Equations of Motion for a Towed Body Moving in a Vertical Plane

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ABSTRACT

Equations of motion are developed for a towed body moving in a plane defined by gravity and the fore-and-aft axis of the ship. Solution of the cross-coupled equations enables predictions to be made of towed body heave, surge, and pitch motions.

The towline is approximated as a spring with one third of its mass added to the body at the towpoint. A discussion of computational techniques to compute masses, damping constants, and spring constants is also presented. Because of the number of terms in the equations for heave, surge, and pitch, the solutions have been programmed in FORTRAN IV.

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EQUATIONS OF MOTION FOR A TOWED BODY
MOVING IN A VERTICAL PLANE

INTRODUCTION

The designer of a ship-towed system must be able to predict the motions of the towed body. In designing a body that should have minimal response to external excitations, he should have such information available in order to avoid a resonance in any of the body motion modes.

This report presents solutions of the cross-coupled equations of motion for a towed body moving with heave, surge, and pitch motions and for various ship-input conditions. By the use of body and cable constants, one can optimize a design for whichever criterion he must satisfy, be it heave, surge, or pitch, or any combination of the three motions. The analysis is limited to motions in the vertical plane, since additional information that could be derived from solving equations of motion for a body with six degrees of freedom did not appear to be significant at the time. In general, towed bodies have symmetry about a plane defined by the vertical and the fore-and-aft axes.

The primary force input to a cable-towed system is taken to be the vertical force at the point of attachment of the cable on the ship. This limitation on the analysis is reasonable, because forces in this plane will not couple into sway, roll, or yaw for a symmetrical body. The solution may be extended, however, to apply to a body moving with all six degrees of freedom.

Lurn solves similar equations by using the Laplace transform technique for the open-loop pitch response of a towed body; however, the results are somewhat difficult to use to find towed-body response for specific input conditions.

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ANALYSIS

A typical towed system is shown in Fig. 1. The dynamic model of the system is presented in Fig. 2.

Fig. 1 - A Towed System

The dimensions, masses, spring constants, and damping factors for Fig. 2 are defined as follows:

- $C_{by}$: damping factor of the body alone for motion in the $y$-direction
- $C_{bz}$: damping factor of the body alone for motion in the $z$-direction
- $C_{br}$: rotational damping factor of the body alone for motion about the $x$-axis
- $C_t$: damping factor of the tail alone for motion in the $z$-direction
- $D$: vertical distance from center of mass to tail center of damping
- $G$: vertical distance from the center of mass to the tail's effective center of resilience and damping.
- $h$: vertical distance from the center of mass to the towpoint
- $j$: vertical distance from the center of mass to the body's center of damping
- $K_b$: spring constant due to change of body lift with angle of attack
- $K_{cy}$: spring constant of deflected tow cable in horizontal ($y$-direction)
Fig. 2 - Dynamic Model of the Towed System

\[ K_{cz} \] - spring constant of deflected tow cable in vertical (z-direction)
\[ K_t \] - spring constant due to change of tail lift with angle of attack
\[ K_{dc} \] - spring constant due to change of tail drag with angle of attack
\[ K_w \] - rotational spring constant due to pendulous effect of the water weight of the body below the towpoint
\[ L_{bt} \] - horizontal distance from front of body to the center of body lift
\[ L_{CD} \] - horizontal distance from front of body to the center of body damping
\[ L_{CM} \] - horizontal distance from front of body to the center of mass
\[ L_t \] - horizontal distance from front of body to the effective center of the tail
\[ L_{TP} \] - horizontal distance from front of body to the towpoint
\[ m_b \] - mass of the body and tail
The mass and hydrodynamic mass terms of the cable are included in Eq. (1), following the usual convention for dynamic problems to include one third of the mass of the spring. A more thorough analysis might be patterned after the work of Dr. L. F. Whicker.2

The rotational spring constant $K_w$ is equal to $w_b n$ if angular displacements are assumed to be small. The spring constants $K_b$ and $K_t$ are actually rotational spring constants but are shown as linear springs in the $z$-direction for ease of computation. The constants can be computed as follows:

$L_{CM} = \frac{y_1 m_b + y_2 m_{h_{bx}} + y_3 m_{fw} + L_t m_{ht} + L_T P \left( \frac{1}{3} m_c + \frac{1}{3} m_{hc_{yz}} \right)}{m_b + m_{h_{bx}} + m_{fw} + m_{ht} + \frac{1}{3} m_c + \frac{1}{3} m_{hc_{yz}}}$. (1)

---

\[ K_b = \frac{\rho}{2} A_b v^2 (\Delta C_{lb}), \quad (2) \]

\[ K_t = \frac{\rho}{2} A_t v^2 (\Delta C_{lt}), \quad (3) \]

and

\[ K_{Dt} = \frac{\rho}{2} A_t v^2 (\Delta C_{Dt}), \quad (4) \]

where

\[ \rho = \text{fluid density}, \]

\[ A_b = \text{horizontal projected area of the body}, \]

\[ A_t = \text{horizontal projected area of the tail}, \]

\[ \Delta C_{lb} = \text{change in lift coefficient of the body with angle of attack}, \]

\[ \Delta C_{lt} = \text{change in lift coefficient of the body with angle of attack}, \quad \text{and} \]

\[ \Delta C_{Dt} = \text{change of drag coefficient of the tail with angle of attack}. \]

A criterion for stability is that the lift moment of the tail must be greater than the lift moment of the body; it can be expressed mathematically by

\[ K_t (L_t - L_{CM}) > K_b (L_{CM} - L_{bt}). \]

Using the equations of motion for a resiliently supported body\(^3\) with six degrees of freedom as presented by Harris and Crede,\(^4\)

---

\(^3\)This form of equation of motion is used because of its linear form. Viscous forces are represented as damping constants and inertial history terms are assumed to be negligible.

