the error signal is supplied from the rotor windings of a control transformer referenced to a second control transformer which is driven by the antenna repeater system. This is shown in Fig. 2.14. When directing the antenna by means of the digital computer, the manual "call for" control transformer is replaced by the control transformer geared to the resolver of the digital servo. Although only one set of control transformers is shown in the figure, the system is actually a two-speed system having a one-speed control transformer for
FIG.2.13 FUNCTIONAL BLOCK DIAGRAM—ANALOG TRACKER AND ANTENNA DRIVE SYSTEM.

FIG.2.12 BLOCK DIAGRAM—ANTENNA DRIVE SYSTEM.

FIG.2.14 BLOCK DIAGRAM—CONTROL SYSTEM.
coarse adjustments, and a 36-speed control transformer for fine adjustments. When the system is being operated in the analog auto-track mode, the input error signal is applied directly to the main servo amplifier from the tracker circuitry and none of the control transformers are used. The velocity feedback is independent of the mode of operation and is obtained from the armature terminals of the DC drive motors. This signal, after being suitably corrected for voltage drops due to resistance in the motor, is used to suppressed-carrier modulate a 60 cps signal which is added to the position error to form the net input to the main servo amplifier.

CHAPTER III
TRACKING PROGRAM

In this chapter the principles of operation used in developing the computer program are discussed. For purposes of comparison, and to point out the differences, a brief description of the operation of the analog tracking circuitry is given.

3.1 Analog Tracker

The output pulses from the crystal filter bank are supplied to the analog tracker since, as was shown earlier, the amplitude modulation of these pulses is a measure of the angular error between target direction and antenna axis. From the pulses a continuous waveform is generated by means of a first order hold or "boxcar" circuit. This has the characteristic of maintaining as its output voltage the peak value of the pulse applied at its input. The train of pulses is thereby transformed into a series of steps. This stepped function is filtered by a very high Q-bandpass filter to extract the fundamental frequency which is equal to the scanning rate. The amplitude of this sinusoidal signal is then proportional to the magnitude of the angular error and the phase of the signal is a measure of its direction. Also supplied to the tracker is a two-phase signal which is obtained from a two-phase reference generator driven directly from the mutating feed horn and at the same speed. This two-phase signal, after suitable phase shifting to compensate for phase shifts inherent in the entire system, is used as a reference signal for a synchronous demodulator circuit. The error signal is rectified by this circuit to produce two DC outputs. The operation of the circuit is such that the DC output produced is proportional to the magnitude
of the input error signal multiplied by the cosine of the angle between its phase and the phase of the reference signal used for that channel. Since the reference generator is aligned in such a way that one phase corresponds to the elevation plane and the quadrature phase to the azimuth plane, the result is that the two DC output voltages produced are the respective azimuth and elevation components of the angular error. These DC signals are then used to modulate suppressed-carrier 60 cps modulators to obtain AC error signals for each axis.

3.2 Rate Memory

The error signal for each axis is used as the input to a positional servo system as shown in Fig. 3.1. The output shaft of this system positions two 10-turn helipots, each supplied from its own variable AC source. One of these is used for position feedback. The time constant for this subsystem is in the order of several seconds so that the output position tends to be a measure of the average of the input error signal. The output voltage from the second helipot is taken as the average error. This signal is added to the actual error signal, and the sum used as the input to the main servo amplifier. The purpose for this is that should the target fade, the analog tracker loses data and its outputs go to zero. Unless other provision were made, the drive system would interpret this condition as meaning that the antenna was right on target and it would stop. Certainly this would not provide satisfactory operation. The addition of the signal from the average error helipot, with its long decay time, means that in the event of loss of data, the antenna will continue to track at a rate somewhat less than when data was available, but nevertheless in the correct direction. At best this system is useful only for short-term
fading since as soon as the input error signal goes to zero, the average value servo system begins to regress toward the zero position. If the target should remain invisible for a period long enough for the antenna to slew to the point that the target passes out of the beam, then the tracker has lost the target and will not reacquire even though it should again become visible.

3.3 Computer Program

The remaining portion of this chapter is devoted to a discussion of the logic involved in the tracking program written for the CG 24 digital computer. Figure 3.2 is a comprehensive flow diagram of the entire program. Figures 3.3 through 3.6 are detailed expansions of individual segments of Fig. 3.2 and will be referred to individually in the following text. The programming for the azimuth axis is identical to that for the elevation axis, hence only the elevation axis will be discussed. Any differences will be noted as they occur.

3.3.1 Data Sorting

The first portion of the program, as illustrated in Fig. 3.3, is common to both axes. It includes facilities for control of the program operation, resetting the program to initial preacquisition conditions, and for selecting and properly channeling data. Toggle switch register 20014 is used to control the operation. Only the zero bit is used: if it is up, the computer proceeds into the main body of the program; if it is down, the computer executes the reset routine and returns to again sample register 20014. As long as it is up, the computer will attempt to ingest data and establish a track. Whenever it is down, the computer will continually cycle through the react routine. At the initial operation of the tracking program, the live

FIG.32 OVERALL FLOW CHART- COMPUTER PROGRAM.
FIG. 3.3 FLOW CHART - DATA READ-IN AND SORTING.

FIG. 3.4 FLOW CHART - GENERATION OF ELEVATION ERROR AND CORRECTED ANGLE.
FIG. 3.5 FLOW CHART - CALCULATION OF POLYNOMIAL COEFFICIENTS.

