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POWER AND SPACE REQUIREMENTS FOR SIMULATION OF MACHINE GUN MOUNTS

Technical Report

David Gelfond

DA Project Title: Development of Aircraft Gun Type Subsystems
DA Project: 11-5-50206-01-M1-M6

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ABSTRACT

The electric power and site requirements are established for a single degree-of-motion-freedom machine gun mount simulator capable of supporting all automatic small arms through 30 millimeter weapons. The concept of using an electromechanical servomechanism to simulate the mass, stiffness, and damping characteristics of the gun mount is shown analytically to be feasible. Experimental verification of the simulation concept is demonstrated by burst-firing of a 5.56 millimeter machine gun on a scale model.
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SUBJECT

Machine gun mount simulation with a displacement feedback servomechanism was studied.

OBJECTIVE

To establish electric power and installation site requirements of a mount simulator for automatic small arms up to and including 30mm weapons.

CONCLUSIONS

1. The feasibility of the basic simulation hypothesis is verified by the test results of both the single-shot and the automatic firing experiments.

2. The influence of the resonant power multiplier (Section 3) on the overall simulator requirements must be reconciled to the capabilities of state-of-the-art motors and controllers. The worst-case power requirements occur at the fundamental resonance and can be moderated by limiting the damping ratio to an acceptable minimum. The minimum damping ratio must, of course, be representative of that ratio which is typically encountered in mounting structures.

RECOMMENDATIONS

Based upon the analyses presented in Sections 3, 4, and 5 of this technical report, the following recommendations are made:

1. Minimum damping ratio should be 0.075.

2. Motor should be a 15-horsepower D.C. unit.

3. Simulator base should have a minimum weight of 32,000 pounds.

4. Available electric power should be 30 KVA at 0.8 power factor.

With respect to basic range dimensions and door sizes, the requirements of the simulator are smaller than those requirements established by the considerations for supporting a helicopter within the range.
1. INTRODUCTION

It is a well-established fact that all automatic weapons will exhibit a performance sensitivity to their mounting conditions. The sensitivity of a weapon to its mounting conditions may manifest itself as change in recoil force only, or, in the extreme, as a complete failure to function. Therefore, it becomes necessary during weapon development to investigate weapon-mount compatibility for installations where the potential mount natural frequencies are in the range of the first several multiples of the weapon firing rate. Over the years, two basic systems, families of helical springs or variable length beams, have been used to simulate weapon mounts. Both of the afore-mentioned approaches to simulation are physically cumbersome, lengthy in the setup and adjustment time, and only rarely have incorporated control over damping ratio. In 1962, scientists at Springfield Armory began a search for simulation techniques that would permit rapid adjustment of mount spring rate and independent control of the damping ratio. The most promising method for mount simulation, that of disturbing a position feedback servosystem at its mechanical output point by the weapon recoil force, has been examined analytically and its feasibility demonstrated experimentally.
2. PRINCIPLES OF OPERATION

a. The Springfield Armory weapon mount simulator concept, depicted in Figure 1, is based upon the hypothesis that a position feedback servo-system disturbed at its mechanical output point exhibits the same response as a classical mass-spring-dashpot network. The unique features of the Springfield Armory concept are in the ease of adjustment of spring rate and independent control of the damping ratio. It will be shown below that the spring rate is a function solely of the time-invariant servoloop gain terms.

From Figure 2:

\[ X(s) = \frac{(\tau n)^2 (1+Ts)}{s^3 + \frac{s^2}{T} + \frac{K_T}{J_T} (K_B + K_v K_a) s + \frac{K_t K_a}{J_T}} F(s) \]  \hspace{1cm} 2-1

When \( F(s) = \frac{F}{s} \), then by the final value theorem Equation 2-1 becomes

\[ X = \frac{(\tau n)^2 F}{K_t K_a} \]  \hspace{1cm} 2-2

The spring rate, \( K \), is given by

\[ K = \frac{F}{X} = \frac{K_t K_a}{(\tau n)^2} \]  \hspace{1cm} 2-3

The factored form of 2-1 is

\[ X(s) = \frac{(\tau n)^2 (1+T) / J_T}{(s + \frac{1}{bT})(s^2 + 2 \xi \omega_1 s + \omega_1^2)} F(s) \]  \hspace{1cm} 2-4
Figure 1

