FILM AND TRANSPERSION COOLING
OF
NOZZLE THROATS

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FOREWORD

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This technical report has been reviewed and is approved.

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ABSTRACT

of

FILM AND TRANSPIRATION COOLING

of

NOZZLE THROATS

Analytical studies were made of liquid film cooling, gas film cooling, and liquid or gas transpiration cooling of hypersonic nozzles. Experimental studies of gas film cooling were performed on a small nozzle using air as both the mainstream and coolant gases. Based on the experimental results obtained and the development of satisfactory calculational techniques to implement the analyses given in the present study, the following conclusions were drawn:

1. Gas film cooling can be used to measurably lower the wall temperatures and the wall heat fluxes in the converging section and at the throat of a high-pressure high-temperature nozzle. Some gross mixing occurs at the injection point, the exact amount being some as yet undetermined function of the injection geometry, the relative velocities of the main gas stream and the coolant stream at the injection point, and the entering velocity profiles and turbulence conditions.

2. A straightforward boundary layer type analysis was developed and programmed which predicts with reasonable accuracy the nozzle wall temperatures and wall heat fluxes in the converging section and at the throat for gas film cooled nozzles.

3. Calculational techniques have been developed and programmed for predicting the effectiveness of liquid film cooling and liquid or gas transpiration cooling in nozzles. These techniques should be checked against experimental data before they are used for design purposes, however.
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CHAPTER I
INTRODUCTION

I. GENERAL DISCUSSION

Recent technological developments have caused an increased interest in analytical and experimental studies of film and transpiration cooling. Military rocket and space research has brought about extensive investigations into the problems of rocket nozzle design. In particular, greater specific thrust can be achieved by using higher combustion chamber pressures and temperatures, however this requires better nozzle throat cooling in order to reduce corrosion, oxidation, and thermal stresses. Another problem involves the surfaces of re-entry vehicles which must be protected from the extremely high heat loads encountered. Experimental studies utilizing wind tunnels which simulate re-entry conditions provide the most direct approach to this problem. Wind tunnel systems designed to provide these test conditions must use air at extremely high stagnation pressures and temperatures resulting in extremely high heat loads at the nozzle throat which must be protected by proper heat transfer design.

Examination of ordinary backside cooling of the nozzle throat shows that it may become inadequate for two principal reasons:
(1) Exceedingly high static pressure of the coolant may be required to prevent boiling (for conventional liquid coolants) with the subsequent danger of burnout, (2) backside cooling cannot reduce the heat load to the nozzle surface so it cannot alleviate the problem of thermal stresses.
It has been proposed, therefore, that rocket combustion chambers and nozzles, re-entry vehicles, high-pressure high-temperature wind tunnel nozzles and other devices with the same problems be protected by film or transpiration cooling or a combination of one of these with backside cooling. Film cooling refers to the injection of a liquid or gas into a system in such a manner that it forms a relatively thin protective layer over the surface to be cooled. The liquid or gas is injected at one or more discrete locations on the surface through holes or slots. Transpiration cooling refers to the injection of a liquid or gas through a porous surface to cool it. Gas film cooling by injection through holes approaches transpiration cooling as the size of the holes decreases and the number increases, however, in actual practice the two are usually well distinguished. Figure 1 shows the two systems diagrammatically.

It is instructive to consider the desirable properties for film and transpiration coolants. In order to maintain a low surface temperature with liquids they should have a low saturation temperature at the static pressure in the chamber. They should also have a high heat of vaporization and a high specific heat capacity in the vapor state. This reduces the amount of coolant required. The primary requirement for a gaseous coolant, from a thermal standpoint, is that it have a high heat capacity. This indicates that high gases, such as helium and hydrogen should make good coolants for gas film and transpiration cooling. There are other requirements to be considered, however. The introduction of the coolant into the system must not produce effects
Fig. 1 Diagram of Liquid Film Cooling and Transpiration Cooling
which destroy the usefulness of the system. This would not severely
limit the choice of coolants for a rocket nozzle or a re-entry vehicle,
but it must be carefully considered for a wind tunnel. If possible
wind tunnels should test vehicles in the atmosphere in which they are
to operate. The main gas stream thus will normally be air. Nitrogen
might be used in some cases. If the cooling process requires in-
jection of an amount which is a significant fraction of the main gas
mass flow rate of the wind tunnel the required coolant would thus have
to be either air or nitrogen.

In both film and transpiration cooling the protective effect of
the heat absorption by the coolant is augmented by the heat transfer
blocking action of the coolant gas as it moves into the main stream.
This "blowing" action at the wall or coolant film surface reduces the
velocity and temperature gradients at the surface and thus decreases the
heat transfer rate.

Backside cooling may be used with film cooling. This combi-
nation reduces the nozzle liner temperature and allows the cooling of a
greater nozzle wall area by the same amount of film coolant. It is more
effective for liquid films under the following conditions:

1. The film saturation temperature is considerably higher
   than the backside coolant temperature.

2. There is a high thermal conductance between the coolant
   film and the backside coolant.
II. REVIEW OF PREVIOUS STUDIES

Analytical and experimental studies of nozzle heat transfer rates have been made by a number of investigators; however, studies of heat transfer with extremely high heat loads using nozzles instrumented to give a good profile of the heat transfer rate are not available. In the present study such a nozzle was used.

Analytical and experimental studies of film and transpiration cooling have also been made by many investigators. The Bibliography contains references to several of these studies. An extensive review of the literature on film cooling studies up to 1957 is given in Graham and Jucrow's "Film Cooling, Its Theory and Application," (16). Librizzi and Cresci's "Transpiration Cooling of a Turbulent Boundary Layer in an Axisymmetric Nozzle" (24) lists a large amount of work which has been done on film and transpiration cooling since 1957. A careful examination of these references, however, disclosed that none of these analyses is directly applicable to the present problem.

Since the present solution to these problems is based on adaptation of previous works (except for gas film cooling), a somewhat more extensive review of these works is in order.

A widely used equation for rapid calculation of heat transfer coefficients in nozzle throats without film or transpiration cooling was developed by Bartz (2). He started from the basic premise, based on more elaborate boundary layer calculations performed earlier (3) that for a turbulent boundary layer the dominant variable factor is the mass flow rate, i.e.
He then rewrote Eq. (1) in the form

\[ \text{Nu} = C(\text{Re})^m (\text{Pr})^n \]  

(2)

which is the same form as the widely used Dittus-Boelter equation for correlation of fully developed turbulent heat transfer in pipe flow.

By regrouping the variables in Eq. (2), assuming \( \mu \sim \frac{T^0}{\rho} \), \( \rho \sim \frac{1}{T} \), \( c_p \) and \( \text{Pr} \) constant, and introducing a factor \( (D_a/r_e) \) to account for curvature effects at the throat, Bartz was able to write Eq. (2) in the form

\[ h = \left( \frac{C}{D_a^2} \right)^{\frac{2}{\text{Pr}^6}} \left( \frac{p_c \alpha g_c}{\nu_e} \right)^{\frac{1}{\text{Pr}^4}} \left( \frac{D_a}{r_e} \right)^{1} \left( \frac{A}{A} \right)^{9} \]  

(3)

in which

\[ \sigma = \frac{1}{\left[ \frac{1}{2} \frac{T_w}{T_o} \left( 1 + \frac{k-1}{2} \frac{M^2}{\text{Pr}^4} + \frac{1}{4} \right) \right]^{1.8-\omega/5} \left( 1 + \frac{k-1}{2} \frac{M^2}{\text{Pr}^4} \right)^{\omega/5}} \]

(4)

The constant \( C \) in the equation was determined from experimental data for heat transfer at nozzle throats to be 0.0265 in a nozzle with contraction and expansion half angles of 30 and 15 degrees, respectively. This is only slightly higher than that usually given for the constant \( C \) when Eq. (3) is applied to fully developed turbulent pipe flow, namely 0.023. An examination of the velocity profiles for turbulent flow with negative axial pressure gradients (39) shows that the pressure gradient has the effect of increasing the wall shear stress, and thus an increase in the heat transfer coefficient is to be expected.
Emmons (13) developed a simplified analysis for liquid film cooling of a cylindrical rocket motor combustion chamber with backside cooling. It was based on the following assumptions:

1. The gas stream pressure and velocity do not vary appreciably in the axial direction.

2. The wall temperature does not vary appreciably in the axial direction.

3. The temperature of the liquid film surface exposed to the hot gas is at all points equal to the saturation temperature of the liquid.

4. Temperature gradients in the radial direction through the liquid film are small.

5. The liquid film temperature does not vary rapidly in the axial direction.

6. The heat transfer coefficient between the gas stream and the liquid film is constant and is based on a steady state analysis.

7. The amount of liquid in the film varies linearly with the distance in the axial direction.

In the case studied by Emmons these were all reasonable assumptions provided the coolant film was injected at the local saturation temperature. It may be noted immediately, however, that several of these assumptions are not reasonable for the case of injection of a coolant liquid well below saturation temperature into a high pressure, high temperature nozzle gas stream. In a supersonic nozzle there are
large axial variations in static pressure, film temperature, heat transfer coefficient, film thickness, and film shear stress. The rate of evaporation is not constant and thus the film mass flow rate does not vary linearly with axial distance. To accurately predict the required film cooling for a nozzle using Emmons' approach it was thus necessary to remove the restrictions of the above assumptions.

III. PLAN OF THE PRESENT INVESTIGATIONS

Objectives. The objectives of the present investigation were to perform analytical and experimental studies of heat transfer rates and required coolant injection rates for a hypersonic nozzle with gas film cooling and to make analytical studies of heat transfer rates and required coolant flow rates for a hypersonic nozzle with liquid film and transpiration cooling.

Theoretical Investigations. For liquid film cooling and transpiration cooling the results of previous investigations, particularly those outlined above, were examined, suitably modified and combined in a finite difference stepwise calculation through the nozzle. This was programmed for solution using an IBM 7040 digital computer.

For gas film cooling a method of solution was programmed which consisted of determining the diffusivities of momentum and heat based on a two-region universal pipe velocity profile for turbulent flow; then performing layerwise momentum and energy balances to determine
the temperature profile and heat transfer rate at the wall. This program was also run on the IBM 7040 digital computer and the results correlated with the experimental results.

**Experimental Investigations.** A temperature instrumented nozzle was constructed for the experimental part of this study. It was instrumented to measure the outside nozzle wall temperatures at one-eighth inch intervals along the nozzle. These temperature readings, along with known flow conditions in the backside cooling annulus, gave sufficient information to calculate local heat transfer rates. The nozzle was also constructed with an air film injector which injected a controlled film of coolant air along the convergent section of the nozzle. Using the experimental information, a comparison could be made to the theoretical heat fluxes and temperature profiles for various nozzle flow conditions.
CHAPTER II
GAS FILM COOLING
I. GENERAL-DISCUSSION

A large number of investigators have worked on solutions of the equations of continuity of mass, momentum, and energy. Bird, Stewart and Lightfoot (7), Bennett and Myers (5), and other authors include in their books many of the available solutions. Other solutions are available in the journals concerning fluid flow and heat transfer. A careful study shows that closed type analytical solutions have been accomplished for only a relatively few rather special cases. With the development of high speed computing equipment numerical methods of solution for the more difficult cases are now feasible. The present investigator decided to make use of such capability in the study of film and transpiration cooling.

For gas film cooling a simplified model was adopted which disregards diffusion or mixing of the coolant gas with the mainstream gas. Since Pr \ll 1 for most gases it was necessary to solve for the developing velocity and temperature profiles simultaneously. This was done by making certain assumptions about the viscosity and thermal conductivity for individual gas layers and then making layerwise momentum and energy balances over each axial increment.
Libby and Pallone (23) worked out and published in 1954 a rather extensive integral method for handling the calculation of velocity and enthalpy profiles for a gas film introduced into a laminar compressible boundary layer. Sixth degree polynomials were used to describe the velocity and enthalpy profiles. Since the species concentration equation was not used the injected fluid was assumed to be the same as the mainstream fluid. The polynomial coefficients were chosen without particular regard for the asymptotic (far downstream) results and thus the solution is not asymptotically correct. The numerical results do indicate a strong effect of the injected fluid for some distance downstream from the point where injection ceases.

Baron (1) worked out and published in 1956 an extension of the Libby and Pallone method. He included the species concentration equation for a binary fluid boundary layer and adjusted the form of the polynomials so as to assure correct asymptotic solutions.

Neither of these solutions is applicable to the present case of nozzle throat cooling with injection into a turbulent boundary layer.
Emmons (13) and Hatch and Papell (17) used a very crude flow model for tangential coolant gas injection into turbulent pipe flow. The coolant gas was assumed to form a layer at the wall with no diffusion or mixing with the mainstream. A heat transfer coefficient $h$ was used to calculate the heat transfer rate to the coolant layer from the mainstream and from the coolant layer to the nozzle wall. The heat transfer coefficient was calculated from the mainstream conditions as though the coolant layer were not present. It may be noted immediately that this model, besides its other deficiencies, is not asymptotically correct. It predicts a downstream heat transfer rate which is only one half the correct rate for the far downstream value.

II. GAS FILM THEORETICAL ANALYSIS

The approach taken in the present investigation was to solve the continuity, momentum, and energy balance equations for the boundary layer written in finite difference form, for turbulent mainstream flow with tangential injection of a turbulent coolant gas.
The necessary assumptions are:

1. The coolant injection is a low percentage of the mainstream flow rate and has little effect on the expansion process of the mainstream.
2. The injected film is turbulent and a laminar sublayer is immediately established.
3. The laminar sublayer thickness and the eddy diffusivities of momentum and heat may be calculated from the two region universal turbulent velocity profile equations.
4. The ratio of the eddy diffusivities of momentum and heat is a constant. As a preliminary assumption this constant is taken to be one.
5. The wall shear stress may initially be calculated using the Blasius turbulent - smooth pipe - flow formula.

Based on the above assumptions the coolant and mainstream gases near the wall may be divided into finite layers parallel to the wall and finite increments axially along the surface. The velocity and temperature profiles are then calculated in a stepwise fashion through the nozzle. The coordinate system is thus one which moves with the fluid.

As was stated above, the analysis depends upon the solution of the continuity, momentum, and energy equations. The continuity equation is satisfied by assigning a fixed amount of fluid to each layer except for the laminar sublayer and the neighboring turbulent layer. The exchange between these layers is calculated at each
increment and thus continuity assured.

A steady state momentum balance for the coolant layer at the coolant-mainstream interface as shown in Figure 2, is written as

\[
\begin{bmatrix}
\text{Pressure forces in } \Delta \text{ direction on coolant layer 1} \\
\text{Inertia forces in } \Delta \text{ direction on coolant layer 1} \\
\text{Shear forces on top and bottom of coolant layer 1}
\end{bmatrix}
\]

Using the notation shown in Figure 2, noting that primed quantities refer to the coolant stream and subscripts refer to layer positions, the resulting mathematical momentum equation is

\[
\pi d\delta \left( p_1 - p_2 \right) + \frac{w \left( v_1' - v_2' \right)}{g_c} + \frac{\pi d\delta \Delta \left( v_1 + v_2 - v_1' + v_2' \right)}{\rho_1 \gamma_1 + \rho_2 \gamma_2} = 0
\]

\[
\pi d\delta \left( v_1'' - v_2'' \right) + \frac{\pi d\delta \Delta \left( v_1' + v_2' - v_1'' - v_2'' \right)}{\rho_1 \gamma_1 + \rho_2 \gamma_2} = 0
\]

*The differential equation and development is shown in Appendix G.*
Solving Eq. (6) for the coolant velocity at the outlet of the first layer, $v'_{21}$ gives

$$v'_{21} = \frac{w_f - 1 + A(v_{i1} + v_{21} - v'_{11}) + B(v'_{12} + v'_{22} - v'_{11}) + P}{\frac{w_f}{g_c} + A + B}$$

(7)

where

$$A = \frac{\pi d_f \Delta}{\delta_1 + \delta \delta}$$

$$= \frac{\delta_1}{\rho_1 \epsilon_i} \frac{\delta \delta}{\rho \epsilon_i}$$

(8)

$$B = \frac{\pi d_f \Delta}{\delta \delta + \delta \delta}$$

$$= \frac{\delta \delta}{\rho \epsilon_i} \frac{\delta \delta}{\rho \epsilon_i}$$

(9)

$$P = \pi (d_f)(\delta \delta)(p_1 - p_2)$$

(10)
Fig. 2 Velocity and Temperature Notation for Gas Film by Layers and Increments
A set of such equations is written for the fluid layers in the coolant from the coolant-mainstream interface to the nozzle wall. Another set is written for the mainstream fluid layers from the coolant-mainstream interface to a point in the mainstream where there is negligible variation in exit velocity between layers. The wall boundary condition requires zero velocity at the wall. The mainstream boundary condition may be handled either by using enough layers of fluid so they contain several times the mass flow rate of the coolant or by specifying a smaller number of layers and adding layers if the chosen number does not cover the velocity boundary layer. The outer mainstream fluid layer is then fixed at the mainstream velocity calculated from frozen flow conditions established by an isentropic expansion with a constant ratio of specific heats. The Gauss-Siedel method of successive substitution is used to solve the equations iteratively.

In order to make the above calculations, the diffusivity of momentum must be evaluated at each layer. In the laminar sublayer, $0 < y^+ < 10$, the molecular diffusivity of momentum is used. In the turbulent core assumption 3 is used and the eddy diffusivities calculated from the universal velocity profile equation for this region, which is:

$$\frac{u}{u^*} = 2.5 \ln \frac{y u^*}{u} + 5.5$$  \hspace{1cm} (11)

Differentiating Eq. (11) with respect to $y$ gives:

$$\frac{du}{dy} = 2.5 \frac{u^*}{y}$$  \hspace{1cm} (12)
Now for the turbulent core

\[ \frac{\tau_g}{\rho} = \varepsilon \frac{du}{dy} = (u^*)^2 \]  \hfill (13)

thus

\[ \varepsilon = \frac{\rho}{\frac{du}{dy}} \]  \hfill (14)

Assuming the velocity boundary layer terminates near the wall

\[ \tau = \tau_w \]  \hfill (15)

so

\[ \varepsilon = \frac{(u^*)}{2.5} \]  \hfill (16)

or

\[ \varepsilon = 0.4 u^*(r_i - r) \]  \hfill (17)

In the notation of Figure 2, page 16, Eq. (17) becomes

\[ \varepsilon_i = 0.4 u^*(r_i - \frac{d_1}{2}) \]  \hfill (18)

or

\[ \varepsilon_i = 0.4 u^*(r_i - \frac{d_2}{2}) \]  \hfill (19)
In order to evaluate the diffusivities using either Eq. (18) or Eq. (19), \( u^* \) and thus \( \tau_w \) is required at each increment. Assumption 5 is used to evaluate \( \tau_w \) for the initial increment, i.e.

\[
\tau_w = 0.053 \left( \text{Re} \right)^{-0.2} \frac{\rho u^2}{2g_c}
\]  

(20)

Following the initial increment, \( \tau_w \) may be evaluated at each successive increment either from Eq. (20) or from the wall velocity gradient at the exit of the previous increment.

Energy balance equations are written for the same fluid layers in order to calculate temperatures at the exit of the increments - (Refer to Figure 2 for notation). The steady state energy equation for the coolant layer at the coolant-mainstream interface is written as

\[
\begin{align*}
\text{Enthalpy change in A direction of coolant layer 1} & \quad + \quad \text{Kinetic energy change in A direction of coolant layer 1} \\
+ \quad \text{Energy transfer from gas layer 1 to coolant layer 1 by diffusion} & \quad + \quad \text{Energy transfer from coolant layer 2 to coolant layer 1 by diffusion} \\
+ \quad \text{Work at boundaries on coolant layer 1. Shear stress effects} & \quad = \quad 0
\end{align*}
\]  

(21)
The mathematical statement of Eq. (21) is

\[
\begin{align*}
\left[ w_{ci}(T'_{11} - T'_{21}) \right] &+ \left[ \frac{w_{fi}(v'_{11}^2 - v'_{21}^2)}{2g_c} \right] \\
&+ \left[ \frac{\pi d_1 \Delta(T_{11} + T_{21} - T'_{11} - T'_{21})}{\delta_1} + \frac{\delta_1}{\rho_1 c_1 \epsilon_1 \psi} \right] \\
&+ \left[ \frac{\pi d_1 \delta_1 \rho_1 c_1 \epsilon_1 (v_{11} + v_{21} - v'_{11} - v'_{21})^2}{J_g c (2\delta_1 + \delta_2 + \delta_1)^2} \right] = 0 \quad (22)
\end{align*}
\]

Solving Eq. (22) for \( T'_{21} \), the exit temperature of the first coolant layer gives:

\[
T'_{21} = \frac{T'_{11}(w^*c^* - A - B) + A(T_{11} + T_{21}) + B(T'_{12} + T'_{22}) + E + F}{w^*c^* - A + B} \quad (23)
\]

where

\[
A = \frac{\pi d_1 \Delta}{\delta_1} + \frac{\delta_1}{\rho_1 c_1 \epsilon_1 \psi} \quad (24)
\]
Since the energy equations contain the entrance and exit velocities, the velocity solution is performed first using layer properties based on the exit temperatures from the preceding increment. The velocity equations are dependent on temperature only through the variation of the fluid properties with temperatures. If desired the fluid properties could be re-evaluated after completion of the temperature iteration.

