
THE DERIVATION OF ELECTRIC AND MAGNETIC FIELD COMPONENTS FOR THE QUASI-STATIC, QUASI-NEAR, AND NEAR-FIELD RANGES

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ABSTRACT

The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic) are derived and presented for the quasi-static, quasi-near, and near-field ranges. The depth, \( h \), of the transmitting dipole is less than or equal to zero; the height, \( z \), of the receiving antenna above the plane, conducting, homogeneous earth varies from zero to some height, \( z \), that is much less than the ionospheric reflecting height (ionospheric effects are neglected). The horizontal separation, \( \rho \), between the transmitting and receiving antennas is comparable to the receiving antenna height, \( z \). The derivations are based upon the quasi-static and near-field approximations to the vector potentials for the vertical and horizontal dipoles and upon application of the reciprocity theorem. The surface-to-surface propagation equations reduce to well-known expressions when \( |y_1 \rho| \gg 1 \) and \( |y_1 \rho| \ll 1 \), as do the subsurface-to-air equations when \( \rho \gg z \).

ADMINISTRATIVE INFORMATION

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DEFINITIONS OF SYMBOLS AND ABBREVIATIONS

c
The velocity of propagation in the upper half-space (free space); \( 3 \times 10^8 \) (meters/sec)

dl
The infinitesimal dipole length (meters)
da
The infinitesimal dipole area (meters^2)
e^{i\omega t}
The time factor assumed

E_p
The horizontal electric field component in the p direction (volts/meter)

E_\phi
The horizontal electric field component in the \( \phi \) direction (volts/meter)

E_z
The vertical electric field component (volts/meter)

G_v
Wait's height gain factor for vertical polarization (valid for \( \rho \gg z \));
\[ 1 + \frac{y_0^2 z}{\gamma_1} \]

h
The depth of burial of the transmitting antenna (h \( \leq 0^+ \)) (meters)

H
The height of the vertical loop antenna above the earth; \( = \frac{1}{\gamma_1} \)

H_\rho
The horizontal magnetic field component in the \( \rho \) direction (amperes/meter)

H_\phi
The horizontal magnetic field component in the \( \phi \) direction (amperes/meter)

H_z
The vertical magnetic field component (amperes/meter)

HED
The horizontal electric dipole

HMD
The horizontal magnetic dipole

HGF
The height gain factor; equal to the field strength at height \( z \) divided by the field strength at height \( z = 0^+ \)

I_0(y_1\rho/2)
The modified Bessel function of the first kind, order zero, and argument \( y_1\rho/2 \)

I_1(y_1\rho/2)
The modified Bessel function of the first kind, order one, and argument \( y_1\rho/2 \)

J_0(\lambda \rho)
The Bessel function of the first kind, order zero, and argument \( \lambda \rho \)

K_0(y_1\rho/2)
The modified Bessel function of the second kind, order zero, and argument \( y_1\rho/2 \)

K_1(y_1\rho/2)
The modified Bessel function of the second kind, order one, and argument \( y_1\rho/2 \)

mks
Meter-kilogram-seconds, the unit employed

R
The radial distance in a spherical coordinate system; \( = (\rho^2 + z^2)^{1/2} \)

T
\[ 16J_1K_1 + 4y_1\rho(I_1K_0 - I_0K_1) + y_1^2 - (I_1K_1 - I_0K_0) \]
\[ (\lambda^2 + y_0^2)^{1/2} \text{(meters}^{-1}) \]

u_0
\[ (\lambda^2 + y_1^2)^{1/2} \text{(meters}^{-1}) \]

VED
The vertical electric dipole

VMD
The vertical magnetic dipole

W
\[ 31J_1K_1 - (y_1\rho/2) (I_0K_1 - I_1K_0) \]

z
The height of the transmitting antenna above the plane conducting earth (\( z \geq 0^+ \)) (meters)
\( Y_0 \) The upper half-space (free space) propagation constant (meters\(^{-1}\)); \( = (\varepsilon_0 \mu_0 \omega^2)^{1/2} \)

\( Y_1 \) The lower half-space (the earth) propagation constant (the displacement currents in the earth are assumed to be negligible); \( = (\varepsilon_0 \mu_0 \omega^2)^{1/2} \) (meters\(^{-1}\))

\( \delta \) The skin depth in the lower half-space (the earth); \( = (2/\omega \mu \sigma_1)^{1/2} \)

\( \varepsilon_0 \) The permittivity of free space; \( \approx 10^{-9}/36 \pi \) (farads/meter)

\( \lambda \) The dummy integration variable

\( \lambda_{\text{air}} \) The wavelength in the upper half-space (free space); \( = c/\lambda \)

\( \lambda_{\text{earth}} \) The wavelength in the lower half-space (the earth); \( = 2\pi \delta \)

\( \mu \approx \mu_0 \) The permeability of free space; \( = 4\pi \times 10^{-7} \) (henries/meter)

\( \rho \) The radial distance in a cylindrical coordinate system; \( = (x^2 + y^2)^{1/2} \)

\( \sigma_1 \) The conductivity of the lower half-space (the earth) (mhos/meter)

\( \phi \) The azimuth angle in a cylindrical coordinate system; \( = \tan^{-1}(y/x) \)