FIG. 3.6 FLOW CHART - "NO DATA" EVALUATION ROUTINE.
registers are read and their contents stored in memory where they may be repeatedly re-examined. The computer will not execute this operation unless the internal circuitry of the computer indicates that the live registers contain valid information. If they do not, the program reverts to the starting location and waits for information to be supplied from the buffer. Once the data has been stored, the zeroth bit of the $W^1$ word is examined. It is the U-pulse that was mentioned earlier. If it is a one, the information contained in the $W^1$ word is range and range rate data and is of no use to the program. All three data words are therefore ignored and the program returns to the starting location to wait for new information. If it is a zero, then the $W^1$ word contains amplitude data and the program continues. The amplitude of the radar return, as contained in the $W^1$ word, is compared with a threshold level. If it is larger, the data are considered the result of a valid radar hit and the program continues to the next step.

If the amplitude data are less than the threshold, then it is considered to be the result of noise and the program ignores all three words of information and returns to the starting location for new data. The zeroth bit of the $W^2$ word is next examined. If it is a zero, the return is in the elevation plane and the program proceeds in the elevation channel. If it is a one, the return is in the azimuth plane and the program transfers to the azimuth channel.

3.3.2 Generation of Error Signal

Presuming that the return is in the elevation plane, the time of this present return is compared with that of the next previous elevation return obtained, as shown in Fig. 3.4. If the time difference is greater than twice the radar interpulse period, then the two returns are not consecutive in the elevation axis and are not compatible with the rest of the program logic.

In this event, the amplitude and time of the present return are stored to be used in connection with succeeding returns, and all previously collected points which have not been used in subsequent operations are discarded. The reason for this discarding of points is that the form of the smoothing routine used later in the program requires seven consecutive points. As will be shown shortly, this requires that the computer ingest eight consecutive radar returns. If the time difference is equal to twice the radar interpulse period, then the two returns are consecutive and the program proceeds to the elevation data processing.

The magnitude of the present return is added to the amplitude of the last return to obtain the sum. Similarly, the magnitude of the present return is subtracted from the magnitude of the previous return to obtain the difference. The sum will always be positive, but the sign of the difference will be dependent on which of the two returns is larger. In order to maintain the convention that when the target is above the antenna axis the sign of the difference will be positive, it is necessary to change the sign of the difference whenever the present pulse is obtained with the beam in the up position. This is done by examining the zeroth bit of the $W^0$ word. If it is a zero, the beam is up and the sign is changed; if it is a one, the beam is down and the sign is not changed. The same procedure is true for the azimuth axis, with right in place of up, and left in place of down. The difference is then divided by the sum and multiplied by a constant. The constant is made up of two factors: the first is twice the maximum possible magnitude, and the second is the linear coefficient relating amplitude difference to angular error. The resulting expression has the form

$$
\epsilon = \left( \frac{a-b}{a+b} \right) X (a+b)_{\text{max}} k
$$
The effect of dividing the difference by the sum of the magnitudes of the two returns and multiplying by the maximum value of the sum is to scale each of the returns so that the average value of any two consecutive returns is a constant; in this case, the maximum value of the magnitude of any one return. The purpose of this operation is to eliminate the effect of fading or any other perturbation of signal level on the overall gain of the system. The quantity obtained by this operation is the angular error in the elevation plane between the target and the antenna axis. This error is added to the antenna pointing angle to obtain the true position of the target, and this value is stored as one point for the subsequent operation of the program. The value of time stored as corresponding to this point is that value associated with the earlier of the two returns used to derive the true point. The reason for this is that the data for each radar return are supplied to the computer just prior to the transmitter pulse which begins the next radar cycle. The time associated with the earlier return is therefore the closest to the true mean time between the two radar hits.

3.3.3 Smoothing Program

When seven such true points have been obtained, an average point is derived according to the formula

\[ y = \frac{1}{21} (-2y_{-3} + 3y_{-2} + 6y_{-1} + 7y_0 + 6y_{+1} + 3y_{+2} - 2y_{+3}) \]

This is the quadratic least squares smoothing formula for seven points as given in Hildebrand. Other smoothing formulas are available and results to date indicate that perhaps experiments with a lower order formula should be tried. The average point obtained is stored for use in the prediction routines.

The time taken as corresponding to the averaged point is the time for the fourth point of the seven used. This places the averaged point directly in the center of the field from which it was derived.

3.3.4 Predicting Polynomials

Three such smoothed points must be accumulated before the program will proceed. Once these points are obtained, two possibilities are available. The first is to fit a parabola to the three points. The second is to consider any deviation of the three points from a straight line to be caused by spurious noise, and as a result fit a straight line to the three points in a least squared error sense. The criterion used to choose between these two possibilities is the value of acceleration. The acceleration of the antenna the second time derivative of its position is a measure of the curvature of the curve required to represent the elevation angle in time. A zero acceleration gives rise to a constant velocity operation where the curve becomes a straight line. For a condition of high acceleration, a parabola would be needed to properly fit the curve. The \( \alpha_2 \) coefficient is the acceleration of the antenna as determined from any group of three smoothed points; therefore, it is compared with a threshold value to form the basis of a machine decision to utilize a linear or a quadratic prediction polynomial.

If the decision calls for a linear polynomial, a straight line of the form

\[ y = a_0 + a_1 t \]

is fitted to the three smoothed points in a least squared error sense. The derivation of the form of the coefficients \( a_0 \) and \( a_1 \) in terms of the three smoothed points is given in Appendix I. In the event that a quadratic formula is called for, the form of the curve would be
\[ y = a_0 + a_1 t + a_2 t^2. \]

The derivation of the form of the three coefficients of this formula is given in Appendix II.