When the velocity iteration is completed the temperature iteration is performed in like manner by successive substitution.

The analysis described above has been programmed in Fortran IV language for execution on a digital computer. The program is included in Appendix C.

Included in the program are provisions for wall shear stress.
calculation either from the Blasius Equation or from the past increment, multiple injection calculations, and provision for approaching a transpiration calculation by injecting a very small amount of coolant at each increment. New coolant layers are added for the multiple injection calculation. The fluid is added at a specified increment, or increments and the wall side coolant layer is recalculated and layers added as needed. Provision is made for backside cooling in the case of single and multiple injection, while an adiabatic wall may be specified for any type injection.

III. EXPERIMENTAL INVESTIGATION

Design Requirements. An examination of some published data for film cooled nozzles (6), (19), (24), (25), indicated that in most cases either insufficient data are available to make a detailed comparison with the present analysis or the fluid stagnation conditions are not in the range of present interests. Therefore an instrumented nozzle was designed and fabricated for this investigation. The general geometrical requirements for this nozzle were that it have an entrance diameter of 1.5 inches, a throat diameter of 0.5 inches, and an expansion angle of 15°. Additional requirements were that it be mechanically designed to handle air as the expanding fluid with stagnation pressure up to 50 atmospheres and stagnation temperatures up to 10,000 °R. In order to make the required correlation with the analytical study, instrumentation for the nozzle was required to produce data necessary for calculation of
the local nozzle wall inside temperature and heat loads as distributed axially through the nozzle.

**Nozzle Design.** Details of the materials study, stress analysis and backside cooling problems have been previously published (27). It was found that the combination of high thermal stresses and hoop stresses in the nozzle could best be handled in the range of interest by using a thin walled nozzle liner turned from copper-zirconium alloy. In higher pressure and heat load regions of the nozzle the wall thickness was 0.040 inches. Downstream from the throat the liner gradually thickened to approximately 0.15 inches near the exit. The liner was surrounded by a steel split sleeve turned to provide a backside coolant flow channel of constant flow area around the liner. The liner and sleeve were surrounded by a steel cylindrical outer casing. The outer casing was provided with passages for thermocouple lead access to the nozzle liner through the split sleeve.

Twenty-four duplex copper-constantan thermocouples were led in on each side of the liner, passing through a narrow slot milled in the split sleeve, through small holes drilled in a positioning rib, and attached to the nozzle liner with 600 °F, softening temperature solder. The thermocouples started 0.875 inches upstream from the point of smallest radius in the nozzle and were spaced 0.125 inches apart on the liner surface. O-ring seals were provided on either side of the thermocouple passage.

A full-scale sectional view of the liner, sleeve, and casing showing the thermocouple installation, is given in Figure 3.
Fig. 3 Nozzle Assembly
Film Injector Design. An injector plate was designed to match the nozzle liner and casing. It consisted of a single piece turned from copper-zirconium and drilled to provide 18 coolant flow passages evenly spaced about the circumference. These coolant passages discharged into a small plenum chamber just before the film entrance annulus. Provision was made for varying the film injection slot width by placing spacer shims between the injector plate and the nozzle casing. Figure 4 shows a sectional view of the injector plate.

Stilling Chamber Design. Experience with the previous instrumented nozzle (27) showed that the nozzle wall heat load was somewhat lower than that anticipated. There was also a swirl component of velocity of the mainstream gas. A possible reason for the low heat flux in the first experiments was a "natural" film cooling effect caused by the peaking of the temperature at the centerline of the arc heater tube. This, together with the centrifugal effect of the swirl flow, would tend to produce lower temperatures near the wall and thus lower heat fluxes. In order to remove the mainstream swirl and provide a more uniform temperature profile at the nozzle entrance, a stilling chamber was designed, fabricated, and installed between the arc air heater and the film injector station. This chamber consisted of a brass liner 0.125 inches thick and a steel outer casing. It was cylindrical in shape with an inside diameter of ~5 inches and an inside length of ~8 inches. The liner consisted of a cylindrical sleeve and two curved end pieces. Cooling water passages were provided between
Fig. 4 Injection Plate
the casing and the liner and a pressure tap was provided at the downstream end. Figure 5 shows a half-scale sectional view of the stilling chamber.

**Arc Air Heater.** The air for expansion through the nozzle was heated by passage through an electric arc heater supplied by the Linde Company, Indianapolis, Indiana. The heater has a capacity of 1.3 pounds per second of air flow at pressures up to ~1400 psia with a 3/8 inch diameter constrictor nozzle. Energy can be supplied to the air up to a maximum rate of ~4 megawatts. The air heater, stilling chamber, injection plate, and nozzle are shown assembled in Figure 6.

**Instrumentation.** The experiment was set up to be operated remotely due to the high pressures and temperatures involved. Instrumentation for the operation of the air supply and air heater was designed by the staff at Arnold Engineering Development Center and will not be further discussed in the present report. Instrumentation for the stilling chamber, injection station and nozzle consisted of temperature and flow measuring equipment for water and air as supplied to the particular pieces of equipment.

Water flow rate to the stilling chamber was measured by a calibrated flow rate meter. The inlet to exit temperature difference of the stilling chamber cooling water was measured by thermistors. The pressure at the stilling chamber exit was measured by a pressure transducer.

Air mass flow rate to the coolant injector was established by choked flow through a venturi. The inlet temperature was measured by
Fig. 5 Stilling Chamber Assembly
Fig. 6 Photograph of Experimental Assembly
a thermocouple.

Water flow rate to the nozzle backside coolant channel was measured by a calibrated flow rate meter. The inlet-to-exit temperature difference was measured by thermistors. The nozzle wall thermocouple leads were attached to an automatic scanning Austin system.

Experimental Procedure. The experimental nozzle was fabricated and assembled at The University of Tennessee, Knoxville, Tennessee, under the supervision of the investigator. It was then taken to the Arnold Engineering Development Center, Tullahoma, Tennessee, where the actual operation of the experiments was performed by a group now called the experimenter.

The investigator specified gas film cooling ratios of approximately 20%, 10%, and 5% with mass flow rates ranging from 0.4 lbm./sec. to 0.6 lbm./sec. and three enthalpy values ranging up to the maximum value available from the heater at the given flow rate. In addition to the cases at the above film cooling ratios the nozzle was operated for the first run with a gas film cooling ratio of 27.4%.

The gas film injection slot was adjusted for each film cooling ratio. In the range of pressures and temperatures of the test cases the mainstream inlet velocity was from 100 to 250 ft./sec. To minimize mixing of the film with the mainstream gas at injection the slot was adjusted to give film inlet velocities of approximately 150 ft./sec.* For air injected at approximately room temperature this required a slot width

*This film velocity approximately matches the mainstream velocity at the location of injection.
of 0.002 inches for each 5% of coolant. Spacing shims 0.004 inches in thickness were provided by the investigator and one shim was inserted for each 5% coolant ratio. Due to the 30° convergent angle of the nozzle each shim increased the slot width 0.002 inches.

When a test case was run an ambient scan of all the thermocouples was first recorded. The heater was then fired and the thermocouples were scanned approximately 20 times during the run. Since steady state for this system was reached very quickly the thermocouple readings during the data taking period were constant to within approximately 5 °F, except for a few runs when the heater was slightly unstable over a few scanning periods.

The stagnation enthalpy of the gas was calculated by the experimenter using an energy balance for the system including the air heater and the stilling chamber. This balance included the input electrical energy to the heater, the energy removed in cooling the heater and the stilling chamber, and the change of energy of the mainstream gas.

The raw instrument readings recorded on tape were reduced to pressure, temperature, and flow rate values by the experimenter using standard programs and then furnished to the present investigator.

IV. DISCUSSION OF RESULTS

The described analysis was programmed in Fortran IV language for execution on a digital computer. The program is included in Appendix C. Experiments with the gas film cooled, instrumented nozzle were run at film cooling ratios of approximately 30%, 20%, 10%, and 5% of the mainstream flow rate. Table I summarizes the experimental conditions for all the cases run.
Experimental Results. From the twenty-eight cases run five were selected for analytical correlation. They were chosen to include cases from each of the film cooling ratios given above, and in addition two pairs were chosen because their stagnation conditions were close to the same while they had different film cooling ratios. From Table I the cases chosen were H1-1, H2-2, H6-2, H5-2, and H9-2. Cases H1-1 and H5-2, while differing some in stagnation pressure and temperature represent approximately the same severity of heat loading on the nozzle.
<table>
<thead>
<tr>
<th>AEDC Case Number</th>
<th>Mainstream Stagnation Pressure (psia)</th>
<th>Mainstream Stagnation Temperature (°K)</th>
<th>Mainstream Mass Flow Rate (lbm./sec.)</th>
<th>Film Coolant Mass Flow Rate (lbm./sec.)</th>
<th>Film Coolant Inlet Temperature (°F)</th>
<th>Backside Coolant Flow Rate (gpm.)</th>
<th>Backside Coolant Inlet Temperature (°F)</th>
<th>AT (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1-1</td>
<td>349</td>
<td>3840</td>
<td>0.394</td>
<td>0.108</td>
<td>69.0</td>
<td>41.5</td>
<td>65.0</td>
<td>2.57</td>
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<td>H2-1</td>
<td>341</td>
<td>3160</td>
<td>0.412</td>
<td>0.080</td>
<td>49.5</td>
<td>32.0</td>
<td>70.0</td>
<td>4.64</td>
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<tr>
<td>H2-2</td>
<td>373</td>
<td>3660</td>
<td>0.419</td>
<td>0.082</td>
<td>52.5</td>
<td>32.0</td>
<td>66.0</td>
<td>7.34</td>
</tr>
<tr>
<td>H2-3</td>
<td>373</td>
<td>3910</td>
<td>0.419</td>
<td>0.092</td>
<td>55.0</td>
<td>32.0</td>
<td>70.5</td>
<td>8.37</td>
</tr>
<tr>
<td>H3-1</td>
<td>409</td>
<td>3290</td>
<td>0.496</td>
<td>0.094</td>
<td>65.5</td>
<td>30.6</td>
<td>71.5</td>
<td>5.31</td>
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<td>3660</td>
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<td>0.096</td>
<td>65.5</td>
<td>30.7</td>
<td>71.5</td>
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<td>0.095</td>
<td>65.5</td>
<td>30.7</td>
<td>71.5</td>
<td>10.13</td>
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<td>34.1</td>
<td>72.0</td>
<td>4.41</td>
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<td>H4-2</td>
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<td>3320</td>
<td>0.605</td>
<td>0.108</td>
<td>69.0</td>
<td>35.7</td>
<td>72.0</td>
<td>6.74</td>
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<td>0.108</td>
<td>69.0</td>
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<td>72.0</td>
<td>8.68</td>
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<td>H5-1</td>
<td>299</td>
<td>3180</td>
<td>0.401</td>
<td>0.039</td>
<td>55.0</td>
<td>30.1</td>
<td>73.0</td>
<td>5.75</td>
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<td>3450</td>
<td>0.409</td>
<td>0.039</td>
<td>57.5</td>
<td>30.1</td>
<td>73.0</td>
<td>8.18</td>
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<td>H5-3</td>
<td>324</td>
<td>3930</td>
<td>0.401</td>
<td>0.039</td>
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<tr>
<td>H6-2</td>
<td>407</td>
<td>3460</td>
<td>0.525</td>
<td>0.049</td>
<td>63.0</td>
<td>49.2</td>
<td>73.0</td>
<td>6.14</td>
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<td>H6-3</td>
<td>413</td>
<td>3760</td>
<td>0.525</td>
<td>0.049</td>
<td>67.0</td>
<td>49.4</td>
<td>73.0</td>
<td>7.03</td>
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<td>H7-1</td>
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<td>2920</td>
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<td>0.059</td>
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<td>55.7</td>
<td>71.0</td>
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<td>3280</td>
<td>0.603</td>
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<td>56.3</td>
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<tr>
<td>H7-3</td>
<td>490</td>
<td>3600</td>
<td>0.603</td>
<td>0.059</td>
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<td>55.8</td>
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<td>H8-1</td>
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<td>H8-2</td>
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<td>3440</td>
<td>0.401</td>
<td>0.020</td>
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<td>51.6</td>
<td>78.0</td>
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<td>H8-3</td>
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<td>5.22</td>
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*Arnold Engineering Development Center
<table>
<thead>
<tr>
<th>AEDC* Case Number</th>
<th>Mainstream Stagnation Pressure (psia.)</th>
<th>Mainstream Stagnation Temperature (°K.)</th>
<th>Mainstream Mass Flow Rate (lbm./sec.)</th>
<th>Film Coolant Mass Flow Rate (lbm./sec.)</th>
<th>Film Coolant Inlet Temperature (°F.)</th>
<th>Backside Coolant Flow Rate (galm.)</th>
<th>Backside Coolant Inlet Temp. (°F.)</th>
<th>ΔT (°F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H9-1</td>
<td>372</td>
<td>3130</td>
<td>0.526</td>
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<td>51.6</td>
<td>78.0</td>
<td>5.74</td>
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<td>H10-3</td>
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<td>3650</td>
<td>0.581</td>
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<td>55.0</td>
<td>51.6</td>
<td>78.0</td>
<td>6.78</td>
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</tbody>
</table>

*Arnold Engineering Development Center
They provide a good comparison of film cooling effectiveness since case H1-1 was run with 27.4% film cooling while case H5-2 was run with only 9.73% film cooling. Cases H6-2 and H9-2 have almost identical stagnation conditions, while being cooled with 9.34% and 4.27% film cooling respectively. The original wall temperature data for the five cases correlated are given in Tables II, III, IV, V, and VI.

Values for the backside cooling heat transfer coefficient and local values of the heat flux were calculated at the various thermocouple positions using the measured wall temperatures, the local dimensions of the coolant flow annulus, the coolant mass flow rate, and the physical properties of the coolant evaluated at the local film temperature. The Dittus-Boelter equation was used for calculation of the backside heat transfer coefficients.

Theoretical calculations of the backside surface temperature and wall heat flux distributions were made for the five cases named above. These are plotted, along with the measured temperatures and with heat flux curves calculated by smoothing the experimental temperature distributions, in Figures 7 through 16.

Comparison of Theoretical and Experimental Results. Attempts were made to perform the theoretical calculations described above using the eddy diffusivities based on wall shear stress calculated from both the Blasius equation and from the past increment value of the wall shear stress. In several cases both calculations were performed successfully, however in other cases the calculation using the wall
**TABLE II**

ORIGINAL TEMPERATURE DATA FOR GAS FILM CASE H1-1

<table>
<thead>
<tr>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F)</th>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88.00</td>
<td>13</td>
<td>193.33</td>
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<tr>
<td>2</td>
<td>122.60</td>
<td>14</td>
<td>159.59</td>
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<td>3</td>
<td>*</td>
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<td>11</td>
<td>*</td>
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<td>12</td>
<td>222.02</td>
<td>24</td>
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</table>

* A dash indicates a thermocouple which was not reading.
TABLE III
ORIGINAL TEMPERATURE DATA FOR GAS FILM CASE H2-2

<table>
<thead>
<tr>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>147.63</td>
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<td>301.16</td>
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<tr>
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<td>197.99*</td>
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<td>7</td>
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<td>117.73</td>
</tr>
<tr>
<td>8</td>
<td>424.68</td>
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<td>23</td>
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</tr>
<tr>
<td>12</td>
<td></td>
<td>24</td>
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* A dash indicates a thermocouple which was not reading
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<th>Thermocouple Number</th>
<th>Averaged Temperatures (°F.)</th>
<th>Thermocouple Number</th>
<th>Averaged Temperatures</th>
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* A dash indicates a thermocouple which was not reading.
**TABLE V**

ORIGINAL TEMPERATURE DATA FOR GAS FILM CASE HS-2

<table>
<thead>
<tr>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
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* A dash indicates a thermocouple which was not reading.
**TABLE VI**
ORIGINAL TEMPERATURE DATA FOR GAS FILM CASE H9-2

<table>
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<tr>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
<th>Thermocouple Number</th>
<th>Averaged Temperature (°F.)</th>
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</thead>
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<tr>
<td>12</td>
<td>163.20</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

* A dash indicates a thermocouple which was not reading.
Fig. 7 Gas Film Temperature Correlation With Data for Case H1-1
Heat Flux

\[ \text{BTU} \times 10^{-6} \]

Stagnation Conditions
\[ P_i = 349 \text{ psia}, \]
\[ T_i = 6450 \text{ F}. \]

Air Film Injected - 27.4 \%

Calculated for 0 \% Film Coolant

Calculated for Total Mixing at Injection

Calculated From Smoothed Experimental Temperature Curve

Calculated for 27.4 \% Film Coolant

Injection Point

Throat

Thermocouple Number

Distance from Injection - inches.

Fig. 8 Gas Film Heat Flux Correlation With Data for Case H1-1
Fig. 9 Gas Film Temperature Correlation With Data for Case H2-2
Stagnation Conditions

\[ P = 373 \text{ psia} \]
\[ T_o = 6128 \text{ }^\circ\text{F} \]

Air Film Injected - 19.5 %

Fig. 10 Gas Film Heat Flux Correlation With Data for Case H2-2
Fig. 11 Gas Film Temperature Correlation With Data for Case H6-2
Stagnation Conditions
\[ P = 407 \text{ psia}, \]
\[ T_0 = 5768 \text{ °F}, \]
Air Film Injected = 9.34 \%
Calculated for 0 \% Film Coolant

Calculated From Smoothed Experimental Temperature Curve
Calculated for 9.34 \% Film Coolant

Injection Point
Throat

Fig. 12 Gas Film Heat Flux Correlation With Data for Case H6-2
Fig. 13 Gas Film Temperature Correlation With Data for Case H5-2
Fig. 14 Gas Film Heat Flux Correlation With Data for Case H5-2
Stagnation Conditions
\[ P_i = 402 \text{ psia}; \]
\[ T_i = 5750 \text{ °F}. \]

Air Film Injected - 4.27 %

Calculated for 0 % Film Coolant

Calculated for 4.27 % Film Coolant

Measured Backside Wall Temperatures

Thermocouple Number

Injection Point

Throat

Distance from Injection - inches.

Fig. 15 Gas Film Temperature Correlation With Data for Case H9-2
Fig. 16 Gas Film Heat Flux Correlation With Data for Case H9-2
shear stress from the past increment became unstable and either the iterative procedures for the velocity profile or the temperature profile would not converge. Reduction of the incremental distance achieved convergence for some of the cases which were initially unstable, however in cases with very low ratios of film coolant even the reduction to increment lengths of the order of 1/10,000 of the nozzle length did not bring about stability of the solutions. Calculations using the eddy diffusivity based on wall shear stress calculated at each increment from the Blasius equation, on the other hand, remained stable for reasonable incremental lengths for all cases. These results are the ones chosen for comparison with the experimental results. Mainstream and coolant fluid properties as functions of temperature were evaluated from the data presented graphically in Appendix E.

The analytical program written gives as output the local wall heat flux and the streamside nozzle surface temperature. The calculated heat flux was used with the wall thickness and thermal conductivity to calculate backside wall temperatures for correlation with the measured backside temperatures.

