PLANNING AND BUDGETING
IN A MULTIPHASE R&D PROCESS

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William Larimer Mellon, Founder
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Efforts to understand, plan and control research and development activities require recognition of differences and relationships among various stages of the research and development process. Often, R&D programs in industrial or governmental laboratories encompass a range of activities, extending from basic research directed to understanding natural phenomena to engineering development of specific products or processes designed to improve the performance of the organization as a whole. Management problems of allocating resources to individual projects differ from straightforward capital investment procedures because of loose coupling among groups of these projects, differences in the end purposes of project results, and the importance of the process by which project alternatives are generated relative to the process of selection from among a given, predetermined set of investment opportunities. Several characteristics of research and development activities contribute to difficulties in design and implementation of management science techniques to aid R&D managers in solving their resource allocation problems. The emphasis in this paper is on one class of such characteristics—interdependencies among projects.

This paper develops a framework for exploring certain dimensions of R&D project selection and budget allocation. The framework includes specification of decision and information flow logic for the identification of project alternatives and on integer programming formulation for selecting an optimum portfolio of projects from among the set of alternatives identified by the project identification.

Without engaging in the usual semantic arguments as to what is "research"
and what is "development" we shall establish a simple dichotomy, defining these terms as they will be used here. Projects may be of two types:

1. "Research" - in which results take the form of information about phenomena, properties, or of technical concepts which increases the technical feasibilities of successfully completing one or more subsequent development projects.

2. "Development," - which uses research results as inputs and yields designs of processes, products, systems and procedures applicable to direct improvement of the performance of the organization which sponsors the "R&D".

In the next section, the resource allocation problems of R&D managers are related to the decision process model developed in the paper. Then, specific types of project interrelationships treated by the model are specified in detail. Third, a mechanism is described for the identification of project alternatives. Fourth, the procedures for evaluating the feasibility of development project proposals and the coupling of research proposals to development activities are outlined. The fifth section of the paper discusses the integer programming formulation for optimal allocation of a total budget among R&D projects. Section six presents implications and conclusions.

**Similarities to Resource Allocation Problems of R&D Managers**

The portrayal of the decision process presented here is not intended to represent an "actual" R&D management process. However, there are situations in governmental and industrial R&D laboratories where directed, mission oriented research projects can be related to subsequent development
projects which employ research results. Frequently, the research program is planned, and its performance is appraised, in terms of how well it will support and make feasible engineering development of products and processes which have direct dollar payoffs to the firm. Usually, there is not a one-to-one correspondence between "R" projects and "D" projects in these circumstances. Rather, an individual research activity may be of such broad scope that its results are information inputs to several ongoing or potential development efforts. This potential for carryover, or joint effects, frequently is a key factor in evaluation and selection of research projects. The hypothetical multiphase process which is subjected to analysis here incorporates some of the significant features of these types of interrelationships among projects.

It will be useful to take a closer look at the actual laboratory management problem which is to be abstracted in the context of this analysis. The first resource allocation problem of management is to choose a portfolio of development projects. Presumably the selection responds to opportunities and problems perceived by the organization, taking into account estimated costs, development time, life span of benefits, and magnitude of benefits of the end-product of the development activity, and probabilities of success of the development project itself. Choice of a development project portfolio implies selection of related research projects. A second management problem therefore is to allocate R and D resources among research and development projects in such a way that investment in increasing the feasibility of development project implementation is balanced against investment to carry out development project implementation, given that feasibility has been established.
A third resource allocation problem is posed by the time lag between research investment and achievement of feasibility of development investment. The R&D manager must make a time phased sequence of investments, allocating funds to research in the current time period with the expectation that this allocation generates opportunities for future investment in development, necessary to exploit research results carryout development projects, and obtain payoffs for the organization.

Another management issue which is highlighted by the formulation of a model which attempts to specify significant features of a multistage research and development process is the problem of generating and testing program alternatives. The program generation and evaluation problem may be triggered by a requirement for calendar review, such as in the annual budgeting cycle, by the perception of a new R&D requirement or opportunity or by a change in the status of a currently active research or development project. For example, assume the trigger event is a requirement for a development project result which is necessary to enable the organization to maintain its present level of performance. If this development project is inserted into the portfolio of active research and development projects, what will be the consequences for allocation of the total resources available to the R&D manager?

Additional research projects also may be required to make the specified development feasible. These research projects in turn generate additional resource requirements. The total new resource commitment necessary for the combined development-research program may be so great, relative to estimated benefits, that the package of activities is not on admissible entry to the active
project portfolio. However, the research project required may have carryover
to one or more different development projects. If the research project
needed for support of development were undertaken, then it would also
support, and add to the feasibility of, other development projects. New
developments either previously identified and assigned inactive status or
previously unidentified and now recognized as opportunities, merit
consideration as potentially attractive investments.

Closer Examination of Project Interrelationships

Several assumptions about interrelationships among "research" and
"development" projects are selected for emphasis in this analysis. In
particular, where the $k^{th}$ development project depends upon engaging in
the $j^{th}$ research project, the relationship may be expressed as:

\[ x_k \leq x_j \]

If both are required to be either 0 or 1, the inequality implies that
the development project (and its attendant rewards) cannot be included
in the plan unless the research project is. 

