THE SPECTRAL STRUCTURE OF TURBULENCE IN A FREE ATMOSPHERE
BASSED ON DATA OBTAINED BY AIRCRAFT

by
G. N. Shur

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by George Kasachkoff

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SUBJECT: Errata for RSIC-543, Title: The Spectral Structure of Turbulence in a Free Atmosphere Based on Data Obtained by Aircraft

TO: Recipients of Subject Report

It is requested that the title page of the subject report be replaced with the enclosed page.

Theodore A. Woerner
Acting Chief, Translation Branch
Redstone Scientific Information Center
On the basis of the analysis of experimental data on turbulence obtained by a TU-104 aircraft, this article examines the energy spectrum of the vertical velocity component of turbulent gusts in a temperature-stratified atmosphere. The relationship between the spectra of the vertical and horizontal component in a stably stratified atmosphere is also examined.

The measurements of the gust loads in a narrow band of frequencies (wave numbers) make it possible to obtain the dispersions in the velocities of gusts in a wide range of scales.


The fully developed theory of local isotropic turbulence makes it possible to obtain a simple expression, known as spectral law "minus five-thirds", for the energy spectrum of turbulence inside the so-called "inertial" range.

\[ S(\Omega) = \epsilon^{\frac{2}{3}} \Omega^{-\frac{5}{3}}, \]  

in which \( \Omega = z \pi / l \) is the wave number (rad/m), \( l \) is the scale (m), \( \epsilon \) is the dissipation rate of the turbulent energy (m²/sec³), \( S(\Omega) \) is the spectral distribution density of turbulent energy as a function of wave number (energy spectrum m/sec²m/rad).

As a consequence of this law, it was assumed that the transfer rate of energy in the spectrum remains constant and equal to the transition rate from the kinetic to the thermal energy of turbulent fluctuations — the dissipation rate of energy.

The tests performed in 1960-61 by TSAO e- turbulence have indicated that the experimental curves of the spectral density substantially differ from the theoretical, especially in the area of small wave numbers (in the area of large scales). The experiment showed that the logarithmic curve has a much greater inclination than it should in accordance with the law "minus five-thirds".

Shur's study contains experimental spectral curves obtained mainly during flights in jet streams and clouds Ci and Cb.

In 1961-62, data on the spectral characteristics of the vertical
component of turbulent fluctuations in jet streams during the winter months were obtained.

Regrettfully, in the majority of flights in jet streams, the bumping of the aircraft, although present, was very slight. In instances when the recorded data were resolved into a spectrum, it was discovered that numerous spectral components have a smaller amplitude than the threshold of sensitivity of the measurement and analysis apparatus. This illustrates the fact that when correctly evaluating the integral energy characteristics (dispersion of the gust velocity or the magnitude of the mean-square gust itself) no essential data were obtained in terms of a spectral density curve.

Actually, valid data may be obtained by means of spectral analyzers described in Shur's work only in those instances when the aircraft experiences bumpiness of not less than $\sigma - \sigma^2$, on the adopted scale.

Continuing our investigation of the spectrum of the vertical component of turbulent fluctuations in wind flow, we have initially decided to perform an experimental verification of the established physical relations (here the change in the inclination of the logarithmic spectral curve is taken into account); then we attempted to show the physical nature of this.

Also of interest is the relationship of the vertical and horizontal component of the spectra as well as the relationship of the turbulent characteristics of the atmosphere to the distribution of the basic thermodynamic parameters.

Results of the observations which were collected in 1961-62 confirm the previously established experimental fact on the presence of different inclinations of the energy spectrum plotted in a logarithmic scale, in different ranges of the spectrum.

As an example, let us examine the turbulence spectrum obtained by the method described in a study by Shur.

Figure 1 shows the energy spectrum of turbulence during a crossing of the jet stream (flight made on 7 February 1962). The flight was at an altitude of 8000 meters. When crossing the jet stream, a moderate bumpiness which was strong at times was observed.

*The corresponding wind profile is shown in Figure 3 in an article by N. Z. Pinus entitled "Some Results of Investigations of the Meso- and Microstructure of the Wind Field at an Altitude of 6-12 km", appearing in the current reference.
Wave numbers $\Omega$ in rad/m and scales $L = 2\pi/\Omega$ in meters were plotted along the abscissa axis, and the spectral density of the energy of a mass unit in $(m/sec^2 m/rad)$ was plotted along the ordinate axis. The curve was plotted in a logarithmic scale.