ELECTROMECHANICAL SERVOLOOP CONCEPT

MOUNT SIMULATOR

Torque Motor

Displacement Feedback

Velocity Feedback

Amplifier

f(t)

Weapon

-4-
\[ F(s) \] = Force input  
\[ r \] = Last gear radius  
\[ n \] = Gear ratio  
\[ J \] = Total moment-of-inertia  
\[ K_t \] = Motor torque constant  
\[ K_b \] = Motor back emf constant  
\[ K_v \] = Velocity feedback constant  
\[ K_a \] = Amplifier gain  
\[ T \] = Motor armature electric time constant  
\[ s \] = Laplace operator  
\[ T(s) \] = Motor torque  
\[ \dot{\theta}(s) \] = Motor speed  
\[ \theta(s) \] = Motor shaft angular displacement  
\[ X(s) \] = Weapon displacement

**Signal Flow Diagram**

Figure 2
2. PRINCIPLES OF OPERATION - Continued

The time response for $F(s) = I$ is

$$X(t) = \frac{(rn)^2 I W_1 e^{-\xi W_1 t}}{K_t K_a (1-\xi^2)^{1/2}} \left( \frac{1-2T\xi W_1 + T^2 W_1^2}{1-2bT\xi W_1 + b^2 T^2 W_1^2} \right)^{1/2}$$

$$+ \frac{(rn)^2 I}{K_t K_a} \frac{T(b-1) e^{-\frac{t}{bT}}}{(1-2bT\xi W_1 + b^2 T^2 W_1^2)}$$

$$\psi = \tan^{-1} \frac{T W_1 (1-\xi^2)^{1/2}}{1-T\xi W_1} - \tan^{-1} \frac{bT W_1 (1-\xi^2)^{1/2}}{1-bT\xi W_1}$$

$$t \leq 0.005, \ \xi \leq 0.05 \ \text{and} \ W_1 \leq 200 \ \text{then} \ 1.1 \geq b \geq 1$$

and 2-5 simplifies to

$$X(t) = \frac{(rn)^2 I W_1}{K_t K_a} e^{-\xi W_1 t}$$

Substituting 2-3 into 2-6 gives

$$X(t) = \frac{I}{(KM)^{1/2}} e^{-\xi W_1 t}$$

Equation 2-7 is identical with the response equation for a mass-spring-dashpot network subjected to an impulse forcing function.

The effect of the velocity feedback can best be seen by applying the Routh-Hurwitz criteria for stability to the characteristic equation of the servoloop.
2. PRINCIPLES OF OPERATION - Continued

The characteristic equation is

\[ s^3 + \frac{s^2}{T} + \frac{K_t}{J} (K_b + K_a K_v) s + \frac{K_t K_a}{J T} \]  

The Routh array is

\[ s^3: \quad 1 \quad \frac{K_t (K_b + K_a K_v)}{J T} \]  

\[ s^2: \quad \frac{1}{T} \quad \frac{K_a K_t}{J T} \]  

\[ s^1: \quad \frac{K_t (K_b + K_a K_v)}{J} - K_a K_t \quad 0 \]  

\[ s^0: \quad \frac{K_a K_t}{J T} \]

For stability, it is necessary that

\[ \frac{K_t}{T} (K_b + K_a K_v) - K_a K_t \geq 0 \]  

The upper limit of loop gain is given by

\[ K_a = \frac{K_b}{T - K_v} \]  

Then as \( K_v \rightarrow T, \ K_a \rightarrow \infty \)  

Without \( K_v \), the upper limit of gain is

\[ K_a = \frac{K_b}{T} \]
2. PRINCIPLES OF OPERATION - Continued

By comparison of Equations 2-12 and 2-13, it is seen that velocity feedback is necessary to obtain the required stability (damping ratio) at the very large values of loop gain that must be used to provide the necessary range of spring rates.

To fulfill the necessary simulation conditions on mass, the total reflected moment-of-inertia must equal the total translational mass. The magnitude of the reflected inertia is controlled by the total gear ratio \( r_n \).