Since the \( a_2 \) coefficient is used as a basis for the linear-quadratic decision, it is the first quantity calculated after the three smoothed points are obtained. Following the decision, either the remaining two quadratic coefficients or the two linear coefficients are calculated. An indicator bit is also set to one if linear is chosen and zero if quadratic is chosen. This bit is used later in the program to determine which polynomial should be evaluated to obtain an antenna command angle. For convenience in handling the data, the time corresponding to the earliest of the three smoothed points is taken as a base for values of time. The time of the first point is then taken as zero and the time for all subsequent points is measured in seconds after this base time. As soon as a new smoothed point becomes available, the time base is updated and the new point is used together with the two most recent of the previous three to recalculate the polynomial coefficients. In order to facilitate study of the character of the predicting polynomials and enable experimental evaluation of the linear-quadratic threshold, provision has been made to store corresponding values of antenna acceleration \( (a_y) \) and antenna pointing angle in consecutive registers in memory. This storage will be accomplished whenever the quadratic prediction polynomial is chosen and the zero bit of toggle switch register 2015 is down. The \( a_2 \) term will be stored in the form \( ax2^3 \) where \( a \) is stored in the first of three consecutive registers and \( x \) is stored in the second. The third register contains the antenna pointing angle in fraction of a revolution where the radix point is to the left of the 13 least significant bits of the register. The first address for the elevation storage file is 4050, and the first address for the azimuth storage file is 6080. These are given as octal numbers as used by the computer. In addition, the number of times that the quadratic polynomial was chosen is stored as a binary number in register 1426, and the number of times linear was chosen is stored in register 1443. The corresponding storage registers for the azimuth axis are 3205 and 3222. The flow chart for the calculation of polynomial coefficients is given in Fig. 5.5.

3.3.5 Generation of Output Command Signals

Once the predicting polynomial has been established, this fact is indicated to the machine operator by displaying an octal number on the computer oscilloscope. A number appearing on the upper half of the scope face indicates that an azimuth prediction polynomial has been formed, while a number appearing on the lower half indicates an elevation polynomial. As soon as it is ready, the polynomial is evaluated for a future instant of time. The present value of time is arrived at by subtracting the time base from the present time as indicated by the \( y^0 \) register. This results in the number of seconds that have elapsed since the time origin of the present predicting polynomial. To this is added a lead time to allow for the response time of the antenna drive system. This lead time is presently taken as 400 milliseconds. The resulting value of future time is used as \( t \) in evaluating either the quadratic or linear predicting polynomial. Whether the linear or the quadratic polynomial is evaluated is dependent on the state of the indicator bit as mentioned earlier. The sine and cosine of the resulting predicted angle are calculated and loaded into the digital servo through its buffer. The digital servo then takes over and generates command signals for moving the
antenna as described above. At this point, the computer program has
traversed one complete cycle and it returns to the starting location to wait
for more data.

3.3.6 Prediction for "no-data" case

The above polynomial evaluation routine is used only immediately
after a new prediction polynomial has been obtained. In the interim, between
the recalculations of the polynomial, provision must be made for successive
evaluations of the present prediction polynomial. This is done by means of
evaluation routines separate from—but identical to—those used above. The
only addition is a time-sensing routine which determines when the polynomial
should be evaluated. This is shown in Fig. 3.6. The time criterion used is
to compare the present value of time, as indicated by the \( V^0 \) register, with
the value of \( V^0 \) time corresponding to the last evaluation. If more than
1/3 of a second has elapsed, the polynomial is re-evaluated for the present
value of time, relative to the time origin of the polynomial, plus the lead
time of 400 milliseconds. In most cases, when the computer exits from the
main stream of the program to return to the starting location, it does so by
way of this "no-data" evaluation routine. This insures that once a predicting
polynomial has been established, the antenna drive system will always receive
position commands every third of a second.

An operator's instruction manual is presented in Appendix IV.
Copies of the coded program as it appears in the computer memory are available
from the Group 12 office.

CHAPTER IV
EXPERIMENTAL WORK

In addition to the development of the computer program required for
the automatic tracking operation, the experimental work of this investiga-
tion included a study of certain characteristics of the remaining elements
of the system. Also, considerable testing and troubleshooting were required
to insure proper operation of the overall closed-loop system.

4.1 Antenna Characteristics

The first series of component tests was to permit the evaluation of the
constant which relates the angular error between target direction and antenna
axis to the difference between the magnitudes of the two diametrical radar
returns in the plane of the error. To do this, it was necessary to carefully
determine the propagation pattern of the antenna and the effect of the
mutating scanner on this pattern. This was done by locating a target trans-
mitter at a range of approximately 3-1/2 miles and sighting on it with the
antenna. The elevation of the antenna was set so that its axis passed
directly through the transmitting dipole. The antenna was then moved in a
horizontal plane and the received signal strength monitored on an antenna
pattern recorder. Two runs were made, one with the feed horn stopped in such
a position that the beam was in the full left position, and the second with
the beam in the full right position. The resulting patterns were superimposed
to form the composite shown in Fig. 2.4. The measurements were made by sensing
the output voltage of a bolometer detector driven by the output of the I.F.
amplifiers. The curve therefore represents received power as a function of
angular error. Three pieces of information may be derived from this curve:
the pattern beamwidth, the squint angle, and most important, the relationship between radar return amplitude difference and angular error.

The beamwidth of the antenna pattern is taken as the included angle between those two points where the power attenuation is 3 dB. These two points are shown by reference marks on Fig. 2.4 and the beamwidth is measured as 2.25°. The squint angle, or vertex angle of the scanning cone, is measured as the included angle between the angles corresponding to the peak value of each of the two patterns. From the plot, this is measured as 1.7°.

