DETERMINING MAXIMUM VALUES OF GRAVITATIONAL DRIFT AND DRIFT FROM UNEQUAL RIGIDITY OF FLOATING INTEGRATION GYROSCOPES

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The moment of interferences, affecting the gyro unit of a floating integration gyroscope, consists in the general case of moments, not depending upon acceleration and moments, depending upon acceleration. To the first one belongs moments, created by flexible current conductors and convectional liquid streams, reactive moments of angle and moment sensing elements, etc. The moments depending upon acceleration, appear to be moments due to unequal suspension and unequal rigidity of the gyro unit. The originations of treatments of the gyroscope because of unequal rigidity of the system of suspension was first mentioned by A. Yu. Ishlinskiy [1].

During the movement of the instrument with linear acceleration \( \ddot{X} \), the lift and weight (more accurately - "apparent weight") of the gyro unit will be accordingly equal [2] \( m_1 \omega \) and \( m \omega (m_1 = \rho V \), mass of the liquid in volume \( V \) of the gyro unit, \( m \) - mass of gyro unit, \( \omega = |\dot{g} - \ddot{a}| \), \( g \) - acceleration of gravitational force, and the floating axis of the gyro unit will be
directed by the "apparent vertical". Consequently at \( a \neq 0 \), the difference between the lifting force and weight force of the gyro unit changes in \( \ddot{g} - \ddot{a} / g \) once in comparison with its value at \( a = 0 \), the differential sign here remains unchanged. As a result of this, the moment, due to unequal suspension of the gyro unit, will be proportional to the first degree of vector projection \( \ddot{w} = \ddot{g} - \ddot{a} \) on the lateral plane of the instrument \( yz_0 \) (\( y \) and \( z_0 \) output and transverse axes of the instrument). Accordingly also the drift, caused by this moment, will also be proportional to the first degree of vector projection \( w \) on plane \( yz_0 \).

Since the drift is due to unequal suspension of the gyro unit, it appears to be an acceleration function, then it should be evaluated in \( \text{deg/hr} \), i.e., to evaluate by the value of angular rate of drift, observed on the ground in the absence of transferable accelerations \( (a = 0) \). The drift due nonequal suspension of the gyro unit, observed on the ground in the absence of transfer accelerations \( (a = 0) \) and during horizontal position of the output axis of the instrument \( x \), will be called gravitational drift. At \( a \neq 0 \) and horizontal position of the axis \( x \) the drift, due to unequal suspension of the gyro unit, will be equal to gravitational drift, multiplied by the expressed in fractions, \( g \) value of vector projection \( \dot{w} = \dot{g} - \dot{a} \) on plane \( yz_0 \).

In floating gyroscopes the moment due to unequal rigidity of the gyro unit, originates because, that in an unequal rigidity gyro unit the elastic displacements of the center of gravity and the center of pressure take place in directions, not coinciding with the direction of action of the forces, causing these displacements. This moment, and consequently, the drift caused by it, are proportional to the second degree of vector projection \( \ddot{w} = \ddot{g} - \ddot{a} \) on the
transverse plane of the instrument $y_{90}$. In conformity with this, the drift due to unequal rigidity of the gyro unit, should be evaluated in $\text{deg/hr} \sqrt{\text{g}}$, i.e., evaluated by the value of angular speed of the drift, observed on the ground in the absence of transferable accelerations ($a = 0$).

At first, the gyro unit of floating integrating gyroscopes consisted of a frame with gyromotor and hollow cylinder, which was fitted over the side sections of the frame, made in the form of disks, and hermetically connected with it. But with the same time of adding to the gyro unit of greater rigidity, the frame was removed and its functions were imposed directly on the float. The float consisted of two parts (each one in the form of a glass) hermetically connected with each other. At first, in one of them the gyromotor was secured. At such gyro unit construction, efforts should be made that the gyro motor should be equi-rigid, and the float—practically absolutely rigid with respect to maximum possible values of the forces affecting same.
Assuming, that the float appears to be practically absolutely rigid, and the construction of the gyro motor does not satisfy the condition of equi-rigidity and it is made with a sational axis, we will bring out expressions for moments $M_2$ and $M_3$, due congruently to unequi-suspension and unequi-rigidity of the gyro unit. For this we will introduce the following designations (see figure).