\( \omega \) The angular frequency; \( = 2\pi f \) (radians/sec)

\( \epsilon^+ \) The infinitesimal distance above the surface of the earth (meters)

\( \epsilon^- \) The infinitesimal distance below the surface of the earth (meters)
SURFACE-TO-SURFACE AND SUBSURFACE-TO-AIR PROPAGATION OF ELECTROMAGNETIC WAVES

The Derivation of Electric and Magnetic Field Components for the Quasi-Static, Quasi-Near, and Near-Field Ranges

INTRODUCTION

Radiation from electric and magnetic dipole antennas in the air above a plane, conducting, homogeneous earth has been considered by many writers, beginning with Sommerfeld. Unfortunately, relatively little attention has been paid to surface-to-surface propagation in the quasi-static range when \( \nu \) is allowed to be any value. (Definitions of the symbols appear in the introductory pages at the front of the report.) In this report the theoretical work that has been accomplished for this range is summarized, and the electric and magnetic field components that have not been derived previously (to the author's knowledge) are derived for the case in which \( \nu \rightarrow \infty \). Surface-to-air propagation in the near-field range has been considered by Norton, who states, however, that some of his formulas are valid only for distances down to a free-space wavelength (i.e., \( R \gg \lambda \)). Subsurface-to-air propagation in the quasi-near and near-field ranges has been treated by Moore, Wait, Baños and Wesley, and Baños (among others) with the restriction that \( \nu \rightarrow \infty \). The field compo...

nents produced by subsurface dipole antennas that are valid for the quasi-near and near-field ranges are presented in this report without the above restriction. Ionospheric effects are neglected.

Four types of dipole antennas will be examined: the vertical electric dipole (VED), vertical magnetic dipole (VMD), horizontal electric dipole (HED), and horizontal magnetic dipole (HMD). The four antennas are situated at the origin of a cylindrical coordinate system and are assumed to carry a constant current, \( I \). The VED and HED are oriented in the \( z \) and \( x \) directions, respectively; the axes of the VMD and HMD are oriented in the \( z \) and \( y \) directions, respectively. The dipole orientations when \( h = 0^+ \) are shown in Fig. 1. The earth occupies the lower half-space \( (z < 0) \); and the air, the upper half-space \( (z > 0) \). The units employed are meter-kilogram-seconds (mks); a time factor of \( e^{-it} \) is assumed.

SURFACE-TO-SURFACE PROPAGATION IN THE QUASI-STATIC RANGE

THE GENERAL RANGE

For the general quasi-static range \( (\rho << \lambda, \rho R < h) \) and \( z = h = 0^+ \), the approximation that the propagation constant in air \( (\gamma_m) \) is equal to zero can be utilized in the derivation of the field components. This is called the quasi-static approximation and was developed by Lien. It is valid for observation distances that are very small with respect to an air wavelength \( (\text{distances} < \lambda_m / 20) \) and for depths, \( h \), much less than the observation distance.

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Fig. 1 - Dipole Orientations when \( h = 0^+ \)

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Several electric and magnetic field-component expressions for the general quasi-static range have been determined previously. The $E_\parallel$ and $H_\perp$ components for the VED have been derived by Norton; the $E_\mu$ component has been derived by Wait and Campbell. All electric and magnetic components for the VMD have been derived by Wait and Gordon; the $H_\mu$, $H_\phi$, and $E_\parallel$ components also have been derived by Wait. Martin has extended Wait's results and has evaluated the $H_\mu$, $H_\phi$, and $E_\parallel$ components. Furthermore, Wait has determined the HMD magnetic field components. These expressions for the general quasi-static range are summarized in Table 1.

The reciprocity theorem (which applies to dipoles in the presence of any linear medium) states that the voltage, $V_2$, induced in antenna 2 by the current, $I_1$, of antenna 1 is the same as the voltage, $V_1$, induced in antenna 1 by an identical current, $I_2$, flowing in antenna 2. Applying the reciprocity theorem, we can then determine the remaining field components for the general quasi-static range. The HMD $E_\mu$ and $E_\phi$ components can be determined from the VED $H_\mu$ and $H_\phi$ components; and the HMD $E_\parallel$ component, from the VED $H_\parallel$ component. Expressions for these field components also are included in Table 1. Note that $d\vec{x} \cdot \vec{V}$ and $d\vec{x} \cdot \vec{A}$ when $\vec{V}$ and $\vec{A}$ are much smaller than $\lambda$ and the measurement distance, $\mu$.

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8 Norton, op. cit. (see footnote 2, above).
12 Wait, op. cit. (see footnote 4, above).
For the quasi-static subrange in which \( \gamma_{wp} < I < \gamma_{vp} \) (i.e., \( \lambda_{\text{earth}} < \mu < \lambda_{\text{air}} \)), which is actually the quasi-near range, the modified Bessel functions of Table 1 can be replaced by the first one or two terms of their asymptotic expansions and the \( e^{-\gamma_{vp}} \) terms approach zero. The resulting field-component expressions are presented in Table 2.

