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THE NATURE OF FLOOD CONTROL BENEFITS AND THE ECONOMICS OF FLOOD PROTECTION

by

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Any errors in this paper are, of course, my sole responsibility.

THE NATURE OF FLOOD CONTROL BENEFITS AND THE ECONOMICS OF FLOOD PROTECTION

INTRODUCTION

This paper is an attempt to extend and clarify several areas of analysis pertaining to the economics of flood protection. More specifically, it addresses the question of the correct measure of the benefits from "land enhancement," and the question of the role of flood insurance in an overall program of flood protection. The first chapter contains a discussion of the nature of benefits from flood protection and the relation of the various types of benefits to five different measures for coping with flood losses. Within this framework the effects of these measures are assessed.

In order to gain insight into the nature of flood control benefits a land-use model is developed in the second chapter. This model incorporates the economic factors which determine the location of various activities in competitive equilibrium. This framework is then used to prove a number of theorems which concern the effects of perturbing an equilibrium by adding or removing new activities and by increasing or decreasing the supply of different types of land. These theorems provide the theoretical framework with which to analyze the benefits from land enhancement.

The third chapter contains an application of these results to the problem of measuring the benefits from the introduction of flood control. The correct measure of benefits from land enhancement is derived, and it is shown that such benefits represent real economic gains and not simply transfers of income. The correct measure of benefits from land enhance-

ment is then used to evaluate the methods employed by the Corps of Engineers to estimate these benefits. Finally, the practical significance of land-enhancement benefits for the planning and justification of flood control projects is discussed.

The final chapter of the paper contains a discussion of the question of whether it is possible to eliminate the cost of risk-bearing through a program of flood insurance. The question arises as to whether the losses from flooding are sufficiently independent for flood insurance to be feasible. It is sometimes argued that there is a high degree of interdependence between occurrences of flood damage, and that flood insurance could be written only if a very large safety-loading charge were included in the premium. It is demonstrated, however, that if the assets subject to flood damage are a small fraction of the total assets of the community, and that if the risks associated with flooding are independent of other risks borne by individuals in the society, then an insurance scheme can be devised where the charge for safety loading is negligible and where everyone underwriting the insurance is at least as well off as before the insurance program was introduced. This result holds, regardless of whether flood losses are interdependent. Similarly, it is proved that if the risks of flooding are pooled among all individuals in society, then the total cost of risk-bearing to society is negligible.

The question of whether flood insurance is a reasonable substitute for structural protection is also discussed in the final chapter. In particular, it is noted that because flood losses are inflicted upon individuals and businesses located outside the flood plain, there may

in practice be some difficulty in identifying these losses and therefore in insuring against them. The problem of identification also arises with respect to losses of income that occur as a result of the interruption of economic activity caused by flooding. Unless these losses can be identified and measured, it will be impossible to develop a flood-insurance program which will provide complete protection against the losses associated with flooding.

CHAPTER I

A SURVEY OF ALTERNATIVE METHODS OF COPING WITH FLOODS

The benefits of a flood-protection measure can be considered as the reduction in the costs from flooding which would result if this measure were implemented. For purposes of this analysis it is convenient to divide these costs into four categories: (1) loss of property and income, (2) risk taking, (3) intangibles, (4) opportunities for use of the flood plain that are foregone because of flood hazards. The nature of the costs in each of these categories is discussed in relation to each of five types of measures for protecting against flood losses. These protective measures are structural transformation of the river bed which includes dams, levees, channel improvements, etc.; flood insurance; flood warning and evacuation systems, flood proofing, and flood zoning.

When floods occur, property in the flood plain is damaged and economic activity is interrupted in firms and households in the flood plain and in firms and households located outside the flood plain which are linked with activity in the flooded area. The costs of flooding include damage to property, losses in receipts and wages, and costs of adjustment such as the expense of temporary housing, evacuation, etc. [3, 117-138]. However, at any point in time or in any given year these costs cannot be predicted with certainty, and it is assumed that flood losses are a random variable with a given distribution. This is assumed for individual losses and for the sum of these losses. The mean of this distribution, or the expected value of flood damage in a

given period, is taken to represent the cost of flooding in that period.

Suppose a flood-protection measure is introduced which alters the distribution of flood losses and in particular lowers the expected value of flood losses. The benefits in each period from damage reduction are measured by the reduction in the expected value of flood losses and the stream of such benefits is discounted to its present value and compared with the present value of costs. This procedure raises some interesting questions about the behavior of individuals under conditions of uncertainty which will be examined in the final chapter in connection with flood insurance. For the present analysis it suffices to accept the reduction in the expected value of flood losses as the measure of benefits from the reduction of property losses. This assumes that a flood-plain occupant would willingly pay an amount equal to the reduction in the expected value of his property losses for protection against these losses.

Structural flood-control measures such as dams and levees alter the stream flow so as to change the distribution function associated with flood losses. Structural protection eliminates the smaller, more frequent floods which account for a large part of total flood losses and, therefore, reduces the expected value of these losses. Flood proofing, on the other hand, does not alter the flood frequency, but lowers the level of flood damage associated with a given level of flooding and thereby reduces the expected value of losses [12, 1-32]. Similarly, flood warning and evacuation systems reduce the expected value of flood damage because less property is left exposed to the hazards of flooding. Thus, structural control systems, flood proofing, and flood

warning and evacuation systems all produce benefits in the form of a reduction in the loss of property and income due to flooding. Therefore, they are alternatives to one another and should be considered as such in the planning process although the optimum program may include a combination of all three measures.

Flood insurance clearly does not reduce the expected value of losses due to flooding and, therefore, does not produce benefits of the first type. The case of flood zoning requires more careful analysis. If zoning is to affect the pattern of development in the flood plain, and thereby reduce the expected value of flood losses, it can do so only by prohibiting some activities from locating in the flood plain which otherwise could have profitably located there. Therefore, while the expected value of flood losses would be decreased it would be at the cost of foregoing uses of the flood plain that are of greater value than the losses that were prevented. This reasoning can be illustrated as follows. Abstract from risk by assuming that a given activity is operated so as to maximize the present value of its stream of expected earnings. Then an activity will locate in the flood plain only if the present value of expected flood losses is less than the increase in the present value of expected earnings that is obtained by locating in the flood plain rather than at the best alternative location outside the flood plain. Therefore, to exclude this activity from the flood plain is to cause a reduction in the present value of its expected earnings which exceeds the present value of expected flood losses.

This argument implicitly assumes that the individuals who decide to move activities into the flood plain are aware of the expected value

of flood losses. Recent studies of flood-plain occupancy have found that ignorance of flood hazards is in some cases an important factor in the choice of a flood-plain location [6]. Flood zoning may reduce flood losses where property is exposed to the hazards of flooding only because of ignorance. However, it is very likely that in practice flood zoning will exclude some activities which could profitably locate in the flood plain as well as some for which it would not be profitable.

Two alternative ways of promoting informed and rational decisions pertaining to the use of the flood plain are programs of public information and mandatory flood insurance. The difficulty with a program of public information is that people may disregard it unless there has been a recent flood to dramatize the situation. Mandatory flood insurance avoids this difficulty because the price of the insurance is incorporated into the cost of operating in the flood plain, and therefore affects the profitability of locating there. The drawback of such a program is that there may be substantial transaction costs associated with a flood-insurance program. Also, in order to make the program acceptable to the insurer a safety-loading charge will have to be included in the premium. Therefore, the premium for flood insurance may exceed by a significant amount the expected value of losses; some activities which would otherwise have found it profitable to locate in the flood plain may not find it profitable if they are required to buy an insurance policy for a premium which exceeds the expected value of their losses. However, before reaching any conclusions about the effects of a mandatory program one must take its effects on risk into account.

In considering the cost of flooding due to the loss of property and income the expected value of these losses is used as the measure of this cost. However, individuals in the flood plain may be interested not only in the mean of the distribution of these losses but in other properties of the distribution as well. For example, one may be very concerned about the probability of sustaining very large losses and be willing to pay a premium to insure against such a contingency. If a flood-control measure removes this contingency, then it creates a benefit equal to the value of this premium. These other properties of the distribution of flood losses with which one can associate a monetary value will be referred to as risks. In the analysis of flood protection the primary interest is in risks which are associated with negative values, that is, where risk-bearing is considered a cost and where an individual is willing to pay a premium in order to change the distribution of his losses in such a way as to reduce or eliminate certain risks. There are two properties of the distribution of flood losses which appear to be particularly important for the analysis of flood-protection systems. First, there is the probability of a catastrophe, which can be defined as losses above a specified level, and secondly, the dispersion of flood losses as characterized by the variance of the distribution. These specific risks will be referred to in the following analysis of the effect of various flood-control measures on risk.

Structural-flood control measures that eliminate the possibility of flooding also eliminate risk. However, few structural systems provide complete protection against flooding. Structural systems generally

eliminate the smaller, more frequent floods and reduce the level of flooding in the case of larger floods. This leads to the conjecture that such measures will reduce the probability of very large losses and also reduce the variance of flood losses because the probability of losses near zero is increased. If this is the case, then structural measures reduce, but in general do not eliminate, both types of risk. Flood proofing and flood warning and evacuation systems can also reduce the probability of very large losses. For example, flood proofing can reduce the probability of heavy losses that occur when structures are washed downstream. A flood warning and evacuation system enables some property to be evacuated from the flood-threatened area and allows time for businesses and households to prepare for the conditions of flooding. Therefore, these systems help to reduce the damage associated with any level of flooding and in particular high levels of flooding.

Of the five types of flood-protection measures, flood insurance is by far the most effective for coping with risks. Assume that a flood-insurance policy which covers all economic losses caused by flooding is offered at a price equal to the expected value of flood losses. By purchasing such a policy the flood-plain occupant adds an amount equal to the expected value of his flood losses to the cost of operating in the flood plain. Therefore, the expected value of costs associated with a flood-plain location is the same with the policy as without it; however, with the policy these costs are known with certainty and risk is eliminated. There are, however, transaction costs associated with a flood-insurance program so that premiums will in general exceed the expected value of losses. If the amount by which the premium

exceeds expected losses is less than the cost of risk-bearing, the flood-plain occupant will purchase the policy and there will be a net benefit equal to the difference between the cost of risk-bearing and the price of the policy. Therefore, when the effect of insurance on risk is taken into account, the adverse effects of a mandatory flood-insurance program are mitigated because the increase in the expected value of costs incurred with the purchase of such a policy is in part or wholly offset by the reduction in the cost of risk-bearing.

Finally, flood-zoning programs can reduce the total cost of risk-bearing by excluding activities from the flood plain. It should be noted that flood zoning in no way reduces the risks of firms and households which actually locate in the flood plain. Flood zoning, by excluding certain activities from the flood plain that otherwise would have located there, prevents these activities from assuming the risks associated with flood-plain occupancy. However, as in the case of expected flood losses, flood zoning prevents the costs of risk at the even greater costs that result from prohibiting these activities to locate in the flood plain. If an individual is aware of the risks of locating in the flood plain, then he will move into the flood plain only if the value of its advantages is greater than the cost of bearing the risks of that location.

Again ignorance of the risks may justify some form of flood zoning; however, a program of public education as to the hazards of flooding, or a program of mandatory flood insurance which would eliminate risk, might be less costly methods of coping with the problem of ignorance. Another case for flood zoning is that society may choose to accept some

responsibility for catastrophic losses, and therefore is justified in regulating individual exposure to the possibility of such losses. For example, when a disastrous flood leaves people homeless and without a source of income, society is often obligated to provide assistance. This may be the case even when the victims of the flood understood the risks and proceeded to occupy the flood plain in spite of them. If society accepts an obligation to pay part of the costs associated with the risks of flooding, and if the attitude of society toward risk differs from that of a given individual, society may be justified in imposing its preference with respect to risk-bearing on the individual. One method of accomplishing this is to establish flood zoning regulations which prevent activities from moving into the flood plain where the risks involved are socially unacceptable. It should be noted, however, that a program of mandatory flood insurance would achieve the same objective, as any individual moving into the flood plain would be required to purchase insurance, thus removing the possibility that society would have to pay part of the losses in the case of a catastrophe. It may be possible to deal with the problem of ignorance and to eliminate the possibility of society having to reimburse individual losses by inaugurating a program of flood insurance that would insure against only part of each individual's losses. Such a program could protect the individual against disastrous losses and thereby remove the need for government relief in cases of disaster. While the premium might be well below the value of expected flood losses, the fact that people would be required to pay a premium for this limited protection would probably alert them to the hazards of flooding.

In concluding the discussion of risk it is appropriate to analyze an argument that is often presented in support of proposals to build larger structural systems than would be justified on the basis of available benefit-cost information. The benefit-cost criterion dictates that the design of a project should be such that the present value of net benefits is maximized. Thus, an incremental increase in the scale of a project should be undertaken only if the increase in net benefits exceeds the costs of that increment. Because of the practical difficulty of measuring the benefits from risk reduction, these benefits are not usually included in benefit estimates. Properly stated, the argument that the scale of structural flood-control measures should be greater than that dictated by benefit-cost information rests on the contention that (1) the increase in the scale of these flood control measures will produce significant benefits by reducing risk, and (2) that if these benefits were included in the benefit estimates, the incremental benefits would exceed the incremental costs.

If one accepts the basic factual assumptions of this argument and the validity of the benefit-cost criterion, then the conclusion follows, provided there are no alternative ways to reduce risk. However, there is an alternative way to reduce risk, namely flood insurance, which will provide complete protection whereas most structural measures will not. Assume that there are no transaction costs of insurance and that the insurance premium is equal to the expected value of flood losses. Then the costs of risk-bearing can be eliminated without incurring any additional costs, because the price of the premium simply replaces the expected value of flood losses in the individual's cost calculation.

In this hypothetical case benefits from the reduction of risk should never be included in benefit estimates for flood-control projects since these benefits can be secured without cost by a flood-insurance program. Thus, the present procedure of omitting them from benefit estimates would be correct if flood insurance were available. In practice there are transaction costs that must be considered and the above statement requires appropriate modification. If the flood-insurance program is large enough to enjoy economies of scale, the transaction cost may be negligible. This is one of the important questions that will have to be investigated before such a program is established.

The third category of benefits is comprised of outputs of a flood-protection system for which it is difficult to assign a meaningful monetary measure. Thus, they are lumped together under the heading of intangible benefits. Some of the items in this category that are considered to be important by the Corps of Engineers are reduction of the loss of life, enhancement of the security of the people, improvement of sanitation, and protection against epidemics [3, 141]. Of these, only the prevention of the loss of life is probably considered by the public at large as one of the primary objectives of flood control although, in fact, only a small number of deaths have been caused by flooding [3, 141-142]. The discussion that follows will be restricted to the effectiveness of different measures in preventing deaths caused by flooding.

It is sometimes argued, as in the case of risk, that the prevention of deaths from large floods is justification for building very large flood-control projects. It is argued, for instance, that a large dam

and reservoir system will eliminate certain floods and reduce the severity of others and thereby prevent deaths. On the other hand, when a floodwall is topped, the rapid inundation of the flood plain may catch the occupants by surprise. This, combined with the tendency of flood-plain occupants to develop a false sense of security when protected by flood control works, may offset the effects of the additional protection [17, 229]. While some floods can be prevented by structural measures, it is seldom economically feasible to build a structural system that will provide complete protection and, therefore, it is necessary for the protection of life to provide a flood warning and evacuation system. There may be some areas where it is not economically feasible to build structural flood-control works, and where hydrologic conditions are such that it is impossible to provide sufficient warning for the evacuation of the flood plain. Such might be the case in a canyon which has a history of flash floods. Under these circumstances flood zoning may be required to prevent utilization of the flood zone that could result in a substantial loss of life.

The fourth category of benefits, land enhancement, is both important and controversial, and therefore will be analyzed in some detail. Its importance stems from the fact that benefits in this category are becoming increasingly important in the justification of flood-control projects [8, 185-86]. There is also a question as to whether there is any real economic gain associated with land enhancement, or whether it only represents a diversion of activity into the flood plain from other locations [8, 186]. Further, there are the questions of what in fact is the correct measurement of benefits from land enhancement

and what relation does the correct measure bear to measures used by federal agencies.

In general there are firms and households which locate off the flood plain because the costs associated with flooding more than offset any possible advantages of flood-plain location. However, some of these firms and households might find it to their economic advantage to locate in the flood plain if the costs of flooding to the flood plain occupants were reduced or eliminated. Assume that a flood-protection measure is introduced which reduces the costs of flooding and therefore makes it profitable for some activities, which had previously located outside the flood plain, to move into the flood plain. The benefit from land enhancement attributable to that measure is defined as the sum of the dollar values of the economic gain of firms and households which now find it profitable to move into the flood plain. Implicit in this definition is the assumption that a firm or household which would gain from a flood-plain location, given protection, would willingly pay an amount equal to or less than the dollar value of this gain in order to secure that protection.

It is clear that any public program which reduces the cost of flooding to the flood-plain occupant can create benefits from land enhancement. This is true whether the program reduces cost by reducing the expected value of flood loss, by reducing risk, or by reducing intangible losses. Therefore, the introduction of almost any program of flood protection can create land-enhancement benefits. There is one notable exception, namely flood zoning. The creation of land-enhancement benefits is critically dependent on the reduction of the

costs of flooding to firms and households which will actually occupy the flood plain; since zoning reduces the cost of flooding only by excluding the activities that would incur these costs, it cannot produce benefits of this type.

There now remain the questions of whether land-enhancement benefits represent real economic gains or are merely a transfer of rents, and whether the methods used by federal agencies to measure these benefits are correct. In order to gain insight into the answers to these questions a model will be developed which incorporates the major economic factors that determine flood-plain utilization. The model and the forthcoming analysis make use of the following assumptions: (1) The conditions of the competitive model are fulfilled; (2) there is a perfect capital market so that borrowing and lending rates are the same and equal the social rate of discount; (3) there is perfect foresight with respect to future states of the economy. To simplify the analysis of flood-control benefits it is further assumed that there are no intangible costs associated with flooding and that each flood-plain occupant buys an annual flood-insurance policy for a premium equal to the expected value of his flood losses. This assumption removes from consideration the cost of risk so that the only cost to the flood-plain occupant is a cost equal to the expected value of his losses.

CHAPTER II

THE THEORY OF RENTS AND THE LOCATION OF ECONOMIC ACTIVITIES

Before developing the analysis of flood-plain use it is necessary to analyze in detail the economic factors that determine the rental value, and therefore the price, of land. This is important for later analysis because the cost of land is a significant factor in determining an activity's choice of location and because changes in land values are sometimes used as a measure of benefits from flood control. Further, it will be demonstrated that under certain conditions the rental value of a parcel of land represents in some sense the social cost of occupying that parcel.