As stated above, Fig. 2.4 shows the attenuation of power in dB as a function of angular error. If a maximum received amplitude of one is assumed, then the corresponding amplitude for all other points may be easily calculated. The difference of the two values thus obtained is then proportional to the error signal that will be developed for any given angular error. A plot of this difference vs angular error is given in Fig. 4.1. It then remains to scale these figures to correspond with the actual value of the maximum received signal. It has been stated earlier that the amplitude input channel to the computer is capable of delivering an eight-bit binary number. If all these bits are one's, then the corresponding decimal number is 255. Therefore, the receivers must be so adjusted that the maximum possible received signal corresponds to a digital output of 255. Since the computer program contains the feature of scaling all received data to a uniform level, only the maximum condition need be considered in arriving at the proportionality constant. If the maximum signal that can occur corresponds to a number of 255, then on the same scale the maximum difference of .76, as read from Fig. 4.1, corresponds to a number of 194. Referring to the equation of Chapter III,
\[ \epsilon = \frac{a - b}{a + b} \left( \frac{a + b}{a - b} \right) \]

Using the values \( a_{\text{max}} = b_{\text{max}} = 255, a - b = 194, a + b = 316 \), and the decimal fraction of a revolution representation for the angle corresponding to the greatest difference \( \epsilon = 36 \), the constant becomes

\[ k = \frac{36}{316} \times \frac{316}{255} = 0.115. \]

When this is multiplied by \( (a+b)_{\text{max}} \), the actual multiplicative constant used in the program is obtained.

\[ K = 0.115 \times 310 = 35.6. \]

In octal form this is stored in the computer memory as \( 1.3264 \times 2^7 \).

4.2 Antenna Drive System

The characteristics of the antenna drive system, insofar as they affect the tracking system, were investigated. The two parameters of principal interest were the linearity and the response time. As pointed out in Chapter II, the main drive system is usually operated in a saturated state and thus behaves as a constant speed drive. Near null, however, it reverts to linear operation and may be treated in a conventional manner. To measure the linear region of the system, a saturation curve was plotted as shown in Fig. 4.2.

The output voltage was measured at the armature terminals of the DC generators after removing the cables connecting them to the antenna drive motors. With the drive motors disconnected, the antenna could not move, and hence a small displacement of the input control transformer would introduce a constant error signal into the system. This was done for a range of error values. It is

FIG. 4.2 - SATURATION CURVE OF ANTENNA DRIVE SYSTEM.
evident that the system departs from linearity when the error reaches a magnitude of 6 minutes and is completely saturated if the error becomes larger than one degree.

The response time of the drive system was measured for one set of conditions. In this case, both the main drive system and the digital servo were used, so the data obtained are characteristic of the entire mechanical portion of the complete system. The sine and cosine of a desired angle were loaded into the buffer of the digital servo by means of a set of test switches. Upon command of a test clock pulse, these values were converted by the digital-to-analog converters used to position the digital servo resolver and in turn position the antenna. The antenna was moved through an arc of 20 minutes on each test and in each direction. The reason for this choice is that at the tracking program's command rate of three per second, this would amount to an antenna travel of 1 degree per second, a slewing rate not uncommon in tracking operations. The response time was taken as the time between initiation of the command by the clock pulse, and the time the system first arrived at a zero error condition. This was measured by using the clock pulse to trigger the sweep of a calibrated oscilloscope and observing that point during the sweep when the magnitude of the main drive error signal first went to zero. The range of values obtained by this method was averaged over 100 trials and the mean response time found to be 775 milliseconds. This value, however, is for a 20-minute traverse from a starting point and would be considerably reduced if the antenna were already in motion. For that reason, a somewhat smaller value of time, 400 milliseconds, was chosen as an initial trial value of lead time. Ideally, the lead time should be adjusted as a function of the target velocity. This feature has not been incorporated into the program but could very easily be done.

4.3 Program Testing

The checking and testing of the computer program required the major portion of the experimental effort. Once the program had been successfully converted to the binary coding required by the Q3 24 computer, the logical and arithmetic operations were checked. This was done by supplying to the program, via the on-line Flexowriter, data previously derived from an arbitrary curve whose equation was known. The operation of the program was checked after each phase by comparing the contents of various storage registers with their hand-calculated counterparts. In this way, several logical and programming errors were detected and corrected. Once it was determined that each of the separate subroutines was performing as intended, the program testing moved into a second phase.

Two real-time satellite operations were conducted with the program operating in the computer. In neither case was control of the antenna transferred to the computer, but valuable experience was gained from observing the passive operation of the program. During the first of these two operations, the program rejected all data supplied from the radar and never developed a prediction polynomial. This difficulty was later found to be caused by the granularity of the digital clock which counts in increments of 8 milliseconds. The tolerance on the criterion used to determine whether two or more interpulse periods had elapsed between returns had been set too closely, and, as a result, a discrepancy of one bit in the least significant place could cause the program to reject the data. Since it is required that eight consecutive radar returns be ingested before one smoothed point is obtained, it is easy to visualize the reason why the program did not acquire sufficient data. This difficulty was resolved by relaxing the tolerances on the time criterion.
During the second real time operation, the program succeeded in reading in sufficient data to establish prediction formulas and proceeded to evaluate them in time. It did not, however, acquire any new points and modify the formulas it had. As a result, the predicted path and the actual satellite path very quickly diverged beyond all reasonable limits. The explanation for this behavior was somewhat more elusive, and it remained for further experimentation to bring it to light.