For an HED antenna there is a diffused return current in the earth corresponding to the current in the horizontal wire. For this reason a horizontal wire above the earth with the ends of the wire grounded is called a "ground return antenna." The current in the horizontal wire of length \( \ell \) and the ground return current act as an HMD (vertical loop) antenna of height \( H = 1/\gamma_{v} \) above the earth. This can be observed easily by comparing the HED and HMD field-component equations in Table 2. That is, the magnitude of the effective area, \( A \), of this equivalent loop antenna is

\[
A = \ell H = \frac{\ell}{\gamma_{v}} = \frac{\ell \delta}{\sqrt{2}}, \tag{1}
\]

where \( \delta \) is the skin depth in the earth. As a matter of interest, a "ground return antenna," either buried or on the surface of a semi-infinite conducting medium, is equivalent to a rectangular one-turn loop of length \( \ell \) and height \( H = 1/\gamma_{v} \) as long as \( \mu >> \delta \).

For the quasi-static subrange in which \( \gamma_{vp} < I >> \gamma_{vp} \) (i.e., \( \lambda_{\text{earth}} >> \mu < \lambda_{\text{air}} \)), the exponential and modified Bessel functions in Table 1 can be replaced by the first two or three terms of their series expansions. The resulting field-component expressions are given in Table 3.

**SOME VALUES OF THE FIELD-COMPONENT FUNCTIONS**

Many of the equations in Table 1 contain modified Bessel functions of order zero or one and argument \( \gamma_{wp} \). The magnitude of the function \( I, K \) versus \( \gamma_{wp} \) is plotted in Fig. 2. For small values of the argument the function approaches the value of 1/2; for large values \( I, K \) is approximately equal to \( 1/\gamma_{wp} \). This asymptotic expansion may be employed for values of \( \gamma_{wp}^2 > 5 \)

The magnitude of the function \( W \) versus \( \gamma_{vp} \) is plotted in Fig. 3. For
Fig. 2 - $|I_{1} K_{1}|$ versus $|\gamma_{1} \rho/2|$

Fig. 3 - $|W|$ versus $|\gamma_{1} \rho/2|$
small arguments this function also approaches 1/2; for large arguments it is approximately equal to 2/γ R. The magnitude of the function T versus |γ R/2| is plotted in Fig. 4. For small arguments T equals approximately γ Rγ/2; for large arguments (R > 78) the asymptotic expansion T ≈ 6/γ R may be employed.

SUBSURFACE-TO-AIR PROPAGATION

THE QUASI-NEAR RANGE

For the quasi-near range (i.e., λ, < < R < < λair) with h ≪ 0 and z ≫ 0, the Lien quasi-static approximation applies as long as the observation distance, R, is less than λ,π/20 and the depth, h, of the transmitting dipole is much less than R. If the earth is a conductor and γ R > 1 (R > 8), then

\[ u_1 ≈ \left( \frac{\gamma^2}{\gamma + \gamma^2} \right)^{\gamma - 1} \left( \gamma + \gamma^2 \right). \]

Therefore, the exponential attenuation-with-depth term, \( e^{\gamma \gamma h} \), may be removed from under the integral sign (in the exact formulas for the field-component functions). By utilizing the above approximations and the integral 15

\[ T \approx \frac{6}{\gamma R}. \]
\[
\int_0^\infty e^{-i\gamma R} J_n(i\mu) \, d\hat{\mu} \cdot \frac{n^2}{R^{n+1}} \frac{P_n\left(\frac{\hat{\mu}}{\gamma}\right)}{R}
\]  

(2)

(where the \( P_n \left( \frac{z}{R} \right) \) are the Legendre polynomials), we have calculated the VED, VMD, and HED field components for subsurface-to-air propagation in the quasi-static range. They are listed in Table 4.

The HMD field components can be obtained with the reciprocity theorem or by a synthesis of the HED field components. They can be found simply by differentiating the HED field components with respect to the depth, \( h \). For \( |\gamma R| >> 1 \) (\( R >> \delta \)) the depth factor is simply \( e^{h/\gamma} \). Therefore, the HMD components can be obtained by replacing \( \mu \) with \( \gamma \mu dA \) in the HED equations. The results of this method agree with those obtained with the somewhat different explanation presented for surface-to-surface propagation in the quasi-static subrange \( |\gamma_\rho| << 1 << |\gamma_\mu| \). The HMD field components also are listed in Table 4.

An examination of the equations presented in Table 4 reveals that the \( E_\mu \), \( E_\phi \), and \( H_\rho \) components have substantial height gain factors. The height gain factor is defined as the field strength at height \( z \) divided by the field strength at height \( z = 0^+ \). However, the \( E_z \), \( H_\rho \), and \( H_\phi \) components are decreased in magnitude when \( z > 0 \). In fact, when \( z_\rho = 1/\sqrt{2} \), the VED \( E_z \) component is essentially zero (to a first-order approximation). Similarly, when \( z_\rho = 1/\sqrt{2} \), the HED \( H_\rho \) components approach zero; and, when \( z_\rho = 1/2 \), the VMD \( H_\rho \) component is essentially zero (to a first-order approximation).