The most common cases assumed to occur in the R&D context are those
in which: (1) a single research project facilitates a number of development
projects; (2) a development project may be facilitated by any one of a number
of research projects; and (3) a development project requires a multiplicity of

\[ \text{This type of interdependency has been discussed by Weingartner [12] in connection with capital project interdependencies in his model, to be discussed in more detail later.} \]
research projects. These may be expressed, respectively, using the $j(k)$ subscript for research (development) projects and $J(k)$ for the applicable set, as:

(a) \[ x_k \leq x_j \quad \text{for all } k \in K \]

(b) \[ x_k \leq \sum_{j \in J} x_j \]

(c) \[ x_k \leq x_j \quad \text{for all } j \in J \]

In addition, mutually exclusive projects—for example where a group of development projects will lead to the same end product and hence cannot simultaneously produce cash inflows—can be represented by:

(5) \[ \sum_{k \in K} x_k \leq 1 \]

assuming that only one from the set of projects will be selected. Similar constraints can be imposed where the talents of a single individual or group are required for a number of competing projects but these people cannot be everywhere at once, or:

(6) \[ \sum_{l \in L_j} x_{kl} \leq 1 \]

where $x_{kl}$ here, can represent a research or a development project which can be undertaken by the $j^{th}$ laboratory member and $L_j$ is the set of projects in which the $j^{th}$ members has the capability of performing.

A few interdependencies are illustrated in figures 1 and 2. In Figure 1, basic relationships are shown. In Figure 2, a modest composite is indicated.
Figure 1

Diagrammatic Interdependency Representations

Research Project 1 is required for Development Projects 1, 2 and 3.

Either Research Project 1 or 2 will facilitate Development Project 1

Research Projects 1 and 2 are both required for Development Project 1

Only one of the three Research Projects, 1, 3, or 4, may be accepted.
Figure 2

An Interdependency Configuration

![Diagram showing interdependency configuration with nodes R₁, R₂, R₃, R₄ linking to nodes D₁, D₂, D₃, D₄.]
Development projects 1 and 2 require research project 1, development project 3 requires research project 1 and either 2 or 3, development project 4 requires either research project 1 or a combination of projects 3 and 4, and development projects 3 and 4 are mutually exclusive.

Relationships among even a relatively small number of projects suggest the value of a formal model to aid in allocation of resources among project combinations.

Mechanism for Identification of Project Alternatives

Two lists of active and inactive projects are assumed to exist at any given time—one for development and another for research activities. The active projects are currently funded, while inactive ones have been proposed but not authorized because at the time they were considered, costs were judged to exceed benefits, or they were relatively less attractive than alternative active projects. Types of events which trigger the project proposal identification process, and types of entries to the active and inactive project lists are summarized in Table 1.

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1/ See Hertz and Carlson [5], p. 6, for a description of a project selection decision process in which a distinction between active and inactive project lists is recognized.
Table 1
Components of the Project Proposal and Identification Process

Sources of "D" Project Triggers
1. New External Problem or opportunity
2. Change in status of active projects
3. Change in status of inactive D projects
4. New idea for a development, conceived by laboratory personnel.

Sources of "R" Project Triggers
1. New External problem in opportunity
2. Change in status of active R projects
3. Change in status of inactive R projects
4. New idea for research, conceived by laboratory personnel.

Inactive List Content

Development
a. proposed projects which are not technically feasible
b. proposed projects which are technically feasible but with relatively unattractive benefits/cost ratios at time of prior evaluation

Research
a. R projects which do not provide technical support for any D projects
b. R projects which provide technical support only for inactive D projects
c. R projects which, although they technically support active D projects, are less attractive than alternative supporting R projects with respect to their affects on cost and benefits.

Active Project List Content
All R and D projects currently funded or designated for future funding according to the plan for resource allocation which exists at that point in time.
It should be noted that this formulation does not include the process by which a "trigger", in the form of a perceived problem, opportunity, or change in program status, is translated into a project proposal, including a statement of a technical problem and a work plan for its solution. Several surveys of R&D laboratories have been made which suggest preliminary information about the characteristics of this process. See for example, Rubenstein and Avery, [10], Rosenbloom and Wolek, [9], and Seiler [11]. However, in contrast to most prescriptive models proposed for selecting research and development projects the framework presented here does include processes for screening project proposals in terms of their interdependencies with other projects, and for successively expanding the scope of this search for interdependencies when proposals fail to meet criteria for insertion into the active program.

Procedure for Evaluation of New Development Project Proposals

Figure 3 portrays the decision and information flow process by which a development project proposal may be examined for feasibility and then tested to see if it merits insertion into the portfolio of active projects. Detailed operation of the "Resource Allocation Procedure" specified at several points in the process logic will be discussed in a later section of this paper.

In the simplest case, a candidate new "D" project will have associated active "R" projects which will yield results making the D project proposal technically feasible. Here the development proposal is submitted directly to the resource allocation procedure. If, however, there are no active R
projects which render the D project feasible, the feasibility search is
expanded to the list of inactive R projects. If there are appropriate
inactive research projects, these projects may in turn contribute to the
feasibility of one or more other D projects, currently on the inactive D
list. In this case the entire package of proposed new D, inactive R's
and associated inactive D's is tested by the resource allocation procedure.
If, as a result of evaluation with respect to resource allocation criteria,
the original D proposal is not acceptable, a "D project generator" is
activated. This generator encompasses activities leading to a
proposal of a new development project, building in the set of inactive
research projects previously identified.
Figure 3: D Project Proposal Evaluation Procedure