Point O—trace of output axis of the instrument x (axes of rotation of gyro unit), directed behind the plane of the drawing;

- $z$ - axis of rotation of gyro motor rotor;
- $y$ - axis, perpendicular to axes $x$ and $z$;
- $n$ and $\xi$ - horizontal and vertical axes;
- $\Theta$ - angle of inclination of axis $z$ to the plane of the horizon;
- $g$ - acceleration of gravitation force;
- $m_1$ - mass of liquid in volume of gyro unit;
- $r_1; \psi_1$ - polar coordinates of center of pressure $O_1$;
- $m_2$ - mass of gyro unit without gyro motor;
- $r_2; \psi_2$ - polar coordinates of point $O_2$ of application of weight force $m_2g$;
- $m_3$ - mass of gyro motor;
- $r_3; \psi_3$ - polar coordinates of point $O_3$ in which the center of mass of the gyro motor would be situated, if the gyromotor would be absolutely rigid;
- $m_4$ - mass of gyromotor rotor unit, subjected to elastic displacement parallel to axis $y$ as well as parallel to axis $z$.
- $m_3-m_4$ - mass of axis unit of gyromotor with stator, subjected to elastic displacement only parallel to axis $y$;
- $O_4$ - point of application of weight force $(m_3-m_4)g$;
- $O_5$ - point of application of weight force $m_4g$;
Figure. For the derivation of expression for moment $M_{2,3}$ due to unequal suspension and unequal rigidity of the unit (gyro unit).

$m = m_2 + m_3$ - weight mass of gyro unit;

$r, \psi$ - polar coordinates of the center of mass of the entire gyro unit at absolutely rigid gyrometer;

$K_{3u}$ - transverse rigidity of the unit of the axis with stator;

$K_{4u}$ and $K_{4z}$ - correspondingly radial and axial rigidity of the unit of rotor with bearings;

\[ \Delta_{1y} = \frac{g_{ms}}{K_{sy}} \cos \theta \quad (1) \]

- elastic displacement of center of unit mass of the axis with stator parallel to axis $y$;

\[ \Delta_{2y} = \frac{g_{ms}}{K_{sy}} \cos \theta; \quad \Delta_r = \frac{g_{ms}}{K_{4r}} \sin \theta \quad (2) \]

- elastic displacement of center of rotor unit mass with bearings relative to the center of mass of the axis unit and stator parallel to $y$ and $z$ respectively.

On the basis of the expressions (1) and (2) we obtain, that the forces, depicted in the figure create around axis $x$, a moment
\[
M_{2,3} = gmr \cos(\theta + \psi) - gm_{1}r_{1} \cos(\theta + \psi) + \\
+ \frac{g^{2}}{2} \left[ \frac{m_{3}^{2}}{K_{y}} + m_{2}^{2} \left( \frac{1}{K_{y}} - \frac{1}{K_{z}} \right) \right] \sin 2\theta ,
\]

where

\[
r = \frac{1}{m} \sqrt{(m_{2}r_{2})^{2} + (m_{3}r_{3})^{2} + 2m_{2}m_{3}r_{2}r_{3} \cos(\psi_{2} - \psi_{3})}.
\]

\[
\tan \psi = \frac{m_{2}r_{2} \sin \psi_{2} + m_{3}r_{3} \sin \psi_{3}}{m_{2}r_{2} \cos \psi_{2} + m_{3}r_{3} \cos \psi_{3}}.
\]

In expression (3) the first two components represent a moment \(M_{2}\), due to unique-suspension of the gyro unit relative to axis \(x\). In this way

\[
M_{2} = g \{mr \cos(\theta + \psi) - mr_{1} \cos(\theta + \psi_{1})\}.
\]

The third component appears to be the moment \(M_{3}\), due to unique-rigidity of gyromotor. Consequently,

\[
M_{3} = \frac{g^{2}}{2} \left[ \frac{m_{3}^{2}}{K_{y}} + m_{2}^{2} \left( \frac{1}{K_{y}} - \frac{1}{K_{z}} \right) \right] \sin 2\theta .
\]