It was decided to conduct another satellite tracking operation, but this time record the data supplied to the computer on punched paper tape. In this way, the testing of the program would not be a one-shot operation but could be repeated and its performance more carefully observed. As the taped data were being read into the computer at a reduced speed, it was noted that the program was accepting very few of the data points as valid radar hits. A visual reading of the data tape provided the explanation and also accounted for the behavior of the program during the second real time trial. During these initial trials, the input to the Transient Intrac analog-to-digital converters had been supplied from an envelope detector at the output of the I.F. amplifiers. At this point in the system, the signal-to-noise ratio is at best in the order of 1:2:1. As a result, the great majority of the points supplied to the computer were rejected as being the result of noise. This was obvious when an inspection of the punched paper tape showed that only one return out of every 15 or 20 exceeded the preset noise threshold. Evidently, during the real time trial, the signal had become strong enough for the program to obtain sufficient points to generate a single prediction equation, and then faded back into the noise. Once discovered, this difficulty was easily cured by supplying the amplitude signal from the output of the crystal filter bank as described earlier, thus taking advantage of the 9:1 signal-to-noise ratio improvement.

A number of raw data tapes were then obtained using the output of the crystal filters for amplitude data. When these were used to test the tracking program, it performed in a manner more nearly like that expected. The predicted points followed the actual path of the satellite quite well, although some perturbations were noticed. When the individual "true points" as obtained from two consecutive radar returns were examined, it was discovered that they were extremely noisy and did not at all lie on any smooth curve. Their long time average, however, displayed some consistency. This pulse-to-pulse noise is attributed to the fact that the assumption of unchanging propagation characteristics from the time of one pulse to the time of the next is not strictly valid. It holds to a degree sufficient to allow the system to operate, but, as observed, introduces considerable noise which degrades the maximum performance possible from such a system. The results of testing the program indicated that it was now suitable for use in real time.

As a final test of the operation of the digital tracking system, several satellite tracks were attempted. The procedure for these tests was to manually control the antenna during the search phase of the operation. Once the target had been sighted, the analog tracker was used to bring the antenna on target and allow the program to collect sufficient data to fit a curve to the trajectory. Once this had been done, as indicated by the computer scope display, control of the antenna was transferred to the computer digital servo. In each case, a track of at least 3 minutes' duration was obtained. From the appearance of the A-scope presentation of the radar returns, the depth of
modulation on the return pulses appeared to be quite slight indicating that
the axis of the antenna was being held directly on the target.

When the target was finally lost, the opinion of site observers was
that in each case the cause was not a tracking fault but the fact that the
target had become invisible due to increasing range or by passing below the
horizon. From a qualitative point of view, these initial digital tracking
operations were considered highly successful. In view of the fact that the
particular satellite passes observed were relatively low in the sky, resulting
in trajectories with very slight curvatures, and since the use of a quadratic
prediction polynomial would tend to exaggerate the noise present on the
averaged points, the program was slightly modified for these trials so that
only linear extrapolation was used. A high elevation pass of a satellite was
tracked using the entire program. The effect of the noise in this case
distorted the generated curve to such an extent that the target was lost after
only a few seconds. The quadratic curve tends to accentuate the effect of
the noise and hence make a bad situation worse. Also, the fact that the
smoothed points are relatively close together in time increases the tendency
toward errors by making the apparent acceleration seem even larger.

4.4 Error Measurement

In order to provide a means of quantitatively evaluating the performance
of the digital tracking system as compared to that of the analog tracker,
a scheme was devised whereby the mean square tracking error could be measured
for each axis. To do this, the error signals, as generated by the analog
tracker, were tape-recorded together with a reference signal on a multichannel
instrumentation tape recorder. For convenience, the error-modulated 60 cps
signals were the ones actually recorded. This tape is then played back and
the output passed through a diode ladder squaring circuit to obtain a
signal proportional to the squared error. This is then integrated over time
by making use of an integrator-amplifier from an Rens Analog Computer. The
resulting output voltage is proportional to the integral square error.
When this is divided by the time duration of the recorded track, the number
obtained is directly proportional to the mean square value of error. Since
this figure is intended only for comparison purposes, it is not necessary to
determine the scaling coefficient relating the value thus obtained to the
actual mean square error. For a demonstration of the fact that the mean
square value of the modulation of a modulated signal may be obtained in the
above manner, refer to Appendix III. The average values of the mean square
tracking error for the analog tracker and the digital tracking system are
4.96 and 5.58, respectively. These two figures are the averages of values
obtained from several trials of each tracking system. In comparing these
figures, it must be remembered that the mean square error criterion places
no penalty on losing the target.
CHAPTER V
DISCUSSION OF RESULTS

The tests of the tracking system to date indicate very strongly that a digital tracking system can be used to considerable advantage in the tracking of satellites and other objects in space. Although the results are not yet perfect, and the system is occasionally plagued by operational difficulties, the results presented earlier are an encouraging preview of what may ultimately be achieved.

5.1 Noise Problem

Perhaps the largest unforeseen problem of this study was pulse-to-pulse noise. It had been presumed that noise interference would be of such a nature as to affect alternate pulses in a similar manner. Furthermore, it was felt that noise differences on alternate pulses would be relatively small. While these statements hold in a very broad sense, it was found that on occasion there existed considerable noise variations between two alternate radar returns. This causes considerable adverse effects since the program compares alternate pulses to obtain an angular correction. If either or both pulses are independently perturbed from their true values, the effect on the angular error obtained will be considerable. This noise disturbance was noticed when the corrected points from the recorded data tests were plotted against time. Instead of lying on a smooth curve, a scattering was obtained about the true curve. The smoothing techniques employed helped to remedy this situation but not perfectly.