Begin by assuming there is a limited supply of homogeneous land which is divided into n identical parcels. In addition, suppose there is a set, $X = \{x^1,\ldots,x^m\}$, of activities which compete for the n parcels of land. Each activity requires one and only one parcel, and is indifferent as to which parcel it occupies. Further assume that the costs of an activity are variable, and that an activity can be initiated and terminated instantaneously. As a result of this assumption, whether or not an activity is operated in a given year depends only on its earnings as compared with the earnings of other activities in that year. The earnings of an activity $x^i \in X$, denoted by S^{x^i}, are defined as the total value of output associated with the operation of x^i minus all costs of production except the cost of land. It is assumed that S^{x^i} $(i = 1,\ldots,m)$ is given and is independent of whatever other activities are in operation.

Let the activities in X be ordered so that $S^{x^1} \geqq \ldots \geqq S^{x^n} \geqq S^{x^{n+1}} \geqq \ldots \geqq S^{x^m} \geqq 0$ and let $m > n$. Suppose the activities in X bid against each other for the n available sites. If the bidding rises above the earnings of a particular activity, that activity will drop from the bidding for that year. The bidding will continue until there are exactly n activities left, at which point it will stop. The bidding will clearly continue until the bids reach $S^{x^{n+1}}$, and therefore the rental value of a unit of land which is established under this particular procedure of bidding is equal to the earnings of activity x^{n+1}. Alternatively, suppose that rents are established in the following way: Each landowner sets the rent on his parcel of land on a take-it-or-leave-it basis, and he raises the rent if one or more activities are willing to rent his parcel at the quoted rental value and lowers the rent if he has no takers. Further, suppose this process continues until an equilibrium is reached. Clearly, in any equilibrium the rent on all parcels of land must be the same; otherwise, all activities which are still seeking land, given the quoted rents, will attempt to locate on the parcel with the lowest rent. This would drive the rent on that parcel up and the rents on other parcels down. It is clear that the common rental value, p, must equal S^{x^n}. Under both systems of bidding, an equilibrium is obtained in which, given the rental value of the land, the demand for land just equals the supply. However, the rental values established are not the same in both cases.

Now assume that $m < n$ so that all [illegible]ities can operate and have parcels of land left vacant. If the first bidding scheme were followed and bids begin at zero, then there would be no further bidding because

all activities can obtain a parcel of land, given $p = 0$. On the other hand, if the second scheme were in operation, rents would also be zero because there is always some parcel of land for which there is no taker, and competition among suppliers of land will drive the rent to zero. Now let $m = n$. Then under the first system of bidding, the equilibrium rental value will be zero, and under the second system the rental value established in equilibrium will equal S^{x^m}.

The results of the preceding can be generalized as follows: Again assume that $m > n$, and in addition assume that $S^{x^{n+1}} < S^{x^n}$. Let p be any rental value which satisfies $S^{x^{n+1}} < p < S^{x^n}$ so that $S^{x^i} - p > 0$ $(i = 1,\ldots,n)$, and $S^{x^i} - p < 0$ $(i = n+1,\ldots,m)$; i.e., given p, activities $x^1,\ldots,x^n$ can earn a profit, and activities $x^{n+1},\ldots,x^m$ can only be operated at a loss. Therefore, given p, $x^1,\ldots,x^n$ will each demand one parcel of land so that any rental value p such that $S^{x^{n+1}} < p < S^{x^n}$ is consistent with an equilibrium in the market for land, and the set of operating activities is $\{x^1,\ldots,x^n\}$. Now suppose that $p = S^{x^n}$ so that $S^{x^i} - p \geqq 0$ $(i = 1,\ldots,n)$, and $S^{x^i} - p < 0$ $(i = n+1,\ldots,m)$. $S^{x^n} - p = 0$ so that it is a matter of indifference whether x^n is in operation; however, if x^n is not in operation, a parcel of land will lie vacant and competition among suppliers of land will drive the rental value of land down and induce x^n to operate. Since an arbitrarily small decrease in p will mean that x^n earns a profit, it is assumed that x^n will demand one unit of land and will operate, given $p = S^{x^n}$. Clearly, $p = S^{x^n}$ is the maximum rental value consistent with equilibrium in the market for land. Now suppose that $p = S^{x^{n+1}}$ so that $S^{x^i} - p > 0$ $(i = 1,\ldots,n)$ and $S^{x^i} - p \leqq 0$ $(i = n+1,\ldots,m)$.

$S^{x^{n+1}} - p = 0$, so it is a matter of indifference whether or not x^{n+1} is in operation. However, if x^{n+1} demands a unit of land, then there will be excess demand in the market for land, and competition among renters will drive the rental value up and force x^{n+1} out of the bidding. Since an arbitrarily small increase in the rental value will mean that x^{n+1} can only be operated at a loss, it is assumed that, given $p = S^{x^{n+1}}$, x^{n+1} does not demand a parcel of land and is not in operation, given the equilibrium which obtains. Clearly, $p = S^{x^{n+1}}$ is the minimum rental value consistent with equilibrium. To summarize, any rental value p such that $S^{x^{n+1}} \leqq p \leqq S^{x^n}$ is consistent with equilibrium, and from the previous discussion it is clear that the minimum rental value consistent with equilibrium will be obtained if the first system of bidding is in effect and the maximum rental value is obtained if the second system is operative. The set of activities in operation, given any equilibrium rental value is $\{x^1,\ldots,x^n\}$.

Now suppose that $S^{x^n} = S^{x^{n+1}}$; then clearly the equilibrium rental value p is uniquely determined, and $p = S^{x^n} = S^{x^{n+1}}$. Given p, some of the activities including x^n and x^{n+1} will just break even so that it is a matter of indifference which of these activities actually operate, provided that all n parcels are occupied. As a result, the set of activities which operate in equilibrium is not uniquely determined. This can be seen as follows: Suppose $S^{x^1} \geqq \ldots \geqq S^{x^{n-1}} > S^{x^n} = S^{x^{n+1}} > S^{x^{n+2}} \geqq \ldots \geqq S^{x^m}$. Then, given $p = S^{x^n} = S^{x^{n+1}}$, the set of activities in operation could be either $\{x^1,\ldots,x^{n-1},x^{n+1}\}$ or $\{x^1,\ldots,x^{n-1},x^n\}$. Notice that each set must contain $x^1,\ldots,x^{n-1}$; however, the nth parcel of land may be occupied by either of the marginal activities x^n or x^{n+1}.

Therefore, the activities which operate in equilibrium are determined by the conditions of equilibrium, except in the case of marginal activities which may be assigned to either the set of activities in operation or to the nonoperative set.

We have discussed the case where $m > n$ in some detail; however, similar results can be obtained in the cases where $m = n$ and $m < n$. First, assume that $m = n$. Then if $0 \leqq p \leqq S^{x^m}$, the demand for land will equal the supply, since all m activities will demand one unit of land. If $S^{x^m} = 0$, then p is uniquely determined and $p = 0$. Suppose that $m < n$; then there is no rental value which will eliminate the excess supply of land. As a result, competition among suppliers will drive the rental value to zero. Therefore, we can conclude that, given any number of activities, m, and any number of parcels of land, n, there is a closed interval such that all rental values within this interval satisfy the conditions of equilibrium. If this interval is degenerate, the equilibrium rental value is uniquely determined.

Suppose we now introduce a new activity, z, to our set of activities so that the new set of activities, X', is given by $X' = X \cup z$. In addition, assume that $S^z > S^{x^{n-1}} > S^{x^n} > S^{x^{n+1}}$. In the initial situation, before the introduction of z, the interval containing rental values consistent with equilibrium is defined by $S^{x^{n+1}} \leqq p \leqq S^{x^n}$, and the set of operative activities in equilibrium is $\{x^1,\ldots,x^n\}$, assuming $m > n$. After the introduction of z, the interval containing rental values consistent with equilibrium is defined by $S^{x^n} \leqq p' \leqq S^{x^{n-1}}$, and the set of operative activities in equilibrium is $\{z,x^1,\ldots,x^{n-1}\}$. Note that the maximum rental value consistent with the initial equilibrium equals

the minimum rental value consistent with the new equilibrium as both are equal to S^{x^n}. When z is introduced, it displaces x^n, which is forced to shut down; therefore, by introducing z an amount equal to S^z was added to the total earnings of all activities in operation; however, there is also loss of earnings equal to S^{x^n}. The net increase in total earnings or, in benefit-cost terms, the net benefit attributable to the introduction of z is $S^z - S^{x^n}$. It follows from the preceding discussion that the cost of introducing z, in terms of the earnings foregone because some activity is displaced, equals the maximum rental value consistent with the initial equilibrium and the minimum rental value consistent with new equilibrium. Therefore, if the mechanism by which equilibrium is obtained is such that the maximum rental value obtains, then the initial rental value of land is the correct measure of the opportunity cost of introducing z. On the other hand, if the minimum rental value obtains, then the new rental value of land is the correct measure of this cost.

Now suppose that the initial rental value, p, is uniquely determined, which implies $p = S^{x^n} = S^{x^{n+1}}$. In this case, p is the proper measure of the cost of introducing z. This statement also holds if the new rental value, p', is uniquely determined so that $p' = S^{x^n} = S^{x^{n-1}}$. From the above statements it follows that if both p and p' are uniquely determined, then they are equal; i.e., in this case the introduction of z does not change the rental value of land. Even if $S^{x^n} \neq S^{x^{n+1}}$, any rental value which obtains in the initial equilibrium is a good approximation of the cost of introducing z if $S^{x^n} - S^{x^{n+1}}$ is small.

Similar results can be derived for the cases where $m = n$ and $m < n$. First, suppose that $m = n$ and that a new activity, z, is introduced for which $S^z > S^{x^{m-1}} > S^{x^m}$ In the initial situation any rental value, p, $0 \leqq p \leqq S^{x^m}$, is consistent with equilibrium. After the introduction of z, there are more activities than parcels of land so that it corresponds to the case where $m > n$; and therefore, the new rental value, p', must satisfy $S^{x^m} \leqq p' \leqq S^{x^{m-1}}$. Since S^{x^m} is displaced by z, the cost of introducing z in terms of its effect on the total earnings of all other activities is equal to S^{x^m}. This cost again equals the maximum rental value consistent with the initial equilibrium and the minimum rental value consistent with the new equilibrium. The case where $m < n$ can be broken into two subcases. First, assume that $m < n-1$. Then even after the introduction of z there will be unused land so that the rental value of land will be zero in both cases. This accurately measures the cost of introducing z because no other activities are affected. If $m = n-1$, then the initial rental value is zero; however, the new rental value p' must satisfy $0 \leqq p' \leqq S^{x^m}$. This is because after the introduction of z, the number of activities equals the number of parcels of land. Since z does not displace any other activity, the opportunity cost of introducing z is zero, and therefore we get the desired result.

The preceding analysis has dealt with the conditions of equilibrium in the market for land and the effects of perturbing the equilibrium by introducing a new activity. Now suppose that there are m activities competing for n parcels of land, $m > n$, and that the system is in equilibrium with a given rental value, p, $S^{x^{n+1}} \leqq p \leqq S^{x^n}$. Further

suppose that the set of operative activities is $Y = \{x^1,\ldots,x^n\}$. Let k be a positive integer, $k < n$, and let $\lambda^1 < \lambda^2 < \ldots < \lambda^k \leqq n$ be positive integers. Now suppose the set $\bar{Y} = \{x^{\lambda^1},\ldots,x^{\lambda^k}\}$ is withdrawn from the initial set, X, so that the new set of activities is $X' = X - \bar{Y}$. In addition, suppose that k parcels of land are withdrawn from the system so that the supply of land is $n-k$ parcels. Then any rental value, p, consistent with the initial situation is consistent with the new equilibrium,and the new set of operative activities is $Y' = Y - \bar{Y}$, provided the marginal activities in Y' are assigned to the operative set. What this means is that if a number of parcels of land, along with the activities which occupy them, are withdrawn, then the remaining activities which were initially in operation will operate in the new equilibrium. However, the rental value of land may change. This can be seen as follows: Clearly, for any p, $S^{x^{n+1}} \leqq p \leqq S^{x^n}$, $S^{x^i} - p \geqq 0$ for any index i ($i = 1,\ldots,n$), such that $x^i \in Y'$, and $S^{x^i} - p \leqq 0$ for any i ($i = n+1,\ldots,m$). There are $n-k$ activities in Y' so that if each activity in Y' occupies one parcel of land, then land is fully utilized. Therefore, p is consistent with the new equilibrium, and since the set of operative activities is determined except for the assignment of marginal activities, we get the desired result. The minimal rental value consistent with the new equilibrium is clearly $S^{x^{n+1}}$; however, the maximum rental value consistent with the new equilibrium is greater than or equal to the maximum rent consistent with the initial equilibrium. In particular, this is true if $S^{x^{n-1}} > S^{x^n}$ and $\lambda^k = n$, so that the earnings of all activities in Y' are greater than S^{x^n}. Therefore, the set of rental values consistent with the initial equilibrium may be a proper subset of

the set of rental values which are consistent with the new equilibrium.

In the model where there is one type of land, an activity either operates on that land or it does not operate. In general, however, there are many types of land on which an activity may locate, and an activity which is bumped from one location may shut down or it may locate on another type of land. While the general model of land use is more complex, it turns out that most of the results derived for the simple case of one type of land hold with only minor modifications in the general case. For example, there is a minimum and maximum rental value for each type of land which is consistent with equilibrium in the markets for land. In addition, suppose that the system is in equilibrium and that a new activity, z, is introduced which locates on a given type of land, thereby displacing some activity previously located there. The displaced activity either shuts down or moves to a new location. If it does the latter, then a second activity may be displaced, which in turn displaces a third activity, etc., until a new equilibrium is attained. It can be shown that the cost of introducing z, in terms of its effect on the earnings of other activities, equals the maximum rental value of the land occupied by z which is consistent with the initial equilibrium. More precisely, if a new activity is introduced on a given parcel of land, the maximum rental value of that parcel which is consistent with the initial equilibrium represents the sum, taken over all other activities, of the changes in earnings which result from the movement to the new equilibrium. Because the results derived from the land-use model with one type of land hold in the general case where there are n types of land, and because these results can be demonstrated more simply, the

foregoing discussion, it is hoped, will clarify the discussion that follows.

The general model can be formulated as follows: There are n types of land, and each type consists of a finite number of identical parcels. In addition, there is a set of activities, X, and each activity $x \in X$ has a given level of earnings on the ith type of land denoted by S_i^x $(i = 1,\ldots,n)$. Again it is assumed that S_i^x is independent of whatever other activities are in operation and of the location of these other activities. For convenience the set X is assumed to be finite; however, this assumption can be relaxed without changing the results of the analysis. The activities in X compete freely for the use of land in a given year, t, and the bidding process continues until an equilibrium is established. Given an equilibrium, a rental value, p_i $(i = 1,\ldots,n)$, is associated with each type of land, and the activities in X are divided among $n+1$ mutually exclusive sets, $A_1,\ldots,A_n,C$ such that $X = A_1 \cup A_2 \cup \ldots \cup A_n \cup C$. An activity, x, is in A_i $(i = 1,\ldots,n)$ if and only if x operates on type i land, and x is in C if and only if x does not operate. The conditions of equilibrium must be such that, given the ordered set of rents $p = (p_1,\ldots,p_n)$, there is no incentive for any activity to move or to reopen the bidding, and in addition if a given type of land is not fully utilized, its rental value is zero.

The conditions of equilibrium are the following:

(1.1) $\quad p_i \geqq S_i^x \qquad$ for any $x \in C \quad (i = 1,\ldots,n)$

(1.2) $\quad p_i \leqq S_i^x \qquad$ for any $x \in A_i \quad (i = 1,\ldots,n)$

(1.3) $\quad S_i^x - p_i \geqq S_j^x - p_j \qquad$ for any $x \in A_i \quad (i,j = 1,\ldots,n)$

(1.4) $\quad p_i \geqq 0 \qquad (i = 1,\ldots,n)$

and the equality holds if conditions (1.1) to (1.3) hold, and the number of activities in A_i is less than the number of parcels of type i land. Condition (1.1) states that in equilibrium it is not profitable for any activity in C to operate, and (1.2) states that no activity in operation is operating at a loss. Condition (1.3) states that, given the set of equilibrium rents, it would not be profitable for an activity operating on type i land to move to type j land, $j \neq i$. The last condition results from the assumptions that a landowner will not pay someone to occupy his land, and that there is price competition among suppliers of land which will drive the rental value of any type of land not fully occupied to zero. The set of equilibrium rental values, p, is restricted by conditions (1.1) to (1.4), but is not in general uniquely determined by these conditions. Therefore, there may be a number of sets of rental values which are consistent with equilibrium in the markets for land. Similarly, given any set of rents which will maintain an equilibrium in the markets for land, the location of the different activities is not completely specified; i.e., the sets $A_1,\ldots,A_n,C$ are not uniquely determined. Suppose, for example, that p satisfies conditions (1.1) to (1.4), given a pattern of location described by the sets $A_1,\ldots,A_n,C$. Further suppose that $S_i^x = S_i^y = p_i$ for some index i, and that $x \in A_i$ and $y \in C$. Then x and y can be interchanged, and p will satisfy conditions (1.1) to (1.4), given the new pattern of location. We get another such example if instead we suppose that $x \in A_i$ and $y \in A_j$, $i \neq j$, and that $S_i^x - p_i = S_j^x - p_j$ and $S_i^y - p_i = S_j^y - p_j$. Again, if the position of x and y were interchanged, the set of rents p would satisfy conditions (1.1) to (1.4), given the new pattern of location.

In both the above cases it is a matter of indifference from the point of view of x and y as to which set they are assigned. An activity which occupies some location, and which, given the set of rents, is either indifferent between operating and shutting down or is indifferent between operating at its present location or at some other type of location, will be said to be marginal to the land it occupies, given these rents. If the rent of any given type of land is increased by an amount, $\epsilon > 0$, no matter how small, the marginal activities either shut down or move to a new location. Since ϵ can be made arbitrarily small, it will in the future be assumed that a marginal activity can be displaced by a supra-marginal activity without effecting a change in the rent. This is important for later analysis where the effects of perturbing an equilibrium by introducing a new activity are investigated. It is also important to note that a marginal activity may be earning a substantial profit. Such an activity may be marginal to a specific type of land, given rents, because it can earn the same profit at some other location.

For any given set of rents, the pattern of location is not uniquely determined; however, for every set of rents consistent with equilibrium, the patterns of location associated with each set are the same. This is stated precisely in the following theorem:

<u>Theorem 1</u>. Let $p = (p_1,\ldots,p_n)$ be a set of rents which satisfies the conditions of equilibrium with a pattern of location described by $A_1,\ldots,A_n,C$. In addition, let $p' = (p_1',\ldots,p_n')$ be any other set of rents which is consistent with an equilibrium in the markets for land. Then conditions (1.1) to (1.4) are satisfied by p', given the pattern of location described by $A_1,\ldots,A_n,C$.