The total moment-of-inertia term \( J \) in Equation 2-1 is

\[
J = J_{\text{motor}} + J_{\text{gear}} + (r_n)^2 M_{\text{weapon}} \tag{2-14}
\]

If \( J_{\text{gear}} \ll (r_n)^2 M_{\text{weapon}} \), then

\[
J = J_{\text{motor}} + (r_n)^2 M_{\text{weapon}} = (r_n)^2 \left[ M_{\text{weapon}} + M_{\text{mount}} \right] \tag{2-15}
\]

The gear ratio is given by

\[n = \left( \frac{J_{\text{motor}}}{M_{\text{mount}}} \right)^{1/2} \tag{2-16}
\]

b. A model of the mount simulator was fabricated to substantiate experimentally the basic hypothesis. The model was tested initially with single-shot firings of ammunition having a 3 pound-second impulse and then tested with automatic firing of ammunition having approximately 1.0 pound-second impulse. In both cases, good agreement was obtained between actual and theoretical values of displacement and natural frequency. Parameter values, test results, and typical time-displacement curves for the single-shot and for automatic firing tests are given in Appendices A and B.

3. MOTOR CHARACTERISTICS

a. The maximum motor torque and speed requirements are, respectively, functions of the peak recoil displacement and peak counterrecoil velocity of the weapon, and the overall gear ratio that provides the conversion from rotary to translational motion. The motor power requirement can be determined from the basic system parameters of mass, stiffness, damping, and the forcing function magnitude and waveform.
GLOSSARY OF SYMBOLS

T = Motor torque
Tp = Peak motor torque
n = Gear ratio
r = Radius of last gear
I = Impulse
K = Spring rate
M = Mass
J = Moment-of-inertia
ξ = Damping ratio
ω₁ = Undamped natural frequency = \( \left( \frac{K}{M} \right)^{1/2} \)
ω = Damped natural frequency = \( ω₁ \left( 1 - ξ^2 \right)^{1/2} \)
X = Displacement
Xₚₚ = Peak recoil displacement
Xₜₜ = Peak counterrecoil velocity
X = Velocity
b = Weighting constant
W = Motor speed
Wₚ = Peak motor speed
s = Laplace operator
c = \( \frac{1}{550} \)
t = time
HP = Horsepower
GLOSSARY OF SYMBOLS - Continued

B = Damper

$f_m$ = Mount natural frequency

$f_g$ = Weapon firing rate

$U_0(t - \lambda)$ = Unit impulse at $t = \lambda$

$U_1(t - \lambda)$ = Unit step function at $t = \lambda$

P = Peak recoil displacement ratio

q = Peak counterrecoil velocity ratio

$k = \frac{f_m}{f_g}$
3. MOTOR CHARACTERISTICS - Continued

Generalized Mechanical Coupling

At peak displacement, the motor torque is
\[ T_p = F_{rn} = r_n K X_{RP} \]  \hspace{1cm} 3-1

At the maximum counterrecoil velocity, the motor speed is
\[ W_p = \frac{X_{C-RP}}{r_n} \]  \hspace{1cm} 3-2

The peak motor power is
\[ T_p W_p = K X_{RP} X_{C-RP} \]  \hspace{1cm} 3-3

General Torque-Speed Characteristics

The motor horsepower requirement is given by
\[ HP = c W T \]  \hspace{1cm} 3-4
3. MOTOR CHARACTERISTICS - Continued

From the general torque-speed characteristic,

\[ T = T_p (1 - \frac{W}{W_p}) \]  

From 3-4 and 3-5,

\[ HP = C T_p W (1 - \frac{W}{W_p}) \]  

Differentiating 3-6 and setting equal to zero, gives

\[ \frac{d HP}{dW} = C T_p (1 - \frac{2W}{W_p}) \]  

\[ W = \frac{W_p}{2} \]

Substituting 3-7 into 3-6 gives maximum horsepower

\[ HP_{\text{max}} = \frac{c T_p W_p}{4} \]  

From 3-3 and 3-8,

\[ HP_{\text{maximum}} = \frac{c K X_{RP} X_{C-RP}}{4} \]  

b. To establish the magnitude of the motor horsepower, two different simulation requirements will be considered.