1. Resource Allocation Procedure
   - Has D₁ been accepted?
     - Yes: Accept New Allocation
     - No: Update Inactive D Project List
   - Are there New Active R Projects to make D₁ technically feasible?
     - Yes: Activate R Project Generator
     - No: Are there New Inactive R Projects made feasible by R₁?
       - Yes: Resource Allocation Procedure
       - No: Update Inactive R Project List
     - Update Inactive D Project List
   - If not feasible, activate D Project Generator
   - Has an appropriate New R Project been generated before stopping?
     - Yes: Accept New Allocation
     - No: Update Inactive D Project List

2. Update Inactive D Project List
   - Has D₁ been Accepted?
     - Yes: Accept New Allocation
     - No: Update Inactive D Project List
   - Resource Allocation Procedure
     - Accept New Allocation
     - Update Inactive D Project List
     - Has Feasible New D Project been generated before stopping?
       - Yes: Resource Allocation Procedure
       - No: Update Inactive D Project List
   - Accept New Allocation
   - Update Inactive R Project List
   - Resource Allocation Procedure
   - Update Inactive R Project List
In the event that the proposed new development project represents such a departure from previous project proposals that neither active nor inactive research projects are available to make the development feasible, an "R project generation activity is initiated, in which new research proposals are sought which would enable technical feasibility of a subsequent development called for in the "triggers" proposal. After feasibility search and resource allocation tests are completed, the active project lists, inactive project lists, or both are updated.

Figure 4 outlines a similar process for the case in which a new research proposal rather than a development proposal is the triggering event. Assuming that such research proposals frequently are very loosely coupled to current R&D programs, all proposals are given a preliminary test for "fit" and degree of association with active and inactive project lists. If the project proposal has current program relevance, as, for example, where the proposed research will be less expensive than other enabling projects for development projects on the active list, subsequent steps are straightforward, and follow the same type of sequences as described above for development proposals. If the proposal has no current program relevance, however, search for new D project opportunities which depend on the proposed new research, is initiated immediately.
The presented scheme for feasibility testing and evaluation in terms of costs of benefits is intended to suggest only one form of decision process. Variations on the relationships between "R" and "D" projects recognized earlier suggest variations in the process logic. For instance, a new research project proposal might make one or more active D projects feasible, but at lower cost than the relevant subset of currently active R projects. Alternatively, the new research proposal might simultaneously contribute to the feasibility of active and inactive D projects. Third, the new research proposal might make one or more active D projects feasible at lower cost only if the new R is combined with one or more currently inactive R projects. Or, the new research proposal might contribute to the feasibility of an inactive D project, but is not in itself sufficient to insure that feasibility. Hence a second new research proposal is required, but, in effect the marginal cost of making the inactive D project feasible has been lowered because the initial research proposal contributes a partial solution to the technical feasibility problem.

To explore certain implications of the decision process logic in further detail, it is necessary to examine how the resource allocation mechanism, imbedded within the process might operate. Project interdependencies are next considered for their consequences to "optimal" allocation of the

\[1/\] For a fuller description of the structure and dynamics of the overall project selection decision process as it is influenced by information and behavioral constraints characteristic of R&D organization, See Brandenburg [1].
organization's research and development budget.

The Allocation of Resources within Search Behavior

We have suggested, above, the existence of an "active" and an "inactive" list for both research and development projects. Having outlined a basic structure of directed search for relevant projects, a search which provides both sets of lists with entries, we shall now specify more concretely the process by which inactive and active projects are distinguished. We include among active projects both those which are ongoing and those whose inclusion in future plans justifies the current project selection. That is, an active project is one which is included in the long-range plan which determines the current allocation of research and development funds.

Thus the current allocation of funds, once the projects which are potential candidates for funds have been provided by the project generators, is assumed to be based on a planning model. For our purposes here we will assume an optimizing model--one which provides the best allocation of resources given the total project lists (and limited information about the projects) provided by the obviously non-optimal search procedure defined above. This optimization model is, of course, a part of the search procedure; by our earlier assumptions, the decision to search is, in the first instance, the result of an allocation program which failed to add a proposed project to the active list.

Thus a combined approach--which uses an optimal allocation device within a structured search procedure, is suggested. The allocation device
provides both the post-search research program and the impetus to enlarge the set of available alternatives should the initially produced program appear unsatisfactory in terms of organizational pressures—e.g., for a development project much wanted by some sub-unit in the organization. We shall now examine a possible research allocation procedure—which, to this point we have treated as a "black box"—in detail.

The Optimal Research Allocation Procedure

The research and development phase of organizational activity is one in which uncertainty and risk are significant features. However, planning for an optimal allocation of research and development resources in the case where complete certainty can be assumed is far from a solved problem. In fact, a well-specified model for resource allocation in the deterministic case remains to be developed.

Most problems arising in a management context are characterized by a reasonable amount of uncertainty as to future outcomes. It is frequently found, however, that a model for optimization which utilizes imperfect data inputs can produce far better performance than that which would have resulted in the absence of the model and only incrementally poorer performance than that which would have occurred using perfect data.

Uncertainties encountered in the capital budgeting problem resemble those in research and development budget allocation problems. It is usually unclear

See, for example, Holt, Modigliani, Muth, and Simon [6]. Also, see Charnes, Dreze, and Miller [4] for an example of a set of situations in which better data can, in fact, produce poorer decisions.
at the time of purchase, whether a machine built to last twenty years will produce a product that is wanted at the end of that time; yet, considerable progress has been made in understanding the nature of the capital budgeting decision using a deterministic model. For one thing it is frequently possible to experiment with alternative estimates of profitability and/or cost to determine just how sensitive a particular (initial) investment plan is to changes in the longer-range (and hence less certain) returns.