In addition to the data taken by the present investigator some recent data were available taken by Lieu (25) for air film cooling of a supersonic nozzle. It was decided to make correlations of the Lieu data for a case with stagnation pressure 315 psia and stagnation temperature 1032 °R. The analytical and experimental results for
Fig. 17  Gas Film Temperature Correlation With Data From the Lieu Nozzle - 4.35% Injection Ratio
Fig. 18 Gas Film Temperature Correlation With Data From the Lieu Nozzle - 8.24% Injection Ratio

Nozzle Conditions

\[ P_0 = 315 \text{ psia} \]
\[ T_0 = 1032 \text{ }^\circ\text{R.} \]

Inlet and Exit Diameters = 1.00 inches.
Throat Diameter = 0.500 inches.

Experimental Wall Temperatures

Calculated Wall Temperatures

Injection Point

Throat
Fig. 19 Gas Film Temperature Correlation With Data From the Lieu Nozzle - 12.33% Injection Ratio
three different coolant injection rates are presented in Figures
17, 18, and 19.

As a check on the changes which would occur in the theoretically predicted wall temperatures and heat fluxes, if it were assumed that the gas film coolant were totally mixed with the free stream gas in a very short distance from the injection point, two additional theoretical calculations were made. Total mixing was assumed for Case H1-1, which had 27.4% gas film coolant, and theoretical calculations were made for wall temperatures and heat fluxes. Case H2-2, which had 9.73% gas film coolant, was also recalculated assuming total mixing at the injection point. These theoretically calculated curves are shown, along with the curves calculated for no film cooling and for full film cooling, in Figures 7, 8, 13, and 14.

Without discussing each of the Figures 7 through 16 in detail several general observations may be made. First, the theoretically predicted temperatures for no film cooling fall well above the measured temperatures for the larger ratios of gas film coolant, but as the amount of film coolant is decreased the theoretical curve approaches the measured temperatures until the correlation is very good for Case H9-2, with approximately 5% film coolant. This indicates that the predicted temperatures for no film cooling using the present analysis should be reasonably accurate. As a further check on the theoretical model, with no film cooling, comparison was made to data published by the Jet Propulsion Laboratory (42) and the results are shown in Figure 20. This figure further demonstrates the validity of the theoretical model to predict temperature or heat flux profiles, at least through the throat. With the accuracy of
Fig. 20. Comparison of Theoretical Model with Data from Jet Propulsion Laboratory.

Wall Temperature °F

- Point for film injection
- Calculated for zero convergence half-angle = 15°, divergence half-angle = 10°, 0 inches entry length
- Case 313
- JPL Data Points
- $P_0 = 201.7$ psia
- $T_0 = 1547°K = 1057°F$.

Throat
the no film cooling calculations established, examination of the above
named figures indicates that the gas film coolant does measurably
lower the wall temperatures and heat fluxes, particularly in the con-
verging section of the nozzle. Third, from Figures 7, 8, 13, and 14,
which contain curves calculated assuming complete mixing of the coolant
with the mainstream at injection, it may be seen that such predicted
values fall well above the measured values of wall temperature and heat
flux.

The effectiveness of the gas film for protecting the nozzle wall
is further shown in Figure 21, which shows photographs of the converging
section of the nozzle and the inside surface of the injection ring.
These photographs were made without cleaning up the surfaces following
the conclusion of the experimental program. The conical converging
section of the nozzle is still relatively smooth and clean. There is a
noticeable change very close to the throat, where the nozzle is somewhat
roughened and discolored. The inside of the injection ring is badly
eroded. In a previous nozzle operated without gas film cooling (27),
the entire convergent portion was eroded during the experiments.

Figures 22 and 23 show a direct comparison of the wall heat
flux for the two pairs of cases with very similar stagnation conditions.
In Figure 22 the larger heat flux for the case with approximately 10 %
cooling as compared with the case for approximately 30 % cooling is
evident. In Figure 23, for cases with approximately 5 % and 10 %
cooling there is very little difference, but an examination of the
tabulated experimental temperatures for the two cases shows that the
temperatures near the nozzle entrance for the 5 % cooling case were
Fig. 21 Photograph of Nozzle Liner and Injection Ring after Termination of Experimentation
Fig. 22 Comparison of Heat Fluxes for Cases H1-1 and H5-2
Fig. 23 Comparison of Heat Fluxes for Cases H6-2 and H9-2
slightly higher. Comparison of the backside coolant flow rates for
the two cases from Table I, pages 33 and 34, shows that the 5 % case
was more strongly cooled. Taken together this is an indication
that the gas film coolant was effective in the converging section.

Study of the measured temperatures in Figures 7 through 16
shows a quick rise in temperature near the nozzle entrance, then
a rather steady temperature until it rises rapidly toward the throat.
The present investigator believes this is due to a certain amount
of gross mixing at or near the injection point. This "mixed film"
then forms the actual film coolant.

The temperature curves also reveal that as the nozzle was
operated for longer periods of time the differential thermal expansion
between the liner and the instrument rib caused the far downstream
thermocouples to loosen from the wall and read low. This was caused
by the freeing of the downstream end of the liner for expansion.
Notice in this connection thermocouples 14 through 24 in Figures 11,
13, and 15. This also explains the poor correlation between the
curves in Figure 22 for the section downstream from the throat.

Comparison of the calculated temperatures with the measured
values in the case of the Lieu nozzle, Figures 17, 18, and 19 on
pages 52, 53, and 54, respectively, show reasonably good correlation
for the 4.35 % injection ratio but poorer correlation for the
higher injection rates. The present investigator noted some apparent
discrepancy between Lieu's data and the present data, however. For
Lieu's nozzle the maximum temperature showed a definite shift upstream
as the film cooling was increased. Both the analytical prediction and
the experimental data in the present investigation show maximum temperatures immediately downstream from the throat with very little shift in position as the injection ratio varies from ~5% to ~30%.
CHAPTER III
LIQUID FILM COOLING

I. THEORETICAL ANALYSIS

As was stated in Chapter I, Bartz's equation for the heat transfer coefficient in a nozzle without coolant injection is given in its final form as

\[ h = \left( \frac{C}{D_x} \right) \left( \frac{u^{*2}}{P_r} \right) \left( \frac{P_o g_c}{u^{*}} \right) \left( \frac{D_x}{A^{*}} \right) \left( \frac{D_x}{A} \right) \sigma \]  

in which

\[ \sigma = \frac{1}{\left[ \frac{1}{2} \frac{T_w}{T_o} + 1 + \frac{1}{2} M^2 + \frac{1}{2} \right]^{0.8 \omega/5} \left[ 1 + \frac{1}{2} M^2 \right]^{0.2 \omega/5}} \]  

The main point of interest for the present investigation is that Bartz based his derivation on the Dittus-Boelter form of correlation for fully developed turbulent pipe flow, i.e.

\[ Nu = C (Re)^m (Pr)^N \]  

Emmons (13) developed a one dimensional analysis for cooling with an evaporating liquid film in fully developed turbulent pipe flow. It was based on the assumptions previously listed in Chapter I. He
started with the following basic equations for a turbulent incompressible boundary layer:

Continuity

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{31}
\]

Momentum

\[
\frac{d}{d\tau} \left( \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = \frac{3}{2} \left[ \bar{u} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right] \tag{32}
\]

Energy

\[
\frac{d}{d\tau} \left( \bar{v} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} \right) = \frac{3}{2} \left[ \bar{v} + \bar{v} \frac{\partial \bar{T}}{\partial y} \right] \tag{33}
\]

Diffusion

\[
\frac{d}{d\tau} \left( \bar{w} \frac{\partial \bar{a}}{\partial x} + \bar{w} \frac{\partial \bar{a}}{\partial y} \right) = \frac{3}{2} \left[ \bar{w} + \bar{v} \frac{\partial \bar{a}}{\partial y} \right] \tag{34}
\]

The barred parameters in these equations represent time averaged quantities, for example:

\[
\bar{u} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau u(t) \, dt \tag{35}
\]

The bars are now dropped, with the understanding that these parameters are time averaged.

Emmons simplified Eqs. (31), (32), and (33) by assuming that terms including gradients in the x-direction are negligible compared with terms involving gradients in the y-direction. From the continuity equation this gave \( \bar{v} = \text{constant} \). Emmons evaluated this constant by assuming that the gas on the surface of the liquid was pure coolant vapor, thus

\[
\bar{v} = \frac{Q}{y} \tag{36}
\]
The momentum and energy equations were then

Momentum

\[ \frac{\partial}{\partial y} \left( \frac{\rho u}{\gamma} \right) = \frac{d}{dy} \left[ (u+ \varepsilon_M) \frac{du}{dy} \right] \] \hspace{1cm} (37)

Energy

\[ \frac{\partial T}{\partial y} = \frac{d}{dy} \left[ (\alpha+ \varepsilon_M) \frac{dT}{dy} \right] \] \hspace{1cm} (38)

At this point Emmons introduced the expression for the eddy diffusivity of momentum as developed by Rannie (29) and modified by Turcotte for the case of wall injection (40).

\[ \varepsilon_M = u \sinh^2 \left( \frac{by}{13.89} \right) \] \hspace{1cm} (39)

Substituting from Eq. (39) into Eq. (37) and integrating assuming zero velocity at \( y = 0 \) gave

\[ u = \frac{\tau_w}{\rho_w Q} \left[ \exp \left( \frac{13.89 \ Q}{by \sqrt{\tau_w / \rho_w}} \right) \tan \left( \frac{by \sqrt{\tau_w / \rho_w}}{13.89 u} \right) - 1 \right] \] \hspace{1cm} (40)

From Turcotte's analysis (40)

\[ \tau_w = \tau_{ni} \left[ \exp \left( \frac{8.13.89 \ Q}{by \sqrt{\tau_w / \rho_w}} \right) \right] \] \hspace{1cm} (41)

so

\[ u = \frac{\tau_{ni} \ e^{-6Q}}{\rho_w Q} \left[ \exp \left( 8Q \ tanh \left( \frac{e \ Q}{6u y} \right) - 1 \right) \right] \] \hspace{1cm} (42)
Evanons then used an extended Reynolds analogy, which \( \text{Pr} = 1 \) and \( \text{Sc} \) to get the temperature distribution as

\[
\frac{T-T_\infty}{T_s-T_\infty} = \frac{1}{\text{Pr}} \frac{e^{-\beta q}}{e^{eta q} - e^{-\beta q}}
\]

\[
\text{St} = - \frac{1}{2} f_1 \frac{u_e}{u} e^{-\beta q}
\]

\[
\frac{u_e}{u} = \sqrt{\frac{1}{\text{Pr}}} e^{\beta q}
\]

where

\[
\beta = 13.89
\]

without further modification.

Use of this temperature distribution gave the following equation for the Stanton Number:

\[
\text{St} = - \frac{1}{2} f_1 \frac{u_e}{u} e^{-\beta q}
\]

Evanons then noted that for the case \( u = u_e \) and \( Q = 0 \) this reduces to the result obtained by the simple Reynolds analogy. He then proceeded to use the heat transfer coefficient obtained from Eq. (44), specifically

\[
\frac{1}{\text{Pr}} \frac{e^{-\beta q}}{e^{eta q} - e^{-\beta q}}
\]
It was noted by the present investigator that the exponential term in Eqs. (46) and (47) could be looked upon as a simple correction term applied to the non-injection heat transfer coefficient to take into account the fluid injection, either by evaporation or by transpiration at the surface.

There are better correlating equations available for predicting the convective heat transfer coefficient without injection than that which comes from the simple Reynolds analogy. One of these is the Dittus-Boelter equation for fully developed turbulent pipe flow, the form of which was used by Bartz in his analysis (2). It was thus decided to program a stepwise finite difference analysis for digital computer solution using the Dittus-Boelter correlation at each step but substituting the coefficient "C" as determined by Bartz. The convective heat transfer coefficient thus determined would then be modified using Emmons' "exponential correction" factor. The program also provides for the direct use of experimentally determined heat transfer rates without injection and modifying these using Emmons' "exponential correction" factor.

Emmons' analysis assumed that the surface of the coolant film was at saturation temperature immediately upon injection. If the coolant has a high heat capacity and high saturation temperature this is not a good assumption, even for very high heat transfer rates. It was thus decided to forego this assumption and calculate the temperature profile of the liquid film starting at the injection point and taking into account the evaporation of the film while it is increasing in
temperature up to the saturation temperature.

In order to calculate the developing temperature profile it is necessary to assume, or calculate, the velocity profile. Since the ratio of the diffusivity of momentum to the diffusivity of heat, i.e., Pr is > 3 at room temperature for water, the main coolant liquid under consideration, it was decided to assume instantaneous development of the velocity profile upon injection. A condition of constant shear stress across the liquid film was also assumed. This latter assumption appears to be reasonable for such a thin film as that used for film cooling.

Consider a fluid layer as shown in Figure 24, with mass flow rate \( w \). Now for constant properties and constant shear stress across this layer

\[
 u_{n+1} = u_n + \frac{\tau_w d_n}{\mu} \quad (48)
\]

and

\[
 w = 2\pi r_n \rho d_n \left( \frac{u_{n+1} + u_n}{2} \right) \quad (49)
\]

Combining Eqs. (48) and (49) and rearranging

\[
 \frac{\rho \tau_w}{(2\mu)} d_n^2 + \left( \rho u_n \right) d_n - \frac{w}{2\pi r_n} = 0 \quad (50)
\]

and

\[
 d_n = \frac{-\rho u_n + \sqrt{(\rho u_n)^2 + \rho \tau_w w/\pi r_n \mu}}{\rho \tau_w / \mu} \quad (51)
\]
Fig. 24 Velocity and Temperature Notation for Liquid Film by Layers and Increments
The positive value on the radical is used since $d_n$ must be positive. Starting with a known value of $u_n$ and estimating initial values of $\rho$, $\mu$, and $\tau_w$, Eq. (51) may be solved for $d_n$, then $u_{n+1}$ calculated from Eq. (19). As better average values of temperatures become known from the iterative solution of the heat balance equations $\rho$ and $\mu$ are corrected and the velocity profile recalculated. It is not necessary to re-evaluate the velocity profile for each temperature iteration as it produces only very slight effects. However, when the convergence criterion for the temperature iteration is satisfied, the velocity profile should be corrected and convergence checked again. This assures the use of values of $\rho$ and $\mu$ which are temperature corrected.

The equations necessary for the temperature profile calculation are derived by taking a heat balance on the nth layer. The heat balance for steady state requires that:

Net Heat Rate Convected In + Net Heat Rate Conducted In = 0  \hspace{1cm} (52)

Referring to Figure 24, page 69.

\[ \text{Net Heat Rate Convected In} = w c_a (T_{1n} - T_{2n}) \]  \hspace{1cm} (53)

\[ \text{Net Heat Rate Conducted In} = 2\pi r A \left[ \frac{T_{1n+1} + T_{2n+1}}{2} - \left( \frac{T_{1n} + T_{2n}}{2} \right) \right] \]

\[ + \frac{T_{1n-1} + T_{2n-1}}{2} - \left( \frac{T_{1n} + T_{2n}}{2} \right) \]

\[ + \frac{d_{n+1}}{2k_{n+1}} + \frac{d_n}{2k_n} \]  \hspace{1cm} (54)
Let

\[
\overline{T}_{n+1} = \frac{(T_{1,n+1} + T_{2,n+1})}{2}
\]  

(55)

and similarly for \(\overline{T}_n\) and \(\overline{T}_{n-1}\).

Also let

\[
K_{n+1} = \frac{2\pi r_{n+1} d_{n+1}}{(\frac{d_{n+1}}{2} + \frac{d_n}{2k_{n+1}})}
\]

(56)

and similarly for \(K_n\) and \(K_{n-1}\).

Now substituting from Eqs. (53), (54), (55), and (56) into (52)

\[
v_n c_n (T_{1,n} - T_{2,n}) + \Delta K_{n+1} (\overline{T}_{n+1} - \overline{T}_n) + \Delta K_{n-1} (\overline{T}_{n-1} - \overline{T}_n) = 0
\]

(57)

Resubstituting from Eq. (55) for \(\overline{T}_n\) and solving for \(T_{2,n}\)

\[
T_{2,n} = \frac{T_{1,n} \left[ w_n c_n - \frac{\Delta (K_{n+1} + K_{n-1})}{2} \right] + \Delta (K_{n+1} T_{n+1} + K_{n-1} T_{n-1})}{w_n c_n + \frac{\Delta (K_{n+1} + K_{n-1})}{2}}
\]

(58)
Now Eq. (58) applies for the interior layers of the film but other equations must be developed for the heat balance on the stream-side and wall-side layers. For the stream-side layer

\[ w_s c_s (T_1 - T_2) + \Delta K_{s-1} (\overline{T}_{s-1} - \overline{T}_s) + 2\pi r_s A h (T - \overline{T}_s) - 2\pi r_s Q_{fg} = 0 \] (59)

and

\[ T_2 = \frac{T_1 \left[ w_s c_s - \frac{\Delta}{2} (K_{s-1} + 2\pi r_s h) \right] + (K_{s-1} \overline{T}_{s-1} + 2\pi r_s h T_s) - 2\pi r_s Q_{fg}}{w_s c_s + \frac{\Delta}{2} (K_{s-1} + 2\pi r_s h)} \] (60)

For the wall-side layer

\[ w_w c_w (T_w - T_{w+1}) + \Delta K_{w+1} (\overline{T}_{w+1} - \overline{T}_w) + \Delta K_b (T_b - T_w) = 0 \] (61)

where

\[ K_b = \frac{2\pi r_w}{\frac{d_w}{2k_w} + \frac{d_1}{k_1} + \frac{1}{h_b}} \] (62)

Thus

\[ T_{2w} = \frac{T_1 w \left[ w_w c_w - \frac{\Delta}{2} (K_{w+1} + K_b) \right] + \Delta (K_{w+1} \overline{T}_{w+1} + K_b T_w)}{w_w c_w + \frac{\Delta}{2} (K_{w+1} + K_b)} \] (63)

Iterative calculation of Q is described starting in paragraph 2, page 77. The coefficient h is determined from the modified Dittus-Boelter equation which will subsequently be discussed in detail. The coefficient h_b is determined from the backside coolant flow conditions.
Since the backside coolant flow will be turbulent for practically all cases, a regular Dittus-Boelter calculation will normally be used. 

Application of Eqs. (58), (60), and (63) across the film results in a set of nonlinear simultaneous algebraic equations. The non-linearity comes from the dependence of \( c, d, \) and \( k \) on \( T \). Since this dependence is relatively weak, however, the equations may be solved by the Gauss-Seidel iteration method with periodic corrections of \( c, d, \) and \( k \) for temperature change. This is done in the same manner as the periodic correction of \( \rho \) and \( \mu \) in the velocity profile calculation.

The temperature profile is first estimated. Substitution is then made into the explicit equations for \( T_2, T_2, \) and the amount of change in \( T_2 \) checked against some convergence criterion until it is satisfied for all the layers. Periodically \( c, d, \) and \( k \) are re-evaluated, with a final evaluation made as a check when \( T_2 \) has converged.

Other methods could be used for the solution of Eqs. (58), (60), and (63). For example, by rearrangement they could be put into the matrix form:

\[
\begin{align*}
\bar{A} \bar{T} &= \bar{B} \\
(64)
\end{align*}
\]

By estimating \( \bar{T} \), \( \bar{A} \) could be calculated. Then
\[
\bar{T} = \bar{A}^{-1} \bar{B}
\]

\( \bar{A} \) could then be recalculated using the new values for \( \bar{T} \) and the method repeated until \( \bar{T} \) converges. The Gauss-Seidel iteration method was used because the coefficient matrix is sparsely populated and this method does not require manipulation of the zero elements.
The incremental film temperature calculation is continued as described above until $T_{2g} > T_{sat}$. Normally after some particular incremental calculation $T_{2g} > T_{sat}$. In order to determine the position where $T_{2g} = T_{sat}$ an interpolation is made. It was found that if the initial incremental steps are small a straight line interpolation here is sufficiently accurate without further iterative calculation of $T_{2g}$.