but restricting the motion to a plane with inputs occurring in that plane only, we can write

\[ M_x = l_{xx} \ddot{a} + \sum (K_{yz} A_y - K_{yy} A_z) \gamma_c + \sum (K_{zz} A_y - K_{yz} A_z) (Z_c - W) \]
\[ + \sum (K_{yy} A_z^2 + K_{zz} A_y^2 - 2K_{yz} A_y A_z) a + \sum (C_{yz} A_y' - C_{yy} A_z') \dot{y}_c \]
\[ + \sum (C_{zz} A_y' - C_{yz} A_z') (\dot{z}_c - \dot{W}) + \sum (C_{yy} A_z'^2 + C_{zz} A_y'^2 - 2C_{yz} A_y' A_z') \dot{a}, \]

(5)

\[ F_y = m_y \ddot{y}_c + \sum K_{yy} \gamma_c + \sum K_{yz} (z_c - W) + \sum (K_{yz} A_y - K_{yy} A_z) a \]
\[ + \sum C_{yy} \ddot{y}_c + \sum C_{yz} (\dot{z}_c - \dot{W}) + \sum (C_{yz} A_y' - C_{yy} A_z') \dot{a}, \]

(6)

and

\[ F_z = m_z \ddot{z}_c + \sum K_{yz} \gamma_c + \sum K_{zx} (z_c - W) + \sum (K_{zx} A_y - K_{yz} A_z) a \]
\[ + \sum C_{yz} \ddot{y}_c + \sum C_{zz} (\dot{z}_c - \dot{W}) + \sum (C_{zz} A_y' - C_{yz} A_z') \dot{a}, \]

(7)

where

- \( A \) = coordinate distance from the body's center of mass to the elastic center of the resilient element;
- \( A' \) = coordinate distance from the body's center of mass to the effective center of the damping element;
- \( C \) = damping factors;
- \( F \) = dynamic input force applied at the body's center of mass;
- \( I \) = mass movement of inertia;
- \( K \) = spring constant;
- \( m \) = body mass;
- \( M \) = dynamic input moment applied at the body's center of mass;
- \( W \) = vertical displacement of the ground point of all vertical springs;
\[ y, \dot{y}, \ddot{y} = \text{displacement, velocity, and acceleration in horizontal direction of the body's center of mass relative to earth coordinates;} \]

\[ z, \dot{z}, \ddot{z} = \text{displacement, velocity, and acceleration in the vertical direction of the body's center of mass relative to earth coordinates;} \]

\[ a, \dot{a}, \ddot{a} = \text{angular displacement, velocity, and acceleration in a vertical plane (defined by } y \text{ and } z) \text{ of the body relative to earth coordinates;} \]

and where

- subscript \( c \) = reference to center of mass;
- subscript \( x \) = reference to \( x \)-direction;
- subscript \( y \) = reference to \( y \)-direction;
- subscript \( z \) = reference to \( z \)-direction; and
- subscript \( a \) = reference to \( a \)-axis of rotation.

The computation of the various coordinates distances \( A \) and \( A' \) is as follows:

for the spring representing the cable,

\[ A_y = L_{CM} - L_{TP} \]

\[ A_z = h \]

for the spring due to tail lift,

\[ A_y = L_{CM} - L_t \]

\[ A_z = G \text{ (negative if effective center of tail lift is below the center of mass);} \]

for the spring due to body lift,

\[ A_y = L_{CM} - L_{bt} \]

\[ A_z = 0 \text{ (it is assumed that the effective center of body lift lies on the horizontal plane of the center of mass);} \]
for the center of damping of the body,

\[ A'_y = L_{CM} - L_{CD} \]

\[ A'_x = \imath \text{ (negative if the center of damping lies below the center of mass)}; \]

for the center of damping of the tail,

\[ A'_y = L_{CM} - L_t \]

\[ A'_x = G \text{ (negative if the center of tail damping is below the center of mass)}. \]

We may now consider the actual towed system and use all masses, elastic elements, damping elements, and input forces in order to apply Eqs. (5), (6), and (7). Rewriting Eqs. (5), (6), and (7) with the above conditions and coordinate distances and assuming that

\[ K_{yx} = K_{xy} = 0 \text{ for the cable, and} \]
\[ C_{yx} = C_{xy} = 0 \text{ for the body,} \]

we have

\[
M_x = (J_h + J_{bh}) \ddot{a} + ( - K_{ch} h) \dot{y}_c + [K_{cv} (L_{CM} - L_{TP})] (z_c - W) \\
+ \left[ K_{ch} h^2 + K_{cv} (L_{CM} - L_{TP})^2 + K_t (L_t - L_{CM}) + K_{bt} (L_{CM} - L_{bt}) + K_w + K_{dt} D \right] \dot{a} \]
\[
- (C_{bh} j) \dot{y}_c + [C_{bv} (L_{CM} - L_{CD}) - C_t (L_t - L_{CM})] \dot{z}_c \\
+ \left[ C_{bh} j^2 + C_t (L_t - L_{CM})^2 + C_{bv} (L_{CM} - L_{CD})^2 + C_{bt} \right] \ddot{a},
\]
\[ F_y = m_{ty} \ddot{y}_c + K_{ch} \dot{y}_c - (K_{ch} h - K_{ah}) \ddot{a} + C_{bh} \dot{y}_c + [C_{br}/L_b - C_{bh}] \ddot{a} + \] (9)

and

\[ F_z = m_{tz} \ddot{z}_c + K_{cv} (z_c - \dot{w}) + [K_{cv} (L_{CM} - L_{TP}) + K_b - K_t] \ddot{a} + (C_{br} + C_t) \dot{z}_c + [-C_t (L_t - L_{CM}) + C_{bv} (L_{CM} - L_{CD}) + C_{br}/L_b] \ddot{a} , \] (10)

where

\[ J_b = \text{mass moment of inertia in the pitch plane about the center of mass;} \]
\[ J_{hb} = \text{hydrodynamic mass moment of inertia in the pitch plane about the center of mass;} \]
\[ F_y = \text{forcing function acting on the center of mass in the y-direction;} \]
\[ F_z = \text{forcing function acting on the center of mass in the z-direction;} \]
\[ M_x = \text{forcing function acting about the center of mass in the pitch plane;} \]
\[ m_{ty} = \text{total oscillating mass in the y-direction;} \]
\[ m_{tz} = \text{total oscillating mass in the z-direction;} \]