Proof: We can assume without loss of generality that the types of land are numbered so that (a) $p_1 - p_1' \leqq p_2 - p_2' \leqq \ldots \leqq p_n - p_n'$. From (a) it follows that (b) $p_k - p_j \leqq p_k' - p_j'$, $k \leqq j$. From (b) and condition (1.3) we get $0 \leqq (S_j^x - p_j) - (S_k^x - p_k) \leqq (S_j^x - p_j') - (S_k^x - p_k')$ for any $x \in A_j$, and $k \leqq j$; and therefore it follows that (c) $S_k^x - p_k' \leqq S_j^x - p_j'$ for any $x \in A_k$ and $k \leqq j$. If the inequality holds in (b), then it also holds in (c). Equation (c) tells us that no activity in A_j $(j = 1,\ldots,n)$ will find it profitable to move to a location with an index $k < j$, given p'. This fact will be used extensively throughout the rest of the proof.

Suppose that, given p', condition (1.1) is not satisfied; i.e., for some $x \in C$, $p_i' < S_i^x$ for one or more indices i. Now choose ℓ such that for this activity, x, $S_\ell^x - p_\ell' \geqq S_i^x - p_i'$ for all i. If more than one index satisfies this condition, let ℓ be the minimum of these. Now suppose that, for some index $i \leqq \ell$, $p_{i-1} - p_\ell < p_{i-1}' - p_\ell'$, and let k be the maximum index for which this is true. Then it follows from (a) and the definition of k that $p_k - p_\ell = p_k' - p_\ell'$, and since clearly $p_\ell' < p_\ell$, it follows that $p_k' < p_k$. From (a) we then get the result that $p_j' < p_j$, $k \leqq j$, so that $S_j^x - p_j' > 0$ for any $x \in A_j$, $k \leqq j$. Moreover, $S_j^x - p_j' > S_i^x - p_i'$ for any $x \in A_j$, $k \leqq j$ and $i < k$. This means that, given p', the number of activities which demand land of types $k,\ldots,n$ is greater than the number of activities in $A_k \cup \ldots \cup A_n$. If the number of activities in A_j $(j = k,\ldots,n)$ equals the number of parcels of type j land, then, given p', there is an excess demand in one of the markets for land so that p' would not be consistent with equilibrium. If the number of activities in A_j is less than the number of parcels of type j

land for some $j \geqq k$, then by condition (1.4) we have $p_j = 0$. However, $p_j > p'_j$, which implies that $p'_j < 0$ and this violates condition (1.4). If we had assumed there was no index $i \leqq \ell$ for which $p_{i-1} - p_\ell < p'_{i-1} - p'_\ell$, then it can be shown that $p'_j < p_j$ for all j, and a similar argument leads to the same contradiction. Therefore, the assumption that the set of rents, p', is not consistent with condition (1.1) leads to a contradiction of the assumption that p' is consistent with an equilibrium in the markets for land.

Now suppose that, given p', condition (1.2) is not satisfied; i.e., for some index, i, $S_i^x - p'_i < 0$ for some $x \in A_i$. Let ℓ be the maximum index for which this is true. Suppose there is an index $i \geqq \ell$ for which $p_\ell - p_{i+1} < p'_\ell - p'_{i+1}$, and let $k \geqq \ell$ be the minimum index for which this holds. By the definition of k, $p_\ell - p_k = p'_k$, and since $p'_\ell > p_\ell$ clearly holds, it follows that $p'_k > p_k$. From (a) we get the result that $p'_i > p_i$, $i \leqq k$, so $S_i^x - p'_i < 0$ for any $x \in C$ and $i \leqq k$. Further, $S_j^x - p'_j > S_i^x - p'_i$ for any $x \in A_j$, $j > k$, $i \leqq k$, and some activity in A_ℓ will either shut down or demand type j land, $j > k$, given p'. Therefore, the number of activities demanding land of types $1,\ldots,k$ is less than the number of activities in $A_1 \cup \ldots \cup A_k$ so that, given p', there will be at least one parcel of land for which there is no demand. If this set of rents is consistent with equilibrium, some rent p'_i, $i \leqq k$, must be zero. However, $p'_i = 0$ for some $i \leqq k$ implies that $p_i < 0$, which is not possible. If there is no index $i \geqq \ell$ for which $p_\ell - p_{i+1} < p'_\ell - p'_{i+1}$, then it follows that $p'_i > p_i$ for all i, and the argument and conclusions follow as before if we let $k = n$.

Now suppose that, given p', condition (1.3) is not satisfied; i.e., for some index i, $S_i^x - p_i' < S_j^x - p_j'$ for some $x \in A_i$ and $i < j$. Now divide the problem into two parts. First, suppose that for some index, k, $p_k' > p_k$ and $S_k^x - p_k' < S_j^x - p_j'$ for some $x \in A_k$ and $k < j$. Then from (a) it follows that $p_i' > p_i$ for $i \leq k$ and the rest of the proof follows from the same argument used in the previous case where condition (1.2) was violated. Now suppose that for every index i for which $S_i^x - p_i' < S_j^x - p_j'$ for some $x \in A_i$, $i < j$, $p_i' < p_i$. Now let ℓ-1 be the minimum index for which the above holds, and let $k \leq \ell$ be defined as in the case where condition (1.1) was violated. The rest of the proof is the same as for that case.

Since p' is assumed to be consistent with the conditions of equilibrium, then condition (1.4) must be satisfied for every pattern of location for which conditions (1.1) to (1.3) hold. Therefore, by assumption and by the fact that conditions (1.1) to (1.3) hold, given p' and the pattern of location described by $A_1,\ldots,A_n,C$, it follows that (1.4) also holds, and this completes the proof.

In the case where there was one type of land, the rental values which were consistent with equilibrium in the markets for land were contained in a closed interval. A similar result holds for the general case as it can be shown that any set, p, of equilibrium rents lies in a closed n-dimensional interval, although the points represented by these sets of rents do not fill the interval as in the case with one kind of land. However, the endpoints of this interval are sets of equilibrium rents so that there is a maximal and minimal set of such rents. This is stated in the following theorem:

Theorem 2. There exist two sets of rental values, $\bar{p} = (\bar{p}_1,\ldots,\bar{p}_n)$ and $\bar{\bar{p}} = (\bar{\bar{p}}_1,\ldots,\bar{\bar{p}}_n)$ such that $\bar{p} \leqq p \leqq \bar{\bar{p}}$, where $p = (p_1,\ldots,p_n)$ is any set of rents which is consistent with equilibrium in the markets for land. In addition, $\bar{p}$ and $\bar{\bar{p}}$ are consistent with equilibrium and will be referred to as the minimal and maximal sets, respectively, of equilibrium rents.

Proof: Let $P_i = \{p_i\}$ $(i = 1,\ldots,n)$ be the set of rental values for type i land such that each $p_i \in P_i$ is contained in an equilibrium set of rents p. Let $\bar{p}_i = \inf. P_i$, and let $\bar{\bar{p}}_i = \sup. P_i$ $(i = 1,\ldots,n)$ so that $(\bar{p}_1,\ldots,\bar{p}_n) \leqq (p_1,\ldots,p_n) \leqq (\bar{\bar{p}}_1,\ldots,\bar{\bar{p}}_n)$ for any set of rents $(p_1,\ldots,p_n)$ which are consistent with conditions (1.1) to (1.4). We must now show that the maximal and minimal sets of rents satisfy conditions (1.1) to (1.4) for any pattern of location described by $A_1,\ldots,A_n,C$. Since by Theorem 1 all sets of equilibrium rents satisfy conditions (1.1) to (1.4) for any pattern of location associated with equilibrium, we can arbitrarily choose any such pattern and demonstrate that the maximal and minimal rents satisfy conditions (1.1) to (1.4) with respect to this pattern of location The proof will be carried out in detail only for the maximal set of rents, as with minor modifications the same line of argument can be used to show that the minimal set of rents satisfies conditions (1.1) to (1.4).

From the definition of $\bar{\bar{p}}_i$ $(i = 1,\ldots,n)$ it follows that $\bar{\bar{p}}$ satisfies condition (1.1). Now suppose that the maximal set of rents does not satisfy condition (1.2); i.e., for some index i, $\bar{\bar{p}}_i > S_i^x$ for some activity x, in A_i. However, since $\bar{\bar{p}}_i = \sup. P_i$, it follows that

there is a $p_i \in P_i$ such that $p_i > S_i^x$ which contradicts the assumption that p_i belongs to the ordered set of rents consistent with equilibrium. Therefore, $\bar{\bar{p}}$ must satisfy condition (1.2). From the definition of $\bar{\bar{p}}_j$ and (1.3), it follows that $S_i^x - p_i \geqq S_j^x - \bar{\bar{p}}_j$ for any $x \in A_i$ $(i,j = 1,\ldots,n)$ and any $p_i \in P_i$. From this inequality, which holds for all $p_i \in P$, it is clear that $S_i^x - \bar{\bar{p}}_i \geqq S_j^x - \bar{\bar{p}}_j$ for any $x \in A_i$ $(i,j = 1,\ldots,n)$ so that condition (1.3) is satisfied. Since $p_i \geqq 0$ for all $p_i \in P_i$, it follows that $\bar{\bar{p}}_i \geqq 0$, and if type i land is not fully occupied, $p_i = 0$ for all $p_i \in P_i$ so that $\bar{\bar{p}}_i = 0$ by definition. Therefore, $\bar{\bar{p}}$ satisfies conditions (1.1) to (1.4), which completes the proof.

The problem as it has been formulated is an integer programming problem; however, this particular problem, known as the assignment problem, can be solved by linear programming techniques as it happens that there is always an integer solution to a linear assignment problem [2, 316-22]. Assuming that the earnings of each activity at each location are exogenously given and independent of the location of other activities, we attempt to locate the activities so as to maximize the total earnings of all activities. The patterns of location which correspond to this maximum can be shown to be the patterns of location corresponding to a competitive equilibrium in the markets for land. There are several advantages to this approach. First, it can be shown, using well-known theorems of linear programming, that there is a set of rents

consistent with equilibrium in the markets for land; second, that the total earnings of all activities, given any pattern of location for which there is a set of equilibrium rents, equals the maximum total earnings attainable, given the conditions of the problem. From this it follows that the total earnings of all activities are the same, given any pattern of location associated with equilibrium in the markets for land. The reasons for not formulating this problem as a linear programming problem are, first, that using the present approach we are able to prove some very strong theorems about the process of substitution when an equilibrium is perturbed and, second, that it is not appropriate for the purposes of this paper to interpret the conditions of equilibrium as the result of the solution to an assignment problem. With regard to the theorems which will be proved, I have not yet been able to derive the results in the context of a linear programming formulation. However, these theorems, including Theorem 2, must hold and would be interesting from the standpoint of the assignment problem alone. I intend to explore the relation between these theorems and the linear programming solution to the assignment problem in a separate paper.

Before extending the formal model and investigating the effects of perturbing a system which is in equilibrium, it is necessary for the general relevance and applicability of this analysis to modify and reinterpret the assumption that the earnings of each activity at each type of location are known in advance of the bidding. If we consider an economy where there are many types of land and a great number of activities, the earnings of any one activity at a particular location will, in general, depend on both the location and the characteristics of other

activities in operation. These factors will affect both the demand and supply functions for the individual activity under consideration. However, assume that a process of competitive bidding takes place and that after adjustment and readjustment an equilibrium is reached where, given the prevailing rents, there is no incentive for any activity in operation to move to another location and there is no incentive for any activity to reopen the bidding. This equilibrium in the markets for land is part of a general competitive equilibrium with which is associated a given set of prices, and these prices are taken as given by the individual activity. Given market prices, rents, and the technical characteristics of an activity, its earnings and profits can be computed for each alternative location. If we interpret S_i^x to be the earnings of activity x on the ith type of land, given the prices which obtain in competitive equilibrium, then conditions (1.1) to (1.4) must be satisfied, given this equilibrium. This is because an individual activity takes market prices and rents as given and will relocate only if it can increase its profits, given current prices.

One cannot, however, be completely comfortable with this interpretation because it assumes that a competitive equilibrium exists and is associated with a given pattern of land use. Beckmann and Koopmans have demonstrated, however, that if the locational interdependence among economic activities takes a particular form, then there is no set of prices, including rents, consistent with equilibrium in the markets for land [7, 69]. That is, given any set of prices, including rents, it will always be profitable for some activity to relocate. In terms of the

present model this means that there may not be a competitive equilibrium which satisfies conditions (1.1) to (1.4). While the results derived by Beckmann and Koopmans suggest that there may not be a set of rents that will sustain an equilibrium in the markets for land, given locational interdependence among economic activities, these results are derived for the very special case where the problem of finding the optimal location of activities is a quadratic assignment problem [7,64-71]. Whether the same results would hold, given the more complicated types of interrelationships found in the real world, is still a matter for conjecture. For purposes of the present analysis it is assumed that a competitive equilibrium exists in the markets for land.

One can either interpret this assumption as meaning that the analysis is applicable only to the case where locational interdependence does not exist, so that the problem corresponds to a linear assignment problem, or as meaning that in the real world the complicated types of locational interdependence which exist are not incompatible with a competitive equilibrium. In either case the starting point for the analysis is an equilibrium which satisfies conditions (1.1) to (1.4).

In the analysis which follows, the effects of perturbing an equilibrium by introducing new activities or by changing the characteristics of some type of land will be studied. When an equilibrium is perturbed in this manner, a process of equilibrium takes place in which a number of activities may relocate and a new equilibrium will be established with a new pattern of location. If the process of adjustment involves a small proportion of the total number of activities in a region,

it is reasonable to assume that prices other than land rents will remain unchanged. Therefore, the earnings of a given activity at each alternative location will be the same in both equilibrium situations; however, the profit realized at each alternative location will be different if rents change. In the analysis that follows it is only essential for the argument that the initial situation is a general equilibrium where, given prices, the earnings of each activity at each alternative location are known and where conditions (1.1) to (1.4) are satisfied in the markets for land. Further, it is assumed that if the original equilibrium is perturbed, the earnings of each activity at each location remain invariant throughout the process of adjustment.

In the case where there was one type of land, the effects of introducing a new activity, z, were discussed. It was demonstrated that if z operates in the new equilibrium, the opportunity cost of introducing z in terms of the decrease in the earnings of other activities is equal to the maximum rental value consistent with the initial equilibrium and the minimum rental value consistent with the new equilibrium. A similar result can be derived in the case where there are n types of land. It will first be demonstrated that if z is introduced and locates on type i land, then the cost of moving to the new equilibrium, in terms of the total decrease in the earnings of all other activities which are forced to relocate, equals the minimum rental value on type i land which is consistent with the new equilibrium. It will then be demonstrated that this rental value is also the maximum rental value on type i land which is consistent with the initial equilibrium.

Suppose that the economy is in equilibrium with a given set of prices, including $p = (p_1,\ldots,p_n)$, the minimal set of equilibrium rents. The set, X, of potential activities is, in equilibrium, divided among n+1 mutually exclusive sets, $A_1,\ldots,A_n,C$. A new activity, z, is introduced so that the set of potential activities is now $X' = X \cup z$. Associated with X' will be a new equilibrium in which a new minimal set of rents, p', is assumed to obtain. All other prices are assumed to remain constant so that the earnings of any activity on any parcel of land are the same before and after the introduction of z. In the new equilibrium the activities in X' will be divided among n+1 mutually exclusive sets, $A_1',\ldots,A_n',C'$.

Suppose that in the new equilibrium $z \in C'$ so that z does not operate. This breaks down into two subcases. First, if $S_i^z \leqq p_i$ $(i = 1,\ldots,n)$, then z does not find it profitable to operate, given the initial set of rents, and therefore the initial equilibrium is not disturbed by the introduction of z. Now suppose that $S_i^z > p_i$ for some i. In this case, z will start bidding for land and the initial equilibrium will be upset. A new set of rents, $p' \geq p$, will be established; however, given these new rents, z, by assumption, does not find it profitable to operate.*/ In other words, z bids up the rental value of land but does not succeed in bidding land away from any of the activities which occupied land in the initial equilibrium. It can easily be shown that $A_i' = A_i$ $(i = 1,\ldots,n)$, provided we assume that activities which are indifferent between two types of land, given both sets of rents,

*/ $p' \geq p$ means $p_i' \geqq p_i$ $(i = 1,\ldots,n)$ and $p_i' \neq p_i$ for at least one value of i.

remain in their initial location. Therefore, the only effect of introducing z in this case is possibly to raise the minimal set of equilibrium rents.

Now suppose $z \in A_i'$ for some index i. Without a loss of generality we can let $i = 1$. If type 1 land is not fully occupied in the initial equilibrium, and if $S_1^z - p_1 \geqq S_i^z - p_i$ $(i = 1,\ldots,n)$, then the initial minimal set of rents will obtain before and after the introduction of z so that $p = p'$. Clearly, $p_1' = p_1 = 0$, which is the desired result since no activities are forced to relocate so that the opportunity cost of introducing z is zero. Now suppose that $S_1^z - p_1 < S_i^z - p_i$ for some i. Then if z is to operate on type 1 land in the new equilibrium, $p_i' > p_i$ for at least one value of i, and therefore the new minimal set of equilibrium rents is such that $p' \geq p$. It can be shown that p' is consistent with the initial equilibrium, from which it follows from condition (1.4) that $p_1' = 0$. This is the desired result since no activities are displaced by the introduction of z. In the new equilibrium the location of some marginal activities may be changed, but this does not affect the general result that the total earnings of all firms other than z are the same in both equilibriums.

Suppose, however, that in the new equilibrium z occupies a type of location which was fully occupied in the initial equilibrium. As a result, a number of firms are forced to relocate, and some activity which previously occupied a type of land which was fully utilized is either forced to shut down or to move to previously unoccupied land. The following analysis will be devoted to the first case, although the case where an activity moves to previously unoccupied land can be analyzed in an analogous manner with identical results.

Without loss of generality we can assume that all land is fully occupied in the initial equilibrium. It follows from this, the conditions of equilibrium, and the fact that a new activity has been added to the set of potential activities that all land will be occupied in the new equilibrium. Clearly, $C \subset C'$ and since z, by assumption, locates on type i_1 land, it follows that one and only one activity previously in operation does not operate in the new equilibrium. It is intuitively clear that z will replace some activity on type i_1 land, that this activity will in turn replace a third activity on i_2, etc., until for some $k \leq n$, an activity which formerly operated on type i_{k-1} land replaces an activity on i_k land which is shut down. If the above description of the effects of the adjustment process is correct and if, for definiteness, we assume $i_1 = 1,\ldots,i_k = k$, then the sets $A_1',\ldots,A_n',C$ can be described as follows:

$$(2)\quad A_1' = (A_1 \cup z) - x^1,\ A_2' = (A_2 \cup x^1) - x^2,\ldots,A_k' = (A_k \cup x^{k-1}) - x^k,$$

$$A_{k+1}' = A_{k+1},\ldots,A_n' = A_n,\ C' = C \cup x^k,$$

where x^i $(i = 1,\ldots,k)$ is a given activity such that $x^i \in A_i$. If (2) holds, then the total decrease in the earnings of activities previously in operation that results from the adjustment to the new equilibrium is

$$(3)\quad \sum_{i=1}^{k-1} (S_i^{x^i} - S_{i+1}^{x^i}) + S_k^{x^k}.$$

In order to prove that (3) equals p_1' it will be demonstrated that:

$$(4.1)\quad S_i^{x^i} - S_{i+1}^{x^i} = p_i' - p_{i+1}' \qquad (i = 1,\ldots,k-1)$$

and

$$(4.2)\quad S_k^{x^k} = p_k',$$

where p' is the new minimal set of rents satisfying the conditions of equilibrium.