Maximum value of moment \(M_{2}\):

\[
M_{2_{\text{max}}} = g \sqrt{(mr)^{2} + (m_{1}r_{1})^{2} - 2m_{1}r_{1} \cos(\psi_{1} - \psi)}
\]

is obtained at an angle \(\Theta_{c}\), equaling

\[
\Theta_{c} = \arctan \frac{m_{1}r_{1} \sin \psi_{1} - mr \sin \psi}{mr \cos \psi - m_{1}r_{1} \cos \psi_{1}}.
\]

Using equalities (6) and (7) we can represent equation (4) in the form of

\[
M_{2} = M_{2_{\text{max}}} \cos(\theta - \Theta_{c}).
\]

Moment \(M_{2}\) principally can be made equal to zero for any given values of angle \(\Theta\) by various methods. But in order, that \(M_{2} = 0\) at any given possible values of mass \(m_{1}\), which appears to be the the function of temperature of the liquid, it is necessary to attain the thing, that \(r = r_{1} = 0\), i. e., that the center of mass and the center of pressure of the gyro unit should
lie on its axis of rotation. Any other given method of turning into the moment \( M_2 \) equality \( M_2 = 0 \) will not be invariant with respect to possible changes in mass \( m_1 \). And so, for example, if at a certain liquid temperature moment \( M_2 \) is turned into zero by bringing the center of mass and center of pressure into one point, not lying on the axis of rotation of the gyro unit \( (m = m_1, \psi = \psi, r = r_1 \neq 0) \) then at a change in temperature the equality \( m_1 = m \) is disrupted because of change in \( m_1 \), in consequence of which \( r = r_1 \neq 0 \), moment \( M_2 \) will cease being zero.

To attain this, that the moment \( M_2 \) should be accurately and stably equal, zero is very difficult. A reduction of moment \( M_2 \) to a value close to zero, is obtained by thorough balancing of the gyro unit by a special technology with the use of highly precision devices.

From equation (5), it is evident that at a change in angle \( \theta \), moment \( M_3 \) changes with dual frequency, and its amplitude value

\[
M_{3, \text{max}} = \frac{\alpha^2}{2} \left[ \frac{m_2^2}{K_{sy}} + m_4^2 \left( \frac{1}{K_{sy}} - \frac{1}{K_{ss}} \right) \right]
\]

(9)

is obtained at angles

\[
\theta = \frac{\pi}{4} (2k + 1); \quad k = 0, 1, 2, \ldots
\]

(10)

In order that moment \( M_3 \) should be equal to zero at any given value of angle \( \theta \), it is necessary to observe the following condition

\[
\frac{m_2^2}{K_{sy}} + m_4^2 \left( \frac{1}{K_{sy}} - \frac{1}{K_{ss}} \right) = 0,
\]

which also appears to be equirigidity condition in the case under question. When an accurate fulfillment of this condition appears to be difficult, the remaining value of the moment, due to unequi-rigidity, may be compensated with the aid of a special compensator, representing a small mass, fastened
Turning the spring around its longitudinal axis, it is also possible to change the direction of its deformation, and consequently, also the direction of mass displacement. Selecting experimentally the position of the spring, it is possible to obtain principally total equirigidity of the system. The given compensator reduces the rigidity of the gyro unit in the direction of that axis, along which it has the greater rigidity.

In the general case, the drift of a floating integration gyroscope is caused by the moment

\[ M = M_1 + M_2 + M_3, \]

which is active around axis \( x \), where \( M_1 \) - moment of interferences, not depending upon acceleration, in other words, a moment due to causes, not connected with the unequi suspension and unequirigidity of the gyro unit. Moments \( M_2 \) and \( M_3 \) are determined by formulas (8) and (5). Correspondingly, in the general case the angular velocity of the drift

\[ \omega_z = \omega_{z1} + \omega_{z2} + \omega_{z3}, \]  

\[ \text{where} \]

\[ \omega_{z1} = \frac{M_1}{H}, \quad \omega_{z2} = \frac{M_2}{H}, \quad \omega_{z3} = \frac{M_3}{H}. \]  

Here \( H \) - natural (kinetic) moment of gyroscope.