The spring constant due to body lift is negative in Eq. (8) because its effect is to upset the body if it is displaced. Other assumptions applied to Eqs. (8), (9), and (10) are that cable damping is small compared with body damping and that the center of tail damping lies along the vertical height of the center of mass. It is also assumed that the flood water mass center, the hydrodynamic mass center, and the center of body heave damping lie in the same horizontal plane.
SOLUTION OF EQUATIONS

In order to change the equations of motion (Eqs. (8), (9), and (10) ) into dimensionless form, the following steps are taken:

1. All a terms are multiplied by $L_b/L_b$, where $L_b$ is the body length.
2. The two force equations are divided by $w_b$, the body weight in water.
3. The moment equation is divided by $w_b L_b$.

Therefore, Eqs. (8), (9), and (10) become

\[
\frac{M_x + K_{cv}(L_{CM} - L_{TP})W}{L_b w_b} - \left[ \frac{J_b + J_{ht}}{L_b w_b} \right] (L_b \ddot{a}) - \left[ \frac{K_{ch} h}{L_b w_b} \right] \dot{c} + \left[ \frac{K_{cv}(L_{CM} - L_{TP})}{L_b w_b} \right] z_c
\]

\[
+ \left[ \frac{K_{ch} h^2 + K_{cv}(L_{CM} - L_{TP})^2 + K_t (L_t - L_{CM}) + K_b (L_{CM} - L_{bt}) + K_w + K_{dt} D}{L_b w_b} \right] (aL_b)
\]

\[
- \left[ \frac{C_{bh} J}{L_b w_b} \right] \dot{y}_c + \left[ \frac{C_{bv} (L_{CM} - L_{CD}) + C_t (L_{CM} - L_t)}{L_b w_b} \right] \dot{z}_c
\]

\[
+ \left[ \frac{C_{bh} h^2 + C_t (L_{CM} - L_t)^2 + C_{bv} (L_{CM} - L_{CD})^2 + C_{br}}{L_b w_b} \right] (L_b \ddot{a})
\]

\[
\frac{F_x}{w_b} = \left[ \frac{m_{ty}}{w_b} \right] \dot{y}_c + \left[ \frac{K_{ch}}{w_b} \right] \dot{y}_c - \left[ \frac{K_{ch} h - K_{dt}}{w_b L_b} \right] L_b \alpha
\]

\[
+ \left[ \frac{C_{bh}}{w_b} \right] \dot{y}_c - \left[ \frac{C_{br}/L_b + C_{bh}}{w_b L_b} \right] L_b \ddot{u}, \quad \text{and}
\]

10
\[
\frac{F_z + \text{W}_{K_{cv}}}{\text{W}_b} = \left[ \frac{a_{tz}}{\text{W}_b} \right] \dot{z} + \left[ \frac{K_{cv}}{\text{W}_b} \right] Z_c + \left[ \frac{K_{cv} (L_{CM} - L_{TD}) + K_b - K_t}{\text{W}_b L_b} \right] L_b a \\
+ \left[ \frac{C_{bv} + C_t}{\text{W}_b} \right] \ddot{z} + \left[ \frac{C_t (L_t - L_{CM}) - C_{br} (L_{CM} - L_{CD}) + C_{br}}{\text{W}_b L_b} \right] \dddot{L}_b. \tag{III}
\]

For Eqs. (I), (II), and (III), let

\[ L_b \alpha = \dot{q}, \]
\[ L_b \ddot{a} = \ddot{q}, \]

and

\[ L_b \dddot{a} = \dddot{q}. \]

In the equations that follow, the terms I, II, and III indicate Eqs. (I), (II), and (III), respectively, and the subscript following these terms represents the variable that multiplies the term.

\[ I_q = \left[ \frac{J_b + J_{hh}}{L_b^2 \text{W}_b} \right]. \]

\[ I_y = \left[ \frac{K_{ch} b}{L_b \text{W}_b} \right]. \]

\[ I_z = \left[ \frac{K_{cv} (L_{CM} - L_{TP})}{L_b \text{W}_b} \right]. \]
\[
I_q = \left[ \frac{K_{ch} b^2 + K_{cv} (L_{CM} - L_{TP})^2 + K_t (L_{c} - L_{CM}) + K_b (L_{CM} - L_b) + K_w + K_{dt} D}{L_b W_b} \right]
\]

\[
I_y = \left[ \frac{C_{bh} j}{L_b W_b} \right]
\]

\[
I_z = \left[ \frac{C_{bv} (L_{CM} - L_{CD}) + C_t (L_{CM} - L_t)}{L_b W_b} \right]
\]

\[
I_q = \left[ \frac{C_{bh} j^2 + C_t (L_{CM} - L_t)^2 + C_{bv} (L_{CM} - L_{CD})^2 + C_{br}}{L_b^2 W_b} \right]
\]

\[
I_y = \left[ \frac{m_{ry}}{W_b} \right]
\]

\[
i_y = \left[ \frac{K_{ch}}{W_b} \right]
\]

\[
I_q = \left[ \frac{K_{ch} b - K_{dt}}{W_b L_b} \right]
\]

\[
I_y = \left[ \frac{C_{bh}}{W_b} \right]
\]

\[
I_q = \left[ \frac{C_{bh} j + C_{br}}{W_b L_b + W_b L_b^2} \right]
\]
\[
III_z = \left[ \frac{m_{rz}}{W_b} \right].
\]

\[
III_z = \left[ \frac{k_{cv}}{W_b} \right].
\]

\[
III_q = \left[ \frac{k_{cv} (L_{CM} - L_{TP}) + k_b - k_t}{W_b L_b} \right].
\]

\[
III_{\dot{z}} = \left[ \frac{c_{bv} + c_t}{W_b} \right].
\]

\[
III_{\dot{q}} = \left[ \frac{c_{bv} (L_{CM} - L_{CD}) - c_t (L_t - L_{CM}) + c_{bv}/L_b}{W_b L_b} \right].
\]

\[
P = \left[ \frac{m_x + k_{cv} (L_{CM} - L_{TP}) W}{L_b W_b} \right].
\]

\[
Y = \left[ \frac{F_y}{W_b} \right].
\]

\[
Z = \left[ \frac{F_z + WK_{cv}}{W_b} \right].
\]
The equations now become

\[ P = I_q \ddot{q} - I_y \dot{y} + I_z \dot{z} + I_q q - I_q \ddot{q} + I_q \dot{q}, \]  