The curve shown in the figure may be approximated by two rectilinear segments. This indicates that the spectral density in different ranges of spectrum is depicted by two power laws appearing as

$$S(\Omega) = \Omega^n.$$ \hfill (2)

For wave numbers, corresponding to scales, less than 600 m, the indicator of power equals 1.67, i.e., the energy spectrum in this scale interval conforms nicely with the law "minus five-thirds" of Kolmogorov-Obukhov. On scales of greater than 700-800 m, the indicator of power $n$ equals 2.7. If the straight line, which approximates the energy spectrum in the small scales, is extended to the left toward the region of large scales, then the experimental value of spectral density obtained for maximum scales will differ from the value lying on this extended straight line by more than an order.

Our analysis on the accuracy of the method indicated that such variations cannot be explained as either being errors of the apparatus or the methods of measurements. Consequently, the obtained distribution of the energy density in the spectrum of turbulent flow in a free atmosphere has to be accepted as real. A typical energy spectrum, shown in Figure 1, has been thoroughly examined by us.

In the above example as well as in Shur's work\(^6\) dealing with experimental curves, the point of inflection is arbitrary. A considerably wide transition zone exists. However, it is possible to notice a change in the slope in all curves with the transition from small to large wave numbers, in the range of wave numbers of order $10^{-2}$ rad/m.

Since the variation in the experimental and theoretical spectra in the area at large scales is a well established fact, it becomes necessary to explore the causes of these variations.

Within the classic inertia range, the existence of which is implied by the theory of locally-isotropic turbulence, the transition rate of energy through the spectrum (the rate of energy transfer from large to smaller formations) should remain constant. Since the total kinetic energy of turbulence transfers to thermal energy in the area of small
scales, the transfer rate of energy in the "inertial" range of the spectrum, within the framework of the theory of locally-isotropic turbulence, is equated to the rate of dissipation.

Figure 1. Energy spectrum of turbulence when crossing the flow (7 February 1962). Strong bumpiness.

We will examine an instance when another mechanism, besides the transfer mechanism of energy from large to smaller vortices, occurs within the so-called "inertial" range and which either transforms the kinetic energy of turbulent vortices in some other form of energy or transforms the other form of energy into a kinetic energy of
turbulence*. At this time we are not making any assumptions as to the mechanism. We will only assume that it is more intense when the vortex scale is larger; and, on the other hand, that by its action in the area of small scales may be disregarded in the range of small scales.

Figure 2 depicts a family of logarithmic energy spectra $S(\Omega) = \frac{2}{3} \Omega^{-\frac{3}{2}}$ for various $\epsilon$, during which $\epsilon_7 > \epsilon_6 > \epsilon_5 > \epsilon_4 > \epsilon_3 > \epsilon_2 > \epsilon_1$. For locally-isotropic turbulence, the rate of the energy input in the "inertial" range is equal to the rate of output from this range interval (curve 1).

If the rate of energy input into the "inertial" range is greater than the output, then the slope of the curve in the range of small $\Omega$ (of the large scales) will be greater than $\frac{3}{2}$ (curve 2). This corresponds to a situation in which a "sink" consuming turbulent energy operates in the range of large scales.

*Geisenberg's work indicates that the introduction of a supplemental mechanism using a kinetic turbulent energy, outside the "inertial" range (in the range of dissipation), leads to an increase in the inclination of the logarithmic spectral curve.
On the other hand, if the rate of energy input is less than the output, then the slope of the curve in the range of small $\Omega$ will be less than $-\frac{9}{3}$ (curve 3). This is an indication that an additional source of energy generated turbulence in the range of small $\Omega$.

It is known that the atmosphere is stratified with regard to density, in which case stratification of the atmosphere, particularly in the upper troposphere, is, as a rule, stable. The turbulent mixing, which is a quasiadiabatic process, tends to reduce the temperature stratification to neutral.

The turbulent formations (vortices) have to act against Archimedean forces; consequently, the transition of the density in kinetic energy of turbulence into potential energy of stratification during a stable stratification (and a reverse transition during an unstable stratification) will be the same mechanism which causes a modification in the spectral curve from the "minus five-thirds" law within the so-called "inertial" range.

Strictly speaking, this range is no longer inertial in the same sense that it was when the term was introduced in connection with assumptions on local isotropy.