We note also that when \( h = 0^- \) and \( z = 0^+ \), the VMD, HED, and HMD field-component expressions in Table 4 reduce to the expressions given in Table 3 (which are valid for the quasi-near range with \( z = h = 0^+ \)). The VED components in Table 4 will be compatible also with those in Table 3 if they are multiplied by the factor \( \gamma_{\mu}^2/\gamma_\rho^2 \) (this is due to the shift from \( h = 0^- \) to \( h = 0^+ \)).

THE NEAR-FIELD RANGE

For the near-field range (\( R \) comparable to \( \lambda_{\text{mi}} \) with \( h < 0^- \) and \( z < 0^+ \), the assumptions that the earth is a conductor (\( \sigma_i \gg \sigma_1 \)), that \( \gamma_i R \gg 1 \) (\( R >> h \)), and that \( R >> h \) still apply. The restriction that \( (\gamma_i^2 \gamma_\rho \rho) < < 1 \) has been assumed also. This limits the range to small "numerical distances," although \( \gamma_\rho \) may exceed unity. Also, the radiation field (\( \gamma_{\mu} > 1 \)) height gain factor for vertical polarization,

\[ \text{16 Norton, op. cit. (see footnote 2, above).} \]
\[ G_r = 1 + \left( \gamma_0 \gamma_1 \right) z. \]  

(3)

is approximately equal to 1. The VED, VMD, and HED field components for this range were evaluated on the basis of the above approximations and with Sommerfeld's integral:

\[ \int_0^\infty e^{-\frac{R z}{u_n}} \frac{J_n(k_i \rho)}{u_n} \, \frac{R}{\rho} \, dk_i = \frac{e^{-\gamma_0 R}}{R}. \]  

(4)

The components are listed in Table 5. The HMD field components (see Table 5) were obtained from the HED field components by setting \( d = \gamma_1 dA \) (as was done for the quasi-near range).

When \( \gamma_0 R \ll 1 \), the near-field components (Table 5) reduce to the quasi-static field components (Table 4). If the VED antenna is placed at the earth's surface \( (h = 0^*) \), the field components will be identical with those previously derived by Norton.\(^{17}\) When \( \rho \gg z \), the near-field components reduce to well-known expressions; these are listed in Table 6. For example, the HED equations are identical with those previously derived by Wait\(^{19}\) except for the following differences:

a. Wait assumed that \( z = 0 \) when he derived the \( E_r \) and \( E_\phi \) components.

b. Wait's height gain factor for the \( E_r \) component \( \left( 1 + \left( \gamma_0 \gamma_1 \right) z \right) \) is negligible for the near-field range \( (\rho \text{ comparable to } \lambda_{sw}); \) therefore, it was not included in Table 6.

When \( \gamma_0 \rho \ll 1 \), the horizontally polarized components (\( E_\phi \), \( H_\rho \), and \( H_\phi \)) also are identical with Wait's. Therefore, the restriction that \( \left| \left( \gamma_0 \gamma_1 \right) \rho \right| \ll 1 \) is not required for these components.

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\(^{17}\) Sommerfeld, op. cit. (see footnote 1, above); and Erdélyi, op. cit. (see footnote 14, above).

\(^{18}\) Norton, op. cit. (see footnote 2, above).

\(^{19}\) Wait, op. cit. (see footnote 4, above).
SUMMARY

The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic types) were derived and presented for:

a. Surface-to-surface propagation in the quasi-static range,

b. Subsurface-to-air propagation in the quasi-near range, and

c. Subsurface-to-air propagation in the near-field range.

When \(|\gamma_{ip}| >> 1\) or \(|\gamma_{ip}| << 1\), the surface-to-surface propagation equations reduce to previously derived expressions, as do the subsurface-to-air equations when \(p >> z\). Also, when \(|\gamma_p R| << 1\), the near-field equations reduce to the quasi-near field formulas. For additional information on the derivation of the field components, see USL Reports Nos. 698, 701, 719, 720, and 729.

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### Table 1
SURFACE-TO-SURFACE PROPAGATION FORMULAS FOR THE GENERAL QUASI-STATIC RANGE

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>$E_{\rho}$</th>
<th>$E_{\phi}$</th>
<th>$E_{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>$-\frac{i \mu_0 \omega Id \eta}{2 \pi \rho} I_1 \left(\gamma_1 \rho \right) K_1 \left(\gamma_1 \rho \right)$</td>
<td>0</td>
<td>$-\frac{Id \eta}{2 \pi \omega \varepsilon \rho^3}$</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
<td>$-\frac{i \mu_0 \omega Id A}{2 \pi \gamma_1^2 \rho^4} \left[3 - (3 + 3 \gamma_1 \rho + \gamma_1^2 \rho^2) e^{-\gamma_1 \rho}\right]$</td>
<td>0</td>
</tr>
<tr>
<td>HED</td>
<td>$\frac{Id \eta}{2 \pi \sigma_1 \rho^3} \left[1 + (1 + \gamma_1 \rho) e^{-\gamma_1 \rho}\right] \cos \phi$</td>
<td>$\frac{Id \eta}{2 \pi \sigma_1 \rho^3} \left[2 - (1 + \gamma_1 \rho) e^{-\gamma_1 \rho}\right] \sin \phi$</td>
<td>$\frac{i \omega \mu_0 Id \eta}{2 \pi \rho} I_1 K_1 \cos \phi$</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{i \omega \mu_0 Id A}{2 \pi \rho^2} I_1 \left(\frac{\gamma_1 \rho}{2}\right) K_1 \left(\frac{\gamma_1 \rho}{2}\right) \cos \phi$</td>
<td>$\frac{i \omega \mu_0 Id A}{2 \pi \rho^2} \sin \phi \left[3 I_1 K_1 - \gamma_1 \rho \left(I_2 K_1 + I_1 K_2 \right)\right]$</td>
<td>$\frac{i \omega \mu_0 Id A}{2 \pi \rho^2} \cos \phi$</td>
</tr>
</tbody>
</table>
### Table 1 (Cont’d)