(IA)

\[ Y = I_y \ddot{y} + I_y y - I_q q + I_q \ddot{q} - I_q \dot{q}, \]  

(IIA)

\[ Z = I_z \ddot{z} + I_z \dot{z} + I_q q + I_z \ddot{z} + I_z \dot{z} \]  

(III A)

or in matrix notation,

\[
\begin{bmatrix}
I_y & 0 & 0 \\
0 & I_z & 0 \\
0 & 0 & I_q
\end{bmatrix}
\begin{bmatrix}
\ddot{y} \\
\ddot{z} \\
\ddot{q}
\end{bmatrix}
+ \begin{bmatrix}
-I_y & -I_q & I_y \\
0 & 0 & -I_z \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{z} \\
\dot{q}
\end{bmatrix}
= \begin{bmatrix}
\ddot{y} \\
\ddot{z} \\
\ddot{q}
\end{bmatrix}
\]

or

\[ M \ddot{Q}(t) + C \dot{Q}(t) + K Q(t) = F(t). \]  

(11)

The solution to Eq. (11) involves a complementary and a particular integral. The complementary solution, which is obtained by setting the forcing function \( F(t) \) of Eq. (11) equal to zero, is

\[ Q_c = e^{-t(C/zm)} \left( C_1 \cos \omega_{nd} t + C_2 \sin \omega_{nd} t \right), \]
where
\[ t = t_{\text{me}} \]

\[ C_1 \text{ and } C_2 = \text{arbitrary constants, and} \]

\[ \omega_{\text{nd}} = \text{damped natural frequency of the system.} \]

It can be seen that the complementary solution is a transient one that diminishes with time. Therefore, the steady-state condition is described by the particular solution.

To effect the particular solution we let
\[ F(t) = F_0 \cos \omega t, \]

and assume that
\[ Q = Q_2 \cos \omega t + Q_1 \sin \omega t, \]

where
\[ \omega = \text{forcing frequency.} \]

Taking derivatives and substituting in Eq. (11) we obtain
\[ \dot{Q} = -\omega Q_2 \sin \omega t + \omega Q_1 \cos \omega t, \]

and
\[ \ddot{Q} = -\omega^2 Q_2 \cos \omega t - \omega^2 Q_1 \sin \omega t; \]

therefore, we have
\[ -\omega^2 M (Q_2 \cos \omega t + Q_1 \sin \omega t) + \omega C(Q_1 \cos \omega t - Q_2 \sin \omega t) \]
\[ + K(Q_2 \cos \omega t + Q_1 \sin \omega t) = F_0 \cos \omega t. \]
Equation (12) must satisfy the condition that the sum of the forces in the vertical and horizontal directions must equal the inertia forces in those directions. Hence, we have

\[-\omega^2 MQ_2 + \omega CQ_1 + KQ_2 = F, \quad (13)\]

and

\[-\omega^2 MQ_1 - \omega CQ_2 + KQ_1 = 0. \quad (14)\]

Because \( M, C, \) and \( K \) are matrices, multiplication of these terms is not commutative.

Solving Eq. (14) for \( Q_2 \) and substituting into Eq. (13), we obtain

\[Q_2 = \frac{1}{\omega} C^{-1} [K - \omega^2 M] Q_1,\]

and

\[\left[ \frac{1}{\omega} C^{-1} (K - \omega^2 M) \frac{1}{\omega} C^{-1} (K - \omega^2 M) + 1 \right] Q_1 = \frac{1}{\omega} C^{-1} F.\]

Solving for \( Q_1 \), we have

\[Q_1 = \left[ \frac{1}{\omega} C^{-1} (K - \omega^2 M)^2 + 1 \right]^{-1} \frac{1}{\omega} C^{-1} F,\]

in which

\[
C = \begin{bmatrix}
I_{y}\, & 0 & -I_{q}\\
0 & I_{z} & I_{q}\\
-I_{y} & I_{z} & I_{q}
\end{bmatrix}
\]
and

\[
C^{-1} = \begin{bmatrix}
C_{yy} & C_{yz} & C_{yq} \\
C_{zy} & C_{zz} & C_{zq} \\
C_{qy} & C_{qz} & C_{qq}
\end{bmatrix},
\]

where

\[
C_{yy} = \Pi_{y}^2 I_{z}^2 - \Pi_{z}^2 I_{y}^2 / A,
\]

\[
C_{zz} = I_{y}^2 \Pi_{y}^2 - I_{z}^2 \Pi_{z}^2 / A,
\]

\[
C_{zy} = -\Pi_{y}^2 I_{z}^2 / A,
\]

\[
C_{qz} = -I_{y}^2 \Pi_{z}^2 / A,
\]

\[
C_{qy} = I_{y}^2 \Pi_{z}^2 / A,
\]

\[
C_{yq} = I_{y}^2 \Pi_{z}^2 / A,
\]

\[
C_{yz} = -\Pi_{y}^2 I_{z}^2 / A,
\]

\[
C_{yq} = -I_{y}^2 \Pi_{z}^2 / A,
\]

\[
C_{qq} = I_{y}^2 \Pi_{z}^2 / A,
\]

where

\[
A = \Pi_{y}^2 (\Pi_{y}^2 I_{z}^2 - \Pi_{z}^2 I_{y}^2) - \Pi_{q}^2 (I_{y}^2 \Pi_{z}^2).
\]