First, however, it is necessary to demonstrate that the new equilibrium is of the general form described by (2). Assume that in the new equilibrium the following situation exists for k activities $2 \leqq k \leqq n$:

$$(5) \quad x^{i_1} \in A_{i_1} \text{ and } x^{i_1} \in A'_{i_2}, \quad x^{i_2} \in A_{i_2} \text{ and } x^{i_2} \in A'_{i_3}, \ldots,$$

$$x^{i_{k-1}} \in A_{i_{k-1}} \text{ and } x^{i_{k-1}} \in A'_{i_k}, \text{ and } x^{i_k} \in A_{i_k} \text{ and } x^{i_k} \in A'_{i_1}.$$

It can be shown that this assumption leads to a contradiction. To simplify the notation again, assume that $i_1 = 1, \ldots, i_k = k$. By (1.3) it follows that

$$(6.1) \quad S_i^{x^i} - S_{i+1}^{x^i} \geqq p_i - p_{i+1} \quad (i = 1, \ldots, k-1)$$

and

$$(6.2) \quad S_k^{x^k} - S_1^{x^1} \geqq p_k - p_1.$$

However, from the same equilibrium condition for the new equilibrium it follows that

$$(7.1) \quad p'_i - p'_{i+1} \geqq S_i^{x^i} - S_{i+1}^{x^i} \quad (i = 1, \ldots, k-1)$$

and

$$(7.2) \quad p'_k - p'_1 \geqq S_k^{x^k} - S_1^{x^k}.$$

Using (6.1) and (7.1), and summing over $i = 1, \ldots, k-1$, we get

$$(8) \quad \sum_{i=1}^{k-1} (p'_i - p'_{i+1}) \geqq \sum_{i=1}^{k-1} (S_i^{x^i} - S_{i+1}^{x^i}) \geqq \sum_{i=1}^{k-1} (p_i - p_{i+1}),$$

and therefore

(9) $p_1' - p_k' \geqq p_1 - p_k$.

From (6.2) and (9), it follows that

(10) $S_k^{x^k} - S_1^{x^k} \geqq p_k - p_1 \geqq p_k' - p_1'$,

which is consistent with (7.2) if and only if equality holds for (6.1), (6.2), (7.1), and (7.2). In other words, given either set of equilibrium prices, x^i is indifferent between a type i and $i+1$ location for $i = 1,\ldots,k-1$, and x^k is indifferent between a type k and type 1 location. Therefore, in either equilibrium situation these activities can be located so that x^i operates on type i land or on type $i+1$ land. The total earnings of these k activities are the same, given either pattern of location, because

$$(11) \quad \sum_{i=1}^{k-1} (S_i^{x^i} - S_{i+1}^{x^i}) + (S_k^{x^k} - S_1^{x^k}) = \sum_{i=1}^{k-1} (p_i' - p_{i+1}') + (p_k' - p_1')$$
$$= \sum_{i=1}^{k-1} (p_i - p_{i+1}) + (p_k - p_1) = 0 .$$

We can therefore assume, without affecting the results of our analysis, that where this special case arises $x^i \in A_i'$.

The foregoing result can be used to show that the new equilibrium is of the general form described by (2). Clearly, if z occupies type 1 land, there has to be a chain of adjustment that results in some activity, which was previously in operation, shutting down. This follows because all land is fully occupied in both equilibriums and because there is a fixed number of parcels of land. However, the question remains: Is there any other relocation that accompanies the adjustment to the new equilibrium? Suppose, in addition to the relocation described by (2), that some

$y^1 \epsilon A_1$, $y^1 \neq x^1$ is in a new location, say on type 2 land, so that $y^1 \epsilon A_2'$. Because y^1 is on type 2 land, some activity $y^2 \epsilon A_2$, $y^2 \neq x^2$, is located on some other type of land, say type 3, so that $y^2 \epsilon A_3'$. Since there is a finite number of types of land, and since all land is occupied in the new equilibrium, one can easily show by proceeding in this manner that at some point on this chain of relocation an activity will be located on the parcel of land vacated by y^1. Therefore, these activities that are relocated in the new equilibrium satisfy (5). which leads to a contradiction except in the special case discussed. Therefore, the new equilibrium may be assumed to be of the form described by (2), provided the land types have been numbered so that z locates on type 1 land, and x^i moves to type i+1 land. The fact that the new equilibrium is of this form will be used extensively in the proof that (4.1) and (4.2) hold.

This proof will be carried out by constructing a set of rental values p' which satisfies (4.1) and (4.2), and then demonstrating that this is the minimal set of rental values satisfying the conditions of equilibrium. First, let $x^k \epsilon A_k$ be an activity such that

(12) $S_k^{x^k} \leqq S_k^{y^k}$ for any $y^k \epsilon A_k$.

Since by assumption some $y^k \epsilon A_k$ does not find it profitable to operate given p', it follows that $p_k' \geqq S_k^{x^k}$ because otherwise all activities in A_k would find it profitable to operate on type k land. Let $p_k' = S_k^{x^k}$, which is the minimum value consistent with the particular equilibrium assumed to exist. Also, from (1.1) it follows that $S_k^{x^k} \geqq p_k$ so that $p_k' - p_k \geqq 0$. Now let $x^i \epsilon A_i$ be such that

(13) $S_{i+1}^{x^i} - S_i^{x^i} \geqq S_{i+1}^{y^i} - S_i^{y^i}$ for any $y^i \epsilon A_i$.

Set p_i' so that

$$(14)\quad S_{i+1}^{x^i} - p_{i+1}' = S_i^{x^i} - p_i' \quad (i = 1,\ldots,k-1).$$

Since p_k' is given, p_i' $(i = 1,\ldots,k-1)$ is determined by (14). In order to understand the significance of this procedure it is helpful to analyze the rental value, p_{k-1}', which is set so that $S_k^{x^{k-1}} - p_k' = S_{k-1}^{x^{k-1}} - p_{k-1}'$. p_{k-1}' is set at the minimum value that is consistent with the new equilibrium because if p_{k-1}' were set so that $S_k^{x^{k-1}} - p_k' < S_{k-1}^{x^{k-1}} - p_{k-1}'$, then it follows that no $y^{k-1} \epsilon A_{k-1}$ would be in A_k', contrary to our initial assumption about the new equilibrium. Since by (1.3) $S_k^{x^{k-1}} - p_k \leqq S_{k-1}^{x^{k-1}} - p_{k-1}$, then

$$(15)\quad p_{k-1}' - p_{k-1} = \gamma_{k-1} \geqq \gamma_k = p_k' - p_k.$$

The same line of reasoning can be used to demonstrate that for any $i = 1,\ldots,k-1$, p_i' is assigned the minimum value consistent with the initial assumption about the characteristics of the new equilibrium. It can also be shown that

$$(16)\quad \gamma_i \geqq \gamma_{i+1} \quad \text{for} \quad i = 1,\ldots,k-1.$$

This last result can be loosely interpreted by saying the minimum rent of land increases most for that land on which the new activity locates, and less for each successive type of land involved in the chain of adjustment.

In order that p' satisfy the conditions of equilibrium,

$$(17)\quad S_i^x - p_i' \geqq S_j^x - p_j' \quad \text{for any} \quad x \in A_i' \quad (i,j = 1,\ldots,k)$$

must hold. It will be demonstrated that the specified set of rents, p', satisfies (17). First, however, it is necessary to show that the activities designated by x^i in (2) satisfy (13) for $i = 1,\ldots,k-1$, and (12) for $i = k$. Suppose that the activity in (2) denoted x^k does not satisfy (12) so that $S_k^{x^k} > S_k^{y^k}$ for some $y^k \in A_k$, $x^k \neq y^k$. Since it is assumed that x^k does not operate in the new equilibrium, $p_k' \geqq S_k^{x^k} > S_k^{y^k}$. Therefore, y^k does not operate on type k land in the new equilibrium, contrary to our initial assumption. A similar line of reasoning can be used to demonstrate that x^i $(i = 1,\ldots,k-1)$ in (2) must satisfy (13).

To show that p' satisfies (17), first assume that $i \geqq j$. Now let $x \in A_i'$ so that either $x \in A_i$ or $x = x^{i-1}$. If $x \in A_i$, we have by (1.3) and (16) that $S_i^x - p_i \geqq S_j^x - p_j$ and $p_i' - p_i \leqq p_j' - p_j$. From this it follows that

(18) $S_i^x - p_i' \geqq S_j^x - p_j'$ for any $x \in A_i$ and $i \geqq j$ $(i,j = 1,\ldots,k)$.

If $x = x^{i-1}$, then it follows from (14) that

(19) $S_i^x - p_i' = S_j^x - p_j'$ for $j = i-1$.

If $j < i-1$, we have the result that

(20) $S_{i-1}^x - p_{i-1}' \geqq S_j^x - p_j'$,

which follows from (18). Combining (19) and (20) we get the desired result that

(21) $S_i^x - p_i' \geqq S_j^x - p_j'$ for $x = x^{i-1}$ and $i \geqq j$ $(i,j = 1,\ldots,k)$.

Now it must be demonstrated that (17) holds for $i < j$. Suppose for some $x \in A_i'$ that (17) does not hold, so that

(22) $S_i^x - p_i' < S_j^x - p_j'$ for some $x \in A_i'$, $i < j$ $(i,j = 1,\ldots,k)$;

it will be shown that the assumption that (22) holds leads to a contradiction. Let $p'' = (p_1'',\ldots,p_n'')$ be some set of rental values which satisfies the equilibrium conditions associated with (2). Then

(23) $S_i^x - S_{i-1}^x \geqq p_i'' - p_{i-1}''$ for any $x \in A_i'$ $(i = 2,\ldots,k)$.

Now by (14) we have

(24) $S_i^{x^{i-1}} - S_{i-1}^{x^{i-1}} = p_i' - p_{i-1}'$ for $x^{i-1} \in A_i'$.

From (23) and (24) it follows that

(25) $p_i' - p_{i-1}' \geqq p_i'' - p_{i-1}''$ $(i = 2,\ldots,k)$.

By using (25) and summing over the inequalities, it follows that

(26) $p_j' - p_i' \geqq p_j'' - p_i''$, $i < j$ $(i,j = 1,\ldots,k)$.

Since p'' satisfies the conditions of equilibrium, we have

(27) $S_i^x - p_i'' \geqq S_j^x - p_j''$ for any $x \in A_i'$;

and from (26) and (27) we get

(28) $S_i^x - p_i' \geqq S_j^x - p_j'$ for any $x \in A_i'$, $i < j$ $(i,j = 1,\ldots,k)$

which contradicts the assumption that (22) holds. This completes the proof that (17) holds. If $k = n$, then it has been demonstrated that p' is the minimal set of rental values consistent with condition (1.3), given the pattern of location described by (2).

However, consider the case where $k < n$ so that the rents $p'_{k+1},\ldots,p'_n$ are yet to be specified. Now define p'_{k+1} to be the minimum rental value which satisfies the following constraints:

(29.1) $S_i^x - p'_i \geqq S_{k+1}^x - p'_{k+1}$ for any $x \in A'_i$ $(i = 1,\ldots,k)$,

(29.2) $p'_{k+1} \geqq 0$

(29.3) $p'_{k+1} \geqq S_{k+1}^x$ for any $x \in C'$.

Suppose that (29.2) and (29.3) are not effective constraints; then it is easily shown that

(30) $S_{k+1}^x - p'_{k+1} \geqq S_i^x - p'_i$ for any $x \in A'_{k+1}$ $(i = 1,\ldots,k)$

so that, given (17), we have

(31) $S_i^x - p'_i \geqq S_j^x - p'_j$ for any $x \in A'_i$ $(i,j = 1,\ldots,k+1)$.

Assume (30) does not hold, so that

(32) $S_{k+1}^x - p'_{k+1} < S_i^x - p'_i$ for some $x \in A'_{k+1}$ and some $i \leqq k$.

From (2) and condition (1.3) it follows that

(33) $S_{k+1}^x - S_i^x \geqq S_{k+1}^y - S_i^y$ for any $x \in A'_{k+1}$ and any $y \in A'_i$

$(i = 1,\ldots,k)$.

Therefore, if (32) holds, it follows from (33) that

(34) $(S_{k+1}^y - p'_{k+1}) - (S_i^y - p'_i) \leqq (S_{k+1}^x - p'_{k+1}) - (S_i^x - p'_i) < 0$

for any $y \in A'_i$ $(i = 1,\ldots,k)$.

This contradicts the assumption that p'_{k+1} is the minimal rental value

satisfying (29.1), and therefore (30) must hold. Now suppose that (29.3) is the effective constraint so that p'_{k+1} is greater than the minimum rental value which satisfies (29.1). Clearly, p'_{k+1} satisfies (29.1), so to demonstrate that (31) holds we must show that p'_{k+1} also satisfies (30). Let $p''_{k+1} = p_{k+1} + (p'_k - p_k) = p_{k+1} + \gamma_k$. Then it follows from (14) and condition (1.3) that (30) holds for $p'_{k+1} \leqq p''_{k+1}$. Since $p''_{k+1} > p_{k+1}$, it also follows that $p''_{k+1} > S^x_{k+1}$ for any $x \in C$. In addition, $p''_{k+1} \geqq S^{x^k}_{k+1}$ because by construction and condition (1.3) $0 = S^{x^k}_k - p'_k \geqq S^{x^k}_{k+1} - p''_{k+1}$. This gives us the result that $p''_{k+1} \geqq S^x_{k+1}$ for any $x \in C'$. Therefore, if $p'_{k+1} \leqq p''_{k+1}$ is the minimum value which satisfies (29.3) when (29.3) is the effective constraint, then p'_{k+1} also satisfies (31). Clearly, if (29.2) is the effective constraint, (31) is satisfied. If we proceed by defining p'_{k+s}, $2 \leqq S \leqq n-k$, so that p'_{k+s} is the minimum value such that $S^x_i - p'_i \geqq S^x_{k+s} - p'_{k+s}$ for any $x \in A'_i$ $(i = 1,\ldots,k+s)$, $p'_{k+s} \geqq 0$, and $p'_{k+s} \geqq S^x_{k+s}$ for any $x \in C'$, we derive a set of rents p' such that

$$(35) \quad S^x_i - p'_i \geqq S^x_j - p'_j \quad \text{for any } x \in A'_i \quad (i,j = 1,\ldots,n).$$

By construction, p' is the minimum set of rents such that $p' \geqq 0$ and such that p' satisfies conditions (1.1) and (1.3) with respect to the pattern of location described by (2). There exists some set of rents p'' which is consistent with the equilibrium described by (2), and clearly $p'' \geqq p'$, and therefore p' satisfies conditions (1.2) and (1.4). Therefore, p' is the minimum set of rents which satisfies the conditions of equilibrium. By construction the set of rents p' satisfies (4.1) and (4.2) which, when substituted into (3), yields the desired result. Thus, the rental value p'_1 equals the total decrease in the earnings of

activities other than z that results from the relocation which accompanies the move to the new equilibrium. If the minimal set of rents actually obtains in the new equilibrium, the increase in the total earnings of all activities equals the profit of z.

It can now be shown that if a new activity, z, is introduced which locates on type i land, then the cost of introducing z in terms of the total reduction of earnings of activities other than z also equals p_i'', where p'' is the maximal set of rents consistent with the initial equilibrium. Suppose z locates on land previously unoccupied. Then no activity in operation is forced to relocate, so there is no loss of earnings. Since the initial rent on the unoccupied land is zero, we get the desired result. Suppose, however, that all land is occupied in the initial equilibrium, and that z replaces some activity on type i land so that a chain of relocation takes place, which results in one activity shutting down. Without loss of generality, it can be assumed that the new equilibrium is described by (2). Careful inspection of the proof that the new equilibrium is of the form given by (2) reveals that this proof depends only on the fact that the initial set of rental values and the new set of rental values are consistent with their respective equilibriums. This is satisfied by the maximal as well as the minimal set of rents. Given the new equilibrium described by (2), the total decrease in the earnings of activities which are forced to relocate is given, as before, by (3). To show that (3) equals p_1'', it is sufficient to show that

$$(36.1) \quad S_i^{x^i} - S_{i+1}^{x^i} = p_i'' - p_{i+1}'' \quad (i = 1,\ldots,k-1)$$

and

$$(36.2)\quad S_k^{x^k} = p_k'' ,$$

where p'' is the initial set of maximal rents.

To prove that (36.1) and (36.2) hold, it will be demonstrated that

$$(37)\quad (p_1'',\ldots,p_k'') = (p_1',\ldots,p_k'),$$

where p'' now represents the maximal set of rents consistent with the initial equilibrium, and p' represents the minimal set of rents consistent with the new equilibrium. It has been demonstrated that p' satisfies (4.1) and (4.2); therefore, if (37) holds, it follows that (36.1) and (36.2) also hold.

First it will be shown that p' satisfies conditions (1.3), given the pattern of location described by $A_1,\ldots,A_n,C$. Suppose $x \in A_i$ for some $i = 1,\ldots,n$; then if $i > k$, $x \in A_i'$; if $i < k$, either $x \in A_i'$ or $x = x^i \in A_{i+1}'$; and if $i = k$, either $x \in A_k'$ or $x = x^k \in C'$. If $x \in A_i'$, then because p' is consistent with the new equilibrium we have

$$(38)\quad S_i^x - p_i' \geqq S_j^x - p_j \quad (i,j = 1,\ldots,n).$$

If $x = x^i$ $(i = 1,\ldots,k-1)$, then by (14) we get

$$(39)\quad S_i^{x^i} - p_i' = S_{i+1}^{x^i} - p_{i+1}',$$

and from (38) it follows that

$$(40)\quad S_i^{x^i} - p_i' \geqq S_j^{x^i} - p_j' \quad (j = 1,\ldots,n).$$

If $x = x^k$, then by the definition of p_k' we have $S_k^x - p_k' = 0$, and because $x \in C'$ it follows that $S_j^x - p_j' \leqq 0$ $(j = 1,\ldots,n)$. Therefore,

(41) $S_k^x - p_k' = 0 \geqq S_j^x - p_j'$ $(j = 1,\ldots,n)$.

From (38), (40), and (41) it follows that

(42) $S_i^x - p_i' = S_j^x - p_j'$ for any $x \in A_i$ $(i,j = 1,\ldots,n)$.