The assumed physical hypothesis may be formulated in the following way. It is apparent that the kinetic energy of the vortex in a free atmosphere will not transfer without losses to vortices of smaller scale. The atmosphere outside the boundary layer has a definite temperature stratification which in most cases is stable, and the vortex during its lifetime has to act against Archimedean forces. It is also apparent that losses of energy in small scale vortices in their action against Archimedean forces are negligible, while large scale vortices may lose a substantial part of their kinetic energy. The kinetic energy of turbulent motion will transfer to potential energy, and by this action the stratification will become neutral. This causes a disruption of the energy equilibrium inside the inertial subregion.

In the theory of locally-isotropic turbulence, it is assumed that the quantity of the energy of an average flow changing at one instance into energy of fluctuational motion is equal to the quantity of energy of fluctuational motion dissipating at one instance into heat. In other words, it is assumed that the transformation rate of energy (according to Obukhov) is equal to the dissipation rate; and this remains constant throughout the whole spectrum.

If the loss of energy counteracting Archimedean forces is considered, then it may be assumed that during a non-neutral stratification of the
atmosphere the transfer rate of energy through point Ω in the spectrum will not remain constant within the "inertial" range and will depend on the scale of the vortex.

Based on this hypothesis, it may be assumed that the trend of the curve of spectral density will correspond with that shown in Figure 2, at which time curve 1 will correspond to a neutral stratification, curve 2 to a stable, and curve 3 to an unstable stratification.

Shur's study presents an analytic expression for the spectral density of the vertical component of turbulent fluctuations of wind velocity, taking into account energy losses against Archimedean forces. The following expression is obtained for the transfer rate of energy through point Ω in the spectrum.

\[ \varepsilon = \varepsilon \left[ 1 + \frac{g}{T} (\gamma_a - \gamma) \varepsilon^{\frac{2}{3}} \Omega^{\frac{4}{3}} \right], \quad (3) \]

in which \( \gamma \) and \( \gamma_a \) are the actual and the adiabatic temperature gradients, \( T \) is the temperature, \( g \) is the acceleration due to gravity, \( \varepsilon \) is the rate of dissipation.

Expression (3) shows that the transfer rate of energy through the spectrum with large values Ω practically coincides with the rate of dissipation. On large scales, i.e., for small values of Ω, the second term on the right side begins to prevail over the first.

We finally get the following:

\[ S(\Omega) = \varepsilon^{\frac{4}{3}} \Omega^{\frac{4}{3}} \left[ 1 + \frac{4}{3} \right] \left( 2 n \right)^{\frac{4}{3}} \Omega^{\frac{4}{3}} - \frac{4}{3} \], \quad (4) \]

in which

\[ b = \frac{g}{T} (\gamma_a - \gamma) \varepsilon^{\frac{2}{3}} \]

Thus it is apparent from expression (4) that the energy spectrum of turbulence during a neutral stratification, i.e., when \( b = 0 \), as follows the law "minus five-thirds" throughout the whole "inertial" range.

During stable stratification, the energy spectrum in the region of large scales within the "inertial" range, which was calculated by means of formula (4), will follow curve 2 (Figure 2), which was plotted on the
basis of the assumptions on the presence of a certain "user" of the turbulent energy. Curve 3 in the same figure may also be analytically expressed by formula (4) on the condition that $b < 0$.

The increase in the slope of the spectral curve was also discussed during investigation of atmospheric turbulence by radar.

Also, A. S. Monin\(^2\) has theoretically arrived at the same results after examining the structure of turbulence in conjunction with Archimedean forces.

![Figure 3. Energy spectrum of turbulence during 1-stable, 2-unstable, 3-neutral stratification.](image)

It is now possible to draw some conclusions on the appearance of the energy spectrum of turbulence. For the purpose of illustrating these conclusions, Figure 3 shows curves of energy spectra for a stable (1), unstable (2) and neutral (3) stratification. For the purpose of comparison, the rate of energy input into the "inertial" range is assumed to be the same.

As may be seen from the figure, the rate of dissipation of energy under different stratifications is not the same. For an unstable stratified atmosphere the dissipation rate is greater and for a stable one it is less than it would be for a neutral stratified atmosphere. In the
area of very large scales outside the "inertial" range, the curves attain their saturation and drop to zero.

The above proposal well agrees with the experimental results obtained by us as well as those obtained theoretically in Shur's study$^6$.