Case I: Ideal Impulse Forcing Function

Generalized Time Response, \( X(b) \)
3. MOTOR CHARACTERISTICS - Continued

The impulse response is

\[ X(t) = \frac{I}{(KM)^{1/2}} e^{-\frac{\xi}{2} \omega_1 t} \sin \omega_1 t, \quad \xi \text{ SMALL} \quad 3-10 \]

\[ X_{RP} = \frac{I}{(KM)^{1/2}} e^{-\frac{\xi}{2} \pi / 2} \quad 3-11 \]

\[ \dot{X}(t) = \frac{I}{M} e^{-\frac{\xi}{2} \omega_1 t} \cos \omega_1 t \quad 3-12 \]

\[ \left| \dot{X}_{C-RP} \right| = \frac{I}{M} e^{-\frac{\xi}{2} \pi} \quad 3-13 \]

From Equations 3-9, 3-11, and 3-13

\[ H P_{\text{MAXIMUM}} = \frac{C K^{1/2} I^2}{4 M^{3/2}} e^{-3 \xi \pi / 2} \quad 3-14 \]

Equation 3-14 represents only the single-shot power requirement.
The effect of resonance will be determined below for various ratios of
mount natural frequency to weapon firing rate.

When \( f_m / f_g \neq 1 \)

\[ I \quad 0 \quad 2\pi / \omega_1 \quad 4\pi / \omega_1 \quad 2\pi / \omega_1 \quad \rightarrow t \]

Generalized Repetitive Impulse Input
Applying the laws of superposition,

\[ X^{(M)}_{RP} = \sum_{n=1}^{\infty} U_0(t-\lambda)x(t), \quad \text{WHERE } \lambda = \frac{(4M-3)\pi}{2\omega}, \quad M = \text{NUMBER OF SHOTS} \]

AND \( x(t) \) IS GIVEN IN 3-10

Expanding the series,

\[ X^{(M)}_{RP} = \frac{I}{(KM)^{1/2}} \left[ e^{-\frac{\pi \xi}{2}} + e^{-\frac{5\pi \xi}{2}} + e^{-\frac{9\pi \xi}{2}} + \cdots \right] \] 3-16

Rewriting Equation 3-16,

\[ X^{(M)}_{RP} = \frac{I}{(KM)^{1/2}} e^{-\frac{\pi \xi}{2}} \sum_{n=0}^{\infty} e^{-2\xi \pi n}, \quad n = m-1 \] 3-17

Equation 3-17 reduces to

\[ X^{(\infty)}_{RP} = \frac{I}{(KM)^{1/2}} \cdot \frac{e^{-\frac{\pi \xi}{2}}}{1-e^{-2\xi \pi}} \] 3-18

The displacement multiplier

\[ P_{1} = \frac{X^{(\infty)}_{RP}}{X^{(1)}_{RP}} = \frac{1}{1-e^{-2\xi \pi}} \] 3-19

\[ X^{(M)}_{C-RP} = \sum_{M=1}^{\infty} U_0(t-\lambda)x(t), \quad \text{WHERE } \lambda = (2M-1)\frac{\pi}{\omega}, \] 3-20
3. MOTOR CHARACTERISTICS - Continued

Expanding Equation 3-20 gives

$$X^{(\infty)}_{CRP} = \frac{I}{M} \left[ e^{-\xi \pi} + e^{-3\xi \pi} + e^{-5\xi \pi} + \ldots \right]$$ 3-21

Equation 3-21 reduces to

$$X^{(\infty)}_{CRP} = \frac{I}{M} \cdot \frac{e^{-\xi \pi}}{1 - e^{-2\xi \pi}}$$ 3-22

The velocity multiplier

$$q = \frac{\dot{X}^{(\infty)}_{CRP}}{\dot{X}^{(1)}_{CRP}} = \frac{1}{1 - e^{-2\xi \pi}}$$ 3-23