Now clearly p_k' is the maximum rental value on type k land which is consistent with the initial equilibrium. Given p_k', p_{k-1}' can be shown to be maximum rent on type k-1 land which is consistent with the initial equilibrium. This can be seen as follows. Assume $p_k'' = p_k'$ and that $p_{k-1}'' > p_{k-1}'$. Then it follows from (13) and (14) that for some $x \in A_{k-1}$

(43) $S_{k-1}^x - p_{k-1}'' < S_k^x - p_k''$,

which is contrary to the assumption that p'' is consistent with the initial equilibrium. Assume the equality holds; then by the previous argument it can be shown that $p_{k-2}'' = p_{k-2}'$. By proceeding in this manner we demonstrate that $p_i'' = p_i'$ $(i = 1,\ldots,k)$. Now let $p_i'' = p_i'$ $(i = 1,\ldots,k)$; then $(p_1'',\ldots,p_k'', p_{k+1}',\ldots,p_n')$ satisfies condition (1.3) with respect to the initial equilibrium. Now assign to p_{k+1}'' the maximum value which satisfies both

(44.1) $S_{k+1}^x - p_{k+1}'' \geqq S_i^x - p_i''$ for any $x \in A_{k+1}$ $(i = 1,\ldots,k)$,

and

(44.2) $S_{k+1}^x - p_{k+1}'' \geqq 0$ for any $x \in A_{k+1}$.

Because $(p_1'',\ldots,p_k'', p_{k+1}',\ldots,p_n')$ satisfies (1.3) and because $p_{k+1}'' \geqq p_{k+1}'$, it follows that

(45) $S_i^x - p_i'' \geqq S_{k+1}^x - p_{k+1}''$ for any $x \in A_i$ $(i = 1,\ldots,k)$.

Therefore, by (44.1) and (45) it follows that

(46) $S_i^x - p_i'' \geqq S_j^x - p_j''$ for any $x \in A_i$ $(k,j = 1,\ldots,k+1)$.

Now assign a value to p_{k+s}'' $(s = 2,\ldots,n-k)$ which is the maximum value that satisfies both

(47.1) $S_{k+s}^x - p_{k+s}'' \geqq S_i^x - p_i''$ for any $x \in A_{k+s}$ $(i = 1,\ldots,k+s)$,

and

(47.2) $S_{k+s}^x - p_{k+s}'' \geqq 0$ for any $x \in A_{k+s}$.

It is easily shown by the argument used to derive (46) that

(48) $S_i^x - p_i'' \geqq S_j^x - p_j''$ for any $x \in A_i$ $(i,j = 1,\ldots,k+s)$.

Setting $s = n-k$, we see that $p''(p_1'',\ldots,p_n'')$ satisfies condition (1.3) with respect to the initial equilibrium. Because $p'' \geqq p' \geqq p$, where p is the minimal set of rents consistent with the initial equilibrium, it follows that p' satisfies condition (1.1). Also, careful inspection of the way in which p'' was constructed will reveal that condition (1.2) is also satisfied, and since by assumption land is fully occupied in the initial equilibrium,condition (1.4) is satisfied. Therefore, p'' is consistent with the initial equilibrium and because of the manner in which it was constructed, it is the maximal set. Further, by construction the components of p'' satisfy (36.1) and (36.2), and by substituting these expressions into (3) we get the desired result that

$$\sum_{k=1}^{k-1} (S_i^{x^i} - S_{i+1}^{x^i}) - S_k^{x^k} = p_1'' .$$

We can restate the previous results as follows:

Theorem 3. Assume there are n types of land, and a set of activities which compete for the available land. In equilibrium the activities of x are divided among the sets $A_1,\ldots,A_n,C$. Suppose a new activity, z, is introduced so the new set of activities is $x' = x \cup z$ and that the activities in x' are divided among the sets $A_1',\ldots,A_n',C'$ in the new equilibrium. Further, suppose $z \in A_i'$. Then the total reduction of the earnings of activities in $A_1 \cup \ldots \cup A_n$ as a result of the adjustment to the new equilibrium is equal to p_i'' and p_i', where p'' is the maximal set of rents consistent with the initial equilibrium, and p' is the minimal set of rents consistent with the new equilibrium.

From a practical standpoint the significance of Theorem 3 would be enhanced if the competitive mechanism by which rents are established in fact insures that the maximal set of rents obtains. In this case the rent which prevails on a parcel of land prior to the introduction of some new activity on that parcel represents the opportunity cost of putting that parcel to this alternative use. If the rents are uniquely determined by the conditions of equilibrium in the initial situation, then the maximal set of rents does obtain. Further, if rents are uniquely determined in the initial equilibrium, and in the equilibrium which obtains after the introduction of z, then we get the following result:

Theorem 4. Assume the situation described in Theorem 3 exists. Then if p' and p'' are both uniquely determined, $p' = p''$.

Proof: It has been demonstrated that p' is consistent with the initial equilibrium. However, p'' by definition is consistent with the initial equilibrium, and since the set of rents consistent with the initial equilibrium is uniquely determined, $p' = p''$.

The question remains, however, as to whether the conditions of a perfectly competitive economy are likely to produce a unique set of rents. To gain some insight into this question, suppose that there are n types of land, and that the number of available units of each type of land is $d_1,\ldots,d_n$, respectively. Let $d = \sum_{i=1}^{n} d_i$; i.e., d is the total number of parcels of all types of land. Suppose the set of potential activities, x, is such that $x = \bigcup_{\ell \in F} x_\ell$, where F is some index set, and that the number of elements in x_ℓ, for any index $\ell \in F$, is greater than d. Further, suppose that if x and y are two elements in x_ℓ, for any index $\ell \in F$, then x and y are identical in the sense that $S_i^x = S_i^y$ for any $i = 1,\ldots,n$, and for any set of market prices. Now assume that a competitive equilibrium is attained and that in this equilibrium some activity x occupies type i land, for some $i = 1,\ldots,n$, so that $x \in A_i$. $x \in X_\ell$ for some index $\ell \in F$, and because the number of activities in X_ℓ is greater than d, it follows that there is an activity $y \in X_\ell$ such that $y \in C$. Because $x \in A_i$ and $y \in C$, it follows that $0 \geqq S_i^y - p_i = S_i^x - p_i \geqq 0$. Therefore, $S_i^x = p_i$ for any $x \in A_i$. If type i land is occupied by one activity, it is fully occupied by activities with identical earnings which just equal the rental value of that land. If we take activities to be firms in a perfectly competitive economy, we have conditions that are identical to those that may hold in

a long-run competitive equilibrium where, because of free entry, profits have been eliminated. The example outlined above makes use of very strong assumptions in order to simplify the demonstration that rents are uniquely determined. The result holds under much weaker assumptions, and even when rents are not uniquely determined by the conditions of equilibrium they may be determined within very narrow bounds by conditions (1.1) to (1.4). In this case the initial rental value of a parcel of land closely approximates the cost of introducing a new activity which locates on that parcel. It is hereafter assumed that the initial rental value represents this cost. This is important because the initial rental value can be observed in advance of the initiation of programs which would alter the pattern of land use, and the information it contains can be used to evaluate whether such programs should be undertaken.

Before proceeding with further analysis, two numerical examples may help to illustrate the content of Theorems 3 and 4. Assume there are two parcels of land, types 1 and 2, and $x = \{x^1,\ldots,x^4\}$. The earnings of these four activities, and of the new activity z, are set forth in Table I.

Table I

	type 1	type 2
s^{x^1}	20	19
s^{x^2}	10	7
s^{x^3}	3	8
s^{x^4}	4	5
s^z	3	11

Careful inspection will show that in equilibrium x^1 will operate on type 2 land, and x^2 will operate on type 1 land. By the conditions of equilibrium it follows that $p_2 \geqq 8$, $p_1 \geqq 4$, $S^{x^1} - p_2 \geqq S^{x^1} - p_1$, and $S^{x^2} - p_1 \geqq S^{x^2} - p_2$. Clearly, the minimum set of rental values consistent with equilibrium is (9,8) and the maximum set of equilibrium rental values is (10,9). Any pair of rental values (p_1,p_2) such that $(9,8) \leqq (p_1,p_2) \leqq (10,9)$ and $p_1 - p_2 \geqq 1$ will satisfy the conditions of equilibrium. Now suppose that z is introduced; then in the new equilibrium the set of rental values is uniquely determined and equals (10,9). z will locate on type 2 land, x^1 will move to type 1 land, and x^2 shuts down. The decrease in the earnings of x^1 is 19 - 20 = -1, and the decrease in the earnings of x^2 is 10, so that the total decrease in the earnings of both x^1 and x^2 is 9. This is the desired result since 9 is the maximum rental value on type 2 land consistent with the initial equilibrium and the minimum rental value consistent with the new equilibrium. This example demonstrates the result in Theorem 3. We can demonstrate the result in Theorem 4 if we change the earning of activity x^3 on type 2 land to 9. In this case the initial set of rental is uniquely determined and is (10,9). The pattern of location associated with the initial equilibrium may either have x^1 located on type 1 land and x^3 located on type 2 land, or x^2 located on type 1 land and x^1 located on type 2 land. It is easily verified that the opportunity cost of introducing z is the same in either case. Also, the sets of equilibrium rents are the same in both situations in accordance with Theorem 4.

The analysis so far has dealt only with the situation where the movement to a new equilibrium is caused by the introduction of a new activity,

z, which locates on some type of land. However, the analysis could have been carried out with identical results if, instead of introducing z onto type 1 land, one unit of type 1 land had been withdrawn from the system. A careful inspection of all the previous analysis will show the maximal and minimal rents associated with the new equilibrium will be the same in both cases. The one difference is that if a unit of type 1 land is withdrawn, $A_1' = A_1 - x^1$ rather than $A_1' = (A_1 \cup z) - x^1$; however, $A_2', \ldots, A_n', C'$ are as described in (2). It is obvious that this difference does not affect the sum given by (3), and the cost, in terms of earnings foregone because a unit of land is withdrawn for some other use, equals the rental value of that land.

Now assume that a unit of type 1 land has been withdrawn from private use and that an equilibrium exists. Consider this equilibrium to be the initial situation in the following discussion. Suppose a unit of type 1 land is now added to the supply available to the activities in the system, and that a new equilibrium is attained. Clearly, this equilibrium corresponds to the initial equilibrium in the previous analysis, and conversely. Therefore, the change in the total earnings of all activities is given, as before, by (3); however, in this case (3) represents an increase in total earnings. Since the initial maximal and minimal sets of rents in this model correspond to the new maximal and minimal sets of rents in the previous model, and conversely, it follows that in this model (3) is equal to the initial rental value of type 1 land if the minimal set of rents obtains and is equal to the new rental value of type 1 land if the maximal set of rents obtains. However, it will be assumed, as before, that the equilibrium conditions given by (1.1)

to (1.4) sufficiently restrict the rental values so that the initial rental value can be used to measure this gain. Further, as before, exactly the same results are obtained if we consider an equilibrium where z occupies type 1 land, and then consider the effect of removing z from the set of potential activities.

So far we have considered only the changes that take place when one activity is added to or subtracted from the system, or when one parcel of land is added to or subtracted from the available supply. Suppose, however, that m activities, $z^1,\ldots,z^m$, are introduced, which locate on type 1 land; or alternatively suppose that m parcels of type 1 land are removed from the system. What is the cost in terms of changes in the earnings of activities forced to relocate? We can answer this question by introducing the new activities one by one and applying the results of the previous analysis. Suppose that in the initial equilibrium the rental value of type 1 land is p_1^0. Further suppose that z^1 is added to the set of potential activities, and that a new equilibrium obtains where z^1 is located on type 1 land and where the new rental value of type 1 land is p_1^1, $p_1^1 \geqq p_1^0$. Now add z^2 to the set of potential activities and assume it locates on type 1 land, and that an equilibrium obtains where the rental value of type 1 land is $p_1^2 \geqq p_1^1 \geqq p_1^0$. By proceeding in this manner one can derive a sequence of rents on type 1 land, $p_1^0, p_1^1, \ldots, p_1^m$, such that

$$(49) \qquad p_1^0 \leqq p_1^1 \leqq p_1^m .$$

The set of potential activities associated with the mth equilibrium is $X^m = X^0 \cup z^1 \cup \ldots \cup z^m$, where X^0 is the set of potential activities associated with the initial equilibrium.

It follows from the preceding analysis that the cost, in terms of the decrease in the total earnings of other activities, of introducing $z^1, z^2, \ldots, z^m$ is

$$(50) \quad \sum_{i=0}^{m-1} p_1^i .$$

From (49) it follows that

$$(51) \quad m\, p_1^0 \leqq \sum_{i=0}^{m-1} p_1^i \leqq m\, p_1^m .$$

In words, the initial rental value of type 1 land multiplied by the number of new activities is a lower bound on the cost of introducing these activities, and the new rental value of type 1 land multiplied by the number of new activities is an upper bound on this cost. If there are a large number of activities located on type 1 land in the initial equilibrium which are marginal to this land, then it is reasonable to expect that $p_1^0 \approx p_1^m$. In this case, $p_1^0 m$ is a good approximation of the opportunity cost of introducing the new activities.

This analysis remains valid if, instead of introducing m new activities which locate on type 1 land, m parcels of type 1 land are withdrawn from the system. Further, if we consider the mth equilibrium to be the initial equilibrium, and withdraw $z^1, \ldots, z^m$ one by one until the equilibrium associated with X^0 is obtained, we can derive the increase in the total earnings of activities in X^0 that results from withdrawing $z^1, \ldots, z^m$. Clearly, this increase is given by (50), and from (51) we get the result that the initial rental value of type 1 land, p_1^m, multiplied by m is an upper bound on this increase, and the new equilibrium rental value of type 1 land, p_1^0, multiplied by m is a lower bound on this increase.

The previous analysis deals with the effect of introducing or withdrawing a unit of land or a new activity from a land-use system which is in equilibrium, and the results of this analysis will be used frequently in the following analysis of the benefits from land enhancement. Also important for the analysis of land enhancement is the following theorem which deals with the effect of simultaneously withdrawing a number of activities and the land they occupy from a land-use system which is in equilibrium. Theorem 5 is a generalization of the result that was demonstrated for the case where there was one type of land.

<u>Theorem 5</u>. Assume there are n types of land and a set, X, of activities which compete for the available land. In equilibrium the activities of X are divided among the sets $A_1,\ldots,A_n,C$. Suppose that $\bar{A}_i$ is a subset of A_i $(i = 1,\ldots,n)$, and let $\bar{a}_i$ be the number of activities in $\bar{A}_i$. Further, suppose that the activities in $\bigcup_{i=1}^{n} \bar{A}_i$ are withdrawn from the system, and that $\bar{a}_i$ units of type i land $(i = 1,\ldots,n)$ are also withdrawn. Given the new set of activities, $X' = X - \bigcup_{i=1}^{n} \bar{A}_i$ and the new supply of each type of land, there will be a new equilibrium with a pattern of location described by $A_1',\ldots,A_n',C'$. Then any set of rents, p, which is consistent with the initial equilibrium is consistent with the new equilibrium, and given any set of rents which is consistent with the new equilibrium, then this set satisfies conditions (1.1) to (1.4) for the pattern of location described by $A_i' = A_i - \bar{A}_i$ $(i = 1,\ldots,n)$ and $C' = C$.

Proof: Let p be any set of rents consistent with the initial equilibrium. Then if we define $A_i' = A_i - \bar{A}_i$ $(i = 1,\ldots,n)$ and $C' = C$ we get the result that, given this pattern of location, p satisfies conditions (1.1) to (1.4). This follows from the fact that p is consistent with the initial equilibrium and the obvious fact that if type i land was fully occupied by the activities in A_i, given the initial situation, then type i land is fully occupied by the activities in A_i' in the new situation. Therefore, any set of rents which is consistent with a new equilibrium is also consistent with the new equilibrium where $A_i' = A_i - \bar{A}_i$ $(i = 1,\ldots,n)$ and $C = C'$. From Theorem 1 we get the desired result that any set of rents, p', which is consistent with the new equilibrium satisfies conditions (1.1) to (1.4), given the pattern of location described above, and that p is consistent with the new equilibrium.

This theorem tells us that if a number of parcels of land are withdrawn along with the activities which occupy them, then the pattern of location of the remaining activities will be left unchanged. This, of course, is subject to the qualification that it may be possible to relocate some of the activities that are marginal in the new equilibrium. However, this relocation will not affect the total earnings of the activities in operation. Also the theorem tells us that any set of rents consistent with the initial equilibrium is consistent with the new equilibrium. Note that if all of one type of land is withdrawn from the system, then the rental value of that type of land is not defined in the new situation. With qualification we can state the following corollary to Theorem 5:

Corollary 1: Let the situation described in Theorem 5 exist, and let $\bar{\bar{p}}$ and $\bar{p}$ be the maximal and minimal set of rents, respectively, which are consistent with the initial equilibrium. Then $\bar{\bar{p}}_i' \geqq \bar{\bar{p}}_i$ and $\bar{p}_i' \leqq \bar{p}_i$ $(i = 1,\ldots,n)$, where $\bar{\bar{p}}'$ and $\bar{p}'$ are the maximal and minimal sets of rents, respectively, which are consistent with the new equilibrium. Further, the strict inequality may hold in either case. If p_i', the rental value of type i land in the new equilibrium, is not defined, then $\bar{p}_i'$ and $\bar{\bar{p}}_i'$ are not defined, and therefore the above inequality is not defined for that index i.

Proof: From Theorem 5 we have p consistent with the new equilibrium if it is consistent with the initial equilibrium. By the definition of $\bar{\bar{p}}_i'$ it follows that $\bar{\bar{p}}_i' \geqq p_i$, and therefore $\bar{\bar{p}}_i' \geqq \bar{\bar{p}}_i$. A similar argument will show that $\bar{p}_i' \leqq \bar{p}_i$. The fact that the inequality can hold for the maximum rental values is demonstrated in the discussion of the case where there is one type of land. To demonstrate that the inequality can hold in the case of the minimum rental values, it suffices to return to the previous numerical example and withdraw activity x^1 and type 2 land from that system. In the initial situation the minimum rental value on type 1 land, $\bar{p}_1$, is 9 and the minimum rental value, given the new situation, is $\bar{p}_1' = 4$.

Suppose that, given the new situation, we introduce a new activity onto type i land; then from Corollary 1 and Theorem 3 it follows that the opportunity cost of introducing this activity is greater than or equal to the opportunity cost of introducing this activity onto type i land, given the initial equilibrium situation. Similarly, the increase in the total earnings of all activities that results from introducing an additional

unit of type i land, given the new equilibrium, is less than or equal to the increase in the total earnings of all activities that results if a unit of type i land is introduced, given the initial situation. These implications of Corollary 1 are important for the analysis that follows.

Before proceeding to apply this model to the measurement of benefits from land enhancement, it is important to examine the assumption that all costs are variable, and therefore that an activity will never operate at a loss. Clearly, this assumption is true only in the long run. Therefore, in order for this model to be applicable to the real world we must interpret S_i^x to be the average annual earnings of activity x on the ith type of land, and the initial equilibrium must be interpreted to be a long-run static equilibrium. When this equilibrium is perturbed the process of adjustment leads us to a new long-run equilibrium. Given this interpretation, all results of the previous analysis hold for the changes that accompany the move from one long-run equilibrium to another.

