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A REPAIRABLE ITEM INVENTORY MODEL

by

J. W. Hatchett
P. F. McNall
D. A. Schrady
P. W. Zehna

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral E. J. O'Donnell, USN
Superintendent

Dr. R. F. Rinehart
Academic Dean

ABSTRACT:

A model is defined to account for an inventory system which carries items that are subject to repair after use, wearout or failure and then reissued. Such a system is called a repairable item inventory system and, important as such systems are to military applications, have received little attention in the scientific literature. A repairable item inventory system would be trivial if it were not for the fact that a used item may not be repairable due to excessive damage either before or during attempted repair and hence new items must be procured from time to time to replenish the system. This report determines decision rules for this dual problem of repair and procurement in a deterministic model.

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Prepared by: J. W. Hatchett
P. F. McNall
D. A. Schrady
P. W. Zehna

Approved by:
J. R. Borsting
Chairman, Department of
Operations Analysis

Released by:
C. E. Menneken
Dean of
Research Administration

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1. Introduction

Inventories of physical goods can be found in every sector of the economy. These inventories exist primarily to make goods available to customers or producers without delay and to increase sales and profits. Although no profit motive exists, military inventory systems carry a diversity of goods in order to satisfy the demands of fleet units without production and transportation delays. Since inventories exist, it is natural to try to classify the types of items carried. Two broad classifications, namely, consumable and repairable, can be used to characterize <u>any</u> item. A consumable item is defined to be one that is either consumed in use or discarded after wear out or failure. Examples of consumable items are paper, pencils, paint, fuel, nails, food, gaskets resistors, to mention only a few. A repairable item, on the other hand, is defined to be one that can be repaired after failure or wear out and subsequently may be available again to the user. Aircraft, refrigerators radios, engines, and hydraulic pumps are all examples of repairable items. They can be repaired by the user, a repair shop, or the manufacturer. In more specific military terms the level of repair can be classified as ship tender, and shipyard, respectively.

A further delineation of the definition of a repairable item is to define it as an item that is returned to a major repair point after use, overhauled or repaired, put back on the shelf in a ready-for-issue (RFI) condition, and reissued to a customer to satisfy a demand. This definition will apply throughout this paper and can be construed as a particular military application of the term repairable.

Consumable and repairable item inventories are worth billions of dollars and are costly to manage. Therefore, once the type of inventory has been established, an effective system to maintain and control the inventory should be developed. In private and commercial concerns, effective control of inventories can result in decreased costs, increased sales and profits and customer satisfaction. In the military, prudent management of inventories may contribute to increased weapons system effectiveness, decreased inventory investment and decreased system costs.

In any system carrying consumable items a set of rules to determine how much of an item to buy and when to buy, i.e., an operating doctrine, must be established. In a repairable system the procurement decision is augmented by an additional decision of how much and when to repair. Thus, the additional repair decision is the basic difference between a purely consumable item and a repairable item inventory system.

Typically, existing inventory control models have applied only to consumable items. "Optimal" order equations resulting from consumable model development have been implemented successfully by both the military and industrial concerns. Although increased management attention has been focused on repairables over the past ten years, repairable inventory decisions have been largely based upon experience and intuition. The main purpose of this report is to define a repairable item inventory model and to develop decision rules for such a model, giving due consideration to the costs associated with a repairable system.

It should be mentioned at this point that the decision to designate an item as repairable or consumable is not perfunctory. A decision to

repair or discard occurs not only in the intermediate step when an item is provisioned for a system, but also in the initial or design stage and finally in the repair or overhaul stage. The most critical stage for an inventory system is initial provisioning. What criteria should be used to designate an item (already in the production phase) as repairable or consumable? What level of repair should be designated? No specific criteria have been developed to answer these questions so far as the authors can determine, and it is not the purpose of this report to do so. However, it should be noted that the rationale behind designating an item as a repairable is that it is more economical to repair than to discard. Basically then, this decision involves the trade-off between costs of repair versus discard.

2. An Existing Repairable Inventory System

To better understand how a repairable system operates, the writers examined the Naval Aviation Supply System. This system exists to support 8,300 aircraft in the Navy. The inventory consists of 393,000 line items valued at 2.1 billion dollars. Of these line items, 31,000 are designated as repairable and these items account for 56% of the inventory value.[1] The inventory manager responsible for procurement and inventory control of all aviation items is the Naval Aviation Supply Office (ASO) located in Philadelphia, Pennsylvania. The items are stored in and issued from a network of stockpoints located throughout the Naval Supply System (e.g. Alameda, Norfolk, San Diego, and Yokosuka, Japan). Additionally there are seven major repair points called overhaul and repair activities (O&R)

[1] ASO Management Resumé - Second Quarter Fiscal Year 1966.

located in the United States. A fourth component, Air Systems Command (a Navy Bureau), provides technical direction and budget policy to ASO and also administers the seven O&R activities located at various air stations throughout the system.

If an item fails or is demolished in the field, a replacement is made from existing stock. The carcass (if suitable) is then considered a non-ready-for-issue (NRFI) item and is sent to an O&R (through a stockpoint) for repair.

Since the seven O&R activities are each juxtaposed to a reporting stockpoint, the latter actually receives and accounts for the NRFI item. This receipt is reported to the inventory manager (ASO) via rapid data transmission facilities. In this sense, the stock points are called reporting activities, i.e., all inventory transactions are reported to ASO who is in turn responsible for inventory control. When ASO determines that an item should be repaired to meet expected demand or to meet existing backorders, an O&R activity is directed to induct the NRFI item(s). Induction scheduling between ASO and the O&R occurs on a weekly basis through a computerized scheduling system. Under present circumstances, most of the items inducted have backorders outstanding. This situation eliminates batching of NRFI items to a great extent. Once the item is repaired, it is returned to the stock point in RFI condition and subsequently issued to meet demand or fill backorders. The system is depicted in Figure 1.

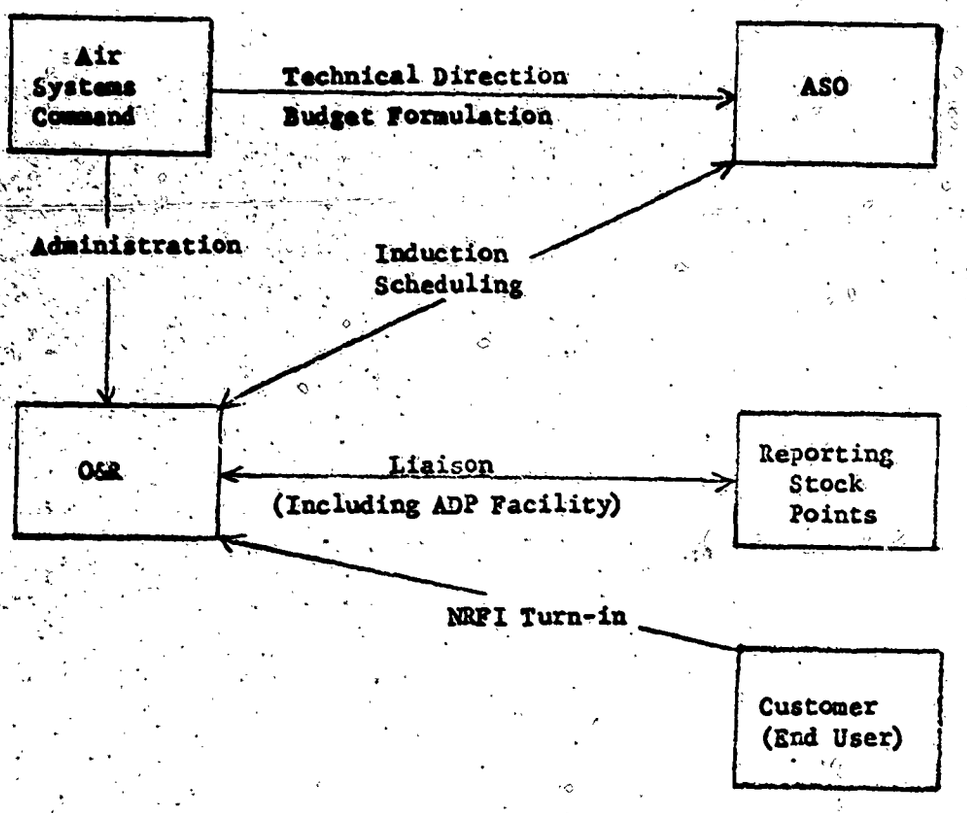

Figure 1. Naval Aviation Supply System (Repairable Items)

3. A Repairable Model

3.1. Model Description

Inventory systems such as the one just described can be classified rather broadly as multiechelon with repair. Analytical studies of multiechelon systems have shown the computations in such models to be such that either simplifying assumptions or approximations are necessary. The introduction of the repair aspect into the system certainly complicates the structure of a model even at single echelon levels. Hence, it was felt that simplifying assumptions would lead to results more in keeping with the objective of the report. Also a search of the literature has

revealed that very little work has been done in the matter of structuring a model for the repair inventory system of the type discussed in Section 2. The model presented here is intended to provide a basis for future studies and represents an initial attempt.