From Equations 3-19 and 3-23, it is seen that q = p; therefore, the power multiplier is

$$P = \frac{1}{(1 - e^{-2\xi \pi})^2}$$ 3-24

When \( \frac{f_m}{f_g} = 2 \)

$$X^{(M)}_{RP} = \sum_{M=1}^{\infty} U_0(t-\lambda) x(t), \lambda = \left( 8M - 7 \right) \frac{\pi}{2W}$$ 3-25

Then by steps of Equations 3-16 through 3-18

$$P_2 = \frac{1}{1 - e^{-4\xi \pi}}$$ 3-26

When \( \frac{f_m}{f_g} = 3 \) then

$$P_3 = \frac{1}{1 - e^{-6\xi \pi}}$$ 3-27
3. MOTOR CHARACTERISTICS - Continued

Generalizing the multiplier,

\[ P = \frac{1}{1 - e^{-2\xi \pi}} \quad \xi = \frac{f m}{f g} = 1, 2, 3, \text{ etc.} \quad 3-28 \]

Then the upper bound of horsepower is

\[ \overline{HP} = \frac{C K^{1/2} I^2 e^{-3\xi \pi/2}}{4 M^{3/2}} \quad P^2 \quad 3-29 \]

Case II: Time-Distributed Forcing Function

Generalized Forcing Function

\[ X(t) = U_i(t) r(t) - U_i(t - \lambda) r(t) \]

WHERE \[ r(t) = \frac{F}{K} \left[ 1 - e^{-\xi \omega_n t} \cos \omega_n t \right] \quad 3-30 \]

Single-Shot Response

-16-
3. MOTOR CHARACTERISTICS - Continued

The peak displacement is given by

\[ X_{RP} = \frac{F}{K} \left[ 1 + e^{-\frac{\pi}{\lambda}} \right], \text{ where } \lambda = \frac{\pi}{\omega_i} \quad 3-31 \]

The peak velocity is given by

\[ |\dot{X}_{c-RP}| = \frac{F}{(KM)^{1/2}} e^{-\frac{\pi}{2\lambda}} \left[ 1 + e^{-\frac{\pi}{\lambda}} \right] \quad 3-32 \]

From Equations 3-9, 3-31, and 3-32,

\[ HP = \frac{c F^2}{4 (KM)^{1/2}} \left[ 1 + e^{-\frac{\pi}{\lambda}} \right]^2 \quad 3-33 \]

Equation 3-33 represents only the single-shot power requirement. The effect of resonance will be determined below.

For burst-firing, the forcing function is

Generalized Repetitive Time-Distributed Impulse

When \( \frac{\epsilon_m}{\epsilon_g} = 1\), \( \lambda = \frac{\pi}{\omega_i} \) AND \( \gamma = 2 \frac{\pi}{\omega_i} \)
3. MOTOR CHARACTERISTICS - Continued

\[ X_{RP}(m) = \sum_{m=1}^{\infty} X_r \left\{(2m-1)\frac{\pi}{\omega}\right\} \quad m = \text{number of shots} \]  
and \( X_r = X(t) \) at \( t = (2m-1) \)

\[ X(t) = \frac{F}{k} \left[ 1 - e^{-\frac{t}{\omega}} \cos\omega t \right] - U_{-1}(t-\lambda) \frac{F}{k} \left[ e^{-\frac{t}{\omega}} \cos\omega(t-\lambda) \right] \]

\[ U_{-1}(t-\lambda) = \begin{cases} 0, & t < \frac{\pi}{\omega}, \\ 1, & t \geq \frac{\pi}{\omega}, \end{cases} \]

\[ X_r = \frac{F}{k} e^{2\xi \pi (1 + e^{-\xi \pi})} e^{-2\xi m \pi} \]

\[ X_{RP}(m) = \frac{F}{k} e^{2\xi \pi (1 + e^{-\xi \pi})} \sum_{m=1}^{\infty} e^{-2\xi m \pi} \]

\[ X_{RP}^{(\infty)} = \frac{F/K(1 + e^{-\xi \pi})}{1 - e^{-2\xi \pi}} \]

\[ P = \frac{1}{1 - e^{-2\xi \pi}} \]
3. MOTOR CHARACTERISTICS - Continued

$$\left| \dot{X}_{C-RP} \right| = \sum_{M=1}^{\infty} \dot{X}_{C-R} \left\{ (4M-1)\frac{\pi}{2} w \right\}$$