This sort of pulse-to-pulse noise disturbance is characteristic of any sequential scanning system where two returns that are to be compared with each other for purposes of developing an error are separated from each other by some interval of time. The most positive cure for this pulse-to-pulse noise problem is the use of a monopulse system of propagation. Under this system, a single-beam pattern is employed for transmission and a four-beam pattern for reception. Each of the four beams is associated with a collector horn, two in each axis. The advantage here is that information is received from all four quadrants simultaneously, hence the problem of noise due to time separation is eliminated. Compensation for long term variations in amplitude may easily be achieved, and this feature has been incorporated in the computer program.

5.2 System Parameters

There are several constants employed by the computer program which control the operation. Among these are the proportionality constant relating angular error to amplitude difference, the lead time for prediction, the time spacing between predictions, and the threshold for choosing quadratic or linear prediction polynomials. The values employed for these various constants were obtained from a cursory analysis of the system fixed parameters and are by no means claimed to be optimum values. As has been pointed out earlier, the system is not a linear system, hence these values may not be optimized by conventional methods. Experimental trials, using perhaps the mean square tracking error as a measure of optimization, appear to be the most feasible manner in which to accomplish these evaluations. The facility for easily changing constants stored in the memory of the computer by means of the toggle switch memory makes this a fairly straightforward operation. Scheduling difficulties, together with a shortage of available time, prevented the writer from carrying out these evaluations.
5.3 System Design

Several modifications of the system, specifically the program format, have suggested themselves during the course of this investigation. First, it is felt that a more powerful smoothing technique should be used in an attempt to overcome the pulse-to-pulse noise problem. This might be accomplished by taking a linear average of corrected points, whether they be consecutive or not, over a period of several seconds and selecting as the time corresponding to this smoothed point the time of maximum density of corrected points. This operation would be very similar to that of the "Data Averaging Program" (DAP). In this way, considerably greater smoothing would be effected than is at present. A second suggested program change would be to modify the curve-fitting operation in such a way that no data are discarded as outdated. Presently all smoothed points earlier than the most recent three are discarded. Provision could be made that once four or more points are available, either a first or second order curve is fitted by means of the least mean squared error method. This would make use of all data collected and after a short while produce a very reliable prediction polynomial. It would also go a long way toward curing the difficulties experienced with the quadratic polynomial. The more points used in the determination of the curve, the less the effect of the noise associated with any one. A foreseeable difficulty with this idea, however, is that as the number of available points increases, the least square fitting formulas become more complex, and the limited word length of the CG 24 computer may prove a stumbling block.

A third suggested improvement involves the value assigned to the antenna lead time. At present this is taken as a constant, but in reality this is not strictly the case. The lead time required to place the antenna on target is a function of how far it is being commanded to move, how fast it is moving when it receives the command, what its acceleration capabilities are, and the range of the target. These parameters are either known constants or may be obtained from the data ingested by the program. It would therefore be possible to write a subroutine that would calculate the optimum value of lead time for each individual prediction evaluation.
APPENDIX I

Derivation of Quadratic Coefficients

Given that a second degree polynomial, \( y(t) \), is to be made to pass through three given points \( (f_0, t_0); (f_1, t_1); (f_2, t_2) \), the coefficients of the desired polynomial may be obtained as follows. Let the desired polynomial be represented by:

\[
y(t) = \sum_{k=0}^{n} L_k(t) f(t_k)
\]

where \( L_k(t) \) is a polynomial function of \( t \) of degree \( n \) or less and \( t_k \) is the value of time at the \( k \)th given point. Since \( y(t) \) is to be of second degree, \( n = 2 \) and \( k \) takes on the values \( 0, 1, 2 \). The polynomial is then given by:

\[
y(t) = L_0(t) f(t_0) + L_1(t) f(t_1) + L_2(t) f(t_2)
\]

or using the notation employed in specifying the given points above

\[
y(t) = L_0(t) f_0 + L_1(t) f_1 + L_2(t) f_2
\]

If the function \( y(t) \) is to pass through the given points \( f_0, f_1, f_2 \), then the conditions placed on the functions \( L_k \) are

\[
L_k = 1 \quad t = t_k \quad 0 \quad t = t_k
\]

These two conditions may be satisfied if \( L_k \) is written in the form

\[
L_k(t) = \frac{(t-t_0) \ldots (t-t_{k-1}) (t-t_{k+1}) \ldots (t-t_n)}{(t_k-t_0) \ldots (t_k-t_{k-1}) (t_k-t_{k+1}) \ldots (t_k-t_n)}
\]

for the case \( n = 2 \) the three \( L \) become

\[
\begin{align*}
L_0(t) &= \frac{(t-t_0) (t-t_2)}{(t_0-t_2)(t_2-t_1)} \\
L_1(t) &= \frac{(t-t_1) (t-t_0)}{(t_1-t_0)(t_2-t_1)} \\
L_2(t) &= \frac{(t-t_2) (t-t_1)}{(t_2-t_0)(t_2-t_1)}
\end{align*}
\]

Expanding the numerators we obtain

\[
\begin{align*}
L_0(t) &= \frac{t^2 - (t_1+t_2) t + t_1 t_2}{(t_1-t_0)(t_2-t_0)} \\
L_1(t) &= \frac{-t^2 - (t_0+t_2) t + t_0 t_2}{(t_1-t_0)(t_2-t_1)} \\
L_2(t) &= \frac{t^2 - (t_0+t_1) t + t_0 t_1}{(t_2-t_0)(t_2-t_1)}
\end{align*}
\]