The evaporation rate $Q$ must be calculated both in the region before $T_{2g} = T_{sat}$ and following this region. The only difference between the two calculations is that in the region before saturation temperature is reached the rate of evaporation is controlled by the available partial pressure of the film vapor and the mass transfer coefficient at the surface, whereas in the region following the rate of evaporation is controlled by the excess thermal energy deposited in the film by heat transfer from the mainstream. In both cases the heat transfer coefficient $h$ must be corrected for the evaporation effect and the loss of liquid film due to evaporation must be calculated. Examination of these effects shows that they are coupled and thus require an iterative solution. The evaporation rate for a given evaporation enthalpy and temperature difference varies with $h$. On the other hand, the following equation for the heat transfer coefficient shows the dependence of $h$ on the evaporation rate:

$$h = 0.0265 \frac{kH}{D} (Re)^{0.8} (Pr)^{0.333} \exp\left(\frac{-13.89 Q}{byu^2}\right)$$  \hspace{1cm} (65)

In Eq. (65) $Q$ is the mass evaporation rate of the film coolant. In the region before saturation temperature is reached $Q$ may be calculated directly using Eq. (69), page 75, and the vapor partial pressure.
For the region following the attainment of saturation temperature, hereafter called the boiling region, there is a simple relationship between $Q$ and $h$.

$$Q_{hfg} = hA(T_{Adw} - \frac{T_1 + T_2}{2}) - Q_s$$  \hspace{1cm} (66)

or

$$Q = \frac{h(T_{Adw} - \frac{T_1 + T_2}{2}) - Q_s}{h_{fg}}$$  \hspace{1cm} (67)

Substituting from Eq. (67) into Eq. (66)

$$h = \frac{.0265k_{fg}B(H(Re), 8(Pr))^{333}}{D^{333}}\exp\left[-13.89h(T_{Adw} - \frac{T_1 + T_2}{2}) - 13.89\frac{Q_s}{A}\right]$$  \hspace{1cm} (68)

In Emmons' analysis it was assumed that the fluid in the boundary layer over the liquid surface is pure coolant vapor and that the thermal conductivity $k_B$ in Eq. (68) should be that of the pure coolant vapor evaluated at $T_{sat}$. It has been demonstrated (12) that constant property solutions for skin friction, Nusselt number and recovery factor may be used even when large variations in physical properties occur provided the properties are evaluated at a reference temperature $T^*$ which is expressed explicitly as

$$T^* = T_g + 0.5(T_w - T_g) + 0.22(T_r - T_g)$$  \hspace{1cm} (69)

It can be readily seen that $T^*$ is the widely used film temperature (or average of $T_w$ and $T_g$) plus a term to account for temperature recovery in high speed flow. This term seems to effect a good property correction even for dissociation effects. In the present solution fluid properties are evaluated at $T^*$. 

75
If the thermal conductivity of the binary mixture differs greatly from that of the coolant vapor an effort should be made to evaluate $k_\text{m}$ for the binary mixture. If the concentration of coolant vapor at the liquid film surface can be calculated the thermal conductivity for a gas mixture at half this concentration should be a better value to use than that of the pure coolant vapor. This conductivity should be evaluated at $T^*$ as indicated above.

Examination of Eqs. (32), (33), and (34) shows that the same analogy may be made between mass transfer and momentum transfer as that which was made between heat transfer and momentum transfer. Experimental results indicate that by use of the proper form for the diffusivity of mass and use of the Schmitt number, $Sc$, in place of $Pr$, a coefficient of mass transfer for turbulent flow may be calculated as

$$h_N = \frac{0.0265 k}{D} \left( Re \right)^{0.8} \left( Sc \right)^{0.333}$$

(70)

It is therefore proposed that in accordance with the above mentioned analogy Eq. (70) be modified and written in the form of Eq. (68) as

$$h_N = \frac{0.0265 k}{D} \left( Re \right)^{0.8} \left( Sc \right)^{0.333} \left[ \exp \left( -13.890 \frac{\text{byu}^2}{x} \right) \right]$$

(71)

If the total coolant injection rate is only a small percentage of the main gas flow rate any concentration of coolant in the main stream may be neglected so for this case

$$\gamma_c = \frac{Q}{h_N}$$

(72)

Now

$$\gamma = \gamma_c + \gamma_s$$

(73)
Substituting Eqs. (72) and (73) into Eq. (71) gives

\[ h_M = \frac{0.0265k_M}{D} \text{ (Re)}^6 \text{ (Sc)}^\frac{8}{333} \left[ \exp \left( \frac{-13.890}{\text{bu} \left( \frac{Q}{h_M} + \gamma_s \right)} \right) \right] \]  \tag{74}

The iterative procedure for determining \( h \) is then as follows:

1. Calculate \( k_M \) and Sc at \( T^* \).
2. Calculate \( Q \) based on value of \( h \) for previous increment.
3. Starting with \( Q \) from step 2 iteratively calculate \( h_M \) using Eqs. (71), (72), (73), and (74).
4. Calculate \( h \) from Eq. (65).
5. Calculate \( Q \) from Eq. (67).
6. Calculate \( \gamma_c \) from Eq. (72).
7. Re-evaluate \( k_M \) based on \( \gamma_c \) from step 6.
8. Re-evaluate \( h \) using Eq. (68) and compare with \( h \) from step 4.
9. Repeat steps 3 through 7 until \( h \) converges.
10. Re-evaluate \( Q \) and compare with \( Q \) from step 2.
11. Repeat steps 3 through 10 until \( Q \) converges.

Two additional problems are encountered in the procedure outlined above. One of these is the determination of the thermal conductivity of a binary gas mixture at a given temperature and the other is the determination of the diffusivity of mass for the coolant vapor through the mainstream gas for a known pressure and temperature.

Thermal conductivities of gas mixtures may be estimated from

\[ k_{\text{mix}} = \sum_{i=1}^{n} x_i k_i \]  \tag{75}
where the $x_i$ and $k_i$ are mole fractions and thermal conductivities, respectively, of the pure gas species.

Given the diffusivity of mass for a binary gas mixture at one condition of pressure and temperature the diffusivity at other pressures and temperatures may be estimated from the following equation (76)

$$D_{AB} = \frac{C T^{3/2}}{P} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]$$

thus

$$\left( D_{AB} \right)_T \frac{T_2}{P_2} = \left( D_{AB} \right)_T \frac{T_2}{P} \frac{T_2}{P_2}$$

(77)

In the present study Eq. (77) was used to predict mass diffusivities at the local reference temperature $T^*$ and the local static pressure. Dependence of the diffusivity on concentration was neglected.

Now for the diffusion of mass

$$I_A = -D_{AB} \frac{\partial \rho_A}{\partial y}$$

(78)

where $I_A$ is the mass current in lbm./ft.$^2$hr. and $\partial \rho_A / \partial y$ is the density gradient in lbm./ft.$^2$. In forced convection mass transfer the diffusivity $D_{AB}$ plays the same role as that played by the thermal conductivity in conduction heat transfer. Eq. (71) may thus be rewritten as...
In order to use the above equations at each axial position the
mainstream velocity, temperature, and density must be calculated.
Frozen flow through the nozzle is assumed with a specific heat ratio
determined by the bulk mean temperature at the nozzle entry. The
equation which relates the local diameter to throat diameter ratio,
specific heat ratio, and Mach number is

\[
\frac{D}{D_e}^2 = \frac{1}{H} \left[ \frac{2}{k+1} + \frac{(k-1)H}{k+1} \right] \left( \frac{k+1}{2(k-1)} \right)
\]

(80)

Now \( \frac{D}{D_e} \) and \( k \) are known and \( H \) is to be calculated. Since Eq. (80)
cannot be solved explicitly for \( H \) some iterative procedure must be
used to solve for \( H \).

To solve Eq. (80) for \( H \) the following rapidly converging
iterative procedures were devised. Designate the Mach numbers in
Eq. (80) differently by subscripts as follows:

\[
\frac{D}{D_t}^2 = \frac{1}{H_a} \left[ \frac{2}{k+1} + \frac{(k-1)H_b}{k+1} \right] \left( \frac{k+1}{2(k-1)} \right)
\]

(81)

Now for iteration in the subsonic portion of the nozzle, i.e.,
upstream from the throat, Eq. (81) is solved explicitly for \( H_a \).
The iteration may be started by substituting any number from 0 to 1 into Eq. (82) for $M_b$. The value of $M_a$ calculated is then substituted back in for $M_b$ and the procedure repeated until the difference between $M_a$ and $M_b$ meets a specified criterion. For the supersonic portion of the nozzle, i.e., downstream from the throat, Eq. (81) is solved explicitly for $M_b$.

$$
M_b = \left[ \left( \frac{M_a}{D/D_t} \right)^2 \frac{2(k-1)}{k+1} - \frac{2}{k+1} \right]^{\frac{1}{2}}
$$

(83)

This iteration is started by substituting 1 or a number greater than 1 into Eq. (81) for $M_a$. After $M_b$ is calculated it is substituted back in for $M_a$ and the procedure repeated as described above.

With the Mach number calculated the local pressure, temperature and density for the main stream gas may be calculated.

The general calculational procedure for liquid film cooling proceeds as follows:

1. For the known axial position calculate the ratio $D/D_t$.
2. Using Eqs. (82) and (83) and the iterative procedure described calculate the Mach number.
3. Using the Mach number calculate the local (free stream) pressure, temperature and density.

4. Evaluating properties at the reference temperature calculate $Re$, $Pr$, Sc, and $D_{AB}$.

5. Calculate $T_{ni}$ and $T_{sat}$ for the film.

6. Check to see if the film surface is up to $T_{sat}$. If not calculate the film velocity and temperature profiles correcting for evaporation effects and go to step (9). If the film surface is at or above $T_{sat}$ go to step (7).

7. Set film surface temperature equal to $T_{sat}$ and calculate film velocity and temperature profiles.

8. Calculate $h$ iteratively correcting for the change in thermal conductivity due to mass transfer.

9. Subtract the amount of film coolant evaporated over the axial increment from the mass flow rate in the film surface layer. (Some fraction of the layer is set as a maximum amount to be evaporated per axial increment and the increment size reduced if this is exceeded).

10. Increment the axial position and return to step (1).

The above procedure has been programmed in Fortran IV language for digital computer execution and the program is included as Appendix B.

The above procedure may also be applied for calculation of coolant evaporation rates if experimental heat transfer values without film cooling are available. If the flow condition is turbulent, so that Turcotte's formula for $e_M$, Eq. (39), is a good approximation
Emmons' exponential correction factor should be directly applicable.

The procedure is as follows:

1, 2, 3, 4, 5. These steps are the same as for calculation of the heat load from stream variables.

6. Check to see if the film surface is up to $T_{\text{sat}}$. If not use the experimental heat load, $q_x$, and calculate film velocity and temperature profiles correcting for evaporation effects and go to step (9). If the film surface is at or above $T_{\text{sat}}$ go to step (7).

7. Set the film surface temperature equal to $T_{\text{sat}}$ and calculate the film velocity and temperature profiles.

8. Using

$$ Q = \frac{q_x - \frac{Q_x}{A}}{h_{fg}} $$  \hspace{1cm} (84)

and

$$ q = q_x \frac{k_H}{(k_H)_{\text{nf}}} \left[ \exp \left( \frac{-13.89 \frac{Q_x}{G_x}}{byu} \right) \right] $$ \hspace{1cm} (85)

calculate the heat load to the film, $q$, iteratively correcting for the change in thermal conductivity due to mass transfer.

9, 10. Same as for direct calculation from stream variables.
II. DISCUSSION OF ANALYTICAL RESULTS

The above analysis of liquid film cooling was programmed in Fortran IV language for execution on a digital computer. The program is included as Appendix B. Since experimental studies of this type cooling were not to be performed a test case was postulated in order to check out the analysis. In this test case the nozzle configuration, mainstream stagnation conditions, and the backside cooling heat transfer coefficient were similar to those for the experimental studies of gas film cooling. It was found that a finite distance is required for the liquid film to rise to saturation temperature, and that a significant fraction of the liquid evaporates during this interval. In the case chosen this amounted to approximately 30 per cent of the injected film mass flow rate. Figure 25 shows the increase of the film surface temperature and the backside wall temperature as the film progresses through the nozzle. The increased heat flux at the throat and the rapidly decreasing pressure cause the liquid at the surface to reach the saturation temperature just downstream from the throat for this particular case. The liquid surface then remains at the saturation temperature in the diverging portion of the nozzle as the pressure continues to drop, and the rate of film evaporation is controlled largely by the heat transfer rate available from the mainstream gas. The sample computer calculation for this case is given in Appendix F.
Stagnation Conditions

P = 300 psia

Mainstream Air Saturated With H₂O

T₀ = 4000 °F.

Film Injected at 70°F.

1.1% H₂O Film Injected at 70°F.

Local Saturation Temperature
For Water Film

Temperature
°F.

Wall Temperature

Film Surface Reached Saturation

Streamside Film Surface Temperature

Injection Point

Throat

Distance From Nozzle Entrance - Inches

Fig. 25 Graph of Sample Liquid Film Calculation Results
Transpiration Based on Emmons' Liquid Film Analysis. A careful comparison of liquid film cooling with transpiration cooling shows that they differ in only two ways, namely, in transpiration cooling:

1. There is no liquid film to be transported or to absorb heat.
2. The coolant flow rate may be controlled locally independently of the heat transfer rate.

The basic analysis for liquid film cooling in the locations where boiling occurs may thus be utilized for transpiration cooling.

In the case of transpiration cooling with a liquid it is necessary for the liquid to be vaporized at the surface or before reaching the surface. The best procedure is to specify a local surface temperature which is equal to or greater than the local saturation temperature. The parameter to be calculated is the coolant mass flow rate per unit area required to maintain the surface at the specified temperature. This may be done iteratively using the following procedure:

1, 2, 3, 4. Same as for liquid film cooling.
5. Calculate $T_{\text{sat}}$ and set the surface temperature either equal to $T_{\text{sat}}$ or to some specified temperature higher than $T_{\text{sat}}$.
6. Starting with the heat transfer rate without transpiration use this to calculate the required rate to hold
the surface at the set temperature.

7. Correct the heat transfer rate for transpiration effects using the procedure outlined for liquid film cooling. Recalculate the required transpiration rate. Repeat this step until either the change in heat transfer rate or the change in transpiration rate meets the set convergence criterion.

8. Increment the axial position and return to step (1).

The procedure is identical for gas transpiration cooling except the calculation of $T_{sat}$ is not required. It has been programmed in Fortran IV language for execution on a digital computer. The program is included in Appendix D.

The same type of calculation may be made using heat transfer rates determined experimentally without transpiration cooling. The changes are the same as those for liquid film cooling except the calculation of film evaporation is not required.

**Analysis Based on Gas Film Cooling.** The analysis given for gas film cooling may be used for gas transpiration cooling provided it is assumed that the same model may be used in the calculation of eddy diffusivities for both types of cooling. This is done by specifying a very small axial increment for the calculation and a small amount of coolant fluid to be added at each increment. A new coolant stream layer is added whenever the second coolant layer from the nozzle wall contains twice as much fluid as it originally contained. The calculation is performed assuming an adiabatic wall.
II. TRANSPIRATION CALCULATION RESULTS

Since experimental studies of transpiration cooling were not to be performed test cases were postulated in order to check out the analyses.

The results of the calculation using the Liquid or Gas Transpiration Cooling Program are shown graphically in Figure 26. Note that as the calculation proceeds into the nozzle an increasing coolant flow rate is required to hold the nozzle wall at 500 °F. The calculation was continued only far enough to insure proper operation of the program and convergence of the iterative procedures. The sample computer calculation for this case is given in Appendix F.

The results of a transpiration calculation using the Gas Film Cooling Program are shown graphically in Figure 27. Note the shape of the wall temperature profile curve for a fixed injection rate at each increment. The maximum temperature occurs, as expected, very near the throat. The sample computer calculation for this case is given in Appendix F.
Fig. 26 Graph of Sample Transpiration Calculation Results Using Liquid or Gas Transpiration Cooling Program
Stagnation Conditions

\[ P = 4400 \text{ psia} \]
\[ T_0 = 3000 \text{ °F} \]

Inlet = 1.5 inches dia.
Throat = 0.5 inches dia.
Exit = 2.0 inches dia.
Injection = 0.02 lbm. per inch of distance

Fig. 27 Graph of Sample Transpiration Calculation Results Using Gas Film Cooling Program
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

Based on the experimental results presented and the development of satisfactory calculational techniques to implement the analyses given in the present study, the following conclusions can be drawn:

1. Gas film cooling can be used to measurably lower the wall temperatures and the wall heat fluxes in the converging section and at the throat of a high-pressure high-temperature nozzle. For this nozzle a change of gas film coolant injection ratio from 5% to 10% produced a measurable decrease in the wall temperatures and the wall heat flux up to a point near the throat. Higher gas film cooling ratios, such as 30%, lower the temperature and heat flux in the converging section and at the throat appreciably and have a measurable effect extending into the supersonic expansion section. Some gross mixing occurs at the injection point, the exact amount being as yet some undetermined function of the injection geometry, the relative velocities of the main gas stream and the coolant stream at the injection point, and the entering velocity profiles and turbulence conditions.

2. A straightforward boundary layer type analysis has been developed and programmed which predicts with reasonable accuracy the nozzle wall temperatures and wall heat fluxes in the converging section and at the throat for gas film cooled nozzles. The calculated wall temperatures and heat fluxes using the full amount of film coolant injected in the experiment and the values calculated assuming no film
coolant bracketed the measured values of temperature and heat flux for all cases correlated. The analysis must be modified in the supersonic portion of the nozzle if accurate prediction of wall temperatures and heat fluxes is required in this region.

3. Calculational techniques have been developed and programmed for predicting the effectiveness of liquid film cooling and liquid or gas transpiration cooling in nozzles. These techniques should be checked against experimental data before they are used for design purposes, however.

It is recommended that gas film cooling experiments in supersonic nozzles be performed with various injection geometries and mainstream to coolant flow velocity ratios in order to evaluate the amount of gross mixing which occurs at injection as some function of these parameters.

It is also recommended that liquid film cooling experiments be performed on a supersonic nozzle operating at high stagnation pressures and temperatures in order to evaluate the accuracy of predictions using the liquid film cooling analysis in the present investigation.

It is further recommended that nozzles fully cooled up to the throat by transpiration be constructed and tested experimentally in order to evaluate the accuracy of predictions using the analyses presented in the present investigation.