Suppose a repairable system, consisting of one inventory control point (ICP), one stock point and one overhaul and repair activity, controls the inventory of a single item. Demands from various customers are placed only at the stock point. The system has continuous updating of records, i.e., transaction reporting. When items wear out or fail, the customer can either scrap the item or return it to the O&R. After inspection the O&R can either scrap the item or repair and return it to the system. Both ready-for-issue and not-ready-for-issue flow of material is depicted in Figure 2.

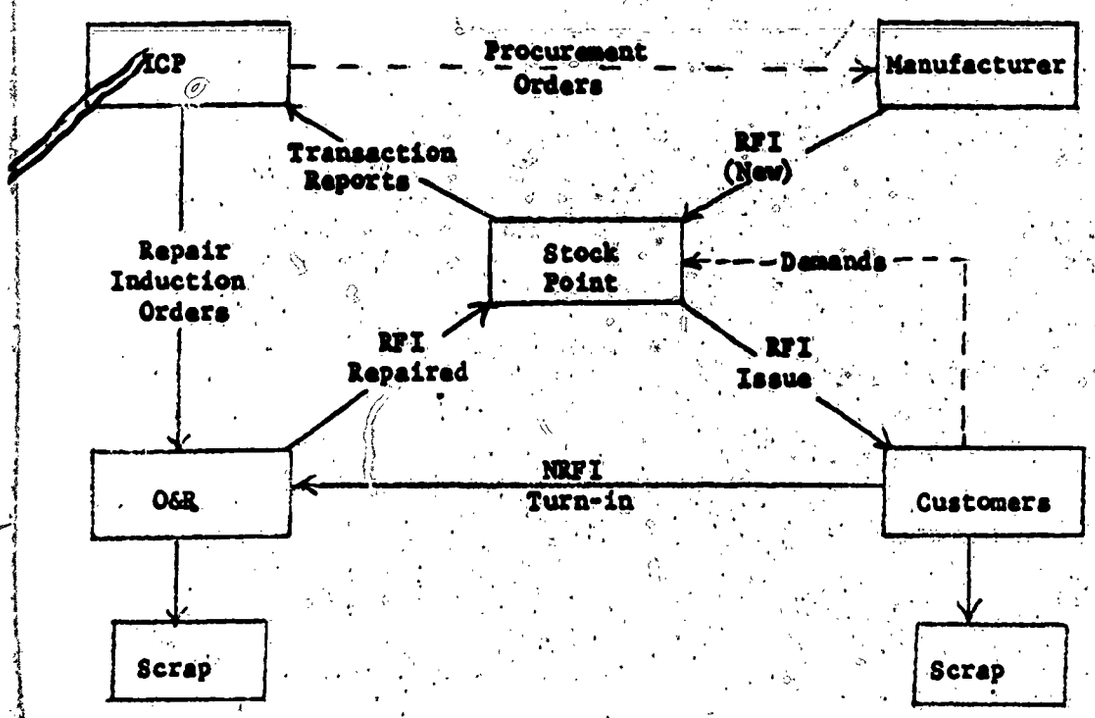

Figure 2. Material Flow in a Repairable System

3.2 Assumptions

Assume that the annual demand rate (D) is known and constant over time. To reiterate, the basic management decisions to be made are when and how much to procure and when and how much to repair. In this model procurement of new items serves to replace items lost by attrition. Suppose that both procurement lead time (τ_1) and repair lead time (τ_2) are known constants independent of the quantity ordered, the quantity inducted for repair, and the annual demand. Furthermore, the rate at which NRFI items are returned to the O&R, called field recovery rate (r_0), and the rate at which the O&R returns RFI to the stockpoint, called O&R recovery rate (r_2), are considered to be known. Items are always procured and repaired in lot sizes, Q_1 and Q_2 respectively, with no price breaks or split deliveries. For the sake of definiteness suppose the system operates indefinitely with the item never becoming obsolete.

The question of when to induct material into the repair operation is partially answered with the assumption that an induction is made whenever a predetermined number of NRFI items have accumulated. In order to return a lot size Q_2 to the stockpoint it is necessary to induct an amount $\frac{Q_2}{r_2}$. This assumption reduces the problem to one of determining Q_2. A more complete pictorial representation of the system is seen in Figure 3.

Under the assumptions of deterministic demand and lead times, it is not necessary to maintain a safety stock. We will require then that whenever the RFI inventory reaches zero, a procurement quantity of size Q_1 will arrive. Thus, the system is never out of stock. Between procurement arrivals, depleting RFI stock is replenished with repaired items.[1]

[1] Under these assumptions it is possible, though extremely unlikely, for a procurement quantity, Q_1, and a repair quantity, Q_2, to arrive at exactly the same instant of time. Accordingly it will be assumed that this case does not arise.

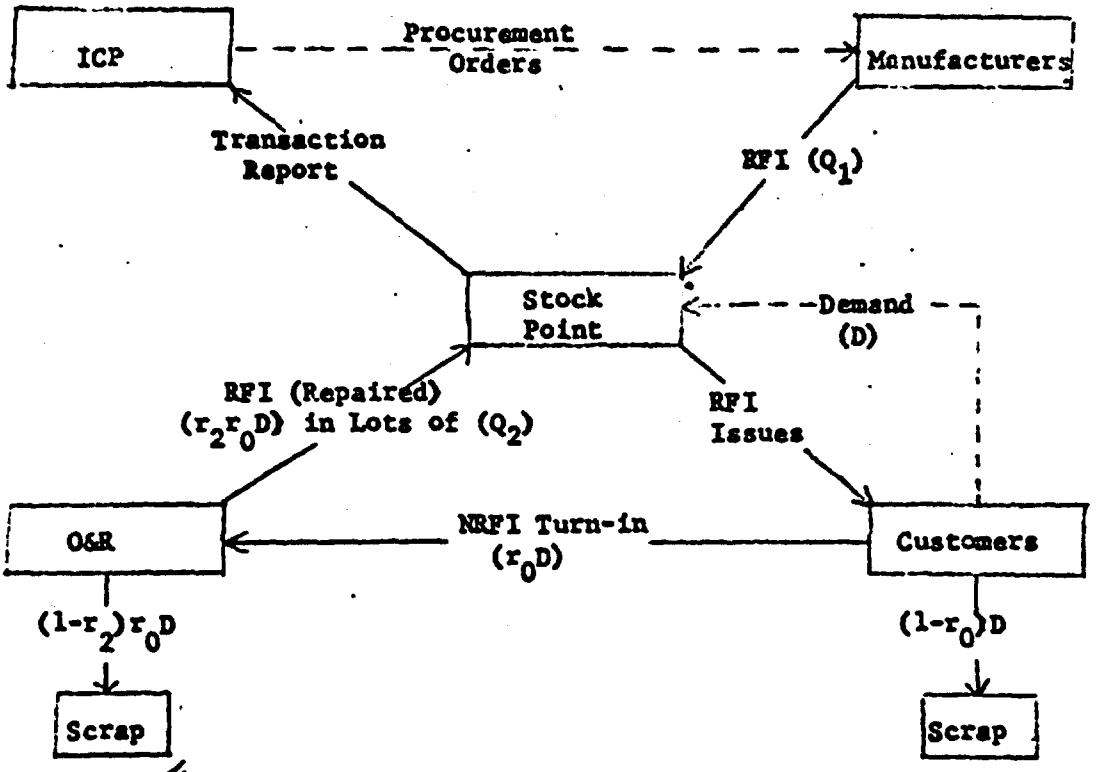

Figure 3. Repair System with Assumptions

Also, a repetitive system will be established regardless of the initial provisioning policy, so that it is sufficient to analyze only one cycle to determine system characteristics. Further, it is advantageous to define a cycle as the length of time between the arrival of two successive procurements. This cycle will be called the procurement cycle and is denoted T_1. A repair cycle, denoted T_2, is defined to be the time between arrival of successive RFI repair quantities. Figure 4 depicts a typical procurement and repair cycle and illustrates the relationship between the RFI and NRFI inventory.