While the research and development planning problem may be inherently more subject to uncertainty it shares many characteristics with the capital budgeting problem. First, both involve outlays now for returns later. Second, both involve distinct projects whose fractionalization is difficult—one cannot buy half a machine or develop half a product. Third, both involve interdependent projects. Finally, some kinds of budgetary constraint characterizes decisions in both areas.

The combination of these similarities suggest using a capital budgeting model as a takeoff point in this investigation. In the process of developing our model here, however, we shall point out critical differences between investing in capital equipment and "investing" in research and development. In so doing we hope to identify directions for development of capital budgeting techniques necessary to meet requirements of the R&D budgeting problem.

\footnote{See, e.g., Weingartner [12], including references.}
A Capital Budgeting Formulation

Before turning specifically to the R&D resource allocation problem, it will be helpful to present Weingartner's "Basic Horizon Model," as a means of specifying an abstraction of the investment problem. A T-period planning horizon is assumed where in each period, t, (t=1, 2, ..., T.), the jth investment requires cash outlay of an amount equal to a(tj). An expenditure is represented by positive a(tj), an incoming cash flow a negative a(tj). At the end of T periods, the sum of the remaining cash flows, for periods T+1, T+2, ..., suitably discounted, is represented by a(T+1), ..., a(T+n) for the jth project, j=1, 2, ..., n. The budget constraint in the tth period is represented by D(t) and the amount of the jth project undertaken by x(tj).

Each project is considered to be a distinct entity which can either be undertaken (x(tj)=1) or not (x(tj)=0) although it is more convenient to state the constraint as 0 ≤ x(tj) ≤ 1 and x(tj) required to be an integer. Each budget constraint must be met precisely although "borrowing" and "lending" are permitted. The mechanism used is that of one-period contracts where the loan is repaid, with interest, in the following period. The amount lent in the tth period is v(t), the amount borrowed, w(t), each at a rate of interest r.

1/ [12], pp. 141-143.

2/ A long-run discount rate can probably be used here. However, using such a rate in the short run may be questionable where there are explicit budget constraints.

3/ Presumably funds invested outside the list of specific available alternatives.
The quantity to be maximized is the standard (deterministic) economic value of the firm: the sum of future discounted cash flows and the amount of available cash less any debt as of the final period examined.

In brief, this problem can be stated as:

(1) Maximize:  
\[
\sum_{j=1}^{n} d_j x_j + v_T - w_T
\]

(2) Subject to:  
(a)  \[
\sum_{j=1}^{n} a_{1j} x_j + v_1 - w_1 \leq D_1
\]

(b)  \[
\sum_{j=1}^{n} a_{tj} x_j - (1+r) v_{t-1} + v_t \\
+ (1+r) w_{t-1} - w_t \leq D_t, \ t=2, \ldots, T
\]

(c)  \[0 \leq x_j \leq 1, \ j=1, 2, \ldots, n\]

(d)  \[v_t, w_t \geq 0, \ t=1, 2, \ldots, T\]

Equation (1) provides the value maximization. Inequalities (2a) and (2b) provide the budget constraints with the former stating implicitly that outstanding borrowing or lending at the beginning of period 1 is subsumed in \(D_1\). Inequalities (2c) and (2d) represent the conditions on projects noted above and the requirement that negative funds can neither be borrowed nor lent. The latter is really vital only where different rates of interest—say, \(r_1\) and \(r_2\)—are assumed for lending and borrowing. An additional constraint is the integer requirement on the \(x_j\).

\[a\] The reader is referred to [10] for further exposition.
Research and Development Variations

Using this abstract representation of the investment model it is possible to illustrate certain critical differences between the typical R&D budgeting situation in practice and the investment problem—even leaving aside the uncertainty issue. The purpose of this exercise is not to show that a formulation similar to that of the capital investment problem is inapplicable. Rather, it is an attempt to state the differences in general terms and to point out the possibly artificial constraints which are inadvertently imposed in the R&D resource allocation decision.

A primary difference between investment and R&D decisions is in the reward functions. In the capital budgeting framework it is possible to assume that all benefits of investment accrue to the firm. Thus the functional (or quantity to be maximized) is global. The organization reaps all of the benefits of the cash flows (or discounted cash flows beyond period $T$) generated by the investments made. The benefits from investment in research and development accrue to the firm, to be sure, but not directly to the research and development laboratory. In the capital budgeting framework, negative $a_{tj}$ (cash inflows) operate as a source for future capital outlays. Such cash inflows are rarely available directly to the R&D organization for the purpose of undertaking new projects.

A second major difference exists in the availability of borrowed funds or, more generally, interperiod transfers. The R&D budget is usually assumed to be fixed, or based on some relatively arbitrary quantity not directly related to the productivity of the laboratory—e.g., a fixed percentage of sales. In fact, it is rarely the case that funds budgeted for one fiscal year, if unexpended, can be applied in the following year.

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At the level of the total R&D effort for the organization, there usually is a ceiling on the rate at which the annual budget can be increased, imposed by limits on professional manpower availability. Even if the R&D director were told that he could have 25% more funds next year, he might not be able to use them because he could not hire enough scientists and engineers with appropriate skills to absorb the budget increase. Such considerations can, of course, be treated as explicit constraints rather than (as appears usually to be the case) translated into organizational behavior patterns which rely on an implicit status quo ante assumption.