Performing matrix operations for the terms in the expressions \( Q_{1} \) and \( Q_{2} \), we obtain

\[
\frac{1}{\omega} C^{-1} (K - \omega^2 M) = \frac{1}{\omega}
\begin{bmatrix}
C_{yy} & C_{yz} & C_{yq} \\
C_{zy} & C_{zz} & C_{zq} \\
C_{qy} & C_{qz} & C_{qq}
\end{bmatrix}
\begin{bmatrix}
\Pi_{y}^2 - \omega^2 \Pi_{y}^2 & 0 & -\Pi_{q}^2 \\
0 & \Pi_{z}^2 - \omega^2 \Pi_{z}^2 & \Pi_{q}^2 \\
-\Pi_{y} & 0 & \Pi_{z} - \omega^2 \Pi_{z}
\end{bmatrix}.
\]
and

\[
\frac{1}{\omega} C^{-1}(K - \omega^2 M) = \frac{1}{\omega} \left[ C_{yy}(I_{yy} - \omega^2 I_{yy}) - I_y C_{yz} C_{yz}(I_{zz} - \omega^2 I_{zz}) + C_{yy} I_z \right. \\
- C_{yz} I_{II} - C_{yy} I_q + C_{yz}(I_{q} - \omega^2 I_{q}) C_{zy} (I_{yy} - \omega^2 I_{yy}) \\
- I_y C_{zz} C_{zz} I_z + C_{zz} (I_{II} - \omega^2 I_{II}) - C_{zy} I_{II} + C_{zz} I_{II} \\
+ C_{zz}(I_{q} - \omega^2 I_{q}) C_{qy} (I_{II} - \omega^2 I_{II}) - C_{aq} I_{II} C_{qz} (I_{II} - \omega^2 I_{II}) \\
+ C_{qq} I_{II} - C_{qy} I_{II} + C_{qz} I_{II} + C_{qq}(I_{q} - \omega^2 I_{q}) \right].
\]

Let

\[
\frac{1}{\omega} C^{-1}(K - \omega^2 M) = \frac{1}{\omega} \begin{bmatrix}
    r_{yy} & r_{yz} & r_{yq} \\
    r_{zy} & r_{zz} & r_{zq} \\
    r_{qy} & r_{qz} & r_{qq}
\end{bmatrix}
\]

Now forming, we have

\[
\begin{bmatrix}
    \left(\frac{1}{\omega} C^{-1}(K - \omega^2 M)^2\right)^2 + 1
\end{bmatrix} = \frac{1}{\omega^2} \begin{bmatrix}
    r_{yy} & r_{yz} & r_{yq} \\
    r_{zy} & r_{zz} & r_{zq} \\
    r_{qy} & r_{qz} & r_{qq}
\end{bmatrix} \begin{bmatrix}
    r_{yy} & r_{yz} & r_{yq} \\
    r_{zy} & r_{zz} & r_{zq} \\
    r_{qy} & r_{qz} & r_{qq}
\end{bmatrix} + 1
\]
\[
\begin{bmatrix}
(r_{yy} + r_{yz} r_{zy}) & (r_{yy} + r_{yz} r_{yz}) + (r_{zy} r_{yy} + r_{zz} r_{yz}) & (r_{yy} r_{yz} + r_{yz} r_{zy}) \\
+ r_{qq} r_{qy} (1/\omega^2) + 1 & + r_{qq} r_{qy} (1/\omega^2) & + r_{qq} r_{qy} (1/\omega^2)
\end{bmatrix}
\]

Let

\[
\left[ \left( \frac{1}{\omega} C^{-1} (K - \omega^2 M) \right)^2 + 1 \right] = \begin{bmatrix}
h_{yy} & h_{yz} & h_{yq} \\
h_{qy} & h_{zz} & h_{qz} \\
h_{qz} & h_{qz} & h_{qq}
\end{bmatrix}
\]

Forming, we have

\[
\left[ (\omega C^{-1} (K - \omega^2 M))^2 + 1 \right]^{-1} = \begin{bmatrix}
J_{yy} & J_{yz} & J_{yq} \\
J_{zy} & J_{zz} & J_{zq} \\
J_{qy} & J_{qz} & J_{qq}
\end{bmatrix}
\]
where
\[
J_{yy} = h_{zz} h_{qq} - h_{zz} h_{qq}/B,
\]
\[
J_{zy} = h_{zz} h_{qq} - h_{zy} h_{qq}/B,
\]
\[
J_{qy} = h_{zy} h_{zz} - h_{zz} h_{qq}/B,
\]
\[
J_{yz} = h_{yz} h_{zz} - h_{y} h_{qq}/B,
\]
\[
J_{zz} = h_{yy} h_{qq} - h_{yy} h_{qq}/B,
\]
\[
J_{qz} = h_{yz} h_{zz} - h_{yy} h_{qq}/B,
\]
\[
J_{yz} = h_{yx} h_{zz} - h_{zz} h_{qq}/B,
\]
\[
J_{zz} = h_{yy} h_{zz} - h_{y} h_{zz}/B,
\]

where
\[
B = h_{yy} (h_{zz} h_{qq} - h_{zz} h_{qq}) + h_{yz} (h_{zz} h_{yy} - h_{yy} h_{qq}) + h_{y} (h_{zy} h_{zz} - h_{zz} h_{yy}).
\]