The astute reader will notice that in the model which has been presented some activities earn profits. The question arises as to whether this is compatible with the competitive assumptions on which the model is based. One can find in any basic text on price theory the statement that competition in the long run eliminates profits. This statement depends on the fact that any special factors of production which allow one firm to produce at a lower average cost than another firm are assumed to earn a rent equal to the difference. Thus, the statement that profits are eliminated is a tautology. In general it is not true that if we add up the costs which a competitive firm has to pay for its inputs the total will equal the value of receipts, even in the long run. The fact that one firm may be able to produce at lower average

cost than another may depend on the particular combination of inputs, including organization, which characterizes that firm. If market prices of these inputs equal their opportunity cost in some other firm, it is very likely that total receipts will exceed total costs calculated at market prices. If this is the case, profits are compatible with a long-run competitive equilibrium. Profits are eliminated only when a special factor of production which is employed by one firm can be transferred to another firm where it is equally productive. The classical case of this is where the special factor is high-quality agricultural land.

CHAPTER III

THE MEASUREMENT OF BENEFITS FROM LAND ENHANCEMENT

The analysis in Chapter II provides an analytical framework with which to analyze the benefits from land enhancement, and using the results derived it is possible to answer the questions that have been raised as to whether such benefits represent real gains and, if so, as to the proper measure of the benefits. Before proceeding with the analysis it is important to note that benefits of the land-enhancement type come into existence whenever some land is made economically more desirable. The introduction of flood control is only one of many public programs which produce such an effect. The introduction of a highway, utilities or other public services, urban renewal programs, etc., may all produce benefits in the form of land enhancement. Therefore, the analysis in this paper, while related specifically to the problem of floods, is applicable to a much wider range of problems.

Begin by assuming that there are $n+1$ types of land denoted by $1,\ldots,n,f$, and that there is a set of potential activities, X, which in competitive equilibrium is divided among $n+2$ mutually exclusive sets $A_1,\ldots,A_n,A_f,C$. Associated with this equilibrium is a set of market prices including the set of rents, $p = (p_1,\ldots,p_n,p_f)$. Land in the flood plain, which is assumed to be unprotected from flooding, is denoted by f, and the set of activities which operates in the flood plain, given the existing equilibrium, is A_f. The cost of operating any activity in the

flood plain includes the cost of a flood-insurance premium equal to the expected value of the flood losses of that activity. This premium buys a policy which covers the full cost of any losses from flooding so that the risks of flooding are eliminated for the activity.

Now, suppose a flood-control project is constructed which protects the flood plain and reduces the expected value of flood losses of any activity which locates in the flood plain. In effect, the flood-control project has turned type f land into a new type of land, denoted by f'. Assuming that all market prices other than rents remain constant, the earnings of any activity are determined at each alternative location, and the earnings of any activity which locates in the flood plain will be greater after the introduction of the flood-control project. More formally,

(52) $\quad S^x_{f'} \geqq S^x_f$ for any $x \in X$.

Since the earnings of all activities at a flood-plain location have increased, the introduction of the flood-control project may perturb the initial equilibrium in the market for land and set off a process of adjustment which will lead to a new equilibrium. The set of rents and the location of activities associated with the new equilibrium may differ from the rents and pattern of location obtained before the flood-control works were constructed. Let the location of activities in X, given the new equilibrium, be described by the sets $A'_1,\ldots,A'_n$, $A_{f'}$, C', and let the new set of rents be $p' = (p'_1,\ldots,p'_n,p_{f'})$. All prices other than rents are assumed to be the same in both equilibriums. Suppose that $A'_1 = A_1,\ldots,A'_n = A_n$, $A_f = A_{f'}$, and $C' = C$; i.e., in both equilibriums each activity occupies the same location, and the same activities are in

operation. Clearly, the earnings of any activity, $x \in X$, on type i land $(i = 1,\ldots,n)$ is the same in both equilibriums, and since $A_i' = A_i$ $(i = 1,\ldots,n)$, it follows that the earnings of each activity located off the flood plain are unchanged. Therefore, the total increase in the earnings of all activities as a result of introducing flood protection is given by the sum of the increases in the earnings of all activities located in the flood plain. This corresponds to the case where the benefits of flood protection take the form of a reduction in the cost of flooding to activities which occupy the flood plain. However, it remains to be shown that the total increase in the earnings of flood-plain occupants is the proper measure of benefits from flood protection.

Assume that land is an intermediate good; i.e., individuals derive no utility from land itself but only from goods such as housing and recreation for which land is an input. Since prices other than rents are assumed to be the same in both equilibriums, the prices of all consumer's goods are the same before and after the introduction of flood protection. Therefore, there is a one-to-one correspondence between the size of an individual's income and his level of utility. If an individual's income is greater after the introduction of flood protection by β dollars, then that individual will be as well off in the new situation as he was originally if he pays a lump-sum tax equal to β dollars. Therefore, assuming this individual seeks to maximize his utility, he will willingly pay an amount less than or equal to β dollars to have the project constructed. In benefit-cost terms the benefit of a project accruing to the individual under consideration is β dollars. Similarly, if an individual's income is β dollars less after the introduction of

flood protection than it was in the initial situation, a lump-sum payment of β dollars would be required to make this individual as well off as before. To this individual the introduction of flood protection imposes a cost, or negative benefit, equal to the decrease in his income. Therefore, given the assumption that the prices of consumer's goods remain constant, the benefits from flood protection equal the change in the total income of all individuals.

Because prices of all factors of production except land are assumed to be the same in both equilibriums, and because these factors are fully employed in a competitive equilibrium, the income received by the owners of these factors of production is unchanged by the introduction of flood protection. This leaves us to consider changes in the income from property and from the ownership of activities. The earnings of an activity are defined as the total value of output minus all costs except the cost of land. Suppose an activity operates on type i land, with earnings S_i^x; then the income accruing to owners of that activity is $S_i^x - p_i \geqq 0$, and the rental income received by the property owner is p_i. Therefore, the total income earned on that parcel of land equals the earnings of the activity which occupies it. Suppose a parcel of land lies vacant; then the rental value of that land is zero and there is no income derived from the ownership of this land. As a result, it follows that the total income of entrepreneurs and property owners, as a class, is just equal to the total earnings of all activities. If the total earnings of all activities increase as a result of the introduction of flood control-works, then the total income of property owners and entrepreneurs increases by an equal amount. Given the assumptions of the

problem, the income derived from the ownership of factors of production other than land is unchanged, so that the total increase in income equals the total increase in the earnings of all activities. Applying this result to the case where flood protection reduces the cost of flooding to activities in the flood plain, thereby increasing the earnings of these activities, it follows that the total increase in income equals the total increase in the earnings of activities located in the flood plain.

Given the assumption that all prices other than land rents are the same in both equilibriums, it is clear from the foregoing discussion that the change in the total income of all individuals equals the change in the total earnings of all activities. Therefore, by presenting the analysis in terms of the change in the total earnings of all activities the correct measure of benefits may be obtained. If rental values change in moving to the new equilibrium, there is a transfer of income between property owners and entrepreneurs; however, the amount of the gains just equals the amount of the losses. By considering the total earnings of all activities these transfers of income are automatically accounted for, as all such transfers are cancelled out in the figure for total earnings. By assuming that all prices other than rents remain constant, it follows that all benefits accrue to producers or property owners in the form of increased incomes. It will be subsequently argued that this case is of the most practical importance in connection with flood control and other projects that affect a small percentage of the land in a given region. This is also the case that has been considered by federal agencies charged with the responsibility of measuring the benefits from land enhancement. However, there are important exceptions where benefits from flood control

accrue to consumers in the form of a reduction in the price of a consumer's good, and this case will be analyzed in part subsequent to the case where the prices of all consumer's goods are held constant.

Given the preceding discussion, we can now proceed to analyze the measurement of benefits from the introduction of flood control in the general case where there are benefits of the land-enhancement type. Such benefits are created whenever activities which, in the absence of flood protection, either locate outside the flood plain or do not operate, find it profitable to locate on the flood plain, given protection. Associated with the relocation of these activities is a general relocation of activities that accompanies the move to a new equilibrium, and the problem is to determine the effect of this relocation on the total earnings of all activities. This problem can be solved by beginning with the initial equilibrium and constructing the new equilibrium in steps, and then by measuring the change in earnings at each step. This is done by beginning with the initial equilibrium and constructing a sequence of artificial equilibriums from which the new equilibrium may be obtained.

In order to carry the construction of this sequence it is necessary to modify the notation. Let the pattern of location in the initial equilibrium be described by $A_1^0,\ldots,A_n^0$, A_f^0, C^0, and let the set of rents that obtains in the initial equilibrium be $p^0 = (p_1^0,\ldots,p_n^0,p_f^0)$. Further, let the pattern of location that is associated with the equilibrium which obtains after the introduction of flood control be described by $A_1^4,\ldots,A_n^4$, $A_{f'}^4, C^4$, and let the set of rents associated with this equilibrium be $p^4 = (p_1^4,\ldots,p_n^4, p_{f'}^4)$. Suppose, for definiteness and simplicity, that $x \in A_1^0$ is the only activity locating in the flood plain,

given flood protection, that would not have located in the flood plain in the absence of flood protection. Since it can be demonstrated that the number of activities in $A^4_{f'}$ is at least as great as the number of activities in A^0_f, it follows from the foregoing supposition that at most one of the activities in A^0_f is not in $A^4_{f'}$. If the number of activities in A^0_f equals the number of parcels of land in the flood plain, then, given the assumption that $x \in A^0_1$ and $x \in A^4_{f'}$, it follows that one activity in A^0_f is not in $A^4_{f'}$. Now we make use of Theorem 5. If $A^0_f \subset A^4_{f'}$, we hypothetically withdraw all the activities in A^0_f and the parcels of land which they occupy from the initial equilibrium, and in addition withdraw all vacant parcels of type f land except one. If some activity in A^0_f is not in $A^4_{f'}$, we then withdraw all activities in $A^0_f \cap A^4_{f'}$ and the parcels of land which they occupy, and in addition withdraw all vacant parcels of type f land. From Theorem 5 it follows that the initial set of rents, p^0, is consistent with the new equilibrium, and the new pattern of location is described by $A^1_1 = A^0_1, \ldots, A^1_n = A^0_n$, $A^1_f = A^0_f - A^4_{f'}$, and $C^1 = C^0$. It is important to note that Theorem 5 holds if a parcel of land which is withdrawn is empty. Clearly, $\bar{p}^1 \leqq p^0 \leqq \bar{\bar{p}}^1$, where $\bar{p}^1$ and $\bar{\bar{p}}^1$ are the minimal and maximal sets of rents, respectively, associated with the equilibrium with index 1. The move to this equilibrium from the initial equilibrium is accompanied by a decrease in total earnings equal to the earnings, in the absence of flood protection, of the activities in $A^0_f \cap A^4_{f'}$.

Suppose that the specified activity, $x \in A^0_1$, is now withdrawn from type 1 land, and that a new equilibrium obtains with a pattern of location described by $A^2_1, \ldots, A^2_n$, A^2_f, C^2, and with a minimal and maximal set of

rents, $\bar{p}^2$ and $\bar{\bar{p}}^2$. respectively. The increase in the total earnings of all activities other than x which accompanies the move from equilibrium 1 to equilibrium 2 is equal to $\bar{p}_1^1 \leqq p_1^0$; however, since x is removed from the system, there is a decrease in the earnings of all activities equal to $S_1^x - \bar{p}_1^1$. Now suppose, given equilibrium 2, that the one remaining unit of type f land is now withdrawn from the system and a new equilibrium is established with a pattern of location described by $A_1^3,\ldots,A_n^3$, A_f^3, C^3. Clearly, $A_f^3 = \emptyset$ and p_f^3 is not defined. The decrease in the earnings of all activities which accompanies the move from equilibrium 2 to equilibrium 3 is equal to $\bar{\bar{p}}_f^2$. However, it was demonstrated in Chapter II that $\bar{\bar{p}}_f^2 = \bar{p}_f^1 \leqq p_f^0$.

Let us now examine the final equilibrium which obtains after the introduction of flood control. The set of potential activities is again X, and the supply of each type of land is the same as in the initial situation. The pattern of location is described by $A_1^4,\ldots,A_n^4$, $A_{f'}^4$, C^4, and the set of rents which obtains is $(p_1^4,\ldots,p_n^4, p_{f'}^4)$. By Theorem 5 it follows that if we withdraw all units of type f' land and the activities which occupy this land, then the set of rents $(p_1^4,\ldots,p_n^4)$ will be consistent with the new equilibrium associated with the set of potential activities, $X-A_{f'}^4$, and with supplies of land which are the same as in the initial equilibrium except that the flood plain has been removed from the system. Careful examination will show that this is identical to the situation associated with equilibrium 3. By Theorem 1 it follows that $(p_1^4,\ldots,p_n^4)$ is consistent with equilibrium 3 so that we can assume $(p_1^3,\ldots,p_n^3) = (p_1^4,\ldots,p_n^4)$. At this point we take the equilibrium with index 3 and perturb it by reintroducing the parcels of land in the flood

plain, which are now assumed to be protected, along with the activities in $A^4_{f'}$. By definition, the activities in $A^4_{f'}$ locate on type f' land and only these activities locate on this land. Since $p^3_i = p^4_i$ $(i=1,\ldots,n)$, we can assume $A^3_i = A^4_i$ $(i=1,\ldots,n)$, i.e., the introduction of the land in the flood plain, which is now protected, along with the activities in $A^4_{f'}$, does not cause any activity outside the flood plain to relocate. Therefore, the increase in the total of the earnings of all activities which accompanies the move from equilibrium 3 to the final equilibrium is equal to the total of earnings of all activities which locate in the flood plain after the introduction of flood protection.

We can get the change in the total of the earnings of all activities which accompanies the move from the initial equilibrium to the final equilibrium by summing the changes which accompany each step. Careful inspection will show that this change in the total earnings equals the total increase in the earnings of activities in $A^0_f \cap A^4_{f'}$ plus $(S^x_{f'} - \bar{p}^1_f) - (S^x_1 - \bar{p}^1_1)$. In other words, there is an increase in the earnings of activities which locate in the flood plain before and after the introduction of flood protection, and this increase is equal to the reduction in the expected value of flood losses of these activities. This component of benefits corresponds to the case discussed previously where all benefits took the form of a reduction in the flood losses of activities located in the flood plain. The other component of the increase in total earnings results from the fact that it is now profitable for x to move to a flood-plain location, and this component of benefits is equal to $S^x_{f'} - \bar{p}^1_f) - (S^x_1 - \bar{p}^1_1)$. In its present form this expression is not very useful since $\bar{p}^1_f$ and $\bar{p}_f$ cannot, in practice, be observed.

However, if we assume that rents are restricted within narrow bounds by the conditions of equilibrium, then $\bar{p}_f^1 \approx p_f^0$ and $p_1^1 \approx p_1^0$, so we can express this component of benefits as $(S_{f'}^x - p_f^0) - (S_1^x - p_1^0)$. It also follows that $(S_{f'}^x - p_f^0) - (S_1^x - p_1^0) = (S_{f'}^x - \bar{p}_f^1) - (S_1^x - \bar{p}_1^1)$ under the alternative assumption that $p_f^0 - \bar{p}_f^1 = p_1^0 - \bar{p}_1^1$. In words, the benefits from land enhancement associated with an activity moving into the flood plain as a result of protection are equal to the difference in the earnings of this activity in the two equilibrium situations plus the difference between the initial rental values of the land occupied by this activity in the two situations. It is important that the only rental values contained in this measure of land-enhancement benefits are rental values which obtain in the initial equilibrium, and therefore they can be observed in advance of flood protection and used to estimate the benefits which would accrue if such protection were provided.

In the case just presented, one activity which was located outside the flood plain in the initial situation moved into the flood plain when flood protection was introduced. If, however, this activity had been in C^0 instead of A_1^0, it can easily be demonstrated by eliminating step 2 from the procedure outlined that the land-enhancement benefits are equal to $S_f^x - p_1^0$. Suppose, however, that when flood control is introduced, k activities, which locate outside the flood plain in the absence of this protection, move into the flood plain. Further, suppose that this set of activities, $\bar{A}_1^0$, is a subset of A_1^0. To construct the equilibrium which obtains after the introduction of flood control we begin by withdrawing all the activities in $A_f^0 \cap A_{f'}^0$, along with the parcels of land they occupy, from the initial equilibrium. Since the number of activities

in $A^4_{f'}$ is no less than the number of activities in A^0_f, it follows that at most k activities in A^0_f are not in $A^0_f \cap A^4_{f'}$. If there are less than k activities in A^0_f which are not in $A^0_f \cap A^4_{f'}$, it follows that there are some vacant parcels of type f land in the initial equilibrium. We also withdraw enough of these vacant parcels so that there are exactly k parcels of type f land remaining in the system. A new equilibrium will be established with a pattern of location described by $A^1_1,\ldots,A^1_n$, A^1_f, C^1. By Theorem 5, the set of rents which obtains in the initial equilibrium satisfies conditions (1.1) to (1.4), given the pattern of location associated with equilibrium 1, and therefore $\bar{p}^1 \leqq p^0 \leqq \bar{\bar{p}}^1$.

Now withdraw the activities in $\bar{A}^0_1$ from the system and obtain a new equilibrium with a pattern of location described by $A^2_1,\ldots,A^2_n$, A^2_f, C^2, and sets of maximal and minimal rents, $\bar{\bar{p}}^2$ and $\bar{p}^1$, respectively. From the expression in equation (51) it follows that the increase in the total earnings of all activities in $X - \bar{A}^0_1$ is bounded above by $\bar{p}^1_1 k \leqq p^0_1 k$. Since the activities in A^0_1 have been removed from the system there is a decrease in the total earnings of all activities in the system which is bounded above by $\sum_{x \in \bar{A}^0_1} S^x_1 - p^0_1 k$. It should be noted that an exact expression for the increase in the earnings of the activities in $X - \bar{A}^0_1$ could have been obtained by withdrawing the activities in $\bar{A}^0_1$ one at a time, and by developing an expression analogous to (50). However, this would greatly complicate the notation and would be of little practical value because the rents associated with the artificial equilibrium situations involved can never in fact be observed.

Now withdraw k units of type f land from the system and obtain equilibrium 3 where, as in the previous case, there are no parcels of

type f land in the system. The decrease in the total earnings which accompanies the move is bounded from below by $\bar{\bar{p}}_f^2 k$. It can be demonstrated that $\bar{p}_f^2 \leq \bar{\bar{p}}_f^1 \leq p_f^0$, and by definition $\bar{\bar{p}}_f^2 \geq \bar{p}_f^2$. Therefore, if we assume that $p_f^0 \approx \bar{p}_f^2$, then $p_f^0 k$ will be a lower bound on the decrease in the total earnings associated with the move from equilibrium 2 to 3. Roughly speaking, $p_f^0 k$ will be a lower bound on this value if the effect on the rent of type f land of withdrawing k activities from type 1 land is less than the effect of withdrawing k units of type f land. This condition appears reasonable. Even if $p_f^0 k$ is somewhat greater than this decrease, it will presently be shown that this is to some extent offset in the expression for the measure of benefits from land enhancement.