SURFACE-TO-SURFACE PROPAGATION FORMULAS FOR THE GENERAL QUASI-STATIC RANGE

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>$H_\rho$</th>
<th>$H_\phi$</th>
<th>$H_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>0</td>
<td>$\frac{\text{Id } \theta}{2\pi \rho^2}$</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>$-\frac{\text{IdA}}{4\pi \rho^3} [16I_1K_1 + \gamma_1^2 \rho^2 (I_1K_1 - I_0K_0)]$</td>
<td>$0$</td>
<td>$-\frac{\text{IdA}}{2\pi \gamma_1^2 \rho^5} [9 - (9 + 9 \gamma_1 \rho + 4 \gamma_1^2 \rho^3 + \gamma_1^3 \rho^5) e^{-\gamma_1 \rho}]$</td>
</tr>
<tr>
<td>HED</td>
<td>$\frac{\text{Id } \theta \sin \phi}{2\pi \rho^2} [3I_1K_1]$</td>
<td>$-\frac{\text{Id } \cos \phi}{2\pi \rho} I_1K_1$</td>
<td>$\frac{\text{Id } \theta}{2\pi \gamma_1^2 \rho^4} [3 - (3 + 3 \gamma_1 \rho + \gamma_1^2 \rho^2) e^{-\gamma_1 \rho}] \sin \phi$</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{\text{IdA}}{2\pi \gamma_1^3} \frac{\sin \phi}{\rho^3} ((\gamma_1 \rho + 5 \gamma_1^2 \rho^2)$</td>
<td>$-\frac{\text{IdA } \cos \phi}{2\pi \gamma_1^2 \rho^3} ((\gamma_1 \rho^2 - 3)$</td>
<td>$\frac{\text{IdA } \sin \phi}{4\pi \rho^3} [\gamma_1^2 \rho^2 (I_1 - I_0K_0)]$</td>
</tr>
<tr>
<td></td>
<td>$+ 12 \gamma_1 \rho + 12) e^{-\gamma_1 \rho} = 12 + 2 \gamma_1^2 \rho^2]$</td>
<td>$+ 3 \gamma_1 \rho + 3) e^{-\gamma_1 \rho} + \gamma_1^2 \rho^2 - 3]$</td>
<td>$+ 4 \gamma_1 \rho (I_1K_1 - I_0K_0) + 16I_1K_1]$</td>
</tr>
</tbody>
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Table 2
SURFACE-TO-SURFACE PROPAGATION FORMULAS FOR THE QUASI-STATIC SUBRANGE $|\gamma_0\rho| << 1 << |\gamma_1\rho|$

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>$E_\rho$</th>
<th>$E_\phi$</th>
<th>$E_z$</th>
<th>$H_\rho$</th>
<th>$H_\phi$</th>
<th>$H_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>$-\frac{i\mu_0 \omega l d t}{2\pi \gamma_1 \rho^3}$</td>
<td>0</td>
<td>$-\frac{1d t}{2\pi i \omega \epsilon_0 \rho^3}$</td>
<td>0</td>
<td>$\frac{1d t}{2\pi \rho^3}$</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
<td>$-\frac{31d A}{2\pi \sigma_1 \rho^6}$</td>
<td>0</td>
<td>$-\frac{31d A}{2\pi \gamma_1 \rho^6}$</td>
<td>0</td>
<td>$-\frac{91d A}{2\pi \gamma_1^2 \rho^6}$</td>
</tr>
<tr>
<td>HED</td>
<td>$\frac{1d t}{2\pi \sigma_1 \rho^3} \cos\phi$</td>
<td>$\frac{1d t}{2\pi \sigma_1 \rho^3} \sin\phi$</td>
<td>$\frac{i\mu_0 \omega l d t \cos\phi}{2\pi \gamma_1 \rho^3}$</td>
<td>$\frac{1d t \sin\phi}{\pi \gamma_1 \rho^3}$</td>
<td>$-\frac{1d t \cos\phi}{2\pi \gamma_1 \rho^3}$</td>
<td>$\frac{31d t}{2\pi \gamma_1^2 \rho^4} \sin\phi$</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{i\omega \mu_0 l d A}{2\pi \gamma_1 \rho^3} \cos\phi$</td>
<td>$\frac{i\omega \mu_0 l d A}{\pi \gamma_1 \rho^3} \sin\phi$</td>
<td>$\frac{i\omega \mu_0 l d A}{2\pi \rho^2} \cos\phi$</td>
<td>$\frac{1d A \sin\phi}{\pi \rho^3}$</td>
<td>$-\frac{1d A \cos\phi}{2\pi \rho^3}$</td>
<td>$\frac{31d A \sin\phi}{2\pi \gamma_1 \rho^4}$</td>
</tr>
</tbody>
</table>
Table 3
SURFACE-TO-SURFACE PROPAGATION FORMULAS FOR THE QUASI-STATIC SUBRANGE \(|\gamma_s \rho| << 1 >> |\gamma_1 \rho|\)