where \( \dot{X}_{C-R} = \dot{X}(t) \) at \( t = (4M-1)\frac{\pi}{2} w \)

$$\left| \dot{X}_{C-R} \right| = \frac{F}{(KM)^{1/2}} e^{-\xi \frac{\pi}{2} (4M-1)} + \frac{F}{(KM)^{1/2}} e^{-\xi \frac{\pi}{2} (4M-3)}$$

$$= \frac{F}{(KM)^{1/2}} e^{3\xi \frac{\pi}{2} (1 + e^{-\xi \pi})} e^{-2\xi \pi M}$$

$$\left| \dot{X}_{C-RP} \right| = \frac{F}{(KM)^{1/2}} \frac{e^{\xi \frac{\pi}{2} (1 + e^{-\xi \pi})}}{1 - e^{-2\xi \pi}}$$

$$q = \frac{1}{1 - e^{-2\xi \pi k}}$$

In Case II, just as in Case I, \( q = P \) and the power multiplier becomes \( P^2 \).

When \( \frac{f_m}{f_0} = 2 \), \( \lambda = \frac{\pi}{w} \) and \( \nu 4.5 \frac{\pi}{w} \).

$$X_{RP}(M) = \sum_{M=1}^{\infty} X_r \left\{ (4M-3)\frac{\pi}{w} \right\}$$
3. MOTOR CHARACTERISTICS - Continued

\[ X(\infty) = \frac{F/K}{1 - e^{-\xi \pi}} \]

\[ P = \frac{1}{1 - e^{-4 \xi \pi}} \]

The multiplier can be generalized to

\[ P = \frac{1}{1 - e^{-2\pi \eta}} K, \quad \xi = \frac{f_m}{f_g} = 1, 2, 3, \text{ etc.} \]

And then the upper bound of horsepower is

\[ H_P = \frac{C F^2 e^{-\xi \pi/2} (1 + e^{-\xi \pi})^2}{4 (K M)^{1/2}} P^2 \]

\text{ c. To establish values of motor horsepower, parameters representing typical limits of small arms ammunition and weapon mass will be considered. Also, various values of structural damping representing typically encountered values will be assumed to provide a range of resonance multipliers. The specific values of spring rates that will be assumed are those that will make the mount natural frequencies represent the fundamental, second, and third harmonics of the firing rate.}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\multicolumn{4}{|c|}{Resonant Power Multiplier, \( P^2 \)} \\
\hline
\( \xi \) & .05 & .075 & .1 \\
\hline
1 & 13.7 & 7.1 & 4.6 \\
2 & 4.6 & 2.7 & 1.96 \\
3 & 2.7 & 1.73 & 1.38 \\
\hline
\end{tabular}
\caption{Table I}
\end{table}

\text{Case I: Ideal Impulse Forcing Function}

Let: \( M = 10 \text{ slugs} \quad I = 30 \text{ lb-sec} \)

\( \xi = .05, .075, .1 \quad k = 1, 2, 3 \)

\( f_g = 10/\text{sec} \)

the stiffness \( K \), will be taken as
3. MOTOR CHARACTERISTICS - Continued

\[ K = k \omega_i^2 M \]

From Equations 3-29 and 3-49, the upper bound of motor horsepower is

\[ \frac{\text{HP}}{4M} = \frac{c \omega_i^2 e^{-3 \psi_{\eta/2}}}{p^2} \]

\[ = \frac{30^2 \times 10 \times 6.28 \times k e^{-3 \psi_{\eta/2}}}{2200 \times 10} \]

\[ = 2.57 k e^{-3 \psi_{\eta/2}} p^2 \]

<table>
<thead>
<tr>
<th>( k/ \psi )</th>
<th>( .05 )</th>
<th>( .075 )</th>
<th>( .1 )</th>
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<tr>
<td>1</td>
<td>27.8</td>
<td>12.75</td>
<td>7.4</td>
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<tr>
<td>2</td>
<td>18.7</td>
<td>9.63</td>
<td>6.26</td>
</tr>
<tr>
<td>3</td>
<td>16.3</td>
<td>9.4</td>
<td>6.65</td>
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Case II: Reduced Mass and Impulse