*A possible modification uses information as shown in Fig. 7-58 of Turbulence by J. O. Hinze, a text published by McGraw-Hill.*
BIBLIOGRAPHY


APPENDIX A
NOTATION

A  nozzle cross sectional area or incremental heat transfer area
A_\text{th} nozzle throat cross sectional area
\bar{A} two-dimensional coefficient matrix
\bar{B} column matrix
b Turcotte's constant to correct diffusivity for mass injection
C constant
c_p specific heat at constant pressure
c_\text{th} sonic velocity at nozzle-throat
c_w heat capacity of fluid in layer at wall
D nozzle inside diameter
D_{AB} binary mass diffusivity
D_\text{th} nozzle throat inside diameter
D_t nozzle throat inside diameter
d_n liquid coolant layer thickness
d_l local nozzle liner thickness
f_{ni} friction factor with no injection or evaporation
g_c gravitational constant
h convective heat transfer coefficient
h_{fg} enthalpy of evaporation
h_M coefficient of mass transfer
J Joules constant = 778 ft lb/Btu
k ratio of specific heats, \( \gamma = \frac{c_p}{c_v} \)
k_L nozzle liner thermal conductivity
k_H thermal conductivity
k_M "conductivity" of mass
M \quad \text{Mach number}

M_A \quad \text{molecular weight of species A}

M_B \quad \text{molecular weight of species B}

Nu \quad \text{Nusselt's number}

p_0 \quad \text{stagnation pressure}

p_1 \quad \text{pressure at increment entrance}

p_2 \quad \text{pressure at increment exit}

Pr \quad \text{Prandtl's number}

Q \quad \text{liquid film evaporation rate}

Q_s \quad \text{energy stored in increment of liquid film}

Re \quad \text{Reynold's number}

r \quad \text{radius}

r_c \quad \text{radius of curvature of nozzle throat}

r_i \quad \text{inside radius of nozzle}

Sc \quad \text{Schmidt number}

St \quad \text{Stanton's number}

T \quad \text{temperature}

T_s \quad \text{temperature of gas stream}

T_o \quad \text{stagnation temperature}

T_r \quad \text{recovery temperature}

T_s \quad \text{temperature at liquid film surface}

T_{\text{sat}} \quad \text{saturation temperature, coolant vapor}

T_w \quad \text{wall temperature}

\bar{T} \quad \text{column temperature matrix}

T_{\text{Adw}} \quad \text{adiabatic wall temperature}

t \quad \text{time}

u \quad \text{time averaged velocity in x-direction}

u^* \quad \text{friction velocity } \sqrt{\frac{T}{\rho}}

u^+ \quad \text{dimensionless velocity } u/u^*

u_w \quad \text{gas stream velocity at throat}
\( \mathbf{u} \)  gas stream velocity

\( \mathbf{u}_m \)  stream mean velocity

\( \mathbf{v} \)  time averaged velocity in the \( y \)-direction

\( \mathbf{W}_a \)  time averaged density of molecular species \( a \)

\( \mathbf{w} \)  mass flow rate

\( \mathbf{w}_w \)  mass flow rate of coolant layer at the wall

\( \mathbf{x} \)  axis parallel to nozzle wall

\( \mathbf{y} \)  axis perpendicular to nozzle wall

\( \psi \)  dimensionless \( y \) variable \( y^* / \mathbf{u} \)

**GREEK**

\( \boldsymbol{\alpha} \)  molecular diffusivity of heat

\( \Delta \)  increment length of liquid film cooling analysis

\( \delta_a \)  molecular diffusivity of mass

\( \epsilon \)  general diffusivity

\( \epsilon_D \)  eddy diffusivity of mass

\( \epsilon_H \)  eddy diffusivity of heat

\( \epsilon_M \)  eddy diffusivity of momentum

\( \gamma \)  gas density at liquid-gas interface

\( \gamma_C \)  coolant gas partial density

\( \gamma_g \)  mainstream gas partial density

\( \mu \)  coefficient of viscosity

\( \nu \)  molecular diffusivity of momentum

\( \rho \)  density

\( \rho_w \)  density at wall or surface

\( \tau_{ni} \)  wall shear stress without injection or evaporation

\( \tau_w \)  wall shear stress

\( \psi \)  ratio of diffusivity of heat to diffusivity of momentum

\( \omega \)  empirical exponent relating viscosity to temperature
APPENDIX B
LIQUID FILM COOLING PROGRAM

FIRST INPUT CARD 40 BLANKS FOR CASE TITLE.
SECOND INPUT CARD
  F1---FS COEFFS FOR FILM VISC, LBM/FT·SEC.
THIRD INPUT CARD
  E1---E5 COEFFS FOR FILM HT. CAP., B/LBM·F.
FOURTH INPUT CARD
  CN1---CN5 COEFFS FOR FILM COND., B/HR·FT·F.
FIFTH INPUT CARD
  G1---G5 COEFFS FOR STREAM VISC., LBM/FT·SEC.
SIXTH INPUT CARD
  R1---R5 COEFFS FOR FILM DENS., LBM/CU·FT.
SEVENTH INPUT CARD
  VK1---VK5 COEFFS FOR VAP. COND., B/HR·FT·F.
EIGHTH INPUT CARD
  AK1---AK5 COEFFS FOR STREAM COND., B/HR·FT·F.
NINTH INPUT CARD
  PR1---PR5 COEFFS FOR MAINSTREAM PRANDTL NO.
TENTH INPUT CARD
  PZ = INLET GAS PRESSURE P-ZERO, LB./IN. SQ.
  T2 = INLET GAS TEMP. T-ZERO, DEGREES F.
  DT = THROAT DIAMETER D*, INCHES
  GI=INITIAL COOLANT FLOW RATE, LBM/SEC.
  TCI = INITIAL COOLANT TEMPERATURE, DEGREES F.
ELEVENTH INPUT CARD
  GK = MAINSTREAM RATIO OF SPECIFIC HEATS.
  RHZ = INITIAL MAINSTREAM DENSITY, LBM./CU·FT.
  B = TURCOTTES CONSTANT FOR WALL SHEAR CALCULATION.
  RG = GAS CONSTANT R AT THROAT CONDITIONS
  PPZ = INITIAL PARTIAL PRESSURE OF COOLANT, PSI.
TWELFTH INPUT CARD
  WCON = NOZZLE LINER THERMAL CONDUCTIVITY, B/HR·FT·F.
  WTMOL = MOLECULAR WEIGHT OF THE FILM LIQUID
  TCOM = CONVERGENCE CRITERION ON TF ITERATION
  TBN = CRITERION FOR THERMAL BOUNDARY LAYER, DEG. F.
  DIF = VAPOR-AIR DIFFUSION COEFF., FT·SQ./SEC.
THIRTEENTH INPUT CARD
  DX= DISTANCE INCREMENT, INCHES.
  SMI = DISTANCE TO NOZZLE THROAT, INCHES
  QDIV=MAX. FRACTION OF LAYER EVAP. PER SECTION
  RGC = COOLANT VAPOR GAS CONST. R
FOURTEENTH INPUT CARD
  M2 = MAXIMUM NUMBER OF INCREMENTS.
  KS = NO. OF TABLE ENTRIES FOR D AND S.
  N = NUMBER OF FILM LAYERS
  KPG = NUMBER OF TABLE ENTRIES FOR TSAT, HF, AND HG.
NEXT KS CARDS
C    S--IN., D--IN., WTHCK--IN., HW--B/HR·SQ·FT·F.
C    NEXT KPG CARDS
C    ON EACH CARD IN ORDER AS FOLLOWS--
C    PG--PSIA, TSAT--DEG·F., HF--BTU/LB, HG--BTU/LB·
C    OUTPUT DESCRIPTION
C    S = DISTANCE ALONG WALL FROM INJECTION, INCHES.
C    Q/A = WALL HEAT LOAD, B/HR·SQ·FT·
C    TB = LOCAL OUTSIDE WALL TEMP., DEG·F.
C    NBOIL = NUMBER TO SHOW BOILING CONDITION
C    0 = NO BOILING
C    1 = BOILING
C    H = MAINSTREAM CONVECTION COEFF., B/HR·SQ·FT·DEG·F.
C    EXP9 = EXPONENTIAL ATTENUATION FACTOR
C    TF2(NI) = TEMP. OF INSIDE COOLANT LAYER, DEG·F.
C    NI = NUMBER OF INSIDE COOLANT LAYER

100 FORMAT (1H,5(E12.3))
101 FORMAT (1H,5(F15))
102 FORMAT (1X,3E12.3,3(E12.3))
103 FORMAT (5X,4HQRAT,14X,2HGI,15X,5HGFL0W)
104 FORMAT (1X,F12.8,5X,F12.8,5X,F12.8)
105 FORMAT (5(E12.3))
106 FORMAT (1H,6HOUTPUT)
107 FORMAT (1H,34H, C*ROLAND*LIQUID FILM COOLING)
108 FORMAT (1H,10HINPUT DATA)
109 FORMAT (5(I5))
110 FORMAT (3(E12.3),3(I4))
111 FORMAT (1X,40H)
112 FORMAT (1X,3E12.3,3(I5))
113 FORMAT (1SHFML EVAPORATED)
114 FORMAT (3(F10.5))
115 FORMAT (1X,3(F10.5))
116 FORMAT (7X,1HS,12X,3HG/A,9X,2HTB,5X,5HNBOIL)
117 FORMAT (1X,25HMAX, DISTANCE IS EXCEEDED)
118 FORMAT (7X,1HS,12X,4HEXP9,5X,7HTF2(NI),4X,2HNI)

DIMENSION TF2(21),V(21),FMU(21),RHF(21),TF(21),CF(21),
       TL(21),CPF(21),RT(21),STAB(40),DTAB(40),WTAB(40),
       2RL(21),QX(21),PGT(40),TSATT(40),HFT(40),HGT(40),
       3HWTAB(40),CT1(21),CT2(21),CT3(21),CT4(21),CT5(21)
I    S=0.0000000001
READ 111
READ 105, F1, F2, F3, F4, F5
READ 105, E1, E2, E3, E4, E5
READ 105, CN, :CN2, CN3, CN4, CN5
READ 105, G1, G2, G3, G4, G5
READ 0L, :1, R2, R3, R4, R5
READ 105, VK1, VK2, VK3, VK4, VK5
READ 105, AK1, AK2, AK3, AK4, AK5
READ 105, PR1, PR2, PR3, PR4, PR5
READ 105, PZ, TZ, DT, QI, TCI
READ 105, GK, RHZ, B, RG, PPZ
READ 105, WCON, WTMOL, TCOM, TBN, DIF
READ 105, DX, SM1, QDIV, RGC
READ 109, M2, KS, N, NTYPE, KPG
DO 150 J=1,KS
150  READ 105, STA8C(J), DTAB(J), WTA8(J), HWTAB(J)
    DO 200 J=1,KPG
200  READ 105, PGT(J), TSATT(J), HFT(J), HGT(J)
    PUNCH 107
    PUNCH 111
    PUNCH 108
    PUNCH 100, F1, F2, F3, F4, F5
    PUNCH 100, E1, E2, E3, E4, E5
    PUNCH 100, CN1, CN2, CN3, CN4, CN5
    PUNCH 100, G1, G2, G3, G4, G5
    PUNCH 100, R1, R2, R3, R4, R5
    PUNCH 100, VK1, VK2, VK3, VK4, VK5
    PUNCH 100, AK1, AK2, AK3, AK4, AK5
    PUNCH 100, PR1, PR2, PR3, PR4, PR5
    PUNCH 100, PZ, TZ, DT, QI, TCI
    PUNCH 100, GK, RHZ, B, RG, PPZ
    PUNCH 100, WCON, WTMOL, TCOM, TBN, DIF
    PUNCH 100, DX, SM1, QDIV, RGC
    PUNCH 101, M2, KS, N, NTYPE, KPG
    DO 230 J=1,KS
230  PUNCH 100, STAB(J), DTAB(J), WTA8(J), HWTAB(J)
    DO 195 J=1,KPG
195  PUNCH 100, PGT(J), TSATT(J), HFT(J), HGT(J)
169  EXP2=GK/(GK-1.)
    EXP3=1./(GK-1.)
    EXP4=2./(GK+1.)
    EXP5=(GK-1.)/(GK+1.)
    EXP6=5/EXP5
    EXP7=2.*EXP5
    EXP8=1.
    DXF=DX/12.
    FN=N
    M=N+1
    QL=QI/FN
    QL2=QL*QDIV
    TSURF=TCI
    NBOIL=0
    NI=1
167  DO 240 J=1,M
    TF(J)=TCI
    TF2(J)=TCI
    QX(J)=QL
    RL(J)=0.
    CT1(J)=0.
    CT2(J)=0.
240  V(J)=0.
S=S+Ox/2
QTOT=0.
DO 500 I=1,M2
IF (S-STAB(KS)) 43,9,550
9 D=DTAB(KS)
WTHCK=WTAB(KS)/12.
HW=HWTAB(KS)
GO TO 11
43 DO 6 J=I+1,KS
IF (S-STAB(J)) 7,8,6
7 D=DTAB(J-1)+(S-STAB(J-1))*(DTAB(J)-DTAB(J-1))/
1(STAB(J)-STAB(J-1))
W=WTAB(J-1)+(S-STAB(J-1))*(WTAB(J)-WTAB(J-1))/
1(STAB(J)-STAB(J-1))
WTHCK=W/12.
HW=HWTAB(J-1)+(S-STAB(J-1))*(HWTAB(J)-HWTAB(J-1))/
1(STAB(J)-STAB(J-1))
GO TO 11
6 CONTINUE
11 DRAT=D/DT
D=D*.083333
VRAT1=1.
IF (S-SM1) 12,12,13
12 VM=((EXP4+EXP5*(VRAT1**2))**EXP6)/(DRAT**2)
GO TO 14
13 VM=SQRT(((VRAT1*(DRAT**2))**EXP7-EXP4)/EXP5)
14 VCRIT=.002*VM
VCOM=ABS(VM-VRAT1)
IF (VCOM-VCRIT) 16,16,15
15 VRAT1=VM
IF (S-SM1) 12,12,13
16 PAR=1+(GK-1.0)/2.*(VM**2)
TG=(TZ+460.0)/PAR
TGS=TG-460.0
RHG=RHZ/((PAR**EXP3)
VG=VM*((GK*32+2*RG*TG)**.5)
IF (S-SM1) 30,30,31
30 TGM=(TG+TSURF+460.0)/2.*22.*445*TM*(GK-1.0)*(VM**2)
TGM=TGM-460.0
TG=TG*(1.0+445*(GK-1.0)*(VM**2))
TG=TG-460.0
GO TO 32
31 TGM=(TG+TSURF+460.0)/2.*22.*430*TM*(GK-1.0)*(VM**2)
TGM=TGM-460.0
TG=TG*(1.0+430*(GK-1.0)*(VM**2))
TG=TG-460.0

32 PG=PZ/(PAR**EXP2)
26 IF (PG-PGT(KPG)) 84,499,499
84 CONTINUE
  DO 19 J=1,KPG
  IF (PG-PGT(J)) 18,17,19
17 TSAT=TSATT(J)
  HFS=HFT(J)
  HG=HGT(J)
  GO TO 20
18 PRAT=(PG-PGT(J-1))/(PGT(J)-PGT(J-1))
  TSAT=TSATT(J-1)+PRAT*(TSATT(J)-TSATT(J-1))
  HFS=HFT(J-1)+PRAT*(HFT(J)-HFT(J-1))
  HG=HGT(J-1)+PRAT*(HGT(J)-HGT(J-1))
  GO TO 20
19 CONTINUE
20 CONTINUE
  DO 680 J=1,KPG
  IF (TF2(NI)-TSATT(J)) 681,682,680
682 PRRES = PGT(J)
  HFGP=HGT(J)-HFT(J)
  GO TO 683
681 TRAT=(TF2(NI)-TSATT(J-1))/(TSATT(J)-TSATT(J-1))
  HFS=HFT(J-1)+TRAT*(HFT(J)-HFT(J-1))
  HG=HGT(J-1)+TRAT*(HGT(J)-HGT(J-1))
  HFGP=HGP-HFS
  GO TO 683
680 CONTINUE
683 CONTINUE
AK=AK1+AK2*TGM+AK3*TGM**2+AK4*TGM**3+AK5*TGM**4
GMU = G1+G2*TGM+G3*TGM**2+G4*TGM**3+G5*TGM**4
PRG=PR1+PR2*TGM+PR3*TGM**2+PR4*TGM**3+PR5*TGM**4
VK=VK1+VK2*TGM+VK3*TGM**2+VK4*TGM**3+VK5*TGM**4
CON=AK
RHY=144.*PG*WTML/(1545.*(TSAT+460.))
REG=(RHG*VG*D)/GMU
DA=3+142*D*DXF
DMF=DIF*(((TGM+460.)/530.))**1.5)*14.7/PG
SCN=GMU/(RHG*DMF)
IF (I-1) 21,21,22
21 GFLOW=RHG*VG*.785*(D**2)
  QRAT=Q1/GFLOW
  PUNCH Q1 PUNCH 106
  PUNCH 103
  PUNCH 104, QRAT, Q1, GFLOW
22 TW1=0.0265*(RHG/32.2)*(VG**2)/(REG**2)
  H=0.0265*CON*(REG**8)*(PRG**333)*EXP9/D
  RHST=QTOT/(+785*D**2*VG)
  PPST=RHST*RG*C*(TG+460.)/144.
  IF (NB00L) 690,690,25
690 QEV5=H*(TG-TF2(NI))/(HFGP*3600.)
QEV = QEV5
HMASS = H*OMF/CON
RHOS = QEV5/HMASS
PPRES = PPST + RHOS*RGC*TF2(N1)/144
DO 940 J=1, KPG
   IF (PPRES - PGT(J)) 941, 941, 940
941 PRAT = (PPRES - PGT(J-1)) / (PGT(J) - PGT(J-1))
   TF2(N1) = TSATT(J-1) + PRAT*(TSATT(J) - TSATT(J-1))
   GO TO 942
940 CONTINUE
942 QTOT = QTOT + QEV*DA
   PRINT 105, PPST, RHST, QTOT, PPRES
   QX(N1) = QX(N1) - QEV*DA
   IF (QX(N1)) 990, 990, 25
   MI = NI + 1
   QX(MI) = QX(MI) + QX(N1)
   NI = NI + 1
25 NCT = 1
   NCT3 = 0
141 NCT2 = 0
   TW = TW1
   LX = M - NI
   DO 62 J = 1, LX
      L = M - J
      TM = (TF(L) + TF2(L)) / 2
      C2 = TM
      C3 = TM**2
      C4 = C2*C3
      C5 = C3*C3
      C6 = C3*C4
      FMU(L) = F1/C3 + F2/C2 + F3 + F4 + F5*C2 + F6*C3
      FMU(L) = FMU(L) / 32.2
      RHF(L) = R1 + R2*C2 + R3*C3 + R4*C4 + R5*C5
      CF(L) = CN1 + CN2*C2 + CN3*C3 + CN4*C4 + CN5*C5
      CPF(L) = E1 + E2*C2 + E3*C3 + E4*C4 + E5*C5
      AV = 1.571 + RHF(L)*TW/FMU(L)
      BV = 3.142*D*V(L+1)*RHF(L)
      CV = QX(L)
      TL(L) = (-BV + SQRT((BV**2) - 4*AV*CV)) / (2*AV)
      VL(L) = V(L+1) + TL(L)*TW/FMU(L)
      IF (L-N) 63, 64, 64
64 RL(L) = TL(N) / (2*CF(N)*DA) + WTHCK/(WCON*DA) + 1 / (Hu*DA)
   GO TO 62
63 RL(L) = TL(L) / (2*CF(L)*DA) + TL(L+1) / (2*CF(L+1)*DA)
   CONTINUE
82 NCT = 1
   DO 80 J = NI, N
      J1 = J - 1
      J2 = J + 1
   IF (NBOIL) 405, 405, 407

106
TF2(N1) = TSAT
GO TO 80

IF (J-N1) 96,96,70

TF2(N1) = TSET
GO TO 80

IF (NCT2-1) 700,701,701

CT1(J) = 1/(QX(J)*7200*CPF(J)*RL(J))
CT2(J) = 1/(QX(J)*7200*CPF(J)*RL(J))
CT3(J) = 1+CT1(J)+CT2(J)
CT4(J) = CT1(J)/CT3(J)
CT5(J) = CT2(J)/CT3(J)

TFCOM=TF2(J)
TF2(J) = TF(J)/CT3(J)+CT4(J)*(TF2(J1)+TF(J1)-TF(J))
1+CT5(J)*(TF2(J2)+TF(J2)-TF(J))

TCT = ABS(TF2(J)-TFCOM)
IF (TCT-TCOM) 80,80,76

NCT=NCT+1
CONTINUE
NCT2=NCT2+1
IF (NCT2-10) 89,141,141
IF (NCT-1) 81,81,82
IF (NCT3) 87,87,92
NCT3=1
GO TO 141

IF (NB0IL) 408,408,409
QST=0.