2. Relationship Between the Energy Spectra of Vertical and Horizontal Components of the Fluctuating Wind Velocity.

As indicated above, the experimentally obtained curves for the spectral density of a vertical component of turbulent fluctuations of wind velocity conform to the law "minus five-thirds" in the altitude range of up to 600-1000 m. The methods and devices used in studies by Shur$^5,7$ did not yield any data on the energy spectrum of a horizontal component.

Figure 4. Observation of a possible trend of a spectral curve of the horizontal component of turbulent fluctuations in a stable stratified atmosphere.
On the basis of theoretical and experimental investigations conducted basically for a surface boundary layer, it is possible to draw the following conclusion: In the area where the distribution of energy in the spectrum of turbulent flow conforms to the law "minus five-thirds" by virtue of local isotropy, a counter confirmation would also be valid. Thus, knowing that the energy spectrum of a vertical component conforms to the law "minus five-thirds", we can state that in ranges of up to 600-1000 m the turbulent vortices are isotropic, i.e., the spectral curves of a vertical and a horizontal component coincide.

Let us examine the variations in the possible trend of the curve of spectral density of a horizontal component of turbulent fluctuations in a stable stratified atmosphere.

Figure 4 illustrates the energy spectrum of vertical gusts (curve 1). Let us assume that the energy spectrum of a horizontal component will coincide with the law "minus five-thirds" within the whole "inertial" range (curve 2). Then an anisotropy of velocities will be valid at the boundary of the "inertial" range for wave numbers Ω corresponding to a large scale:

\[
\overline{V}_{B_1} = \sqrt{S_B(\Omega_1)} \Omega_1 > \overline{V}_{r_1} = \sqrt{S_r(\Omega_1)} \Omega_1,
\]

where \(S_B(\Omega_1)\) is the energy spectrum of a vertical component at point \(\Omega_1\), \(S_r(\Omega_1)\) is the energy spectrum of a horizontal component at point \(\Omega_1\), \(\overline{V}_{B_1}\) is the mean-square velocity of a vertical gust of dimension \(l_1 = 2\pi/\Omega_1\), and \(\overline{V}_{r_1}\) is the mean-square velocity of a horizontal gust of dimension \(l_1 = 2\pi/\Omega_1\), i.e., the mean-square velocity of vertical gusts of large magnitudes must be greater than the mean-square velocity of horizontal gusts.

Apparently, in the case of stable stratification when the Archimedean forces hinder the movement of the particle along the vertical, this is physically impossible; consequently, the assumption made is incorrect. Along the same considerations, the curve of the energy spectrum of a horizontal component cannot appear as curve 3. We can now only assume that the energy spectrum of a horizontal component is of the same nature as the energy spectrum of a vertical one, i.e., it will appear as either curve 3 or 4. Curve 4 is valid in the case when there is an isotropy of velocities inside the whole "inertial" range even for
small values $\Omega$ (of large magnitudes). Curve 5 indicates that there is
an anisotropy of velocities for large magnitudes:

\[
\bar{v}_r = \sqrt{S_r(\Omega_1)\Omega_1} > \bar{v}_B = \sqrt{S_B(\Omega_1)\Omega_1},
\]

which is physically possible.

As to the trend of the spectral curve of a horizontal component, it
is necessary to resolve this by means of special investigations of
horizontal fluctuations in a free atmosphere.

3. Derivation of Integral Characteristics of Turbulence in a Wide Frequency Range of
Scales, Through Data Obtained from Measurements of Overloads in a Narrow Band
of Frequencies.

At the present time it is an established fact that when an air-
craft encounters a turbulent zone and experiences bumpiness, the
spectrum of turbulence is continuous and reaches dimensions of several
thousand meters. It is also known that the transfer functions of air-
craft during the decrease of frequency tend to zero.

An aircraft, even flying at a speed of 200-250 m/sec, will react
with a weak overload to a sinusoidal gust with a scale greater than
1000 m. Thus, turbulent disturbances with dimensions of up to 1000 m
have the greater effect on subsonic aircraft. At the same time, the
energy spectrum of turbulent fluctuations of a vertical component of
velocity with a scale up to 600-1000 m well agrees with the law "minus
five-thirds". On the basis of this, it is possible to arrive at some
practical conclusions from which we can obtain data on atmospheric
turbulence by means of a very simple measuring technique.

By knowing any frequency component of the gust load and a numeri-
cal multiplier which is determined by the transfer function of the air-
craft, it is possible, after performing a few simple procedures, to
obtain and register directly the dispersion of the velocity of turbulent
gusts directly on board the aircraft, calculating there in a range of the
spectrum which clearly obeys the law "minus five-thirds".