Prior models designed to develop decision rules for repairable inventory systems have treated the repair and procurement decisions independently. In some models the repairable decision rules have been cast in the framework of a consumable model with modified parameters. It

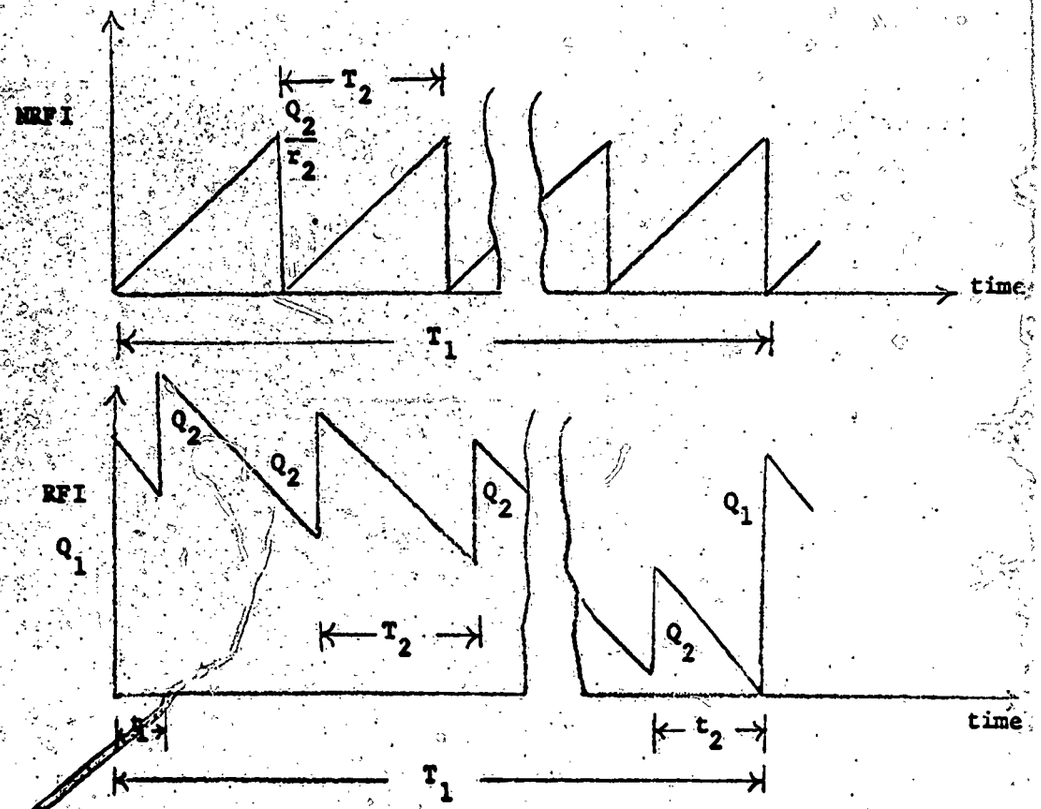

Figure 4. Procurement and Repair Cycles

is a major point of this report that the two problems should be treated simultaneously. Accordingly, the decision rule regarding "when" and "how much" to procure as well as "when" and "how much" to repair is deriv by minimising the total average annual variable cost of operating the system accounting for both repair and procurement.

3.3 Discussion of Costs

Successful management of a repairable inventory system is dependent on proper identification of relevant costs, i.e., costs which affect the operating doctrine. Five possible broad cost categories are listed below and discussed subsequently:

1. Information and Issue Systems
2. Procurement
3. Repair
4. Holding
5. Shortage

The costs associated with maintaining financial and inventory records represent the largest segment of the information system costs. Typical among these costs are: (a) data processing equipment and related personnel, (b) commodity analysts, (c) financial inventory control, (d) AUTODIN and (e) quality control. Issue system costs are primarily: (a) requisition processing, (b) warehousemen, (c) transportation and (d) disposal. It can be rationalized that these costs are a function of demand and not the operating doctrine. For this reason information and issue systems costs will not be included in defining the total annual variable cost for determining operating doctrines.

The only real procurement cost is the cost of the items ordered, frequently called the variable order cost. If there is a price break for large quantities, then this cost is a function of the quantity ordered and the unit cost. If there is no price break, as assumed in this paper, then this cost is simply the unit cost times the quantity ordered. The traditional fixed order cost involves salaries of purchase and receiving personnel, material inspectors, telephones, paper, pencils, duplicating machines, etc. It is easily seen that both the fixed and variable order costs affect the operating doctrine and will be included as a part of the cost formulation.

The usual approach to repair costs considers in detail a set-up or tooling cost.[1] Strictly speaking, repairables cannot be handled on a production line basis simply because each item may have a different "ailment." Each item must be checked and tested separately, therefore there is no fixed set-up or tooling cost. However, there is a repair cost associated with placing an induction order. Repair costs can then be divided into direct labor and material and overhead, i.e., the cost of repairing an item can be considered to be a function of direct labor and material plus some overhead cost. It follows that the fixed repair cost is really the cost of making and carrying out the decision to induct a given quantity into the repair cycle. It is assumed here that some "average" cost, C_2, of direct labor and material per item can be found. Thus the variable cost of repair will be formulated at C_2 times the quantity inducted. Both variable and fixed repair costs will be included in the total cost formulation since they affect operating doctrine.

The costs associated with maintaining items in inventory include among others, obsolescence, opportunity, deterioration, breakage and normal warehousing costs. This "holding cost" is quite intangible and has proven difficult to evaluate. In the past, holding cost has been expressed as a function of unit cost (C_1) or more specifically as $h_1 = IC_1$, where I is a holding rate. The holding rate I incorporates for the main part, opportunity, obsolescence, and warehousing costs and is usually expressed as cost per unit time per monetary unit invested in inventory. However, for purposes of this model it is assumed that there exists a holding cost, h_1, for each RFI item and a similar cost, h_2, for a NRFI

[1] J. F. Magee, Production Planning and Inventory Control, (New York: McGraw-Hill Book Co., Inc., 1958) p. 11.

item. Both h_1 and h_2 are defined in terms of cost per unit per unit time, which for this model is measured in dollars per unit-year. Of course both holding costs affect the operating doctrine.

Finally, the shortage cost represents the cost of being unable to meet customer demands. In general, this is a very important cost, though again quite difficult to measure. By assumption of deterministic demand and lead times it is never necessary to "go short" of stock, and so shortage costs have been excluded from the model presented in this paper.

4. Solutions

As previously stated, the objective of the model is to determine an optimal procurement quantity Q_1* and an optimal repair quantity Q_2* that minimizes the average annual cost. These optimal values, coupled with respective reorder points, X_1 and X_2, constitute an optimal operating doctrine.

To determine the average annual variable cost, the costs per cycle must be investigated. The product of per cycle costs and the number of cycles per year will then yield the average annual cost. The total cost for any given cycle T_1 is the sum of procurement, repair and holding costs.

Since there is only one procurement per cycle, the variable order cost is the actual cost of the items ordered and can be expressed as Q_1C_1. The fixed procurement cost for one cycle is A_1.

To determine the repair costs per cycle it is necessary to compute

the number of repair cycles per procurement cycle. Let $\frac{T_1}{T_2} = n$. It will be shown that n is an integer. It is sufficient to show that $t_1+t_2 = T_2$ where $t_1 = kT_2$ for some $0 \le k \le 1$ and $t_2 = (1-k)T_2$ (cf. Figure 6). This follows because $T_1 - (t_1 + t_2)$ is an integer number of repair cycles by definition. By assumption a quantity Q_2 will arrive at intervals of length T_2 as long as there is sufficient NRFI material to induct. But also, by assumption, the arrival of a quantity Q_1 insures that there will always be enough NRFI to allow for induction. Moreover, it was assumed that a quantity Q_1 will arrive only when RFI on hand balance reaches zero and this is prior to the arrival of a scheduled repair delivery. Let the time between the arrival of a procurement and the very next repair arrival be t^*. Now, $t_2 + t^* = T_2$ since a repair quantity arrives at every interval T_2. But $t^* = t_1$, since demand and leadtimes are deterministic, and the cycles are periodic. Therefore the cost of items repaired per procurement cycle is C_2Q_2 and the fixed repair cost is A_2n, where A_2 is defined to be the fixed repair cost per induction.