Needless to say, recourse to the usual capital markets for funds is usually limited except in organizations where research and development approaches the sole function of the organization. In such cases, the mechanism is however, usually different; the organization applies for funds to finance specific projects. Although this resembles the flotation of a stock or bond issue for financing a specific capital expansion, the concept of obtaining capital at a prevailing rate of interest is usually absent.

Some Implications

The differences noted above between the capital and research budgeting situations represent some combination of the real and the self-imposed. There is no particularly cogent reason for a long-term research and development venture not to be considered as a capital outlay made for the purpose of obtaining future cash inflows. While it is permitted, for accounting and tax purposes, to consider research and development outlays as a deduction from current revenues, such institutional considerations (aside from the obvious tax advantage accruing from investment in R&D rather than fixed plant)
should not mask the basic issue: current investment in R&D is undertaken (or should be) for the purpose of producing future cash flows to the organization as a whole.

Viewed in this light, the typical case in R&D budget allocations—assuming a fixed budget, ignoring costs of obtaining funds in the usual capital markets, even disallowing later use of unexpended budget funds—severely limits the degree to which resource allocation among competing R&D projects is rational in the light of costs and benefits accruing to the organization as a whole.

What may be even more critical, however, is the failure to specify a criterion function for the allocation of R&D funds. If the technical expertise of the R&D segment of the organization is to be relied upon in the allocation of resources allotted to it, the criterion used by the laboratory should be highly relevant. It is possible to conjecture that the reason for the typically observed lack of concern with precise formulation of a laboratory criterion function is a parallel absence of a model for resource allocation sufficiently well-defined as to make this precision critical.

In formulating and investigating the resource allocation model that follows we shall attempt to explore certain ramifications of the structure

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1/ At minimum, funds used for R&D cannot be invested in fixed plant; if money is borrowed, or earnings retained for the latter, a related cost for the former can reasonably be assumed to apply.

2/ See Mansfield and Brandenburg [8] for a case study of the extent to which a expected net profit maximization criterion explains project selection decisions in a R&D laboratory.
of laboratory criteria and constraints. While we recognize that reducing the allocation problem to manageable proportions necessitates a certain degree of abstraction, it is suggested that the problem of resource allocation in the absence of any structural framework requires a far greater degree of abstraction. To say that the expenditure of research and development funds is "in the best interests of the company" or "in the best interests of the laboratory" without specifying what one means by either generality would seem to represent a most formidable obstacle to establishing comprehensible rules for action.

**Formal Specification of the Model**

In addition to the interdependency relations of the type represented by inequalities (4), (5) and (6), the investment model must be adapted for the research and development situation. Because the laboratory is a subunit within the organization as a whole funds available to each must be separately specified although the overall function to be maximized will be assumed to be the firm's value at the end of period T. (It is difficult to rationalize some other function from the standpoint of the firm's welfare although it is conceivable that a decentralized decision-making apparatus might result in the laboratory's maximizing its own welfare at the expense of the firm as a whole.)

In adopting Weingartner's model to the research and development case, it will be convenient to distinguish cash flows which are charges against the laboratory budget and those flows which are outside the laboratory budget. Let us designate the laboratory flow for a particular project in the t^{th} period
as $a_{tj}$, the non-laboratory flow as $b_{tj}$. Also, let us denote $y_t$ as "lending" and $z_t$ as "borrowing" by the laboratory (with, as before, a one-period repayment) with interest rates $r_1$ and $r_2$, respectively. These may or may not be the same rates as the corporate lending and borrowing rates assumed. The laboratory budget may be designated as $B_t$ for the $t^{th}$ period. The budget constraints (for the laboratory) may then be expressed as:

$$\sum_{j} a_{tj} x_{j} + y_{1} - z_{1} \leq B_{1}$$

$$\sum_{j} a_{tj} x_{j} - (1 + r_1) y_{t-1} + y_{t}$$

$$+ (1 + r_2) z_{t-1} - z_{t} \leq B_{t}, \quad t=2, \ldots, T$$

The amount of funds made available to the firm as a whole from the laboratory operation could be expressed in a variety of ways, although perhaps most straightforwardly as:

$$v_{1} - w_{1} = -\sum_{j} a_{1j} x_{j} - \sum_{j} b_{1j} x_{j}$$

$$v_{t} - w_{t} = -\sum_{j} a_{tj} x_{j} - \sum_{j} b_{tj} x_{j} + (1 + r_1) v_{t-1}$$

$$- (1 + r_2) w_{t-1}, \quad t=2, \ldots, T.$$
The assumption implicitly made here is that funds generated by laboratory projects are available to the firm for other purposes or, if negative, must be borrowed by the firm financed from sale of stock or obtained from other operations (retained earnings). It should also be apparent that the sum of the cash outflows, $\sum a_{tj}x_j$, is equivalent to the laboratory budget adjusted for the internal budget overruns and/or slack permitted. The amount of funds available to the firm at the end of period $T$ (and hence the functional to be maximized) can be expressed also in terms of the project outflows, say $\hat{a}_j$ and $\hat{b}_j$ and the final net credit balance of the firm. Thus, the maximization problem for optimal resource allocation can be summarized as:

\begin{align}
(5) \quad \text{Maximize} & \quad \sum \hat{a}_j x_j + \sum \hat{b}_j x_j + v_T - w_T \\
(6a) \quad \text{Subject to} & \quad \sum a_{1j} x_j + y_1 - z_1 - B_1 \\
(6b) & \quad \sum a_{tj} x_j - (1+r_1) y_{t-1} + y_t + (1+r_2) z_{t-1} - z_t - B_t , \quad t=2, \ldots, T \\
(6c) & \quad \sum a_{1j} x_j + \sum b_{1j} x_j + v_1 - w_1 = 0 \\
(6d) & \quad \sum a_{tj} x_j + \sum b_{tj} x_j - (1+r_1) v_{t-1} + v_t + (1+r_2) w_{t-1} - w_t = 0 , \quad t=2, \ldots, T \\
(6e) & \quad 0 \leq x_j \leq 1 , \quad j=1, \ldots, n \\
& \quad v_t, w_t, y_t, z_t \geq 0, \quad t=1, \ldots, T \end{align}

\footnote{This is clearly not the most compact form in which the model might be stated.}
Additional constraints for project interdependencies must be added as well as where desired, limits on the lending and borrowing variables where desired or necessitated by inherent inflexibilities in possible rate of growth.

Examination of Budget Policies

It is not presumed that one can construct a model of this form for the research and development allocation problem in all instances and proceed in a straightforward manner with well-known linear programming techniques. In the form stated it is, indeed a linear programming problem or, if partial projects are disallowed, mixed-integer programming problems (requiring that some or all of the $x_j$'s must be either 0 or 1). The integer programming problem presents many computational difficulties and it is probable that, for possible, application in the foreseeable future fractional projects be eliminated by other means—e.g., budget relaxation—rather than resorting to integer programming codes. The current state of the art in the solution of integer programming problems does not allow large problems to be handled except in special cases.

What may be more interesting at this stage of the research, however, is investigation of various budgeting policies via the model. The policies typically followed are: (1) no lending or borrowing—i.e., a fixed research budget; or (2) lending allowed at no interest—i.e., allowing the carrying forward of unexpended funds. A third form of interest, although rarely used, is the case where some fraction of the yields of the research laboratory (as
well as prior slack) are fed back to the laboratory—a form of borrowing in our general model—for simplicity, assumed to be returned in the following period.

In these three typical cases, the effect is to strengthen (in decreasing amount) the constraints on the $B_t$. In the first, the constraints require $y_t = z_t = 0$ or, rewriting the relevant inequalities, we obtain

$$
\sum_j a_{tj} x_j \leq B_t, \quad t=1, \ldots, T.
$$

(7)

The second case may be written as

$$
\sum_j a_{1j} x_j + y_1 \leq B_1
$$

(8)

$$
\sum_{j} a_{tj} x_{j} - y_{t-1} + y_t \leq B_t \quad t=2, \ldots, T
$$

where, in effect, $r_1 = 0$, $z_t = 0$ are assumed. Finally, the third case can be comprehended within the general model (with $r_1 = r_2 = 0$) but constraining the $z_t$:

$$
z_1 = 0
$$

(9)

$$
z_t \leq -\alpha \sum_{t=1}^{t-1} \sum_{j \in t-1,j} b_{t-1,j} x_j \quad t=2, \ldots, T.
$$

so that the cumulative budget overrun permitted in any period is limited by the "profitability" of the laboratory where $\alpha$ is the proportion of "profit returned" to it.

Actually, any or all of the constraints on the $B_t$ may be viewed as constraints on firm profit. Putting the resource allocation problem in the
form of a maximization model makes (at least in the deterministic case) exceedingly clear that if the firm is limiting the research and development budget to below that which would be expended using straightforward maximization model:

(10) \[
\text{Maximize } \sum_{j} (a_j + \hat{b}_j) x_j + v_T - w_T
\]

(11) Subject to: \[
\sum_{j} (a_{1j} + b_{1j}) x_j + v_1 - w_1 = 0 \\
\sum_{j} (a_{tj} + b_{tj}) x_j + (1+r_1) v_{t-1} + v_t + (1+r_2) w_{t-1} - w_t = 0,
\]

\[t=2, \ldots, T\]

\[0 \leq x_j \leq 1 \quad j=1, \ldots, n\]

\[v_t, w_t \geq 0 \quad t=1, \ldots, T\]

then it is simply reducing profit. In fact, solving the general problem, the dual variables associated with the constraints on B_t will provide the amount of additional profit which could be obtained for each dollar of budget relaxation.

Viewed in another sense, the explicit budget constraints are implicit limitations on the amount the firm is willing to invest in the uncertain returns of the research and development operation. The amount that the firm pays for this risk limitation in the way of foregone profits may readily be estimated by comparing budget-constrained and unconstrained solutions.

1/ This is a somewhat oversimplified picture since interpretation of the marginal return to additional budget will depend upon just how much relaxation and on the period interdependencies assumed. See, e.g., Charnes, Cooper, and Miller [3]. Also, if integer routines are used, the interpretation of the dual variables is not straightforward [12].
Budget Policies and Project Selection

It is reasonable to assume that the imputed interest rate for laboratory borrowing is greater than that for the firm as a whole (r₂ > r₂) and that the imputed lending rate of the laboratory is at least as great as that for the firm where such rates are used. Presumably the risks associated with research expenditure are higher than those for the firm as a whole; if a budget overrun is permitted or if research expenditures can be saved the rates of return associated with these higher risk expenditures would reflect a risk premium. Also, for simplicity, let us assume that the \( A \) and \( J \) are all zero—i.e., rule out the case of negative discounted returns after period T.