We will designate

\[ \omega_{z1}^{\max} = \frac{M_1^{\max}}{H} \]  

- maximum value of drift velocity, due to the moment of interferences, not depending upon acceleration:

\[ \omega_{z2}^{\max} = \frac{M_2^{\max}}{H} \]  

- maximum value of drift velocity, due to unequisuspension of gyro unit;
- maximum value of drift velocity, due to unequirigidity of the gyro unit (in our case, gyromotor).

When planning a floating integration gyrooscope, the kinetic moment $J$ and quality $D$ of the gyromotor ($D = H/P_3$, $P_3$ - weight of gyro motor) should be selected in dependence upon the given values $d_{2\text{max}}$ and $d_{3\text{max}}$ by the following formulas, obtained from expressions (14), (6) and (15), (9);

$$\frac{a}{D} \left(1 + bc\right) = \omega_{a3\text{max}};$$

$$\frac{H}{D^2} \left[\frac{1}{K_{xy}} + d^2 \left(\frac{1}{K_{xy}} - \frac{1}{K_{xa}}\right)\right] = \omega_{a3\text{max}},$$

where $a = P/P_3$ ($P$ - weight of gyro unit); $b = Q/PQ$ - weight of liquid in volume of gyro unit); $c = r_1/r$; $d = P_4/P_3$ ($P_4$ - weight of gyromotor-rotor).

Remaining designation as before.

Formula (16) was obtained in the assumption, that $\psi_1 - \psi = \psi'$, because in this case moment $H_{2\text{max}}$, determinable by formula (6), has at other equal conditions the maximum value.

In order that the component of angular drift velocity, which does not depend upon acceleration, should not exceed the maximum permissible value $d_{l\text{max}}$, it is necessary to observe the following inequality

$$M_{l\text{max}} < H\omega_{a1\text{max}}.$$

At an equirigid gyromotor $H$ and $D$ are recommended to be determined by formulas (18) and (16).

As is known, the angular velocity of the drift of floating integrating gyroscopes is determined with the aid of a special dynamic stand, which allows to measure same, at horizontal position of axis $x$ and various values of angle
0, as well as in the case, when axis x is vertical. We will assume that the angular velocity of the drift, measured at angles $\theta$ equal to 0, $\frac{\pi}{4}$ and $\frac{\pi}{2}$, equals correspondingly $\omega_d(0)$, $\omega_d\left(\frac{\pi}{4}\right)$ and $\omega_d\left(\frac{\pi}{2}\right)$. The angular velocity of the drift, measured at vertical position of axis x, will be designated by $\omega_d_{vert}$. This velocity can be adopted as velocity $\omega_d\left(\frac{\pi}{2}\right)$, because at vertical position of axis x, $M_2 = M_3 = 0$ and, consequently, the drift is caused only by moment $M_1$.

We will show how, knowing $\omega_d_{vert}$; $\omega_d(0)$; $\omega_d\left(\frac{\pi}{4}\right)$ and $\omega_d\left(\frac{\pi}{2}\right)$, it is possible to determine $\omega_{d2}^{\text{max}}$ and $\omega_{d3}^{\text{max}}$.

From equation (4) and (6) it is evident, that

$$M_2^{\text{max}} = \sqrt{M_2^2(0) + M_2^2\left(\frac{\pi}{2}\right)}, \quad (19)$$

where $M_2(0)$ and $M_2\left(\frac{\pi}{2}\right)$ - values of moment $M_2$ respectively at $\theta = 0$ and $\theta = \frac{\pi}{2}$.