In Section 3, h_1 and h_2 were defined as RFI and NRFI holding costs per unit per unit time respectively. Therefore the holding costs per cycle will be $h_1\mathcal{A}_{T_1} + h_2\mathcal{A}_{T_2}$, where $\mathcal{A}_{T_1}$ is the area under the RFI curve and $\mathcal{A}_{T_2}$ is the area under the NRFI curve. To compute the area under the RFI curve consider Figure 6 showing RFI inventory for one procurement cycle. Since D is known and constant, the area of U, $\mathcal{A}_U$, is simply $\frac{1}{2}t_1(Q_1 + a)$, where $t_1 = kT_2$ and $a = Q_1 - Dt_1$. This reduces to

$$(4\text{-}1)\quad \mathcal{A}_U = t_1Q_1 - \frac{Dt_1^2}{2} .$$

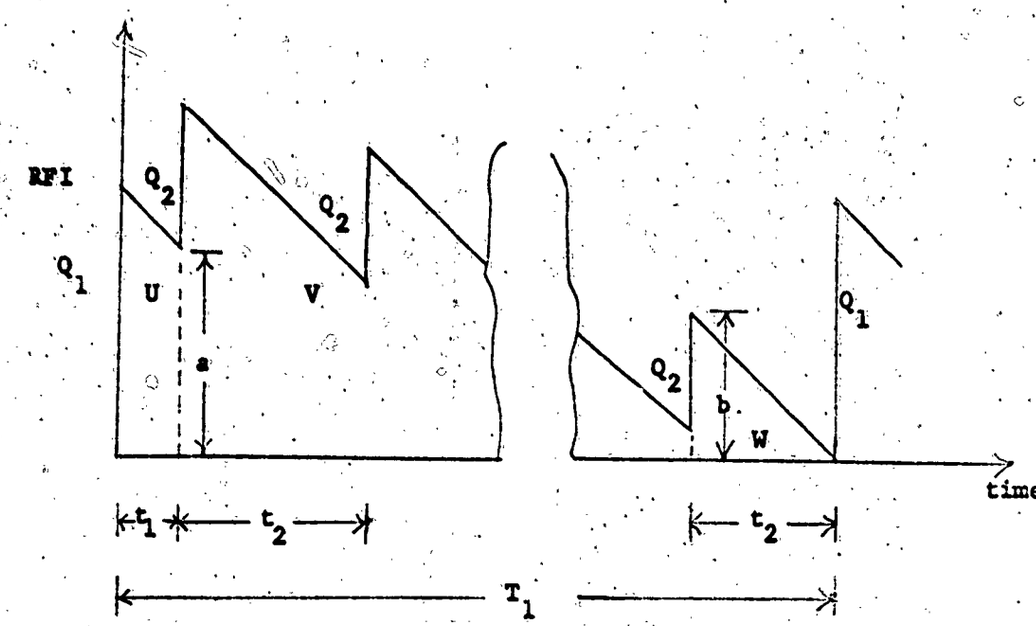

Figure 6. Procurement Cycle

Since n is an integer, the area of V is the sum of n-1 trapezoids each having one side of length T_2. To determine the area of each trapezoid a recursive relation will be developed. Let $\mathcal{A}_{V_i}$ denote the area of the ith trapezoid in V (Figure 6). Then,

$$\mathcal{A}_{V_1} = \tfrac{1}{2}T_2\left[2Q_2 + 2(Q_1 - t_1D) - DT_2\right],$$

$$\mathcal{A}_{V_2} = \tfrac{1}{2}T_2\left[4Q_2 + 2(Q_1 - t_1D) - 3DT_2\right],$$

and, proceeding recursively,

$$\mathcal{A}_{V_i} = \tfrac{1}{2}T_2\left[2iQ_2 + 2(Q_1 - t_1D) - (2i - 1)DT_2\right], \quad i = 1, \ldots, n - 1.$$

Therefore, $\mathcal{A}_V = \sum_{i=1}^{n-1} \mathcal{A}_{V_i}$ so that

$$\mathcal{A}_V = \tfrac{1}{2}T_2\left[2Q_2\sum_{i=1}^{n-1} i + 2(n-1)(Q_1 - t_1D) - DT_2\sum_{i=1}^{n-1}(2i-1)\right]$$

$$= \tfrac{1}{2}T_2\left[Q_2 n(n-1) + 2(n-1)Q_1 - 2(n-1)t_1D - DT_2 n(n-1)\right.$$

$$\left. + DT_2(n-1)\right]$$

Finally,

$$(4\text{-}2)\quad \mathcal{A}_V = \tfrac{1}{2}T_2\left[2(n-1)(Q_1 - t_1D) + (n^2 - n)Q_2 - (n-1)^2 DT_2\right].$$

$\mathcal{A}_W$ is easily calculated as $\frac{1}{2}t_2 b$, where $t_2 = (1-k)T_2$ and, b, from the above recursive relation of trapezoidal bases, is $nQ_2 + (Q_1 - t_1D) - (n-1)DT_2$. Thus,

$$(4\text{-}3)\quad \mathcal{A}_W = \tfrac{1}{2}t_2\left[nQ_2 + Q_1 - t_1D - (n-1)DT_2\right].$$

Finally the total area under the RFI curve is

$$\mathcal{A}_{T_1} = \mathcal{A}_U + \mathcal{A}_V + \mathcal{A}_W = t_1Q_1 - \tfrac{1}{2}Dt_1^2 + \tfrac{1}{2}T_2\left[2(n-1)(Q_1 - Dt_1)\right.$$

$$(4\text{-}4)\qquad \left. + (n^2 - n)Q_2 - (n-1)^2 DT_2\right] + \tfrac{1}{2}t_2\left[Q_1 - Dt_1 + nQ_2 - (n-1)DT_2\right].$$

To compute the area under the NRFI curve consider a single repair cycle as shown in Figure 7. Since n is an integer and the buildup rate of NRFI items, $r_0 D$, is constant, the area under the NRFI curve is simply n times the area under the repair cycle curve.

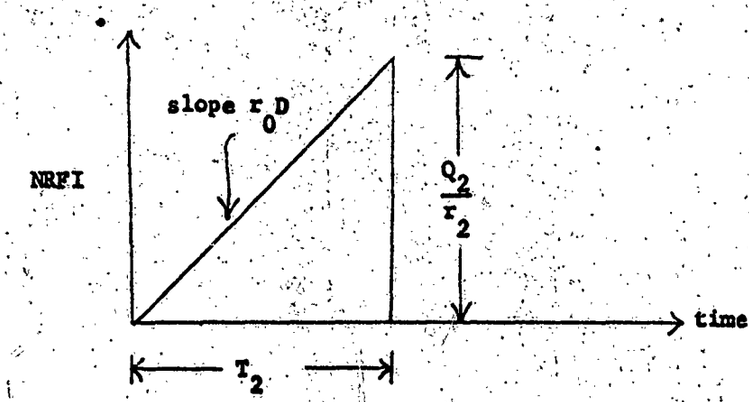

Figure 7. Repair Cycle

The area under the repair cycle curve (Figure 7) is $\frac{T_2 Q_2}{2r_2}$ so that the NRFI holding cost per procurement cycle is $\frac{nT_2 Q_2 h_2}{2r_2}$.

The total cost per procurement cycle becomes

$$(4\text{-}5) \quad \mathcal{K}_{T_1} = Q_1 C_1 + A_1 + \frac{A_2 T_1}{T_2} + \frac{C_2 Q_2 T_1}{T_2} + \frac{h_2 Q_2 T_1}{2r_2} + h_1 \mathcal{A}_{T_1} .$$

The total average annual cost is then

$$(4\text{-}6) \quad \mathcal{K} = \frac{\mathcal{K}_{T_1}}{T_1} = \frac{Q_1 C_1}{T_1} + \frac{A_1}{T_1} + \frac{A_2}{T_2} + \frac{C_2 Q_2}{T_2} + \frac{h_2 Q_2}{2r_2} + \frac{h_1 \mathcal{A}_{T_1}}{T_1} .$$

To simplify $\mathcal{K}$ the following relations are helpful:

$$(1)\quad T_2 = \frac{Q_2}{r_2 r_0 D}; \qquad (2)\quad \frac{T_1}{T_2} = n,\ n \in \{1,2,3,\ldots\};$$

$$(3)\quad t_1 = kT_2 = \frac{kQ_2}{r_2 r_0 D} \quad \text{for } 0 \le k \le 1;$$