In the least constrained case, given in (10) and (11), where R&D outlays and returns are treated in the same way as other expenditures of the firm, the timing of both outlays and returns is clearly relevant. Indeed, in the event that \( r₁ = r₂ = r \), the conditions for a perfect capital market is met and any project combination which produces a rate of return at least equal to r will be accepted. Indeed, one would substitute a higher interest rate, \( r' \), which included a risk premium and solve the problem in this form, where

---

1/ An exception to this assumption about lending rates will be investigated below.

2/ This, in effect, assumes that any project which might have outlays beyond period T will at least pay for itself. Actually no such project would be selected in an optimal program unless some other project which had positive cash flows, depended upon it; in most cases of this type, an equivalent reduction in the \( B \) of the dependent project would effect the appropriate compensation for eliminating the negative value.
the optimization technique simply becomes a device for avoiding examination of the project interdependencies by enumeration of all possible combinations. It should be emphasized, however, that the embedded assumption of a perfect capital market is a strong one; the firm is assumed to be able to obtain unlimited funds at the specified rate of return, the latter remaining unchanged in the process.

It will be observed that the system given in (5) and (6) is merely that of (1) and (11) with the addition of constraints (6a) and (6b). Clearly, if the budgeted amounts, $B_L$, are sufficiently large, the addition of these constraints will not affect the solution; the amount of money saved (or lent) by the laboratory will only enter the solution through the firm-wide relationships, inasmuch as the $y_L$ do not enter the functional directly. As the budgets are reduced, however, the relevant rates of return for selected investments become increasingly dependent on the internal (laboratory) rates of return, $r_1$ and $r_2$. However, only the outlays for projects are constrained by the laboratory budget (as is the usual case) so that the effect is, loosely speaking, to discount the timing of outlays at the imputed (higher) laboratory rates and income at the lower firm rate of return. The selected projects will, of course, be influenced by some combination of rates but it is clear that as the laboratory rates increase relative to the firm rates and/or budgets are reduced, the timing of outlays becomes increasingly more important.

\[1/\]

The budget, in effect, is an amount which can be borrowed from a separate source; since it need not be returned, its effective one-period interest rate is $-1$. As the amount of this "free good" diminishes, the dependence on obtaining funds at the imputed rate $r_L$ must increase. The lending rate, $r$, will usually diminish in importance also since lending becomes less and less likely as budgets are diminished. Since $r_2 \geq r_1$, the effective rate of return pertinent to discounting outlays will thus increase.
than that of the positive returns, $b_{tj}$.

Presumably then, tightening budgets and increasing the imputed laboratory rates will tend to discourage the selection of projects with high initial outlays. The timing of incoming flows resulting from project completion will tend to diminish in importance. A project currently in the development stage which requires a high initial outlay for completion but which will produce incoming flows very shortly thereafter may well be supplanted by another whose outflows and inflows are postponed.

At the extreme, although with a quirk, is the usual budgeting procedure where laboratory "lending", insofar as the laboratory is concerned, is equivalent to giving it away or lending at an effective rate of interest of -1. "Borrowing," by contrast, takes place at an infinite rate of interest--i.e., it is prohibited. This may be expressed formally by replacing the inequalities in (6b) and (6c) with those in (8). Thus, although the fixed budget implies $r_2 > r_2$, the laboratory rate for lending is less than the firm rate. This last violates our usual assumption and, particularly where $r_1$ is small relative to $r_2$, will result in "using up" the budgeted amount in some periods on projects whose effective rates of return are less than the firm's lending rate only in later periods, to reject projects whose returns are greater than the firm's lending rate. This phenomenon will generally be observed to a limited extent in any program of this type because of the project indivisibility (integer) constraints. However the usual pattern in constrained capital budgeting models requires that early outlays be more stringently examined than later ones, while the general reversal
here is possibly because of the peculiar interest rate condition implied by the inability to apply saved funds to laboratory operations in later periods. This anomaly can be eliminated by allowing carry-forward without imputed interest—i.e., replacing the constraints in (7) by those in (8); then, at least, money not spent now is available to the laboratory at a later date.

In either case, however, when the number of "good" projects (i.e., those which would be accepted in the least constrained case) is large relative to the available budgeted funds, the fixed budget, with or without carryover permitted, will tend to produce a solution heavily influenced by the arrangement of outlays. Since the imputed interest rates for laboratory "borrowing" to finance outlays can be much larger than the firm rates of return, the latter, through which the timing of inflows is reflected in the functional, diminish in importance. Indeed, the selection would seem to be made, in heuristic terms, by choosing a set of projects whose outlays seem to fit the budget pattern.

Suppose one has a four-period horizon and three development projects to choose from, one with an outlay of 4 in the first two periods, one with an outlay of 2 in the first two periods and one with an outlay of 5 in the last two periods, and a budget of 5. Obviously, if the rate of return on the third

1/ Actually, an even more extreme case is found in some forms of institutional budgeting where unexpended funds result in budget cuts in later periods. In our model, this practice would be represented by making \( r_1 = -2 \), a condition calculated to use up budgeted funds even if the only projects available have negative rates of return.
project is at least that of the firm's borrowing rate (and possibly even less, if inflows from one of the others makes the lending rate relevant instead) it will be accepted. However, only one of the first two projects can be accepted even if both have rates of return far greater than that of the third. Assuming that both rates exceed the firm's borrowing rate the rate of return on the $2 per period outlay must be practically twice that of the $4 per period outlay to be accepted since the net dollar flow rather than the rate of return becomes the relevant criterion. While this case is somewhat extreme, it is clear that the fixed budget situation can be quite heavily influenced by the outlay pattern with the inflows and returns acting as limiters or requiring vast differences to be influential.