Forming, we have
\[
\frac{1}{\omega} C^{-1} F = \frac{1}{\omega} \begin{bmatrix} C_{yy} & C_{yz} & C_{qy} \\ C_{zy} & C_{zz} & C_{zq} \\ C_{qy} & C_{qz} & C_{qq} \end{bmatrix} \begin{bmatrix} \bar{Y} \\ \bar{Z} \\ \bar{P} \end{bmatrix}
\]

and
\[
\frac{1}{\omega} C^{-1} F = \frac{1}{\omega} \begin{bmatrix} C_{yy} \bar{Y} + C_{yz} \bar{Z} + C_{qy} \bar{P} \\ C_{zy} \bar{Y} + C_{zz} \bar{Z} + C_{zq} \bar{P} \\ C_{qy} \bar{Y} + C_{qz} \bar{Z} + C_{qq} \bar{P} \end{bmatrix} \begin{bmatrix} E \\ F \\ G \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} \bar{Y} \\ \bar{Z} \\ \bar{P} \end{bmatrix}.
Finally, we obtain

\[
\left[ \left( \frac{1}{\omega} C^{-1} (K - \omega^2 M) \right)^2 \cdot I \right]^{-1} \frac{1}{\omega} C^{-1} F = Q_1 = \begin{bmatrix} y_1 \\ z_1 \\ q_1 \end{bmatrix}
\]

\[
\begin{bmatrix} y_1 \\ z_1 \\ q_1 \end{bmatrix} = \frac{1}{\omega} \begin{bmatrix} j_{yy} & j_{yz} & j_{yq} \\ j_{zy} & j_{zz} & j_{zq} \\ j_{qy} & j_{qz} & j_{qq} \end{bmatrix} \begin{bmatrix} E \\ F \\ G \end{bmatrix}
\]

\[
Q_1 = \frac{1}{\omega} \begin{bmatrix} j_{yy} & j_{yz} & j_{yq} \\ j_{zy} & j_{zz} & j_{zq} \\ j_{qy} & j_{qz} & j_{qq} \end{bmatrix} \begin{bmatrix} y_1 \\ z_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ z_1 \\ q_1 \end{bmatrix}.
\]  

Therefore, we obtain

\[
Q_2 = \frac{1}{\omega} \begin{bmatrix} r_{yy} & r_{yz} & r_{yq} \\ r_{zy} & r_{zz} & r_{zq} \\ r_{qy} & r_{qz} & r_{qq} \end{bmatrix} \begin{bmatrix} y_1 \\ z_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} y_2 \\ z_2 \\ q_2 \end{bmatrix}
\]
\[
\begin{bmatrix}
y_2 \\
z_2 \\
q_2
\end{bmatrix} = \frac{1}{\epsilon} \begin{bmatrix}
\gamma \gamma_1 + \gamma \gamma_2 + \gamma q_1 \\
\gamma \gamma_1 + \gamma \gamma_2 + \gamma q_1 \\
\gamma \gamma_1 + \gamma \gamma_2 + \gamma q_1
\end{bmatrix}
\]

and

\[
y = y_1 \sin \omega t + y_2 \cos \omega t,
\]

\[
z = z_1 \sin \omega t + z_2 \cos \omega t, \text{ and}
\]

\[
q = q_1 \sin \omega t + q_2 \cos \omega t.
\]

COMMENTS

A simple computer program has been written to effect these complex solutions. This program with necessary inputs is shown in Appendix A.

In working with these equations, one finds that the most difficult task is determining the cable spring constants, hydrodynamic masses, and damping factors since there is very little information in the literature dealing with the computation of these factors for towed bodies. Appendix B discusses the computations.

The solutions have been used to study the motions of the AN/SQA-10 towed body for various ship speeds and sea conditions. The results agree favorably with data taken at Sea States 2 and 5 with a full-size AN/SQA-10 towed body.
SUMMARY

The differential equations of motion have been written for the case of a cable-supported towed body. The solution for a sinusoidal forcing function input is shown in Eqs. (15) and (16). Computation of the inertial terms, spring constants, and damping factors required for Eqs. (12), (13), (14), (15), and (16) enables one to solve for the magnitude and phase angle of motions in surge, heave, and pitch resulting from vertical displacements of the cable end on the ship. A trial-and-error approach can be used to determine the natural frequencies of the system by substituting various frequencies into the solution.
Appendix A

COMPUTER PROGRAM FOR COMPUTATION OF TOWED BODY MOTIONS IN SURGE, HEAVE, AND PITCH FOR A VERTICAL SINUSOIDAL INPUT AT THE SHIP

INPUT TO COMPUTE SURGE, HEAVE, AND PITCH OF A TOWED BODY*

- $m_b$ - mass of body and tail
- $W_{ba}$ - weight (includes tail) in air
- $m_{hby}$ - hydrodynamic mass of body in fore-and-aft direction
- $m_{hbz}$ - hydrodynamic mass of body in vertical direction
- $m_{fw}$ - flood water mass
- $m_{ht}$ - hydrodynamic mass of tail in vertical direction
- $W_b$ - body weight (includes tail) in water
- $y_1$ - distance from front of body to body center of gravity in air
- $y_2$ - distance from front of body to body center of vertical hydrodynamic mass
- $y_4$ - distance from front of body to flood water mass center of gravity
- $L_t$ - distance from front of body to tail hydrodynamic mass center of gravity and tail damping
- $L_{Tp}$ - distance from front of body to towpoint
- $L_{CD}$ - distance from front of body to body center of damping
- $L_{bt}$ - distance from front of body to center of body lift
- $n$ - distance from towpoint to body center of gravity in air
- $n$ - distance from towpoint to body center of mass in water (positive if above center of mass)
- $j$ - distance from center of mass to center of damping (minus if center of damping below center of mass)

*Units must be kept homogeneous.
The surge, heave, and pitch will be printed out in the following order.

Real part of surge
Imaginary part of surge
Vector sum of real and imaginary parts of surge
Phase angle between real and imaginary parts of surge
Real part of heave
Imaginary part of heave
Vector sum of real and imaginary parts of heave
Phase angle between real and imaginary parts of heave
Real part of pitch
Imaginary part of pitch
Vector sum of real and imaginary parts of pitch
Phase angle between real and imaginary parts of pitch.