If each of these terms is multiplied by its corresponding value of \( f_k \) and the three products added, the result is an expression for \( y(t) \). If this expression is appropriately factored and regrouped, the following form is obtained.

\[
y(t) = \left( \frac{f_0}{(t_1-t_0)(t_2-t_0)} - \frac{f_1}{(t_1-t_0)(t_2-t_1)} + \frac{f_2}{(t_2-t_0)(t_2-t_1)} \right) t^2
\]

\[
+ \left( \frac{-f_1(t_1+t_2)}{(t_1-t_0)(t_2-t_0)} + \frac{f_0(t_0+t_2)}{(t_1-t_0)(t_2-t_1)} - \frac{f_2(t_0+t_1)}{(t_2-t_0)(t_2-t_1)} \right) t
\]

\[
+ \left( \frac{f_2 t_2}{(t_1-t_0)(t_2-t_0)} - \frac{t_0 f_2}{(t_1-t_0)(t_2-t_1)} + \frac{t_1 f_2}{(t_2-t_0)(t_2-t_1)} \right)
\]

If, for convenience, the time origin of the system is considered to be coincident with the first point, then \( t_0 = 0 \) and the above expression may be reduced to

\[
y(t) = \left( \frac{f_0}{t_1 t_2} - \frac{f_1}{t_1 (t_2-t_1)} + \frac{f_2}{t_2 (t_2-t_1)} \right) t^2
\]

\[
+ \left( \frac{-f_1(t_1+t_2)}{t_1 t_2} + \frac{t_2 f_1}{t_1 (t_2-t_1)} - \frac{t_1 f_2}{t_2 (t_2-t_1)} \right) t
\]

\[+ f_0\]
The desired coefficients that must be calculated by the program are therefore

\[ a_2 = \frac{r_0}{t_1 t_2} - \frac{r_1}{t_1 (t_2 - t_1)} + \frac{r_2}{t_2 (t_2 - t_1)} \]

\[ a_1 = \frac{(t_1 + t_2) f_0}{t_1 t_2} + \frac{t_2 f_1}{t_1 (t_2 - t_1)} - \frac{t_1 f_2}{t_2 (t_2 - t_1)} \]

\[ a_0 = f_0 \]

**APPENDIX II**

**Derivation of Linear Coefficients**

Given the three points \((t_0, y_0); (t_1, y_1); (t_2, y_2)\), the coefficients of a first degree polynomial which will fit these three points in such a way that the sum of the squared errors is a minimum may be found as follows.

Let the desired polynomial be represented by

\[ y(t) = a t + b \]

The sum squared error is then given by

\[ E = [r_0 - y(t_0)]^2 + [r_1 - y(t_1)]^2 + [r_2 - y(t_2)]^2 \]

It is desired to find those values of \(a\) and \(b\) which minimize the functional \(E\). To do this, \(E\) must be expanded in terms of \(a\) and \(b\).

\[ E = f_0^2 - 2f_0 y(t_0) + y^2(t_0) + f_1^2 - 2f_1 y(t_1) + y^2(t_1) + f_2^2 - 2f_2 y(t_2) + y^2(t_2) \]

\[ E = f_0^2 - 2f_0 a t_0 + 2f_0 b + a^2 + b^2 + 2a b t_0 + b^2 \]

\[ + f_1^2 - 2f_1 a t_1 + 2f_1 b + a^2 + b^2 + 2a b t_1 + b^2 \]

\[ + f_2^2 - 2f_2 a t_2 + 2f_2 b + a^2 + b^2 + 2a b t_2 + b^2 \]

To minimize this with respect to \(a\) and \(b\), the partial derivatives of \(E\) with respect to \(a\) and \(b\) are extracted and equated to zero to attain two simultaneous equations in \(a\) and \(b\)

\[ \frac{5 E}{a} = -2f_0 t_0 + 2a t_0^2 + 2bt_0 \]

\[ -2f_1 t_1 + 2a t_1^2 + 2bt_1 \]

\[ -2f_2 t_2 + 2a t_2^2 + 2bt_2 = 0 \]

\[ a (t_0^2 + t_1^2 + t_2^2) + b (t_0 + t_1 + t_2) = f_0 t_0 + f_1 t_1 + f_2 t_2 \]
\[
\frac{5 E}{5 b} = -2f_2 + 2at_2 + 2b \\
-2f_1 + 2at_1 + 2b \\
-2f_2 + 2at_2 + 2b = 0 \\
a(t_0^2 + t_1^2) + b(3) = f_0^2 + f_1^2 + f_2^2
\]

The simultaneous solution of these two equations provides the desired values of \(a\) and \(b\). These expressions may be obtained in the form
\[
a = \frac{(f_0^2 + f_1^2 + f_2^2) - 3b}{t_0^2 + t_1^2 + t_2^2}
\]
\[
b = \frac{(f_0^2 + f_1^2 + f_2^2)(t_0^2 + t_1^2 + t_2^2) - (f_0^2 + f_1^2 + f_2^2)(t_0^2 + t_1^2 + t_2^2)}{(t_0^2 + t_1^2 + t_2^2) - (t_0^2 + t_1^2 + t_2^2)}
\]