In order to increase the influence of the inflows within the planning horizon, a constraint such as (9) must be imposed on the general model given in (6). The effect of such a constraint, other things being equal, will be to encourage the selection of projects which have early inflows. Compared with a situation where no return to the laboratory is provided and the budgeted amounts are unchanged, the "profit sharing" will permit additional projects whose outflows occur after inflows begin to be selected.

If, however, budgets are reduced over time so that the laboratory becomes "self-supporting," the long-run tendency must be for the laboratory to take projects with more rapid payoffs rather than eventual long-run gains. In some sense their projects with long-term payoffs are a luxury, not to be selected if one's future support depends upon early results. Permitting the laboratory to borrow might mitigate the situation somewhat but the borrowing
would be likely to finance projects with high initial outlays and early returns rather than the "basic" research project characterized by a long period of modest expenditures which eventually leads to a large gain.

Thus, the budget policy followed can serve to influence the kind of projects which would be selected using a constrained optimization technique of the type described in the previous section. While reasoning from the technique provides sharper insights than would a less structured approach, the results do not seem to be at variance with intuitive reasoning.

A fixed budget may produce certain anomalies such as taking on low-return projects early. But the very thing that causes the anomaly—i.e., some money left over which is lost if not spent—may serve to finance the long-term research project. Making the laboratory a profit-sharer increases its flexibility but tends to encourage short-term-payoff projects. Allowing the laboratory to "borrow" and "lend" funds will undoubtedly increase the profits accruing from laboratory operations but, as long as the outlays are treated at higher interest rates than inflows, timing of the former will have greater influence than the latter. Only if borrowing to finance outlays is tied to inflows directly will the full effect of inflow timing be felt on the project selection process.

Feedback on the Total Process

Since the project selection phase feeds back into the project generation search procedure we outlined above, it is clear that the budget policy selected must influence the whole process. A feasible development project with high initial outlays, for example, is much more likely to be selected by the
optimization routine if borrowing is permitted. Its acceptance with or without further search is greatly enhanced if it has high early payoffs only if these payoffs are shared by the laboratory. Similarly, suggested R projects are more or less likely to be accepted depending upon the budget system embedded in the project selection process.

The optimization model also provides guides for stop rules for search. It is generally possible to estimate, from the optimal program solution, just how close a project is to acceptance. For example, if a suggested project is initially unacceptable, a "dummy" project which depends upon one or more of the same R projects as the subject D project may be inserted. If some outflows and inflows can be reasonably estimated for this project the long-term payoff for the dummy which allows both it and the subject project to be selected can be found. The dummy project may be allowed to vary in size by constraining its corresponding $x_j$ to some figure other than 1. Thus a "profile" for a required D project can be sketched and its reasonableness determined. Clearly the outlay for the project must include initial search cost so that, by varying the first-period outlay with an assumed fixed return can give insight into the amount of search for this project that can be justified.

Such manipulations are simple enough within the allocation model framework. Obviously, some estimates of what is likely to be found through project search must be made and these may not be accurate. However, at least in gross terms, if the dummy project must have a rate of return which is twice as great as the suggested one to put them both on the active list,
a good chance exists that search should be directed elsewhere.

Clearly the types of projects searched for will be dependent on the budgeting-planning procedure used. Depending upon the type of budgeting procedure, the kind of additional project which will facilitate the acceptance of a suggested project will differ. Also, the effect on other projects will differ. Using some budgetary schemes, a new project may be accepted without cutting back on others—indeed if gains are fed back to the laboratory, the acceptance of the new project may actually transfer others from the inactive to the active list. On the other hand, the fixed budget case must invariably result in canceling other ongoing or planned projects to introduce a new one. In short the operation of the project initiation and search procedures is inseparable from the planning and budgeting process used in project selection.

Conclusion

We have outlined a system for project generation and selection. It is not intended in its present form as a blueprint for action but rather as a framework for analyzing the interrelationships between structured planning and budgeting phases of the process and the less-structured organizational search activities which surround them.

In the process of analyzing this model the dependence of the project search procedure on the evaluation procedure has been stressed. The model which serves as the project selector effectively accepts or rejects newly

See Brandenburg and Stedry [2] for a discussion of the diminished likelihood of finding improved projects through search as the length of search increases.
suggested projects. By this means it also serves to indicate the need for search by rejection. Using the model as a "pre-evaluator" for hypothetical projects it is possible to gain insight into the kind of projects which it is worthwhile to search for and whether search is worthwhile at all. Since the possibilities for interaction are multifarious it is difficult to spell them out in detail. Indeed, further specifications of these couplings is considered a subject for further research.
References


An attempt is made to develop a combined optimizing-satisficing model of the research and development process. The focus is on the selection of both research and development projects for investment of research funds. A "quasi-rational" fund allocation process is assumed wherein the allocation of funds among proposed projects is made by an integer programming technique but the development of the list of projects which are candidates for receipt of funds is assumed to be the result of an ancillary heuristic process. The allocation model is not intended as a firm prescription for behavior but rather as a device to gain insight into the project selection process.
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