Let us assume that in the proximity of value $\Omega_1$ in the energy
spectrum of gust loads in a narrow band $\pm \Delta \Omega$, the value of the spectral
density of gust loads \( S_n(\Omega) \) will remain constant. Then

\[
S_n(\Omega_1)2\Delta\Omega = \sigma_n^2(\Omega_1),
\]

(8)

where \( \sigma_n^2 \) is the dispersion of gust loads determined in a band of \( \Omega_1 \pm \Delta\Omega \) wave numbers. By knowing the transfer function of the aircraft \( \Phi(\Omega) \), we will obtain its value for a given wave number \( \Omega_1 \). We will also assume that within the limits of the change in wave numbers from \( \Omega_1 - \Delta\Omega \) to \( \Omega_1 + \Delta\Omega \) the transfer function, by virtue of the smallness of \( \Delta\Omega \), remains constant and equals \( k\Omega_1 \). Thus for the spectral density of the gust velocities

\[
S_w(\Omega_1) = S_n(\Omega_1) \frac{1}{k^2\Omega_1}.
\]

(9)

Considering expression (8) we get

\[
\sigma_n^2(\Omega_1) = k^2\Omega_1 S_w(\Omega_1)2\Delta\Omega.
\]

(10)

Technically, it is necessary to have a device which would produce a signal (of stress) \( U_1 \), proportional to an average square of gust loads in a narrow band near the frequencies corresponding to wave number \( \Omega_1 \):

\[
U_1 = k_1 \sigma_n^2(\Omega_1) = k_1 k^2\Omega_1 S_w(\Omega_1)2\Delta\Omega.
\]

(11)

Stress \( U_1 \) uniquely determines the value of the spectral density \( S(\Omega_1) \). Based on the assumption that the law "minus five-thirds" is satisfied, we get

\[
S_w(\Omega_1) = k_2 \Omega_1^{-\frac{5}{3}}.
\]

(12)

We determined \( k_2 \) from expressions (11) and (12):

\[
k_2 = \frac{U_1}{k_1 k^2 \Omega_1 2\Delta\Omega_1 \Omega_1^{-\frac{5}{3}}}.
\]

(13)

The dispersion of the velocities of turbulent gusts \( \sigma_w^2 \) in the range of wave numbers \( \Omega_0 \) to \( \Omega_2 \) is determined by relationship...
Thus by measuring the average square of gust loads in a narrow band of frequencies (wave numbers), it is possible to obtain the dispersion of turbulent velocities in a wide diapason of wave numbers, corresponding to scales of up to 600-1000 m.

Technically, this is reduced to having an electro-accelerometer with a narrow-band filter. This unit should have a quadratic detector with an averaging circuit on the outlet. If such an accelerometer is connected to any recorder, it will register the value of $\sigma^2_w$, and coefficient $k$ will determine the scale of the recording. For operation of this device, it is essential to select correctly the frequency to be admitted by the filter as well as the average time in the output of the quadratic detector.

Findings

The appearance of the vertical component of the velocity of gusts depends on the stratification of the atmosphere.

Its appearance as proposed by us coincides nicely with experimental data and theoretical findings of other investigators.

Assumption as to the trend of the spectral curve of a horizontal component lacks experimental confirmation.
The experimental confirmation which had earlier verified the agreement of the turbulence spectra in scales up to 600-1000 m with the law "minus five-thirds" has enabled us to solve easily the problem of the dispersion value of the velocity by means of data obtained on the gust loads in a narrow band of frequencies.

It will be very interesting to perform further studies on the diversion of the spectral density curve from the law "minus five-thirds" in the area of large scales. In this case, the spectral measurements have to be complemented by correct measurements of thermodynamic parameters of a free atmosphere and especially by the gradients of these parameters. Determination of the relationships between the average characteristics of the temperature fields and the wind, as well as the spectral characteristics of the fluctuations of these fields will, in our opinion, give a clearer understanding of the dynamics and energies of the atmosphere.

In conclusion, the author would like to express his thanks to M. M. Kulik, A. F. Epishev, V. C. Alexandrov, N. A. Titov and V. V. Kozlov for their management and performance of complex flight experiments.
LITERATURE CITED


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Spectral structure  
Turbulent gusts  
Vertical component  
Horizontal component  
Stratified atmosphere  
Gust loads  
"Inertial" range  
Wave number  
Turbulent energy
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29 August 1966

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