Let \( M = 3 \) slugs \( I = 6 \) lb-sec

<table>
<thead>
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<th>( .075 )</th>
<th>( .1 )</th>
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<td>.85</td>
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<td>3</td>
<td>2.18</td>
<td>1.25</td>
<td>.9</td>
</tr>
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TABLE III

Motor Horsepower for Case II
3. MOTOR CHARACTERISTICS - Continued

Case III: Time - Distributed Forcing Function

Let: \( F = 1000 \) pounds for .05 seconds, all other parameters are the same as in Case I.

From Equations 3-48 and 3-49, the upper bound of motor horsepower is

\[
\text{HP} = \frac{CF^2 e^{-\xi \pi/2}}{4k} \left(1 + e^{-\xi \pi}ight) \quad \text{p}^2
\]

\[
= \frac{1000^2 e^{-\xi \pi/2}}{2200 \times k \times 10 \times 6.28} \left(1 + e^{-\xi \pi}ight) \quad \text{p}^2
\]

\[
= \frac{7.25 e^{-\xi \pi/2}}{k} \left(1 + e^{-\xi \pi}ight) \quad \text{p}^2
\]

### TABLE IV

Motor Horsepower for Case III

<table>
<thead>
<tr>
<th>( k^{\xi} )</th>
<th>.05</th>
<th>.075</th>
<th>.1</th>
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<tr>
<td>1</td>
<td>31.5</td>
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### TABLE V

Spring Rates in Pounds/Inch for Cases I, II and III

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<td>985</td>
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4. ELECTRIC POWER REQUIREMENTS

The total electric power requirement is determined from the motor horsepower requirement, the motor efficiency, and the controller efficiency. Rotary power amplifiers of the Ward-Leonard or amplitidyne type must be considered as well as an electronic controller in the power calculation.
4. ELECTRIC POWER REQUIREMENTS - Continued

Motor Watts = \( \frac{746 \text{ HP}}{\text{Motor Eff.}} = \frac{746 \times 15}{0.8} = 14,000 \)\(^{4-1}\)

Electronic Controller Input = \( \frac{\text{Motor Watts}}{\text{Controller Eff.}} = \frac{14000}{0.9} = 15,500 \)\(^{4-2}\)

Two-Stage Rotary Amplifier = \( \frac{14000}{0.6} = 23,300 \) watts\(^{4-3}\)

For the two-stage rotary amplifier, the prime mover is generally a three-phase motor with a 0.8 power factor. Therefore, the KVA requirement is

\[ \text{KVA} = \frac{\text{KW}}{\text{Power factor}} = \frac{23.3}{0.8} = 29.2 \] \(^{4-4}\)

5. PHYSICAL SIZE OF COMPONENTS

Motor Weight: Approximately 320 pounds

Motor Dimensions:
- Length, 20 inches
- Height, 14 inches
- Depth, 24 inches

Controller Weight: 500 pounds (total)

Operator's Console:
- Length, 5 feet
- Height, 4 feet
- Depth, 3 feet

Power Units:
- Length, 4 feet
- Height, 6 feet
- Depth, 2 feet

Simulator Base: The simulator base is considered as an isolated mass large enough in value so that it does not affect the frequency of oscillation or amplitude of displacement of the simulator.
Generalized Installation Dynamics

\[ \begin{align*}
\left[ M_1 s^2 + B s + K \right] X_1(s) - \left[ K + B s \right] X_2(s) &= F(s) \\
- \left[ K + B s \right] X_1(s) + \left[ M_2 s^2 + B s + K \right] X_2(s) &= 0
\end{align*} \]

\[ X_1(s) - X_2(s) = \frac{M_2}{K(M_1 + M_2)} \cdot \frac{F(s)}{\frac{M_1 M_2}{K(M_1 + M_2)} s^2 + \frac{B}{K} s + 1} \]

For \( M_2 = 100 M_1 \),

\[ X_1(s) - X_2(s) = \frac{1}{1.01 K} \cdot \frac{F(s)}{\frac{100 M_1}{K} s^2 + \frac{B}{K} s + 1} \]