(4-7)

$$(4)\quad t_1 + t_2 = T_2; \quad (5)\quad T_1 = \frac{Q_1}{(1-r_0 r_2)D} = \frac{Q_1}{RD};$$

$$(6)\quad R = (1-r_0 r_2)$$

Substituting the above relations in (4-6) and simplifying terms yields

$$(4\text{-}8)\quad K = \frac{A_1 RD}{Q_1} + C_1 RD + \frac{A_2 r_2 r_0 D}{Q_2} + C_2 r_2 r_0 D + \frac{h_2 Q_2}{2r_2}$$

$$+ \frac{h_1 Q_1}{2} - h_1 k Q_2 + \frac{h_1 Q_2}{2}$$

Note that the terms $C_1 RD$ and $C_2 r_0 r_2 D$ are independent of Q_1 and Q_2, hence do not affect the operating doctrine. Therefore, it appropriate to redefine the average annual cost of ordering, repairing and holding as

$$(4\text{-}9)\quad K = \frac{A_1 RD}{Q_1} + \frac{A_2 r_0 r_2 D}{Q_2} + \frac{h_2 Q_2}{2r_2} + \frac{h_1 Q_1}{2} - h_1 k Q_2 + \frac{h_1 Q_2}{2}$$

From differential calculus the optimal Q_1 and Q_2 will be those valu that satisfy the equations

$$(4\text{-}10)\quad \frac{\partial K}{\partial Q_1} = 0 \quad \text{and} \quad \frac{\partial K}{\partial Q_2} = 0.$$

Since $r_2 > 0$ by assumption, and non-positive values of Q_1, Q_2 are meaningless, K is continuous for all other values of Q_1 and Q_2 and is differentiable. Taking the partial derivatives results in

$$\frac{\partial K}{\partial Q_1} = \frac{-A_1 RD}{Q_1^2} + \frac{h_1}{2} \quad ; \quad \frac{\partial K}{\partial Q_2} = \frac{-A_2 r_0 r_2 D}{Q_2^2} + \frac{h_2}{2r_2} - kh_1 + \frac{h_1}{2}$$

Solving the equations (4-10) yields

$$(4\text{-}11) \qquad Q_1^* = \sqrt{\frac{2A_1 RD}{h_1}}$$

$$(4\text{-}12) \qquad Q_2^* = \sqrt{\frac{2A_2 r_0 r_2^2 D}{h_2 + r_2 h_1 (1-2k)}}$$

where Q_1^* and Q_2^* are the optimal procurement and repair quantities respectively. Strictly speaking, Q_1^* and Q_2^* are not optimal quantities in the sense that the restriction $\frac{T_1}{T_2} = n$, where n is a positive integer, has not been considered as a constraint in the solution. Since $T_1 = \frac{Q_1}{RD}$ and $T_2 = \frac{Q_2}{r_0 r_2 D}$, the equation $\frac{T_1}{T_2} = n$ is equivalent to the constraint $\frac{Q_1 r_0 r_2}{Q_2 r} = n$. In Section 5 a method will be presented for adjusting Q_1^* and/or Q_2^* such that the constraint is satisfied. In addition, a slight inaccuracy will result from round-off of Q_1^* and Q_2^* for the purpose of ordering integer quantities.

Notice that the optimal value, Q_1^*, is independent of k and in fact if $R = 1$ there is no repair, and Q_1^* is the standard EOQ formula for consumables. However, Q_2^* is a function of the parameter k, where $0 \le k \le 1$. Therefore, it is desirable to determine how sensitive the model is to k. By assumption, simultaneous receipt of a Q_1 and Q_2 is not possible so that $0 < k < 1$. By definition $t_1 = kT_2$. It has been shown that $t_1 + t_2 = T_2$ which implies $T_2 - t_2 = kT_2$ or

$$(4\text{-}13) \qquad k = 1 - \frac{t_2}{T_2}$$

Consider the length of time t_2. Since $t_2 < T_2$ by assumption, $t_2 < \frac{Q_2}{r_2 r_0 D}$, by equation (4-7 (1)). But t_2 is at least the length of time necessary to issue Q_2, i.e., $\frac{Q_2}{D} \le t_2$, and $\frac{Q_2}{D} \le t_2 < \frac{Q_2}{r_2 r_0 D}$. From equation (4-13), when t_2 takes on its minimum value, $\frac{Q_2}{D}$,

$$k = 1 - \frac{\frac{Q_2}{D}}{\frac{Q_2}{r_2 r_0 D}} = 1 - r_2 r_0 = R.$$

As t_2 approaches $\frac{Q_2}{r_2 r_0 D}$, k approaches 0. Thus $0 < k \le R$. Substituting Q_1^* and Q_2^* in (4-9) gives the minimum average annual cost of the system, K^* as a function of k:

$$(4\text{-}14) \qquad K^* = \frac{A_1 RD}{Q_1^*} + \frac{A_2 r_0 r_2 D}{Q_2^*} + \frac{h_2 Q_2^*}{2r_2} + \frac{h_1 Q_1^*}{2} - h_1 k Q_2^* \quad \frac{h_1 Q_2^*}{2}$$

Actually (4-14) represents a family of minimum costs indexed by k where $0 < k \leq R$. To determine a k that yields a minimum in this family observe that (4-14) can be written as

$$K^* = f(Q_1^*, Q_2^*) - kh_1Q_2^*$$

where f is not a function of k. Moreover, k is a free variable in the sense that k is not determined by Q_1^* and Q_2^* and hence the optimization procedure is independent of k. Clearly, the value of k that minimizes (4-14) is $k = R = 1 - r_0r_2$ so that the particular solution is determined by

$$(4\text{-}15) \qquad Q_2^* = \sqrt{\frac{2A_2r_0r_2^2D}{h_2 + r_2h_1(1-2R)}}$$

The repair reorder point, X_2, is simply $\frac{Q_2}{r_2}$ (in terms of NRFI) by the original induction policy as previously discussed on page 17.

In order to determine the procurement reorder point, X_1, first consider the case $\tau_1 \leq T_1$. To ensure that an order arrives when on hand RFI inventory reaches zero and no repair lot is due, i.e., the end of a cycle, a procurement must be placed τ_1 time units prior to reaching the end of the cycle. Since cycles do repeat, this implies that an order should be placed at time $T_1 - \tau_1$ after the beginning of every cycle. See Figure 8.

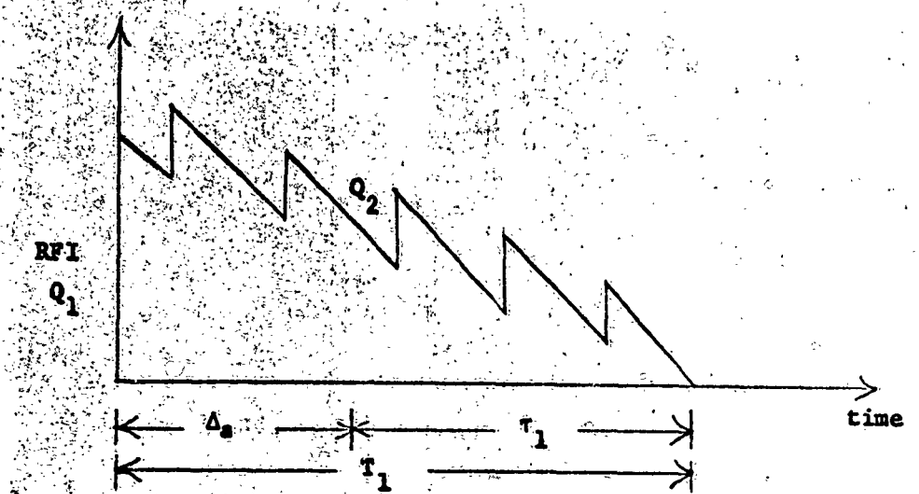

Figure 8. Procurement Reorder Point ($\tau_1 \leq T_1$)

When $\tau_1 > T_1$, as in Figure 9, when there is more than one cycle in a procurement leadtime.

Figure 9. Procurement Reorder Point ($\tau_1 > T_1$)

Let m be the smallest integer such that $mT_1 > \tau_1$. Define Δ_a to be the positive quantity $mT_1 - \tau_1$. Then Δ_a is the amount of time between the least number of cycles greater than τ_1 and τ_1, and an order should be placed Δ_a units of time after the beginning of a cycle. This order will arrive τ_1 units of time later and will coincide with the end of a cycle. Note that if $\tau_1 \leq T_1$, $m = 1$, and this is the preceding case. Thus, in any case, if we define $\Delta_a = mT_1 - \tau_1$ where m is the smallest integer such that $mT_1 > \tau_1$, the reorder rule will be to place an order Δ_a units of time after the beginning of each cycle.