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>(E_\rho)</th>
<th>(E_\phi)</th>
<th>(F_z)</th>
<th>(H_\rho)</th>
<th>(H_\phi)</th>
<th>(H_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>(-\frac{i \mu_e \omega \hat{a}_l}{4 \pi \rho})</td>
<td>0</td>
<td>(-\frac{\hat{a}<em>l}{2 \pi \omega \epsilon</em>\rho^3})</td>
<td>0</td>
<td>(-\frac{\hat{a}_l}{2 \pi \rho^2})</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
<td>(-\frac{i \pi \mu_e \omega}{4 \pi \rho^3})</td>
<td>0</td>
<td>(-\frac{i \rho \gamma_1^3}{16 \pi \rho})</td>
<td>0</td>
<td>(-\frac{i \rho \gamma_1}{4 \pi \rho^3})</td>
</tr>
<tr>
<td>NED</td>
<td>\frac{\hat{a}_l}{4 \pi \rho^3 \cos \phi}</td>
<td>\frac{\hat{a}_l}{2 \pi \rho \sin \phi}</td>
<td>\frac{i \pi \mu_e \omega \hat{a}_l \cos \phi}{4 \pi}</td>
<td>\frac{\hat{a}_l}{4 \pi \rho^2 \sin \phi}</td>
<td>\frac{i \rho \gamma_1 \hat{a}_l \cos \phi}{4 \pi}</td>
<td>\frac{i \rho \gamma_1^2 \sin \phi}{4 \pi \rho^2}</td>
</tr>
<tr>
<td>NMD</td>
<td>\frac{i \rho \gamma_1 \hat{a}_l}{4 \pi \rho^2 \cos \phi}</td>
<td>\frac{i \rho \gamma_1 \hat{a}_l \sin \phi}{4 \pi \rho^2}</td>
<td>\frac{i \rho \gamma_1 \hat{a}_l \cos \phi}{2 \pi \rho^2}</td>
<td>\frac{\hat{a}_l}{4 \pi \rho^3 \cos \phi}</td>
<td>\frac{i \rho \gamma_1^2 \sin \phi}{16 \pi \rho}</td>
<td></td>
</tr>
</tbody>
</table>
Table 4
SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE QUASI-NEAR RANGE

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>$E_{\rho}$</th>
<th>$E_{\phi}$</th>
<th>$E_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>$\frac{1dI e^{j\phi}}{2\pi \sigma_1} \frac{\rho}{R^2} \left[ 3 \frac{z}{R} - \frac{y_1^2}{R^2} \right]$</td>
<td>0</td>
<td>$- \frac{1dI e^{j\phi}}{2\pi \sigma_1} \frac{\rho}{R^3} \left( 1 - \frac{3z^2}{R^2} \right)$</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
<td>$- \frac{3MA e^{j\phi}}{2\pi \sigma_1} \frac{\rho}{R^2} \left( 1 + \frac{y_1^2}{R^2} - \frac{3z^2}{R^2} \right)$</td>
<td>0</td>
</tr>
<tr>
<td>NED</td>
<td>$\frac{1dI \cos \phi e^{j\phi}}{2\pi \sigma_1} \frac{1}{R^3} \left[ 1 - \frac{y_1^2}{R^2} \right]$</td>
<td>$\frac{1dI \sin \phi e^{j\phi}}{2\pi \sigma_1} \frac{1}{R^2} \left( 2 + \frac{y_1^2}{R^2} - \frac{3z^2}{R^2} \right)$</td>
<td>$\frac{i\mu_0 I dA \cos \phi e^{j\phi}}{2\pi \gamma_1} \frac{\rho e^{j\phi}}{R^3}$</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{i\mu_0 I dA \cos \phi e^{j\phi}}{2\pi \gamma_1 R^2} \left( 1 - \frac{y_1^2}{R^2} \right)$</td>
<td>$\frac{i\mu_0 I dA \sin \phi e^{j\phi}}{2\pi \gamma_1 R^3} \left( 2 + \frac{y_1^2}{R^2} - \frac{3z^2}{R^2} \right)$</td>
<td>$\frac{i\mu_0 I dA \cos \phi e^{j\phi}}{2\pi R^3}$</td>
</tr>
<tr>
<td>Dipole Type</td>
<td>$H_\rho$</td>
<td>$H_\phi$</td>
<td>$H_z$</td>
</tr>
<tr>
<td>-------------</td>
<td>---------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>VED</td>
<td>0</td>
<td>( \frac{\text{ld} , \gamma_1^2}{2\pi \gamma_1} e^{\gamma_1 h} \frac{\rho}{R^3} )</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>(- \frac{3\text{ldA} , e^{\gamma_1 h}}{2\pi \gamma_1} \frac{\rho}{R^3} \left(1 - \frac{5x^2}{R^2}\right))</td>
<td>0</td>
<td>(- \frac{3\text{ldA} , e^{\gamma_1 h}}{2\pi \gamma_1} \frac{3(1 + \gamma_1 n)}{R^5} - \frac{5x^2}{R^3} (6 + \gamma_1 n) - \frac{35x^4}{R^6})</td>
</tr>
<tr>
<td>HED</td>
<td>(\frac{\text{ldA} , \sin \phi , e^{\gamma_1 h}}{2\pi \gamma_1} \frac{2 - \frac{3x^2}{R^2}}{R^3})</td>
<td>(- \frac{\text{ldA} , \cos \phi , e^{\gamma_1 h}}{2\pi \gamma_1} \frac{1 + \gamma_1 n - \frac{5x^2}{R^2}}{R^3})</td>
<td>(\frac{3\text{ldA} , \sin \phi , pe^{\gamma_1 h}}{2\pi \gamma_1} \frac{1 + \gamma_1 n - \frac{5x^2}{R^2}}{R^3})</td>
</tr>
<tr>
<td>HMD</td>
<td>(\frac{\text{ldA} , \sin \phi , e^{\gamma_1 h}}{2\pi R^3} \left[2 - \frac{3x^2}{R^2}\right])</td>
<td>(- \frac{\text{ldA} , \cos \phi , e^{\gamma_1 h}}{2\pi R^3})</td>
<td>(\frac{3\text{ldA} , \sin \phi , pe^{\gamma_1 h}}{2\pi \gamma_1 R^3} \left(1 + \gamma_1 n - \frac{5x^2}{R^2}\right))</td>
</tr>
</tbody>
</table>
### Table 5

**SURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE**

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>$\frac{1}{2\pi\sigma_1} \int \frac{e^{-\gamma_1 h}}{\rho e^{-\gamma_0</td>
<td>R</td>
<td>}} \left[ z (3 + 3 \gamma_0</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
<td>$\frac{1}{2\pi\sigma_1} \int \frac{e^{-\gamma_1 h}}{\rho e^{-\gamma_0</td>
<td>R</td>
</tr>
<tr>
<td>HED</td>
<td>$\frac{1}{2\pi\sigma_1} \int \frac{e^{-\gamma_1 h}}{\rho e^{-\gamma_0</td>
<td>R</td>
<td>}} \left[ 1 - \gamma_1 x \right] (1 + \gamma_0</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{i \mu_0 \omega dA \cos \phi e^{-\gamma_1 h}}{2\pi \gamma_1} e^{-\gamma_0</td>
<td>R</td>
<td>} \left[ 1 - \gamma_1 x \right] (1 + \gamma_0</td>
</tr>
<tr>
<td>Dipole Type</td>
<td>$H_\rho$</td>
<td>$H_\phi$</td>
<td>$H_z$</td>
</tr>
<tr>
<td>-------------</td>
<td>---------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>VED</td>
<td>0</td>
<td>$\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} e^{\gamma_1 k} \rho e^{-\gamma_0 R} \left(1 + \gamma_0 R\right) e^{-\gamma_0 R}$</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>$-\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} e^{\gamma_1 k} \rho e^{-\gamma_0 R}$</td>
<td>0</td>
<td>$-\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} e^{\gamma_1 k} \rho e^{-\gamma_0 R} \left{ 9 \left(1 + \gamma_1 z\right) - \frac{15z^2}{R^2} \left(6 + \gamma_1 z\right) + \frac{105z^4}{R^4} \left(1 + \gamma_0 R\right) \right}$</td>
</tr>
<tr>
<td>HED</td>
<td>$\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \sin \phi e^{\gamma_1 k} e^{-\gamma_0 R} \left(2 - \frac{3z^2}{R^2}\right) \left(1 + \gamma_0 R\right) - \gamma_0^2 z^2}$</td>
<td>$-\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \cos \phi e^{\gamma_1 k} e^{-\gamma_0 R} \left(1 + \gamma_0 R + \gamma_0^2 R^2\right)$</td>
<td>$-\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \sin \phi e^{\gamma_1 k} e^{-\gamma_0 R} \left(3 + 3\gamma_1 z \right)$</td>
</tr>
<tr>
<td>HMD</td>
<td>$\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \sin \phi e^{\gamma_1 k} e^{-\gamma_0 R} \left(2 - \frac{3z^2}{R^2}\right) \left(1 + \gamma_0 R\right) - \gamma_0^2 z^2$</td>
<td>$-\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \cos \phi e^{\gamma_1 k} e^{-\gamma_0 R} \left(1 + \gamma_0 R + \gamma_0^2 R^2\right)$</td>
<td>$\frac{1}{2\pi} \frac{\gamma_1}{\gamma_2} \sin \phi \rho e^{\gamma_1 k} e^{-\gamma_0 R} \left(3 + 3\gamma_1 z - \frac{15z^2}{R^2}\right) \left(1 + \gamma_0 R\right)$</td>
</tr>
</tbody>
</table>

Table 5 (Cont'd)

SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE
<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>( E_\rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k}}{\rho^4} \left( \frac{\gamma_0^2}{\gamma_1} \right) (1 + \gamma_0 \rho)^2 ]</td>
</tr>
<tr>
<td></td>
<td>[-z (3 + 3 \gamma_0 \rho + \gamma_0^2 \rho^2) e^{-\gamma_0 \rho} ]</td>
</tr>
<tr>
<td>VMD</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k} e^{-\gamma_0 \rho}}{\rho^4} (3 + 3 \gamma_0 \rho + \gamma_0^2 \rho^2) (1 + \gamma_1 z) ]</td>
</tr>
<tr>
<td>HED</td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k} e^{-\gamma_0 \rho}}{\rho^3} \left( \frac{1 - \gamma_1 z}{(1 + \gamma_0 \rho + \gamma_0^2 \rho^2)} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k} e^{-\gamma_0 \rho}}{\rho^3} \left( \frac{2 + \gamma_1 z}{(1 + \gamma_0 \rho)} \right) ]</td>
</tr>
<tr>
<td>HMD</td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k} e^{-\gamma_0 \rho}}{\rho^3} \left( \frac{1 - \gamma_1 z}{(1 + \gamma_0 \rho + \gamma_0^2 \rho^2)} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[-\frac{1}{2 \pi \sigma_1} \frac{e^{\gamma_1 k} e^{-\gamma_0 \rho}}{\rho^3} \left( \frac{2 + \gamma_1 z}{(1 + \gamma_0 \rho)} \right) ]</td>
</tr>
</tbody>
</table>
**Table 6 (Cont'd)**

**SUBSURFACE-TO-AIR PROPAGATION FORMULAS FOR THE NEAR-FIELD RANGE WITH \( \rho >> z \)**

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>( H_\rho )</th>
<th>( H_\phi )</th>
<th>( H_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VED</td>
<td>0</td>
<td>[ \frac{1}{\gamma_1} \frac{\gamma_1^2}{2\pi \rho^2} (1 + \gamma_6 \rho) e^{\gamma_1 \rho} e^{-\gamma_6 \rho} ]</td>
<td>0</td>
</tr>
<tr>
<td>VMD</td>
<td>[ \frac{1}{2\pi \gamma_1} \frac{\gamma_1^2}{\rho^4} (3 + 3 \gamma_6 \rho + \gamma_6^2 \rho^2) ]</td>
<td>0</td>
<td>[ \frac{1}{2\pi \gamma_1^2} \frac{\gamma_1 \gamma_6^2}{\rho^3} (9 + 9 \gamma_6 \rho + 4 \gamma_6^2 \rho^2 + \gamma_6^3 \rho^3) ]</td>
</tr>
<tr>
<td>HED</td>
<td>[ \frac{1}{\pi \gamma_1} \frac{\gamma_1^2}{\rho^3} ] [1 + \gamma_6 \rho]</td>
<td>[ \frac{1}{\pi \gamma_1} \frac{\gamma_1 \gamma_6}{\rho^4} ] [1 + \gamma_6 \rho + \gamma_6^2 \rho^2]</td>
<td>[ \frac{1}{\pi \gamma_1} \frac{\gamma_1 \gamma_6}{\rho^4} ] [(1 + \gamma_1 \rho) (3 + 3 \gamma_6 \rho + \gamma_6^2 \rho^2)]</td>
</tr>
<tr>
<td>HMD</td>
<td>[ \frac{1}{2\pi} \frac{\gamma_1^2}{\rho^3} [2(1 + \gamma_6 \rho)] ]</td>
<td>[ \frac{1}{2\pi} \frac{\gamma_1 \gamma_6}{\rho^4} ] [1 + \gamma_6 \rho + \gamma_6^2 \rho^2]</td>
<td>[ \frac{1}{2\pi \gamma_1} \frac{\gamma_1 \gamma_6}{\rho^4} ] [(3 + 3 \gamma_6 \rho + \gamma_6^2 \rho^2) (1 + \gamma_1 \rho)]</td>
</tr>
</tbody>
</table>
The electric and magnetic field components produced by vertical and horizontal dipoles (both electric and magnetic) are derived and presented for the quasi-static, quasi-near, and near-field ranges. The depth, $h$, of the transmitting dipole is less than or equal to zero; the height, $z$, of the receiving antenna above the plane, conducting, homogeneous earth varies from zero to some height, $z$, that is much less than the ionospheric reflecting height (ionospheric effects are neglected). The horizontal separation, $p$, between the transmitting and receiving antennas is comparable to the receiving antenna height, $z$. The derivations are based upon the quasi-static and near-field approximations to the vector potentials for the vertical and horizontal dipoles and upon application of the reciprocity theorem. The surface-to-surface propagation equations reduce to well-known expressions when $|\gamma_1 p| >> 1$ and $|\gamma_1 p| << 1$, as do the subsurface-to-air equations when $p >> z$. 
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<table>
<thead>
<tr>
<th>Key Words</th>
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<th>Link B</th>
<th>Link C</th>
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<tr>
<td>Electromagnetic fields</td>
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<td></td>
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<tr>
<td>Near field</td>
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<td></td>
</tr>
</tbody>
</table>

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