DO 410 J=N1:N
QWT=QST+QX(J)*CPF(J)*(TF2(J)-TF(J))
QWT=(TF(N)+TF2(N)-2*TC1)/(7200*RL(N))
TW=TW1*EXP9
UST=(TW*32.2/RHV)**.5
HM1=0.0265*DMF*(REG**.8)*(SCN**.333)*EXP9/D
QEVI=((H1*(TG-TSAT)/3600)-QST/DA-QWT/DA)/(HG-HFS)

GMAS=(PG*144.0*QEVI*RGC*(TSAT+460.0)/HM1)/(RG*
1*(TSAT+460.0))
EXP9=EXP(-1.3*89*QEVI/(B*UST*(QEVI/HM1+GMAS)))
HM=0.0265*DMF*(REG**.8)*(SCN**.333)*EXP9/D
IF (ABS(HM-HM1)-02*HM) 420,420,421

HM1=HM
TW=TW1*EXP9
UST=(TW*32.2/RHV)**5
GO TO 422

CONTINUE

H1=0.0265*CON*(REG**.8)*(PRG**.333)*EXP9/D
QEVI=((H1*(TG-TSAT)/3600)-QST/DA-QWT/DA)/(HG-HFS)
QEVI=QM/GM
PV=GMAS*(TSAT+460.0)/(144.0*WTMOL)
IF (PV-PG) 430,431,431
PV=PG
GV=PV*144.0/(RGC*(TSAT+460.0))
GVM = GV / 2
PVM = GVM * RGC * (TGM + 460) / 144
PSM = PG - PVM
GSM = PSM * 144 / (RG + (TGM + 460))
GVM = GVM / 1545
GSM = GSM * RG / 1545
XC = GVM / (GVM + GSM)
XS = GSM / (GVM + GSM)
CON = XC * VK + XS * AK
H = 0.0265 * CON * (REG ** 8) * (PRG ** 333) * EXP9/D
QEVI = (H * (TG - TSAT) / 3600 - QST / DA - QWT / DA) / (HG - HFS)

GO TO 432

430 GVM = GMA / 2
PVM = GVM * RGC * (TGM + 460) / 144
PSM = PG - PVM
GSM = PSM * 144 / (RG + (TGM + 460))
GVM = GVM / 1545
GSM = GSM * RG / 1545
XC = GVM / (GVM + GSM)
XS = GSM / (GVM + GSM)
CON2 = XC * VK + XS * AK
H = H1 * CON2 / CON
IF (ABS (H - H1) > 0.02 * H) 432, 432, 434

434 CON = CON2
GO TO 436

432 IF (ABS (QEVI - QEVI1) > 0.02 * QEVI) 495, 495, 494

494 QEVI1 = QEVI
GO TO 422

495 QT = QEVI * (3 * 14 * D * DXF)
IF (QT - QL2) 440, 440, 441

441 S = S - (1 - QL2 / QT) * DX
DX = QL2 * DX / QT
DXF = DX / 12
DO 443 J = NI + N

443 TF2 (J) = TF (J) + (QL2 / QT) * (TF2 (J) - TF (J))

440 IF (QT - QX (NI)) 446, 447, 448

446 QX (NI) = QX (NI) - QT
GO TO 300

447 QX (NI) = C
NI = NI + 1
IF (NI - N) 300, 300, 575

448 QEX = QT - QX (NI)
QX (NI) = 0
NI = NI + 1
QX (NI) = QX (NI) - QEX
IF (NI - N) 300, 300, 575

408 IF (TF2 (NI) - TSAT) 300, 302, 302

302 RAT = (TSAT - TF (NI)) / (TF2 (NI) - TF (NI))
NBOIL = 1
DO 305 J=NI,N

305   TF2(J)=TF(J)+RAT*(TF2(J)-TF(J))
     S=S-DX+RAT*DX

300   TM=(TF2(N)+TF(N))/2.
     QA=(TM-TCI)/(RL(N)*DA)
     TB=TCI+QA/HW
     PUNCH 116
     PUNCH 112, S, QA, TB, NBOIL
     PUNCH 118
     PUNCH 112, H, EXP9, TF2(NI)*NI
     S=S+DX

450   DO 455 J=NI,N
     TDIF=TF2(J)-TF(J)
     TF(J)=TF2(J)

455   TF2(J)=TF(J)+TDIF

500   CONTINUE
     GO TO 1

499   PUNCH 102
     GO TO 1

575   PUNCH 113
     GO TO 1

550   PUNCH 117
     GO TO 1

END
APPENDIX C
GAS FILM COOLING COMPUTER PROGRAM

First Input Card: 40 Blanks for Case Title.

Second Data Input Card:
- PZ = inlet gas pressure, lb/sq.in
- TZ = inlet gas temperature, degrees F
- DT = throat inside diameter, inches
- N = number of coolant layers
- M = initial number of stream layers
- NSH = no. to specify type wall shear calculation:
  0 = wall shear from Blasius equation
  1 = wall shear from previous increment profile

Third Data Input Card:
- GK = gas constant, cp/cv
- RHZ = initial gas density, lbm/cu.ft
- GRAT1 = injection ratio at entrance
- GRAT2 = second injection ratio
- GRAT3 = third injection ratio

Fourth Data Input Card:
- RG = gas constant, R
- WCON = liner thermal conductivity, B/hr*ft*deg.F
- PR = Prandtl no. of coolant gas
- M2 = max. no. of increments
- KS = no. of table entries for D and WTHCK from S
- NT = no. to designate type wall cooling:
  0 = backside cooled
  1 = adiabatic wall

Fifth Data Input Card:
- DX = distance increment, inches
- EHM = ratio of thermal to momentum diffusivities
- SM1 = distance to nozzle throat, inches
- NTYPE = case type:
  0 = single injection
  1 = multiple injection
  2 = injection at each increment
- NI1 = increment no. for second injection
- NI2 = increment number for third injection

Sixth Data Input Card:
- RGC = coolant gas constant, R
- GAP = coolant input slot width, inches
- VCRIT = convergence criterion on iterations
- QINJ = injection ratio at entrance
- VINJ = injection velocity

Seventh Data Input Card:
- TCI = initial coolant temperature, degrees F
- DXPT = distance between printouts, inches

Eighth Data Input Card
AK1---AK5  COEFFS FOR AK (MOLECULAR),B/HR•FT•DEG.F.

NINTH DATA INPUT CARD

G1---G5  COOLANT GMU COEFFS (MOLECULAR),LB M/HR•FT.
TENTH DATA INPUT CARD

VI---VS  STREAM GMU COEFFS (MOLECULAR),LB M/HR•FT.

ELEVENTH DATA INPUT CARD

CS1---CS5  COEFFS FOR CPS, B/LBM•DEG.F.

TWELFTH DATA INPUT CARD

CC1---CC5  COEFFS FOR CPC, B/LBM•DEG.F.

THIRTEENTH DATA INPUT CARD

NPT = CONTROL NUMBER FOR PRINTOUT

0 = VELOCITY AND TEMPERATURE PROFILES PRINTED
1 = NO PROFILE PRINTOUT

NEXT KS INPUT CARDS.

S--IN., D--IN., WTHCK--IN., HW--B/HR•SQ•FT•F.

NEXT N INPUT CARDS

VC1---FT/SEC.  TC1---DEG. F.

NEXT M INPUT CARDS

VS1---FT/SEC.  TS1---DEG. F.

OUTPUT DESCRIPTION

QINJ = COOLANT INJECTION RATE, LBM/SEC.
GFLOW = MAINSTREAM FLOW RATE, LBM/SEC.
S = DISTANCE ALONG LINER, INCHES.
TSURF = LINER SURFACE TEMPERATURE, DEG.F.
Q/A = HEAT TRANSFER RATE TO LINER, B/HR•SQ•IN.
DISP.THCK. = DISPLACEMENT THICKNESS AT S, INCHES.
DIMENSION STAB(60),DTAB(60),WTAB(60),TC1(60),TS1(20),1TC2(60),TS2(20),DC(60),DS(20),QS(20),QC(60),VS1(20),2VC1(60),VC2(60),RHS(20),RHC(60),EDS(20),3EDC(60),DIC(60),DIS(20),CPC(60),CPS(20),AC(60),BC(60),4AS(20),BS(20),EC(60),ES(20),FC(60),FS(20),PC(60),5PS(20),DYC(60),DYS(20),YC(60),YS(20),HWTAB(60)

100 FORMAT (5X,4H01QINJ,16X,SHGFLOW)
101 FORMAT (5X,1HS,9X,5HTSURF,9X,3H0_Q/A,6X,10H0DISP•THCK.)
102 FORMAT (5(E12.3))
103 FORMAT (1H•10HINPUT DATA)
104 FORMAT (3E12.3,3I5)
105 FORMAT (F12.5,8X,F12.5)
106 FORMAT (1H1•6HOUTPUT)
107 FORMAT (1H1,3IH•8HROLANO • GAS FILM COOLING)
108 FORMAT (1X,19HADIABATIC WALL CASE)
109 FORMAT (1X,20HBACKSIDE COOLED CASE)
110 FORMAT (1X,4OH)
111 FORMAT (1H•5(E12.3))
112 FORMAT (1X,3E12.3,3I5)
113 FORMAT (1X,30H WALL SHEAR FROM PAST INCREMENT)
114 FORMAT (1X,32H WALL SHEAR FROM BLASIUS EQUATION)
115 FORMAT (3F10.5)
116 FORMAT (1H•3F10.5)
FORMAT (1X, 25H MAX. DISTANCE IS EXCEEDED)
FORMAT (1X, 16H SINGLE INJECTION)
FORMAT (1X, 18H MULTIPLE INJECTION)
FORMAT (1X, 27H INJECTION AT EACH INCREMENT)
FORMAT (1X, 6E12.3)
FORMAT (1X, 35H TRANSPIRATION WITH BACKSIDE COOLING)
FORMAT (1H, 15, 5X.E12.3)
S*0.00000000001
SPT=0.0
READ  110
READ  104, PZ, TZ, DT, N, M, NSH
READ  102, GK, RHZ, QRAT1, QRAT2, QRAT3
READ  104, RG, WCON, PR, M2, KS, NT
READ  104, DX, EHM, SM1, NTYPE, NI1, NI2
READ  102, RGCI, GAP, VCRIT, QINJ, VINJ
READ  102, TCI, DXPT
READ  102, AK1, AK2, AK3, AK4, AK5
READ  102, G1, G2, G3, G4, G5
READ  102, V1, V2, V3, V4, V5
READ  102, CS1, CS2, CS3, CS4, CS5
READ  102, CC1, CC2, CC3, CC4, CC5
READ  640, NPT
PRINT  107
PUNCH  107
PRINT  110
PUNCH  110
IF (NSH)  572, 572, 573
572 PRINT  114
PUNCH  114
GO TO  574
573 PRINT  113
PUNCH  113
574 IF (NT)  570, 570, 575
570 PRINT  109
PUNCH  109
GO TO  580
575 PRINT  108
PUNCH  108
580 IF (NTYPE=1)  820, 825, 830
820 PRINT  118
PUNCH  118
GO TO  850
825 PRINT  119
PUNCH  119
GO TO  850
830 IF (NT)  150, 150, 151
151  PRINT 120
PUNCH 120
850  PRINT 103
PUNCH 103
PRINT 112, PZ, TZ, DT, N, M, NSH
PRINT 111, GK, RHZ, QRAT1, QRAT2, QRAT3
PRINT 112, RG, WCON, PR, M2, KS, NT
PRINT 112, DX, EHM, SM1, NTYPE, N11, N12
PRINT 111, RGC, GAP, VCRIT, QINJ, VINJ
PRINT 111, TCI, DXPT
PRINT 111, AK1, AK2, AK3, AK4, AK5
PRINT 111, G1, G2, G3, G4, G5
PRINT 111, V1, V2, V3, V4, V5
PRINT 111, CS1, CS2, CS3, CS4, CS5
PRINT 111, CC1, CC2, CC3, CC4, CC5
PRINT 647, NPT
PUNCH 112, PZ, TZ, DT, N, M, NSH
PUNCH 111, GK, RHZ, QRAT1, QRAT2, QRAT3
PUNCH 112, RG, WCON, PR, M2, KS, NT
PUNCH 112, DX, EHM, SM1, NTYPE, N11, N12
PUNCH 111, RGC, GAP, VCRIT, QINJ, VINJ
PUNCH 111, TCI, DXPT
PUNCH 111, AK1, AK2, AK3, AK4, AK5
PUNCH 111, G1, G2, G3, G4, G5
PUNCH 111, V1, V2, V3, V4, V5
PUNCH 111, CS1, CS2, CS3, CS4, CS5
PUNCH 111, CC1, CC2, CC3, CC4, CC5
PUNCH 647, NPT
DO 4 J=1, NS
2  READ 102, STAB(J), DTAB(J), WTAB(J), HWTAB(J)
PUNCH 111, STAB(J), DTAB(J), WTAB(J), HWTAB(J)
4  PRINT 111, STAB(J), DTAB(J), WTAB(J), HWTAB(J)
DO 904 J=1, N
902  READ 102, VCI(J), TCI(J)
PUNCH 111, VCI(J), TCI(J)
904  PRINT 111, VCI(J), TCI(J)
DO 905 J=1, M
903  READ 102, VS1(J), TSI(J)
PUNCH 111, VS1(J), TSI(J)
905  PRINT 111, VS1(J), TSI(J)
GK1=GK-1,
GK2=GK+1,
EXP2=GK/GK1
EXP3=1./GK1
EXP4=2.*GK2
EXP5=GK1/GK2
EXP6=.5/EXP5
EXP7=2.*EXP5
CINJ=CC1+CC2*TCI+CC3*TCI**2+CC4*TCI**3+CC5*TCI**4
TCRIT=VCRIT
ML=M-1
DO 500 I=1,M2
NI=I

65 DI=Dx
   IF (S-STAB(KS)) 5,5,550
5    DO 6 J=1,2
   IF (S-STAB(KS)) 43,9,550
9   D=DTAB(KS)
   WTHCK=WTAB(KS)*.083333
   HW=HWTAB(KS)
   GO TO 11

43 DO 26 K=1,KS
   K1=K-1
   IF (S-STAB(K)) 17,18,26
17 D=DTAB(K1)+(S-STAB(K1))*(DTAB(K)-DTAB(K1))/(STAB(K)-STAB(K1))
   WTHCK=(WTAB(K1)+(S-STAB(K1))*(WTAB(K)-WTAB(K1)))/(STAB(K)-STAB(K1))
   HW=HWTAB(K1)+(S-STAB(K1))*(HWTAB(K)-HWTAB(K1))/(STAB(K)-STAB(K1))
   GO TO 11

18 D=DTAB(K)
   WTHCK=WTAB(K)*.083333
   HW=HWTAB(K)
   GO TO 11

26 CONTINUE

11 DRAT=D/DT
   D=D+.083333
   VRAT1=1.
   IF (S-SM1) 12,12,13
12 VM=((-EXP4+EXP5*(VRAT1**2))**EXP6)/(DRAT**2)
   GO TO 14
13 VM=SQRT((VRAT1*(DRAT**2)**EXP7-EXP4)/EXP5)
14 VCOMP=.002*VM
   VCOMP=ABS(VM-VRAT1)
   IF (VCOMP-VCOMP) 16,16,15
15 VRAT1=VM
   IF (S-SM1) 12,12,13
16 PAR=1.+5.*GK1*(VM**2)
   TG=(TZ+460*)/PAR
   TGS=TG
   RHG=RHZ/(PAR**EXP3)
   VG=VM*((GK*32.2*RG*TG)**5)
   IF (S-SM1) 30,30,31
30 TGM=(TG+TC1(N)+460*)**5+22*445*TG*G'1*VM**2)
   TG=TG*(1.+445*GK1*(VM**2))
   GO TO 32
31 TGM=(TG+TC1(N)+460*)**5+22*430*TG*GK1*(VM**2)
TG = TG * (1 + 430 * GK1 * (VM**2))

32 PG = PZ / (PAR**2 * EXP2)

IF (J-1) 7, 7, 8

7 PG1 = PG

TG1 = TG - 460*

TGS1 = TGS - 460*

TGM1 = TGM - 460*

VG1 = VG

RHG1 = RHG

D1 = D

WTHCK1 = WTHCK

IF (NTYPE = 1) 940, 943, 635

943 IF (NI = N11) 940, 941, 942

941 S = S + DI / 5*

SI = S - DI / 10*

GO TO 6

942 IF (NI = N11 - 5) 941, 941, 944

944 IF (NI = N12) 635, 941, 946

946 IF (NI = N12 - 5) 941, 941, 635

940 IF (NI = 5) 630, 630, 635

630 S = S + DI / 5*

SI = S - DI / 10*

GO TO 6

635 S = S + DI

SI = S - DI / 2*

GO TO 6

8 PG2 = PG

TG2 = TG - 460*

TGS2 = TGS - 460*

TGM2 = TGM - 460*

VG2 = VG

RHG2 = RHG

D2 = D

WTHCK2 = WTHCK

6 CONTINUE

IF (NI = 1) 71, 71, 72

71 GFLOW = RHG2 * VG2 * 785 * (D2**2)

QI = ORAT1 * GFLOW

PRINT 106

PRINT 100

PRINT 105, QI, GFLOW

PUNCH 106

PUNCH 100

PUNCH 105, QI, GFLOW

PUNCH 101

IF (NPT) 72, 72, 672

672 PRINT 101

72 RHG = (RHG1 + RHG2) / 2*

PG = (PG1 + PG2) / 2*
$$\begin{align*}
TG &= \frac{(TG1+TG2)}{2}; \\
VG &= \frac{(VG1+VG2)}{2}; \\
TGM &= \frac{(TGM1+TGM2)}{2}; \\
RHI_{N1J} &= \frac{PG\times 144}{(RGC\times (TC1+460))}; \\
D &= \frac{(D1+D2)}{2}; \\
WTHCK &= \frac{(WTHCK1+WTHCK2)}{2}; \\
RHGC &= \frac{PG\times 144}{(RGC\times (TGM+460))}; \\
GNUS &= \frac{(V1/TGM+V2+V3\times TGM)}{(3600\times RHGC)}; \\
QI &= Q_{RGR1}\times GFLOW; \\
REG &= VG\times D/GNUS; \\
\text{IF} \ (N\times 1-1) = 610, 610, 605 \quad \text{IF} \ (N\times 1H) = 610, 610, 615; \\
610 \quad TSS &= \frac{0.53\times RHG\times (VG\times 2)}{(64\times 4\times REG\times 2)}; \\
620 \quad USTS &= \sqrt{\frac{TSS\times 32\times 2}{RHGC}}; \\
615 \quad TSS &= 2\times EDC\times (N)\times VC2\times (N)\times RHC\times (N)/(32\times 2\times DC\times (N)); \\
\text{DO} 10 \ J = 1, \ M; \\
35 \quad RHS\times (J) &= PG\times 144\times (RG\times (TS1\times (J)+460)); \\
CPS\times (J) &= CS1+CS2\times TS1\times (J)+CS3\times TS1\times (J)\times 2+CS4\times TS1\times (J)\times 3+ \\
1CS5\times TS1\times (J)\times 4; \\
TS1\times (J) &= TS1\times (J); \\
VS2\times (J) &= VS1\times (J); \\
\text{GO TO} \ 10; \\
36 \quad TDI\times F &= TS2\times (J)-TS1\times (J); \\
TS1\times (J) &= TS2\times (J); \\
TS2\times (J) &= TS1\times (J)+TDIF; \\
CPS\times (J) &= CS1+CS2\times TS1\times (J)+CS3\times TS1\times (J)\times 2+CS4\times TS1\times (J)\times 3+ \\
1CS5\times TS1\times (J)\times 4; \\
RHS\times (J) &= PG\times 144\times (RG\times (TS1\times (J)+460)); \\
VDIF &= VS2\times (J)-VS1\times (J); \\
VS1\times (J) &= VS2\times (J); \\
VS2\times (J) &= VS1\times (J)+VDIF; \\
10 \text{ CONTINUE; } \\
TS1\times (M) &= TGS1; \\
TS2\times (M) &= TGS2; \\
VS2\times (M) &= VG2; \\
\text{DO} \ 33 \ J = 1, \ N; \\
81 \quad RHC\times (J) &= PG\times 144\times (RG\times (TC1\times (J)+460)); \\
CPC\times (J) &= CC1+CC2\times TC1\times (J)+CC3\times TC1\times (J)\times 2+CC4\times TC1\times (J)\times 3+ \\
1CC5\times TC1\times (J)\times 4; \\
TC2\times (J) &= TC1\times (J); \\
VC2\times (J) &= VC1\times (J); \\
\text{GO TO} \ 33; \\
93 \quad TD1 &= T_{D1}\times (J)\times TC1\times (J); \\
TC1\times (J) &= TC2\times (J); \\
TC2\times (J) &= TC1\times (J)+TDIF.
CPC(J) = CC1 + CC2 * TC1(J) + CC3 * TC1(J) ** 2 + CC4 * TC1(J) ** 3 +
CC5 * TC1(J) ** 4
RHC(J) = PG * 144 / (RGC * (TC1(J) + 460))
VDIF = VC2(J) - VC1(J)
VC1(J) = VC2(J)
VC2(J) = VC1(J) + VDIF

CONTINUE
NL = N - 1
FNJ = NL
IF (NL > 1) 19, 19, 760
19 VC1(N) = 5 * USTS
DC(N) = D
DC(N) = 10 * GNUS / USTS
YC(N) = DC(N) / 2
QC(N) = RHC(N) * 3.142 * DIC(N) * DC(N) * VC1(N)
IF (QI > 2 * QC(N)) 910, 911, 911
911 QL = (QI - QC(N)) / FNJ
DO 50 J = 1 + NL
50 QC(J) = QL
DO 21 J = 1 + M
21 QC1(J) = 2 * QC(J)
GO TO 33