5. Examples

To illustrate the use of the decision rules developed in Section 4, two examples will be presented. Consider first a repairable system which carries an item experiencing relatively low demand. The item has parameters with the following values:

$D = 120$ units per yr	$k = R = 1 - r_0 r_2 = 0.0975$
$r_0, r_2 = 0.95$	$C_2 = \$250.00$ per unit
$C_1 = \$500.00$ per unit	$A_2 = \$50.00$ per induction
$A_1 = \$100.00$ per order	$h_2 = \$50.00$ per unit per yr
$h_1 = \$100.00$ per unit per yr	$\tau_2 = 0.25$ yr
$\tau_1 = 1.0$ yr	

From (4-11) and (4-12) the unconstrained optimal order and repair quantities are:

$$Q_1^* = \sqrt{\frac{2A_1RD}{h_1}} = \sqrt{\frac{2(100)(0.0975)(120)}{100}} = 4.838 \text{ units}$$

$$Q_2^* = \sqrt{\frac{2A_2r_0r_2^2D}{h_2 - 2r_2h_1k + h_1r_2}} = \sqrt{\frac{2(50)(0.95)^3(120)}{50-2(0.95)(100)(0.0975)+100(0.95)}}$$

$$= 9.017 \text{ units}$$

To examine the constraint $\frac{T_1}{T_2} = n$ (a positive integer), compute

$$T_1 = \frac{Q_1^*}{RD} = \frac{4.838}{(0.0975)(120)} = 0.4135 \text{ yr}$$

$$T_2 = \frac{Q_2^*}{r_0r_2D} = \frac{9.017}{(0.95)^2(120)} = 0.0833 \text{ yr}$$

$$\frac{T_1}{T_2} = 4.97$$

Thus the constraint is not satisfied and in order to obtain a consistent policy it is necessary to adjust T_1 or T_2 to make $\frac{T_1}{T_2} = n$, a positive integer. It is reasonable to select the integer nearest 4.97, i.e., choose $n = 5$. Adjusting T_1 will yield the following values:

$Q_1^*(\text{adj}) = 4.873$; $Q_2^* = 9.017$

$T_1(\text{adj}) = 0.4165$; $T_2 = 0.0833$

To check the new solution we obtain

$$\frac{T_1}{T_2} = \frac{Q_1 r_2 r_0}{Q_2 R} = \frac{4.873(0.9025)}{9.017(0.0975)} = 5.0026 \approx 5$$

Thus the policy is consistent with the model of Section 4. However, an operating system cannot deal with fractional units so Q_1^*(adj) and Q_2^* must be rounded off to 5 and 9 units respectively. This results in further interval computations as follows:

$$T_1 = 0.427 \text{ yr}; \quad T_2 = 0.0831 \text{ yr}; \quad \frac{T_1}{T_2} = 5.14$$

But in this case the system will be out of stock after five repair cycles, i.e., for a period $(.14)T_2 = .0116$ year. To compensate for this period so as not to allow shortages, the reorder point must be adjusted. Recall from Section 4 that the rule is to reorder $\Delta_a = mT_1 - \tau_1$ units of time after the beginning of each cycle. To ensure a procurement arrival at the point a zero balance is reached, an order for a quantity $Q_1 = 5$ must be placed $\Delta_a - .14T_2$ time units after the commencement of a procurement cycle. Since $\tau_1 = 1$ year, $m = 3$, $\Delta_a = .281$ year, and $\Delta_a - .14T_2 = .2694$ year. Thus a workable policy is to procure 5 units .2694 year after the beginning of each procurement cycle. The repair quantity is $Q_2 = 9$ units.

Substituting the appropriate values from all three sets of computations in

$$K^* = \frac{A_1 RD}{Q_1^*} + \frac{A_2 r_0 r_2 D}{Q_2^*} + \frac{h_2 Q_2^*}{2r_2} + \frac{h_1 Q_1^*}{2} - h_1 k Q_2^* + \frac{h_1 Q_2^*}{2},$$

results in three cost comparisons as follows:

$$K^*(Q_1 = 4.838,\ Q_2 = 9.017) = 241.84 + 600.53 + 237.33 + 241.90 - 87.92 + 450.85 = \underline{\$1684.53}$$

$$K^*_{(adj)}(Q_1 = 4.873,\ Q_2 = 9.017) = 240.10 + 600.53 + 237.33 + 243.65 - 87.92 + 450.85 = \underline{\$1684.54}$$

$$K^*_{(round\ off)}(Q_1 = 5,\ Q_2 = 9) = 234.00 + 601.67 + 236.88 + 250.00 - 87.75 + 450.00 = \underline{\$1684.80}$$

As a second example consider a system carrying a repairable item with the following parameters:

$D = 12{,}000$ units per yr	$k = R = 1 - r_0 r_2 = 0.19$
$r_0, r_2 = 0.90$	$C_2 = \$50.00$ per unit
$C_1 = \$100.00$ per unit	$A_2 = \$50.00$ per induction
$A_1 = \$50.00$ per order	$h_2 = \$10.00$ per unit per yr
$h_1 = \$20.00$ per unit per yr	$\tau_2 = 0.25$ yr
$\tau_1 = 0.75$ yr	

The results of the computations are indicated below

	1st Computation	Q_1 Recomputed	Q_1, Q_2 Rounded-off
Q_1^*	106.77	95.30	95
Q_2^*	203.33	203.33	203
T_1	0.0468	0.0418	0.0417
T_2	0.0209	0.0209	0.0209
n	2.239	2	1.995
K^*	\$6,914.75	\$6,928.55	\$6,929.33

6. Constraints

In general, it is very unlikely that an inventory system would be established to manage a single line item. In applying the theory developed in Section 4 it becomes apparent that consideration must be given to inventory systems that manage many line items. In a multi-item inventory there can be many types of interactions between items. Notable among these are the interactions of items competing for limited resources. For example, there would most likely be an upper limit to the number of repair inductions directed per year. There most assuredly is a limit on funds that can be used for procurement of new items. These limits are called constraints. The remainder of this section will investigate the effect that certain constraints have on the repairable model. The constraints to be considered are; (a) number of procurements per year, (b) number of repair inductions per year, (c) dollar investment in inventory and (d) annual repair budget, i.e., repair dollars that can be spent for direct labor and materials.

Consider first the constraint on the number of procurements per year, P. Assume there are M items in the inventory system and let j denote the jth item. The constraint can then be expressed as

$$(6\text{-}1) \qquad \sum_{j=1}^{M} \frac{D_j}{Q_{1j}} \leq P$$

since quantity Q_{1j} is ordered each time an order is placed. Let K_1 be those terms of equation (4-9) involving the variables in the constraint. Thus, if

$$(6\text{-}2) \qquad K_{1j} = \frac{A_{1j}R_jD_j}{Q_{1j}} + \frac{h_{1j}Q_{1j}}{2}, \quad j = 1,\ldots,M$$

is that portion of the cost of the jth item that is affected by the quantity procured, then the cost for the system is

$$(6\text{-}3) \qquad K_1 = \sum_{j=1}^{M} K_{1j}$$

To find the optimal Q_{1j}, $j = 1,\ldots,M$, it is desirable to minimize K_1 subject to equation (6-1). The reader is referred to Hadley and Whitin (2) for the necessary mathematical background needed to solve problems of this type. Briefly, the procedure is to first solve the unconstrained problem by using equation (4-11) for each item. Then substitute these Q_{1j} into equation (6-1). If (6-1) is satisfied the constraint is said to be inactive and the problem is solved. Assuming the constraint is active, i.e., quantities computed by using equation (4-11) do not satisfy equation (6-1), the technique of Lagrange multipliers is employed to determine optimal order quantities. Form the function

$$(6\text{-}4) \qquad J_1 = K_1 + \lambda_1\left(\sum_{j=1}^{M}\frac{D_j}{Q_{1j}} - P\right)$$

where λ_1 is a Lagrange multiplier. The optimal Q_{1j} must satisfy the equations

$$(6\text{-}5) \qquad \frac{\partial J_1}{\partial Q_{1j}} = 0 = \frac{-A_{1j}R_jD_j}{Q_{1j}^{\,2}} + \frac{h_{1j}}{2} - \frac{\lambda_1 D_j}{Q_{1j}^{\,2}}, \quad j = 1,\ldots,M$$

$$(6\text{-}6) \qquad \frac{\partial J_1}{\partial \lambda_1} = 0 = \sum_{j=1}^{M} \frac{D_j}{Q_{1j}} - P$$

A solution to the set of M+1 equations in (6-5) and (6-6) may not exist in closed form. In this case, a numerical procedure should be used to approximate the optimal Q_{1j}. In any case, solving for Q_{1j} in equation (6-5) yields

$$(6\text{-}7) \qquad Q_{1j}^{*} = \sqrt{\frac{2D_j}{h_{1j}}(A_{1j}R_j + \lambda_1^{*})}, \; j = 1,\ldots,M$$

where λ_1^{*} is that value of λ_1 such that the Q_{1j}^{*}, $j = 1,\ldots,M$, satisfy

$$(6\text{-}8) \qquad P = \sum_{j=1}^{M} \frac{D_j}{Q_{1j}^{*}} = \sum_{j=1}^{M} \sqrt{\frac{D_j h_{1j}}{2(A_{1j}R_j + \lambda_1^{*})}}$$

Clearly, λ_1^{*} can not be expressed in closed form without making further assumptions so that a numerical procedure must be used to solve for the Q_{1j}^{*}.