For \( M_2 = \infty \) \( s - 4 \) becomes

\[ X_1(s) = \frac{1}{K} \frac{F(s)}{\frac{M_1}{K} s^2 + \frac{B}{K} s + 1} \]
5. PHYSICAL SIZE OF COMPONENTS - Continued

By comparison of Equations 5-4 and 5-5, it is seen that, for $M_2 = 100 M$, there is a reduction in natural frequency of 0.5 per cent and in amplitude of 1.0 per cent. Considering the above indicated differences as negligible leads to the conclusion that the minimum acceptable simulator base weight is 100 times the weight of the weapon and mount.

- Simulator Base Weight: 32,000 pounds
- Working Surface: 7 feet by 7 feet
- Height Above Floor: 2 feet, 6 inches
- Minimum Depth Below Floor: 3 feet
REPORT
SA-TR20-2906

APPENDICES

A - Results of Single-Shot Firing (Typical Record)
B - Results of Automatic Firing (Typical Record)
C - Distribution
RESULTS OF SINGLE-SHOT FIRINGS

Experimental Model Parameters
Test Results

Time-Displacement Curve
  Measured Displacement
  Predicted Displacement

Spring Simulator Electromechanical Servoloop
  (Right side view)

Spring Simulator Electromechanical Servoloop
  (Left side view)
EXPERIMENTAL MODEL PARAMETERS

\[ J = 0.03 \text{ lb-ft-sec}^2 \]
\[ K_t = 0.158 \text{ lb-ft/volt} \]
\[ K_b = 1.56 \text{ volt/rad/sec} \]
\[ K_a = 20, 50, 100, 200 \]
\[ r = 1.125 \text{ in.} \]
\[ I = 3.0 \text{ lb-sec} \]
\[ T = 0.004 \text{ sec} \]

TEST RESULTS

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Measured Displacement

300 16/in
Scale 1/4

0.3"

Displacement

Predicted Displacement

0.300"

300 16/in
Scale 1/4

TIME-DISPLACEMENT CURVE
Test Conditions

Typical Time-Displacement Curves

Photographs
Automatic firing tests were conducted to demonstrate that a reasonable level of mount dynamics similitude can be obtained with an unsophisticated model. The level of similitude that was obtained can be seen by comparing the burst firing time-displacement curves of the simulator with those curves obtained from the four leaf cantilever beam mount. The observable differences in the time-displacement records are related primarily to the test-to-test variation in weapon rate-of-fire and to the differences in damping forces. The cantilever beam mount exhibits a viscous friction damping, whereas the simulator damping is composed of both viscous and coulomb frictions. The coulomb friction portion of simulator damping is related to the motor commutator brush and gear antibacklash loads.

Test Conditions:

- Weapon: 5.56mm MG
- Ammo Impulse: 1.0 pound-second (nominal)
- Mount Stiffness: 1800 pounds/inch
- Total Mass: 3.2 slugs
CANTILEVER BEAM TIME-DISPLACEMENT CURVE
4 BEAM CONFIGURATION

MOUNT RATE: 1800 POUNDS/INCH
WEAPON: 5.56mm MG
FIRING RATE: 765 SPM

COUNTER-RECOIL
0.05"

RECOIL

MOUNT RATE: 1800 POUNDS/INCH
WEAPON: 5.56mm MG
FIRING RATE: 740 SPM

COUNTER-RECOIL
0.05"

RECOIL

SIMULATOR TIME-DISPLACEMENT CURVE
ELECTROMECHANICAL SERVOLOOP CONCEPT

0.05"
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Director of Defense Research and Engineering
The Pentagon
Washington, D.C. 20310

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The electric power and site requirements are established for a single degree-of-motion-freedom machine gun mount simulator capable of supporting all automatic small arms through 30 millimeter weapons. The concept of using an electromechanical servomechanism to simulate the mass, stiffness, and damping characteristics of the gun mount is shown analytically to be feasible. Experimental verification of the simulation concept is demonstrated by burst-firing of a 5.56 millimeter machine gun on a scale model.
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### KEY WORDS

1. Gun mounts
2. Recoil mechanisms
3. Structural properties

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