The units of surge and pitch will be commensurate with the units of the input constants. The phase angles are in degrees. The pitch angles are in radians times the body length.
COMPUTER PROGRAM

EQUATIONS OF MOTION FOR A TOWED BODY

100 FORMAT(54E15.5)
110 FORMAT(54E15.5)
2 WRITE OUTPUT TAPE 4,101,1,JOB
101 FORMAT(14,A5//7TH EQUATIONS OF MOTION FOR A TOWED BODY//)
4 IF(SWITCH.EQ.2) G00,A01
A00 READ INPUT TAPE 3,991,XSER,MODE,40
991 FORMAT(7A)
996 FORMAT(7A)
997 READ INPUT TAPE 3,997,(CUT(1),I=1,12)
998 FORMAT(7A)
999 READ INPUT TAPE 3,998,MT,II1,II2,II3,II4,II5,II6,II7,II8,II9,II10,II11,II12

DO 200 I = 1,N,JOB
200 READ INPUT TAPE 3,120,J(TK,KK),KK=1,4
DO 201 I = 1,10
201 READ INPUT TAPE 3,120,TTHH,KK,TKH=1,4
DO 202 I = 1,5
202 READ INPUT TAPE 3,120,TTHC,J,J=1,5
DO 203 I = 1,5
203 READ INPUT TAPE 3,120,TMHC,J,J=1,5
DO 204 I = 1,4
204 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 205 I = 1,4
205 READ INPUT TAPE 3,120,TMHC,J,J=1,4
DO 206 I = 1,4
206 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 207 I = 1,4
207 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 208 I = 1,4
208 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 209 I = 1,4
209 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 210 I = 1,4
210 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 211 I = 1,4
211 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 212 I = 1,4
212 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 213 I = 1,4
213 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 214 I = 1,4
214 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 215 I = 1,4
215 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 216 I = 1,4
216 READ INPUT TAPE 3,120,TMKC,J,J=1,4
DO 217 I = 1,4
217 READ INPUT TAPE 3,120,TMKC,J,J=1,4
COMPUTER PROGRAM (Cont'd)

TTY = (TKCH*TH) / A
T17 = (TKC* (TLCM - T1)) / A
T1Q = (TKCH*TH#2 + TKCV* (TLCM - TLTP) #2 + TKT#TI T - TCLM) * TKD
T10 = TKB* (TLC - TLAL) + TKW/* (A*TLB)

TTY1 = (CBH*TJ) / A
T171 = (CBV* (TLCM - TLC) + CT*(TLCM - TLTP)) / A
T1C1 = (CBH*TJ*2 + CBV* (TLCM - TLD)) #2 + CT*(TLCM - TLTP) #2 +
1 CRP / (A*TLB)

TTY2 = TMV/WB
TTY3 = TKC/WR

TYY = (T1101* T1101 = T1101* T1101) = T1101* (T110* T1101)
CYY = (T1112* T1112 - TI101* T1112) / A
CYY = - T1101* T1101 / A
CYY = (T1101* T1112) / A
CYY = (T1112* T1112) / A
CYY = (T1112* T1112) / A
CYY = (T1112* T1112) / A
RYY = CYY* (T112 - OMFGA**2) * T1122 = T112* CYY
RYY = CYY* (T112 - OMFGA**2) * T1122 = T112* CYY
RYY = CYY* (T112 - OMFGA**2) * T1122 = T112* CYY
RYY = CYY* (T112 - OMFGA**2) * T1122 = T112* CYY

HYY = (RYY* RYY + RYY* RYY + RYY* RYY + RYY* RYY + RYY* RYY) / OMFGA**2

BR = HYY* (HYY* HYY + HYY* HYY) + HYY* (HYY* HYY) + HYQ* (HYY*
COMPUTER PROGRAM (Cont'd)

156 FORMAT(11H0, 5H Y) *F15.4, 5H X =F15.4
168 5H C1 =F15.4, 5H C2 =F15.4
27H QMP =F15.4, 4H YAAG =F15.4
39A IF (SENSF SWITCH 1) 620, A21

A20 YID = -TY1
       CONST = N
       YD1 = -TY1
       OID1 = -TI101
       OITD = -TI1C

603 FORMAT(3F20.4) WRITE OUTPUT TAPE 6, 603, YID, CID, CID: CINST, YD1, YD2, CINST, TI1C

WRITE OUTPUT TAPE A, 160, CINF, CINF, CINF, CINF, CINF, CINF, CINF

WRITE OUTPUT TAPE 6, 603, YID, CID, CID: CINST, YD1, YD2, CINST, CINF

IF (SENSF SWITCH 2) 602, A20

602 CALL SFTUP( 11(1), 5, 15(1), 1D(1), 155(1), 10)
       TINC = INCR
       TINCR = 1.0/TINC
       TMAX = 6.281854/OMFGA
       T = TMAX/TINCR + 1.
       D0 995 JJ = 1:1J
       T1 = JJ - 1
       ANG = T1*TINCR*OMFGA
       N = Y1*COSF(ANG) + Y2*INF(ANG)
       J = Z1*CSNF(ANG) + Z2*INF(ANG)
       O = CI*COSF(ANG) + Q2*INF(ANG)
       X = COSF(ANG)
       WORD(1) = ANG
       WORD(2) = N
       WORD(3) = J
       WORD(4) = 0
       WORD(5) = X
       IF (SENSF SWITCH 3) 500, 501

500 WRITE CLOUTPUT TAPE 4, 500, (WORD(I1), 1I1 = 1*5)

502 FORMAT(1H, 5H 1F15.4) 901 CALL PLOT(WORD(1), 15F(1), NXSF, YOFF, ICM(1), 10)

995 CONTINUE CALL C15SF(10, 5)

201 CONTINUE

204 CONTINUE 700 CONTINUE

IF (SENSF SWITCH 1) 622, 623

622 END FILE 6

END FILE 4

623 READ INPUT TAPE 3, 109, ISTAR

109 FORMAT(A11)

IF (IND = ISTAR) 555, 5

55 PRINT 110, 1.I0

110 FORMAT(1H, 15F, 3/304, SHOULD BE END CARD ALT IS NOT)

END FILE 4

STOP 555

6 END FILE 4

STOP 5

END(1+1+0+1)