Next, consider the constraint on the number of repair inductions per year, L. This constraint can be expressed as

$$(6\text{-}9) \qquad \sum_{j=1}^{M} \frac{r_{2j} r_{0j} D_j}{Q_{2j}} \leq L$$

Let K_2 be those terms of K involving the variables in the constraint. Thus, if

$$(6\text{-}10) \qquad K_{2j} = \frac{A_{2j} r_{2j} r_{0j} D_j}{Q_{2j}} + \frac{h_{2j} Q_{2j}}{2r_{2j}} - h_{1j} k_j Q_{2j} + \frac{h_{1j} Q_{2j}}{2},$$

$$j = 1, \ldots, M$$

is that portion of the cost of the jth item that is affected by the quantity inducted for repair, then the system cost is

$$(6\text{-}11) \qquad K_2 = \sum_{j=1}^{M} K_{2j}$$

To determine the set of Q_{2j} which minimize K_2 subject to equation (6-9) the function

$$(6\text{-}12) \qquad J_2 = K_2 + \lambda_2 \left(\sum_{j=1}^{M} \frac{r_{0j} r_{2j} D_j}{Q_{2j}} - L \right)$$

is formed where λ_2 is a Lagrange multiplier. The optimal Q_{2j} must satisfy the equations

$$(6\text{-}13) \qquad \frac{\partial J_2}{\partial Q_{2j}} = 0 = \frac{-A_{2j} r_{0j} r_{2j} D_j}{Q_{2j}^2} + \frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{1j} k_j$$

$$- \frac{\lambda_2 r_{0j} r_{2j} D_j}{Q_{2j}^2}, \quad j = 1, \ldots, M$$

$$(6\text{-}14) \qquad \frac{\partial J_2}{\partial \lambda_2} = 0 = \sum_{j=1}^{M} \frac{r_{2j} r_{0j} D_j}{Q_{2j}} - L$$

Solution of equation (6-13) yields

$$(6\text{-}15) \qquad Q_{2j}^{*} = \sqrt{\frac{r_{2j}r_{0j}D_j(A_{2j} + \lambda_2^{*})}{\frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{1j}k_j}}, \quad j = 1,\ldots,M$$

where λ_2^{*} is that value of λ_2 such that the Q_{2j}^{*} satisfy

$$(6\text{-}16) \qquad L = \sum_{j=1}^{M} \frac{r_{2j}r_{0j}D_j}{Q_{2j}^{*}} = \sum_{j=1}^{M} \sqrt{\frac{r_{2j}r_{0j}D_j\left(\frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{ij}k_j\right)}{A_{2j} + \lambda_2^{*}}}$$

Notice the assumption that $A_{21} = A_{22} = A_{23}$ yields a solution of λ_2^{*} in equation (6-16), and thus Q_{2j}^{*} in equation (6-15), by elementary methods. Practically speaking, this assumption is very reasonable since it is unlikely that the cost of making a repair induction would depend on the item.

Now consider the constraint on dollar inventory investment, I_1. This requires that

$$(6\text{-}17) \qquad \sum_{j=1}^{M} C_{1j}Q_{1j} \le I_1$$

Letting K_3 be the terms of K involving the variables in the constraint, it is seen that $K_{3j} = K_{1j}$ and $K_3 = \sum_{j=1}^{M} K_{3j} = K_1$. Define the function

$$(6\text{-}18) \qquad J_3 = K_3 + \lambda_3 \left(\sum_{j=1}^{M} C_{1j}Q_{1j} - I_1\right)$$

where λ_3 is a Lagrange multiplier. The optimal Q_{1j} must then satisfy the equations

$$(6\text{-}19) \qquad \frac{\partial J_3}{\partial Q_{1j}} = 0 = \frac{-A_{1j}R_jD_j}{Q_{1j}^2} + \frac{h_{1j}}{2} + \lambda_3 C_{1j}, \; j = 1,\ldots,M$$

$$(6\text{-}20) \qquad \frac{\partial J_3}{\partial \lambda_3} = 0 = \sum_{j=1}^{M} C_{1j}Q_{1j} - I_1$$

Solution of equation (6-19) yields

$$(6\text{-}21) \qquad Q_{1j}^* = \sqrt{\frac{A_{1j}R_jD_j}{\frac{h_{1j}}{2} + \lambda_3^* C_{1j}}}, \; j = 1,\ldots,M$$

where λ_3^* is that value of λ_3 such that the Q_{1j}^* satisfy

$$(6\text{-}22) \qquad I_1 = \sum_{j=1}^{M} C_{1j} \sqrt{\frac{A_{1j}R_jD_j}{\frac{h_{1j}}{2} + \lambda_3^* C_{1j}}}$$

Finally, consider the constraint on the annual repair budget, I_2. This constraint is expressed as

$$(6\text{-}23) \qquad \sum_{j=1}^{M} C_{2j}Q_{2j} \le I_2$$

If K_4 represents the terms of K involving the variables in the constraint, it is seen that $K_{4j} = K_{2j}$ and $K_4 = \sum_{j=1}^{M} K_{4j} = K_2$. To determine the optimal

Q_{2j} subject to (6-23), the function

$$(6\text{-}24) \qquad J_4 = K_4 + \lambda_4 \left(\sum_{j=1}^{M} C_{2j} Q_{2j} - I_2 \right)$$

is formed where λ_4 is a Lagrange multiplier. It follows that the optimal Q_{2j} must satisfy the equations

$$(6\text{-}25) \qquad \frac{\partial J_4}{\partial Q_{2j}} = 0 = \frac{-A_{2j} r_{2j} r_{0j} D_j}{Q_{2j}^2} + \frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{1j} k_j$$

$$+ \lambda_4 C_{2j} \ , \ j = 1, \ldots, M$$

$$(6\text{-}26) \qquad \frac{\partial J_4}{\partial \lambda^4} = 0 = \sum_{j=1}^{M} C_{2j} Q_{2j} - I_2$$

Solution of (6-25) yields

$$(6\text{-}27) \qquad Q_{2j}^* = \sqrt{\frac{A_{2j} r_{2j} r_{0j} D_j}{\frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{1j} k_j + \lambda_4^* C_{2j}}} \ , \ j = 1, \ldots, M$$

where λ_4^* is that value of λ_4 such that the Q_{2j}^* satisfy

$$(6\text{-}28) \qquad I_2 = \sum_{j=1}^{M} C_{2j} \sqrt{\frac{A_{2j} r_{2j} r_{0j} D_j}{\frac{h_{2j}}{2r_{2j}} + \frac{h_{1j}}{2} - h_{1j} k_j + \lambda_4^* C_{2j}}}$$

The following example illustrates the employment of a single constraint. Consider a repairable inventory system of the type presented here which stocks three items, i.e., $M = 3$. The management wishes to restrict the number of repair inductions per year to 50, thus $L = 50$. It can be rationalized that the fixed repair cost, A_{2j}, is the same for all items thus it will be assumed here that $A_{21} = A_{22} = A_{23} = \$50.00$. The remainder of the pertinent data is listed below.

Item	1	2	3
Field recovery rate, r_1	0.95	0.90	0.95
O&R recovery rate, r_2	0.95	0.90	0.95
Demand rate, D	240	2,000	200
NRFI holding cost, h_2	\$60	\$10	\$100
RFI holding cost, h_1	\$120	\$20	\$200
Cycle constant, k	0.0975	0.19	0.0975
Unit cost, C_1	\$600	\$100	\$1,000
Unit repair cost, C_2	\$300	\$60	\$700

Without the constraint, $L \leq 50$, the optimal repair quantities are given by equation (4-12). Computing we get $Q_{21}^* = 11.64$, $Q_{22}^* = 83.01$ and $Q_{23}^* = 8.23$. With these values of Q_{2j}, the number of repair inductions per year would be 60.04. Thus the constraint is active and equations (6-15) and (6-16) are used to compute the Q_{2j}. Under the assumption $A_{21} = A_{22} = A_{23}$, equation (6-16) yields $\lambda_2^* = 21.98$. Substituting this value in equation (6-15) gives $Q_{21}^* = 13.97$, $Q_{22}^* = 99.84$ and $Q_{23}^* = 9.88$. As expected, constraining the number of repair inductions increases the size of the repair quantities for each item.