The rest of the procedure for constructing the final equilibrium is exactly the same as in the previous case. We reintroduce the activities of $A_{f'}^4$, along with land in the flood plain, which is now assumed to be protected, and it can be shown that the activities in operation in equilibrium 3 remain in operation in the final equilibrium at the same location. The increase in the total of earnings which accompanies this move to the final equilibrium equals the total earnings of the activities in $A_{f'}^4$, on type f' land. The increase in the total of earnings which accompanies the entire process of adjustment involved in the move from the initial equilibrium to the equilibrium which obtains after flood control has been introduced is again made up of two components. The first component is the increase in the earnings of activities in $A_f^0 \cap A_{f'}^4$, which represents the reduction in the expected value of flood losses to activities which locate in the flood plain with and without flood

protection. The second component represents the benefits from land enhancement. There has been a change in the earnings of the activities in $\bar{A}_1^0$ which is given by

$$(53) \qquad \sum_{x \in \bar{A}_1^0} (S_{f'}^x - S_1^x) ,$$

and this expression represents part of the total change in the earnings of activities in X associated with the move to the new equilibrium. In order to arrive at the benefit figure from land enhancement we must subtract from (53) an amount no less than $p_f^0 k$ and add an amount no greater than $p_1^0 k$. Therefore,

$$(54) \qquad \sum_{x \in \bar{A}_1^0} (S_{f'}^x - S_1^x) - p_f^0 k + p_1^0 k$$

is an upper bound on the benefits from land enhancement. This argument assumes that $p_f^0 k$ is less than the decrease in the total of earnings which accompanies the move from equilibrium 2 to 3. Even if this assumption does not hold, (54) may still be an upper bound on the benefits from land enhancement if $p_1^0 k$ exceeds by a sufficient amount the increase in the total of earnings of activities in $X-\bar{A}_1^0$ which accompanies the move from equilibrium 1 to 2. In this case, (54) will more nearly approximate the correct measure of benefits from land enhancement. The expression given by (54) can be rewritten as

$$(55) \qquad \sum_{x \in \bar{A}_1^0} [(S_{f'}^x - p_f^0) - (S_1^x - p_1^0)] .$$

Therefore, to get an upper bound on the measure of land-enhancement benefits one would calculate for each activity that moves into the flood plain the profit which that activity would earn if it moved into the flood

plain, given flood protection and the initial rental value p_f^0, and subtract the profit that this activity earns in the initial situation.

In the previous analysis the activities which moved into the flood plain were all located initially on the same type of land; however, in general the activities which will move into the flood plain will come from a number of different locations. This more general case can be analyzed in an analogous manner with the result that, given assumptions similar to those made previously, the component of benefits attributable to land enhancement is bounded from above by the sum $\Sigma(S_{f'}^x - p_f^0) - (S_i^x - p_i^0)$ taken over all activities x which move into the flood plain. For each x, i represents the index of the initial location of this activity.

The point of departure in the foregoing analysis is a static equilibrium which we then perturb by changing the economic characteristics of type f land. However, in many cases flood-control works are undertaken in anticipation of new activities moving into the area. This situation can be represented by supposing that k new activities, $z,\ldots,z_k$, will be introduced into the system which will locate outside the flood plain in the absence of flood protection and on the flood plain, given protection. We could handle this case by introducing the activities $z^1,\ldots,z^k$ without flood protection and then perturbing this equilibrium by introducing flood protection. The disadvantage of this procedure is that we cannot in practice observe the rental values which would obtain if the set $Z = \{z^1,\ldots,z^n\}$ were introduced in the absence of flood protection, and therefore the expression for the benefits from land enhancement would be of little practical value.

However, we can solve this problem as follows. Suppose that in the initial equilibrium the activities in X are divided among the sets $A_1^0,\ldots,A_n^0$, A_f^0, C^0, and the set of rents which obtains is p^0. Now suppose that the activities in Z are introduced into this system and that in the absence of flood protection these activities locate on type 1 land. It follows that the resultant decrease in the total of earnings of all activities in X, E_1, is bounded below by $p_1^0 k$. If $p_1^0 \approx p_1^1$, where p_1^1 is the new rental value on type 1 land, then $p_1^0 k$ is a good measure of E_1. Since the activities in Z have been added to the system, the increase in the total of earnings of all activities is bounded above by

$$(56) \qquad \sum_{j=1}^{k} (s_1^{z^j} - p_1^0).$$

Now we calculate what this increase will be if flood protection is also introduced. Suppose that flood protection were introduced at the same time the activities in Z were introduced into the system. A new equilibrium would be obtained in which the activities in $Z \cup X$ would be divided among the sets $A_1^4,\ldots,A_n^4$, $A_{f'}^4, C^4$, and a set of rents p^4 would obtain. By assumption, $Z \subset A_{f'}^4$. Now suppose we return to the initial equilibrium and withdraw all the activities in $A_f^0 \cap A_{f'}^4$, along with the parcels of land occupied by these activities, from the system. In addition we withdraw enough vacant parcels of type f land, if any exist, so that exactly k parcels of type f land remain in the system. A new equilibrium will be established with a pattern of location described by $A_1^1,\ldots,A_n^1$, A_f^1, C^1. From Theorem 5 it follows that the initial set of rents, p^0, is consistent with this equilibrium. Now we withdraw k units of type f land from the system and obtain equilibrium 3. The decrease in

the total earnings, E_2, which accompanies the move from equilibrium 1 to equilibrium 3 is bounded from below by $p_f^0 k$. At this point we reintroduce the activities in $A_{f'}^4$ and the supply of type f' land into the system to obtain the final equilibrium. The increase in the total of earnings of all activities equals the increase in the earnings of the activities in $A_f^0 \cap A_{f'}^4$ plus

$$(57) \qquad \sum_{j=1}^{k} S_{f'}^{z^j} - E_2.$$

The difference between the increase in the total of earnings when the activities in Z are introduced along with flood protection, and the increase when they are introduced without it, is equal to the increase in the earnings of the activities in $A_f^0 \cap A_{f'}^4$ plus

$$(58) \qquad \sum_{j=1}^{k} (S_{f'}^{z^j} - S_1^{z^j}) + E_1 - E_2.$$

If we assume that

$$(59) \qquad p_1^0 k - E_1 \approx p_f^0 k - E_2,$$

then (58) is approximately

$$(60) \qquad \sum_{j=1}^{k} (S_{f'}^{z^j} - S_1^{z^j}) + p_1^0 k - p_f^0 k,$$

or equivalently

$$(61) \qquad \sum_{j=1}^{k} (S_{f'}^{z^j} - p_f^0) - (S_1^{z^j} - p_1^0).$$

If $A_f^0 \cap A_{f'}^4 = \emptyset$, then all the benefits are of the land-enhancement type and are given by (61).

The question arises as to how this analysis can be applied. Further, there are questions about the types of information which would be needed to

carry out an actual cost-benefit study and about the particular advantages of the present formulation. In order to calculate the benefits from land enhancement, one would need fairly accurate information about what types of activities would move into the flood plain if flood protection were introduced, and in addition one would need to know the rental values of the land occupied by these activities and of land in the flood plain. Further, one would need information about the difference in the earnings of each activity at the two locations. Often this can be measured by the difference in the operating costs, excluding the cost of land, which would be realized at each location. In order to calculate the benefits from land enhancement one would calculate $(S_{f'}^{x} - p_f^0) - (S_i^x - p_i^0)$ for each activity which would move into the flood plain, and sum over all such activities. From the previous discussion it follows that this procedure would give an upper bound on the correct measure of these benefits. There are a number of cases where we may presume this measure is very close to the correct measure. As we have seen, this is true when the activities moving into the flood plain are new to the system. Also, if the number of activities involved in the process of relocation is a small percentage of the activities on any given type of land, then the changes in the rental values that accompany steps 2 and 3 will probably be small, which will give us the desired result.

There are a number of advantages to this approach to the problem. First, and probably most important, the model on which the present measure is based is a general equilibrium model, and all the effects of the readjustment which takes place as a result of changing the economic characteristics of type f land are accounted for. Second, the measurement

procedure makes use only of rental values which obtain in the initial equilibrium, which means that the needed information can be obtained in advance of the introduction of the project, and therefore can be used in the evaluation procedure. This eliminates the need to evaluate what future rents would be given the project. Incidentally, if it were possible to accurately estimate the rents that would obtain after the system was perturbed, then one could get a perfectly accurate measure of the benefits from land enhancement, using the present procedure. This would be done by carrying out steps 2 and 3 by withdrawing the activities and parcels of land involved one at a time, and by making use of the results summarized in (50). Finally, as a result of Theorem 3 and its extensions, one does not have to calculate changes in land values, and therefore profits, throughout the system. This relates to the previous point, as all the necessary information is contained in the initial set of rental values.

However, given the advantages of this approach, there remain a number of practical difficulties. First, in order to estimate the benefits from land enhancement one needs to know which activities would move into the flood plain. This may be difficult to determine, and it could be argued that before one could determine which activities would locate in the flood plain, given protection, one would have to know the set of rents which would obtain in the new equilibrium. If this were the case, then the problem of determining the new set of rents in advance of the project would still be with us, only under a different guise. While it is true that the rental values which obtain in the new equilibrium are intimately connected with pattern of location, in many cases one may be

able to determine the general pattern of location without knowing the precise set of rents that will obtain. For example, one may be able to predict that an area will be used for apartments, offices, industrial uses, etc., without being able to specify the precise rent that will obtain. While the problem of land-use forecasting is difficult, I believe it is less difficult than forecasting future land values under these conditions.

Second, there is the problem of determining where an activity will come from which moves into the flood plain. The question arises as to whether such an activity moved from a type i location or whether it is new to the system. In practice one has no way of distinguishing between these two situations; however, it turns out that this is not a serious problem. In order to calculate the benefits from land enhancement, first determine which types of activities will occupy the flood plain if flood control is introduced. Then determine the difference between what the earnings of these activities would be on the flood plain and at the best alternative location outside the flood plain, assuming that the present rents obtain and that there is flood protection. Then if we take the sum of these differences we get the desired measure, which is the same in both cases.

There are two cases where the expression for the measure of benefits takes a particularly simple form. Consider the case where the activities which will move into the flood plain if flood control is provided are new to the system. In the absence of flood protection these activities will locate outside the flood plain at some alternative location. Suppose both the land in the flood plain and land at the alternative location are vacant, so that $p_f^0 = p_1^0 = 0$. Then from (60) it follows that the benefits

from land enhancement are given by $\sum_{j=1}^{k} S_{f'}^{z^j} - S_1^{z^j}$; i.e., these benefits are measured by the difference in the earnings of these activities if they locate on the flood plain, given flood protection, and their earnings at the best alternative location. One can imagine such a situation arising in cases where an urban area is growing and the alternative is to build in the flood plain or on the vacant land on the outskirts of the city. The second case is where, with flood protection, the earnings of the new activities which would move into the flood plain, given flood protection, are the same on the flood plain as at the best alternative location outside the flood plain; i.e., in equation (59), $S_{f'}^{z^j} - S_1^{z^j} = 0$ $(j = 1,\ldots,k)$. In this case the benefits from land enhancement are equal to $(p_1^0 - p_f^0)k$, or, in other words, the difference between the initial rental value of land in the flood plain and the initial rental value of land at the alternative location multiplied by the number of parcels of land that are involved. Since the price of a parcel of land under competitive conditions is equal to the discounted value of the stream of rents, it follows that if the benefits of flood protection accrue forever, then in the previous case the present value of benefits is

$$(62) \qquad (\rho_1^0 - \rho_f^0)k,$$

where ρ_1 and ρ_f are the current prices of type 1 and type f land, respectively. Even if the appropriate time horizon is not infinite, if the benefits accrue over a sufficiently long period of time (62) gives a good approximation of the benefits in this case.

In practice, however, there are some hazards involved with using the price of land as the discounted value of the stream of rents. In our model we assume that the rate of interest is the same in all markets and is equal to the social rate of discount. In fact, rates vary in the economy and are different for different individuals. As a result the rate which a buyer or seller of a piece of property uses in determining the value of that property may differ from the rate considered appropriate from a social point of view. In addition, there is the difficulty that the price of land depends on what people expect the rental value of this land to be in the future. These expectations may vary greatly and may not reflect what the land will be worth if present conditions continue into the future. For example, vacant land in a flood plain may be priced on the basis of the expectation that flood protection will, in fact, be provided in the future. From the previous analysis it follows that the rental value of a parcel of a given type of land cannot exceed the earnings of the least profitable activity which occupies this land, and this may be a good surrogate in cases where the rental value cannot be observed.

Given the previous discussion and the theoretical framework developed in Chapter II, we can now analyze the procedures which federal agencies are instructed to use in measuring the benefits from land enhancement. Senate Document 97, which contains the most current statement of federal planning practices in the water-resources field, states that the "increase in the net return from higher use of property made possible as a result of lowering the flood hazard" should be included in the benefits from flood control [14,10]. An expanded version of this statement can be found in the "Green Book," which is the predecessor of Senate Document 97.

It states:

> The benefit resulting from changes in use of property made possible by flood control should be measured as the increase, in excess of the estimated reduction of flood damage, in the net income of the affected property under conditions expected with and without flood control. . . .
>
> As an alternative method, an approximation of the difference in net return from more intensive use may be made by estimating the increase in market value of the affected property and converting it to an average annual basis by applying a rate of return applicable to private investment in the type of activity involved, adjusted for flood reduction benefits.
>
> . . . . . .
>
> Under either method, the associated costs (i.e., all costs other than project costs) necessary to increase the net return of the property must be deducted to obtain the amount of benefit attributable to the project.
>
> . . . . . .
>
> When flood control results in both prevention of flood damage and change in land use on the same piece of property, care must be taken to avoid double counting of the benefit. In such cases, the entire benefit may be measured as the increase in net income from the property with and without the project or part of the benefit may be measured as flood damage prevention and the remainder as a benefit of more intensive use [13,39].

From the preceding statements it is clear that the benefits from land enhancement are to be measured as the increase in the net income of property in the flood plain. To arrive at this figure, associated costs are subtracted from the increase in receipts, where associated costs include all the costs of operating an activity except the rental value of land. Therefore, the land-enhancement benefit associated with a given

parcel of land in a given year corresponds to the difference in the earnings, exclusive of the rental value of land, of the activities which would occupy that parcel with and without flood protection. If for some reason the difference in net earnings cannot be measured directly, it is suggested that the change in the rental value of the land is a reasonably good approximation of the change in net earnings.

If an activity locates in the flood plain before and after the introduction of flood control, then the correct measure of the benefits from flood protection associated with the parcel of land which this activity occupies is the increase in the earnings of this activity. In most cases this increase may be considered equal to the reduction in the cost of flooding to the activity. With respect to this case, the present analysis and the statement of federal procedures are consistent. Suppose, however, that a given parcel of land is occupied by one activity before flood control is introduced and by a second activity when flood control is provided. The measure of benefits used by federal agencies would be the difference between the earnings of the second and the first activity. Suppose x is an activity which locates off the flood plain in the absence of flood protection, say on type 1 land, but which moves into the flood plain when flood control is introduced. Further, suppose that y is an activity which locates in the flood plain in the absence of flood protection, but which is displaced by x when flood control is introduced and either moves to a location outside the flood plain or shuts down. According to federal practices, the benefits associated with activity x moving into the flood plain and replacing activity y as a result of the introduction of flood control is measured by $S_{f'}^{x} - S_{f}^{y}$. To get the total

of benefits of the land-enhancement type, the sum is taken over all such activities which move into the flood plain. It is easily shown that this measure of benefits may be greater than or less than the correct measure of the benefits from land enhancement. First, it is possible that $S^x_{f'} < S^y_f$ so that $S^x_{f'} - S^y_f < 0$. This is clearly incorrect, as it has been demonstrated that there are positive benefits associated with land enhancement. The foregoing result might arise where a very profitable activity was displaced by a less profitable one because the more profitable activity, y, is equally profitable at some alternative location. This is analogous to the case where an activity may be marginal to a given type of land and still be earning a substantial profit. The measure of the benefits associated with an activity moving into the flood plain which was developed previously is $(S^x_{f'} - p^0_f) - (S^x_1 - p^0_1)$. Now $S^x_{f'} - S^y_f \geqq (S^x_{f'} - p^0_f) - (S^x_1 - p^0_1)$ if, and only if, $S^y_f \leqq S^x_1 + p^0_f - p^0_1$. There is no reason why this inequality could not hold so that theoretically $S^x_{f'} - S^y_f$ could grossly overstate the benefits involved. Thus, in general, the method for computing the benefits from land enhancement which is suggested for use by federal agencies is simply incorrect. The benefits may be understated or overstated, depending on the circumstances. There is one important case where the measure suggested for use by federal agencies is identical to the measure developed in the preceding analysis. Suppose that the economy is in a long-run equilibrium and that, in fact, all profits are eliminated. In this case, $S^x_1 - p^0_1 = 0$ and $S^y_f - p^0_f = 0$. From this it follows that $S^x_{f'} - p^0_f) - (S^x_1 - p^0_1) = S^x_{f'} - p^0_f = S^x_{f'} - S^y_f$.

The alternative measure of benefits which is suggested for use by federal agencies is the increase in the value of land in the flood plain,

i.e., $(P^4_{f'} - p^0_f)$, multiplied by the number of parcels of land. This case is of particular importance because the change in the value of land is often held to be the correct measure of benefits when, as the result of some program, the economic properties of a given type of land are improved. It can be easily demonstrated that in some cases this measure will understate the benefits that accrue as a result of flood protection. In other cases it will overstate the benefits. Suppose that, in the absence of flood protection, land in the flood plain is not fully occupied. Further suppose that when flood control is introduced new activities move into the flood plain and the earnings of activities which remain located in the flood plain are increased. If we assume, given the new equilibrium, that there are still vacant parcels of land in the flood plain, then $p^4_{f'} = p^0_f = 0$. In this case, if the change in land values is used as a measure of benefits, the benefits will clearly be understated. Now suppose instead that there is only one type of land in the flood plain, and that the activities in $X = \{x^1,\ldots,x^m\}$ compete for the n parcels of land, $m > n$. The earnings of each activity in the absence of flood protection is given by $S^{x^i}_f$ $(i = 1,\ldots,m)$, and $S^{x^i}_f = \ldots = S^{x^{n-1}}_f > S^{x^n}_f = \ldots = S^m_f$. Clearly, in the initial equilibrium the total earnings of all activities equals $(n-1)\, S^{x^1}_f + S^{x^n}_f$ and the rental value is $p_f = S^{x^n}_f$. Now suppose that flood control is introduced and that $S^{x^1}_{f'} = \ldots = S^{x^{n-1}}_{f'} = S^{x^n}_{f'} = \ldots = S^m_{f'}$. The new rental value, $p_{f'}$, will equal $S^{x^n}_{f'}$, and the increase in the total of earning of all activities which accompanies the introduction of flood protection is equal to $S^{x^n}_{f'} - S^{x^n}_f = p_{f'} - p_f$. However, the increase in the rental value of land is $n(p_{f'} - p_f)$, which grossly overstates the benefits.