NL = N - 1
FNJ = NL
IF (NL > 1) 19, 19, 760
19 VC1(N) = 5 * USTS
DC(N) = D
DC(N) = 10 * GNUS / USTS
YC(N) = DC(N) / 2
QC(N) = RHC(N) * 3.142 * DIC(N) * DC(N) * VC1(N)
IF (QI > 2 * QC(N)) 910, 911, 911
911 QL = (QI - QC(N)) / FNJ
DO 50 J = 1 + NL
50 QC(J) = QL
DO 21 J = 1 + M
21 QC1(J) = 2 * QC(J)
GO TO 33

...
CPc(N) = (QINJ*INJ+(QC(N)-QEX)*CPc(N))/Qcom
RHC(N) = (QINJ*RHINJ+(QC(N)-QEX)*RHC(N))/Qcom
VC1(N) = (QINJ*VINJ+(QC(N)-QEX)*VC1(N))/Qcom
TC1(N) = (QINJ*INJ*TC1+(QC(N)-QEX)*CPc(N)*TC1(N))/
1(Qcom*CPc(N))

QC(NL) = QC(NL) + QEX
QC(N) = QC(N)
IF (QC(NL)/QC(N)-2.) < 538, 861, 861

M5 = N + 1
TC1(M5) = TC1(N)
TC2(M5) = TC2(N)
VC1(M5) = VC1(N)
VC2(M5) = VC2(N)
QC(M5) = QC(N)
CPc(M5) = CPc(N)
RHC(M5) = RHC(N)
DIc(M5) = 0
DC(M5) = DC(N)
QC(NL) = QC(NL)/2.*
TC1(N) = TC1(NL)
TC2(N) = TC2(NL)
VC1(N) = VC1(NL)
VC2(N) = VC2(NL)
QC(N) = QC(NL)
CPc(N) = CPc(NL)
RHC(N) = RHC(NL)
N = M5
NL = N - 1
GO TO 53

QST = QC(NL) + QC(N) + QEX
CPST = QC(NL) + CPc(NL) + QC(N) + CPc(N) + QEX*INJ
TC1(NL) = (QC(NL) + CPc(NL) + TC1(NL) + QC(N) + CPc(N)*TC1(N) +
1QEX*INJ*TC1)/CPST
TC2(NL) = TC1(NL)
VC1(NL) = (QC(NL) + VC1(NL) + QC(N) + VC1(N) + QEX*INJ)/QST
VC2(NL) = VC1(NL)
CPc(NL) = CPST/QST
RHC(NL) = (QC(NL) + RHC(NL) + QC(N) + RHC(N) + QEX*INJ)/QST
QC(NL) = QST
IF (QC(NL)/QC(N)-2.) < 538, 851, 851

QC(NL) = QC(NL)/2.*
TC1(N) = TC1(NL)
TC2(N) = TC2(NL)
VC1(N) = VC1(NL)
VC2(N) = VC2(NL)
QC(N) = QC(NL)
CPc(N) = CPc(NL)
RHC(N) = RHC(NL)
M5 = N + 1
DIC(M5)=D
DC(M5)=DC(N)
N=M5
NL=N-1

795 QC(N)=QCOM
TC1(N)=TC1
TC2(N)=TC1
RHC(N)=RHNJ
CPC(N)=CINJ
VC1(N)=VINJ
VC2(N)=VINJ
GO TO 53

927 QEX=QINJ-QCOM
NXL=QEX/QC(1)
FNXL=NXL

IF (NXL) 928,928,123

123 QST=QC(NL)+QC(N)
CPST=(QC(NL)*CPC(NL)+QC(N)*CPC(N))/QST
RHC(NL)=QC(NL)*RHC(NL)+QC(N)*RHC(N)/QST
TC1(NL)=(QC(NL)*CPC(NL)*TC1(NL)+QC(N)*CPC(N)*TC1(NL))/
        (QST*CPST)
VC1(NL)=(QC(NL)*VC1(NL)+QC(N)*VC1(NL))/QST
QC(NL)=QST
CPC(NL)=CPST
M5=N+NXL

DO 85 J=N,M5
QC(J)=QEX/FNXL
RHC(J)=RHNJ
TC1(J)=TC1
TC2(J)=TC1
VC1(J)=VINJ
VC2(J)=VINJ
CPC(J)=CINJ

85 CONTINUE
QC(M5)=QCOM
DC(M5)=DC(N)
DIC(M5)=D
N=M5
NL=N-1
GO TO 53

20 QC(NL)=QC(NL)+QC(N)-QCOM
QC(N)=QCOM

53 EDC(N)=G1+G2*TC1(N)+G3*TC1(N)**2+G4*TC1(N)**3+G5*
       1TC1(N)**4/(3600*RHC(N))
DO 24 J=1,N,L
JL=J-N
JL1=JL+1
DIC(JL)=DIC(JL1)-2*DC(JL1)
DC(JL)=QC(JL)/(RHC(JL)*3*142*DIC(JL)*VC1(JL))
$Y_C(JL) = Y_C(JL1) + DC(JL) + DC(JL)/2 + DC(JL)/2$

24  $E D C(JL) = 4\times U S T S \times Y_C(JL)$

810  DO 75  J=1,N

J1=J-1

22  DIS(J)=DIC(1)-2*DC(1)

DS(J)=DS(J)/(RHS(J)*3.142*DIS(J)*VS(J))

812  NIT=0

63  NCT=0

DO 34  J=1,N

J1=J-1

J2=J+1

VC0M=VC2(J)

IF (NIT) 51,51,52

51  PC(J)=3.142*(DI/12.)*DIC(J)/(RHC(J)*EDC(J))

52  IF (J1 37,37,38

37  IF (NIT) 61,61,62

61  AC(J)=3.142*(DI/12.)*DIS(J)/(32.2*(DS(J)/(RHS(J)*

1EDS(J)))+DC(J)/(RHC(J)*EDC(J))))

62  BC(J)=3.142*(DI/12.)*DIC(J)/(32.2*(DC(J)/(RHC(J)*

1EDC(J)))+DC(J2)/(RHC(J2)*EDC(J2))))

63  VC2(J)=(QC(J)*VC1(J)/32.2+AC(J)*(VS2(J1)=VC1(J))

1+BC(J)*(VC1(J2)+VC2(J2)-VC1(J1)+PC(J))/QC(J)/32.2

2+AC(J)+BC(J))

IF (VC2(J)=VG2) 300,300,301

301  VC2(J)=VG2

300  IF (ABS(VC2(J)-VC0M)-VCRIT*VC2(J)) 34,34,39

38  IF (J=N) 41,42,42

41  IF (NIT) 171,171,172

171  AC(J)=3.142*(DI/12.)*DIC(J)/(32.2*(DC(J1)/(RHC(J1)*EDS(J1))

1+DC(J1)/(RHC(J)*EDC(J)))+DC(J2)/(RHC(J2)*EDC(J2))))

172  BC(J)=3.142*(DI/12.)*DIC(J)/(32.2*(DC(J)/(RHC(J)*

1EDC(J)))+DC(J2)/(RHC(J2)*EDC(J2))))

172  VC2(J)=(QC(J)*QC1(J)/32.2+AC(J)*(VC1(J1)+VC2(J1)-

1VC1(J1)+BC(J)*(VC1(J2)+VC2(J2)-VC1(J1)+PC(J))/

2(QC(J)/32.2+AC(J)+BC(J))

IF (VC2(J)-VG2) 302,302,303

303  VC2(J)=VG2

302  IF (ABS(VC2(J)-VC0M)-VCRIT*VC2(J)) 34,34,39

38  IF (J=N) 41,42,42

41  IF (NIT) 181,181,182

181  AC(J)=3.142*(DI/12.)*DIC(J)/(32.2*(DC(J1)/(RHC(J1)*EDS(J1))

1+DC(J1)/(RHC(J)*EDC(J)))+DC(J2)/(RHC(J2)*EDC(J2))))

182  BC(J)=3.142*(DI/12.)*DIC(J)/(32.2*(DC(J)/(RHC(J)*

1EDC(J)))+DC(J2)/(RHC(J2)*EDC(J2))))
AEDC-JJ)

\[
\begin{align*}
\text{(1)} & \quad \text{VC2(J)} = \left( \frac{QC(J) \times VC1(J)}{32.2} + AC(J) \times (VC1(J) + VC2(J)) - \right. \\
& \quad \left. VC1(J) \right) - BC(J) \times VC1(J) + PC(J) / (QC(J) / 32.2 + AC(J) + BC(J)) \\
\end{align*}
\]

**IF** (VC2(J) - VG2) \( \leq 304,304,305 \)

\[
\begin{align*}
\text{VC2(J)} &= \text{VG2} \\
\end{align*}
\]

**IF** (ABS(VC2(J) - VCOM) - VCRIT * VC2(J)) \( \leq 34,34,39 \)

**NCT = NCT + 1**

**CONTINUE**

816 DO 70 J = 1, N

\[
\begin{align*}
J1 &= J - 1 \\
J2 &= J + 1 \\
\end{align*}
\]

**VCOM = VS2(J)**

**IF** (NIT) \( \leq 91,91,92 \)

91 \[
\begin{align*}
\text{PS(J)} &= 3.142 \times \text{DIS(J)} \times \text{DS(J)} \times (PG1-PG2) \times 72 \times (1 + (D2/D1)^2) \\
\end{align*}
\]

92 **IF** (J1) \( \leq 88,88,89 \)

88 **IF** (NIT) \( \leq 121,121,122 \)

121 \[
\begin{align*}
\text{AS(J)} &= 2.618 \times D1 \times \text{DIS(J)} / (32.2 \times (DS(J) / (RHS(J) \times EDS(J)))) \\
\text{BS(J)} &= 3.142 \times (D1 / 12) \times \text{DS(J)} / (32.2 \times (DS(J) / (RHS(J) \times EDS(J))) \\
\end{align*}
\]

122 \[
\begin{align*}
\text{VS2(J)} &= (QS(J) \times VS1(J) / 32.2 + AS(J) \times (VS1(J) + VS2(J) - VS1(J)) + BS(J) \times (VS1(J) + VS2(J) - VS1(J)) + PS(J)) / (QS(J) / 232.2 + AS(J) + BS(J)) \\
\end{align*}
\]

**IF** (VS2(J) - VG2) \( \leq 306,306,307 \)

307 **VS2(J) = VG2**

306 **IF** (ABS(VS2(J) - VCOM) - VCRIT * VS2(J)) \( \leq 70,70,69 \)

89 **IF** (NIT) \( \leq 131,131,132 \)

131 \[
\begin{align*}
\text{AS(J)} &= 2.618 \times D1 \times \text{DIS(J)} / (32.2 \times (DS(J) / (RHS(J) \times EDS(J)))) \\
\text{BS(J)} &= 3.142 \times (D1 / 12) \times \text{DS(J)} / (32.2 \times (DS(J) / (RHS(J) \times EDS(J))) \\
\end{align*}
\]

132 \[
\begin{align*}
\text{VS2(J)} &= (QS(J) \times VS1(J) / 32.2 + AS(J) \times (VS1(J) + VS2(J) - VS1(J)) + BS(J) \times (VS1(J) + VS2(J) - VS1(J)) + PS(J)) / (QS(J) / 232.2 + AS(J) + BS(J)) \\
\end{align*}
\]

**IF** (VS2(J) - VG2) \( \leq 308,308,309 \)

309 **VS2(J) = VG2**

308 **IF** (ABS(VS2(J) - VCOM) - VCRIT * VS2(J)) \( \leq 70,70,69 \)

69 **NCT = NCT + 1**

70 **CONTINUE**

818 **NIT = NCT**

**IF** (NCT) \( \leq 80,80,63 \)

80 **CONTINUE**

**NIT = 0**

213 **NCT = 0**

DO 205 J = 1, N

\[
\begin{align*}
J1 &= J - 1 \\
J2 &= J + 1 \\
\end{align*}
\]

**TCOM = TC2(J)**

**IF** (J1) \( \leq 215,215,220 \)
215 IF (NIT) 216,216,217
216 AC(J)=.2618*DI*DIS(J)/(DS(J)/(RHS(J)*EHM*EDS(J)* 
1*CPS(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
BC(J)=3.142*(DI/12.)*DIC(J)/(DC(J)/(RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
EC(J)=QC(J)*VC1(J)**2-VC2(J)**2)/(2*778.*32.2)
FC(J)=(.2618*DI*DIC(J)/DC(J)/RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))

217 TC2(J)=TC1(J)*(QC(J)*CPC(J)-AC(J)-BC(J))+AC(J)* 
1*(TS1(J)+TS2(J))+BC(J)*EC(J)+FC(J)
2/(QC(J)*CPC(J)+AC(J)+BC(J))
IF (ABS(TC2(J)-TCOM)-TCRIT*TC2(J)) 205,205,235

220 IF (J-N) 221,222,222
221 IF (NIT) 223,223,224
222 AC(J)=.2618*DI*DIC(J)/(DC(J)/(RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
BC(J)=.2618*DI*DIC(J)/(DC(J)/(RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
EC(J)=QC(J)*(VC1(J)**2-VC2(J)**2)/(2*778.*32.2)
FC(J)=(.2618*DI*DIC(J)/DC(J)/RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))

224 TC2(J)=TC1(J)*(QC(J)*CPC(J)-AC(J)-BC(J))+AC(J)* 
1*(TC1(J)+TC2(J))+BC(J)*EC(J)+FC(J)
2/(QC(J)*CPC(J)+AC(J)+BC(J))
IF (ABS(TC2(J)-TCOM)-TCRIT*TC2(J)) 205,205,235

222 IF (NIT) 223,223,224
225 AC(J)=.2618*DI*DIC(J)/(DC(J)/(RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
BC(J)=3.142*(DI/12.)*DIC(J)/(DC(J)/(RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))
GO TO 460

450 EC(J)=QC(J)*VC1(J)**2-VC2(J)**2)/(2*778.*32.2)
FC(J)=.2618*DI*DIC(J)/DC(J)/RHC(J)*EHM*EDC(J)* 
1*CPC(J)) + DC(J)/(RHC(J)*EHM*EDC(J)*CPC(J))

226 TC2(J)=TC1(J)*(QC(J)*CPC(J)-AC(J)-BC(J))+AC(J)* 
1*(TC1(J)+TC2(J))+BC(J)*EC(J)+FC(J)
2/(QC(J)*CPC(J)+AC(J)+BC(J))
IF (ABS(TC2(J)-TCOM)-TCRIT*TC2(J)) 205,205,235

235 NCT=NCT+1
205 CONTINUE
821 DO 280 J=1,ML
-J1=J-1
J2=J+1
TCOM=TS2(J)
IF (J1 ) 241,241,242
241 IF (NIT) 243,244
243 AS(J) = *2618*DS1*DS(J2)/(DS(J2)/(RHS(J2)*EHM*EDS(J2))
1*CPS(J2)) + DS(J)/(RHS(J)*EHM*EDS(J)*CP(J)
) )
BS(J) = 3.142*(DI1) / DIS(J)/(DS(J)/(RHS(J)*EHM*EDS(J))
1*CPS(J)) + DS(J)/(RHS(J)*EHM*EDS(J)*CP(J))
) )
ES(J) = QS(J)*(VS1(J)**2-VS2(J)**2)/(2*778*32.2)
FS(J) = (-2168*DS1*DS(J)*DS(J)*RHS(J)*EHS*EDS(J)/4*(778*
132.2)*(-VS1(J)**2+VS2(J)**2)/(DS(J)+DS(J2)
2/2+DC(J)/2))**2

244 TS2(J) = (TS1(J)*(QS(J)*CP(J)-AS(J)-BS(J))+AS(J)*
1(TS1(J2)+TS2(J2)+BS(J)*(TC1(J)+TC2(J)))*ES(J)*FS(J)/
2(QS(J)*CP(J)+AS(J)+BS(J))
) )
IF (ABS(TS2(J)-TCOM)-TCR*TS2(J)) 280,280,279

242 IF (NIT) 247,247,248
247 AS(J) = *2618*DS1*DS(J2)/(DS(J2)/(RHS(J2)*EHM*EDS(J2))
1*CPS(J2)) + DS(J)/(RHS(J)*EHM*EDS(J)*CP(J)
) )
BS(J) = 3.142*(DI1) / DIS(J)/(DS(J)/(RHS(J)*EHM*EDS(J))
1*CPS(J)) + DS(J)/(RHS(J)*EHM*EDS(J)*CP(J))
) )
ES(J) = QS(J)*(VS1(J)**2-VS2(J)**2)/(2*778*32.2)
FS(J) = (-2168*DS1*DS(J)*DS(J)*RHS(J)*EHS*EDS(J)/4*(778*
132.2)*(-VS1(J)**2+VS2(J)**2)/(DS(J)+DS(J2)
2/2+DC(J)/2))**2

248 TS2(J) = (TS1(J)*(QS(J)*CP(J)-AS(J)-BS(J))+AS(J)*
1(TS1(J2)+TS2(J2)+BS(J)*(TS1(J)+TS2(J)))*ES(J)*FS(J)/
2(QS(J)*CP(J)+AS(J)+BS(J))
) )
IF (ABS(TS2(J)-TCOM)-TCR*TS2(J)) 280,280,279

279 NCT=NCT+1
280 CONTINUE
826 NIT=NCT
1 IF (NCT) 290,290,213
290 CONTINUE
TAV=5*(TC1(N)+TC2(N))
1 IF (NT) 410,410,420
410 CON1 = PR*DC(N)/(EDC(N)*CP(N)*RHC(N)*7200)
CON2 = WTHK/WCON
CON3 = 1./HW
QA = (TAV-TC1)/((CON1+CON2+CON3)*144)
TSURF = TC1+QA*(CON2+CON3)*144
GO TO 426
420 TSURF = TAV
QA = 0
426 NL=N-1
DYN(N) = 6.*DC(N)
DO 660 J=1,NL
JL=N-J
JL1=JL+1
660 DYC(JL) = DYC(JL1)+6.*DC(JL1)+6.*DC(JL)
DO 670 J=1,M
J1=J-1
IF (J1) 675.675,680
675  DYS(J)=DYC(1)+6.*DC(1)+6.*DS(1)
   GO TO 670
680  DYS(J)=DYS(J1)+6.*DS(J1)+6.*DS(J)
670  CONTINUE
   DSPTK=DYC(1)+6.*DC(1)
   DO 720 J=1,M
720  DSPTK=DSPTK+(1.-RHS(J)*VS2(J)/(RHG2*VG2))*DIS(J)*12.*
1DS(J)/DIS(1)
   IF (NPT) 641.641,642
641  IF (S1-SPT) 500,501,501
501  PRINT 621, TC2
   PRINT 621, VC2
   PRINT 621, QC
   PRINT 101
642  IF (S1-SPT) 500,502,502
502  PRINT 102, SI, TSURF, QA, DSPTK
   PUNCH 102, SI, TSURF, QA, DSPTK
   SPT=SPT+DXPT
500  CONTINUE
550  PRINT 117
   GO TO 995
910  PRINT 990
   GO TO 995
150  PRINT 140
995  GO TO 1
END
APPENDIX D
TRANSPIRATION COOLING COMPUTER PROGRAM