It is interesting to see how the constraint affects system operating cost. Consider only that portion of the total cost that is affected by the repair quantity, as given by equation (6-11). The unconstrained Q_{2j}^*, i.e., $L = 60.04$, yields $K_2 = \$6004.08$. For $L = 50$, $K_2 = \$6104.90$. Thus the constraint forces the system operating cost to exceed the optimal cost by \$100.82.

It is quite possible that more than one constraint could be imposed at one time. For example, suppose that all four of the constraints previously considered separately are now imposed simultaneously. It is desired to minimize system variable cost subject to (6-1), (6-9), (6-17) and (6-24). Denote system variable cost by K. It is seen that

$$K = K_1 + K_2 \tag{6-29}$$

To minimize K the following procedure is used: First solve the unconstrained problem to find Q_{1j} and Q_{2j} for $j = 1,\ldots,M$, by using (4-12). With these values check to see if any of the four constraints are active. If all constraints are inactive, the problem is solved. If one or more constraints are active, use the method of Lagrange multipliers, as described in this section, to find new values for Q_{1j} and Q_{2j}. Again the constraints are checked to determine if they are active. The process is repeated for each constraint as long as any constraint is active. When all four constraints have been used singly, two constraints are tested at a time by employing two Lagrange multipliers. Again the remaining constraints are checked to see if they are active. If either

constraint is active, another set of two constraints is used employing two Lagrange multipliers to find new values of Q_{1j} and Q_{2j}. This process is repeated until all possible combinations of two constraints are used or until solutions result for which no constraint is active. If solutions cannot be found by using two Lagrange multipliers, then the technique is extended to three multipliers and finally four. In the case where all four constraints are active under the aforementioned conditions, form the function

$$(6\text{-}30) \qquad J = K + \lambda_1 \left(\sum_{j=1}^{M} \frac{D_j}{Q_{1j}} - P \right) + \lambda_2 \left(\sum_{j=1}^{M} \frac{r_{2j} r_{0j} D_j}{Q_{2j}} - L \right)$$

$$+ \lambda_3 \left(\sum_{j=1}^{M} C_{1j} Q_{1j} - I_1 \right) + \lambda_4 \left(\sum_{j=1}^{M} C_{2j} Q_{2j} - I_2 \right)$$

where λ_i, $i = 1,\ldots,4$, are the Lagrange multipliers. It follows that the optimal Q_{1j} and Q_{2j} must satisfy the equations

$$(6\text{-}31) \qquad \frac{\partial J}{\partial Q_{1j}} = 0, \; j = 1,\ldots,M; \quad \frac{\partial J}{\partial Q_{2j}} = 0, \; j = 1,\ldots,M$$

$$(6\text{-}32) \qquad \frac{\partial J}{\partial \lambda_i} = 0, \; i = 1,\ldots,4.$$

It is worth remarking that, in general, equations (6-31) and (6-32) will be extremely difficult to solve. In fact, in most cases it will be

necessary to resort to numerical procedures that give approximations to the optimal Q_{1j} and Q_{2j}.

7. Conclusions

This report has derived a decision rule in the form of optimal quantities and reorder points for a repairable item inventory system by considering the procurement and repair decisions simultaneously. For the deterministic model developed, equations (4-11) and (4-12) show that the optimal procurement and repair quantities are independent of each other. In addition, this model can be applied to a consumable item inventory system by taking $r_0 = r_2 = 0$. In this sense, the model developed is a natural generalization of a consumable model.

Although the formulation of a cost equation was essential in deriving optimal operating doctrine, costs per se were not discussed in detail. To actually use this model holding cost relationships must be given. In addition the fixed costs of procurement and repair (A_1 and A_2) must be known. Although the main interest of the report is not the analysis of costs, it is evident that further research in this area would be desirable prior to application of the model.

It should be pointed out that this model considered a single item and when applied to a multi-item inventory system will result in trade-offs between items and costs. These interactions between items lead to competition among items for limited resources expressed in this model as constraints. As illustrated in Section 6, one constraint considering just a few items presented difficulties in calculations. A feasible

method different from the one discussed for handling constraints in a multi-item system must be developed.

The deterministic model developed here is by no means unique nor necessarily the most suitable one to apply to a given type of item. In a forthcoming report, a variation of this model is to be discussed in detail. As further research in this area progresses, suggestions for modifications to formulate still different models may arise.

Since only deterministic assumptions have been made in this report, shortages and hence shortage costs never arose. A natural step in the direction of further research would be to define a probabilistic version of the model to be more realistic and thereby consider shortages and related ideas such as demand forecasting. Such considerations will be the concern of further project reports.

8. APPENDIX

8.1 DEFINITIONS OF TERMS

Consumable	An item that is either consumed in use or discarded after wearout or failure.
Field recovery rate	Percent of items issued that are subsequently returned to the overhaul and repair activity.
Holding costs	The costs associated with the physical maintenance of an inventory.
Information and Issue systems costs	The costs associated with operating an inventory system excluding procurement, repair and holding costs.
Inventory control point	An inventory manager in the United States Naval Supply System (for example ASO).
Inventory Manager	An activity responsible for the procurement and inventory control of items in an inventory system.
Non-ready-for-issue	Condition of an item that is not capable of providing complete flow of services in its designed use.
Operating doctrine	A set of rules which prescribe procurement and repair quantities and respective reorder points (when and how much to procure and repair).
Overhaul and Repair activity	An industrial activity responsible for testing, checking, repairing, etc. components and equipments designated as repairable by the inventory manager on either a scheduled or emergency basis.
Overhaul and Repair recovery rate	Percent of items returned to the overhaul and repair activity that are subsequently returned in an RFI condition to the stock point.
Procurement costs	The costs associated with placing a procurement order including the cost of the items.

Procurement cycle	Time between arrival of successive procurement quantities.
Procurement leadtime	Time between the placement of a procurement order and receipt of the procurement quantity.
Ready-for-issue	Condition of an item that is capable of providing complete flow of services in its designed use.
Repairable	An item that can be repaired after wearout or failure and subsequently provide some flo of services.
Repair costs	The costs associated with placing a repair induction including the cost of repairing the items.
Repair cycle	Time between the placement of an induction order and receipt of the repair quantity.
Shortage costs	The cost incurred by the system when a deman cannot be filled from stock.
Stock point	An activity responsible to the inventory manager for the receipt, storage and issue of material and the report of transactions.

8.2. TABLE OF SYMBOLS AND ABBREVIATIONS

A_1	Fixed cost to place a procurement order
A_2	Fixed cost to place a repair order
ADP	Automatic data processing
ASO	Aviation Supply Office, Philadelphia, Pa.
AUTODIN	Automatic digital network
C_1	Unit cost of a new item
C_2	Unit cost of repair
D	Demand rate
Δ_a	The number of units of time after the beginning of a procurement cycle for placing a procurement order.

h_1	Holding cost for RFI material ($/unit-yr)
h_2	Holding cost for NRFI material ($/unit-yr)
ICP	Inventory control point
n	$\frac{T_1}{T_2}$, the number of repair cycles in a procurement cycle.
NRFI	Non-ready-for-issue
O&R	Overhaul and repair activity
Q_1	Procurement quantity
Q_2	Repair quantity
Q_1^*	Optimal procurement quantity
Q_2^*	Optimal repair quantity
r_0	Field recovery rate
r_2	Repair recovery rate
R	$1 - r_0 r_2$
RFI	Ready-for-issue
T_1	Procurement cycle
T_2	Repair cycle
τ_1	Procurement leadtime
τ_2	Repair leadtime
X_1	Procurement reorder point
X_2	Repair reorder point

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McNall, Phillip F.
Schrady, David A.

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A model is defined to account for an inventory system which carries items that are subject to repair after use, wearout or failure and then reissued. Such a system is called a repairable item inventory system and, important as such systems are to military applications, have received little attention in the scientific literature. A repairable item inventory system would be trivial if it were not for the fact that a used item may not be repairable due to excessive damage either before or during attempted repair and hence new items must be procured from time to time to replenish the system. This report determines decision rules for this dual problem of repair and procurement in a deterministic model.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Repairable Recoverable Inventory Decision Rules Deterministic Repairable Inventory Model Deterministic						

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