If the time origin of the system is again considered to be coincident with the first point, then \(t_0 = 0\) and the above expressions reduce to
\[
a = \frac{(f_0^2 + f_1^2 + f_2^2) - 3b}{t_1^2 + t_2^2}
\]
\[
b = \frac{(f_0^2 + f_1^2 + f_2^2)(t_1^2 + t_2^2) - (f_0^2 + f_1^2 + f_2^2)(t_1^2 + t_2^2)}{(t_1^2 + t_2^2) - (t_1^2 + t_2^2)}
\]

APPENDIX III

Mean Square Value of Amplitude Modulation

Given a suppressed carrier, amplitude-modulated signal whose carrier frequency is \(w\), it may be shown as follows that under certain conditions the mean square value of the modulating signal is twice the mean square value of the modulated signal. Let the modulated signal be represented by
\[
f(t) = A(t) \sin wt.
\]

The mean square value of this signal is then by definition
\[
\overline{f^2} = \frac{1}{T} \int_0^T f^2(t) \, dt.
\]

Substituting from above for \(f(t)\)
\[
\overline{f^2} = \frac{1}{T} \int_0^T A^2(t) \sin^2 (wt) \, dt
\]

but
\[
\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)
\]

therefore
\[
\overline{f^2} = \frac{1}{2T} \int_0^T A^2(t) \left[1 - \cos (2\omega t)\right] dt
\]
\[
= \frac{1}{2T} \int_0^T A^2(t) dt - \frac{T}{2\omega T} \int_0^T A^2(t) \cos (2\omega t) dt
\]

The first term of this expression is recognized as one-half the mean square value of the modulation. If the function \(A(t)\) may be assumed slowly varying with respect to \(2\omega\), such that it is essentially a constant over any single
cycle of the $\cos 2\omega t$ oscillation, then the value of the second integral goes to zero over each complete cycle, and its contribution over any sufficiently long period $T$ will be negligible. The result is that the mean square value of the modulation may be expressed in terms of that of the total signal by the following expression

$$\frac{1}{T} \int_0^T A^2(t)dt = \frac{2}{T} \int_0^T R^2(t)dt$$

$$A^2 = 2R^2$$

APPENDIX IV

Operating Instructions for Auto Track Program

1. Make certain that amplitude data from the transcon converters is being supplied to the computer. The input to the transcons should be adjusted such that the background noise lights no more than the three least significant bits.

2. Read in auto track program. If it is desired to use only the linear extrapolation, read in the short auxiliary program entitled "Quadratic Disable". This changes the two transfer on negative instructions involved in the quadratic-linear decision to transfer instructions and insures linear operation.

3. Set the toggle switches as follows:

   | 20006 | 0.00000007 |
   | 20007 | 0.00000007 |
   | 2001\# | 0.00000000 |
   | 20015 | 1.00000000 |

4. Start the program with the "Start Program" push button. The program will continually cycle through the reset routine and a constant note will be audible from the speaker.

5. When a target has been acquired, lift the zeroth bit of 2001\# to enter the main portion of the program. It will now proceed with the data collection and tracking unaided.

6. The program is ready for tracking whenever the number $0.7777777$ is displayed on the face of the oscilloscope. Its appearance on the upper half indicates readiness in azimuth; likewise, its appearance on the lower half
indicates readiness in elevation. Control of the antenna may be transferred to the computer at any time after these indications are obtained.

7. The gain of the azimuth channel may be adjusted by changing the contents of 20006 and similarly the gain of the elevation channel may be adjusted by changing the contents of 20007. Adding one to the numbers contained in these registers increases the gain by a factor of two, and subtracting one decreases the gain by a factor of two.

8. The value of $Q_2$ for azimuth and elevation, together with the antenna pointing angle to which it corresponds, may be consecutively stored in the unused remainder of memory zero by depressing the zeroth bit of 20015. This storage will continue as long as this bit is down; however, the allotted space will be overflowed if it is continued for greater than 150 seconds. This information may be read out at the end of the run by means of Sim or any other convenient method.

9. A track may be broken by depressing the zeroth bit of 20014. This returns the program to the reset routine and clears all stored data except the values of $Q_2$. It does, however, reset the storage instruction for $Q_2$ back to its initial value so that if this storage feature is used after a reset, the new data stored will be written over the old, thus erasing it. If control of the antenna is not taken from the computer before breaking track, the antenna will stop at the last position to which it was driven by the program.

BIBLIOGRAPHY


Digital Tracking Loop for High-Powered, Experimental Radar

The problem of using a digital computer as an element of a radar system for tracking of space objects, such as satellites, is investigated. The basic principles of curve extrapolation for digital tracking are introduced and the theory of linear and second order curve to three points is developed.

The facility of the Millennium Radar site, where the experimental work was done, is described in detail, with emphasis on its role as an element of a closed-loop tracking system. For purposes of illustration, a brief analysis of the operation of the present analog tracking system is presented.

A computer program for the digital computer is written and each phase of the program explained in detail. The emphasis is on the methods and techniques for tracking rather than on mathematical operations.

The system is tested with simulated and real-time data. The arithmetic and logical tests indicated perfect operation of the program. Testing with real data showed the presence of a noise problem. No actual operational data were available to test the system. A comparison was made with a digital computer covered in a previous work.

A relation is developed for measuring the mean square tracking error of either the analog or digital tracking system using curve-fitted error signals and an analog computer. The ratio of the errors obtained for the digital tracking system to the analog system is 1:12.

It is recommended that further work be done in several aspects of this field. Since tests showed that some tracking errors were caused by the use of inaccurate analog computers, the system should be modified to include a greater number of analog computers. The system could be the standard but further tests should be made. Further work is now in progress.

Key words:
- Digital tracking radar
- Millstone radar
- Digital computers
- Noise
- Space tracking
- Analog computer