Having replaced expression (19) in (14), we will obtain

$$\omega_{d2}^{\text{max}} = \sqrt{\omega_{d2}^2(0) + \omega_{d2}^2\left(\frac{\pi}{2}\right)}, \quad (20)$$

where $\omega_{d2}(0)$ and $\omega_{d2}\left(\frac{\pi}{2}\right)$ - angular velocities of gravitational drift respectively at $\theta = 0$ and $\theta = \frac{\pi}{2}$. From expression (5) and formula (12) for $\omega_{d3}$, it is evident that at these values of angle $\theta$, the angular velocity of the drift $\omega_{d3} = 0$ and, consequently, by formula (11)

$$\begin{align*}
\omega_{d2}(0) &= \omega_A(0) - \omega_A^{\text{rep}}, \\
\omega_{d2}\left(\frac{\pi}{2}\right) &= \omega_A\left(\frac{\pi}{2}\right) - \omega_A^{\text{rep}}.
\end{align*} \quad (21)$$

Substituting (21) in (20), we obtain a final formula for the calculation of maximum value of gravitational drift:
From (8) it is evident, that \( \omega_{d2 \text{max}} \) will be observed at angles \( \Theta \), equal \( \Theta^* \) and \( \pi + \Theta^* \). At both angles the direction of the drift will be identical.

\[
\theta^* = \arctg \frac{M_2 \left( \frac{\pi}{2} \right)}{M_2(0)}.
\]

Using equality (12) for \( \omega_{d2} \) and formula (21), we will obtain

\[
\theta^* = \arctg \frac{\omega_{d2} \left( \frac{\pi}{2} \right) - \omega_{d2 \text{mpt}}}{\omega_{d2}(0) - \omega_{d2 \text{mpt}}}.
\]

(23)

From expressions (5), (9) and (15), it is directly evident that \( d_{3 \text{max}} \) will take place at angle \( 0 \), determinable by equality (10) and, particularly at \( \Theta = \frac{\pi}{4} \). Consequently, on the basis of equation (11) we can write

\[
\omega_{A3 \text{max}} = \omega_{A} \left( \frac{\pi}{4} \right) - \omega_{A} \left( \frac{\pi}{4} \right) - \omega_{A \text{mpt}}.
\]

(24)

where \( \omega_{d2} \left( \frac{\pi}{4} \right) \) - the value \( \omega_{d2} \) at \( \Theta = \frac{\pi}{4} \).

Having adopted in expression (4) \( 0 = \frac{\pi}{4} \), we will obtain

\[
M_2 \left( \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \left[ M_2(0) + M_2 \left( \frac{\pi}{2} \right) \right].
\]

Substituting this value \( M_2 \left( \frac{\pi}{4} \right) \) in equation (12), for \( \omega_{d2} \) we obtain, that

\[
\omega_{A2} \left( \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \left[ \omega_{A2}(0) + \omega_{A2} \left( \frac{\pi}{2} \right) \right].
\]

Having substituted the given expression and equation (21) in (24) we obtain finally a formula for the calculation of maximum drift value, due to unequivalence of the gyro unit:
\[ \omega_{A3,\text{max}} = \omega_A\left(\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2} \left[\omega_A(0) + \omega_A\left(\frac{\pi}{2}\right)\right] + (\sqrt{2} - 1)\omega_A, \text{vert. vert.} \]  \hfill (25)

In this way, we calculate values \( \omega_{d2,\text{max}} \), \( \omega_{d3,\text{max}} \) and \( \Theta^* \) it is necessary to know, as is evident from formulas (22), (25) and (23), the angular velocities of the drift \( \omega_d(0) \), \( \omega_d\left(\frac{\pi}{4}\right) \), \( \omega_d\left(\frac{\pi}{2}\right) \) and \( \omega_d, \text{vert.} \). It should however, be mentioned that at horizontal position of axis \( x \), the rate \( \omega_d \) may differ somewhat from \( \omega_d, \text{vert.} \). Furthermore, together with the mentioned simple method of determining \( \omega_{d2,\text{max}} \) and \( \omega_{d3,\text{max}} \), another method should also be recommended, which consists in measuring \( \omega_d \) at a horizontal position of the axis \( x \) and various values of angle \( \Theta \) from 0 to 360°, within equal intervals, e. g., within 30°. Then process the dependence graph \( \omega_d \) from \( \Theta \) by the harmonic balance method.

Literature