31, 32
Appendix B

COMPUTATION OF HYDRODYNAMIC MASS, DAMPING,
AND SPRING CONSTANTS

COMPUTATION OF HYDRODYNAMIC
MASS CONSTANTS

As shown in Eq. (11), the mass matrix

\[
\begin{bmatrix}
  m_{ty} & 0 & 0 \\
  0 & m_{tx} & 0 \\
  0 & 0 & J_b + J_{hb}
\end{bmatrix}
\]

must be evaluated. If the body has a plane of symmetry defined by
gravity and the fore-and-aft direction, and if the coordinate axes
are along the principal axes of inertia, this matrix in its complete
form would be

\[
\begin{bmatrix}
  m_{tyy} & 0 & m_{tya} \\
  0 & m_{txx} & m_{txa} \\
  m_{tya} & m_{txa} & m_{tza}
\end{bmatrix}
\]

where

\[ m_t \]

is the total mass in a given direction.

The total mass is composed of the body mass (or rotational inertia),
the flood-water mass (or rotational inertia), and the hydrodynamic
mass (or rotational inertia). The cross-coupling "masses," \( m_{tya} \),
Hydrodynamic masses for bodies of various shapes moving in translation have been determined experimentally by Patton.\textsuperscript{B1} The effects of frequency and displacement amplitude have been investigated by Miller.\textsuperscript{B2} Hydrodynamic mass moments of inertia can be computed for ellipsoids in accordance with the methods of Zahm.\textsuperscript{B3} Information can not be found in the literature that enables one to compute the cross-coupled hydrodynamic masses. Consequently, the mass matrix used in Eq. (12) has all off-diagonal elements equal to zero.

**COMPUTATION OF DAMPING CONSTANTS**

It is extremely difficult to find data in the literature that enable one to compute the damping constants for a towed body. Newman\textsuperscript{B4} presents an analytical technique to compute damping coefficients of ellipsoids. Patton\textsuperscript{B5} presents experimental data for model towed bodies moving in translational motion. These data can be scaled to a full-size body if scaling laws for size and frequency can be determined. This method was used in the computation of damping constants for the AN/SQA-11 VDS towed body.\textsuperscript{B6} It was assumed that the damping varies with the cube of the body size.


\textsuperscript{B3} A. F. Zahm, Flow and Force Equations for a Body Revolving in a Fluid, National Advisory Committee for Aeronautics Report No. 323, 1929.


\textsuperscript{B5} K. T. Patton, op. cit. (see footnote B1 above).

Experimentally measured rotational damping constants and cross-coupled damping constants can not be found in the literature. The damping of the AN/SQA-11 towed body oscillating in the pitch mode was approximated by computing the damping on an equivalent ellipsoid after the method of Newman.

**COMPUTATION OF SPRING CONSTANTS**

The spring constants due to the body lift and the tail lift have been considered in the text. A certain amount of judgment is called for at this point because the springs may be nonlinear. \((\Delta C_{lb} \text{ and } \Delta C_{lt})\) may be nonlinear.) One must estimate the pitch amplitude and substitute the best approximate linear values for the actual nonlinear values.

The cable spring constants \(K_{ch}\) and \(K_{cv}\) can be computed by considering two springs in series. The first spring represents the axial extension of the cable under some load. This spring constant is given by

\[
K_1 = \frac{AE}{L},
\]

where

- \(A =\) cross-sectional area of the cable,
- \(E =\) cable modulus of elasticity, and
- \(L =\) length of the cable.

The second spring in the series combination is due to the changes in configuration of the towcable because of changes in load at the towed body. First, the equilibrium configuration of the system is computed by using suitable tables or, possibly, the computer.
solution programmed by Cuthill. The drag of the body is then increased by some amount typical of the dynamic loads existing in the cable. The new configuration is computed, and the change in trail distance is used to compute a spring constant $K_{yy}$. The change in depth can be used to compute a cross-coupled spring constant $K_{yz}$. The same process is undertaken to compute the spring constants $K_{zz}$ and $K_{zy}$ by increasing the weight of the body.

Equivalent spring constants $K_{ch}$ and $K_{cv}$ are computed as follows:

$$K_{ch} = \frac{K_{yy} K_z \sin \phi_0}{K_{yy} + K_z \sin \phi_0},$$

and

$$K_{cv} = \frac{K_{zz} K_z \cos \phi_0}{K_{zz} + K_z \cos \phi_0},$$

where

$\phi_0$ is the angle (from the vertical) of the towline at the towed body.

The spring constants $K_{ch}$ and $K_{cv}$ are not constant because the springs that they represent are nonlinear. One should compute a number of deflections for different increments of drag and weight to approximate the nonlinear springs in the yy- and zz-directions with linear spring constants $K_{ch}$ and $K_{cv}$.

---

\footnotesize

$^{\text{B7E Cuthill, A FORTRAN Program for the Calculation of the Equilibrium Configuration of a Flexible Cable in a Uniform Stream, David Taylor Model Basin Report No. 182, May 1963 (UNCLASSIFIED).}}$
Equations of motion are developed for a towed body moving in a plane defined by gravity and the fore-and-aft axis of the ship. Solution of the cross-coupled equations enables predictions to be made of towed body heave, surge, and pitch motions.

The towline is approximated as a spring with one third of its mass added to the body at the towpoint. A discussion of computational techniques to compute masses, damping constants, and spring constants is also presented. Because of the number of terms in the equations for heave, surge, and pitch, the solutions have been programmed in FORTRAN IV.
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Ship-Towed Systems
Cross-Coupled Equations of Motion
Ship-Towed Bodies
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