There is one important case where the increase in the earnings of all activities equals the change in land values. Suppose all the activities which occupy the flood plain are identical and that $S_f^x - p_f^0 = 0$ for each activity, x, in the flood plain. Now suppose that when flood control is introduced the same set of activities occupies the flood plain and that each activity now earns $S_{f'}^x > S_f^x$. If, in the new equilibrium, profits are eliminated, i.e., $S_{f'}^x - p_{f'} = 0$, then $S_{f'}^x - S_f^x = p_{f'} - p_f$ for each x in the flood plain. If we take the sum, $\Sigma(S_{f'}^x - S_f^x)$, over all x which locate in the flood plain, we get the correct measure of the benefits. This equals $(p_{f'} - p_f)$ multiplied by the number of parcels of land in the flood plain.

So far we have assumed that in moving from one equilibrium to another all prices other than land rents remain constant. By structuring the problem in this way all benefits accrue to the owners of activities and property. It is interesting to note that the assumption of constant prices is implicit in the procedure for calculating benefits suggested in the "Green Book" and in Senate Document 97. There, benefits are to be measured by the increase in the earnings of activities in the flood plain. Clearly, this measure would be incorrect if a decrease in the price of products produced in the flood plain offset the savings in costs which resulted from flood protection. The assumption that prices other than rents remain constant simplifies the problem of measuring benefits, as the problem of measuring consumer's surplus does not arise. The question which remains, however, is how well the assumption of constant prices fits the conditions of the problem. If the activities which occupy the flood plain sell their output in national or large regional markets, the

effect of a change in their costs is not likely to change the price of their output. Also, if these activities purchase their inputs in large regional markets, any adjustment in the demand for factors by these activities will have a negligible effect on factor prices. However, there is one particularly important case where these conditions do not hold, and a price other than rents can be expected to change. This case is exemplified by housing, where the price is significantly affected by the value of land in any locality.

The problem of calculating benefits when a number of prices change is exceedingly difficult because of the problem of dealing with the substitution that takes place. One of the achievements of the foregoing analysis is that the substitution effects in the markets for land are accounted for in measuring the benefits from land enhancement. However, in a number of cases we can reasonably assume that there are no substitution effects, and in such cases we get the same measure of benefits as before. Suppose, as is the case with housing, that land is a variable input in producing the given output. Further suppose that, given flood control, activities relocate and rents change and that, as a result of the fact that the value of some land decreases, the price of a given product also decreases. If we assume that the quantity purchased of this product is the same in the new equilibrium as in the initial equilibrium, then the saving to consumers just equals the reduction in total earnings below what they would have been if the price of the product had remained constant.

If we assume that the increase in real income to the consumers who benefit from this decrease in price does not affect the prices of other goods or the earnings of other activities, then the previous measure of

benefits is the correct one. This would be the case if other commodities were produced at constant marginal cost within the relevant range. In this case, the correct measure of benefits would be the total increase in earnings of all activities, with the specification that the earnings of each activity are calculated on the basis of the initial set of prices. This takes care of the most important case where a price changes as the result of the introduction of flood control. It is probably not unreasonable to assume that the demand for a product such as housing is, in a given area, fixed. Since the measurement of benefits is at best imprecise, it does not seem that the foregoing assumption will introduce an intolerable amount of error into the process of measurement.

CHAPTER IV

FLOOD INSURANCE AND THE COST OF RISK-BEARING

In the preceding chapters it is assumed that each occupant of the flood plain has the opportunity to purchase a flood insurance policy, providing full coverage, for a premium equal to the expected value of his flood losses. In addition, it is assumed that individuals are risk averters and, therefore, everyone who locates in the flood plain purchases such a policy. As a result of these assumptions, it is possible to eliminate risk from the discussion in Chapters II and III. In addition, it is suggested in Chapter I that a program of flood insurance may be an effective and relatively inexpensive method of eliminating the risks associated with flooding. In this chapter we examine in some detail the questions of how such an insurance scheme might be designed, and how the total costs of risk-bearing would be affected. In addition, some of the practical problems that might arise in connection with flood insurance are discussed.

To begin, suppose that the flood losses of everyone affected by flooding are independent random variables which are identically distributed, and let the random variable X, with mean μ and variance σ^2, represent the losses of any individual. Given these circumstances, the amount of risk borne by any individual can be made very small by an arrangement whereby the risks are pooled. Under such an arrangement the cost of flooding to any individual in a given year would equal the total of flood losses in that year divided by n, where n is the number of

individuals participating in the pool. It is easily demonstrated that under this arrangement the expected value of the cost of flooding to any individual is μ, and the variance of this cost is $\frac{\sigma^2}{n}$. If n is large, then, although the expected cost of flooding remains the same, the risk to each individual is largely eliminated. In reality the losses of all individuals are not identically distributed, and each individual's contribution to the pool would be proportional to his expected losses. Under such an arrangement, risk, for the most part, can be eliminated without costs other than transaction costs. This arrangement would be, in all essential respects, like a mutual insurance company, which differs from a program where each individual purchases an insurance policy for a fixed premium.

The effect of either pooling or insurance on the position of the flood-plain occupant would be similar. Given either arrangement, the expected value of the cost of flooding remains the same; the variance of this cost to the individual is made very small by pooling, and is eliminated by the purchase of a policy providing full coverage for a fixed premium. In the limiting case where n approaches infinity, the effects of pooling and of insurance are the same from the point of view of the flood-plain occupant.

It is assumed above that the flood losses of each individual are statistically independent, and this assures that risk can be significantly reduced through pooling. In reality, individual losses are not statistically independent, and there may be high positive correlation between the losses of any two individuals. For example, in any given flood everyone in the flood plain is flooded. More generally, river-basin

systems are interconnected, and flood losses in one part of the basin may exhibit a high positive correlation with losses in other parts of the basin. In cases where such a correlation exists it may not be possible to significantly reduce the risk of each individual through a mutual insurance company. More empirical research is needed to determine whether such a program would be an effective way of coping with the risks of flooding; at the same time it is important to investigate other types of insurance programs which would be applicable even where flood losses are highly correlated.

In the case where flood losses are pooled, the only individuals involved are those who bear the risks of flooding; therefore, such a program imposes no costs on individuals outside the flood plain. Now consider an insurance program providing full coverage of flood losses. Under such a program, the individual flood-plain occupant bears no risk; however, this risk is now transferred to the underwriters of the policy. Assuming these are risk averters, they will voluntarily accept this risk only if the premium equals the expected value of losses plus a loading charge. The question investigated here is whether it would be possible to construct a program of insurance where this loading charge can be made very small, thereby eliminating risk-bearing for each flood-plain occupant for a cost equal to the small loading charge. The risk is now borne by the individuals underwriting the insurance; however, they are fully compensated for bearing this risk.

One might consider an alternative program of insurance where each policy is sold for a premium equal to the expected value of losses. In this case, risk would be eliminated without cost to the flood-plain

occupant; however, the individuals who underwrite this insurance now bear the risk and costs associated with it. This situation might occur if the government were to sell flood insurance for a premium equal to the expected value of flood losses and include receipts and claims in general revenues and disbursements. One might suppose that the budget would be balanced so that if claims exceeded receipts from premiums, taxes would be raised; if receipts exceeded claims, taxes would be lowered. In this case, each taxpayer would bear part of the cost of the risk. The question investigated here is whether the total cost of risk-bearing will become very small if the risk is spread among a large number of people. It will be shown that if the number of people with assets subject to the hazards of flooding is a small percentage of the total population, then by spreading the risk among all individuals the total cost of risk-bearing will be very small. Similarly, under these conditions it is possible to design an insurance scheme where the insurance can be underwritten for a premium which includes a small loading charge, and yet to have the underwriters fully compensated for the risks they bear.

Suppose that each individual's utility is a function of his income and is given by $U(Y)$. Further, assume that U is a bounded function and is an increasing function of income so that $U'(Y) > 0$. Assume also that, given a choice among alternatives, the consequences of which are uncertain, the individual selects that action for which the expected value of his utility, $E[U(Y)]$, is maximized. In addition, we assume that the individual is a risk averter so that $U''(Y) < 0$. The utility function simply gives a preference ordering; however, it can be shown that the utility function is defined only up to positive linear transformations,

so that regardless of the particular function, U, selected to represent this ordering of preferences, the signs of the first and second derivatives will be the same.

Now suppose that an individual's income is a random variable, A. In addition, suppose that X is a random variable, independent of A, which represents the profits of an insurance company. The profits may be positive or negative. Each individual is given the option of becoming a stockholder free of charge, subject to the restriction that his share of any losses can be covered by his component of income represented by A. This assumption is not of great practical importance for this case; however, it eliminates the difficult analytical problem of accounting for situations where individuals may default on their obligations. The income of an individual can be written as $Y = A + aX$, where a represents the percentage of the total stock of the insurance company which the individual chooses to hold. The question arises as to the circumstances under which a will be positive [1,38-41].

Let $W(a) = E[U(Y)] = E[U(A + aX)]$, so that by the assumption that the individual seeks to maximize the expected value of his utility it follows that he will choose a so that $W(a)$ is maximized. Clearly, $W'(a) = E[U'(Y)X]$ and $W''(a) = E[U''(Y)X^2]$. Since $U''(Y) < 0$, it follows that $W''(a) < 0$. $W(a)$ has a maximum at $a = 0$ if, and only if, $W'(0) \leqq 0$. However, if $a = 0$, then $Y = A$ and $U'(Y) = U'(A)$, and $W'(0) = E[U'(A)X]$. Since A and X are independent random variables, $W'(0) = E[U'(A)]\,E[X]$, so $a = 0$ if, and only if, $E[X] \leqq 0$. Thus, unless the insurance company charges the insured a premium equal to the expected value of their losses, plus a positive loading charge, no

individual will choose to accept a share in the company. On the other hand, if $E(X) > 0$, then $a > 0$. This says that if the value of receipts exceeds the expected value of claims by any amount, however small, then each individual will choose to hold some share in this insurance arrangement. The exact value of a for a given individual will in general depend both on his utility function and on the distribution of component A of his income. If A were known with certainty, it could be shown under reasonable assumptions that a increases with A. This model might have been formulated more appropriately in terms of wealth rather than income; however, the result is the same in either case.

This result can now be applied to the case of flood insurance. Suppose that flood insurance is offered at a premium slightly greater than the expected value of flood losses. Then the total of receipts will be greater than the expected value of claims. It follows that each individual whose income is not subject to fluctuations because of flooding will accept a share in this insurance arrangement. If the number of these individuals is large, and if their incomes are large, then the sum of the a's will be greater than or equal to one. In this case, an insurance arrangement can be worked out where the loading charge paid by each policyholder is very small, and where the shareholders in the insurance company are at least as well off as before the company was organized. Roughly speaking, such an arrangement is possible if the number of individuals affected by flooding and the total of incomes subject to the hazards of flooding are small relative to the total population and the total of all incomes, respectively.

Now suppose that instead of charging an insurance premium greater than the expected value of flood losses a premium is set which equals

expected losses. In this case, $E(X) = 0$ so that no individual would choose to hold any part of the risky asset X. From this it follows that $W'(0) = 0$, or equivalently that

$$\lim_{a \to 0} \frac{E[U(A) - U(A + aX)]}{a} = 0 .$$

Now let $a = \frac{1}{n}$, so that the previous expression becomes

$$\lim_{n \to \infty} nE[U(A) - U(A + 1/n\ X)] = 0 .$$

There exists a unique number, $k(n) > 0$, for each value of n, such that

$$E[U(A + 1/n\ X)] = E[U(A - k(n)]$$

or, in words, an individual would be indifferent between paying an amount equal to $k(n)$ and accepting the asset represented by $1/n\ X$; $k(n)$, therefore, is said to be the cost of risk-bearing associated with the risk of holding the asset $1/n\ X$. It can be easily demonstrated that

$$\lim_{n \to \infty} k(n) = 0$$

or, in other words, the cost of holding the risky asset goes to zero as the amount of this asset which is held goes to zero. From the previous results it follows that

$$\lim_{n \to \infty} nE[U(A) - U(A - k(n)] = 0 .$$

However,

$$\lim_{n \to \infty} \frac{E[U(A) - U(A - k(n))]}{k(n)} = E[U'(A)] > 0$$

and, therefore, it follows that

$$\lim_{n \to \infty} n\ k(n) = 0.$$

This result can be interpreted as follows: Suppose that all individuals are identical in the sense that they have identical utility functions and the same certain income, A. Suppose in addition that there is a certain amount of risk represented by the random variable X, $E(X) = 0$, which has to be borne by someone. The cost of risk-bearing to any individual is defined above. Now suppose that the risk is divided equally among n people; i.e., each individual holds the asset $1/n\ X$. Then the total cost of risk-bearing is $n\ k(n)$. It follows from the result derived above that if n is large, then $n\ k(n)$ is very small. Therefore, given a certain amount of risk, the total cost of risk-bearing can be made very small by spreading this risk over a large number of individuals. This, of course, ignores the transfers of income that will occur in the process of spreading the risk.

Again consider the case of a flood insurance plan where the government sells flood insurance for a premium equal to the expected value of flood losses. The risk is now transferred to the taxpayers so that the effect of such a program is to take the risk borne by a few individuals who face flood hazards and spread it among the whole population. It follows that the total cost of risk-bearing becomes very small although there has been a small transfer of income from the general taxpayer to those individuals who buy flood insurance. However, for the purposes of benefit-cost analysis this transfer can be ignored. Therefore, a program of flood insurance sponsored by the government, where flood insurance is sold for a premium equal to expected flood losses, would essentially eliminate the cost of risk-bearing. If it were desired that any cost of risk-bearing that remains be paid by the flood-plain occupants, then a

small loading charge could be included in the premium.

The discussion to this point has assumed that in any pooling arrangement or insurance scheme the losses of any individual affected by flooding are fully covered. This assumption is necessary if the effect of flood insurance on risk is to be comparable to the effect of structural measures which completely eliminate flooding. In practice it may be difficult to achieve this objective. In the first place, there is the difficulty of identifying flood losses when they take the form of a loss of income. In this case, there is the problem of estimating the income that an activity or individual would have earned in the absence of flooding. Related to this is the problem of estimating the losses incurred by individuals and firms outside the flood plain. In this case, there is the problem of identifying the individuals and firms affected and then estimating the loss of income which is the result of flooding. In this paper these questions can only be raised, as it will take some experience with such programs to determine the feasibility of providing full coverage.

Related to the problem of estimating losses is that of establishing the correct premiums. In order to establish the correct premiums one needs information about the expected value of flood losses. While one can get fairly accurate information on the probability of floods of various sizes, it is difficult to relate this to losses. Losses of property caused by flooding may be reasonably easy to estimate, given the knowledge that a flood of a certain size occurs; however, it is much more difficult to estimate losses due to interruption of economic activities on the basis of this information.

In practice, any program of flood insurance will probably not provide complete protection, and the premium will not bear exactly the desired

complete protection, and the premium will not bear exactly the desired relation to the expected value of losses. If one wishes to recapture all the costs of providing the insurance—including a return to the individuals who underwrite the insurance—then the premium will be set at a level clearly above the expected value of losses. This is essentially the procedure used by private insurance companies. Of course, the premium cannot be set too high or no one will buy insurance. On the other hand, if the government runs the program, the premium might be set equal to the most reliable estimate of expected losses, with the government absorbing any difference between this estimate and the true value of expected losses.

In the previous discussion it has been implicitly assumed that there were no transaction costs associated with the provision of flood insurance. In practice, these costs must be considered, as they may be a significant part of the total cost of the premium. There is reason to believe that if flood insurance is made mandatory, then the transaction costs will be significantly reduced. First of all, if insurance were mandatory, the costs associated with selling it would be greatly reduced. Second, there are probably economies of scale in the estimation of flood losses. The topographical and hydrologic information necessary for the estimation of flood losses and, therefore, for the determination of premiums has to be collected whether one hundred or several thousand policies are sold. Thus, in addition to the reasons established in Chapter I for making flood insurance mandatory, the reduction in transaction costs provides cause why this should be considered.

BIBLIOGRAPHY

[1] Arrow, Kenneth J., Aspects of the Theory of Risk-Bearing. Yrjö Jahnsson Lectures. Helsinki: Yrjö Jahnssonin Säätiö, 1965.

[2] Dantzig, George B., Linear Programming and Extensions. Princeton: Princeton University Press, 1963.

[3] Eckstein, Otto., Water Resource Development: The Economics of Project Evaluation. Cambridge: Harvard University Press, 1958.

[4] Ford, L. K., and D. K. Fulkerson, Flows in Networks. Princeton: Princeton University Press, 1962.

[5] Hoyt, William G., and Walter B. Langbein, Floods. Princeton: Princeton University Press, 1954.

[6] Kates, Robert William, Hazard and Choice Perception in Flood Plain Management. Department of Geography Research Paper no. 78. Chicago: University of Chicago Press, 1962.

[7] Koopmans, Tjalling C., and Martin Beckmann, "Assignment Problems and the Location of Economic Activities," Econometrica, XXV (January, 1957), 53-76.

[8] Krutilla, John V., "An Economic Approach to Coping with Flood Damage," Water Resources Research, II (Second Quarter, 1966), 183-90.

[9] Langbein, Walter B., "Annual Floods and the Partial Duration Series," Transactions of the American Geophysical Union, XXX (1949), 879-81.

[10] Langbein, Walter B., and William G. Hoyt, Water Facts for the Nation's Future. New York: Ronald Press, 1954.

[11] Savage, Leonard J., The Foundations of Statistics. New York: John Wiley and Sons, 1954.

[12] Sheaffer, John Richard, Flood Proofing: An Element in a Flood Damage Reduction Program. Department of Geography Research Paper no. 65. Chicago: University of Chicago Press, 1960.

[13] United States Inter-Agency Committee on Water Resources, Sub-Committee on Evaluation Standards, Proposed Practices for Economic Analysis of River Basin Projects. Revised report, May, 1958. Washington: Government Printing Office, 1958.

[14] United States Congress, Senate. Policies, Standards, and Procedures in the Formulation, Evaluation, and Review of Plans for Use and Development of Water and Related Land Resources. Report by the President's Water Resources Council, 87th Cong., 2nd Sess., S. Doc. No. 97, May 29, 1962. Washington: Government Printing Office, 1952.

[15] White, Gilbert F., Choice of Adjustment to Floods. Department of Geography Research Paper no. 93. Chicago: University of Chicago Press, 1964.

[16] White, Gilbert F.(ed.), Papers on Flood Problems. Department of Geography Research Paper no. 70. Chicago: University of Chicago Press, 1961.

[17] White, Gilbert F., et al., Changes in Urban Occupance of Flood Plains in the United States. Department of Geography Research Paper no. 57. Chicago: University of Chicago Press, 1958.

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This paper deals with the problem of assessing the benefits from various types of flood protection. The first chapter contains a discussion of the nature of the benefits from flood protection and relates the various types of benefits to five different measures for coping with flood losses. The second chapter develops a theory of land use as it relates to the structure of rents. This theoretical framework is used in Chapter III to develop the proper measure of benefits from "land enhancement." The final chapter deals with the effects of alternative programs of insurance on the cost of risk-bearing associated with flooding.

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