C FIRST INPUT CARD 40 BLANKS FOR CASE TITLE.
C SECOND INPUT CARD
   G1---G5 COEFFS FOR STREAM VISC.*LBM/FT.*SEC.*
C THIRD INPUT CARD
   VK1---VKS COEFFS FOR VAP. COND.*B/HR.*FT.*F.*
C FOURTH INPUT CARD
   AK1---AKS COEFFS FOR STREAM COND.*B/HR.*FT.*F.*
C FIFTH INPUT CARD
   PRI---PRS COEFFS FOR STREAM PRANDTL NO.*
C SIXTH INPUT CARD
   PZ = INLET GAS PRESSURE P-ZERO.*LB/IN.*SQ.*
   TZ = INLET GAS TEMP. T-ZERO.*DEG.*F.*
   DT = THROAT DIAMETER D.* INCHES.*
   TSURF = SPECIFIED SURFACE TEMP
C SEVENTH INPUT CARD
   CPf = HEAT CAPACITY OF TRANSP.* LIQ.*B/LBM.*F.*
C EIGHTH INPUT CARD
   GK = MAINSTREAM RATIO OF SPECIFIC HEATS
   RHZ = INITIAL MAINSTREAM DENSITY.*LBM/CU.*FT.*
   B = TURCOTTES CONSTANT FOR WALL SHEAR CALC.*
   RG = STREAM GAS CONSTANT R.*
   CPV = HEAT CAPACITY OF TRANSP. GAS OR VAP.*B/LBM.*F.*
C NINTH INPUT CARD
   WTMOL = COOLANT MOLECULAR WEIGHT.*
   DIF = COOLANT-AIR DIFFUSION COEFF.* FT.* SQ./SEC.*
   DX = DISTANCE INCREMENT.* INCHES.*
   SMI = DISTANCE TO NOZZLE THROAT,* INCHES.*
C NEXT KS CARDS
C NEXT KPG CARDS
C Output Description
   S = DISTANCE ALONG WALL FROM INJECTION.* INCHES.*
   QEV = INJECTION RATE REQUIRED.* (LBM/SEC)/SQ.* FT.*
   QTQ = TOTAL INJECTION TO S,* LBM/SEC.*
   QRAT = RATIO OF QTQ/(GAS FLOW) TO S.*
   TG = MAINSTREAM STATIC TEMPERATURE,* DEG.* F.*
   PG = LOCAL STATIC PRESSURE,* PSIA.*
EXP9 = EXPONENTIAL ATTENUATION FACTOR

100 FORMAT (1H *5(E12.3))
101 FORMAT (1H *5(I5))
102 FORMAT (1X,34H PRESSURE EXCEEDS CRITICAL PRESSURE)
103 FORMAT (1X,20HLIQUID COOLANT TABLE)
104 FORMAT (1X,12HNUZZLE TABLE)
105 FORMAT (5(E12.3))
106 FORMAT (1H1,6HOUTPUT)
107 FORMAT (1H1,39HM*, C* ROLAND*Liq* AND GAS TRANSPERSION)
108 FORMAT (1H0,10HINPUT DATA)
109 FORMAT (5(I5))
110 FORMAT (3(E12.3), 3(I5))
111 FORMAT (1X,4CH)
112 FORMAT (1X,34H INCOMPATIBLE--TSURF LESS THAN TSAT)
113 FORMAT (7X,1HS, 11X,3HQEV, 9X, 4HQTOT, 8X, 4HQRAT)
114 FORMAT (1H0, 5X, SHGFLOW)
115 FORMAT (7X, 2HTG, 10X, 2HPG, 9X, 4HEXP9)
117 FORMAT (1X, 25HMAX. DISTANCE IS EXCEEDED)
DIMENSION STAB(40), DTAB(40), WTAB(40), PGT(40), TSATT(40),
1HFT(40), HGT(40)
1
S=0.0000000001
READ 111
READ 105, G1, G2, G3, G4, G5
READ 105, VK1, VK2, VK3, VK4, VK5
READ 105, AK1, AK2, AK3, AK4, AK5
READ 105, PR1, PR2, PR3, PR4, PR5
READ 105, PZ, TZ, DT, TSURF, CPL
READ 105, GK, RHZ, B, RG, CPV
READ 105, WTMOL, DIF, DX, SMJ
READ 109, M2, KS, KPG, NTYPE
DO 200 J=1, KS
200 READ 105, STAB(J), DTAB(J), WTAB(J)
DO 201 J=1, KPG
201 READ 105, PGT(J), TSATT(J), HFT(J), HGT(J)
PUNCH 107
PUNCH 111
PUNCH 108
PUNCH 100, G1, G2, G3, G4, G5
PUNCH 100, VK1, VK2, VK3, VK4, VK5
PUNCH 100, AK1, AK2, AK3, AK4, AK5
PUNCH 100, PR1, PR2, PR3, PR4, PR5
PUNCH 100, PZ, TZ, DT, TSURF, CPL
PUNCH 100, GK, RHZ, B, RG, CPV
PUNCH 100, WTMOL, DIF, DX, SM1
PUNCH 101, M2, KS, KPG, NTYPE
PUNCH 104
DO 202 J=1, KS
202 PUNCH 100, STAB(J), DTAB(J), WTAB(J)
PUNCH 103
DO 203 J=1, KPG
PUNCH 100: PGT(J), TSATT(J), HFT(J), HGT(J)
PUNCH 106
EXP2= GK/(GK-1)
EXP3=1/(GK-1)
EXP4=2/(GK+1)
EXP5= (GK-1)/(GK+1)
EXP6= .5/EXP5
EXP7=2*EXP5
EXP9=1
S= S+DX/2
DXF=DX/12
QTOT=0
DO 500 I=1,M2
IF (S-STAB(KS)) 43,9,550
D=DTAB(KS)
WTHCK=WTAB(KS)/12
GO TO 11
DO 6 J=1,KS
J1= J-1
IF (S-STAB(J)) 7,8,6
D=DTAB(J1) + (S-STAB(J1)) *(DTAB(J)-DTAB(J1))/
1(STAB(J)-STAB(J1))
W=WTAB(J1) + (S-STAB(J1)) *(WTAB(J)-WTAB(J1))/
1(STAB(J)-STAB(J1))
WTHCK=W/12
GO TO 11
DO 6 J=1,KS
J1= J-1
IF (S-STAB(J)) 7,8,6
D=DTAB(J)
WTHCK=WTAB(J)/12
GO TO 11
CONTINUE
11 DRAT=D/DT
D=D*.08333
VRAT1=1
IF (S-SM1) 12,12,13
VM=((EXP4+EXP5*(VRAT1**2))**EXP6)/(DRAT**2)
GO TO 14
13 VM=SQRT(((VRAT1*(DRAT**2))**EXP7-EXP4)/EXP5)
14 VCRIT=.002*VM
VCOM=ABS(VM-VRAT1)
IF (VCOM>VCRIT) 16,16,15
VRAT1=VM
IF (S-SM1) 12,12,13
16 PAR=1 + ((GK-1)/2)*(VM**2)
TG=(TZ+460)*PAR
TGS=TG-460
RHG=RHZ/(PAR**EXP3)
VG=VM*( (GK*32*2*RG*TG)**.5)
IF (S-SM1) 30,30,31
30 TGM=(TG+TSURF+460)/2 + 22*.445*TG*(GK-1)*(VM**2)

127
TGM = TGM - 460
TG = TG*(1 + 445*(GK-1)*(VM**2))
TG = TG - 460
GO TO 32

31 TGM = (TG + TSURF + 460) / 2 + 22 * 430 * TG*(GK-1)*(VM**2)
TGM = TGM - 460
TG = TG*(1 + 430*(GK-1)*(VM**2))
TG = TG - 460

32 PG = PZ/(PAR**EXP2)

26 IF (PG = PGT(KPG)) 84, 499, 499
84 CONTINUE
DO 19 J = 1, KPG
IF (PG = PGT(J)) 18, 17, 19

17 TSAT = TSATT(J)
HFS = HFT(J)
HG = HGT(J)
GO TO 20

18 PRAT = (PG - PGT(J-1))/(PGT(J) - PGT(J-1))
TSAT = TSATT(J-1) + PRAT*(TSATT(J) - TSATT(J-1))
HFS = HFT(J-1) + PRAT*(HFT(J) - HFT(J-1))
HG = HGT(J-1) + PRAT*(HGT(J) - HGT(J-1))
GO TO 20

19 CONTINUE
20 CONTINUE

IF (NTYPE) 700, 700, 701
700 AK = AK1 + AK2*TGM + AK3*TGM**2 + AK4*TGM**3 + AK5*TGM**4
GMI = G1 + G2*TGM + G3*TGM**2 + G4*TGM**3 + G5*TGM**4
PRG = PR1 + PR2*TGM + PR3*TGM**2 + PR4*TGM**3 + PR5*TGM**4
VK = VK1 + VK2*TGM + VK3*TGM**2 + VK4*TGM**3 + VK5*TGM**4
CON = AK
CON = (DT*DRAT)/12
RHV = 144.*PG*WTMOL/(1545.*(TSURF + 460.))
REG = (RHG*VG*D)/GMI
DMF = DIF*((TGM + 460.)/530.)*1.5*(14.7/PG)
SCN = GMI/(RHG*DMF)
IF (1-1) 21, 21, 22
21 GFLOW = RHG*VG*785*(D**2)
PUNCH 114
PUNCH 100, GFLOW

22 TW = 0.0265*(RHG/32.2)*(VG**2)/(REG**2)
H = 0.0265*CON*(REG**8)*(PRG**3.333)*EXP9/D
TW = TW*1*EXP9
UST = (TW*32.2/RHV)**9.5
HM1 = 0.0265*DMF*(REG**8)*(SCN**3.333)*EXP9/D
IF (NTYPE) 470, 470, 471
470 QEV1 = (H*(TG-TSURF)/3600.)/(CPV*(TSURF-TCI))
GO TO 422
471 QEV1 = (H*(TG-TSURF)/3600.)/(CPV*(TSAT-TCI) + CPV)
1(TSURF=TSAT))

422 GMAS=(PG*144.-QEV1*RGC*(TSURF+460.)*/HM1)/\(RG^*(TSURF+460.)*\))
\[ \text{EXP9}=\text{EXP}(-13.89*QEV1/(B*UST*(QEV1/HM1+GMAS)))) \]
\[ \text{HM}=.0265*DMF*(REG**.8)*(SCN**.333)*\text{EXP9}/D \]
\[ \text{IF (ABS(HM-HM1)-.02»HM) 420,420,421} \]

421 HM=HM
\[ \text{TW}=\text{TW1}^*\text{EXP9} \]
\[ \text{UST}=(\text{TW}*32.2/RHV)**.5 \]
\[ \text{GO TO 422} \]

420 CONTINUE

436 H1=.0265*CON*(REG**.8)*(PRG**.333)*\text{EXP9}/D
\[ \text{IF (NTYPE) 450,450,451} \]

450 QEV1=(H1*(TG-TSURF)/3600.)/(CPV*(TSURF-TCI))
\[ \text{GO TO 452} \]

451 QEV1=(H1*(TG-TSURF)/3600.)/(CPL*(TSAT-TCI)+CPV*(TSURF-1-TSAT))

452 GMAC=QEV1/HM
\[ \text{PV}=\text{GMA}^*1545.(*(TSURF+460.)*)/(144.**WTMOL) \]
\[ \text{IF (PV-PG) 430,431,431} \]

431 PV=PG
\[ \text{GV}=\text{PV}^*144.**WTMOL/(1545.**(TSAT+460.)) \]
\[ \text{GVM}=\text{GV}^/2. \]
\[ \text{PVM}=\text{GVM}^*1545.**(TGM+460.)/(144.**WTMOL) \]
\[ \text{PSM}=\text{PG}-\text{PVM} \]
\[ \text{GSM}=\text{PSM}^*144./(RG^*(TGM+460.)) \]
\[ \text{GVM}^=\text{GVM}/\text{WTMOL} \]
\[ \text{GSM}=\text{GSM}^*RG^/1545. \]
\[ \text{XC}^=\text{GVM}/(\text{GVM}+\text{GSM}) \]
\[ \text{XS}^=\text{GSM}/(\text{GVM}+\text{GSM}) \]
\[ \text{CON2}^=\text{XC}^*\text{VK}+\text{XS}^*\text{AK} \]
\[ \text{H1}=\text{H1}^*\text{CON2}/\text{CON} \]
\[ \text{GO TO 433} \]

430 GVM=GM/2.
\[ \text{PV}=\text{GVM}^*1545.**(TGM+460.)/(144.**WTMOL) \]
\[ \text{PSM}=\text{PG}-\text{PVM} \]
\[ \text{GSM}=\text{PSM}^*144./(RG^*(TGM+460.)) \]
\[ \text{GVM}^=\text{GVM}/\text{WTMOL} \]
\[ \text{GSM}=\text{GSM}^*RG^/1545. \]
\[ \text{XC}^=\text{GVM}/(\text{GVM}+\text{GSM}) \]
\[ \text{XS}^=\text{GSM}/(\text{GVM}+\text{GSM}) \]
\[ \text{CON2}^=\text{XC}^*\text{VK}+\text{XS}^*\text{AK} \]
\[ \text{H1}=\text{H1}^*\text{CON2}/\text{CON} \]
\[ \text{GO TO 433} \]

433 IF (NTYPE) 460,460,461

460 QEV=(H1*(TG-TSURF)/3600.)/(CPV*(TSURF-TCI))
\[ \text{GO TO 462} \]

461 QEV=(H1*(TG-TSURF)/3600.)/(CPL*(TSAT-TCI)+CPV*(TSURF-1-TSAT))
\[ \text{IF (ABS(QEV-QEV1)-.02*QEV) 465,465,464} \]

129
464  QEV1=QE V
   GO TO 422
465  CONTINUE
   QTOT=QTOT+QEV*3.14*D*DXF
   QRAT=QTOT/GFLOW
   PUNCH 113
   PUNCH 100, S, QEV, QTOT, QRAT
   PUNCH 115
   PUNCH 100, TG, PG, EXP9
   S=S+DX
500  CONTINUE
550  PUNCH 117
   GO TO 1
720  PUNCH 112
   GO TO 1
499  PUNCH 102
   GO TO 1
END
APPENDIX E
PHYSICAL PROPERTIES

The viscosity, thermal conductivity, and specific heat at constant pressure for air are given in graphical form in Figures 28, 29, and 30, respectively. The data for temperatures below 3000 °F. were taken from tables in Kreith's *Principles of Heat Transfer*. The data above 3000 °F. were taken from references (15) and (28) in the Bibliography.
Fig. 28 Viscosity of Air as a Function of Temperature
Conductivity

\[ \frac{x_2}{10^2} \]

\[ \frac{B}{(\text{hr. ft. } ^\circ F)} \]

Temperature, °F.

Fig. 29 Thermal Conductivity of Air as a Function of Temperature
Fig. 30 Specific Heat of Air as a Function of Temperature
APPENDIX F
SAMPLE COMPUTER CALCULATIONS

This Appendix includes Tables VII, VIII, which are the sample computer calculation outputs for transpiration cooling using the gas film cooling program, transpiration cooling using the liquid or gas transpiration program, and the liquid film cooling calculation, respectively.
### TABLE VII
SAMPLE CALCULATION FORTRANSPIRATION USING GAS FILM PROGRAM

**INPUT DATA**

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SAMPLE CALCULATION FOR TRANSPIRATION USING LIQUID OR GAS TRANSPIRATION PROGRAM

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### TABLE IX

**SAMPLE CALCULATION FOR LIQUID FILM COOLING**

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141
### TABLE IX (Concluded)

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APPENDIX G
DEVELOPMENT OF DIFFERENCE EQUATIONS
FOR GAS FILM COOLING ANALYSIS

The difference equation for the momentum (or force) balance, Eq. (6), page 14, is developed by referring to Fig. 2 and evaluating the terms in Eq. (5).

The pressure force is the difference in pressure multiplied by the annular area of the layer projected in the axial direction, thus

\[ F_p = \pi \delta \delta \left( p_1 - p_2 \right) \]  (86)

Pressure forces in Δ direction on coolant layer 1

The inertia forces are the net rate of momentum flow into the volume element as a result of the axial mass flow rate and the velocity of the fluid. Thus,

\[ F_i = \frac{\omega}{\delta_c} \left( v'_{11} - v'_{21} \right) \]  (87)

Inertia forces in Δ direction on coolant layer 1

The shear forces on the top and bottom of the coolant layer are calculated from the shear stress equation

\[ \tau = \rho \varepsilon \frac{dy}{dy} \]  (88)
Considering \( \tau \) as the current of momentum and \( dv \) as the potential for momentum flow, Eq. (88) may be written as

\[
\tau = \frac{dv}{(dy/\rho c)}
\]  (89)

By analogy, the resistance to momentum flow is given by the denominator of Eq. (89). Since resistances in series are additive the shear stress on the top of the first coolant layer is given in finite difference form as

\[
\tau_{\text{top}} = \frac{v_{11} + v_{21}}{2} - \frac{v'_{11} + v'_{21}}{2}
\]

\[
\frac{\delta_1}{2\rho_1} \frac{\delta_1}{\rho_1} + \frac{\delta_1}{2\rho_2} \frac{\delta_1}{\rho_2}
\]  (90)

Evaluation of the shear stress on the bottom of coolant layer 1 is determined in a similar manner. The shear forces are found by multiplying the stresses by appropriate areas.

Shear forces on the top and bottom of coolant layer 1

\[
\pi df \Delta \frac{(v_{11} + v_{21} - v'_{11} - v'_{21})}{\delta_1 + \delta_1}
\]

\[
\frac{\delta_1}{\rho_1} \frac{\delta_1}{\rho_1} \frac{\delta_1}{\rho_2} \frac{\delta_1}{\rho_2}
\]  (91)
Equating the sum of Eqs. (86)(87)(91) to zero yields the momentum balance equation in finite difference form.

The difference equation for the steady state energy balance, Eq. (22), page 20, is developed in a similar manner. Refer to Fig. 2, page 16.

The enthalpy change in the axial direction of a layer is determined from the ideal gas laws as

Enthalpy change in
\[ \Delta \text{ direction of } = w_i c_i \left(T'_{1} - T'_{2}\right) \quad (92) \]
coolant layer 1

The change in kinetic energy is determined from the square of the velocities as

Kinetic energy change
in \( \Delta \) direction of \( = \frac{w_i}{\rho \gamma g_c} \left(v'_{1}^2 - v'_{2}^2\right) \quad (93) \)
coolant layer 1

The thermal energy conduction term is calculated using the temperature difference between layer. The resistance to the flow of heat is

\[ \text{Thermal resistance} = \frac{\delta}{\rho c e \psi} \]
Thus

\[
\text{Thermal energy transfer from gas layer 1 to coolant layer 1 by diffusion} = \frac{n d_1 \Delta}{2} \left( \frac{T_{l1} + T_{l2}}{2} - \frac{T'_{l1} + T'_{l2}}{2} \right)
\]

and a similar equation is written for the thermal energy transfer between coolant layer 1 and coolant layer 2.

In evaluating the work at the boundaries of the increment only shear stress effects are considered.

Thus

\[
\text{Viscous work/unit volume} = \mu \left( \frac{dv}{dy} \right)^2
\]

Apply Eq. (95) to the first coolant layer, noting that the velocity gradient in difference terms is established using three layers

\[
\text{work/unit volume} = \mu \left[ \frac{v_{l1} + v_{l2}}{2} - \frac{v'_{l1} + v'_{l2}}{2} \right]^2
\]

Thus evaluating \( \mu \) and multiplying by the layer volume of coolant layer 1 gives

\[
\text{work at boundaries on coolant layer 1 from shear stress effects} = \frac{n d_1 \Delta p \delta_1 c_i (v_{l1} + v_{l2} - v'_{l1} - v'_{l2})^2}{J g_c (2 \delta_1 + \delta_2 + \delta_1)^2}
\]
FILM AND TRANSPERSION COOLING OF NOZZLE THROATS

Analytical studies were made of liquid film cooling, gas film cooling, and liquid or gas transpiration cooling of hypersonic nozzles. Experimental studies of gas film cooling were performed on a small nozzle using air as both the mainstream and coolant gases. Based on the experimental results obtained and the development of satisfactory calculational techniques to implement the analyses given in the present study, the following conclusions were drawn: (1) Gas film cooling can be used to measurably lower the wall temperatures and the wall heat fluxes in the converging section and at the throat of a high-pressure high-temperature nozzle. Some gross mixing occurs at the injection point, the exact amount being some as yet undetermined function of the injection geometry, the relative velocities of the main gas stream and the coolant stream at the injection point, and the entering velocity profiles and turbulence conditions. (2) A straightforward boundary layer type analysis was developed and programmed which predicts with reasonable accuracy the nozzle wall temperatures and wall heat fluxes in the converging section and at the throat for gas film cooled nozzles. (3) Calculational techniques have been developed and programmed for predicting the effectiveness of liquid film cooling and liquid or gas transpiration cooling in nozzles. These techniques should be checked against experimental data before they are used for design purposes, however.
nozzles
cooling - gas film
cooling - liquid film
transpiration

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